Does $\rho - \gamma$ mixing solve the e^+e^- vs. τ spectral function puzzle ?

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Abstract
The energy dependence of the $\rho-\gamma$ mixing in the 2×2 $\gamma-\rho$ propagator matrix, is shown to be able to account for the e^+e^- vs. τ spectral function discrepancy.

Work in collaboration with Robert Szafron [e-Print: arXiv:1101.2872]

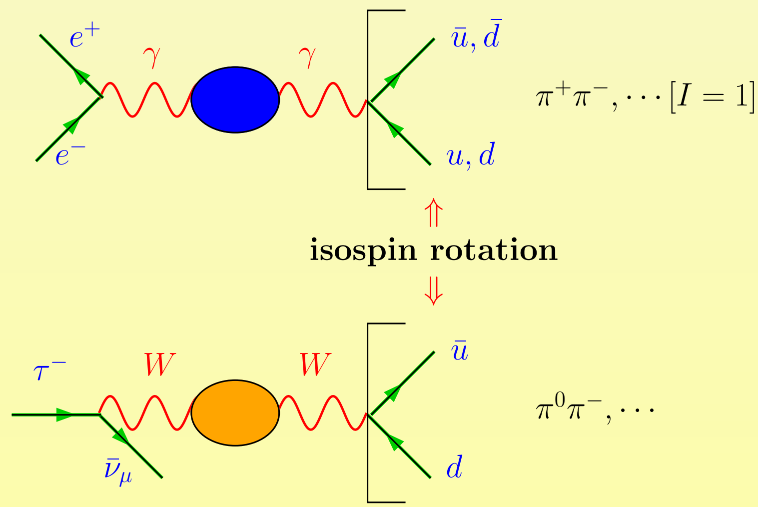
Outline of Talk:

- The τ vs. e^+e^- problem (as known)
- A minimal model: VMD + sQED
- $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop
- Applications: a_μ and $B^{\rm CVC}_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$
- Summary and Outlook

❑ The τ vs. e^+e^- problem

Concerns: calculation of hadronic vacuum polarization from appropriate hadron production data.

① A good idea: enhance e^+e^-–data by isospin rotated/corrected τ–data + CVC



ALEPH–Coll., (OPAL, CLEO), Alemany, Davier, Höcker 1996,
Belle–Coll. Fujikawa, Hayashii, Eidelman 2008

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+e^- \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

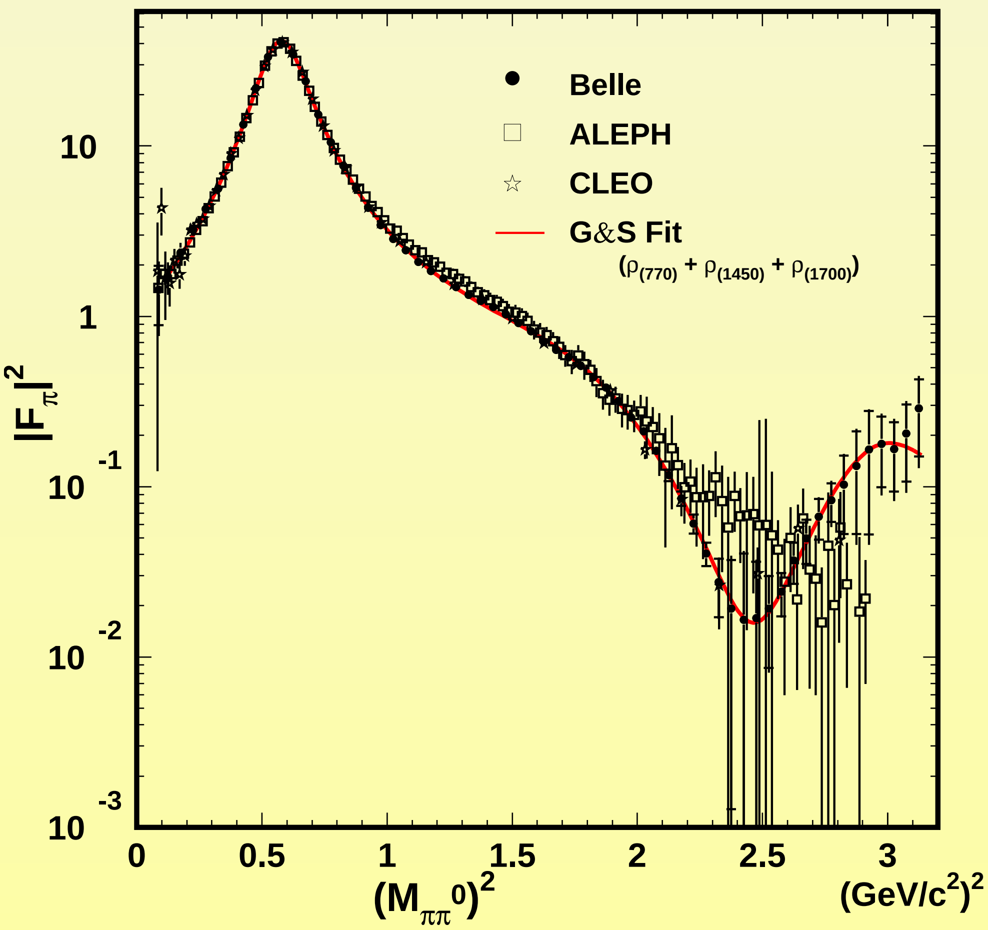
in terms of the τ spectral function v_1.

- mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)
- which is dominating in $a_\mu^{\rm had}$ (72%)

$I = 1 \sim 75\%$; $I = 0 \sim 25\%$ τ–data cannot replace e^+e^-–data

$$\begin{array}{lcccccc} \delta a_\mu & : & 15.6 \times 10^{-10} & \to & 10.2 \times 10^{-10} & & \\ \delta\Delta\alpha & : & 0.00067 & \to & 0.00065 & & (ADH1997) \end{array}$$

Data: ALEPH 97, ALEPH 05, OPAL, CLEO and
most recent measurement from *Belle* (2008):

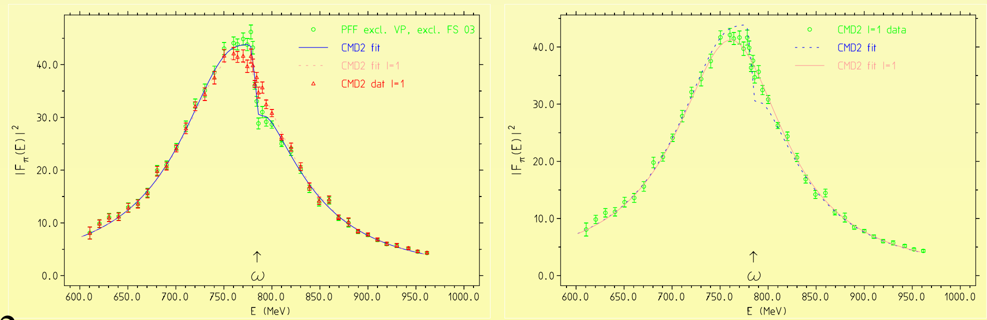


e^+e^-–data*= data corrected for isospin violations:

In e^+e^- (neutral channel) $\rho - \omega$ mixing due isospin violation be quark mass difference $m_u \neq m_u$ $\Rightarrow$
I=0 component; to be subtracted for comparison with τ data

$$|F(s)|^2 = (|F(s)|^2\text{–data}) \; / \; |\left(1 + \frac{\epsilon s}{(s_\omega - s)}\right)|^2 \quad \text{with } s_\omega = (M_\omega - \tfrac{i}{2}\Gamma_\omega)^2$$

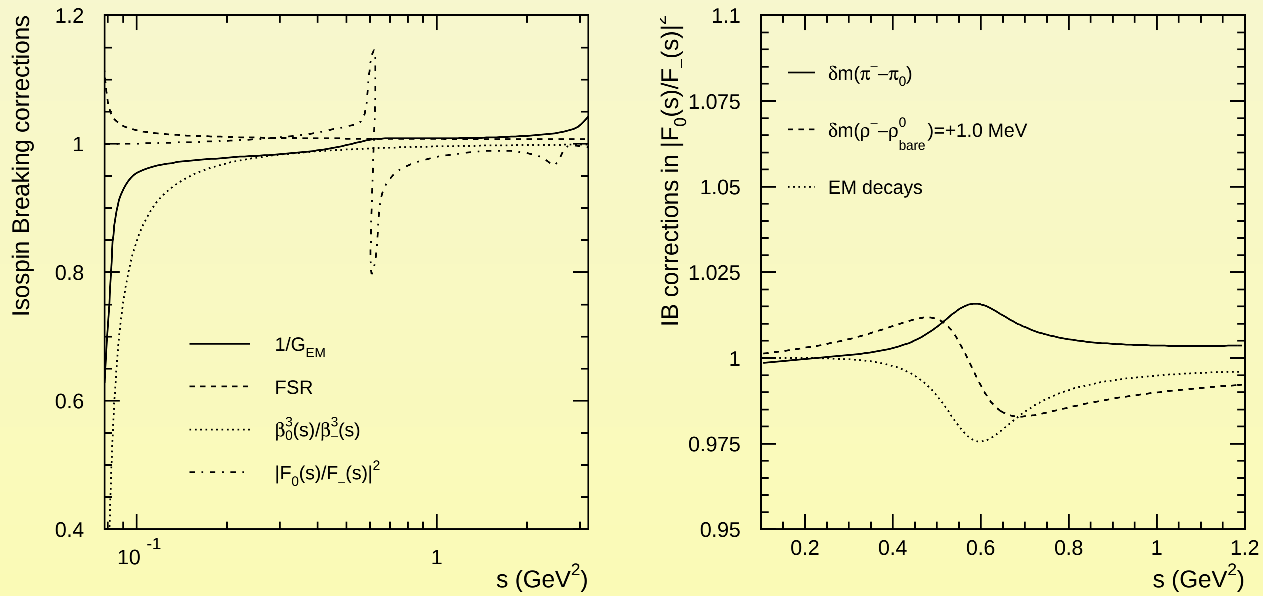
ϵ determined by fit to the data: $\epsilon = 0.00172$



CMD-2 data for $|F_\pi|^2$ in $\rho - \omega$ region together with Gounaris-Sakurai fit. Left before subtraction right after subtraction of the ω.

I=0 component to be added to τ data for calculating $a_\mu^{\rm had}$!

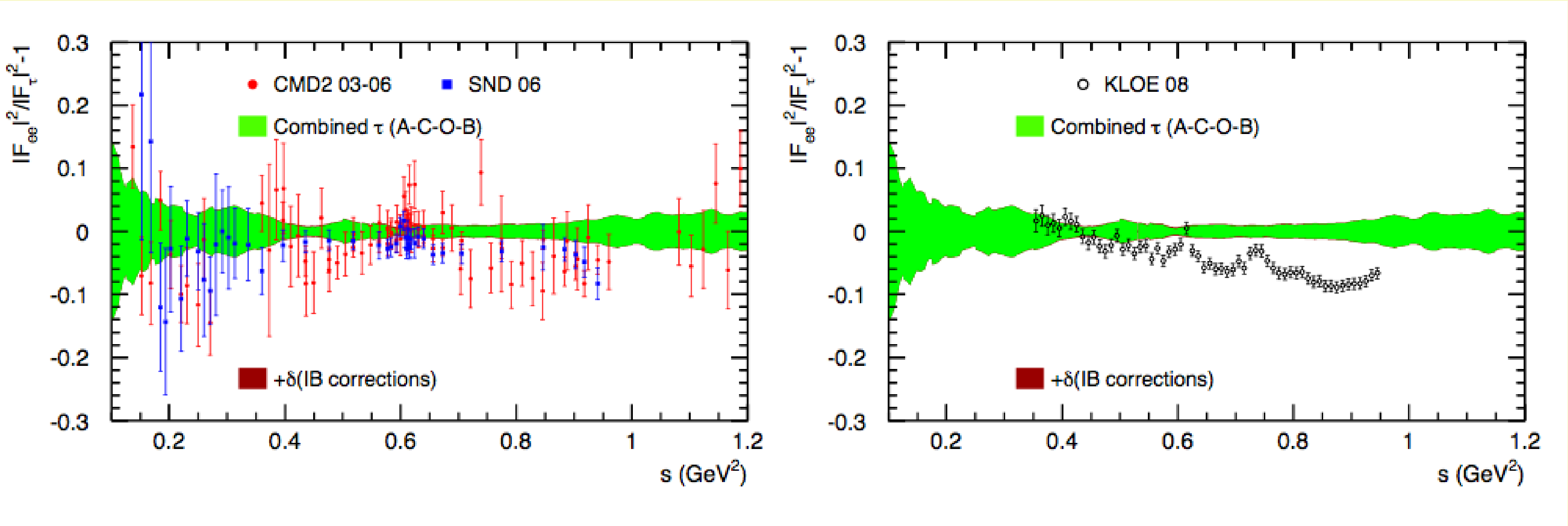
Other isospin-breaking corrections Cirigliano et al. 2002, López Castro el al. 2007



Left: Isospin-breaking corrections $G_{\rm EM}$, FSR, $\beta_0^3(s)/\beta_-^3(s)$ and $|F_0(s)/F_-(s)|^2$.
Right: Isospin-breaking corrections in $I=1$ part of ratio $|F_0(s)/F_-(s)|^2$:

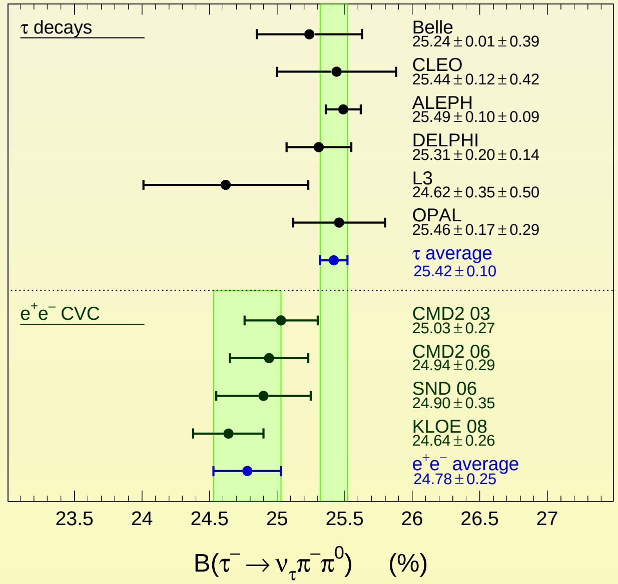
– π mass splitting $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$,
– ρ mass splitting $\delta m_\rho = m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$, and
– ρ width splitting $\delta \Gamma_\rho = \Gamma_{\rho^\pm} - \Gamma_{\rho^0}$.

