What can data provide for HLbL ?

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INT Workshop on
The Hadronic Light-by-Light Contribution to the Muon Anomaly,
Seattle, February 28 - March 4, 2011

Abstract

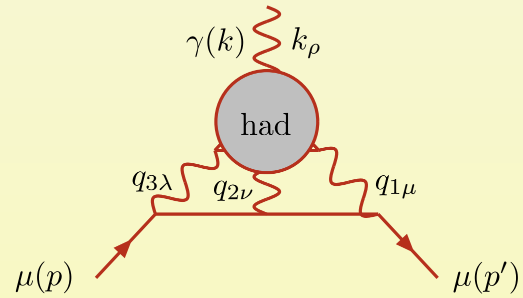
Experimental data form $\gamma\gamma \to$ hadrons, $\gamma\gamma^* \to$ hadrons, $\gamma^*\gamma^* \to$ hadrons, are giving more and more constraints to hadronic $\gamma\gamma^* \to \gamma^*\gamma^*$ amplitudes. Status, problems and possibilities are discussed.

Outline of Talk:

- The hadronic LbL: setup and problems
- Pseudoscalar exchanges: π^0, η, η'
- Axial exchanges: a_1, f_1', f_1
- Scalar exchanges: $a_0, f_0', f_0, \cdots$
- Present & Future

The hadronic LbL: setup and problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



Key object full rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int d^4x_1\, d^4x_2\, d^4x_3\, e^{i\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\, 0\,|\,T\{j_\mu(x_1)\,j_\nu(x_2)\,j_\lambda(x_3)\,j_\rho(0)\}\,|\,0\,\rangle\;.$$

- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- 32 can contribute to $g-2$

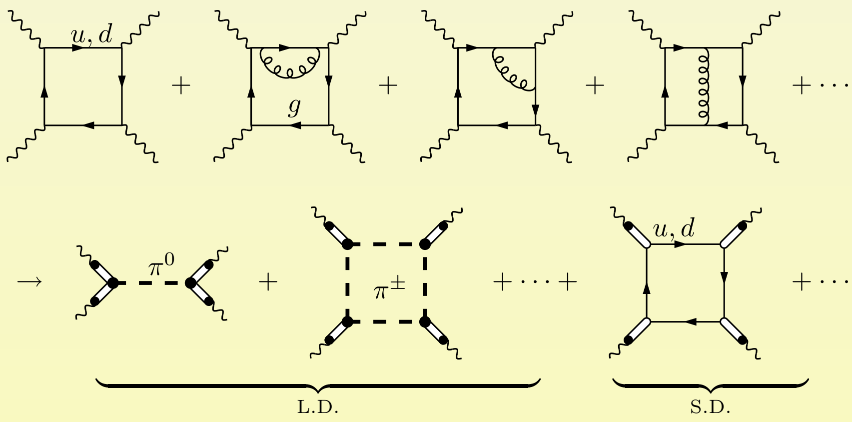
- fortunately, dominated by the pseudoscalar exchanges $\pi^0, \eta, \eta', ...$ described by the effective **Wess-Zumino Lagrangian**

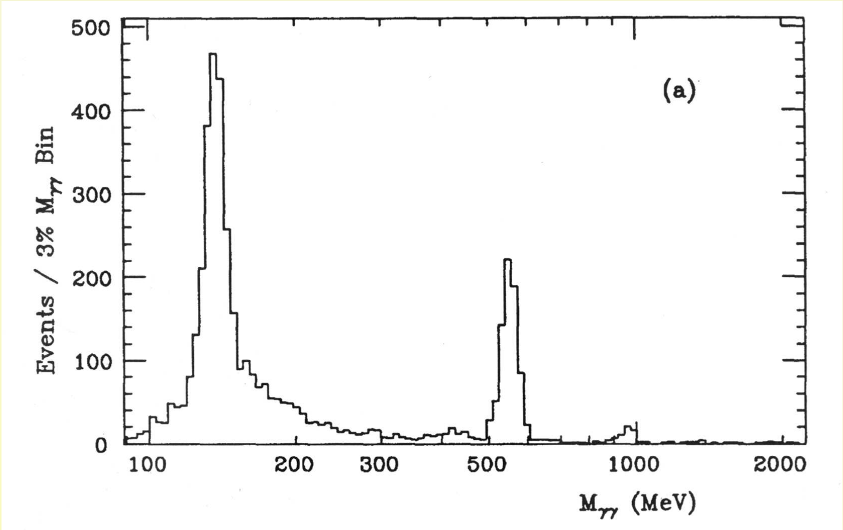
- generally, pQCD useful to evaluate the short distance (S.D.) tail

- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N_c inspired ansätze, and others

Need appropriate low energy effective theory ⇒amount to calculate the following type diagrams

Crystal Ball 1988

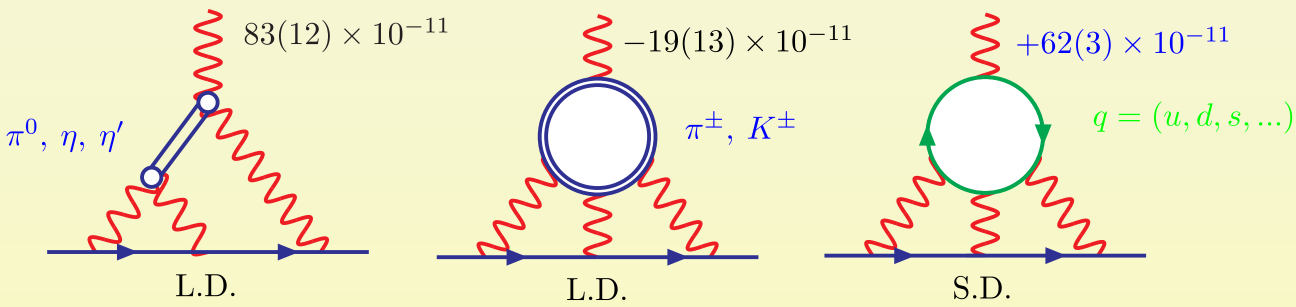




Data show almost background free spikes of the PS mesons! Substantial background form quark loop is absent (seems to contradict large quark-loop contribution as obtained in SDA). Clear message from data: fully non-perturbative, evidence for PS dominance. However, no information about axial mesons (Landau-Yang theorem). Illustrates how data can tell us where we are.

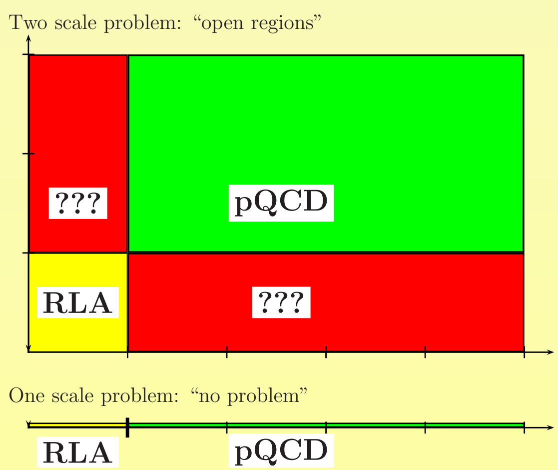
Low energy expansion in terms of hadronic components: theoretical models vs experimental data

➠ KLOE, KEDR, BES, BaBar, Belle, ?



LD contribution requires low energy effective hadronic models: simplest case $\pi^0\gamma\gamma$ vertex

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



??? – Data, OPE,
– QCD factorization,
– Brodsky-Lepage approach

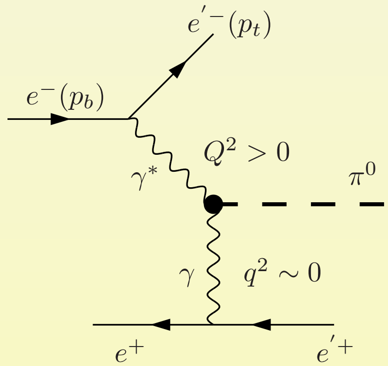
Novel approach: refer to quark–hadron duality of large-N_c QCD, hadron spectrum known, infinite series of narrow spin 1 resonances 't Hooft 79 $\Rightarrow$no matching problem (resonance representation has to match quark level representation) De Rafael 94, Knecht, Nyffeler 02

