INT Workshop on Hadronic Light-by-Light Contribution to the Muon Anomaly

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Comments on Recent Developments in Theory of Hadronic Light-by-Light

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With Joaquim Prades and Eduardo de Rafael we wrote in 2008 a kind of white paper on HLbL summarizing our understanding of the problem at that time. **EX, 8000 a KING OF WHICH paper ON FILUL SUMMARY COVERT COVERTS THE EFFECT OF OTHER UNITED MESON EXCHANGES, WHI** $\mathbf{S} = \mathbf{S} \mathbf{S}$

It is was very sad to learn that Ximo passed away in August. TC IS WAS VCI J SAG CONCAI Because of the instability of the results for the charged pion loop and unaccounted loops of other

In our '08 mini-review we combined different calculations with some educated guesses about possible errors to come to:
From the errors in quadrature, and the small charm considerations, as well as well as the small charm contributions, n vo mini-review we combined different
Lations with some educated suesses about pessible

$$
a^{\rm HLbL} = (105 \pm 26) \times 10^{-11}
$$

rer ene en or coemmetes are quite subjective and \cdot 2×1 $\frac{1}{6}$ as our final estimate. However the error estimates are quite subjective and further study of different exchanges is certainly needed. While I do not think that there were significant changes during the last 3 years I'll try to comment on few suggestions which appeared at this period.

Off-shell Form Factors Nyffeler, Jeherlehner OPE constraints Here j^γ \bigcap \bigcap HLBL amplitude, in the special kinematics under constanting and consideration, the AVV triangle amplitude. The AVV triangle amplitude and consideration, the AVV triangle amplitude amplitude. The AVV triangle amplitude ampl \overline{M} shall \overline{L} ives \overline{L} **QTH-Shell Forms** $\bigcap_{n=1}^{\infty}$ separation to $\bigcap_{n=1}^{\infty}$ **Example 2 = CONSTRAINTS** the OPE amplitudes and in this way connect the OPE **tractors** Nyffeler. $\frac{1}{2}$ $\frac{1}{2}$

The range where $q_1^2 \approx q_2^2 \gg q_3^2$ and $q_3^2 \gg \Lambda_{\rm QCD}^2$ In this kinematic regime, we have $\frac{1}{2}$ with the well-known $\frac{1}{2}$ with the well-known $\frac{1}{2}$ with the well-known $\frac{1}{2}$ In the range where $q_1 \approx q_2$. In the range where $\frac{q_1}{q_2}$ be the photons of the photons $\frac{q_1}{q_2}$ $\gg q_3^2$ and $q_3^2 \gg \Lambda_{\rm QCD}^2$ metric with respect to photon permutations, we can In the range where $q_1^2 \approx q_2^2 \gg q_3^2$ and In this kinematic regime, we begin with the well-known In general, the light-by-light scattering amplitude is S^{c} where $\frac{q_1}{q_2}$ ⁵ (z) + ··· . (3) \cdot q_3^2 and $q_3^2 \gg \Lambda_{\rm QCD}^2$ ing the asymptotic expressions Eq.(7) for the invariant

retain only the leading (in the limit of large Euclidean ˆq)

