Green's functions and form factors

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INT Workshop
``Hadronic light-by-light scattering"

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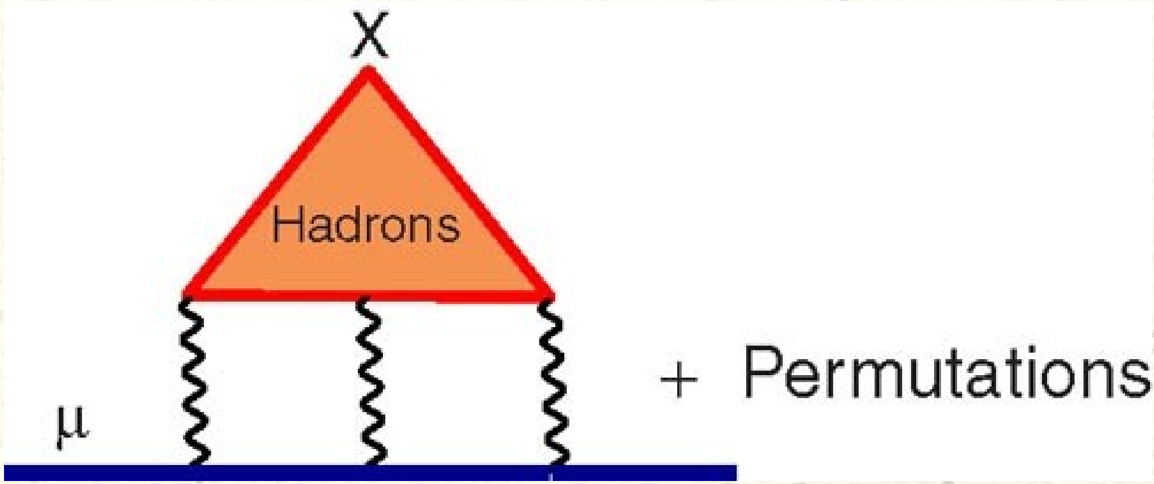
Hadronic light-by-light scattering

- What we would like to know is the hadronic light-by-light scattering contribution



$$\Pi_{\mu\nu\alpha\beta} \sim \langle 0 | T j_\mu(x_1) j_\nu(x_2) j_\alpha(x_3) j_\beta(0) | 0 \rangle$$

Hadronic light-by-light

- Correlator of four electromagnetic currents is known in two limits
  - short distances – perturbative regime → quark loop
  - long distances – meson poles
- All we can do is to come up with a formula for the correlator of four electromagnetic currents which isn't too dumb to violate any of the above constraints
- The success isn't guaranteed since the interpolation between what we call loong-distance and what we call short-distance may be important

General expression for the amplitude

- The short-distance constraints are complicated, they are described by nineteen invariant form-factors

$$
\begin{aligned}
\mathcal{A} \;=\;& G_1^{(1,2,3)}\,\{f f_1\}\{f_2 f_3\} + G_1^{(2,3,1)}\,\{f f_2\}\{f_3 f_1\} + G_1^{(3,1,2)}\,\{f f_3\}\{f_1 f_2\} \\
+\;& G_2^{(1,2,3)}\left\{f \tilde{f}_1\right\}\left\{f_2 \tilde{f}_3\right\} + G_2^{(2,3,1)}\left\{f \tilde{f}_2\right\}\left\{f_3 \tilde{f}_1\right\} + G_2^{(3,1,2)}\left\{f \tilde{f}_3\right\}\left\{f_1 \tilde{f}_2\right\} \\
+\;& G_3^{(1,2,3)}\,\{\eta_{23} f\, f_1\, \eta_{23}\}\{f_2 f_3\} + G_3^{(2,3,1)}\,\{\eta_{31} f\, f_2\, \eta_{31}\}\{f_3 f_1\} + G_3^{(3,1,2)}\,\{\eta_{12} f\, f_3\, \eta_{12}\}\{f_1 f_2\} \\
+\;& \tilde{G}_3^{(1,2,3)}\,\{\eta_{23} f\, f_1\, q_1\}\{f_2\, f_3\} + \tilde{G}_3^{(2,3,1)}\,\{\eta_{31} f\, f_2\, q_2\}\{f_3\, f_1\} + \tilde{G}_3^{(3,1,2)}\,\{\eta_{12} f\, f_3\, q_3\}\{f_1\, f_2\} \\
+\;& G_4^{(1,2,3)}\,\{\eta_{23} f\, f_1\, \eta_{23}\}\{q_2 f_2\, \eta_{31}\}\{q_3 f_3\, \eta_{12}\} + G_4^{(2,3,1)}\,\{\eta_{31} f\, f_2\, \eta_{31}\}\{q_3 f_3\, \eta_{12}\}\{q_1 f_1\, \eta_{23}\} \\
+\;& G_4^{(3,1,2)}\,\{\eta_{12} f\, f_3\, \eta_{12}\}\{q_1 f_1\, \eta_{23}\}\{q_2 f_2\, \eta_{31}\} \\
+\;& \tilde{G}_4^{(1,2,3)}\,\{q_1 f\, f_1\, q_1\}\{q_2 f_2\, \eta_{31}\}\{q_3 f_3\, \eta_{12}\} + \tilde{G}_4^{(2,3,1)}\,\{q_2 f\, f_2\, q_2\}\{q_3 f_3\, \eta_{12}\}\{q_1 f_1\, \eta_{23}\} \\
+\;& \tilde{G}_4^{(3,1,2)}\,\{q_3 f\, f_3\, q_3\}\{q_1 f_1\, \eta_{23}\}\{q_2 f_2\, \eta_{31}\} + G_5^{(1,2,3)}\,\{q_1 f\, q_3\}\{q_1 f_1\, \eta_{23}\}\{q_3 f_3\, \eta_{12}\}\{q_2 f_2\, \eta_{31}\}\,.
\end{aligned}
$$

