Hadronic LBL: Insights from χ Symmetry

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Outline

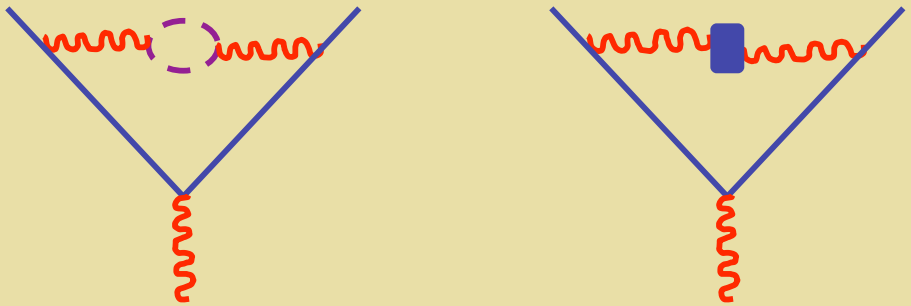
- *Uses & limits of chiral perturbation theory*
- *Pion pole contribution*
- *Charged pion contribution (in progress)*
- *Numerics*

Uses of Chiral Perturbation Theory

- *Exploit hierarchy of scales:* m_π , m_μ , $p_{ext} << \Lambda_{had}$
- *Calculable: non-analytic dependence on* m_q , m_μ , p_{ext} *("very long distance effects")*
- *Parameterize ignorance: organize "somewhat long distance" contributions according to approx* χ *sym of QCD* $\rightarrow$ *LEC's*
- *Determine LECs from measurements*
- *Systematic (consistent power counting in scale ratios) & model independent*

Limits of Chiral Perturbation Theory

- *Contributions from momentum scales ~ Λ_{had}*
- *Example: HVP*



Pure χPT

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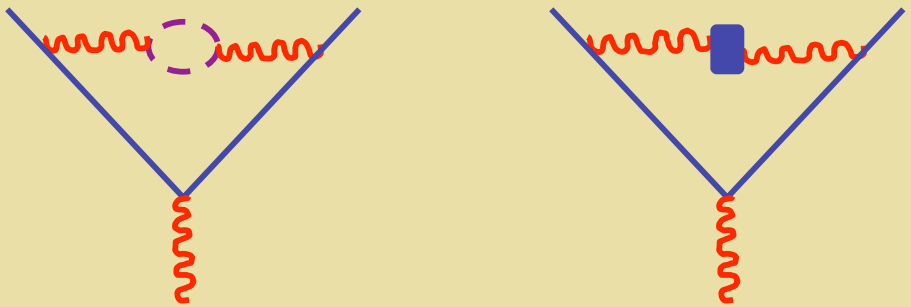

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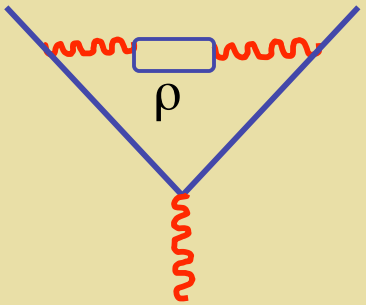
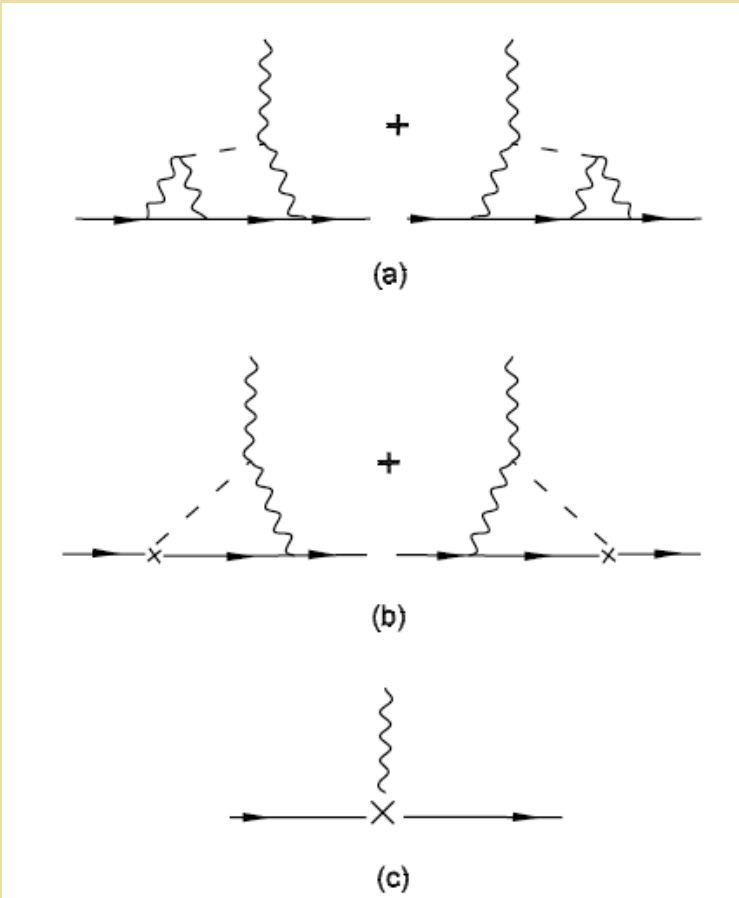


