INT Workshop on Hadronic Light-by-Light Contribution to the Muon Anomaly

February 28 -March 4, 2011, Seattle

Comments on Recent Developments in Theory of Hadronic Light-by-Light

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With Joaquim Prades and Eduardo de Rafael we wrote in 2008 a kind of white paper on HLbL summarizing our understanding of the problem at that time.

It is was very sad to learn that Ximo passed away in August.

In our '08 mini-review we combined different calculations with some educated guesses about possible errors to come to:

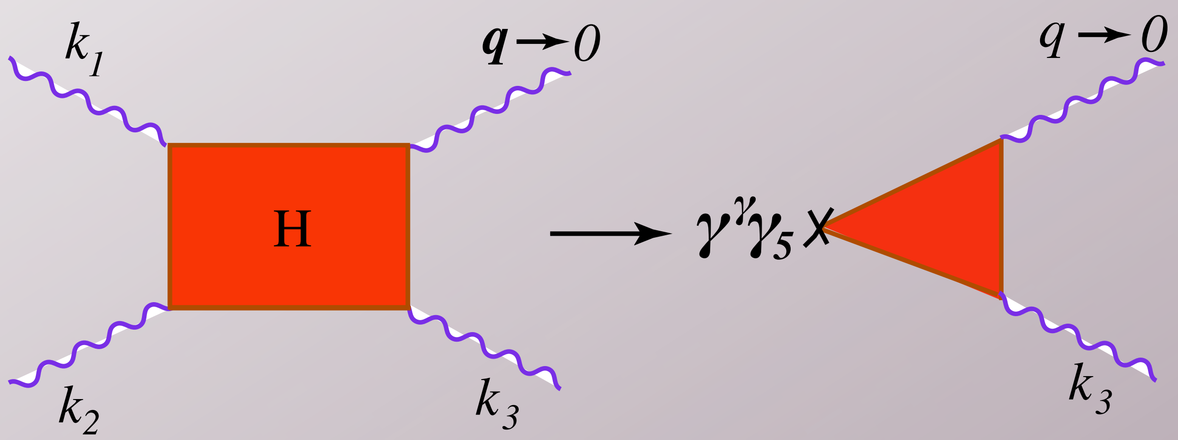
$$a^{\mathrm{HLbL}} = (105 \pm 26) \times 10^{-11}$$

However the error estimates are quite subjective and further study of different exchanges is certainly needed.

While I do not think that there were significant changes during the last 3 years I'll try to comment on few suggestions which appeared at this period.

Off-shell Form Factors Nyffeler, Jeherlehner

OPE constraints



In the range where $q_1^2 \approx q_2^2 \gg q_3^2$ and $q_3^2 \gg \Lambda^2_{\mathrm{QCD}}$

$$i \int \mathrm{d}^4 x \, \mathrm{d}^4 y \, \mathrm{e}^{-iq_1 x - iq_2 y} \, T \{ j_{\mu_1}(x), j_{\mu_2}(y) \} = \int \mathrm{d}^4 z \, \mathrm{e}^{-i(q_1+q_2)z} \frac{2i}{\hat{q}^2} \epsilon_{\mu_1 \mu_2 \delta \rho} \, \hat{q}^\delta j_5^\rho(z) + \cdots$$

$$\hat{q} = (q_1 - q_2)/2 \approx q_1 \approx -q_2$$

we get for the HLbL amplitude

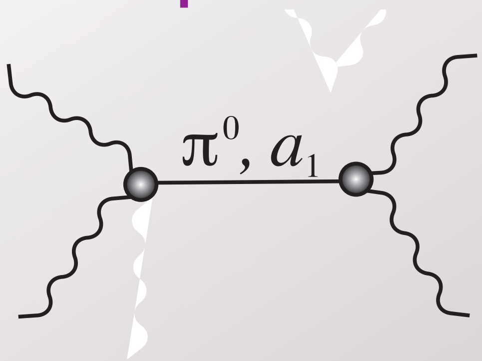
$$\mathcal{M} \;=\; \alpha^2 N_c\,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}$$

$$\mathcal{A} = \frac{4}{q_3^2\,\hat{q}^2}\{f_2\tilde{f}_1\}\{\tilde{f}f_3\}$$

$$-\frac{4}{q_3^2\,\hat{q}^4}\Big(\{q_2 f_2\tilde{f}_1\tilde{f}f_3 q_3\} + \{q_1 f_1\tilde{f}_2\tilde{f}f_3 q_3\} + \frac{q_1^2+q_2^2}{4}\{f_2\tilde{f}_1\}\{\tilde{f}f_3\}\Big) + \cdots$$

where $f_i^{\mu\nu} = q_i^\mu\epsilon_i^\nu - q_i^\nu\epsilon_i^\mu$ are field strengths of photons. Thus, the amplitude is unambiguously fixed in the range $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda^2_{\mathrm{QCD}}$ Note an absence yet of any reference to the pion pole. By quantum numbers the first line refers to pseudoscalar exchange, the second -- pseudovector.

Compare this now with the meson exchange near the meson pole. For the pion pole we have



$$\mathcal{A}_{\pi^0} = -\frac{N_c W^{(3)}}{2\pi^2 F_\pi^2} \frac{F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)}{q_3^2 + m_\pi^2} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}$$

$W^{(3)} = \frac{1}{4}$ accounts for projection to the isovector part of the axial current.

