INT-11-2d, Seattle, August 10, 2011

Recent applications of Green's function methods to link structure and reactions

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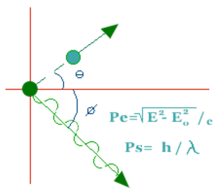

- Motivation
- Green's function method as a framework to analyze experimental data (and extrapolate)

  --> dispersive optical model (DOM)
- Recent developments
  - Nonlocality in "HF" potential
  - Recent DOM fits
  - Predictions from extrapolations
  - DOM and transfer reactions
  - Link with ab initio Green's function results
- Conclusions

--> drip line

Drip-line nuclear physics

- Many reactions necessarily involve strongly interacting particles
  - (p,2p) perhaps (p,pn)
  - (d,p) or (p,d)
  - HI knock-out reactions
- Interactions of “projectiles” with “target” are not experimentally constrained at this time --> no unambiguous information

- Present Green’s function project
  - intends to provide a frame work for such constraints
  - simultaneous treatment of negative (structure) and positive energies (reactions)
  - linking information below and above the Fermi energy such as elastic scattering cross sections, level structure, charge densities, knock-out cross sections etc.

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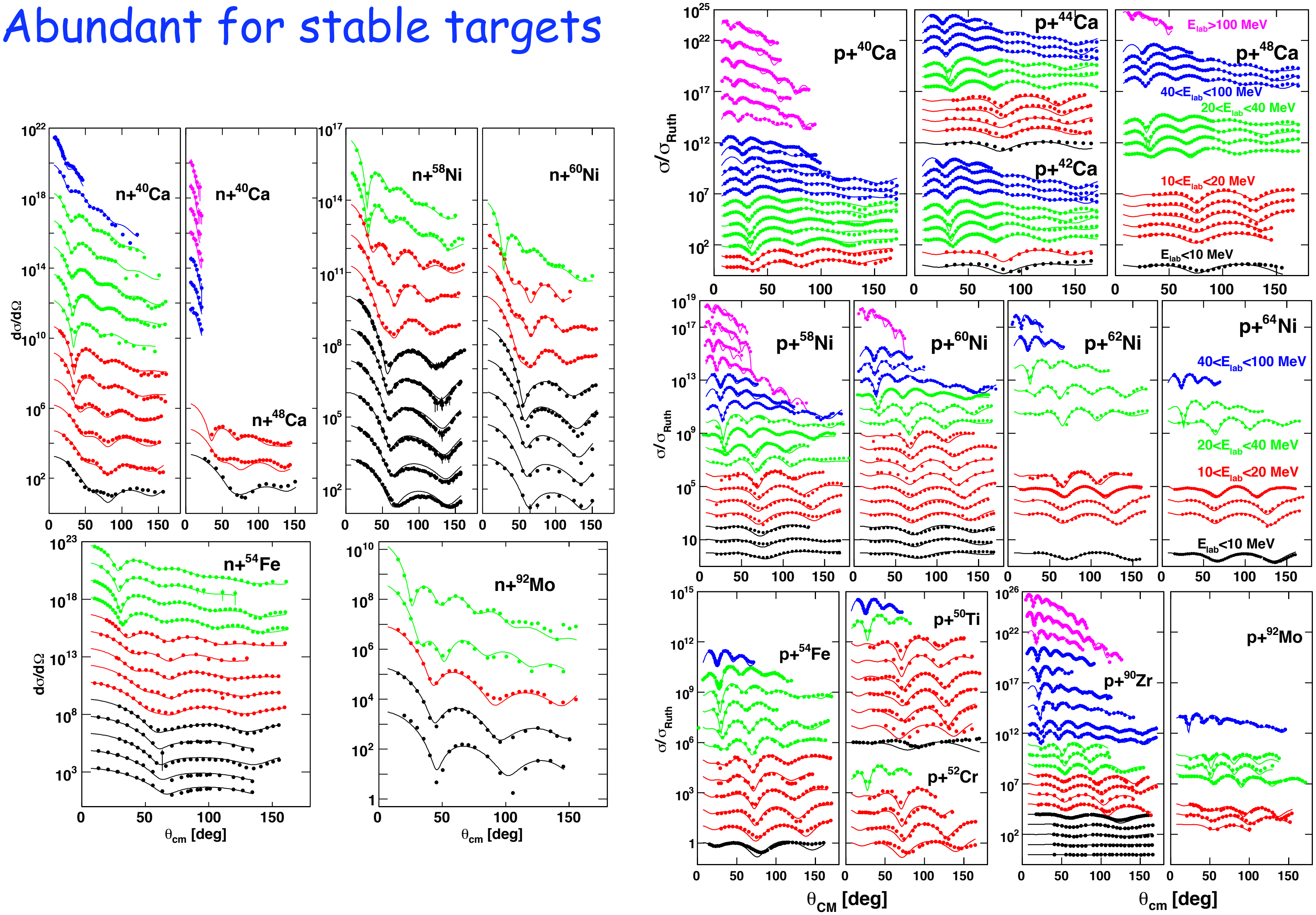
Elastic scattering data for protons and neutrons

- Abundant for stable targets



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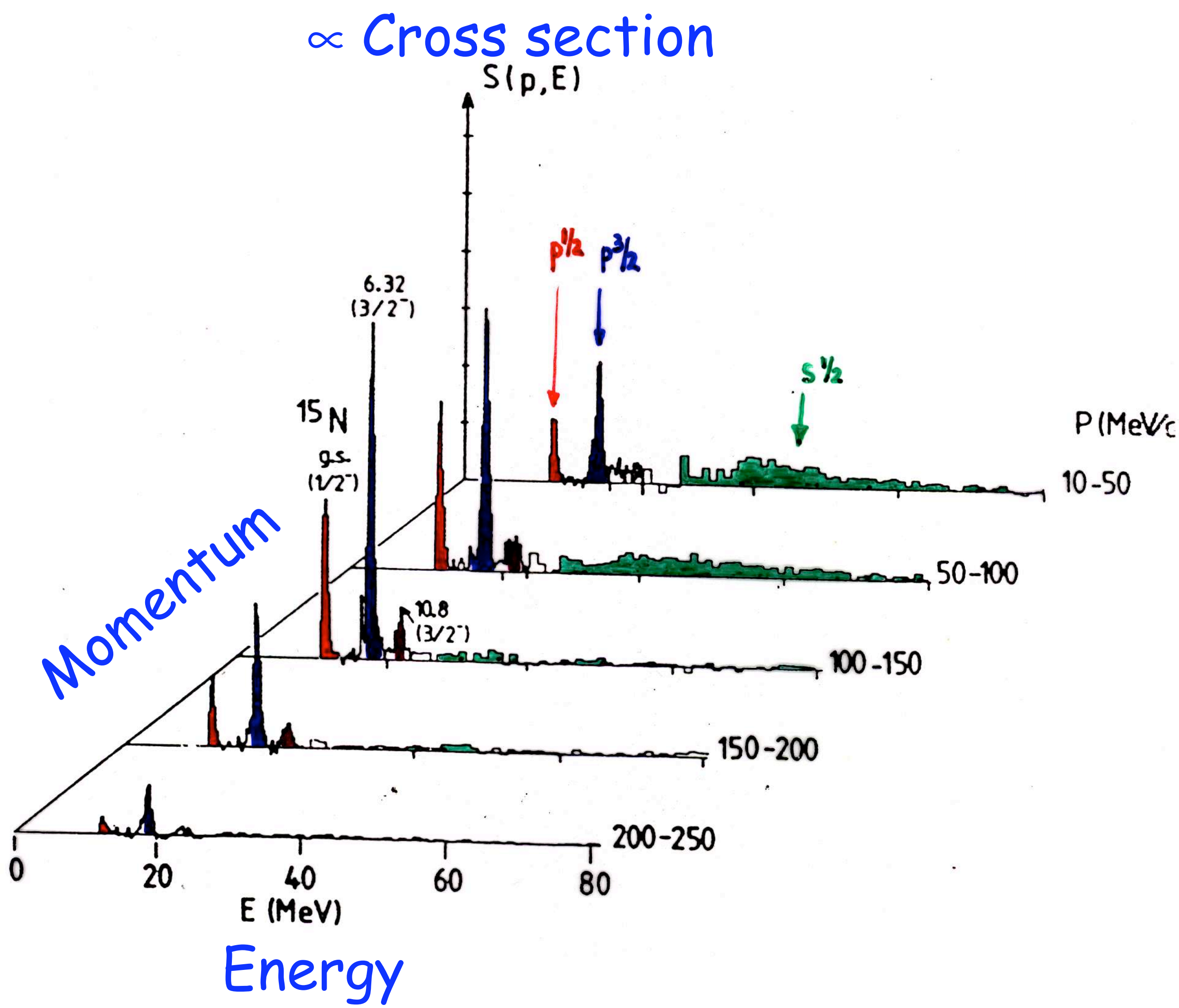
Mougey et al., Nucl. Phys. A335, 35 (1980)

$^{16}O(e,e'p)$



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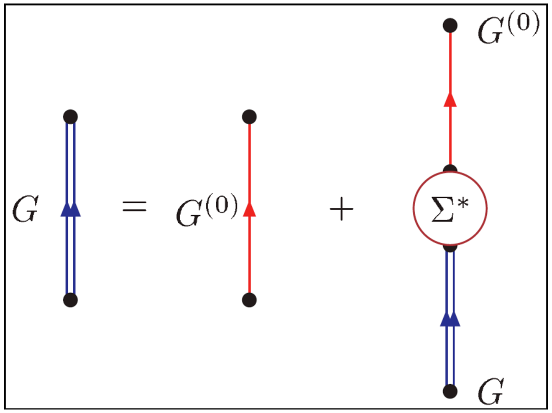
Propagator / Green's function

- Lehmann representation
$$G_{\ell j}(k,k';E) = \sum_m \frac{\langle\Psi_0^A|\, a_{k\ell j}\, |\Psi_m^{A+1}\rangle \langle\Psi_m^{A+1}|\, a^\dagger_{k'\ell j}\, |\Psi_0^A\rangle}{E-(E_m^{A+1}-E_0^A)+i\eta} + \sum_n \frac{\langle\Psi_0^A|\, a^\dagger_{k'\ell j}\, |\Psi_n^{A-1}\rangle \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle}{E-(E_0^A-E_n^{A-1})-i\eta}$$
- Any single-particle basis can be used
- Overlap functions --> numerator
- Corresponding eigenvalues --> denominator
- Spectral function
$$S_{\ell j}(k;E) = \frac{1}{\pi}\,\mathrm{Im}\ G_{\ell j}(k,k;E) \qquad E \leq \varepsilon_F^-$$
$$= \sum_n \left| \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle \right|^2 \delta(E-(E_0^A-E_n^{A-1}))$$
- Spectral strength in the continuum
$$S_{\ell j}(E) = \int_0^\infty dk\ k^2\ S_{\ell j}(k;E)$$
- Discrete transitions $\sqrt{S_{\ell j}^n}\ \phi_{\ell j}^n(k) = \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle$

Propagator from Dyson Equation and "experiment"



Equivalent to ...