Quark loop contribution

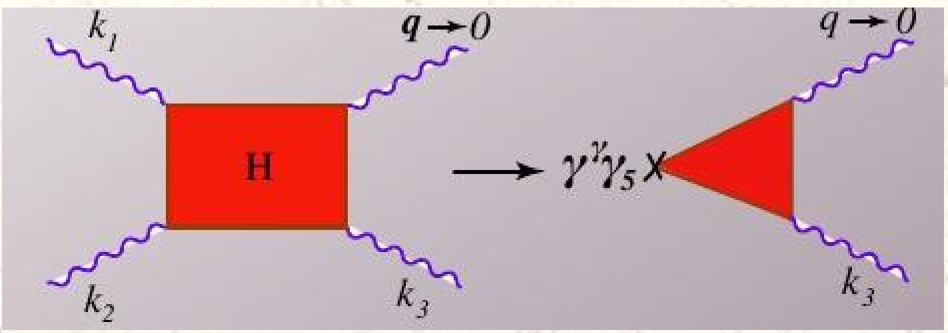
- Each of these invariant form-factors is also complex; for example

$$
\begin{aligned}
G_1(s_1,s_2,s_3) = &-\left(4s_3s_2^4 - 4s_3^3s_2^2 + 6\,s_3^2s_1^3 - 4s_3s_1^4 - 22s_1^3s_2^2 + 28s_3s_2^2s_1^2 + 26\,s_3^2s_2^2s_1 - 11s_2^4s_1 + 32s_1^2s_2^3\right.\\
&\left.-48s_3s_2^3s_1 - 2s_2^5 + s_3^4s_1 + 2s_1^4s_2 - 4s_3s_2s_1^3 - 32s_3^2s_2s_1^2 + 32s_3^3s_2s_1 + 2s_3^4s_2 + s_1^5 - 4s_3^3s_1^2\right)\frac{\ln(s_3)}{D^2s_1(s_1-s_3-s_2)s_2}\\
&+\left(s_2^5 - 4s_2^4s_1 + 6\,s_1^2s_2^3 + 6\,s_3^3s_1^2 + 21s_3s_2^4 - 22s_3^2s_2^3 - 22s_3^3s_2^2 - 26\,s_3s_2^2s_1^2 + 56\,s_3^2s_2^2s_1 - 4s_1^3s_2^2 + 21s_3^4s_2\right.\\
&\left.+4s_3s_2s_1^3 - 26\,s_3^2s_2s_1^2 + s_3^5 + s_1^4s_2 - 4s_3^2s_1^3 - 4s_3^4s_1 + s_3s_1^4\right)\frac{\ln(s_1)}{D^2s_3(s_1-s_3-s_2)s_2}\\
&-\left(-4s_1^4s_2 - 32s_3s_2^2s_1^2 + 28s_3^2s_2s_1^2 + 32s_3s_2^3s_1 + 26\,s_3^2s_2^2s_1 - 4s_1^2s_2^3 - 2s_3^5 + 6\,s_1^3s_2^2 - 4s_3^2s_2^3 + s_2^4s_1 + 2s_3s_2^4\right.\\
&\left.+4s_3^4s_2 - 11s_3^4s_1 + s_1^5 - 4s_3s_2s_1^3 - 48s_3^3s_2s_1 - 22s_3^2s_1^3 + 32s_3^3s_1^2 + 2s_3s_1^4\right)\frac{\ln(s_2)}{D^2s_1s_3(s_1-s_3-s_2)}\\
&-2\left(18s_3^2s_2^2 + 7s_2^3s_1 - 7s_1^3s_2 + 18s_3s_2s_1^2 - 19s_3s_2^2s_1 - 4s_3^3s_2 + 3s_1^2s_3^2 - 4s_3s_2^3 - 7s_1^3s_3 + 2s_1^4 + 3s_1^2s_2^2\right.\\
&\left.+7s_1s_3^3 - 19s_3^2s_2s_1 - 5s_3^4 - 5s_2^4\right)\frac{J(s_1,s_2,s_3)}{D^2(s_1-s_3-s_2)} - 4\frac{(-s_2^2 - 3s_1s_3 + s_1^2 - s_3^2 - 3s_2s_1 + 2s_3s_2)}{(s_1-s_3-s_2)s_1D},
\end{aligned}
$$