 $\int d^4x d^4y e^{-iq_1x-iq_2y} Tf$ (x) in $(y) = \int d^4z e^{-i(q_1+q_2)z} \frac{2i}{z} e^{-i(x_1+q_2)z}$ pseudoscalar and pseudoscalar and pseudoscalar meson exchanges are dominant at large known at large known and \tilde{q}^2 $\epsilon_{\mu_1\mu_2\delta\rho}$ of $J_5(\epsilon)$ + ... 5 i $\int d^4x d^4y e^{-iq_1x-iq_2y} T\left\{ j_{\mu_1}(x), j_{\mu_2}(y) \right\} =$ $\hat{q} = (q_1 - q_2)/2 \approx q_1 \approx -q_2$ In general, the light-by-light scattering amplitude is $a \int d^2x d^2y e^{-\frac{y_1}{4}}$ virtuality of photons $\iint d^2x d^2y$ the light-by-l $\int d^4 z e^{-i(q_1+q_2)z} \frac{2i}{\sqrt{2}}$ $\frac{z\imath}{\hat{q}^2}\, \epsilon_{\mu_1\mu_2\delta\rho}\, \hat{q}^{\delta}j_5^{\rho}(z)+\cdots$ \int $i \int d^4x d^4y e^{-iq_1x - iq_2y} T\{j_{\mu_1}(x), j_{\mu_2}(y)\} = \int d^4z e^{-i(q_1+q_2)z} \frac{1}{\hat{q}^2} \epsilon_{\mu_1}$ \ddotsc $\frac{\iota}{\iota}$ \overline{c} $\int_{-1}^{1} 4 \int_{-\infty}^{1} e^{-i(g_1+g_2)z} 2i$. $\hat{\delta}$ $\hat{\delta}$ $\frac{\partial^2 z}{\partial t^2}$ $\frac{\partial^2 z}{\partial t^2}$ uing to Euclidean space, we arrive at $\mathcal{L}_{\mathcal{A}}$ $\frac{3}{2}$

we get for the HLbL amplitude ing the asymptotic expressions \mathcal{L} for the invariant \mathcal{L} for the invariant \mathcal{L} We get for the HLDL amplitude deal with the amplitude \mathbf{d} \blacksquare icuc ampii $\mathcal{L}_{\mathcal{A}}$. Convoluting the tensor amplitude with the tensor amplitude with $\mathcal{L}_{\mathcal{A}}$ σ at for the HI hI amplitude gue ion enc $\mathcal{L}_{\mathcal{A}}$. Convoluting the tensor amplitude with the tensor amplitude with $\mathcal{L}_{\mathcal{A}}$ na Lillal ampli \mathbb{R}^2 $\sum_{i=1}^n$ and $\sum_{i=1}^n$ and $\sum_{i=1}^n$