Dynamics

Pseudoscalar Pole Contribution



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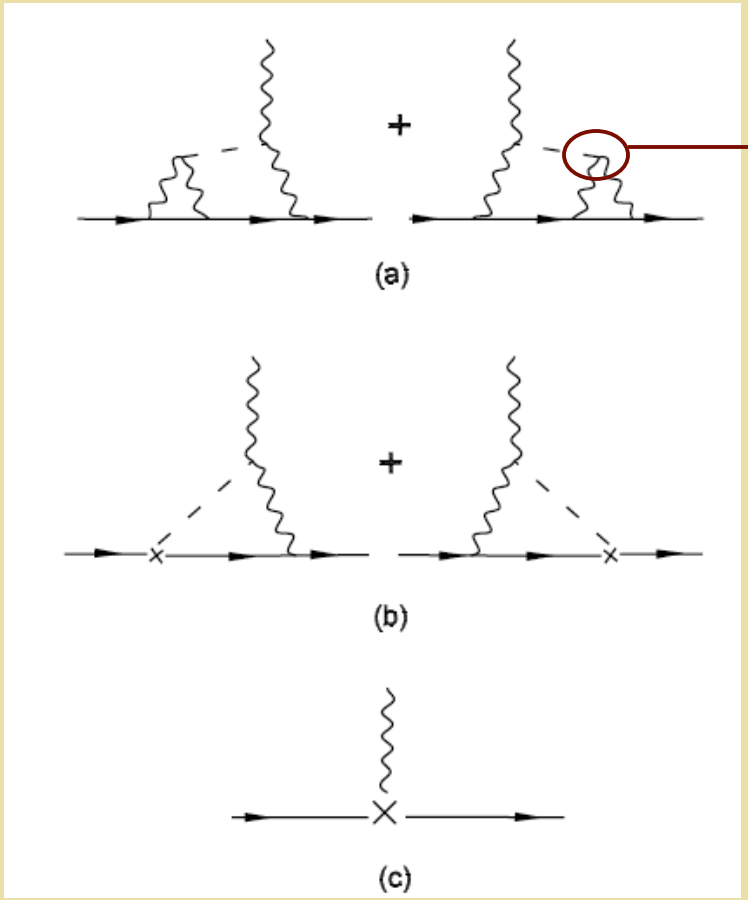