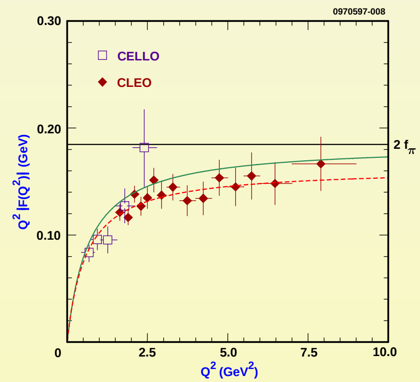
Constraints for on-shell pions (pion pole approximation)

- General form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$ is largely unknown

- The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025$ GeV^{-1} well determined by $\pi^0 \to \gamma\gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

- Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ from $e^+e^- \to e^+e^-\pi^0$ experiments





CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2. outdated now by BaBar?

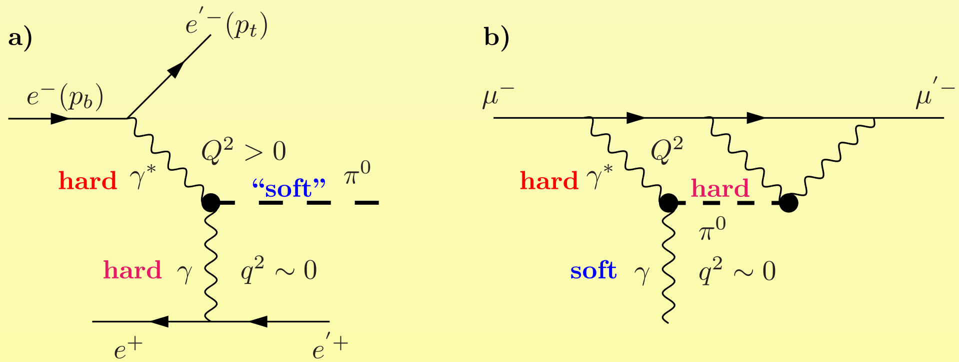
Brodsky–Lepage interpolating formula gives an acceptable fit.

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0) \simeq \frac{1}{4\pi^2 f_\pi} \frac{1}{1 + (Q^2/8\pi^2 f_\pi^2)} \sim \frac{2 f_\pi}{Q^2}$$

Inspired by pion pole dominance idea this FF has been used mostly (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

❒ Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$, which avoids eventual kinematic inconsistency, thus no VMD damping ⇒result increases by 30% !

❒ In $g-2$ external photon at zero momentum ⇒ only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ not $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply!. Obsolete far off-shell pion (in space-like region).



Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2, needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$.

- I still claim using $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0,0,0)$ in this case is not a good approximation!

Need realistic "model" for off–shell form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$!

Is it really to be identified with $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0,0,0)$?

Can we check such questions experimentally or in lattice QCD?

Evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework

- Knecht & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-N_c $\pi^0\gamma\gamma$–form-factor

- FJ & A. Nyffeler: relax from pole approximation, using KN off-shell LDM+V form-factor