New isospin corrections applied shift in mass and width [as advocated by S. Ghozzi and FJ in 2003!!!] plus changes [López Castro, Toledo Sánchez et al 2007] below the ρ which Davier et al say are not understood! The discrepancy now substantially reduced but with the KLOE data persists. New BaBar radiative return $\pi\pi$ spectrum in much better agreement, in particular with *Belle* τ spectrum!



e^+e^- vs τ spectral functions: $|F_{ee}|^2/|F_\tau|^2 - 1$ as a function of s. Isospin-breaking (IB) corrections are applied to τ data with its uncertainties included in the error band.

CVC prediction of $\mathcal{B}_{\pi\pi^0}$ normalization of BELLE, CLEO and OPAL not fixed by the experiment itself

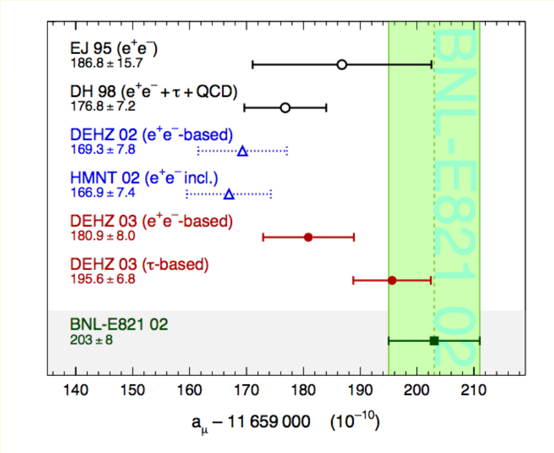


The measured branching fractions for $\tau^- \to \pi^-\pi^0\nu_\tau$ compared to the predictions from the $e^+e^- \to \pi^+\pi^-$ spectral functions (after isospin-breaking corrections). (Named e^+e^- results for $0.63-0.958$GeV). The long and short vertical error bands correspond to the τ and e^+e^- averages of 25.42 ± 0.10 and 24.76 ± 0.25, respectively.

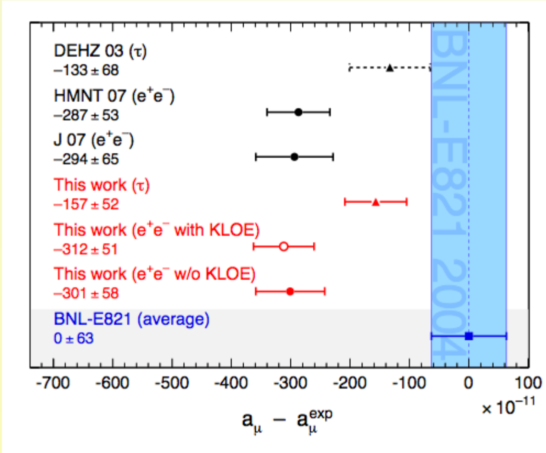
Note -2% in *Belle* τ data means $25.42 \to 24.91$ in agreement with e^+e^-

$$[|F_\tau(0)|^2 = 1.02 \to |F_\tau(0)|^2 = 1]$$

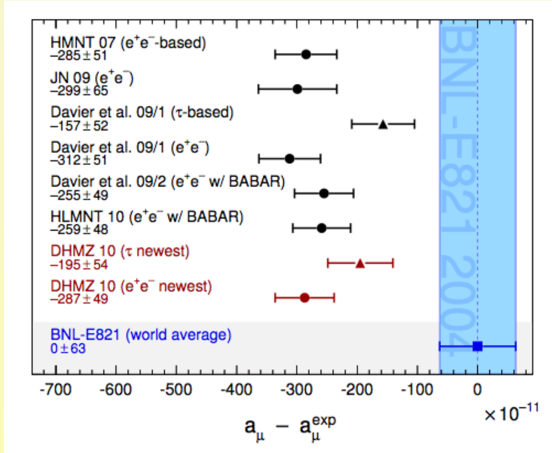
History:



EJ 95 (e⁺e⁻)
186.8 ± 15.7
DH 98 (e⁺e⁻ + τ + QCD)
176.8 ± 7.2
DEHZ 02 (e⁺e⁻-based)
169.3 ± 7.8
HMNT 02 (e⁺e⁻ incl.)
166.9 ± 7.4
DEHZ 03 (e⁺e⁻-based)
180.9 ± 8.0
DEHZ 03 (τ-based)
195.6 ± 6.8
BNL-E821 02
203 ± 8
BNL-E821 02
140 150 160 170 180 190 200 210
a_μ − 11 659 000 (10⁻¹⁰)



DEHZ 03 (τ)
−133 ± 68
HMNT 07 (e⁺e⁻)
−287 ± 53
J 07 (e⁺e⁻)
−294 ± 65
This work (τ)
−157 ± 52
This work (e⁺e⁻ with KLOE)
−312 ± 51
This work (e⁺e⁻ w/o KLOE)
−301 ± 58
BNL-E821 (average)
0 ± 63
BNL-E821 2004
-700 -600 -500 -400 -300 -200 -100 0 100
× 10⁻¹¹
a_μ − a_μ^exp



HMNT 07 (e⁺e⁻-based)
−285 ± 51
JN 09 (e⁺e⁻)
−299 ± 65
Davier et al. 09/1 (τ-based)
−157 ± 52
Davier et al. 09/1 (e⁺e⁻)
−312 ± 51
Davier et al. 09/2 (e⁺e⁻ w/ BABAR)
−255 ± 49
HLMNT 10 (e⁺e⁻ w/ BABAR)
−259 ± 48
DHMZ 10 (τ newest)
−195 ± 54
DHMZ 10 (e⁺e⁻ newest)
−287 ± 49
BNL-E821 (world average)
0 ± 63
BNL-E821 2004
-700 -600 -500 -400 -300 -200 -100 0
× 10⁻¹¹
a_μ − a_μ^exp

Possible origin of problems:

- Radiative corrections involving hadrons fully under control?
- IB in parameter shifts: $m_{\rho^+} - m_{\rho^0}$, $\Gamma_{\rho^+} - \Gamma_{\rho^0}$ fully known?

Key problem: on basis of commonly used Gounaris-Sakurai type parametrizations

e^+e^- vs. τ fit with same formula $\Rightarrow$ differ in parameters only: NC vs. CC process δM_ρ, $\delta \Gamma_\rho$, mixing coefficients etc.

Other possible source: do we really understand quantum interference?

- e^+e^-: $|F_\pi^{(e)}(s)|^2 = |F_\pi^{(e)}(s)[I=1] + F_\pi^{(e)}(s)[I=0]|^2$ what we need and measure
- τ: $|F_\pi^{(\tau)}(s)[I=1]|^2$ measured in τ-decay
- $ee+\tau$: $|F_\pi^{(e)}(s)|^2 \simeq |F_\pi^{(e,\tau)}(s)[I=1]|^2 + |F_\pi^{(e)}(s)[I=0]|^2$??? usual approximation

Need theory $\to$ specific model for the complex amplitudes

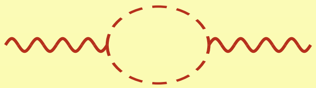
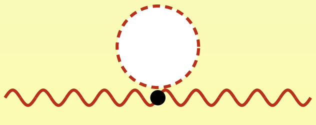
❑ A minimal model: VMD + sQED

Effective Lagrangian $\mathcal{L} = \mathcal{L}_{\gamma\rho} + \mathcal{L}_{\pi}$

$$\mathcal{L}_{\pi} = D_\mu \pi^+ D^{+\mu} \pi^- - m_\pi^2 \pi^+ \pi^- \; ; \quad D_\mu = \partial_\mu - \mathrm{i}\, e\, A_\mu - \mathrm{i}\, g_{\rho\pi\pi}\, \rho_\mu$$

$$\mathcal{L}_{\gamma\rho} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{M_\rho^2}{2} \rho_\mu \rho^\mu + \frac{e}{2\, g_\rho} \rho_{\mu\nu} F^{\mu\nu}$$

Self-energies: pion loops to photon-rho vacuum polarizations

$$-\mathrm{i}\, \Pi_{\gamma\gamma}^{\mu\nu\;(\pi)}(q) =$$  $+$  .

bare $\gamma - \rho$ transverse self-energy functions

$$\Pi_{\gamma\gamma} = \frac{e^2}{48\pi^2} f(q^2)\,, \quad \Pi_{\gamma\rho} = \frac{e\, g_{\rho\pi\pi}}{48\pi^2} f(q^2) \quad \text{and} \quad \Pi_{\rho\rho} = \frac{g_{\rho\pi\pi}^2}{48\pi^2} f(q^2)\,,$$

Explicitly, in the $\overline{\mathrm{MS}}$ scheme (μ the $\overline{\mathrm{MS}}$ renormalization scale)

$$h(q^2) \equiv f(q^2)/q^2 \quad = \quad 2/3 + 2\,(1-y) - 2\,(1-y)^2\,G(y) + \ln\frac{\mu^2}{m_\pi^2}\;,$$

where $y = 4m_\pi^2/s$ and $G(y) = \frac{1}{2\beta_\pi}\,(\ln\frac{1+\beta_\pi}{1-\beta_\pi} - \mathrm{i}\,\pi)$, for $q^2 > 4\,m_\pi^2$.

Mass eigenstates, diagonalization: renormalization conditions are such that the matrix is diagonal and of residue unity at the photon pole $q^2 = 0$ and at the ρ resonance $s = M_\rho^2$, [$\Pi_{..}(0) = 0\,, \Pi'_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(q^2)/q^2$]

$$\Pi^{\mathrm{ren}}_{\gamma\gamma}(q^2) \quad = \quad \Pi_{\gamma\gamma}(q^2) - q^2\,\Pi'_{\gamma\gamma}(0) \doteq q^2\,\Pi'^{\mathrm{ren}}_{\gamma\gamma}(q^2)$$

$$\Pi^{\mathrm{ren}}_{\gamma\rho}(q^2) \quad = \quad \Pi_{\gamma\rho}(q^2) - \frac{q^2}{M_\rho^2}\,\mathrm{Re}\,\Pi_{\gamma\rho}(M_\rho^2)$$

$$\Pi^{\mathrm{ren}}_{\rho\rho}(q^2) \quad = \quad \Pi_{\rho\rho}(q^2) - \mathrm{Re}\,\Pi_{\rho\rho}(M_\rho^2) - (q^2 - M_\rho^2)\,\mathrm{Re}\,\frac{\mathrm{d}\Pi_{\rho\rho}}{\mathrm{d}s}(M_\rho^2)$$

Propagators = inverse of symmetric 2×2 self-energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}$$

inverted $\Rightarrow$

$$\begin{aligned} D_{\gamma\gamma} &= \frac{1}{q^2 + \Pi_{\gamma\gamma}(q^2) - \frac{\Pi_{\gamma\rho}^2(q^2)}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)}} \\ D_{\gamma\rho} &= \frac{-\Pi_{\gamma\rho}(q^2)}{(q^2 + \Pi_{\gamma\gamma}(q^2))(q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)) - \Pi_{\gamma\rho}^2(q^2)} \\ D_{\rho\rho} &= \frac{1}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) - \frac{\Pi_{\gamma\rho}^2(q^2)}{q^2 + \Pi_{\gamma\gamma}(q^2)}} . \end{aligned}$$

Resonance parameters $\Leftrightarrow$ location s_P of the pole of the propagator

$$s_P - m^2_{\rho^0} + \Pi_{\rho^0\rho^0}(s_P) - \frac{\Pi^2_{\gamma\rho^0}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0\;,$$

with $s_P = \tilde{M}^2_{\rho^0}$ complex.

$$\tilde{M}^2_\rho \equiv \left(q^2\right)_{\rm pole} = M^2_\rho - {\rm i}\; M_\rho\;\Gamma_\rho$$

Diagonalization $\Rightarrow$ physical ρ acquires a direct coupling to the electron

$$\begin{array}{rcl} \mathcal{L}_{\rm QED} & = & \bar{\psi}_e\gamma^\mu(\partial_\mu - {\rm i}\,e_b\,A_{b\mu})\,\psi_e \\ & \Downarrow & \\ \mathcal{L}_{\rm QED} & = & \bar{\psi}_e\gamma^\mu(\partial_\mu - {\rm i}\,e\,A_\mu + {\rm i}\,g_{\rho ee}\rho_\mu)\,\psi_e \end{array}$$

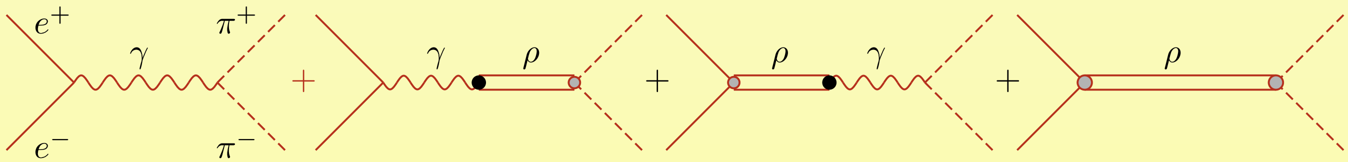
with $g_{\rho ee} = e\,(\Delta_\rho + \Delta_0)$, where in our case $\Delta_0 = 0$.