$\mathcal{L}_{wzw}$: ln^2 term $\rightarrow$

Sign error discovered by Knecht et al

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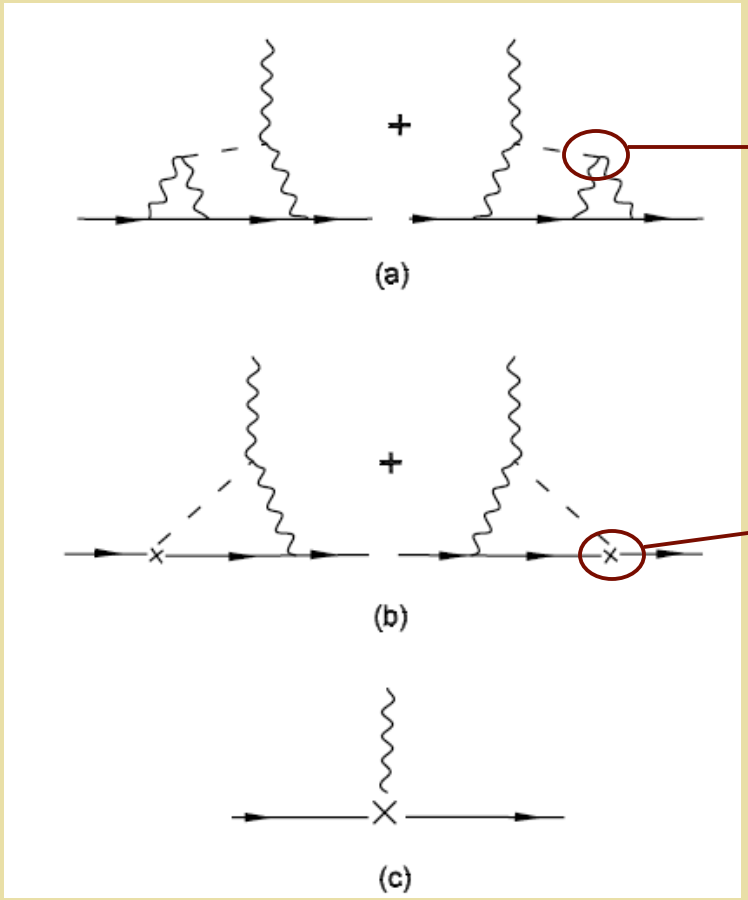

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Exp't (MRM, Wise)

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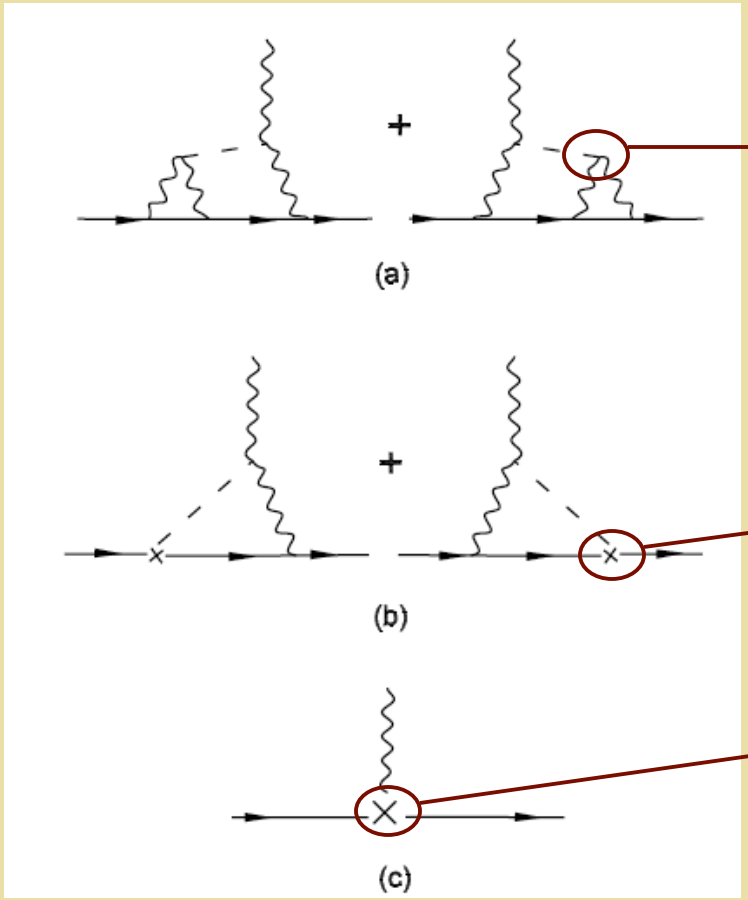