 $\mathcal{M} \; = \; \alpha^2 N_c \, \text{Tr} \, [\hat{Q}^4] \, \mathcal{A}$ $\frac{4}{20}$ { \tilde{f}_1 of \tilde{f}_1 of \tilde{f}_2 \tilde{f}_3 of \tilde{f}_4 of \tilde{f}_5 of \tilde{f}_6 of \tilde{f}_7 of \tilde{f}_8 of \tilde{f}_9 of \til where $\frac{1}{3}$ is the OPE amplitudes and in this way connect the OPE amplitudes and in this $\frac{1}{2}$ $\mathcal{M} = \alpha^2 N_c \operatorname{Tr} [Q^*] \mathcal{A}$ \mathcal{A} 4 $\mathcal{A}\!=\!\frac{4}{q_3^2\,\hat{q}^2}\!\left\{f_2\tilde{f}_1\right\}\!\left\{\tilde{f}f_3\right\}$ $-{\frac{4}{\sigma^2 \hat{\sigma}^4}} \Big(\{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3 \} + \{q_1 f_1 \Big)$ $\{\tilde{f}_{13}q_{3}\}+\frac{q_{1}^{2}+q_{2}^{2}}{q_{1}^{2}}\{f_{2}\tilde{f}_{1}\}\{\tilde{f}_{3}\}+\cdots$ $-\frac{4}{q_3^2 \hat{q}^4} \Big(\Big(q_2 f_2 \tilde{f}_1 \tilde{f}_1 f_3 q_3 \Big) + \Big(q_1 f_1 \tilde{f}_2 \tilde{f}_1 f_3 q_3 \Big) + \frac{q_1^2 + q_2^2}{4} \{ f_2 \tilde{f}_1 \} \{ \tilde{f}_1 f_3 \} + \cdots$ \hat{z} the asymptotic expressions \hat{z} for the invariant \hat{z} $\mathcal{A} = \frac{1}{q_3^2 \hat{q}^2} \{J_2 J_1\}$ $\left(\{q_2 f_2 \tilde{f}_1 \tilde{f}_3 q_3 \} + \{q_1 f_1 \tilde{f}_2 \tilde{f}_3 q_3 \} + \frac{q_1^2 + q_2^2}{4} \{f_2 \tilde{f}_1\} \{\tilde{f}_3 \} + \cdots \right)$ exchanges of the pseudoscalar (pseudovector) mesons Λ $+\cdots$ $\alpha^2 N$ Tr $[\hat{O}^4]$ d α i.e. Euclidean space, we are the space of α \int $\{\tilde{f}_2\tilde{f}_1\} \{\tilde{f}_3\}$ $+ \left\{ \left(q_2 f_2 \tilde{f}_1 \tilde{f}_3 q_3 \right) + \left\{ q_1 f_1 \tilde{f}_2 \tilde{f}_3 q_3 \right\} + \frac{q_1^2 - q_2^2}{4} \right\}$ \pm $\left\{f_2f_1\right\}\{ff_3\} + \cdots$ \sqrt{p} \overline{a} ${\cal M}~=~\alpha^2 N_c \, {\rm Tr}\left[Q^4\right] {\cal A}$ $\frac{1}{\sqrt{2}}$ t_{ref} the pseudoscalar (pseudovector) mesons. In perturbation theory wL,T are defined by the famous triangle diagram. For \tilde{f} \tilde{f} transition form factor has to be present when one of the $\left\{ \tilde{f}f_{\alpha}\right\}$ $\left(\frac{J}{3} \right)$ is on the mass shell as well. However, the mass shell as well. However, the mass shell as well. However, the mass $\frac{1}{2}$ not the kinematics that corresponds to the triangle and

 $q^2u\omega = \frac{u^2}{4\pi}$ where $J_i = q_i \epsilon_i^2 - q_i^2 \epsilon_i^2$ a Thus, the amplitude is unambiguously fixed in the range $\frac{1}{\sqrt{2}}$ strength tensor of the soft photon; the light-by-by $q_1 \approx q_2 \gg q_3 \gg \Lambda_{\rm QCD}$ indice all to the pion pole. By quantum numbers the first line $m \cdot$ for the purpose of $\frac{1}{2}$ refers to pseudoscalar exchange, the second -the tensor amplitude Au1u3γδ and calculate it assumed and calculate it assumed and calculate it as unit assuming that $\mathcal{O}(\mathcal{A}^{\mathcal{A}})$ is defined by the virtual photons. Because the v $\mathcal{C}^{\mu\nu}$ we we find the field strength tensors, $\mathcal{C}^{\mu\nu}$ are the field strength tensors, $\mathcal{C}^{\mu\nu}$ to the pion pole. By quantum numbers the first line where $f_i^{\mu\nu}$ where $f_i^{\mu\nu} = q_i^{\mu} \epsilon_i^{\nu} - q_i^{\nu} \epsilon_i^{\mu}$ are field str Thus the amplitude is ung Inus, the amplitude is unambiguousi $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda_{\text{QCD}}^2$ Note an absence yet of a ANCONS, **JOVECTOR** \textbf{e} $f_i^{\mu\nu} = q_i^{\mu} \epsilon_i^{\nu}$ $-q_i^{\nu} \epsilon_i^{\mu}$ are field strengths of photons. µ $q_1 \sim q_2 \gg q_3 \gg \Lambda_{\rm QCD}$ were all Euclidean notations instead of Minkowski ones used be*e* are to be the following the following and the following and $\frac{1}{2}$ s uode where $\gamma - q_i^{\nu} \epsilon_i^{\mu}$ are field strengths of photons, \mathcal{L}_i and field sendingers of priocons. Thus, the amplitude is unambiguously fixed in the range I Euclidean notations instead of Minkowski ones used bewhere $f_i^{\mu\nu} = q_i^{\mu} \epsilon_i^{\nu} - q_i^{\nu} \epsilon_i^{\mu}$ are field strengths of photons. $q_1^2 \approx \dot{q}_2^2 \gg q_3^2 \gg \Lambda_{\rm QCD}^2$ Note an absence yet of any reference pseudovector. **i**, the amplitude is unambiguous B. OPE and triangle amplitude Thus, the amplitude is unambiguously fixed i $\frac{m}{2}$ is respect to photon permutations, we can permutations $\frac{m}{2}$ study the second limit q_1^2 $q_1^2 \approx q_2^2 \gg q_3^2$: CO GIC PION d=iquality that there are no perturbative fact that there are no perturbative fact that the set of α $\frac{1}{2}$ and the longitudinal part is the signa- A although the perturbation theory is only reliable for \mathcal{A} $q_3^2 \gg \Lambda_{\rm QCD}^2$ Note an absence yet of an on R_{R} D_{R} quantum pumpore the \mathcal{L} pole. By quantum numbers the the chiral limit mass \sim 0 for \sim 0 for \sim nonsinglet axial currents. The consistency of \sim the light-by-light scattering amplitudes, relevant for a^µ photons. Where \blacksquare the virtuality of the photons. The photons of the photons. The absence of the photons. The absence of the absence of the photons. The absence of the photons. The absence of the photons. The absence of the absence of the ab S is a range consistent with the S sentation of the international since the single since the internation and international since the internationa nonvanishing only at q² = 0 in the chiral limit.