❑ $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop

The $e^+e^- \to \pi^+\pi^-$ matrix element in sQED is given by

$$\mathcal{M} = -\mathrm{i}\, e^2\, \bar{v}\gamma^\mu u\, (p_1 - p_2)_\mu\, F_\pi(q^2)$$

with $F_\pi(q^2) = 1$. In our extended VMD model we have the four terms



Diagrams contributing to the process $e^+e^- \to \pi^+\pi^-$.

$$F_\pi(s) \propto e^2\, D_{\gamma\gamma} + e g_{\rho\pi\pi}\, D_{\gamma\rho} - g_{\rho ee} e D_{\rho\gamma} - g_{\rho ee} g_{\rho\pi\pi}\, D_{\rho\rho}\,,$$

Properly normalized (VP subtraction: $e^2(s) \to e^2$):

$$F_\pi(s) = \left[e^2\, D_{\gamma\gamma} + e\,(g_{\rho\pi\pi} - g_{\rho ee})\, D_{\gamma\rho} - g_{\rho ee} g_{\rho\pi\pi}\, D_{\rho\rho}\right] / \left[e^2\, D_{\gamma\gamma}\right]$$

Typical couplings

$g_{\rho\pi\pi\,\mathrm{bare}} = 5.8935$, $g_{\rho\pi\pi\,\mathrm{ren}} = 6.1559$, $g_{\rho ee} = 0.018149$, $x = g_{\rho\pi\pi}/g_\rho = 1.15128$.

We note that the precise s-dependence of the effective ρ-width is obtained by evaluating the imaginary part of the ρ self-energy:

$$\mathrm{Im}\,\Pi_{\rho\rho} = \frac{g^2_{\rho\pi\pi}}{48\,\pi} \beta^3_\pi\, s \equiv M_\rho\, \Gamma_\rho(s)\ ,$$

which yields

$$\Gamma_\rho(s)/M_\rho = \frac{g^2_{\rho\pi\pi}}{48\,\pi} \beta^3_\pi \frac{s}{M^2_\rho}\ ;\ \ \Gamma_\rho/M_\rho = \frac{g^2_{\rho\pi\pi}}{48\,\pi} \beta^3_\rho\ ;\ \ g_{\rho\pi\pi} = \sqrt{48\,\pi\,\Gamma_\rho/(\beta^3_\rho\, M_\rho)}\ .$$

In our model, in the given approximation, the on ρ-mass-shell form factor reads

$$F_\pi(M_\rho^2) \;=\; 1 - \mathrm{i}\,\frac{g_{\rho ee}\, g_{\rho\pi\pi}}{e^2}\,\frac{M_\rho}{\Gamma_\rho}\;;\;\; |F_\pi(M_\rho^2)|^2 = 1 + \frac{36}{\alpha^2}\frac{\Gamma_{ee}}{\beta_\rho^3\,\Gamma_\rho}\;,$$

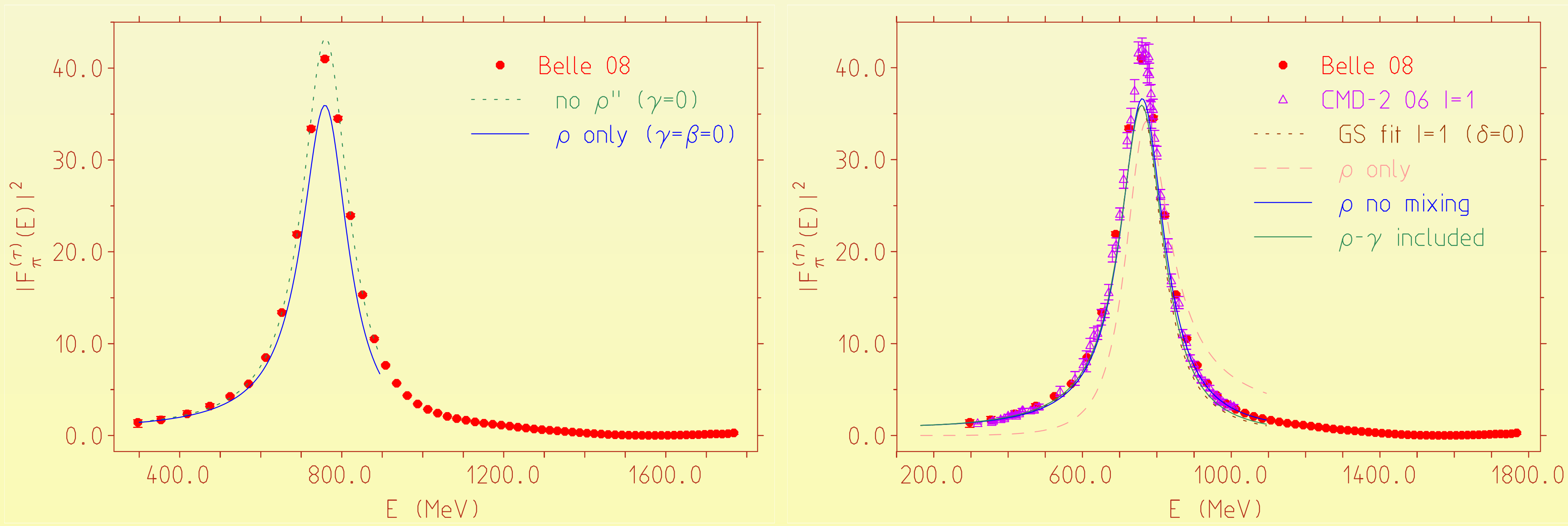
$$\Gamma_{\rho ee} \;=\; \frac{1}{3}\,\frac{g_{\rho ee}^2}{4\pi}\,M_\rho\;;\;\; g_{\rho ee} = \sqrt{12\pi\,\Gamma_{\rho ee}/M_\rho}\;.$$

Compare: Gounaris-Sakurai (GS) formula

$$F_\pi^{\mathrm{GS}}(s) = \frac{-M_\rho^2 + \Pi_{\rho\rho}^{\mathrm{ren}}(0)}{s - M_\rho^2 + \Pi_{\rho\rho}^{\mathrm{ren}}(s)}\;;\;\; \Gamma_{\rho ee}^{\mathrm{GS}} = \frac{2\alpha^2\,\beta_\rho^3 M_\rho^2}{9\,\Gamma_\rho}\,\left(1 + d\,\Gamma_\rho/M_\rho\right)^2 \;.$$

GS does not involve $g_{\rho ee}$ resp. $\Gamma_{\rho ee}$ in a direct way, as normalization is fixed by applying an overall factor $1 + d\,\Gamma_\rho/M_\rho \equiv 1 - \Pi_{\rho\rho}^{\mathrm{ren}}(0)/M_\rho^2 \simeq 1.089$ to enforce $F_\pi(0) = 1$ (in our approach “automatic” by gauge invariance).

Relation to data:

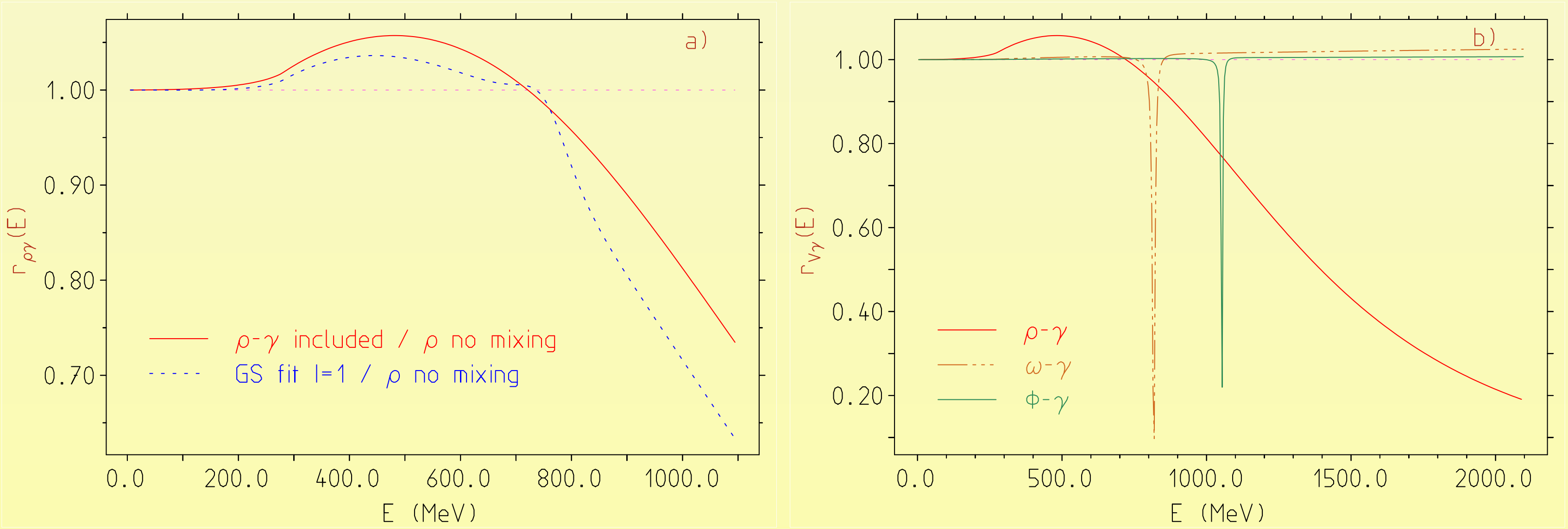


Left: GS fits of the Belle data and the effects of including higher states ρ' and ρ'' at fixed M_ρ and Γ_ρ. Right: Effect of $\gamma - \rho$ mixing in our simple EFT model

Parameters: $M_\rho = 775.5$ MeV, $\Gamma_\rho = 143.85$ MeV,
$\mathcal{B}[(\rho \to ee)/(\rho \to \pi\pi)] = 4.67 \times 10^{-5}$, $e = 0.302822$, $g_{\rho\pi\pi} = 5.92$, $g_{\rho ee} = 0.01826$.

Detailed comparison, in terms of the ratio:

$$r_{\rho\gamma}(s) \equiv \frac{|F_\pi(s)|^2}{|F_\pi(s)|^2_{D_{\gamma\rho}=0}}$$



a) Ratio of $|F_\pi(E)|^2$ with mixing vs. no mixing. Same ratio for GS fit with PDG parameters. b) The same mechanism scaled up by the branching fraction $\Gamma_V/\Gamma(V \to \pi\pi)$ for $V = \omega$ and ϕ. In the $\pi\pi$ channel the effects for resonances $V \neq \rho$ are tiny if not very close to resonance.