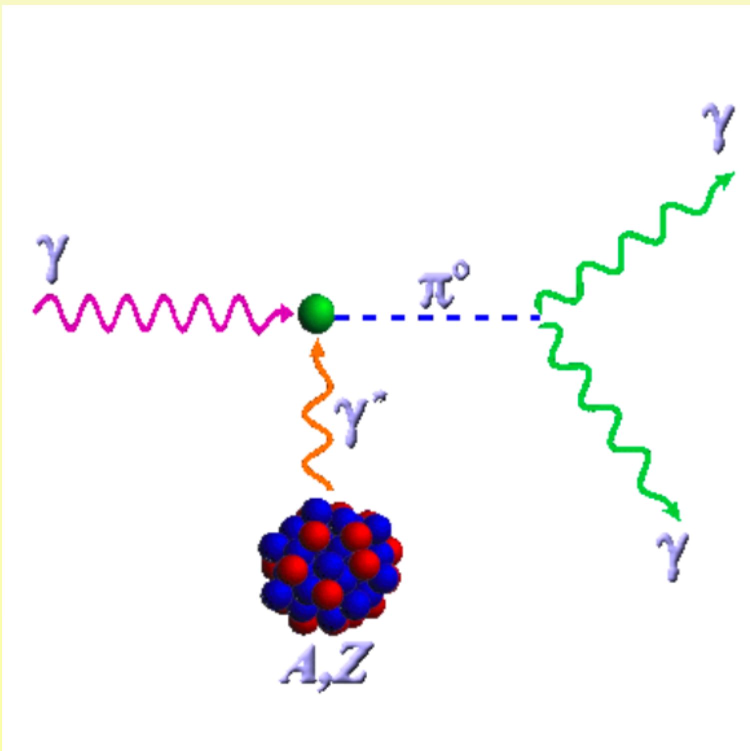
$$\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2,q_1^2,q_2^2) &= \frac{F_\pi}{3}\frac{\mathcal{P}(q_1^2,q_2^2,p_\pi^2)}{\mathcal{Q}(q_1^2,q_2^2)} \\
\mathcal{P}(q_1^2,q_2^2,p_\pi^2) &= h_7 + h_6\,p_\pi^2 + h_5\,(q_2^2+q_1^2) + h_4\,p_\pi^4 + h_3\,(q_2^2+q_1^2)\,p_\pi^2 \\
&\quad + h_2\,q_1^2\,q_2^2 + h_1\,(q_2^2+q_1^2)^2 + q_1^2\,q_2^2\,(p_\pi^2+q_2^2+q_1^2)) \\
\mathcal{Q}(q_1^2,q_2^2) &= (q_1^2-M_1^2)\,(q_1^2-M_2^2)\,(q_2^2-M_1^2)\,(q_2^2-M_2^2)
\end{aligned}$$

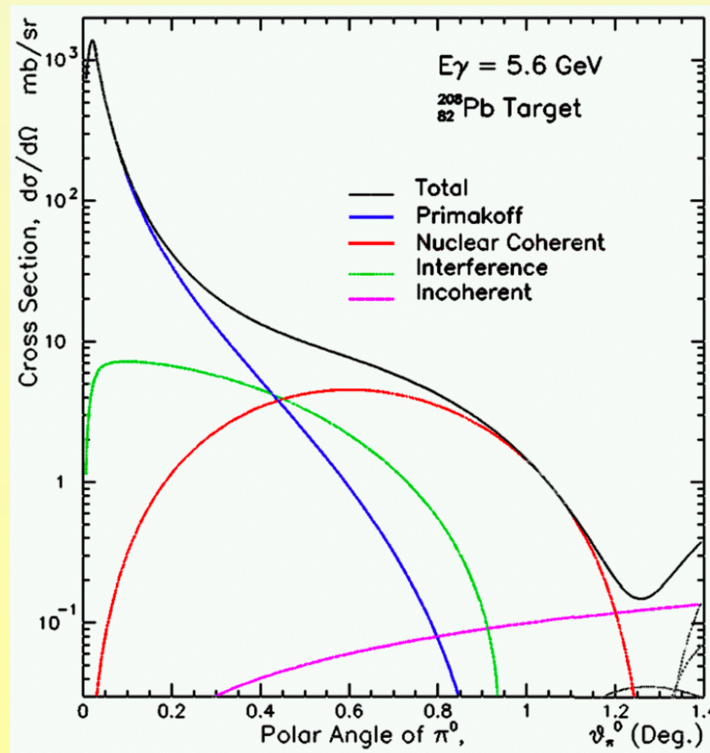
all constants are constraint by SD expansion (OPE). Again, need data to fix parameters! Looking for new ideas to get ride of model dependence

- ❒ Need better constrained effective resonance Lagrangian (e.g. HSL and ENJL models vs. RLA of Ecker et al.). “Global effort” needed!
- ❒ Lattice QCD will provide an answer [take time (“yellow” region only?)]!
- ❒ Try exploiting possible new experimental constraints:

$\pi^0\gamma\gamma$ form-factor: experimental facts and possibilities

- time-like ($q_\pi^2 > 0$) phenomenology (single tag data) versus space-like ($q_\pi^2 < 0$) phenomenology poorly investigated, Primakoff-effect (π^0 production by high energetic photons in Coulomb field of atomic nuclei) PRIMEX JLAB experiment





- relation between the off-shell (needed for a_μ) and the on-shell (measured) from-factor is not a priory clear

Note: $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ is a one-scale problem. Self-energy type of problem $\Rightarrow$ can get via dispersion relation from appropriate data

Existing data for $F(m_\pi^2,Q^2,0)$: $e^+e^- \to e^+e^-\pi^0$ single tag data $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}$

➠CELLO: 0.5 GeV$^2 < Q^2 < 2.17$ GeV^2 [Z. Phys. C49 (1991) 401]

➠CLEO: 1.5 GeV$^2 < Q^2 < 9$ GeV^2 [Phys. Rev. D57 (1998) 33]

➠BaBar: 4 GeV$^2 < t_2 < 40$ GeV^2 [Phys. Rev. D80 (2009) 052002]

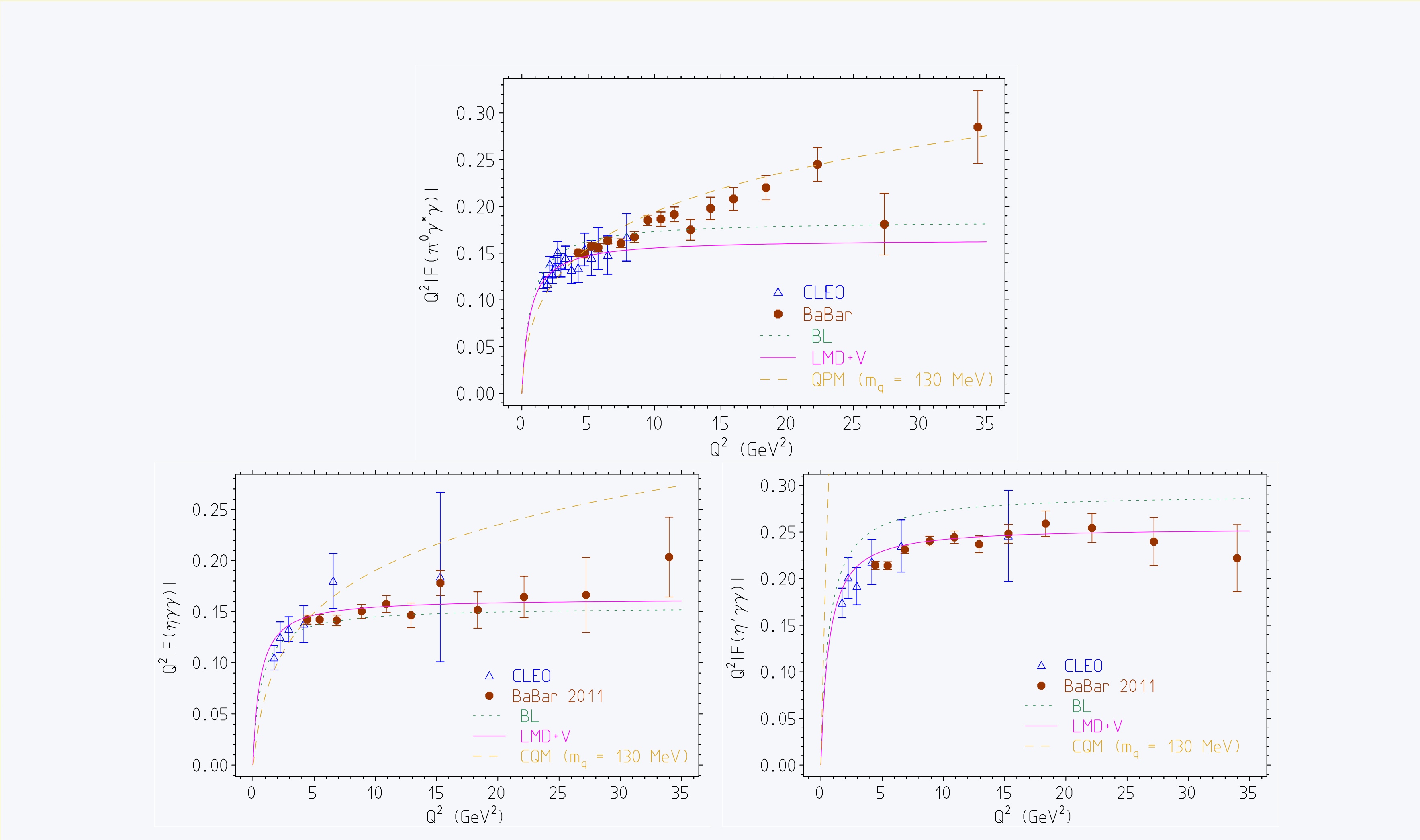
➠ new quest for theory

- before BaBar: consensus about large Q^2 behavior; π^0, η and η' consistent

➠Brodsky-Lepage (BL) $\sim 1/Q^2$

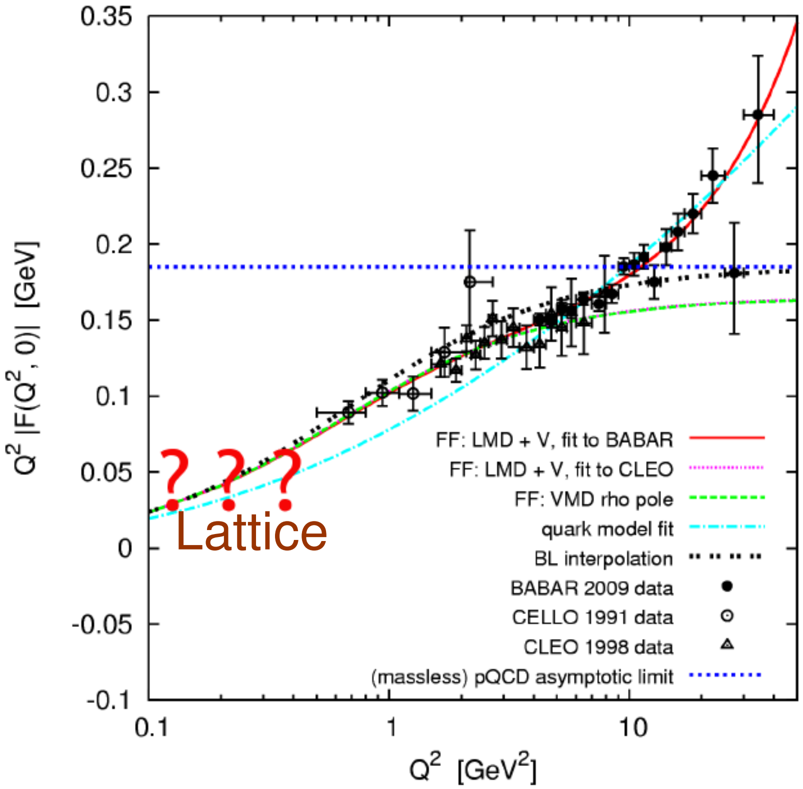
- with BaBar: goes to higher $Q^2 \to$ violating Brodsky-Lepage behavior

- BaBar: π^0, η and η' not consistent in the sense: expect same behavior for all pseudoscalars



- asymptotic behavior is not understood ??? data consistent ???

$Q^2 F_{\pi^0\gamma^*\gamma}(m_\pi^2, Q^2, 0)$



Theory:

[A. Nyffeler, 0912.1441]

[M. Knecht and A. Nyffeler,

Phys. Rev. D**65**, 073034 (2002)]

[ibid.]