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

$\Rightarrow$ as observed in (e,e'p)

$$\frac{k^2}{2m}\phi_{\ell j}^n(k) + \int dq\ q^2\ \Sigma_{\ell j}^*(k,q;E_n^-)\ \phi_{\ell j}^n(q) = E_n^-\ \phi_{\ell j}^n(k)$$

Spectroscopic factor $\quad S_{\ell j}^n = \int dk\ k^2\ \left|\langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle\right|^2 < 1$

Dyson equation also yields $\quad \chi_c^{A+1}(\boldsymbol{r}\sigma;E) = \langle\Psi_E^{c,A+1}|\, a_{\boldsymbol{r}\sigma}^\dagger\, |\Psi_0^A\rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

Dispersive optical model provides:

Link between scattering and structure data from **dispersion relations**

--> drip line

Optical potential and nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\mathrm{Re}\ \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1})\right\}$

$$\mathrm{Re}\ \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'}$$

Subtract

$$\mathrm{Re}\ \Sigma(E) = \mathrm{Re}\ \Sigma^{\widetilde{HF}}(\varepsilon_F)$$

$$- \quad \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

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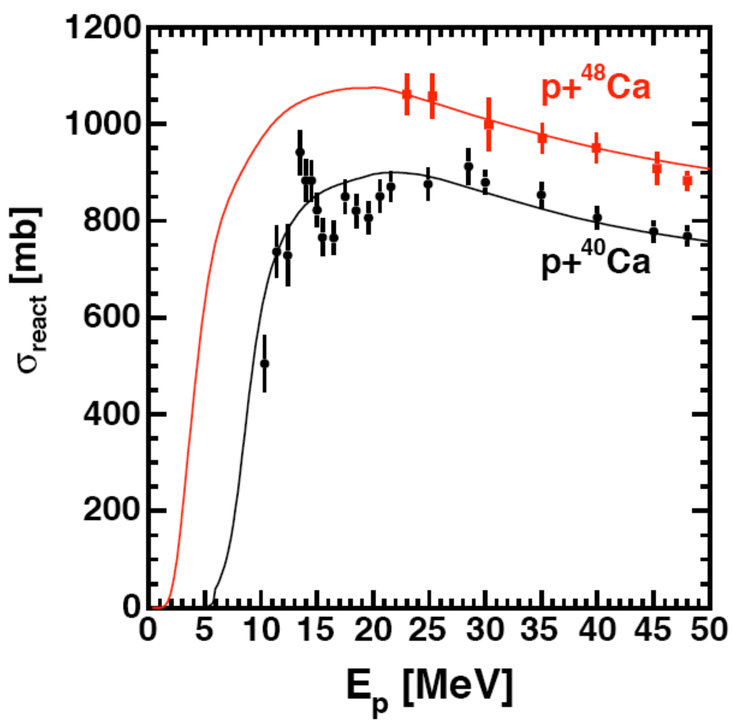
Does the nucleon self-energy also have an imaginary part above the Fermi energy?



Loss of flux in the elastic channel

Answer: YES!

Potentials assumed to have standard forms: including surface and volume absorption; parameters determined by fit to data. Potentials assumed local or “made” local. --> drip line

DOM = Dispersive Optical Model

C. Mahaux and R. Sartor, Adv. Nucl. Phys. **20**, 1 (1991)

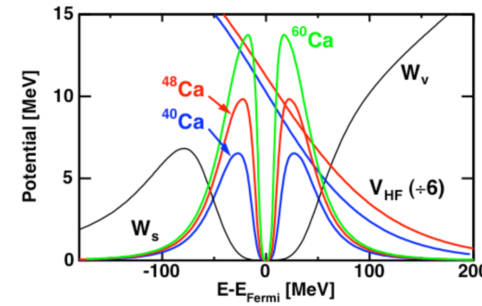
Goal: extract "propagator"/"self-energy" from data

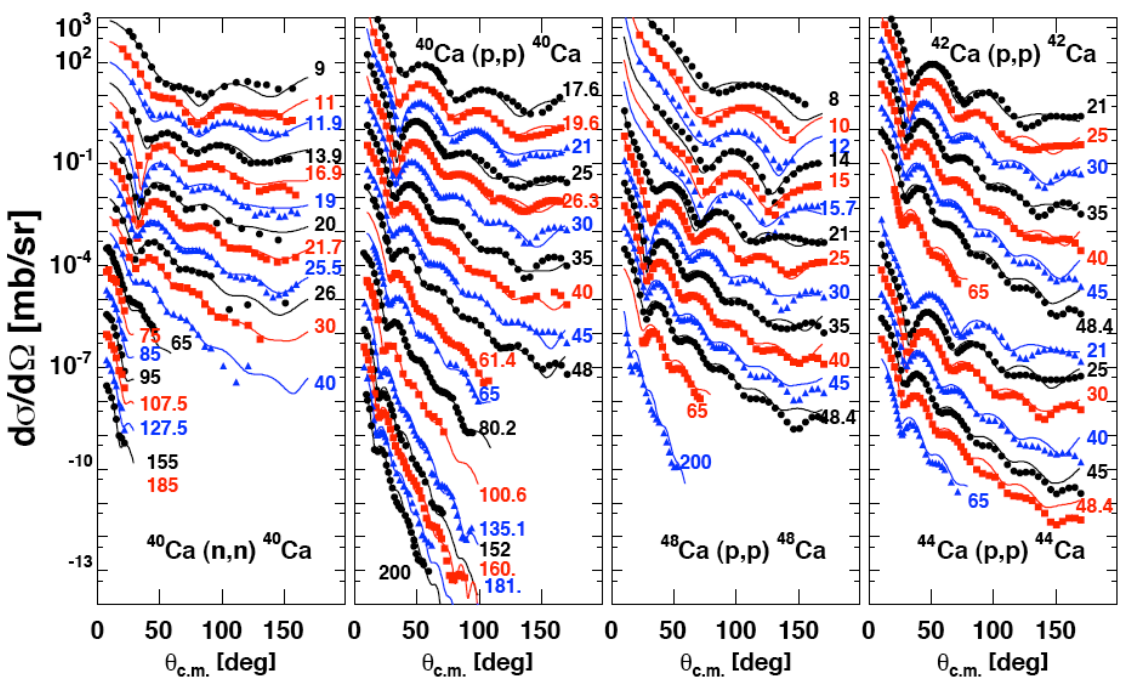
Vehicle for data-driven extrapolations / predictions to the drip lines

Combined analysis of protons/neutrons in ^{40}Ca and ^{48}Ca

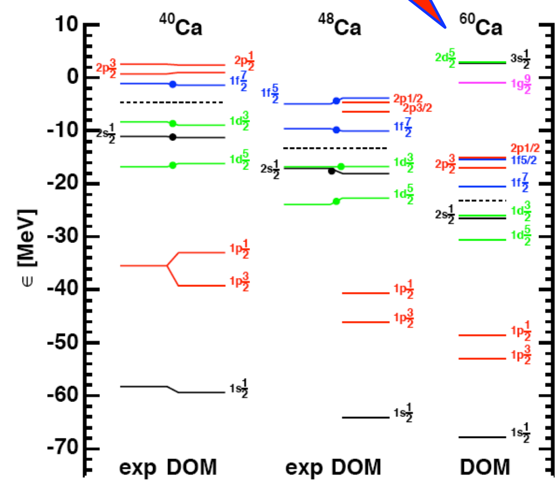
Charity, Sobotka, & WD, PRL **97**, 162503 (2006)

Charity, Mueller, Sobotka, & WD, PRC**76**, 044314 (2007)





Predict



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Correlations for nuclei with N very different from Z?

$\Rightarrow$ Radioactive beam facilities

Nuclei are TWO-component Fermi liquids

- SRC about the same between pp, np, and nn
- Tensor force disappears for n when N >> Z
- New surface effects?

- Any surprises?

- Ideally: quantitative predictions based on solid foundation

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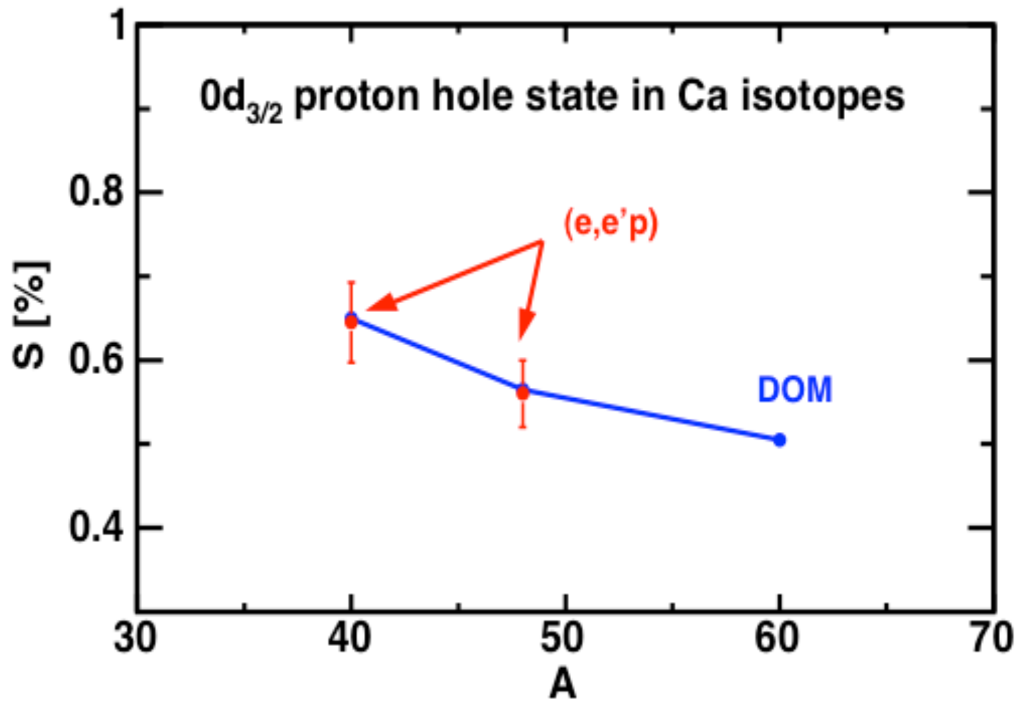
Spectroscopic factors as a function of δ

Below ε_F from DOM



Protons more correlated with asymmetry

Modest asymmetry effect

but not at the drip line yet...