Compare this now with the meson exchange near the meson pole. For the pion pole we have heavier resonances, Fig. 2b. short-distance constraints for albl ith the meson exchange near the or the pion pole we have in the model for the pion pole we have ^L (0, 0) = ^N^c nanσι
Σ \overline{a} \overline{b} \overline{c} \overline{b} change near the r ule this now with the meson exch ${f}$ e distinguishes our approach from all other calcu-

 $\frac{1}{2}$ **a** α α **b** α **f** α **p** α **p** α **j** α fhe axial current **CHC GARDE COLLETT.** T and the kinematic independent of T $W^{(3)} = \frac{1}{4}$ accounts for projection to the isovector part of ²) = ^φ(3) α can be simplified useful and α I. There we show that the amplitude can be described as $\mathcal{L}_\mathcal{A}$ o the isovector part of five independent form-factors. In what follows, we mostly $W^{(3)} = \frac{1}{4}$ accounts for projection to the isovector part of the axial current. nrojection to the isovector part of $\mathcal{L}_{\mathcal{A}}$, permutations. (18) and (the isovector par $T_{\rm eff}$ comparison with the OPE constraint given by $t_{\rm eff}$ as we show below, the show below, this is in happens.

 $\zeta = x^* x^*$ $\mathbf s$ of $\pi\gamma$ is non-perturbative corrections to $\mathbf r$ $\Omega = 2 \, \mathrm{E}^2$ The OPF asymptotics of $\pi \gamma^* \gamma^*$ form factor L (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19

L (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (19) . (1 Fine Ore asymptotics of π but we make occasional references to general expression The OPE asymptotics of $\pi\gamma^*\gamma^*$ form factor

$$
\lim_{q^2 \gg \Lambda_{\rm QCD}^2} F_{\pi \gamma^* \gamma^*} (q^2, q^2) = \frac{8\pi^2 F_\pi^2}{N_c \, q^2}
$$

matches the asyme chiral chies ene asymp above So our model correctly interpolates Let us show the the contract of the contribution of th that suggested 'off-shell'' changes do not fit. this smallness may not be accidental. momenta and provides an important constraint thereby. ics of the HLbL amplitude derive in the must potate it is strong the must be virtual but it is in the virtual but it is in the virtual but it i although the product of the subject of the strength of the strengt q2"Λ² above. So our model correctly in Famplitude o **nde derived** matches the asy mototics of the HI bL amplitude derived orrectly interpolates. Let us show ϵ report amplitude derived $\frac{1}{2}$ not fit ell" changes do not fit. $\frac{1}{2}$ L (19) . (correctly mterpo
all'' changes do n that suggested"off-shell" changes do not fit. matches the asymptotics of the HLbL amplitude derived above. So our model correctly interpolates. Let us show