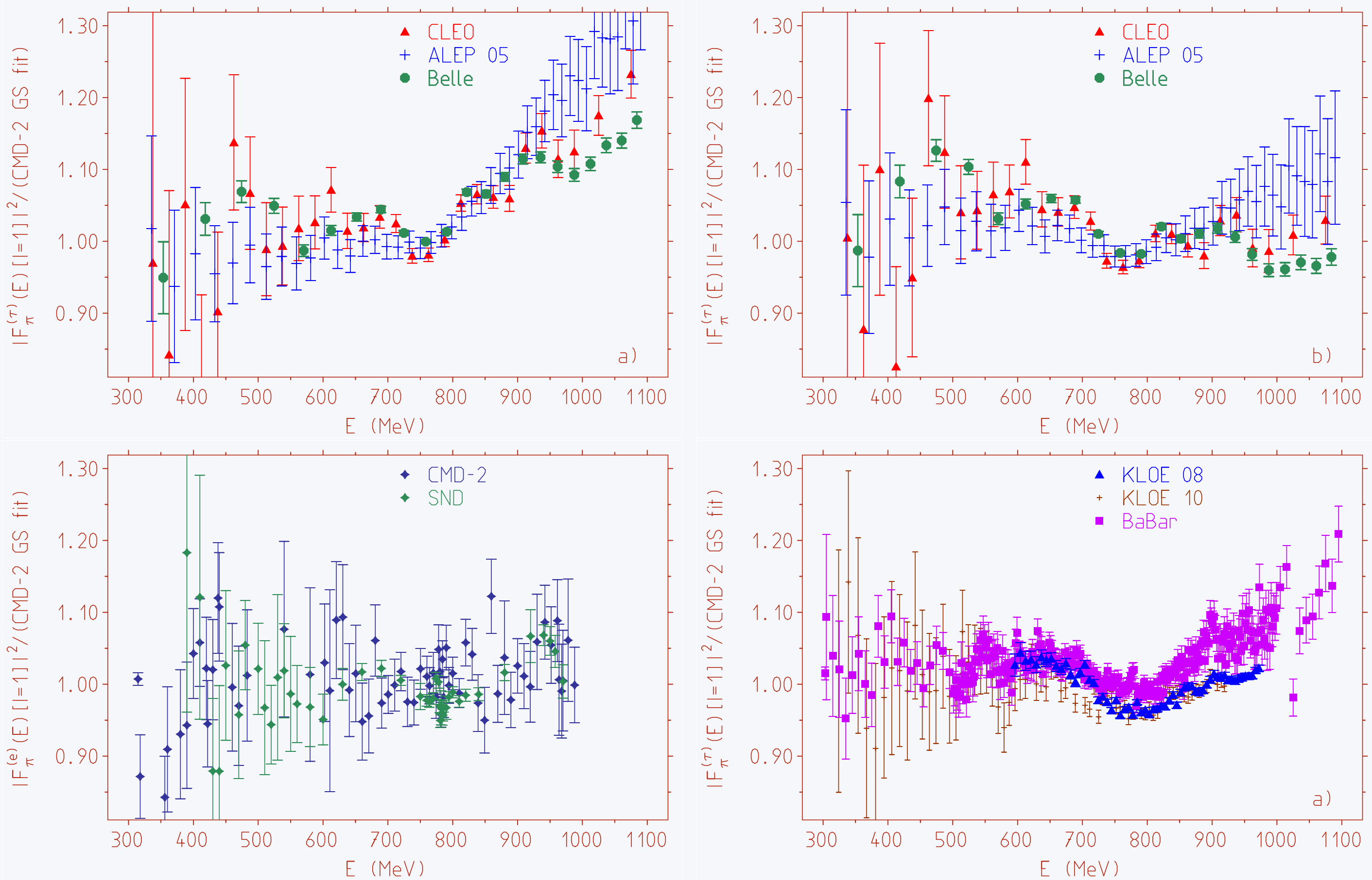
If mixing not included in $F_0(s)$ $\Rightarrow$ total correction formula on spectral functions

$$v_0(s) = r_{\rho\gamma}(s)\, R_{\rm IB}(s)\, v_-(s)$$

$$R_{\rm IB}(s) = \frac{1}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left|\frac{F_0(s)}{F_-(s)}\right|^2$$

- $G_{\rm EM}(s)$ electromagnetic radiative corrections
- $\beta_0^3(s)/\beta_-^3(s)$ phase space modification by $m_{\pi^0} \neq m_{\pi^\pm}$
- $|F_0(s)/F_-(s)|^2$ incl. shifts in masses, widths etc

Final state radiation correction FSR(s) and vacuum polarization effects $(\alpha/\alpha(s))^2$ and I=0 component $(\rho - \omega)$ we have been subtracted from all e^+e^--data.



$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit): a) τ data uncorrected for $\rho - \gamma$ mixing, and b) after correcting for mixing. Lower panel: e^+e^- energy scan data [left] and e^+e^- radiative return data [right]

❏ Applications: a_μ and $B^{\rm CVC}_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$

How does the new correction affect the evaluation of the hadronic contribution to a_μ ? To lowest order in terms of e^+e^--data, represented by $R(s)$, we have

$$a_\mu^{\rm had,LO}(\pi\pi) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty {\rm d}s\, R^{(0)}_{\pi\pi}(s) \frac{K(s)}{s}\;,$$

with the well-known kernel $K(s)$ and

$$R^{(0)}_{\pi\pi}(s) = (3s\sigma_{\pi\pi})/4\pi\alpha^2(s)) = 3v_0(s)\;.$$

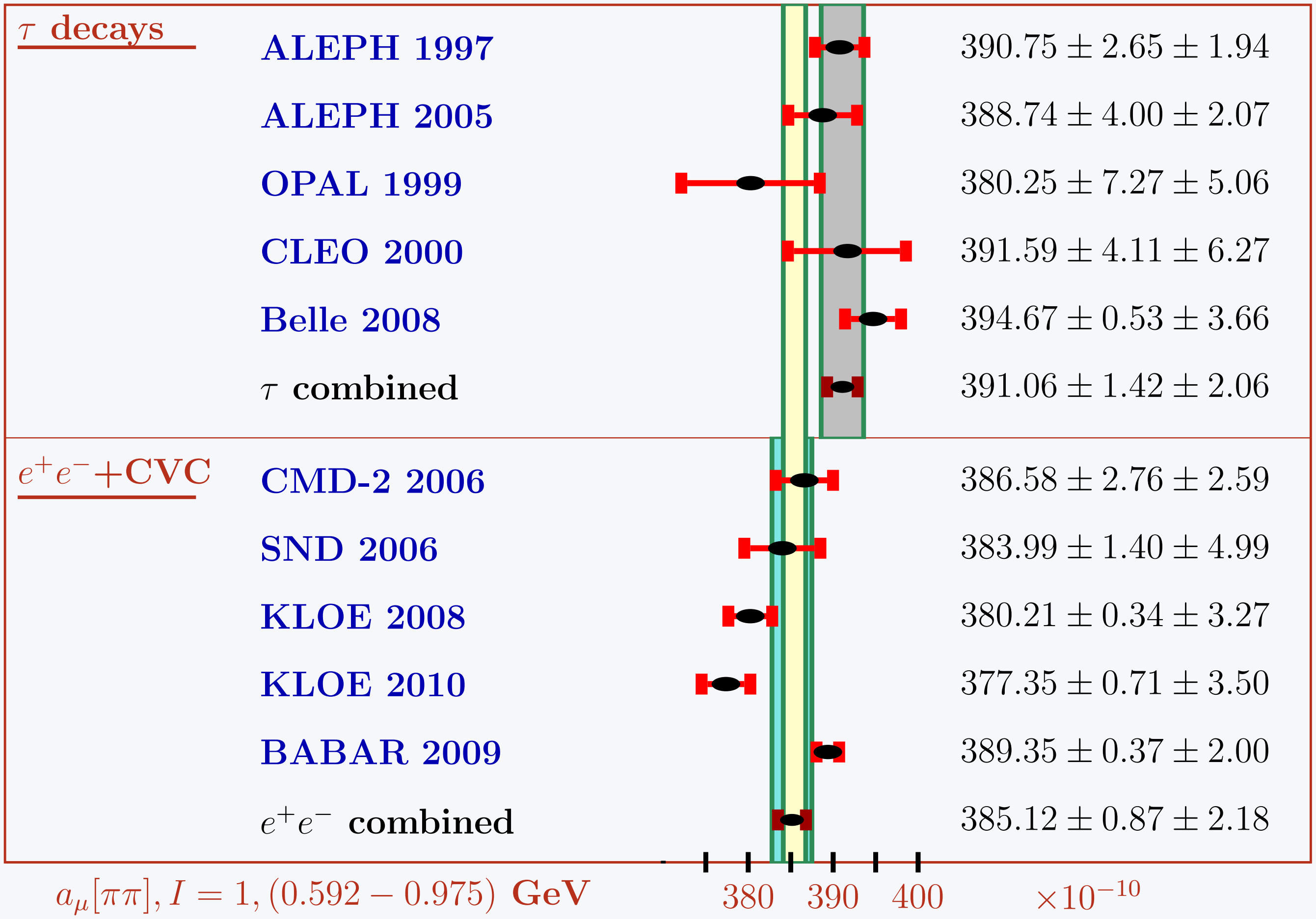
Note that the $\rho - \gamma$ interference is included in the measured e^+e^--data, and so is its contribution to $a_\mu^{\rm had}$. In fact $a_\mu^{\rm had}$ is intrinsic an e^+e^--based “observable” (neutral current channel).

How to utilize τ data: subtract CVC violating corrections

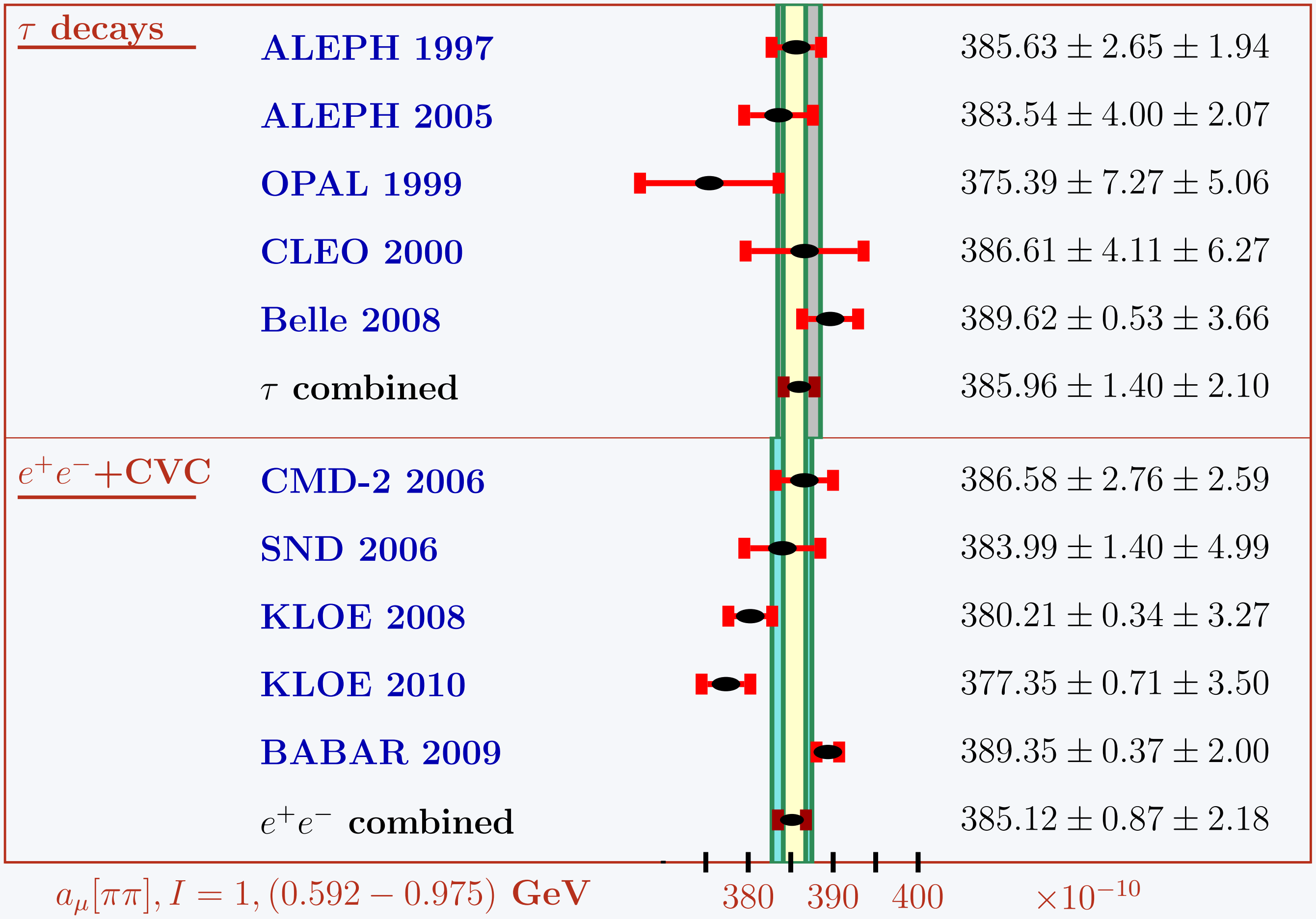
❖traditionally $v_-(s) \to v_0(s) = R_{\rm IB}(s)\, v_-(s)$

❖our correction $v_-(s) \to v_0(s) = r_{\rho\gamma}(s)\; R_{\rm IB}(s)\, v_-(s)$

Result for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$: $\delta a_\mu^{\rm had}[\rho\gamma] \simeq (-5.1 \pm 0.5) \times 10^{-10}$