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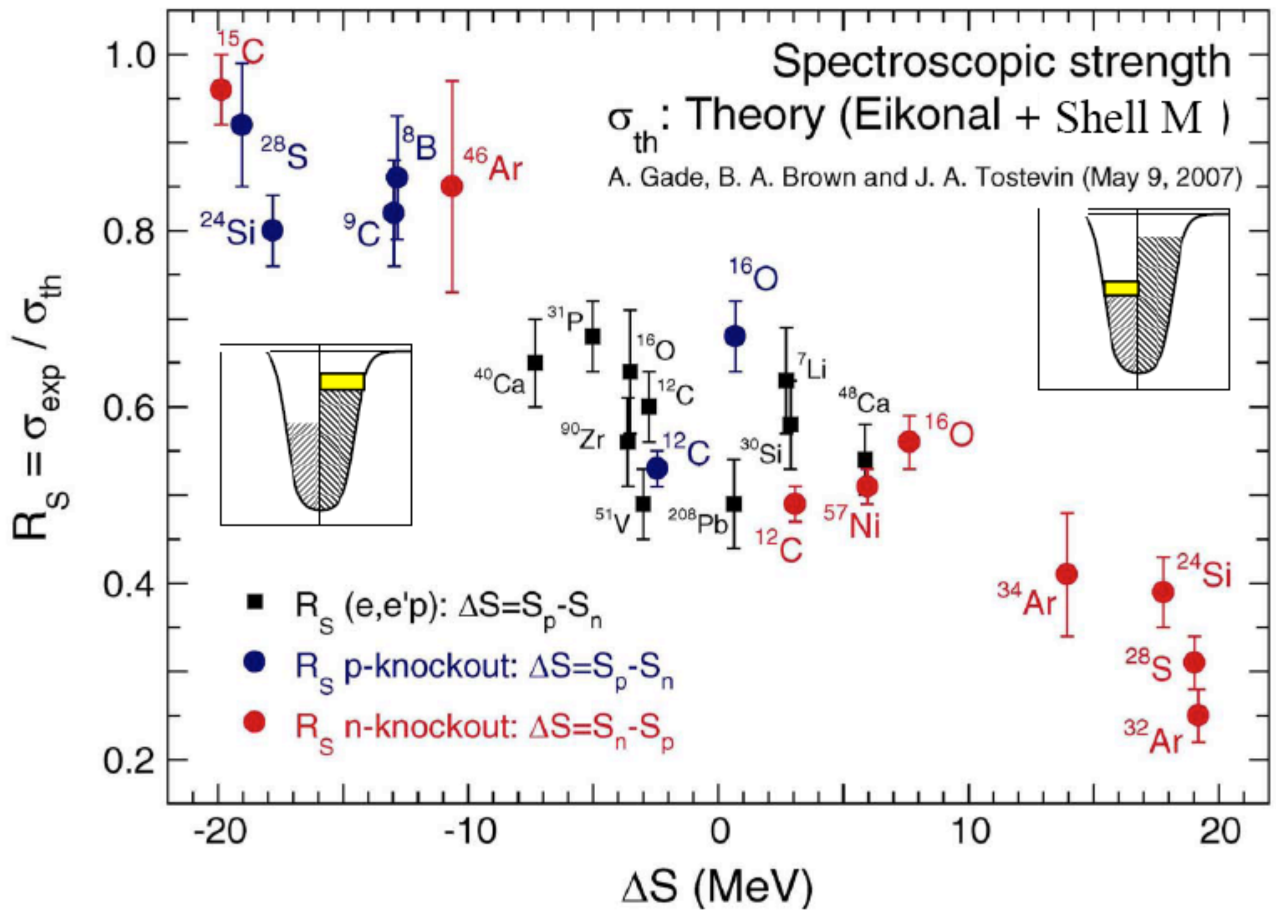
Gade et al. Phys Rev C77, 044396 (2008)

Deeply-bound systems



$R_S \neq$ not spectroscopic factor

Reduction w.r.t. shell model

neutrons more correlated with increasing proton number and accompanying increasing separation energy & vice versa

$\Rightarrow$ Spectroscopic factors become very small; way too small?

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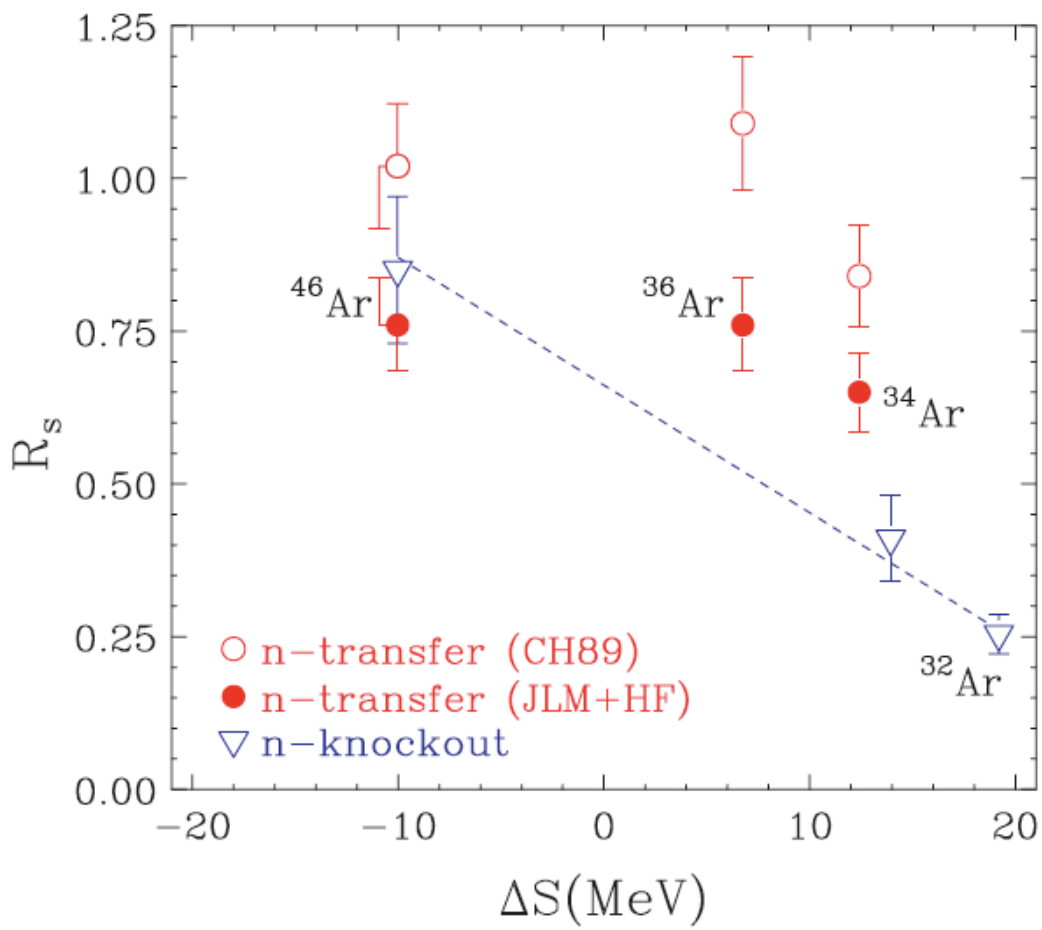
Transfer vs. HI Knockout Reactions



Different optical potentials --> different reduction factors

J. Lee et al. PRL 104, 112701 (2010)

Considerably different asymmetry dependence between transfer and HI knockout!

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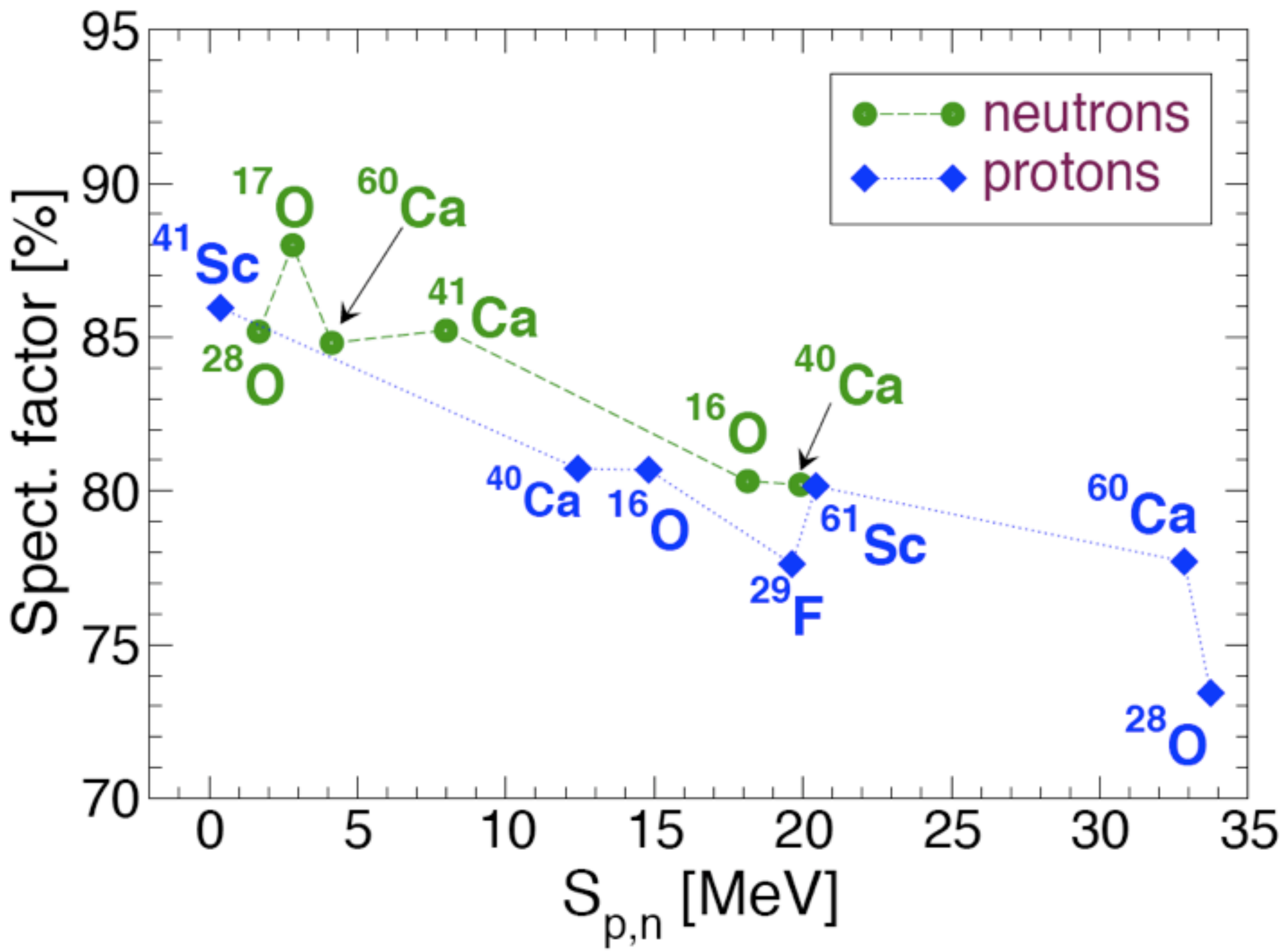
Ab initio: Faddeev-RPA ⇒ Barbieri, WD IJMPA24, 2060 (2009)



Trend similar to HI knockout but magnitude very different ...

