Models Discussion

Eduardo de Rafael

Centre de Physique Théorique
CNRS-Luminy, Marseille

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Reference Value for Models

$$a_\mu^{\mathrm{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}$$

Prades-de Rafael-Vainshtein '10

Reference Behaviour for Theory

$$a_\mu^{\mathrm{HLbL}}(\chi\mathrm{PT}, \mathrm{leading}) = \left(\frac{\alpha}{\pi}\right)^3 N_c \, \frac{m_\mu^2 N_c}{48\pi^2 F_\pi^2} \, \log^2 \frac{M_\rho}{m_\pi} \; (\, 9.3 \times 10^{-10} \,)$$

Knecht-Nyffeler-Perrottet-de Rafael '02

We also want to have a **Simple Reference Model**

Constituent Chiral Quark Model

The CχQM Revisited

This goes back to work by: *Manohar-Georgi '84,..., Weinberg '10*

Lagrangian in the presence of external sources

$$\mathcal{L}_{\mathrm{C}\chi\mathrm{QM}}(x) = \underbrace{i\bar{Q}\gamma^{\mu}\left(\partial_{\mu}+\Gamma_{\mu}\right)Q - \frac{i}{2}g_A\,\bar{Q}\gamma^{\mu}\gamma_5\xi_{\mu}Q}_{M-G} - \frac{1}{2}\bar{Q}\left(\Sigma - \Gamma_5\Delta\right)Q - \underbrace{M_Q\bar{Q}Q}_{M-G}$$

$$+ \underbrace{\frac{1}{4}{F_{\pi}}^2\mathrm{tr}\left[D_{\mu}UD^{\mu}U^{\dagger} + U^{\dagger}\chi + \chi^{\dagger}U\right]}_{M-G} + e^2 C\ \mathrm{tr}(Q_R U Q_L U^{\dagger})$$

$$+ L_5\ \mathrm{tr}D_{\mu}U^{\dagger}D^{\mu}U(\chi^{\dagger}U + U^{\dagger}\chi) + L_8\ \mathrm{tr}(U\chi\dagger U\chi\dagger + U^{\dagger}\chi U^{\dagger}\chi)$$

With Q *Goldstone-free quark fields*: $Q = \xi q$

$$D_{\mu}U = \partial_{\mu}U - ir_{\mu}U + iUl_{\mu}\,, \quad U = \xi\xi$$

$$\Gamma_{\mu} = \frac{1}{2}\left[\xi^{\dagger}(\partial_{\mu} - ir_{\mu})\xi + \xi(\partial_{\mu} - il_{\mu})\xi^{\dagger}\right] \quad \xi_{\mu} = i\left[\xi^{\dagger}(\partial_{\mu} - ir_{\mu})\xi - \xi(\partial_{\mu} - il_{\mu})\xi^{\dagger}\right]$$

$$\Sigma = \xi^{\dagger}\mathcal{M}\xi^{\dagger} + \xi\mathcal{M}^{\dagger}\xi \qquad \Delta = \xi^{\dagger}\mathcal{M}\xi^{\dagger} - \xi\mathcal{M}^{\dagger}\xi$$

Weinberg's PRL '11

- OBSERVATION
  In the Large-N_c limit, the Lagrangian of the CχQM is renormalizable.
- CLAIM
  The CχQM *could be* the effective low energy Lagrangian of Large-N_c QCD at low energies.

- The OBSERVATION is trivial, but it is CORRECT
- The CLAIM in my opinion is unfortunately DOUBTFUL

This model, however, with $g_A = 1$ has very good features

It reproduces rather well the phenomenological values of the $\mathcal{O}(p^4)$ Gasser-Leutwyler constants:

$$2L_1 = L_2 = \frac{1}{12}\frac{N_c}{16\pi^2}\,, \quad L_3 = \frac{1}{6}\frac{N_c}{16\pi^2} \quad \text{and} \quad 2\mathrm{L}_{10} = -\mathrm{L}_9 = -\frac{1}{3}\frac{\mathrm{N_c}}{16\pi^2}$$

Notice that with $g_A \neq 1$ the predicted constants are logarithmically divergent!

What is the value of M_Q?

