HLbl from a Dyson–Schwinger Approach

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INT Workshop on Hadronic Light-by-Light contribution to the Muon Anomaly

February 28th–March 4th 2011

We haven't yet presented our work...

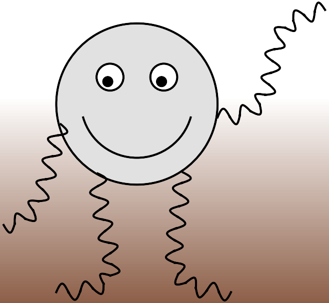
...already there are demands that we:

calculate a different contribution to a_μ

- Hadronic Vacuum Polarisation

significantly improve the calculation

- consider rho-pole suppression in quark-loop



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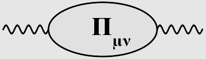
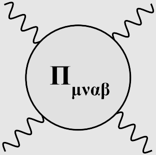
The muon $g-2$

Photon vacuum polarisation tensors





How to proceed?

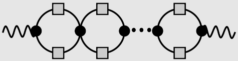
- Ideally solve using ONE approach
- one-scale problem vs. two-scale problem.

existing models large N_c plus chiral counting / effective description of QCD / scale matching.

‘Little brother’ of $\Pi_{\mu\nu\alpha\beta}$: $\Pi_{\mu\nu}$

Two-photon vacuum polarisation tensor

Extended NJL



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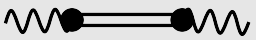
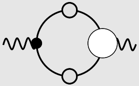


Dyson-Schwinger



Quarks are different

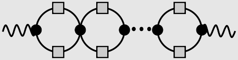
non-renormalisable
constituent-like

renormalisable
momentum dependent

‘Little brother’ of $\Pi_{\mu\nu\alpha\beta}$: $\Pi_{\mu\nu}$

Two-photon vacuum polarisation tensor

Extended NJL



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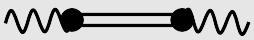
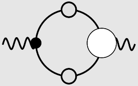


Dyson-Schwinger



Rho-poles are the result of

- **ENJL:** Bubble sum of constituent-like quarks
- **DSE:** QCD corrections to quark-photon vertex (QPV)

NOTE QPV satisfies Ward-Takahashi identity. Vector meson contained in structures transverse to photon momentum

Diagrammatic content of $\Pi_{\mu\nu}$

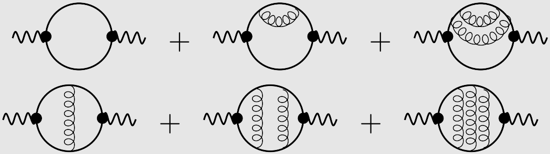
anticipate future truncation

- large N_c
- rainbow-ladder
- only planar diagrams
- no gluon-self interactions

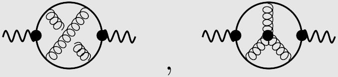
Restrict topologies of resummed diagrams.

rainbow

ladder



neglect/model



Diagrammatic content of $\Pi_{\mu\nu}$

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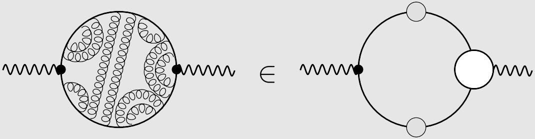
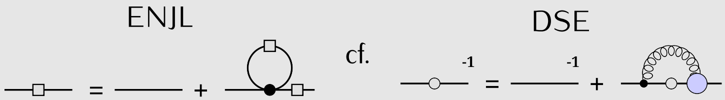


achieved by appropriate restrictions on the Green's functions:

Ingredients I – Quark Propagator



Parameterisation and Solution

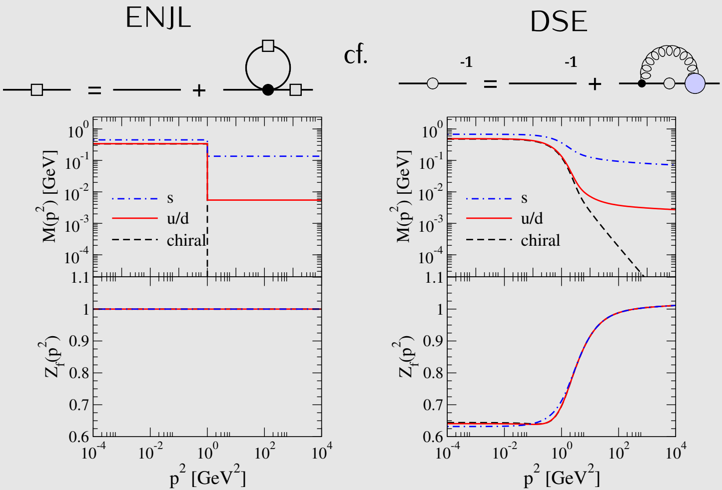
$$S_F^{-1}(p;\mu) = i\not{p}\ \mathcal{A}(p^2;\mu^2) + \mathbb{1}\ \mathcal{B}(p^2;\mu^2)$$

- $Z_f = 1/\mathcal{A}$, $M = \mathcal{B}/\mathcal{A}$: $\mathcal{B}(p^2)$, $\mathcal{A}(p^2)$ scalar, vector dressings
- momentum p, renormalisation point μ

DSE Specified by model/truncation:

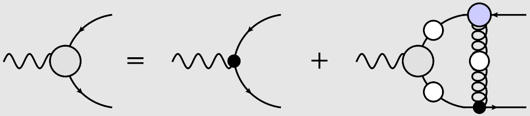
- quark-gluon vertex
- gluon propagator

Ingredients I – Quark Propagator



Ingredients II

Quark-photon vertex (ladder-truncation)



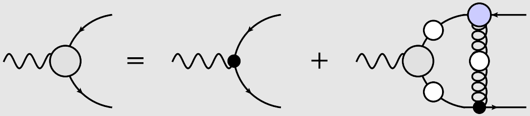
basis decomposition: $(\mathbb{1}, \not{k}, \not{P}, [\not{k}, \not{P}]) \otimes (\gamma^\mu, k^\mu, P^\mu)$

$$\Gamma_\mu(P,k) = \sum_{i=1}^{12} V_\mu^{(i)} \lambda^{(i)}(P,k) = \Gamma_\mu^{\mathrm{L}} + \Gamma_\mu^{\mathrm{T}}$$

- covariant tensor $V_\mu^{(i)}$, scalar dressing function $\lambda^{(i)}(P,k)$
- total momentum P, relative momentum k
- Γ_μ^{T} **transverse** to P, Γ_μ^{L} is **non-transverse**

Ingredients II

Quark-photon vertex (ladder-truncation)



solution

- Ward-Takahashi identity constrains Γ_μ^L in terms of S_F^{-1}

$$\Gamma_\mu^L = \gamma_\mu \Sigma_A + 2\not{k}\, k_\mu \Delta_A + i2k_\mu \Delta_B$$

Σ_A, Δ_A functions of A , Δ_B function of B

- Γ_μ^T solved numerically. Contains dynamical ρ-pole

the story thus far...

photon two-point function

We have

- (large N_c inspired) truncation of $\Pi_{\mu\nu}$
- non-perturbative quark propagator / quark-photon vertex

Can determine a_μ from the HVP:

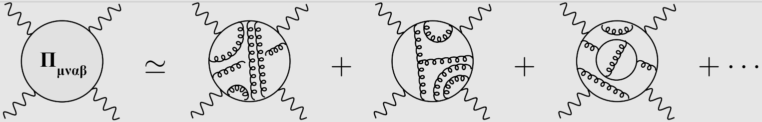
- No scale matching
- Do not distinguish short/long distances
- Separate contributions by topology

Next: photon four-point function

For consistency apply **same** methods of truncation

‘Big brother’ of $\Pi_{\mu\nu}$: $\Pi_{\mu\nu\alpha\beta}$

general diagrammatic content



procedure and truncation

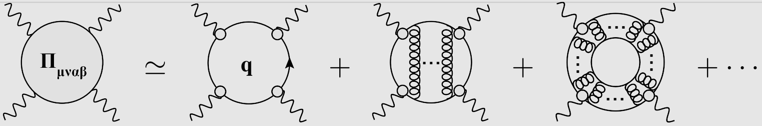
- Order diagrams according to large-N_c counting.
- Neglect non-planar diagrams/gluon self-interactions.
- Resummation using Dyson-Schwinger equations.

classify according to the topology of resummed diagram

no separation into short-distance/long-distance!