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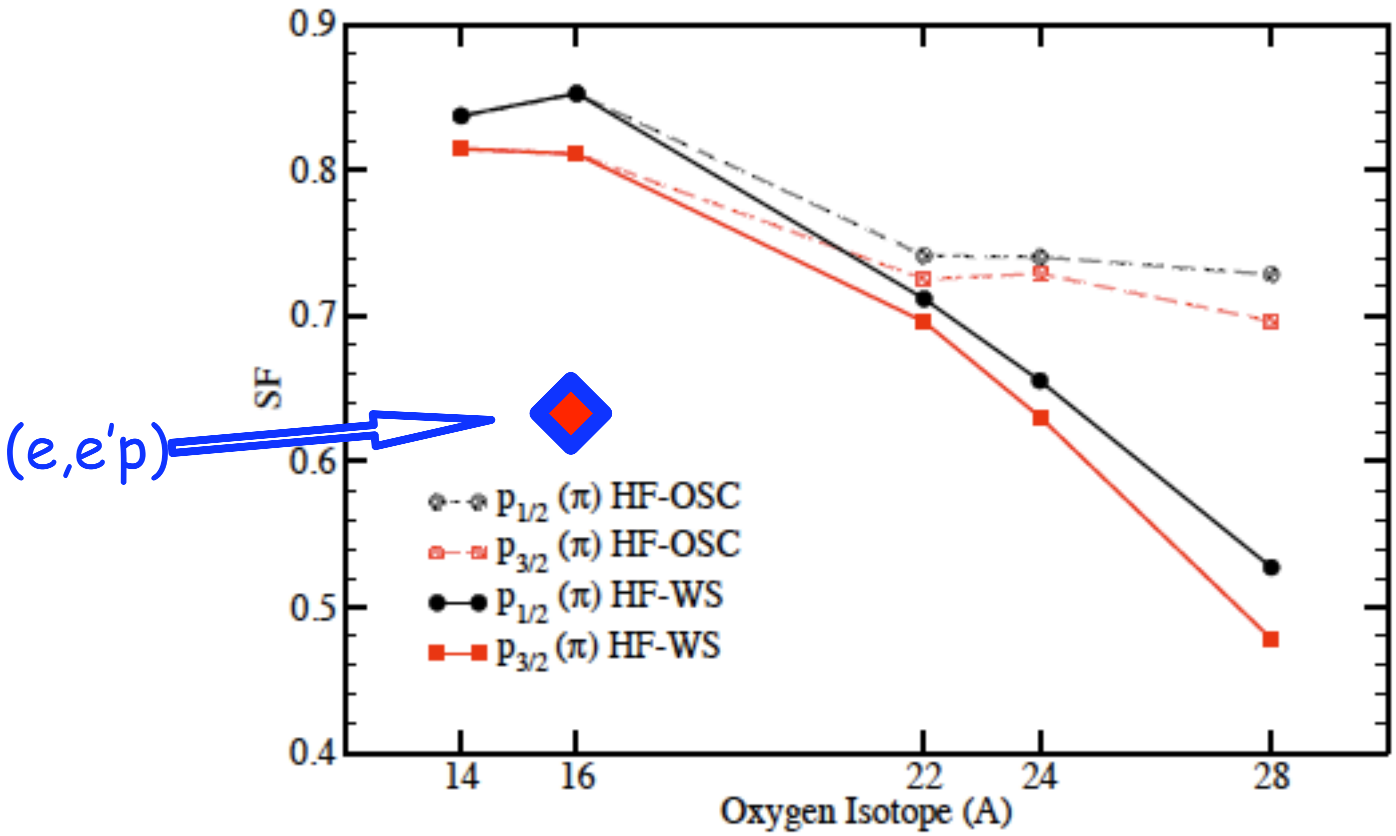
Role of the continuum...

- Jensen et al., arXiv:1104.1552v1--> PRL107,032501(2011)
- Coupled-cluster calculation with coupling to the continuum



--> drip line

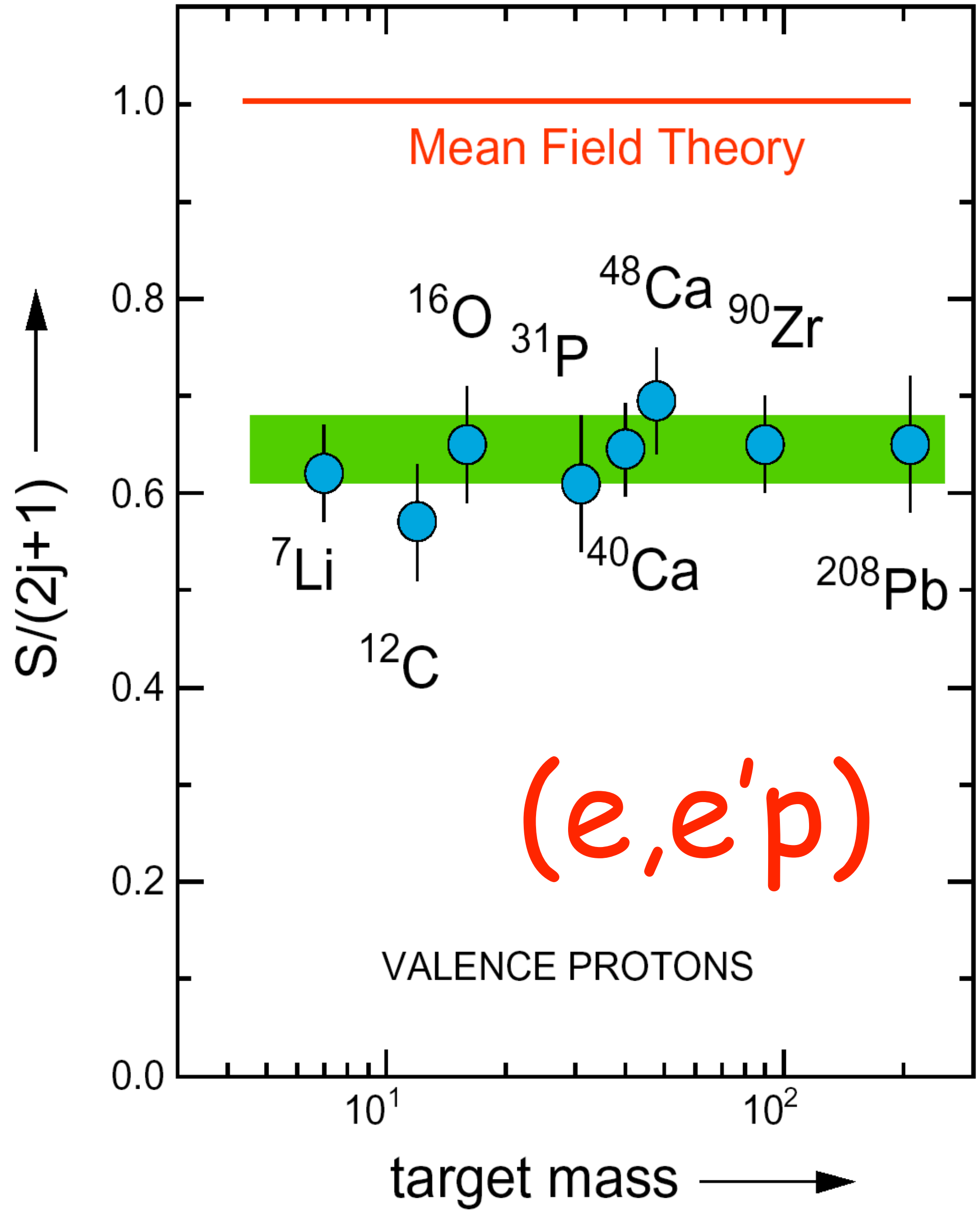
Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

S ≈ 0.65 for valence protons
Reduction ⇒ both SRC and LRC

Weak probe but propagation in the nucleus of removed proton using standard optical potentials to generate distorted waves --> associated uncertainty ~ 5-10%

Why: details of the interior scattering wave function uncertain since non-locality is not constrained (so far)



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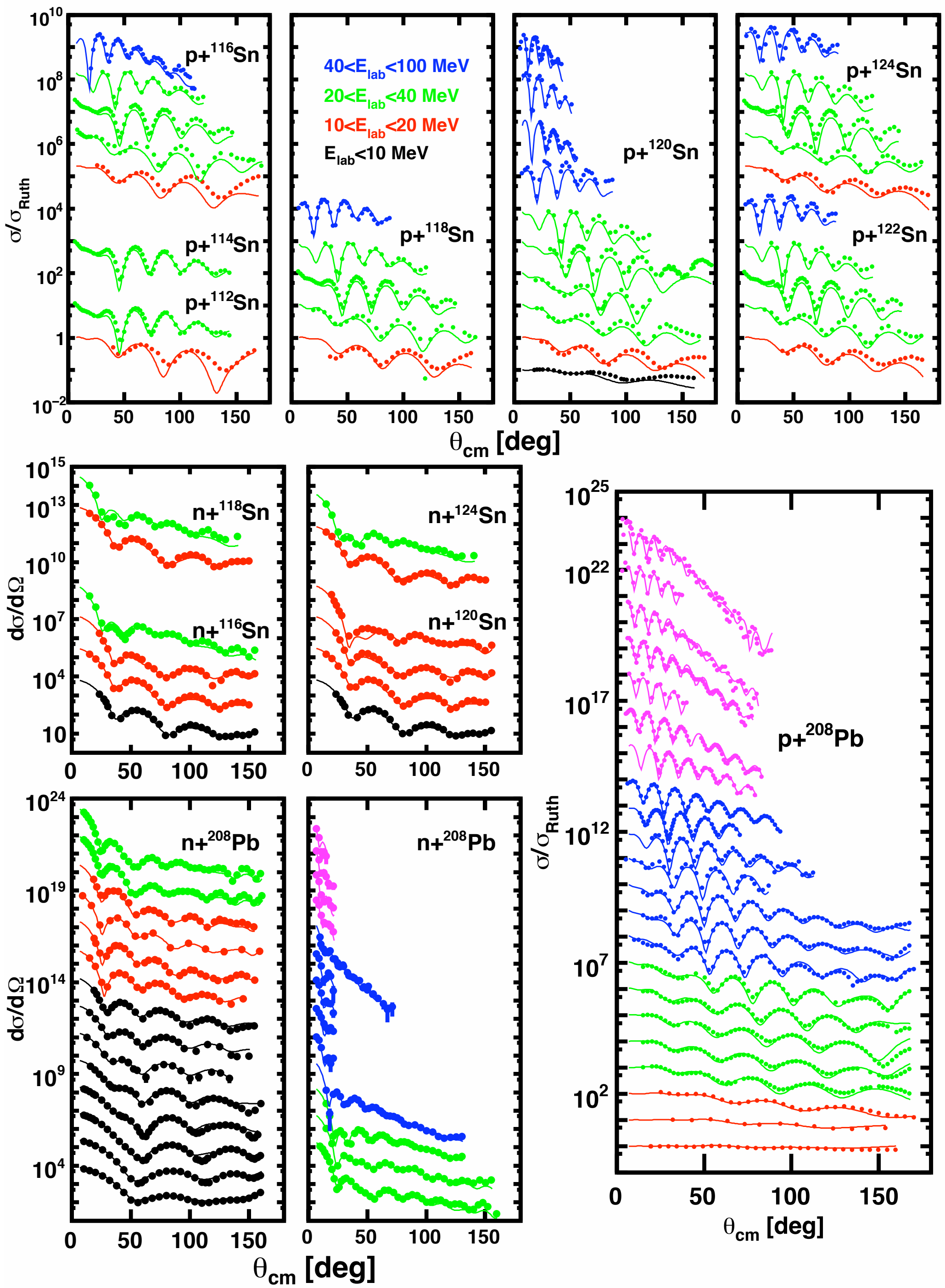
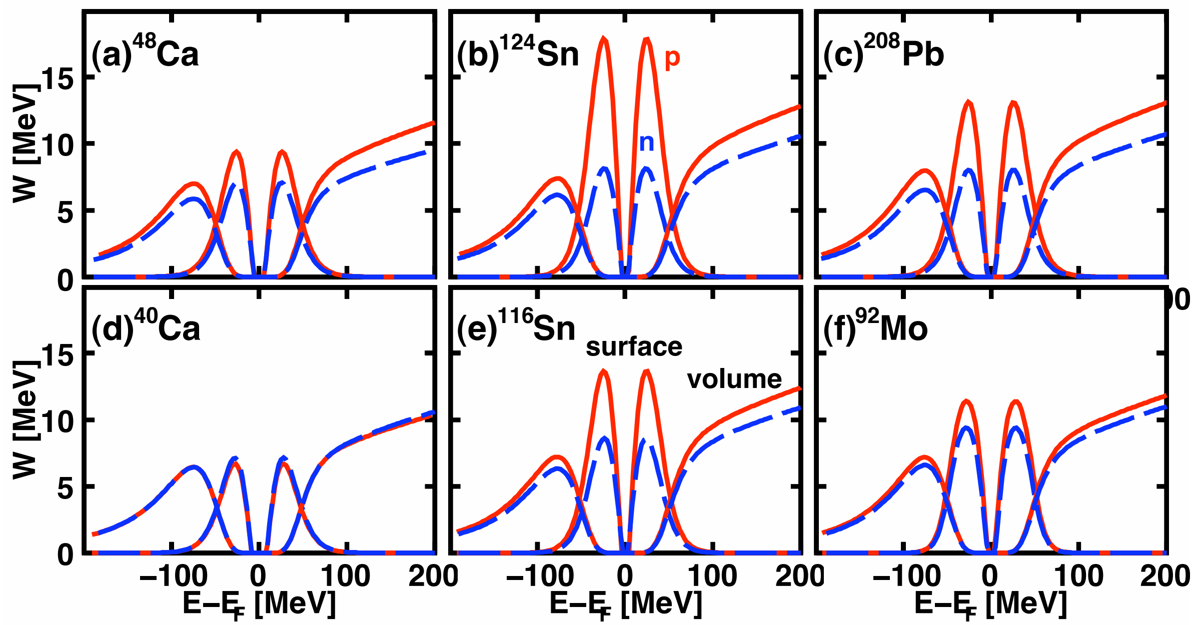
Recent DOM analysis --> towards global