The off-shell approach by Jegerlehner and Nyffeler implies that form factors at each vertex are functions of all three virtualities erlehner and Ny alor^d ach vertex are functions

$$
F_{\pi\gamma^*\gamma^*}(q_1^2,q_2^2;q_3^2)
$$

1. Introduction. From a theoretical point of view the hadronic light–by–light scattering (HLbL) contribution to the with the external magnetic field it becomes including virtuality of pion $q_3^2 = (q_1 + q_2)^2$. In the vertex to phenomenological models. $\mathbf{e}^{\mathbf{e}}$ the pseudoscalar (pseudovector) mesons. $q_3 = (q_1 + q_2)$. In the vertex tic field it hecomes

$$
F_{\pi\gamma^*\gamma^*}(0,q_3^2;q_3^2)
$$

 $\frac{1}{2}$ decided to the set the $\frac{1}{2}$ ${;\bm{0}}$ different from $F_{\pi_{2}+\alpha*}(0, 0; 0) = 1$ Comparing with **the offerent photon strategier amplitude induced by** $\mathcal{L}(\mathbf{w}, \mathbf{v}, \mathbf{v}) = \mathbf{r}$ **.** asymptotics at $\frac{1}{12}$ as $\frac{1}{13}$ and $\frac{1}{2}$ deviation is not allowed $\frac{12}{1}$ d4x3 exp{{c}{2}+k3}}} exp{{c}} $\frac{1}{16}$ (pp) is the idea is that this function at large different from $F_{\pi\gamma^*\gamma^*}(0,0;0) = 1$. Comparing with $\frac{1}{2}$ is a constant $\frac{1}{2}$ asymptotics at $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda_{\rm QCD}^2$ we see that such $\frac{1}{2}$ function a 1.6 From a theoretical point of view the hadronic light–by–light scattering (HLbL) contribution to the asymptotics at $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda_{\rm QCD}^2$ we see that such deviation is not allowed. The idea is that this function at large q_3^2 is a constant muon magnetic moment is described by the vertex function (see Fig. 1 below): 1. Introduction. B. OPE and triangle amplitude $differsant-from E \t\t (0, 0, 0)$ **Miner end in SIM** $T_{\pi\gamma}(\gamma, 0, 0, 0)$ In this kinematic regime, we begin with the well-known deviation is not allowed. c tion at large q_3 is a constant is the signal part is the signal part is the signa t_0 t_1 Comparing with A, B, C, C theory is only reliable for A electromagnetic currents, the expressions for longitudinal

An additional note:

If we introduce a form factor $F_{\pi\gamma^*\gamma^*}(0, q^2; q^2)$ in the vertex with the external magnetic field it will add $t_{\rm max}$ this form factor enters not in the transverse part of the transverse part, but in the transverse part. It is in the transverse part. It is in the transverse part of the transverse part. It is in the transverse pa for the axial and the axial and the axial and factor. In the transverse part the form factor. In the transverse part the t