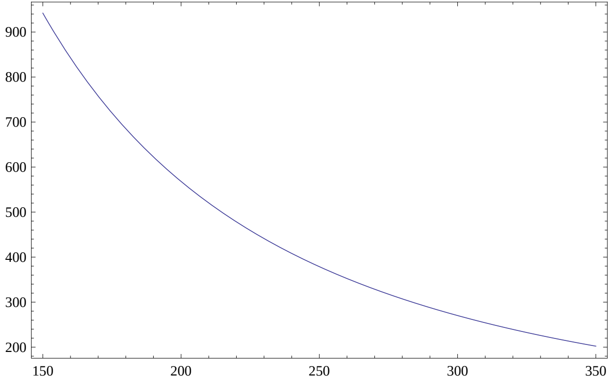
I claim it has to be rather low for this Lagrangian to be effective

$$2M_Q < M_\rho \qquad M_Q \simeq 200\ \mathrm{MeV}$$

This low value reproduces well the known $\mathcal{O}(p^6)$ low energy constants:

- The slope of Hadronic Vacuum Polarization at the origin.
- The C_{87} constant of the $\Pi_{\mathrm{LR}}(Q^2)$ Correlation Function.
- The $\chi(\mu)$ constant which governs The $\pi^0 \to e^+e^-$ Decay

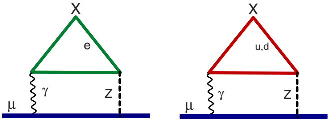
Predicted Value in 10^{10} units versus M_Q in MeV



Phenomenological Value from *Davier et al '09*

$$a_\mu^{\rm hvp} = (687.3 \pm 4.2_{\rm exp} \pm 1.9_{\rm rad} \pm 0.7_{\rm QCD}) \times 10^{-10} \quad [e^+e^- - \text{data}]$$

This is the contribution from the first and second generations induced by the Feynman diagrams:



The CχQM does well in evaluating this contribution because, with $g_A = 1$, *Vainshtein's* leading short-distance-behaviour in $\frac{1}{Q^2}$ is guaranteed.

This short-distance-behaviour is at the origin of the cancellation of $\log M_Z$ in each generation (*Marciano's dixit*)

$$a_\mu^{\mathrm{EW}}(e,\mu,u,d,s,c) = \frac{G_{\mathrm{F}}}{\sqrt{2}}\,\frac{m_\mu^2}{8\pi^2}\,\frac{\alpha}{\pi} \times (-24.6 \pm 1.8)$$

The CχQM predicts two contributions:

- The Constituent -Goldstone Free Quark- Loop

$$a_\mu^{\mathrm{HLbL}}(\mathrm{Qloop}) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right) \left\{\left[\frac{3}{2}\zeta(3) - \frac{19}{16}\right] \frac{m_\mu^2}{{M_Q}^2} + \mathcal{O}\left(\frac{m_\mu^4}{{M_Q}^4} \log^2 \frac{m_\mu^2}{{M_Q}^2}\right)\right\}$$

(which goes as $\sim \frac{1}{{M_Q}^2}$**)**

- The Goldstone Exchange with CχQM Form Factors

$$\mathcal{F}^{(\mathrm{C}\chi\mathrm{QM})}_{\pi^{0*}\gamma^*\gamma^*}\left(q_2^2, q_1^2, q_3^2\right) =$$

$$-\frac{N_c}{12\pi^2 f_\pi} \int_0^1 dxx \int_0^1 dy \frac{2{M_Q}^2}{{M_Q}^2 - x(1-x)(1-y)q_1^2 - x^2 y(1-y)q_2^2 - xy(1-x)q_3^2}$$

(which goes as $\sim \log^2 \frac{4{M_Q}^2}{m_\pi^2}$**)**

Light-by-Light Hadronic Contribution to $g_\mu - 2$

- There is a numerical calculation by *Bartős et al '02* with $M_Q = 280\ \mathrm{MeV}$:

$$a_\mu^{\mathrm{HLbL}}(\mathrm{C}\chi\mathrm{QM}) = [\underbrace{(8.2 \pm 1.8)}_{\pi^0} + \underbrace{(0.6 \pm 0.2)}_{\eta} + \underbrace{(6.2 \pm 1.9)}_{\mathrm{Loop}}] \times 10^{-10}$$
$$= (15.0 \pm 2.6) \times 10^{-10}$$