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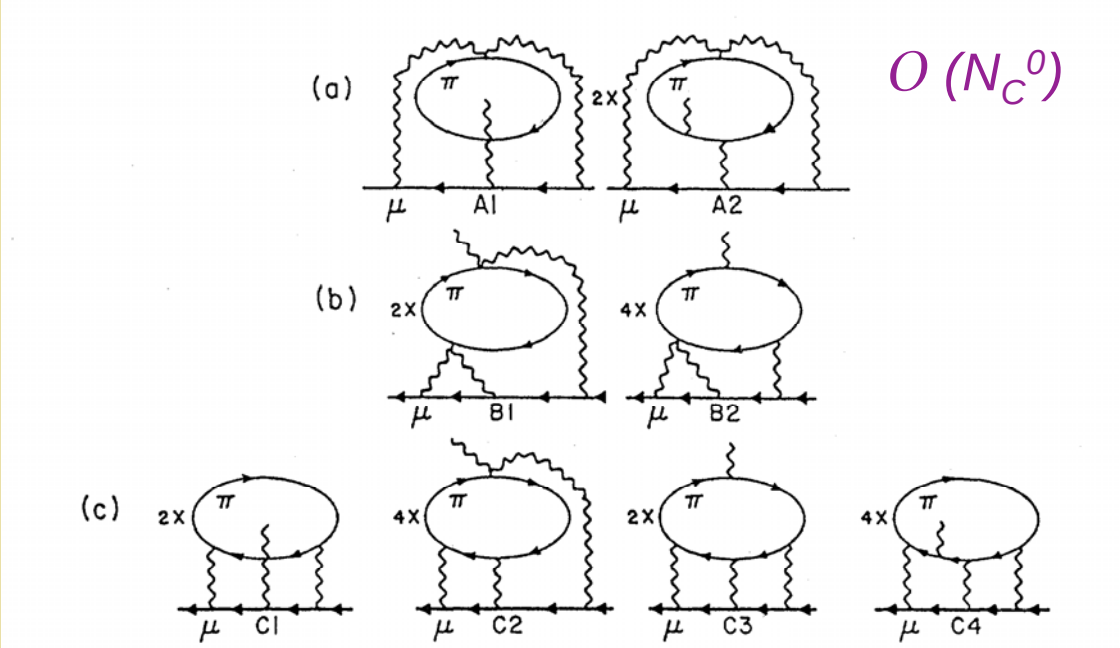
Significantly reduced: KTeV '07

Charged Pion Contribution

Kinoshita, Nizic, Okamoto '85 ; Hayakawa, Kinoshita, Sanda '95



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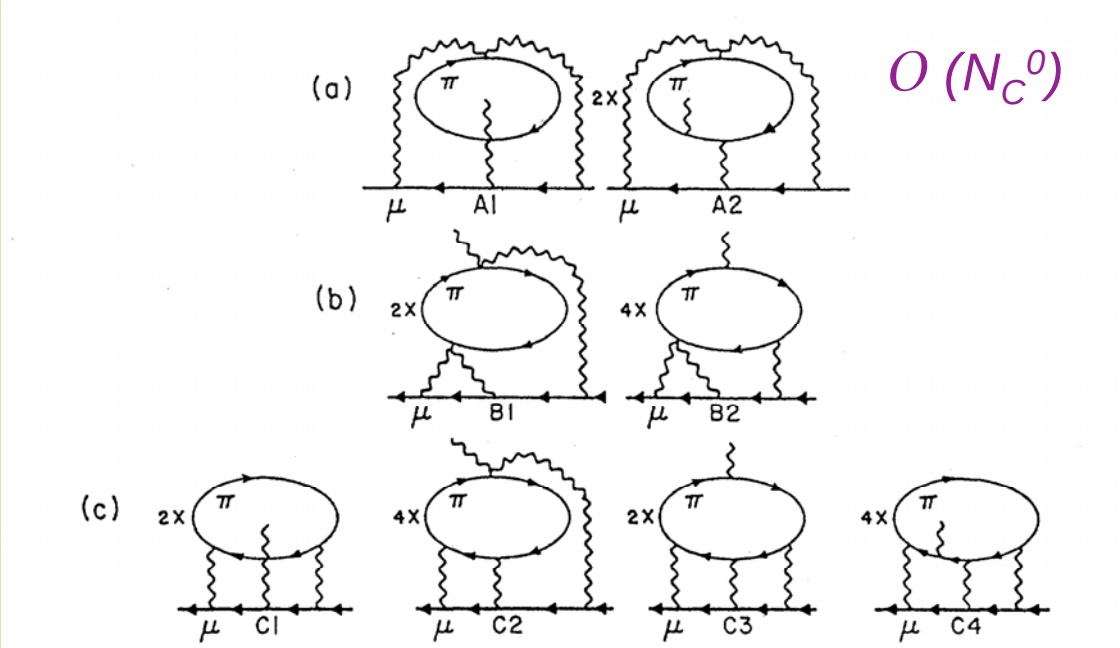

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Point-like pions: $-0.0383\ (19)\ (\alpha/\pi)^3$