PRC83,064605 (2011)

--> drip line

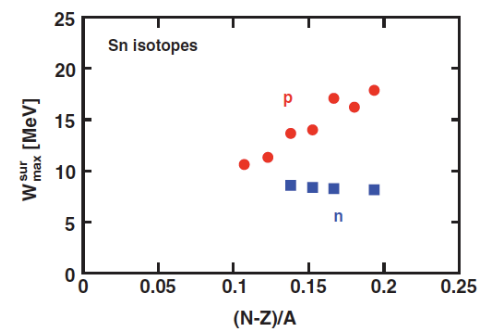
Asymmetry dependence of imaginary potentials



- Volume -> small asymmetry dependence determined in ^{208}Pb

$$W_{volume} = W^0_{volume} \pm \frac{N-Z}{A} W^1_{volume}$$

- Neutron surface -> no strong dependencies on A or (N-Z)/A
- Proton surface absorption -> increases with increasing neutron number



--> drip line

DOM improvements

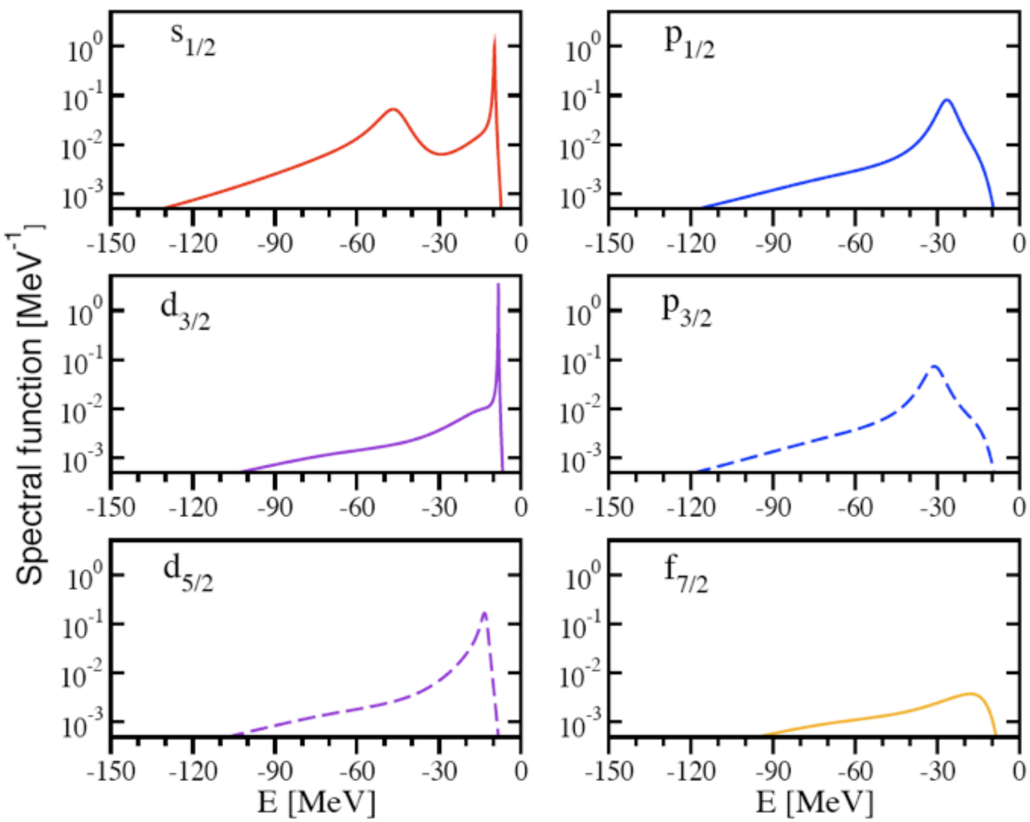
- Replace local energy-dependent HF potential by non-local (energy-independent potential) in order to calculate more properties below the Fermi energy like the charge density and spectral functions --> PRC82, 054306 (2010)

--> drip line

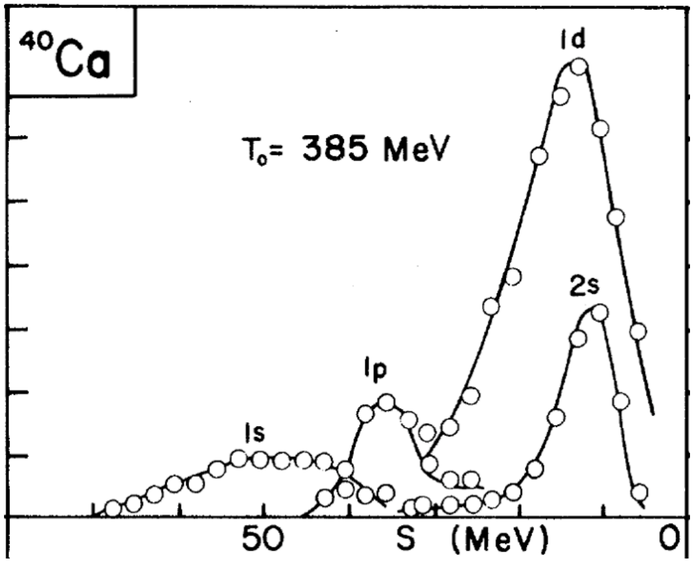
Below ε_F

^{40}Ca spectral function

Recent theoretical development: nonlocal "HF" self-energy --> below the Fermi energy
WD, Van Neck, Charity, Sobotka, Waldecker, PRC82, 054306 (2010)



Old (p,2p) data from Liverpool



--> drip line

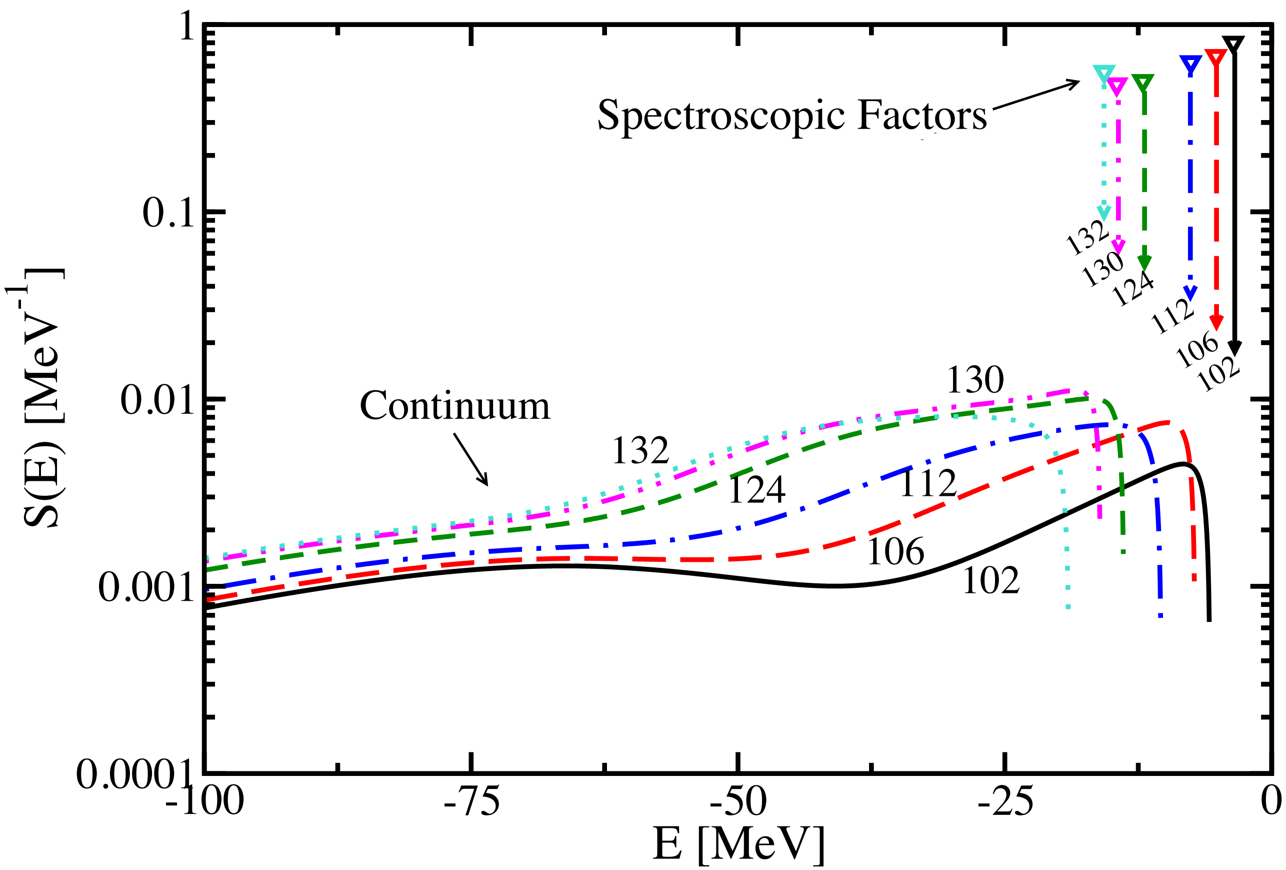
DOM predictions

- Use non-local “HF” potential and dispersive DOM potential to extrapolate to unstable Sn isotopes and predict (e.g.) properties of the last proton (based on the analysis of elastic scattering data on STABLE Sn nuclei)