'Big brother' of $\Pi_{\mu\nu}$: $\Pi_{\mu\nu\alpha\beta}$

general diagrammatic content – (collected by topology)



(NB: quark propagators are fully dressed!)

procedure and truncation

- Order diagrams according to large-N_c counting.
- Neglect non-planar diagrams/gluon self-interactions.
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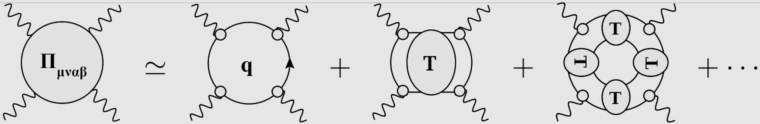
‘Big brother’ of $\Pi_{\mu\nu}$: $\Pi_{\mu\nu\alpha\beta}$

general diagrammatic content – (resummed)

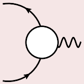


(NB: quark propagators are fully dressed!)

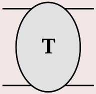
necessary ingredients

quark propagator

quark-photon vertex

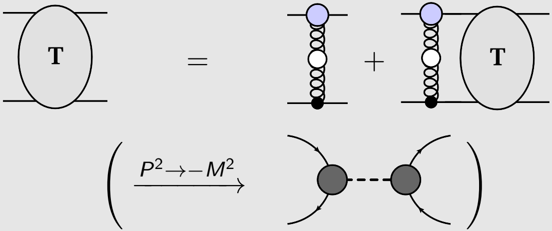


T-matrix



T-matrix

ladder truncation



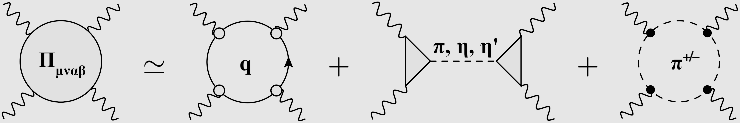
infinite ladder summation of non-perturbative gluons

- dynamically generates all $q\bar{q}$ bound-state poles
- encodes **all** on- and off-shell information unambiguously

T-matrix: hard to calculate in practice (work-in-progress)

Truncation scheme

resonance expansion of T-matrix



obtain picture similar to existing approaches.

Eduardo de Rafael.
Hadronic contributions to the muon g-2 and low-energy QCD.
Phys.Lett., B322:239–246, 1994.

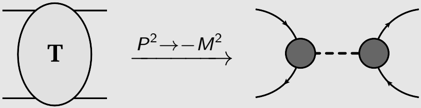
G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael.
Chiral Lagrangians for Massive Spin 1 Fields.
Phys.Lett., B223:425, 1989.

G. Ecker, J. Gasser, A. Pich, and E. de Rafael.
The Role of Resonances in Chiral Perturbation Theory.
Nucl.Phys., B321:311, 1989.

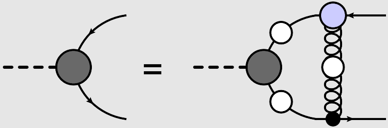
T-matrix: bound-state poles

homogeneous Bethe-Salpeter amplitude

- postulate existence of particle pole

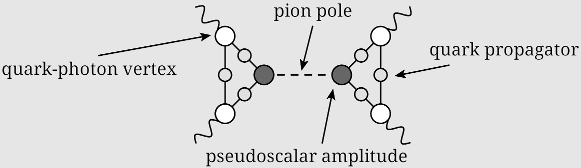


- obtain equation for ‘amplitude’ on-shell



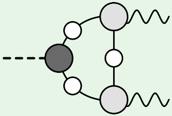
Pion-pole approximation

assume pole dominance



analogous to existing approaches

form factor

$$\Lambda_{\mu\nu}^{\pi\gamma^*\gamma^*} =$$ 

caveat: pion here is defined on-shell.