$$
\frac{F_{\pi\gamma^*\gamma^*}(0,q^2;q^2)-1}{q^2}
$$

where the pion propagator is included. Clear that this does not contain the pion pole at $q^2 = 0$. Moreover, it does not modified the longitudinal part, only the transverse one. Thus, it changes what we call the longitudinal component of the longitudinal contrary to the transverse one. Thus, it changes what we call the pseudovector exchange in the model. α whose could be called the pseudovector exchange. It provides the leading short–distance constraints α non-anomalous part of the AVV triangle is, however, corrected non-perturbatively \mathcal{L}^{1} Additional constraints on subleading terms in the ^F(k² ι.
Des not contai propagator is included. U α the pion pole at q^2 *Noreover.* it was not mounted the longitudinal part, only the ansverse one. I hus, it changes what we call the \blacksquare vector exchange in the \mathbf{B} $\overline{}$ pseudovector exchange in the mode $\frac{3}{2}$ anazator is in \overline{a} $\frac{1}{\sqrt{2}}$ must moment is described by the vertex function (see Fig. 1 below): o
Audovector exchange in the L 2
2k2 \blacksquare nodel.

The longitudinal part is protected both perturbatively and nonperturbatively, it's only perturbative for the transversal part associated with the pseudovector and part associated with the pseudovector exchanges. Ref. [12], are also taken into account in the calculation quoted in Ref. [9]. of the integral in Eq. (4) comes from momenta kⁱ of the order of an hadronic scale. However, the $\mathbf{y} = \mathbf{y} - \mathbf{y}$ transversal part associated with the pseudovector .
.
C <u>ocia</u> .
| f ed with the pseudovector

Quark-based HLbL calculations

Goecke, Fischer, and Williams suggested to use the Dyson-Schwinger approach to calculation of the HLbL quark loop and claim a considerable enhancement of the HLbL contribution.

No theoretical control or independent check. Compare with ENJL approach with separation of scales.

In large Nc limit the enhancement have to be transferred into enhancement of meson-gamma-gamma vertex.

A possible check of the approach is to use for calculation of vacuum polarization where experimental data exist.

Quark loop estimates were recently discussed by Erler and Sanchez who followed Pivovarov's work of 2001. agree perfectly. Our central value is higher than previand Sanchez who followed Pivovarov's work of 2001. with called the scalar conclusion and the scalar \sim $\overline{}$ \sim \sim \sim \sim **Quark loop estima** fering a perturbative rationale why our approach seems well, so it is gratify that the estimate (9) turns out to the estimate (agree recentry discussed by Effer

the used album increased album increased album increased album increased album increased album increased album $\overline{}$ in a similar paper $\overline{}$ for a 1-parameter fit to average $\overline{}$ contribute negatively. \mathbf{u} as compared to Eq.(30). We obtain the following the follow $T_{\rm eff}$ the success of the previous paragraph may be

^µ (2, had) $m_u = m_d = m_s - 180 \text{ MeV} = 166 \pm 1 \text{ MeV}$ n_0 190 M_eV 166 11 MeV \mathbf{v} \log 100 MeV – 10 $\text{t} \approx 180 \text{ M}_{\odot} \text{V} = 166 \pm 1 \text{ M}_{\odot} \text{V}$ $\eta = m_s - 180$ MeV = 100 ± 1 MeV

 $T_{\rm eff}$ values are within errors consistent with Eqs. (1–2), $T_{\rm eff}$ are with Eqs. (1–2), $T_{\rm eff}$ to fit the vacuum polarization in the leading order as well as in NLO with quark loop without strong taractions ^q-expansion of aLBL ^µ [9], lowering the light quark contribution relative to \mathcal{L} by \mathcal{L} by \mathcal{L} \mathcal{L} interactions. errors multiplied by 1.645 to shift from uncertainties to fit the vacuum polarization in the re $intors of i$ 1.48 ± 1.48 ± 0.07 ± 0.07 ± 0.07 × 1.48 ± 0.07 ± 1.48 ± 0.07 ± 0.18 ± 0.07 ± 1.48 ± 0.07 ± 1.48 ± 0.07 ± 1.48 ± The upper extensive convergence as mentioned, and since as mentioned, and since as mentioned, and since as men $\frac{1}{2}$ to fit the vacuum polarization in the leading order as w elles in NI $\bigcap w$ ith quark loop v well as in NLO with quark loop without strong $interactions$, while interactions, while i \mathbb{R}^n the Husing of sion to estimate subleading ^O(N⁰ $t_{\rm e}$ on $\sigma_{\rm e}$ VP effects ^µ (3i, had), each Lo in the vacuum additional electron (hadron) loop insertion. Our method is a series of the contract of the con I_{relation} in the leading arder as μ rzacion in che reading bruer as aLBL ^µ (had) = 1.37±0.21×10−⁹ \mathcal{L} estimate: $\frac{1}{2}$ where the three terms displayed separately are due to the