--> drip line

Last proton in Sn nuclei ($g_{9/2}$)

Spectral function for different Sn isotopes

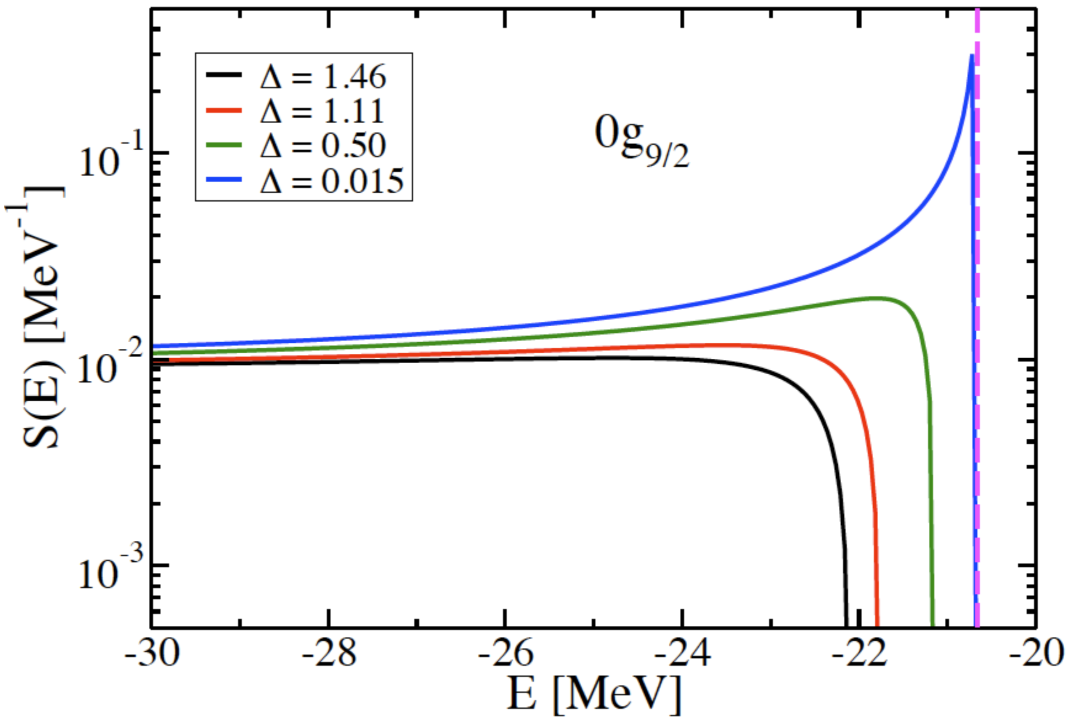


Sn	S	n
102	0.80	0.91
106	0.68	0.85
112	0.63	0.83
124	0.50	0.78
130	0.48	0.78
132	0.56	0.81

--> drip line

How about even larger asymmetry?

- Extrapolate to Meyers-Swiatecki estimate of drip line -> ^{154}Sn
- Vary distance to the continuum in ^{153}In
- Spectral function

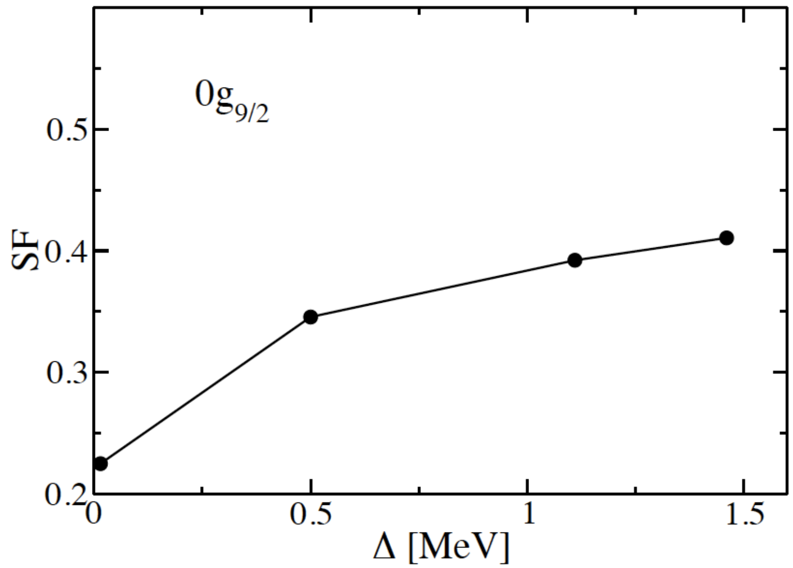


- Strength moves into the continuum

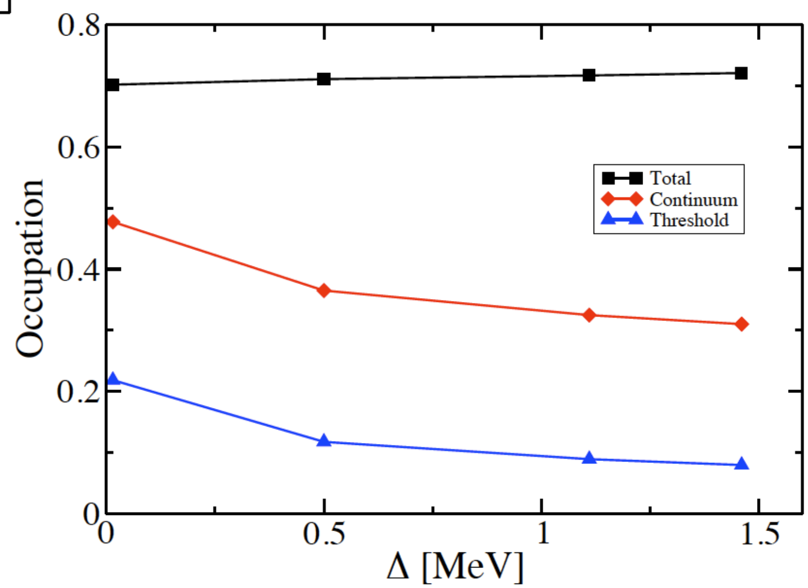
--> drip line

Occupation vs. spectroscopic factor

- Spectroscopic factor



- Occupation number

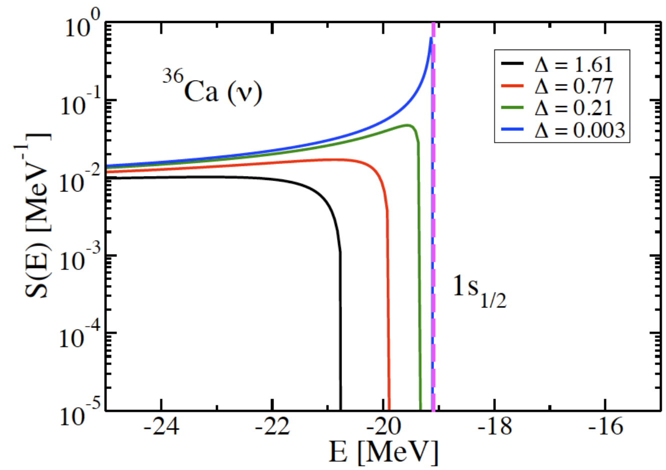


- STRENGTH nearby

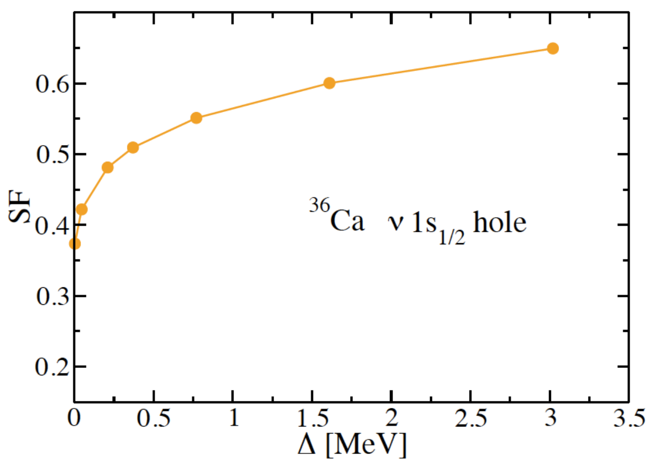
--> drip line

Other drip line --> knockout from ^{36}Ca (WU group)