- It would be nice to have an analytic calculation of the Goldstone Exchange contribution. Here the following Mellin-Barnes representation of Form Factors may be useful:

$$\mathcal{F}^{(\chi\mathrm{QM})}_{\pi^{0*}\gamma^*\gamma^*}\left(q_2^2, q_1^2, q_3^2\right) =$$
$$-\frac{N_c}{12\pi^2 f_\pi}\left(\frac{1}{2\pi i}\right)^3 2\int_{c_1-i\infty}^{c_1+i\infty} ds_1 \left(\frac{[-q_1^2]}{M_Q^2}\right)^{-s_1} \int_{c_2-i\infty}^{c_2+i\infty} ds_2 \left(\frac{[-q_2^2]}{M_Q^2}\right)^{-s_2} \int_{c_3-i\infty}^{c_3+i\infty} ds_3 \left(\frac{[-q_3^2]}{M_Q^2}\right)^{-s_3}$$
$$\times \frac{\Gamma(1-s_1-s_2)\Gamma(1-s_1-s_3)\Gamma(1-s_2-s_3)}{\Gamma(3-2s_1-2s_2-2s_3)}\Gamma(s_1)\Gamma(s_2)\Gamma(s_3)\Gamma(1-s_1-s_2-s_3)$$

- Impressive Progress in Lattice QCD!

Various Suggestions:

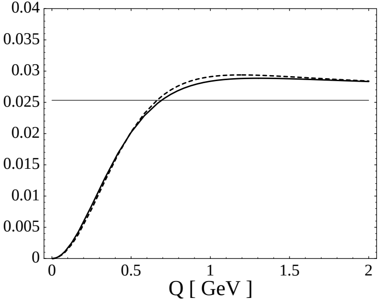
- HVP Contribution

$$a_{\mu\mathrm{h}} = \frac{\alpha}{\pi}\frac{1}{2}\int_0^1 \frac{dx}{x}(1-x)(2-x)\mathcal{A}\left(\frac{x^2}{1-x}m_\mu^2\right)$$

where

$$\mathcal{A}(Q^2) = -Q^2\frac{\partial\Pi(Q^2)}{\partial Q^2} = \int_0^\infty dt\,\frac{Q^2}{(t+Q^2)^2}\frac{1}{\pi}\mathrm{Im}\Pi(t)\,,$$

and show us the Adler function plot.



Comments from Discussions at this Workshop

- $\pi^0 \to e^+e^-$

  Here we need the form factor $\mathcal{F}_{\pi^0\gamma*\gamma*}(0, Q^2, Q^2)$. There is an integral over Q^2 with a simple kernel, which defines the coupling χ needed for the Log coefficient in the χPT approach to a_μ^{LbyL}.

- EW Hadronic Contribution

$$Q^2\left[w_L(Q^2) - 2w_T(Q^2)\right] = \frac{16\pi^2}{\sqrt{3}}\int d^4x \int d^4y\, e^{iq\cdot x}(x-y)_\lambda \epsilon^{\mu\nu\rho\lambda}\langle 0|\hat{T}\left\{L_\mu^3(x)V_\nu^3(y)R_\rho^8(0)\right\}|0\rangle$$

$$L_\mu^3(x) = \bar\psi(x)\frac{\lambda_3}{2}\gamma^\mu\frac{1-\gamma_5}{2}\psi(x)\,, \quad R_\rho^8(x) = \bar\psi(0)\frac{\lambda_8}{2}\gamma^\rho\frac{1+\gamma_5}{2}\psi(0)\,, \quad V_\nu^3(y) = \bar\psi(y)\frac{\lambda_3}{2}\gamma^\nu\psi(y)$$

$$w_L(Q^2) = 2\frac{N_c}{Q^2}$$

$$w_T(Q^2) \underset{Q^2\to\infty}{\sim} \frac{N_c}{Q^2} - 32\pi^4\left(\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right)\langle\bar\psi\psi\rangle\,\Pi_{\mathrm{VT}}(0)\frac{1}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right)$$

$$w_T(Q^2) \underset{Q^2\to 0}{\sim} 128\pi^2 C_{22}^W + \mathcal{O}(Q^2)$$

- From Presentations and Discussions about Models
  - There is conservation of energy-momentum!
  - Melnikov-Vainshtein's OPE Constraint
    Discrepancies $\Rightarrow$ Misunderstandings
    We should be more careful in applying this OPE constraint to Models
  - Express contributions, in as much as possible, as convolutions of a known QED kernel with unknown Hadronic Functions (models)
  - Holographic Approach:
    Appears to me as a useful **MODEL** parameterization of Large-N_c
    More effort to generalize to other contributions (axials)
  - Bethe-Salpeter Approach: Better understanding of qualitative aspects; better understanding of Asymptotic Behaviour of Model
    Where do Big contributions come from?