[A. E. Dorokhov, 0905.4577]

[G. P. Lepage and S. J. Brodsky,

Phys. Rev. D **22**, 2157 (1980)]

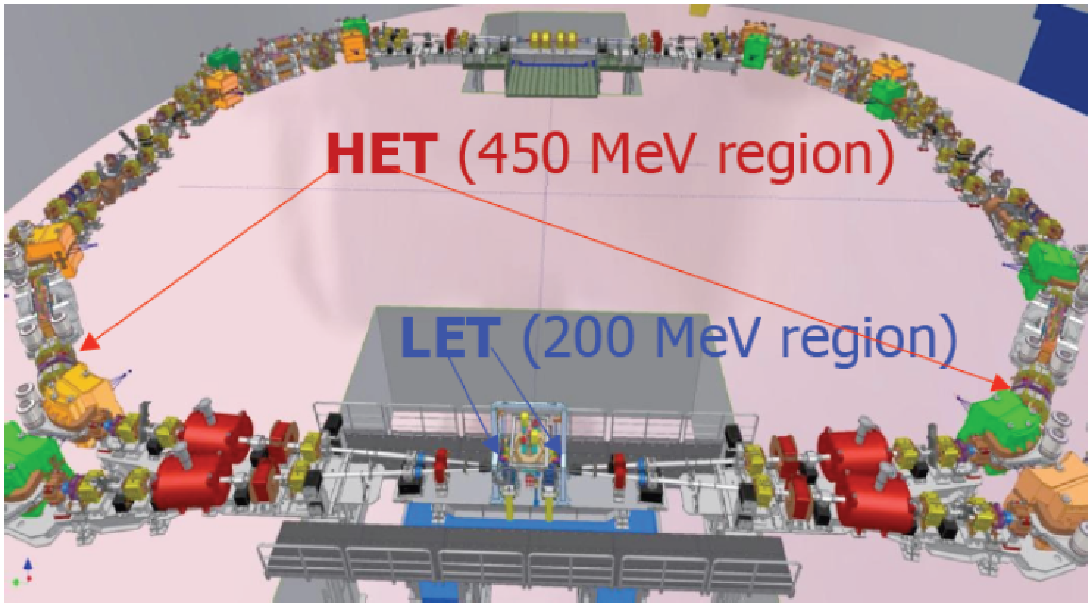
No data at $0.02\ \mathrm{GeV}^2 < Q^2 < 0.4\ \mathrm{GeV}^2$

Sergiy IVASHYN (Katowice, Kharkov) | $\pi^0\gamma\gamma$ | 21 / VI / 2010 @ Mainz | 22 / 66

Lattice (Shoij Hashimoto)

$\gamma^*\gamma^*\pi^0$ at KLOE-2

KLOE-2 experiment (PROJECT)



The $\phi(1020)$ meson factory DAΦNE (Frascati)
+ KLOE detector + small angle taggers

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MC Simulation with EKHARA Generator

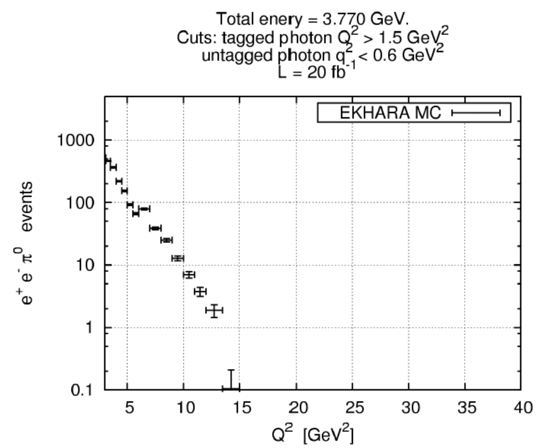
H. Czyż, S. Ivashyn [http://prac.us.edu.pl/ẽkhara]

Tagging:

- single tagging LET: tagged invariant t_1 close to zero, promising range $0.05\ \text{GeV}^2 < t_2 < 0.4\ \text{GeV}^2$
- LET-LET and LET-HET double tagging is not possible
- LET + central: promising range $0.18\ \text{GeV}^2 < t_2 < 0.4\ \text{GeV}^2$
- single tagging HET: tagged invariant t_1 close to zero $\Rightarrow$ t_2 also close to zero
- HET-HET double tagging is possible but both photons quasi-real $\Rightarrow$ good for measurement of $\pi^0 \to \gamma\gamma$ width, pion practically at rest