Off-shell prescription

rough outline

- start with axial-vector Ward-Takahashi identity in χ-limit

$$2P_\mu \Gamma^{a=3}_{\mu 5}(k,P) = iS^{-1}(k_+)\gamma_5 + i\gamma_5 S^{-1}(k_-)$$

- Relates S and $\Gamma_{\mu 5}$. Note

$$\Gamma^{a=3}_{\mu 5}(k,P) \simeq \frac{P_\mu f_\pi \Gamma_\pi(k,P)}{P^2+M^2} + \text{reg.}$$

- contains BS amplitude as pseudoscalar pole, with

$$\Gamma_\pi(k;P) = \gamma_5\Big[E + \cdots\Big] \qquad S^{-1}(k) = i\not{p}\,A(k) + B(k)$$

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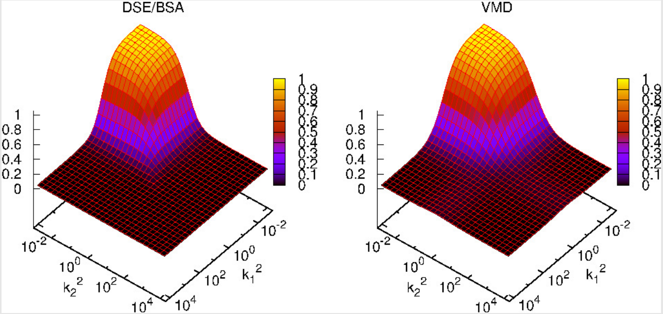
$$\Gamma_\pi(k;P) = \gamma_5\left[E + \cdots\right] \qquad S^{-1}(k) = i\not{p}A(k) + B(k)$$

on-shell	off-shell
$E(k, k\cdot P) = B(k^2)/f_\pi$,	$E(k,P) = (B(k_+^2) + B(k_-^2))/2f_\pi$

$$\boxed{E(k,P) = \left(E(k_+, k_+\cdot P) + E(k_-, k_-\cdot P)\right)/2}$$

Pseudoscalar form-factor

form-factor (use full quark-photon vertex)

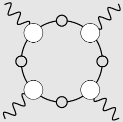


comments

- similar behaviour to VMD and thus comparable results
- approximation: pole dominance + off-shell pion BSA

Quark-loop

Calculation (exploit Ward-Takahashi identity)

$$\widetilde{\Pi}_{(\rho)\mu\nu\alpha\beta} = \frac{\partial}{\partial k_\rho}\left[\right.$$  $$\left. + \text{permutations}\right]$$

Projector

$$a_\mu = \frac{1}{48 m_\mu} \mathrm{tr}\left[\left(i\not{P} + m_\mu\right)\left[\gamma_\sigma, \gamma_\rho\right]\left(i\not{P} + m_\mu\right)\widetilde{\Gamma}_{\sigma\rho}\right]\Bigg|_{k\equiv 0}$$

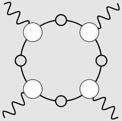
with $\widetilde{\Gamma}_{\sigma\rho}$ related to muon vertex via

$$ie\Gamma_\mu = iek_\rho\widetilde{\Gamma}_{\rho\mu}$$

Quark-loop

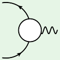
Calculation (exploit Ward-Takahashi identity)

$$\widetilde{\Pi}_{(\rho)\mu\nu\alpha\beta} = \frac{\partial}{\partial k_\rho} \Bigg[$$



$$+ \text{permutations} \Bigg]$$

Algebraically challenging

 2 terms ,  12 terms

- 331 776 terms × Dirac trace algebra **before derivative**.

Full quark-photon vertex Highly non-trivial!