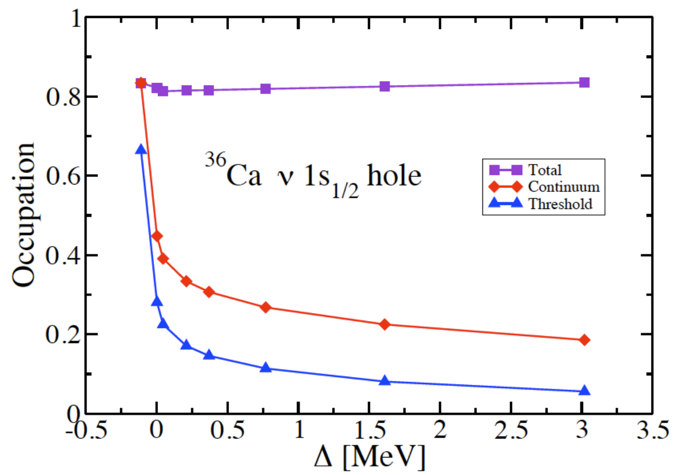
- Spectral function neutron $1s_{1/2}$



- Spectroscopic factor



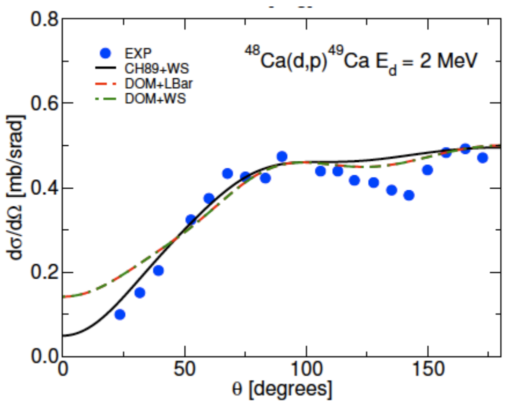
- Occupation number

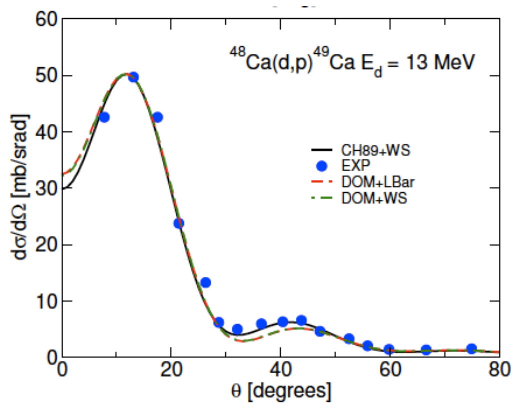


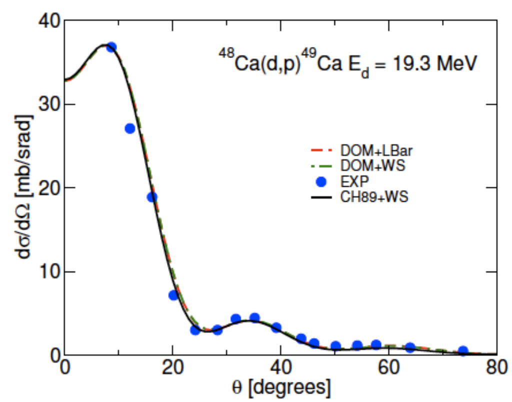
- MAY HAVE BEEN SEEN... --> drip line

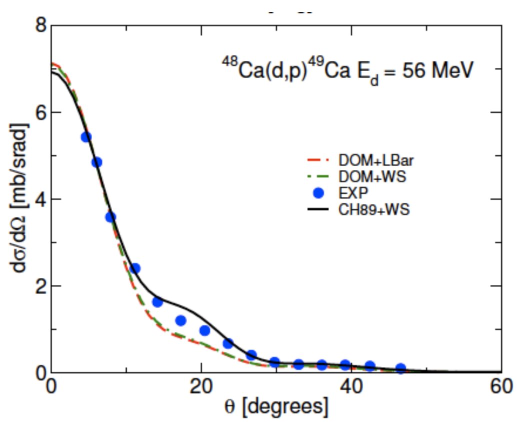
DOM ingredients and transfer reactions

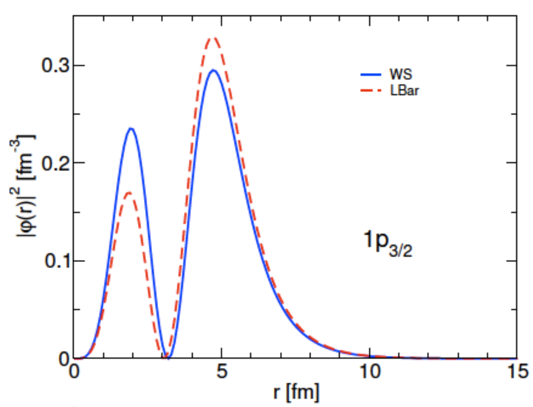
- Overlap function
- p and n optical potential
- ADWA (developed by Ron Johnson)
- Collaboration MSU-WU: Nguyen, Nuñes, Waldecker, Charity,WD --> submitted to PRC
- ^{40,48}Ca,^{132}Sn,^{208}Pb(d,p)









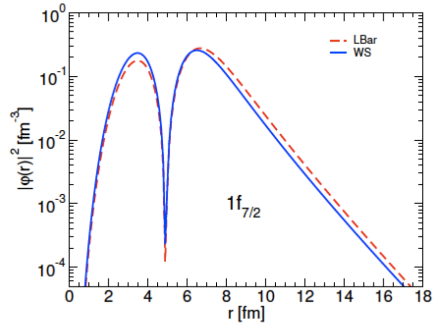


E	CH+ws	DOM
2	0.94	0.72
13	0.82	0.67
19.3	0.77	0.68
56	1.1	0.70

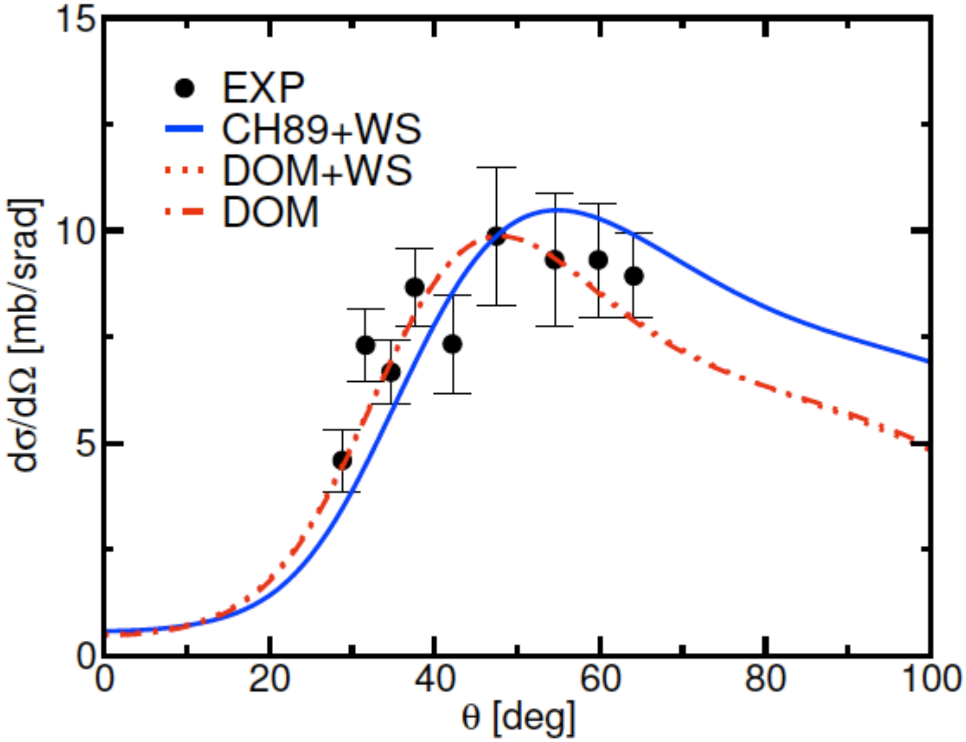
--> drip line

^{132}Sn(d,p)

- Does it work when the potentials are extrapolated?



- Data: K.L. Jones et al., Nature 465, 454 (2010)
- E_d = 9.46 MeV ^{132}Sn(d,p)^{133}Sn
- CH89+WS --> $S_{1f7/2}$ =1.1
- DOM --> $S_{1f7/2}$ =0.72



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DOM extensions linked to ab initio FRPA

- Employ microscopic FRPA calculations of the nucleon self-energy to gain insight into future improvements of the DOM --> submitted to PRC

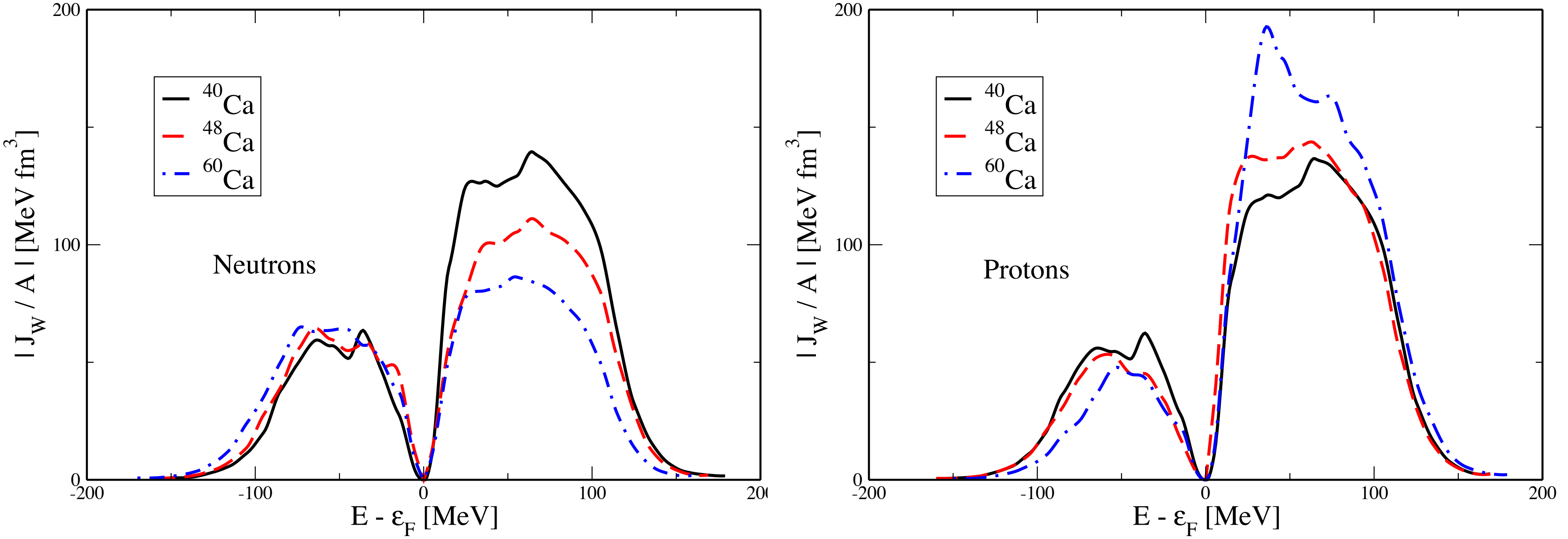
- FRPA = Faddeev RPA --> Barbieri for a recent application see e.g. PRL103,202502(2009)

--> drip line

Volume integrals from microscopic FRPA
relevant up to ~ 75 MeV

Volume integral for local imaginary potentials

$$J_W(E) = 4\pi \int dr\ r^2\ W(r,E)$$



Microscopic potentials: nonlocal --> depend on ℓ

Here averaged

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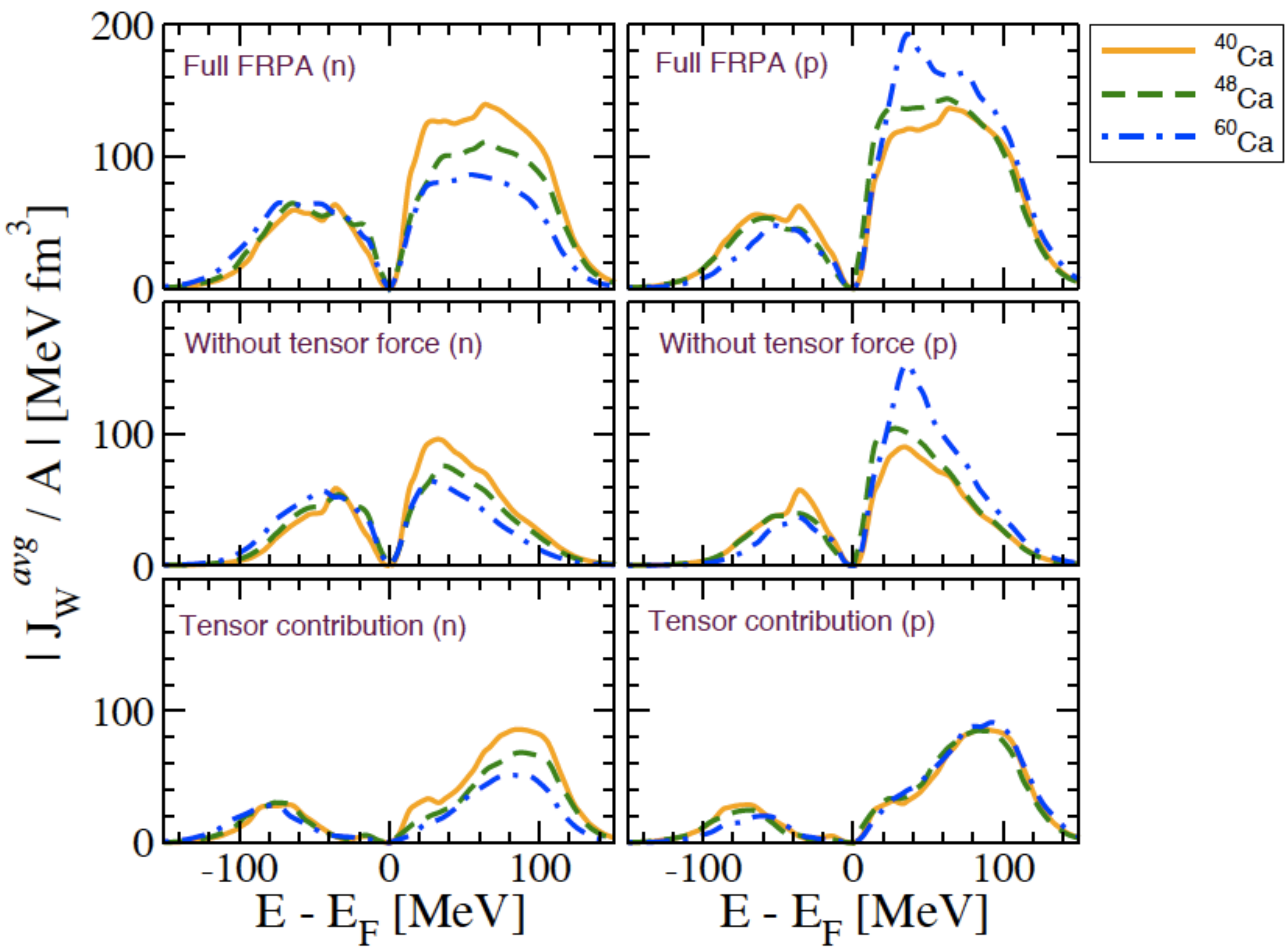
Tensor force