I=1 part of $a_\mu^{\rm had}[\pi\pi]$

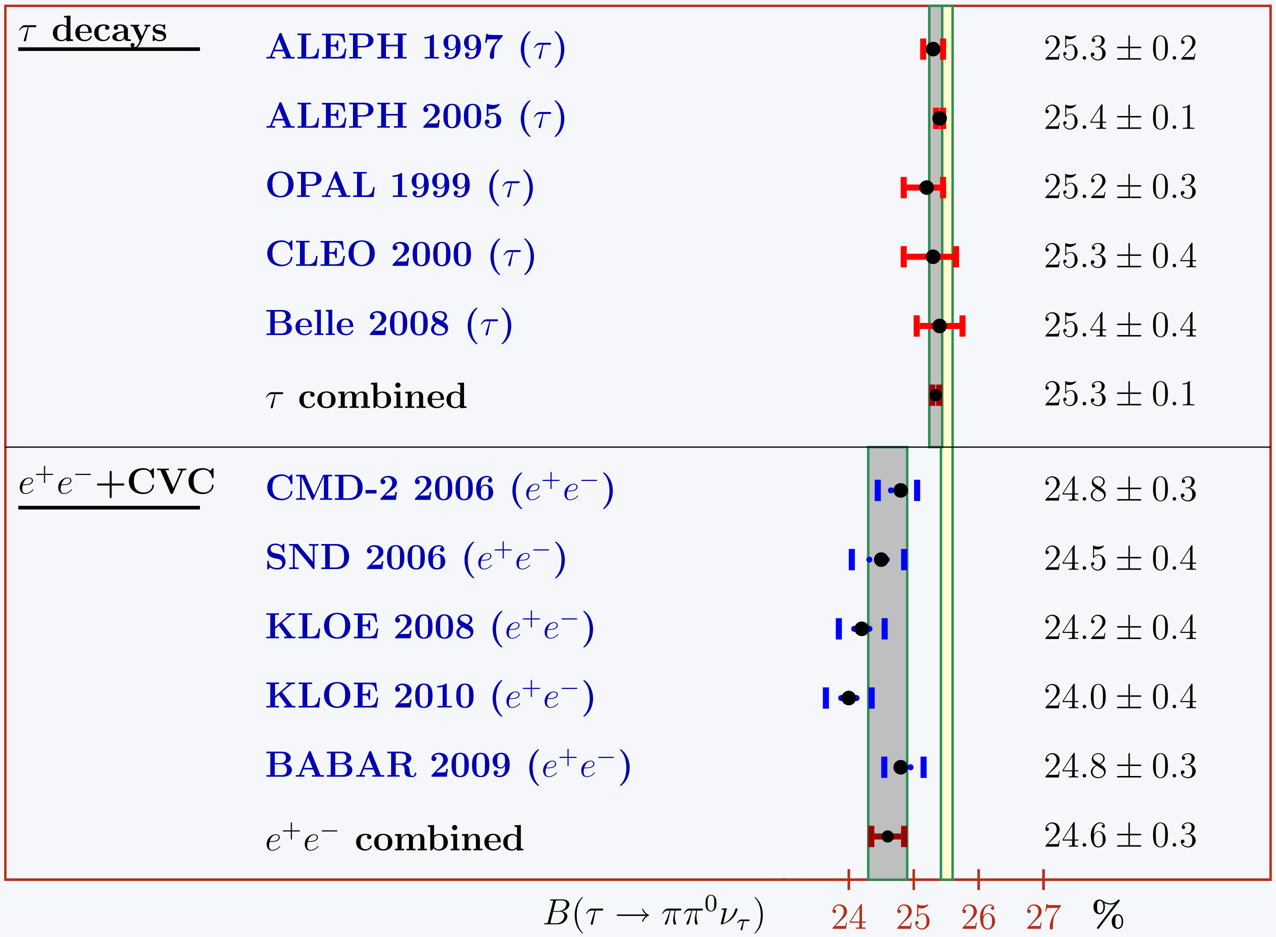


I=1 part of $a_\mu^{\rm had}[\pi\pi]$

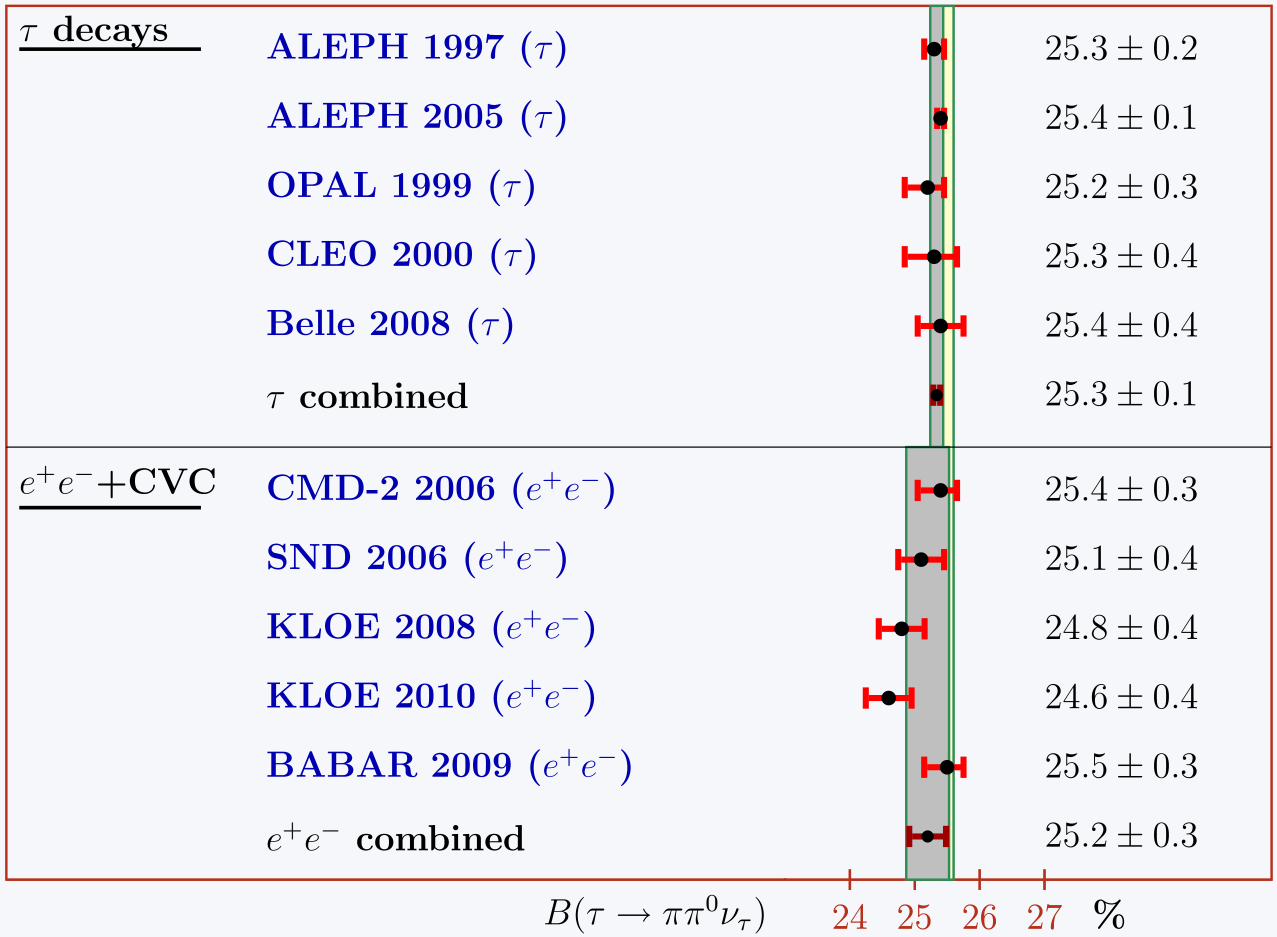
The $\tau \to \pi^0 \pi \nu_\tau$ branching fraction $B_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$ is another important quantity which can be directly measured. This "τ-observable" can be evaluated in terms of the I=1 part of the $e^+e^- \to \pi^+\pi^-$ cross section, after taking into account the IB correction $v_0(s) \to v_-(s) = v_0(s)/R_{\rm IB}(s)$ $/r_{\rho\gamma}(s)$,

$$B^{\rm CVC}_{\pi\pi^0} = \frac{2S_{\rm EW} B_e |V_{ud}|^2}{m_\tau^2} \int_{4m_\pi^2}^{m_\tau^2} {\rm d}s\, R^{(0)}_{\pi^+\pi^-}(s) \left(1-\frac{2}{m_\tau^2}\right)^2 \left(1+\frac{2s}{m_\tau^2}\right) \frac{1}{r_{\rho\gamma}(s)\, R_{\rm IB}(s)} \,,$$

where here we also have to "undo" the $\rho - \gamma$ mixing which is absent in the charged isovector channel. The shift is $\delta B^{\rm CVC}_{\pi\pi^0}[\rho\gamma] = +0.62 \pm 0.06$ %



Branching fractions $B(\tau \to \pi\pi^0\nu_\tau)$

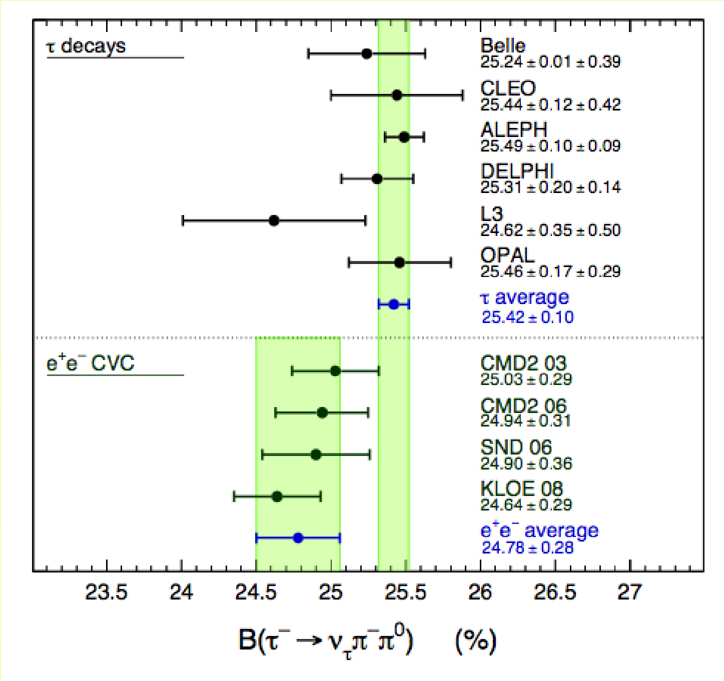


Branching fractions $B(\tau \to \pi\pi^0\nu_\tau)$

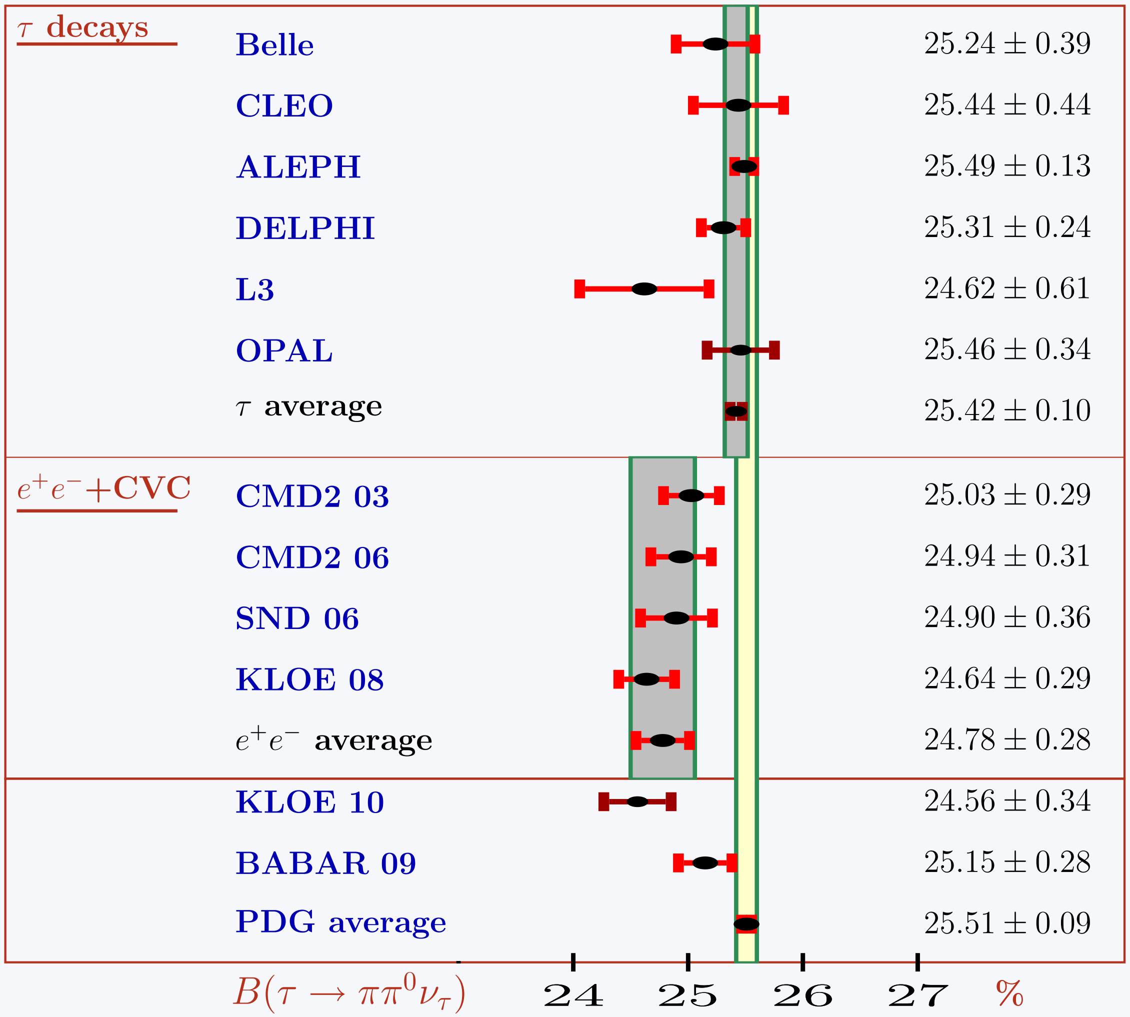
Most recent results of Davier et al:

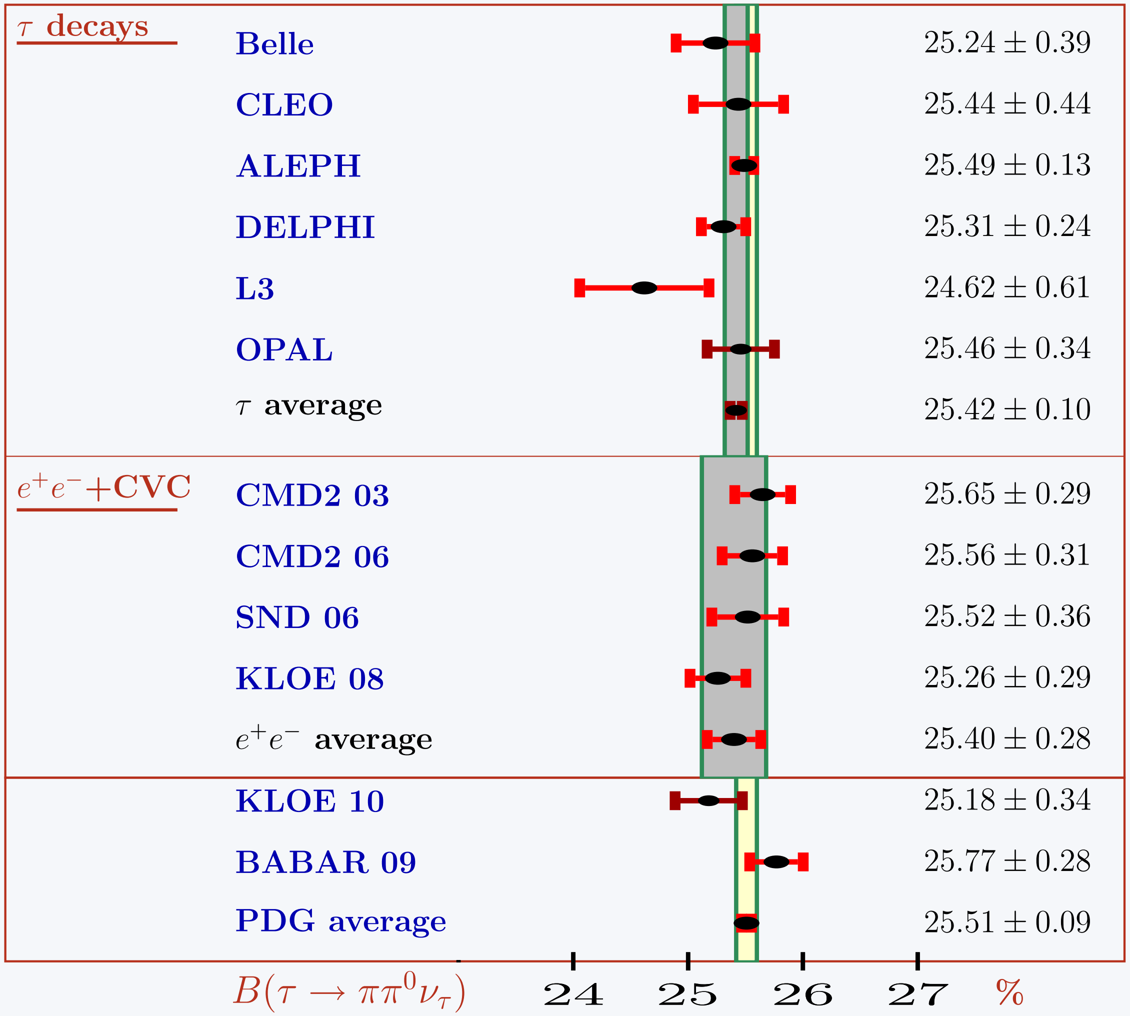
- Pre BaBar: 25.42 ± 0.10 % for τ

  24.78 ± 0.28 % $\overset{+\rho\gamma}{\Rightarrow}$ $25.40 \pm 0.28 \pm 0.06$ % for e^+e^- + CVC
- New BaBar: 25.15 ± 0.28 % $\overset{+\rho\gamma}{\Rightarrow}$ $25.77 \pm 0.28 \pm 0.06$ % for e^+e^- + CVC



shift $\delta B^{\mathrm{CVC}}_{\pi\pi^0}[\rho\gamma] = +0.62 \pm 0.06$ %


τ decays
Belle
CLEO
ALEPH
DELPHI
L3
OPAL
τ average
e+e− +CVC
CMD2 03
CMD2 06
SND 06
KLOE 08
e+e− average
KLOE 10
BABAR 09
PDG average
25.24 ± 0.39
25.44 ± 0.44
25.49 ± 0.13
25.31 ± 0.24
24.62 ± 0.61
25.46 ± 0.34
25.42 ± 0.10
25.03 ± 0.29
24.94 ± 0.31
24.90 ± 0.36
24.64 ± 0.29
24.78 ± 0.28
24.56 ± 0.34
25.15 ± 0.28
25.51 ± 0.09
B(τ → ππ0ντ)
24
25
26
27
%


τ decays
Belle
CLEO
ALEPH
DELPHI
L3
OPAL
τ average
e^+e^-+CVC
CMD2 03
CMD2 06
SND 06
KLOE 08
e^+e^- average
KLOE 10
BABAR 09
PDG average
25.24 ± 0.39
25.44 ± 0.44
25.49 ± 0.13
25.31 ± 0.24
24.62 ± 0.61
25.46 ± 0.34
25.42 ± 0.10
25.65 ± 0.29
25.56 ± 0.31
25.52 ± 0.36
25.26 ± 0.29
25.40 ± 0.28
25.18 ± 0.34
25.77 ± 0.28
25.51 ± 0.09
$B(\tau \to \pi\pi^0\nu_\tau)$
24
25
26
27
%

Summary and Conclusions

➠VMD+sQED EFT understood as the tail of the more appropriate resonance Lagrangian approach (Ecker et al. 1989) in low energy $\pi\pi$ production yields

- proper ρ propagator self-energy effects for GS form factor ($\rho \to \pi\pi$)
- pion-loop effects in $\rho - \gamma$ mixing contributes sizable interferences

Note: so far PDG parameters masses, withs, branching fractions etc. of resonances like ρ^0 all extracted from data assuming GS like form factors (model dependent!)

Pattern:

❒ moderate positive interference (up to +5%) below ρ,
substantial negative interference (-10% and more)
above the ρ (must vanish at $s = 0$ and $s = M_\rho^2$)

- remarkable agreement with pattern of e^+e^- vs τ discrepancy
- shift of the τ data to lie perfectly within the ballpark of the e^+e^- data

Lesson: effective field theory the basic tool (not ad hoc pheno. ansätze)

- $\rho - \gamma$ correction function $r_{\rho\gamma}(s)$ entirely fixed from neutral channel
- τ data provide independent information

What does it mean for the muon $g-2$?

- it looks we have fairly reliable model to include τ data to improve $a_\mu^{\rm had}$
- there is no τ vs. e^+e^- alternative of $a_\mu^{\rm had}$

For the lowest order hadronic vacuum polarization (VP) contribution to a_μ we find

$$a_\mu^{\rm had,LO}[e,\tau] = 690.96(1.06)(4.63) \times 10^{-10} \quad (e+\tau)$$

$$a_\mu^{\rm the} = 116591797(60) \times 10^{-11}$$ $$a_\mu^{\rm exp} = 116592080(54)(33) \times 10^{-11}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm the} = (283 \pm 87) \times 10^{-11}$$

$$3.3\,\sigma$$

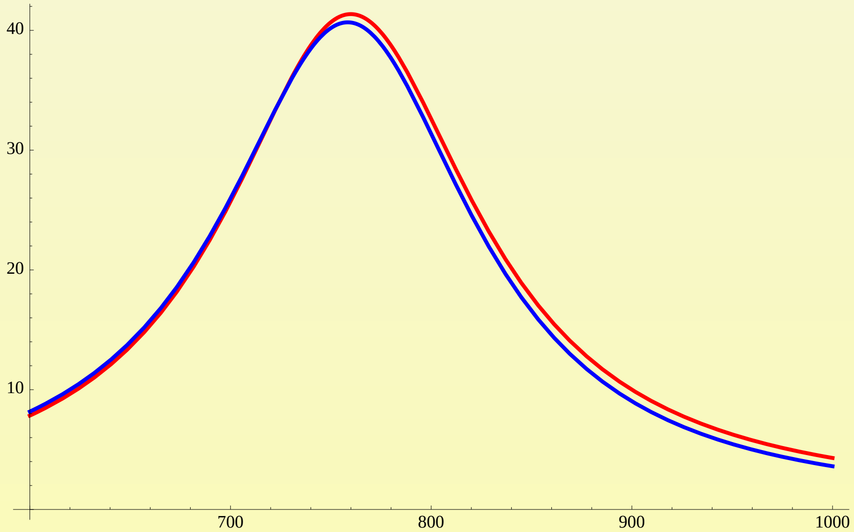
Höcker 2010 (theory-driven analysis)

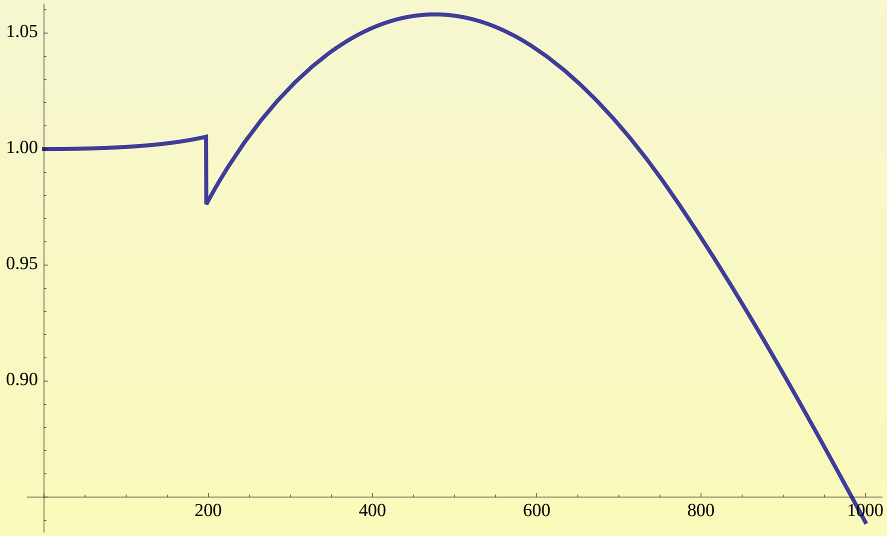
$a_\mu^{\rm had,LO}[e] \quad = (692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10}$ (e^+e^- based),
$a_\mu^{\rm had,LO}[e,\tau] = (701.5 \pm 3.5 \pm 1.9 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10}$ ($e^+e^- + \tau$ based),

❒ Note: ratio $F_0(s)/F_-(s)$ could be measured within lattice QCD, without reference to sQED or other hadronic models. Do it!

❒ Including $\omega, \phi, \rho', \rho'', \cdots$ requires to go to appropriate Resonance Lagrangian extension (e.g HLS model Benayoun et al.)

Backup slides





Rober Szafrons first attempt to $\rho - \gamma$ mixing (based on my QCD lectures at Katowice (see: `http://www-com.physik.hu-berlin.de/~fjeger/books.html`)).

Real parts and moduli of the individual terms normalized to the sQED photon exchange term are displayed in Fig. 1.

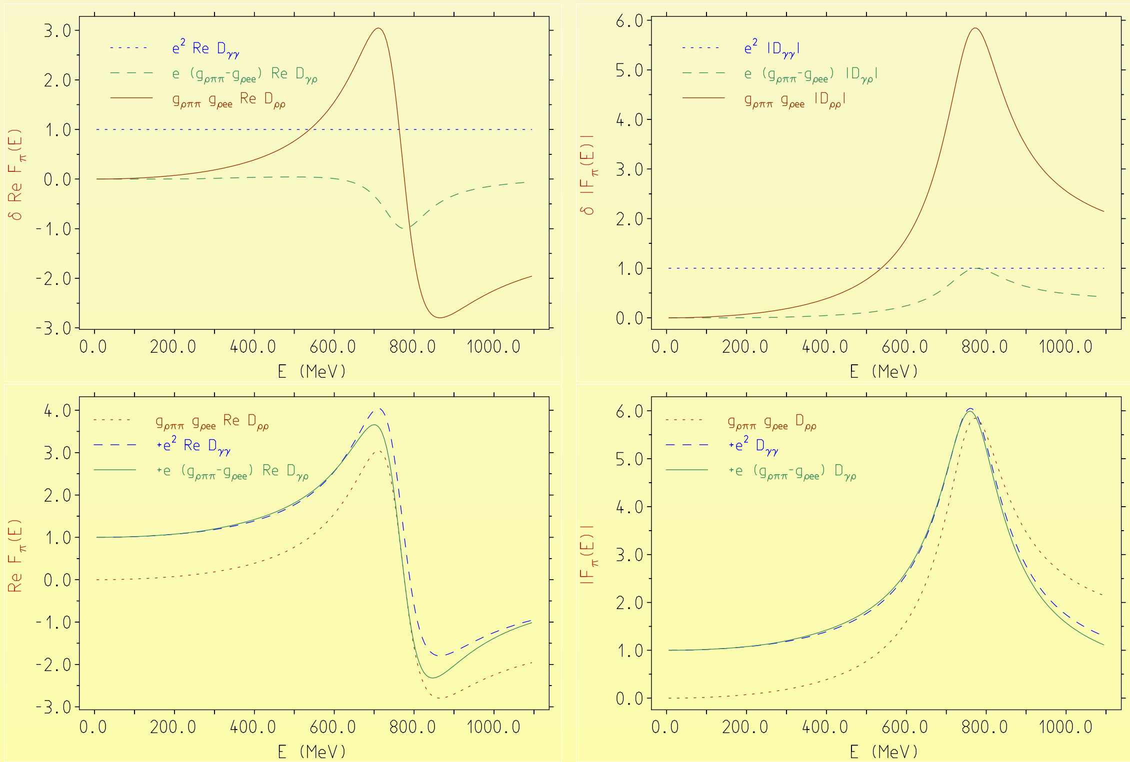


Figure 1: The real parts and moduli of the three terms of (**??**), individual and added up.