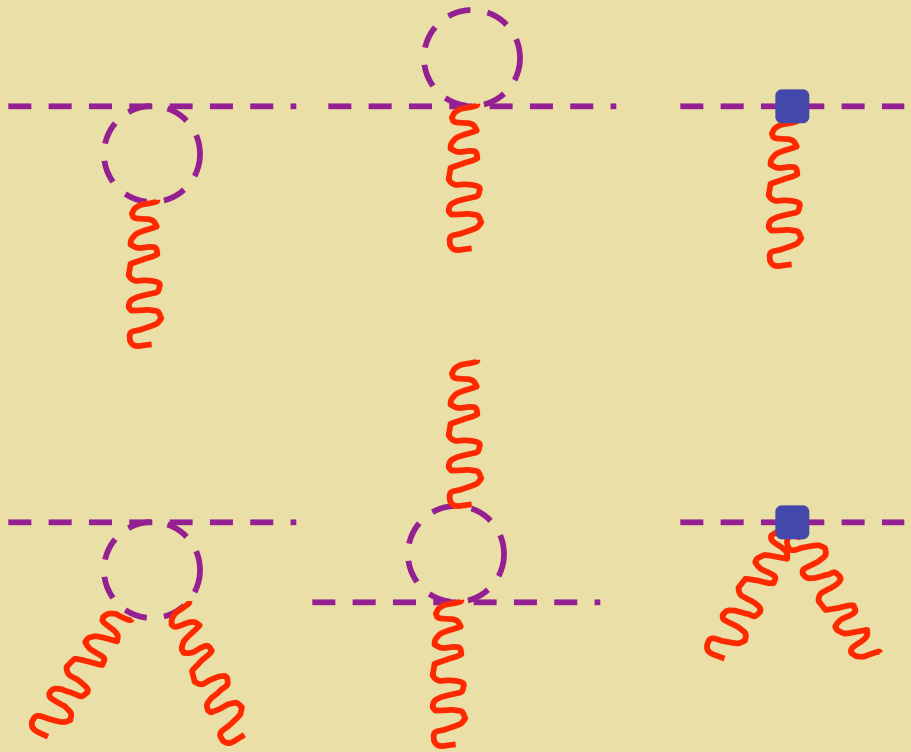
Include $F_\pi(q^2)$: $-0.0125\ (19)\ (\alpha/\pi)^3$

"HLS": $-0.00355\ (12)\ (\alpha/\pi)^3$

Charged Pion Contribution: χPT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: subgraphs



+ ...