Specifics: model interaction

effective quark-gluon interaction (rainbow-ladder)

phenomenologically successful

model: **gluon** and **quark-gluon vertex**

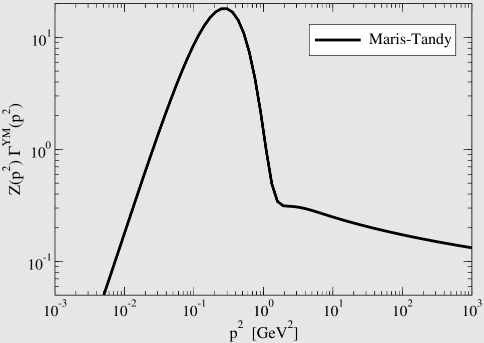
- meson and baryon spectroscopy
- EM form-factors, pion charge radius
- decay constants, widths.

Parameters of model are tuned for meson phenomenology.

use values from literature **without fine-tuning**

Specifics: model interaction

effective quark-gluon interaction (rainbow-ladder)



- IR enhancement provides dynamical chiral symmetry breaking.
- UV tail matches perturbation theory

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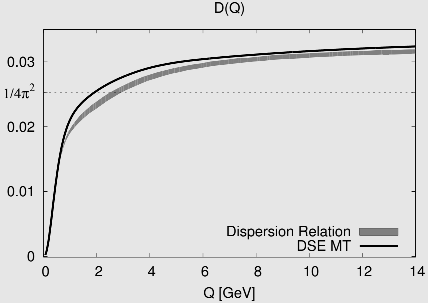
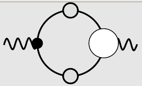
Results: Adler function ($N_f = 5$)

preliminary/unpublished



$$\left[\frac{1}{3q^2} T_{\mu\nu}(q) \right.$$  $$\left. \right] = \Pi(q^2), \quad D(q) = -q^2 \frac{d\Pi(q^2)}{dq^2}$$

(dispersion data from Jegerlehner, 2008)

Results: Adler function ($N_f = 5$)

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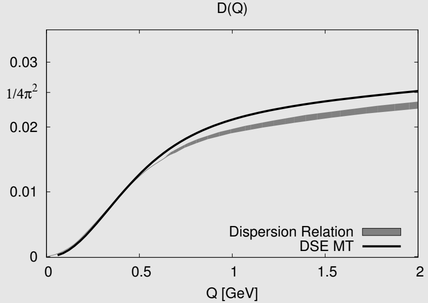
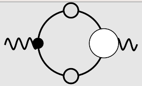


$$\left[\frac{1}{3q^2}\,T_{\mu\nu}(q)\right.$$  $$\left.\right] = \Pi(q^2), \quad D(q) = -q^2\frac{d\Pi(q^2)}{dq^2}$$

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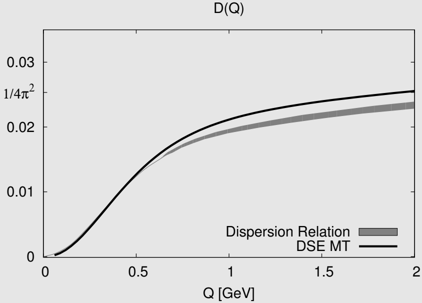
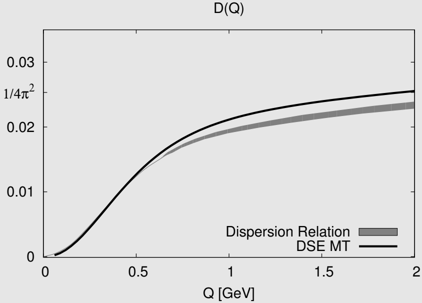


$$a_\mu^{\mathrm{HVP}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\left[-e^2\Pi\left(\frac{x^2}{1-x}m_\mu^2\right)\right]$$

(de Rafael, 1994)

Results: Adler function ($N_f = 5$)

preliminary/unpublished

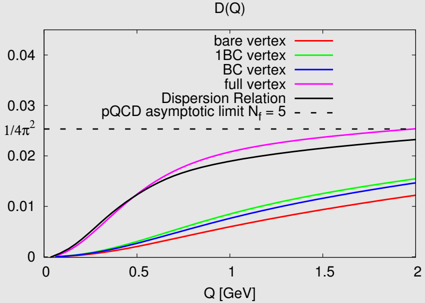


$$a_{\mu,\mathrm{DSE}}^{\mathrm{HVP,LO}} = 6\,700 \times 10^{-11} \quad \text{cf.} \quad a_{\mu,\mathrm{expt}}^{\mathrm{HVP,LO}} = 6\,903.0(52.6) \times 10^{-11}$$

approach gives consistent results for HVP!