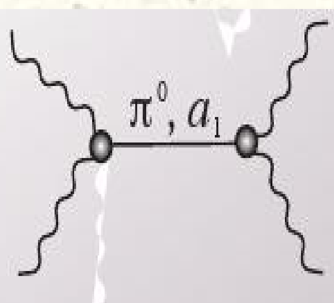
Matching any non-pertrubative model to pQCD prediction is guaranteed to be highly non-trivial

The model that fits the box

- We simplify the problem by picking up a particular part in the phase-space $q_1^2 \gg q_2^2 \gg q_3^2 \gg \Lambda^2_{\mathrm{QCD}}$ However, we require that in that part of the phase-space the amplitude is reproduced ``exactly"



$$\mathcal{M} = \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}$$



$$\mathcal{A}_{\pi^0} = -\frac{N_c W^{(3)}}{2\pi^2 F_\pi^2}\,\frac{F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)}{q_3^2 + m_\pi^2}\,\{f_2\tilde{f}_1\}\{\tilde{f} f_3\}$$

$$\mathcal{A} = \frac{4}{q_3^2\,\hat{q}^2}\{f_2\tilde{f}_1\}\{\tilde{f} f_3\}$$

$$-\frac{4}{q_3^2\,\hat{q}^4}\Big(\{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3\} + \{q_1 f_1 \tilde{f}_2 \tilde{f} f_3 q_3\} + \frac{q_1^2 + q_2^2}{4}\{f_2\tilde{f}_1\}\{\tilde{f} f_3\}\Big) + \cdots$$

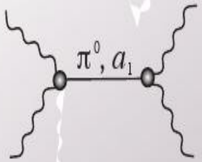
Phase-space dominance

- The key point here is that we fit full correlator of four currents
- The pion (pole and off-pole) naturally appears in this construction
- We are not advocating pion pole dominance – if anything, we are saying that an asymmetric part of the phase-space gives large(r) contribution than everything else
- This point can be discussed/critisized but this isn't the pion pole dominance story

Off-shell form-factors

- Off-shell form factors is a way to account for additional pieces in the correlator of four electromagnetic currents

$$F_{\pi^*\gamma\gamma^*}(q_1^2, q_2^2, m_\pi^2) \rightarrow F_{\pi^*\gamma\gamma^*}(q_1^2, q_2^2, q_3^2)$$



$$\mathcal{A}_{\pi^0} = -\frac{N_c W^{(3)}}{2\pi^2 F_\pi^2} \frac{F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)}{q_3^2 + m_\pi^2} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}$$

$$\mathcal{A} \sim \frac{F(q_1^2, q_2^2, q_3^2)}{q_3^2 + m_\pi^2} \left\{f_2 \tilde{f}_1\right\} \left\{\tilde{f} f_3\right\} = \left(\frac{F(q_1^2, q_2^2)}{q_3^2 + m_\pi^2} + C\right) \left\{f_2 \tilde{f}_1\right\} \left\{\tilde{f} f_3\right\}$$

As long as the off-shell form-factors are treated and interpreted as additional pieces in the photon scattering amplitudes, everything is fine and the issue is merely linguistics

Off-shell form factors

- However, I think it is somewhat dangerous because when we say ``off-shell form-factors", we make it sounds as if we can extract useful pieces of the four-photon amplitude without measuring it
- For example – off-shell (for pion) Primakoff. We require effects that are suppressed by the ratio $\Gamma_{\pi_0\gamma\gamma}/M_\rho \sim 10^{-8}$ My guess is that this is simply not feasible.