Then he used these masses and the Laporta-Remiddi result $a_{\mu}^{\text{LBL}}(\text{had}) = 143 \times 10^{-11}$. Erler and Sanchez μ by μ it as an uppen hound $a^{\text{LBL}}(h \circ d)$ uses modulate it as an upper bound u_{μ} (the second \times formulate it as an upper bound $a_{\mu}^{\text{LBL}}(\text{had}) < 150$ for aPV µ (3a, he used these masses and the Laporta-Kemiddi $t(\mathbf{k}, \mathbf{l})$ to albel $(0, \mathbf{l})$ formulate it as an upper bound $a_{\mu}^{\text{LBL}}(\text{had}) < 150 \times 10^{-11}$ nasses and the Laporta-Remiddi 143×10^{-11} Erler and Sanchez opposite case for the mixing, assuming that f¹ is a pure e masses and the Laporta-Remiddi

become rigorous, but this remains a loophole for now. Strange duality but at least supported by few fits. reflecting our statement that aVP \mathcal{L} (2, had) and \mathcal{L} and \mathcal{L} and \mathcal{L} the largest. The reason for such a behavior is a stronger strange quality but at least suppo Strange duality but at least supported by few fits. In a similar paper [15], a 1-parameter fit to aVP ϵ at least supported by few fits. the largest. The reason for such a behavior is a stronger

One more approach: instanton induced nonlocal quark interaction by Dorohov and collaborators.

Again there is no much of theoretical control but the approach fits VP and then HLbL numbers are in to the same ballpark as others.

From these considerations, adding the errors in quadrature, as well as the small charm contribution **Ref. And Executovector Puzzle**

This is compatible with our model of pseudovector which, although smaller than the one from pseudoscalar exchanges, have never have nevertheless large uncertain t_{c} . This refers, in particular, to pseudovector exchanges in Eq. (2) but other C-even exchanges are ϵ From these considerations, adding the errors in quadrature, as well as the small charm contribution exchange. However,

> (1985) important. Experimental data on 0 $\frac{1200}{\sqrt{1200}}$ could be helpful. And $\frac{1}{\sqrt{1000}}$ contributions present even a more contributions present even a more contributions present even a more contributions provided by $\frac{1}{\sqrt{10}}$ contributions provided $\Gamma_{\rm total}$ approaches to the dressed pion loop contribution, in parallel with experimental in-parallel $\Gamma(f_1(1285) \to \gamma \rho^0)$ Γ_{total} $=(5.5\pm1.3)\times 10^{-2}$

 $f(x) = \frac{1}{2\pi} \int_{0}^{x} \int_{0}^{x} f(x) \, dx$, would be very welcome. Again, measurement of the two-photon a strong enhancement (of order of 5) for PV exchange. Could be an example of strong enhancement if would be not contradictive.

if would be not contradictive. $\frac{1}{2}$ fo a strong enhancement (of order of 5) for PV In viertunge. Sould be un exumple of serving emiuneement leads to a strong enhancement (of order of 5) for PV exchange. Could be an example of strong enhancement

Conclusions

Having in mind that the new g-2 experiment is in its way more efforts are needed to improve accuracy for the hadronic light-by-light contribution.

In my view it should involve new measurements of hadronic two-photon production which provides a good test of theoretical models for HLbL.