The OPE asymptotics of $\pi\gamma^*\gamma^*$ form factor

$$\lim_{q^2 \gg \Lambda_{\rm QCD}^2} F_{\pi\gamma^*\gamma^*}(q^2, q^2) = \frac{8\pi^2 F_\pi^2}{N_c\, q^2}$$

matches the asymptotics of the HLbL amplitude derived above. So our model correctly interpolates. Let us show that suggested"off-shell" changes do not fit.

The off-shell approach by Jegerlehner and Nyffeler implies that form factors at each vertex are functions of all three virtualities

$$F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2; q_3^2)$$

including virtuality of pion $q_3^2 = (q_1 + q_2)^2$. In the vertex with the external magnetic field it becomes

$$F_{\pi\gamma^*\gamma^*}(0, q_3^2; q_3^2)$$

The idea is that this function at large q_3^2 is a constant different from $F_{\pi\gamma^*\gamma^*}(0, 0; 0) = 1$. Comparing with asymptotics at $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda^2_{\rm QCD}$ we see that such deviation is not allowed.

An additional note:

If we introduce a form factor $F_{\pi\gamma^*\gamma^*}(0, q^2; q^2)$ in the vertex with the external magnetic field it will add

$$\frac{F_{\pi\gamma^*\gamma^*}(0, q^2; q^2) - 1}{q^2}$$

where the pion propagator is included. Clear that this does not contain the pion pole at $q^2 = 0$. Moreover, it does not modified the longitudinal part, only the transverse one. Thus, it changes what we call the pseudovector exchange in the model.

The longitudinal part is protected both perturbatively and nonperturbatively, it's only perturbative for the transversal part associated with the pseudovector exchanges.

Quark-based HLbL calculations

Goecke, Fischer, and Williams suggested to use the Dyson-Schwinger approach to calculation of the HLbL quark loop and claim a considerable enhancement of the HLbL contribution.

No theoretical control or independent check. Compare with ENJL approach with separation of scales.

In large Nc limit the enhancement have to be transferred into enhancement of meson-gamma-gamma vertex.

A possible check of the approach is to use for calculation of vacuum polarization where experimental data exist.

Quark loop estimates were recently discussed by Erler and Sanchez who followed Pivovarov's work of 2001.

He used

$$m_u = m_d = m_s - 180 \text{ MeV} = 166 \pm 1 \text{ MeV}$$

to fit the vacuum polarization in the leading order as well as in NLO with quark loop without strong interactions.

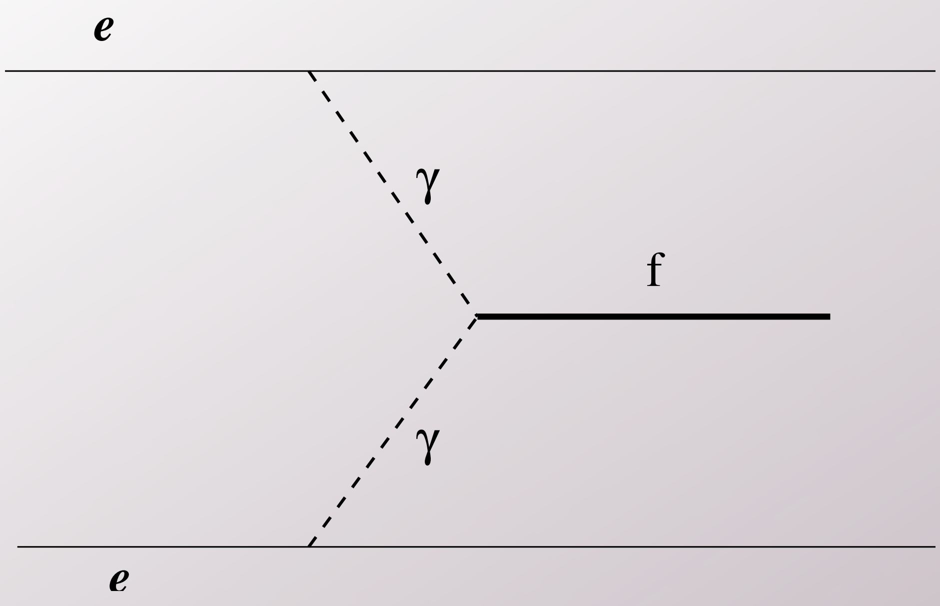
Then he used these masses and the Laporta-Remiddi result $a_\mu^{\mathrm{LBL}}(\mathrm{had}) = 143 \times 10^{-11}$. Erler and Sanchez formulate it as an upper bound $a_\mu^{\mathrm{LBL}}(\mathrm{had}) < 150 \times 10^{-11}$.

Strange duality but at least supported by few fits.

One more approach: instanton induced nonlocal quark interaction by Dorohov and collaborators.

Again there is no much of theoretical control but the approach fits VP and then HLbL numbers are in to the same ballpark as others.

Pseudovector Puzzle



$$\Gamma(f_1(1285) \to \gamma\gamma^*) = (2.8 \pm 0.8)\ \mathrm{keV}$$

This is compatible with our model of pseudovector exchange. However,

$$\frac{\Gamma(f_1(1285) \to \gamma\rho^0)}{\Gamma_{\mathrm{total}}} = (5.5 \pm 1.3) \times 10^{-2}$$

leads to a strong enhancement (of order of 5) for PV exchange. Could be an example of strong enhancement if would be not contradictive.

Conclusions

Having in mind that the new g-2 experiment is in its way more efforts are needed to improve accuracy for the hadronic light-by-light contribution.

In my view it should involve new measurements of hadronic two-photon production which provides a good test of theoretical models for HLbL.