Pion charge radius: first non-trivial term in expansion of $F_\pi(q^2)$

$O(p^4)$ LEC: α_9

Pion polarizability: distinct physics from ff

$O(p^4)$ LEC: α_9 & α_{10}

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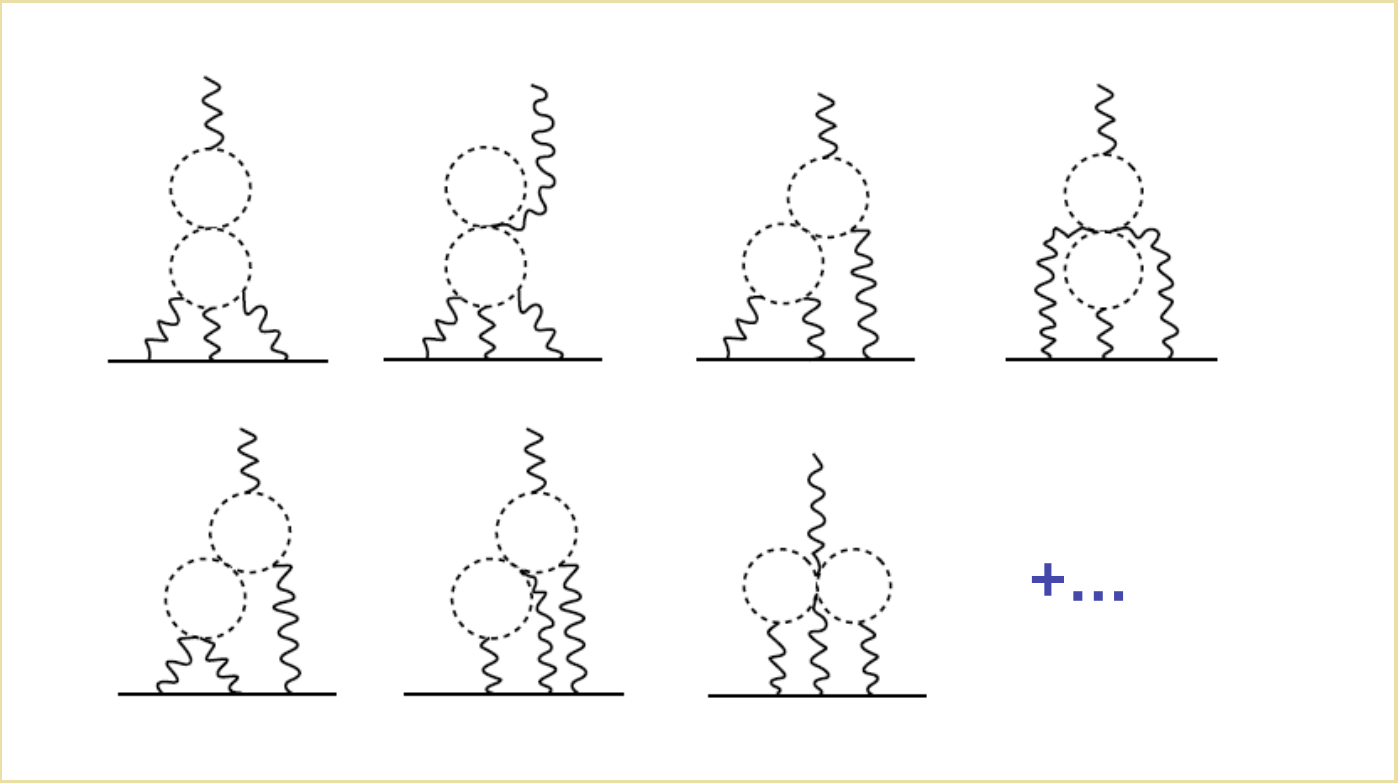
Beyond leading order: embedding subgraphs in full HLBL contribution



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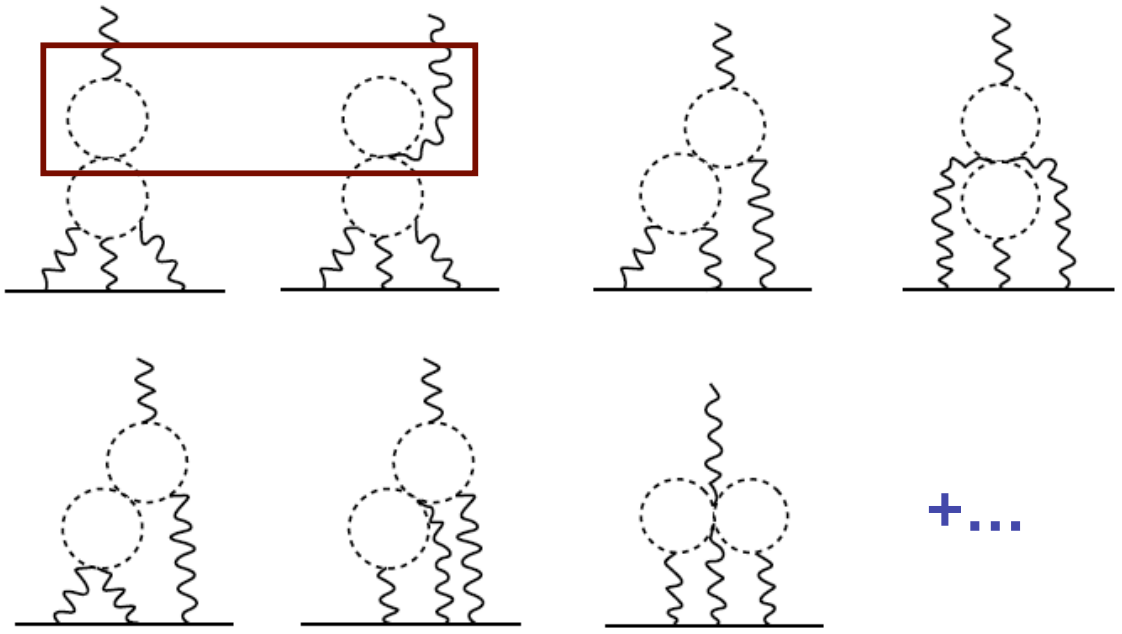