An improved theory of the pion form factor has been developed in [**?**]. One of the key ingredients in this approach is the strong interaction phase shift $\delta_1^1(s)$ of $\pi\pi$ (re)scattering in the final state. In Fig. 2 we compare the phase of $F_\pi(s)$ in our model with the one obtained by solving the Roy equation with $\pi\pi$-scattering data as input. We notice that the agreement is surprisingly good up to about 1 GeV. It is not difficult to replace our phase by the more precise exact one.

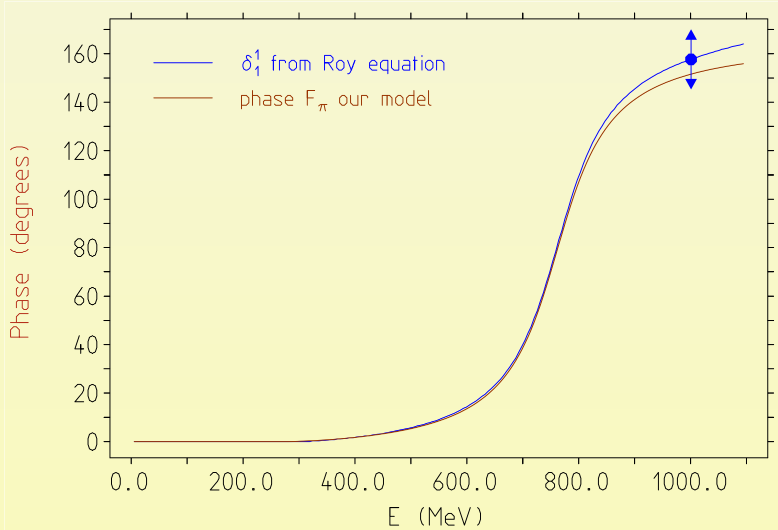
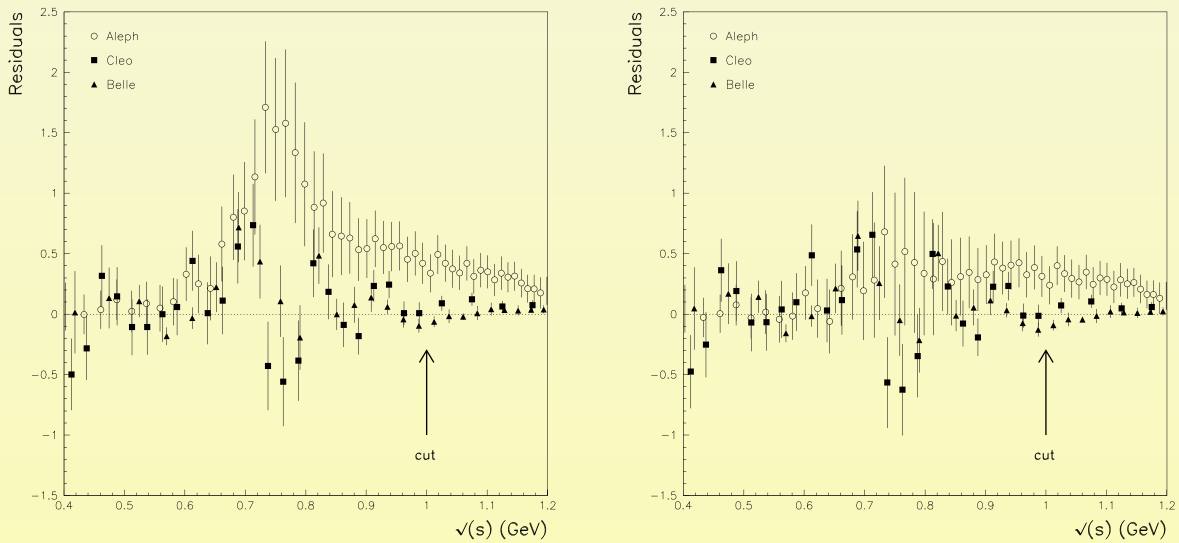


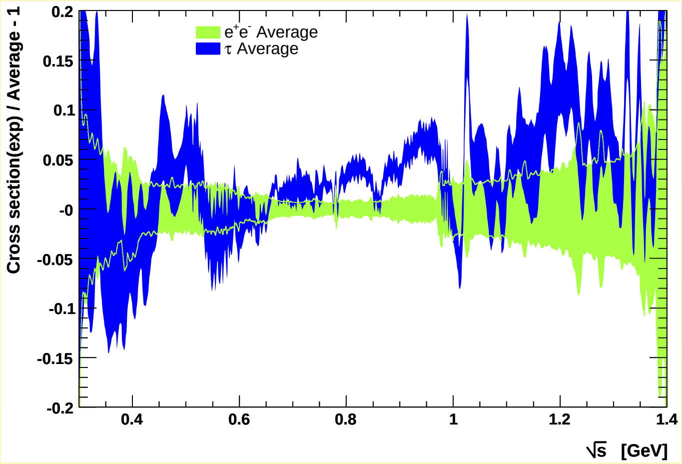
Figure 2: The phase of $F_\pi(E)$ as a function of the c.m. energy E. We compare the result of the elaborate Roy equation analysis of Ref. [**?**] with the one due to the sQED pion-loop. The solution of the Roy equation depends on the normalization at a high energy point (typically 1 GeV). In our calculation we could adjust it by varying the coupling $g_{\rho\pi\pi}$.



τ data vs. residual distribution in the fit of τ data: Left: BELLE+CLEO, Right: ALEPH+BELLE+CLEO (from Benayoun et al 09))

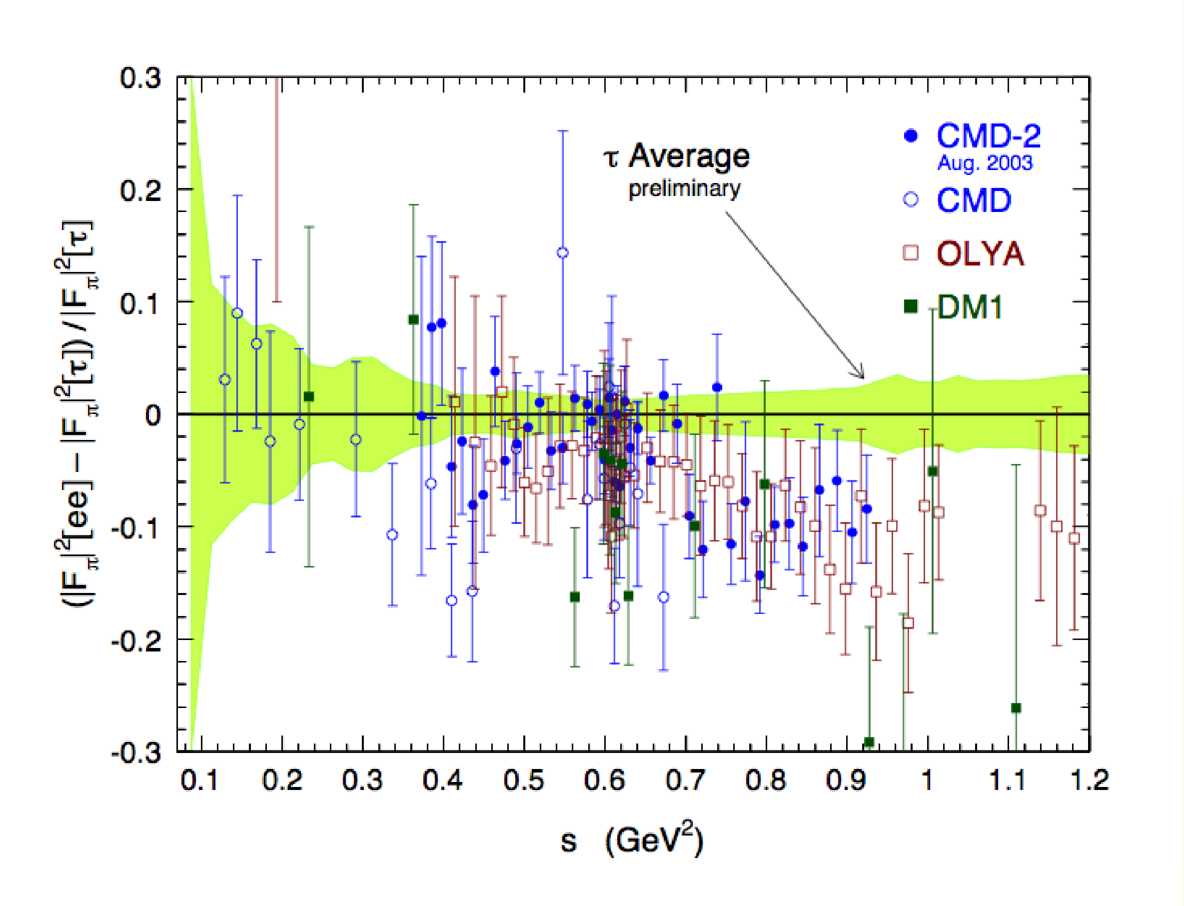
BELLE: best fit of $|F_\tau(s)|^2$ yields $F_\tau(0) = 1.02 \pm \pm 0.01 \pm 0.04$
$\Rightarrow$ this violates em current conservation. Benayoun et al. 2009 suggest that normalization may be wrong $\rightarrow$ shift down data by 2%; actually with global shift by -4.5 % perfect agreement with Novosibirsk e^+e^- data (as a distribution). Is the main problem that ALEPH lies very high ???

- Needed what is measured in e^+e^-: $|A_{I=1}(s) + A_{I=0}(s)|^2 \neq |A_{I=1}(s)|^2 + |A_{I=0}(s)|^2$;
- τ evaluations based on $|A^{\tau}_{I=1}(s)|^2 + |A^{e^+e^-}_{I=0}(s)|^2$ which may overestimate the effects; separation of $|A^{e^+e^-}_{I=0}(s)|^2$ using Gounaris-Sakurai fit of the $\rho - \omega$ [$\varepsilon_{\rho\omega} = (2.02 \pm 0.1) \times 10^{-3}$]; (see HLS model calculation by Benayoun et al. which claims large diminution by interference).
- hadronic final state photon radiation not under quantitative control, in τ–decay enhanced short distance sensitivity (UV-log modeled by quark model, rest by sQED)

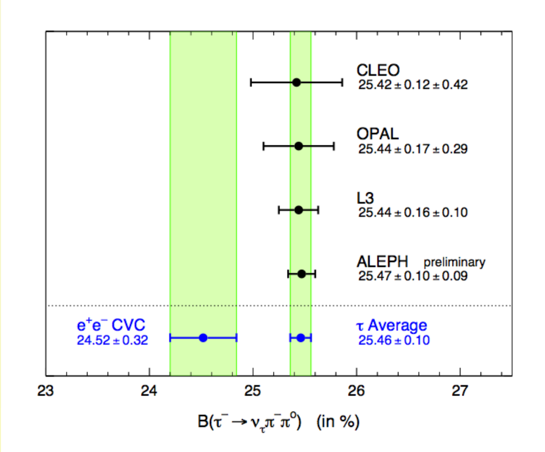


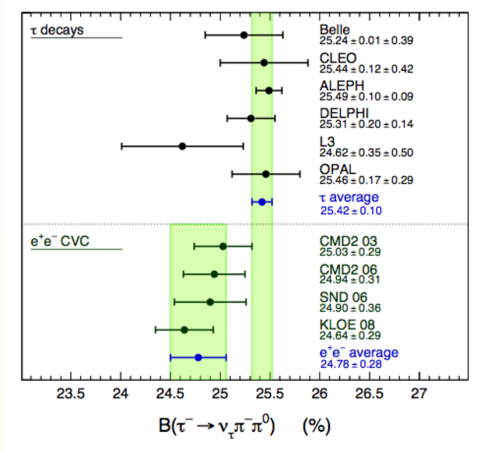
Relative comparison between the combined τ (dark shaded) and e^+e^- spectral functions (light shaded), normalized to the e^+e^- result.

M. Davier et al. 2009



τ Average
preliminary
CMD-2
Aug. 2003
CMD
OLYA
DM1
$(|F_\pi|^2[ee] - |F_\pi|^2[\tau]) / |F_\pi|^2[\tau]$
s (GeV^2)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2





Isovector (I=1) contribution to $a_\mu^{\rm had} \times 10^{10}$ from the range [0.592 - 0.975] GeV from selected experiments. First entry: results from τ-data after standard isospin breaking (IB) corrections. Second entry: results from τ-data after applying in addition the $\rho-\gamma$ mixing corrections $r_{\rho\gamma}(s)$, with fitted values for M_ρ, Γ_ρ and $\Gamma_{\rho ee}$ [$M_\rho = 775.65$ MeV$, \Gamma_\rho = 149.99$ MeV$, \mathcal{B}[(\rho \to ee)/(\rho \to \pi\pi)] = 4.10 \times 10^{-5}$]. For the $\rho-\omega$ mixing we subtracted 2.67×10^{-10}. Errors are statistical, systematic, isospin breaking and $\rho-\gamma$ mixing, assuming a 10% uncertainty for the latter. Final state radiation is not included.