Results: Adler function ($N_f = 5$)

Dependence on vertex truncation



vertex	a_μ
bare	800×10^{-11}
1BC	1200×10^{-11}
BC	1500×10^{-11}
full	6700×10^{-11}

- vector meson pole necessary
- **importance depends on kinematics of problem at hand**
  - one-scale problem vs. two-scale problem for $\Pi_{\mu\nu\alpha\beta}$

Now: Hadronic Light-by-Light Scattering

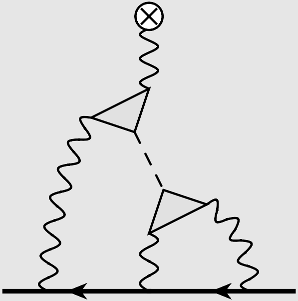
photon two-point function

- Approach gives reasonable prediction for HVP
  - leading-order result at 5%-level!
- No additional fine tuning of model parameters

Justifies application of DSE approach to the four-point function

Results: pseudoscalar-pole

pion exchange



- leading π^0-pole

$$a_\mu^{\mathrm{LBL}} = 58(7) \times 10^{-11}$$

phenomenological mixing for η, η'

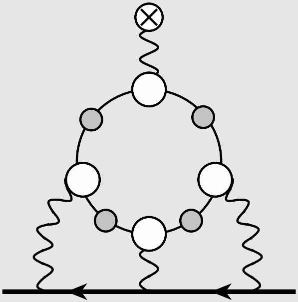
- Total result for pseudoscalar-pole

$$a_\mu^{\mathrm{LBL}} = 80.7(12) \times 10^{-11}$$

Marc Knecht and Andreas Nyffeler.
Hadronic light by light corrections to the muon g-2: The Pion pole contribution.
Phys.Rev., D65:073034, 2002.

Results: quark-loop

quark loop



bare vertex

$$a_\mu^{\text{LBL}} = (\ 61 \pm 2) \times 10^{-11}$$

1BC

$$a_\mu^{\text{LBL}} = (107 \pm 2) \times 10^{-11}$$

full BC

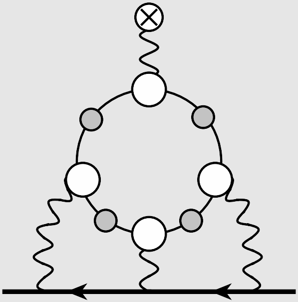
$$a_\mu^{\text{LBL}} = (176 \pm 4) \times 10^{-11}$$

Dressing effects from Ward-Identity enhance: $A \neq 1$, $B \neq M$.

Size of suppression from rho-pole?

Results: quark-loop

quark loop



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full BC

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full vertex (unpublished)

$$a_\mu^{\mathrm{LBL}} = (106 \pm 5) \times 10^{-11}$$

Dressing effects from Ward-Identity enhance: $A \neq 1$, $B \neq M$.

Size of suppression from rho-pole? -40%

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Summary and Outlook

Summary

First DSE calculation of g-2 for

- HVP
  - **predicted** within our approach!
  - 6700×10^{-11} (cf $\simeq 6903$ expt)
- hLBL
  - **pseudoscalar-pole** $81(12) \times 10^{-11}$
  - **quark-loop** $106(5) \times 10^{-11}$

Find: vertex dressing important

- transverse – dominated by **rho-pole**
- non-transverse – constrained by **Ward-Identity**

preliminary estimate: $\boxed{a_\mu = 116\,591\,861.0(71.0)10^{-11}}$ (2.4σ)

Summary and Outlook

Outlook

Check quality of pion-pole approximation via DSEs

- Melnikov–Vainshtein constraint
- calculate **pion propagator**
- calculate quark-antiquark scattering matrix T