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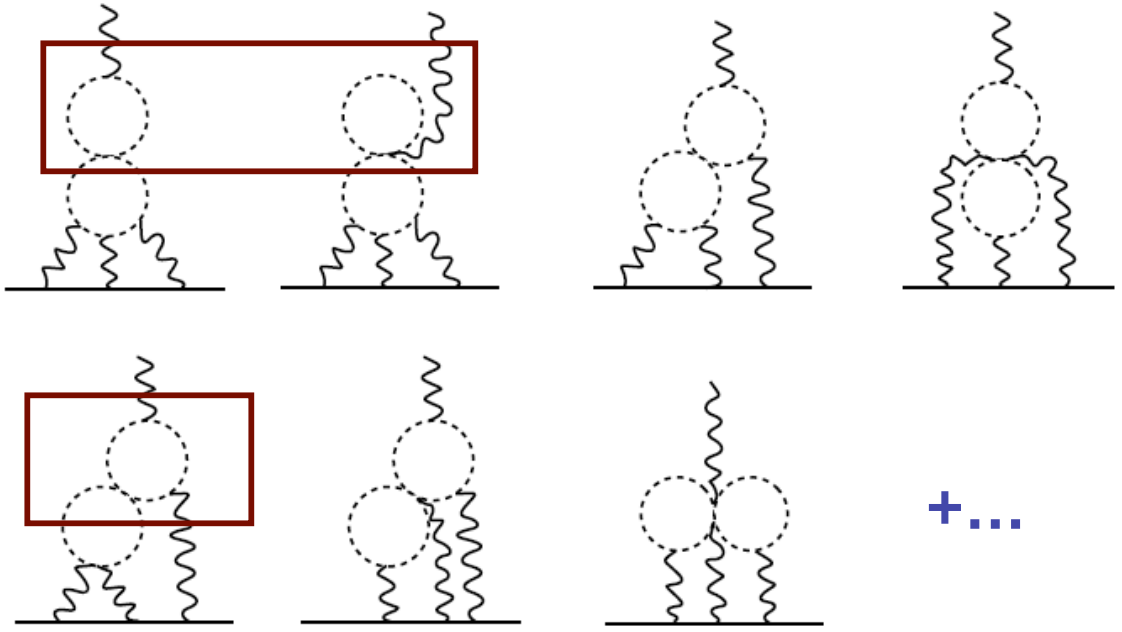
Pol'bility



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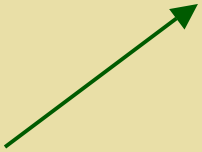

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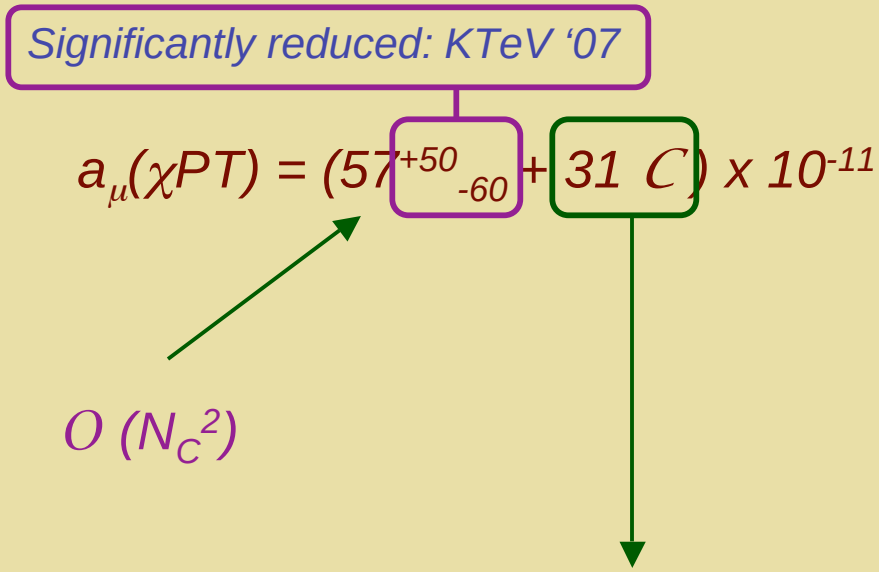

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χ*PT:* ?

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Proposed error: $^{+}_{-}$ 15