Data	standard IB corrections	incl. $\rho-\gamma$ mixing
ALEPH 1997	390.75(2.69)(1.97)(1.45)	385.63(2.65)(1.94)(1.43)(0.50)
ALEPH 2005	388.74(4.05)(2.10)(1.45)	383.54(4.00)(2.07)(1.43)(0.50)
OPAL 1999	380.25(7.36)(5.13)(1.45)	375.39(7.27)(5.06)(1.43)(0.50)
CLEO 2000	391.59(4.16)(6.81)(1.45)	386.61(4.11)(6.72)(1.43)(0.50)
BELLE 2008	394.67(0.53)(3.66)(1.45)	389.62(0.53)(3.66)(1.43)(0.50)
average	391.06(1.42)(1.47)(1.45)	385.96(1.40)(1.45)(1.43)(0.50)
CMD-2 2006		386.34(2.26)(2.65)
SND 2006		383.99(1.40)(4.99)
KLOE 2008		380.24(0.34)(3.27)
KLOE 2010		377.35(0.71)(3.50)
BABAR 2009		389.35(0.37)(2.00)
average		385.12(0.87)(2.18)
all e^+e^- data		385.21(0.18)(1.54)
$e^+e^- + \tau$		385.42 (0.53)(1.21)

Calculated branching fractions in % from selected experiments. Experimental data completed down to threshold and up to m_τ by corresponding world averages where necessary. The experimental world average of direct branching fractions is $B^{\mathrm{CVC}}_{\pi\pi^0}$ = 25.51 ± 0.09 % .

τ data	$B_{\pi\pi^0}$[%]	e^+e^- data	$B^{\mathrm{CVC}}_{\pi\pi^0}$[%]
ALEPH 97	25.27 ± 0.17 ± 0.13	CMD-2 06	25.40 ± 0.21 ± 0.28
ALEPH 05	25.40 ± 0.10 ± 0.09	SND 06	25.09 ± 0.30 ± 0.28
OPAL 99	25.17 ± 0.17 ± 0.29	KLOE 08	24.82 ± 0.29 ± 0.28
CLEO 00	25.28 ± 0.12 ± 0.42	KLOE 10	24.65 ± 0.29 ± 0.28
Belle 08	25.40 ± 0.01 ± 0.39	BaBar 09	25.45 ± 0.18 ± 0.28
combined	25.34 ± 0.06 ± 0.08	combined	25.20 ± 0.17 ± 0.28

For the direct τ branching fractions the first error is statistical the second systematic. For e^+e^-+CVC the first error is experimental the second error includes uncertainties of the IB correction +0.06 from the new mixing effect. Remaining problems seem to be experimental.

$\rho - \omega$ mixing

In order to include the I=0 contribution form $\omega \to \pi^+\pi^-$ we need to consider the corresponding symmetric (γ, ρ, ω) 3×3 matrix propagator, with new entries $\Pi_{\gamma\omega}(q^2)$, $\Pi_{\rho\omega}(q^2)$ and $q^2 - M_\omega^2 + \Pi_{\omega\omega}(q^2)$, supplementing the inverse propagator matrix (1) by a 3rd row/column. Treating all off-diagonal elements as perturbations (after diagonalization) to linear order the new elements in the propagator read:

$$
\begin{aligned}
D_{\gamma\omega} &\simeq \frac{-\Pi_{\gamma\omega}(q^2)}{(q^2+\Pi_{\gamma\gamma}(q^2))\,(q^2-M_\omega^2+\Pi_{\omega\omega}(q^2))} \\
D_{\rho\omega} &\simeq \frac{-\Pi_{\rho\omega}(q^2)}{(q^2-M_\rho^2+\Pi_{\rho\rho}(q^2))\,(q^2-M_\omega^2+\Pi_{\omega\omega}(q^2))} \\
D_{\omega\omega} &\simeq \frac{1}{q^2-M_\omega^2+\Pi_{\omega\omega}(q^2)}\,.
\end{aligned}
$$

The self-energies again are the renormalized ones and in the two pion channel $e^+e^- \to \pi^+\pi^-$ given up to different coupling factors by the same self-energy functions as in the $\gamma - \rho$ sector. Thus, the bare self-energy functions read

$$\Pi_{\gamma\omega} = \frac{e g_{\omega\pi\pi}}{48\pi^2} f(q^2) \;, \quad \Pi_{\rho\omega} = \frac{g_{\rho\pi\pi} g_{\omega\pi\pi}}{48\pi^2} f(q^2) \quad \text{and} \quad \Pi_{\omega\omega} = \frac{g^2_{\omega\pi\pi}}{48\pi^2} f(q^2) \;,$$

and they are renormalized analogous to (1,1) subtracted at the ω mass shell. The $\rho - \omega$ mixing term is special here because if we diagonalize it on the ρ mass shell the matrix is no longer diagonal at the ω-resonance, where

$$\Pi^{\mathrm{ren}}_{\rho\omega}(q^2) = \Pi_{\rho\omega}(q^2) - \frac{q^2}{M_\rho^2} \,\mathrm{Re}\, \Pi_{\rho\omega}(M_\rho^2) \stackrel{q^2=M_\omega^2}{\rightarrow} \Pi_{\rho\omega}(M_\omega^2) - \frac{M_\omega^2}{M_\rho^2} \,\mathrm{Re}\, \Pi_{\rho\omega}(M_\rho^2) \neq 0 \;,$$

and which yields the leading I=0 contribution to the pion form factor[1]. The ω

[1]Typically, $\Pi^{\mathrm{ren}}_{\gamma\rho}(M_\omega^2) = \frac{e g_{\rho\pi\pi}}{48\pi^2} M_\omega^2 \left(h(M_\omega^2) - \mathrm{Re}\, h(M_\rho^2)\right)$ and $D_{\gamma\rho}(M_\omega^2) = -\frac{e g_{\rho\pi\pi}}{48\pi^2} \frac{\left(h(M_\omega^2) - \mathrm{Re}\, h(M_\rho^2)\right)}{M_\omega^2 - M_\rho^2 + \mathrm{i}\, M_\rho \Gamma_\rho}$. Similarly, $D_{\rho\rho}(M_\omega^2) = \frac{1}{M_\omega^2 - M_\rho^2 + \mathrm{i}\, M_\rho \Gamma_\rho}$ taking $\Gamma_\rho(M_\omega^2) \sim \Gamma_\rho$.

induced terms contribute to the pion form factor

$$\Delta F_\pi^{(\omega)}(s) = \left[e\ (g_{\omega\pi\pi} - g_{\omega ee})\ D_{\gamma\omega} - (g_{\rho ee}g_{\omega\pi\pi} + g_{\omega ee}g_{\rho\pi\pi})\ D_{\rho\omega}\right] / \left[e^2\ D_{\gamma\gamma}\right]\ ,$$

which adds to (**??**). The direct $e^+e^- \to \omega \to \pi\pi$ term given by $-g_{\omega\pi\pi}g_{\omega ee}\ D_{\omega\omega}$ by convention is taken into account as part of the complete ω-resonance contribution.

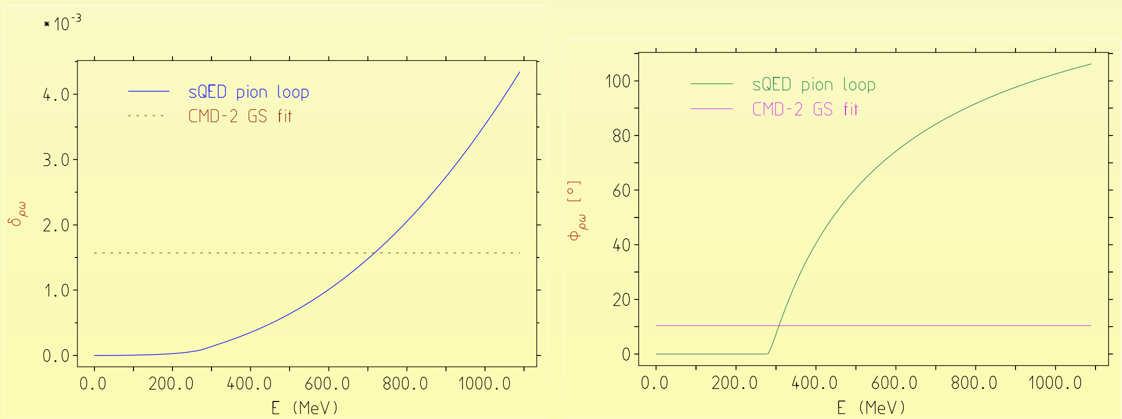


Figure 3: Dynamical mixing parameter $\delta(E)$ obtained in our EFT, in contrast to the approximation by a constant. The latter seems justified by the narrow width of the ω.

So far we have extended our effective Lagrangian by including direct $\rho-\omega$, $\gamma-\omega$,

$\omega\pi\pi$ and ωee vertices only, such that at the one-loop level only the previous pion loops show up. Missing are $\omega\pi^+\pi^-\pi^0$ and $\omega\pi^0\gamma$ effective vertices, which are necessary in order to obtain the correct full ω-width in place of the $\omega \to \pi\pi$ partial width only. Since the ω is very narrow we expect to obtain a good approximation if we use the proper full width in Im $\Pi_{\omega\omega} = \mathrm{i}\, M_\omega \Gamma_\omega(s)$, namely,

$$\Gamma_\omega \to \Gamma_\omega(s) \quad = \quad \sum_X \Gamma(\omega \to X, s) = \frac{s}{M_\omega^2}\Gamma_\omega \left\{ \sum_X \; Br(\omega \to X) \frac{F_X(s)}{F_X(M_\omega^2)} \right\},$$

where $Br(V \to X)$ denotes the branching fraction for the channel $X = 3\pi, \pi^0\gamma, 2\pi$ and $F_X(s)$ is the phase space function for the corresponding channel normalized such that $F_X(s) \to$ const for $s \to \infty$ [**?**].

If we include $\omega - \rho$ mixing in the usual way (see (**??**)) by writing

$$F_\pi(s) = \left[e^2 D_{\gamma\gamma} + e\,(g_{\rho\pi\pi} - g_{\rho ee})\, D_{\gamma\rho} - g_{\rho ee} g_{\rho\pi\pi}\, D_{\rho\rho} \cdot \left(1 + \delta \frac{s}{M_\rho^2} BW_\omega(s) \right) \right] / \left[e^2 D_{\gamma\gamma} \right] \; .$$

with $\mathrm{BW}_\omega(s) = -M_\omega^2/((s - M_\omega^2) + \mathrm{i}\, M_\omega \Gamma_\omega(s))$ in our approach $\delta_{\mathrm{eff}}(s)$ is given by

$$\delta_{\mathrm{eff}}(s) = \frac{(g_{\rho ee} g_{\omega\pi\pi} + g_{\omega ee} g_{\rho\pi\pi})\, D_{\rho\omega} - e\,(g_{\omega\pi\pi} - g_{\omega ee})\, D_{\gamma\rho}}{(g_{\rho\pi\pi} g_{\rho ee})\, D_{\rho\rho} \cdot \mathrm{BW}_\omega(s)}$$

which is well approximated by

$$\begin{aligned} \delta_{\mathrm{dyn}} \quad &= \quad -\frac{(g_{\rho ee} g_{\omega\pi\pi} + g_{\omega ee} g_{\rho\pi\pi})}{g_{\rho\pi\pi} g_{\rho ee}} \frac{\Pi^{\mathrm{ren}}_{\rho\omega}(s)}{M_\omega^2} \\ &\overset{s \sim M_\omega^2}{\sim} \quad -\frac{(g_{\rho ee} g_{\omega\pi\pi} + g_{\omega ee} g_{\rho\pi\pi})}{g_{\rho\pi\pi} g_{\rho ee}} \frac{g_{\rho\pi\pi} g_{\omega\pi\pi}}{48\pi^2} \left(h(M_\omega^2) - \mathrm{Re}\, h(M_\rho^2) \right) \; . \end{aligned}$$

The second term $g_{\omega ee} g_{\rho\pi\pi} \sim 0.03$ is an order of magnitude larger than than the first one $g_{\rho ee} g_{\omega\pi\pi} \sim 0.003$ and thus is sensitive to $g_{\omega ee}$ once the $g_{\rho\pi\pi}$ has been fixed in the ρ-sector. In leading approximation $\delta \propto g_{\omega ee}/g_{\rho ee} \cdot g_{\rho\pi\pi} g_{\omega\pi\pi}$. The phase is actually fixed by the pion loop alone as we take couplings to be real (unitarity). We have $|\delta| = 1.945 \times 10^{-3}$ and $\phi_\delta = 90.49°$.

A complete EFT treatment of the $\rho - \omega$ mixing, as well as the proper inclusion of the higher ρ's, requires the extension of our model, e.g. in the HLS version as performed in [**?**, **?**]. This is beyond the scope of the present study. Nevertheless, the discussion of the $\rho - \omega$ mixing presented above illustrates the need for a reconsideration of the subject.