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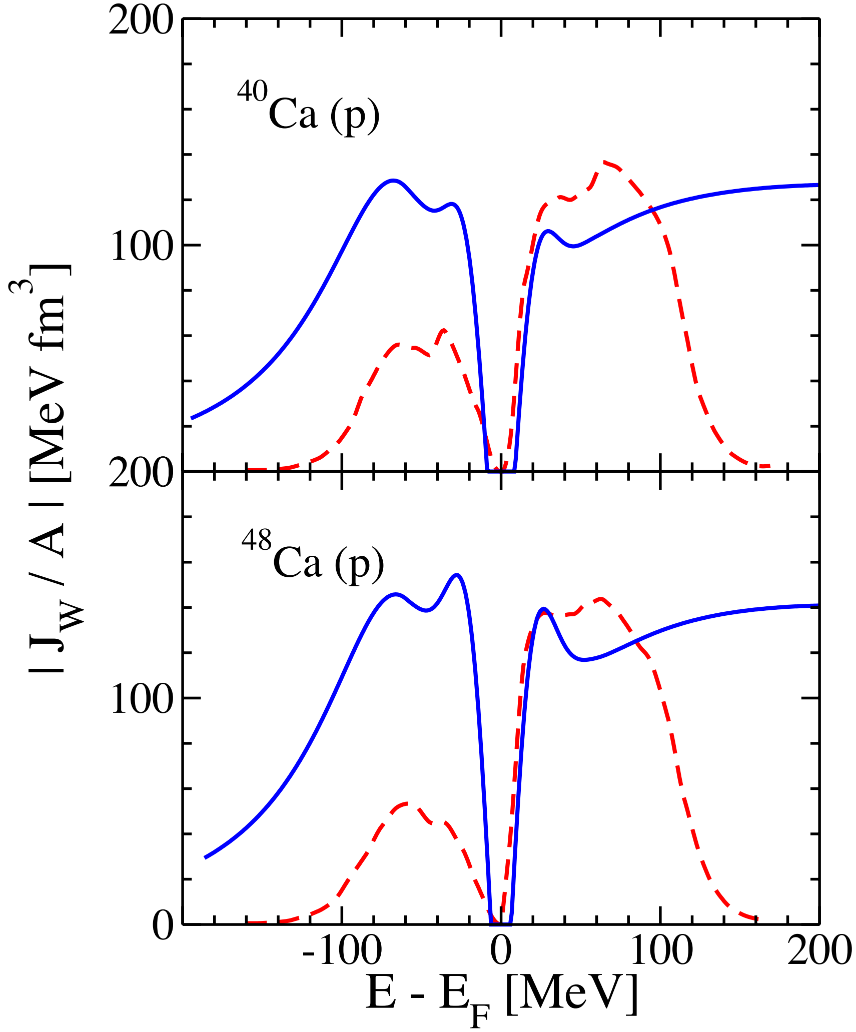
Comparison with DOM for ^{40,48}Ca



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DOM extensions linked to ab initio treatment of SRC

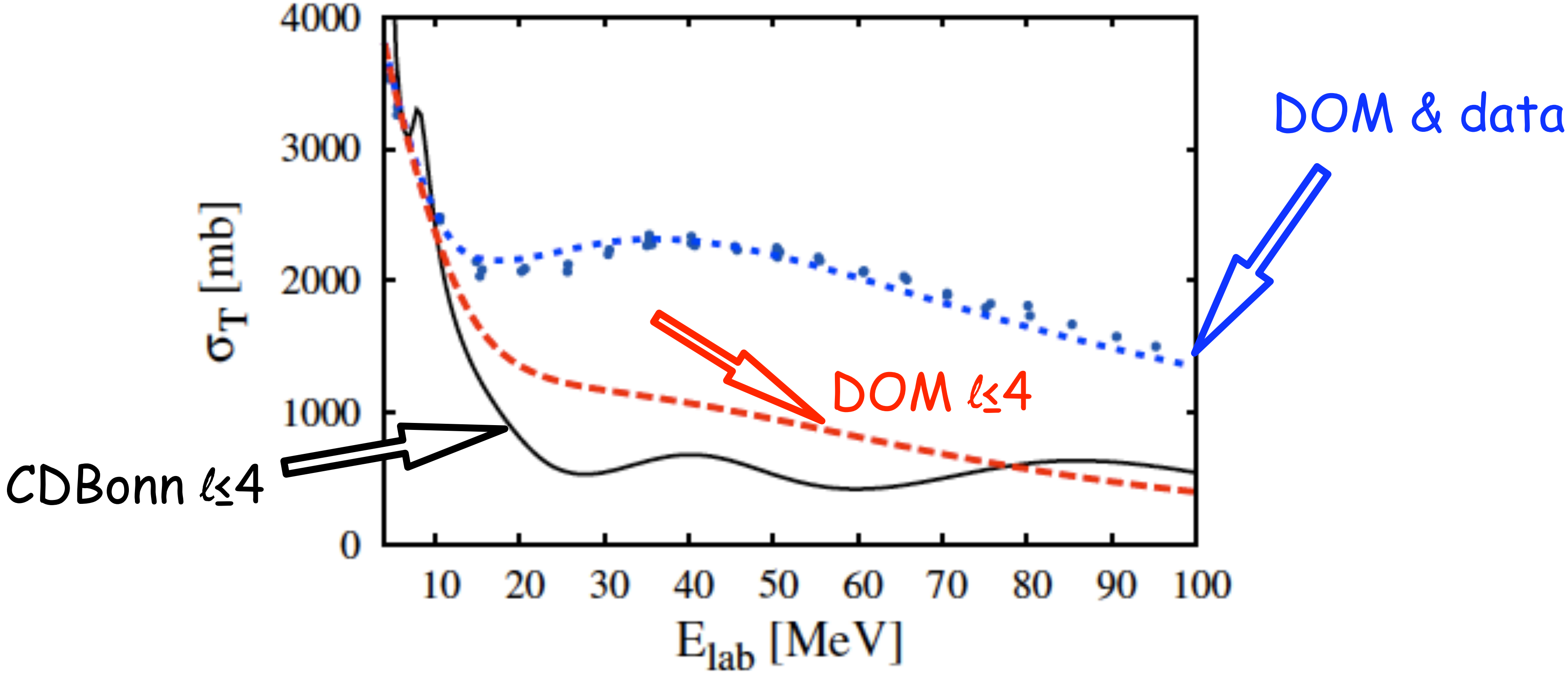
- Employ microscopic calculations of the nucleon self-energy to gain insight into future improvements of the DOM --> submitted to PRC

- CDBonn --> self-energy in momentum space for ^{40}Ca

--> drip line

Ab initio calculation of elastic scattering n+^{40}Ca

- Dussan, Waldecker, Müther, Polls, WD --> submitted to PRC
- Also generates high-momentum nucleons below the Fermi energy
- ONLY treatment of short-range and tensor correlations



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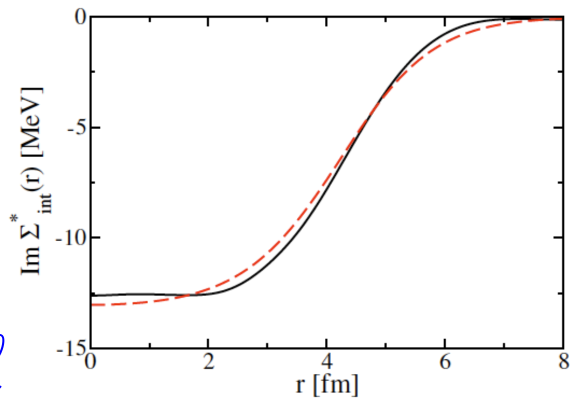
Non-locality of imaginary part

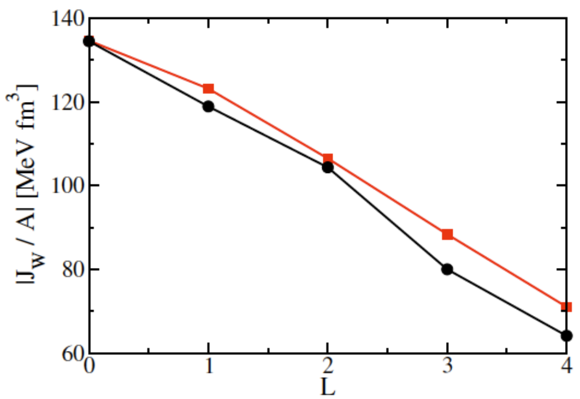
- Fit non-local imaginary part for ℓ=0

$$W_{NL}(\boldsymbol{r},\boldsymbol{r}') = W_0\sqrt{f(r)}\sqrt{f(r')}H\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{\beta}\right)$$

- Integrate over one radial variable



- Predict volume integrals for higher ℓ



Parameters

Energy MeV	W_0	r_0	a_0	β	$\lvert J_W/A\rvert$	$\lvert J_W/A\rvert$ CDBonn
-76	36.30	0.90	0.90	1.33	193	193
49	6.51	1.25	0.91	1.43	73	73
65	13.21	1.27	0.70	1.29	135	135
81	23.90	1.22	0.67	1.21	215	215

--> drip line

Conclusions

★ DOM based on Green's function framework useful vehicle to analyze elastic scattering data and level structure information

★ DOM allows data-driven extrapolation to the drip line

★ Check predictions experimentally --> improve predictions

★ Important pointers from ab initio theory (FRPA & SRC treatment) about non-locality etc.

★ Other reactions can also be described employing DOM for example (p,d) and (d,p) --> collaboration with MSU group

--> drip line