Single-tag measurement at BES-III

e.g. for extraction of the form-factor $F(m_\pi^2, Q^2, 0)$



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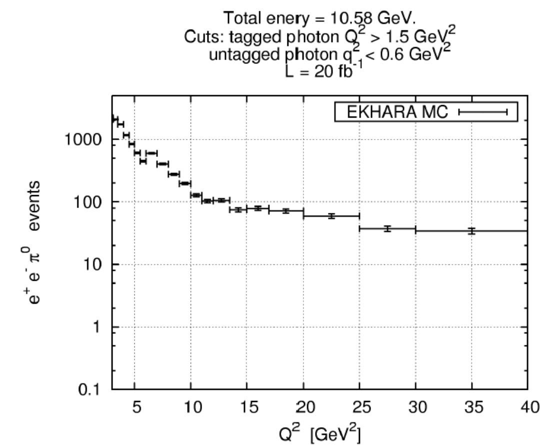
cf. Single-tagging at BaBar energy

for illustration: the same integrated luminosity



Sergiy IVASHYN (Katowice, Kharkov) $\pi^0\gamma\gamma$ 21 / VI / 2010 @ Mainz 53 / 66

Cross check of BaBar only possible by *Belle*!

Expected contribution from PS mesons:

$$a_\mu[\pi^0, \eta, \eta'] \sim (93.91 \pm 12.40) \times 10^{-11}$$

Axial exchanges: a_1, f_1', f_1

Axial exchanges

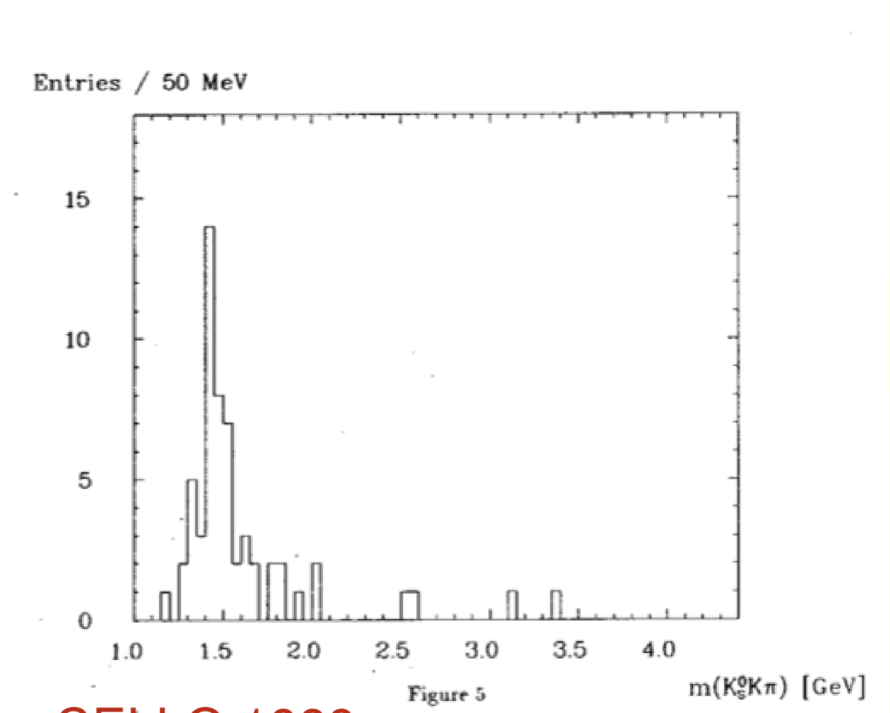
Landau-Yang Theorem: $\mathcal{A}$ (axial meson $\to \gamma\gamma$)=0

e.g. $Z^0 \not\to \gamma\gamma$, while $Z^0 \to \gamma e^+ e^-$ ✓

Why $a_\mu[a_1, f_1', f_1] \sim 25 \times 10^{-11}$ so large?

- untagged $\gamma\gamma \to f()$ no signal!
- single-tag $\gamma^*\gamma \to f()$ strong peak is $Q^2 \gg m_f^2$

$\sigma(\gamma^*\gamma \to f_1 \to K_s^0 K\pi)$



CELLO 1989

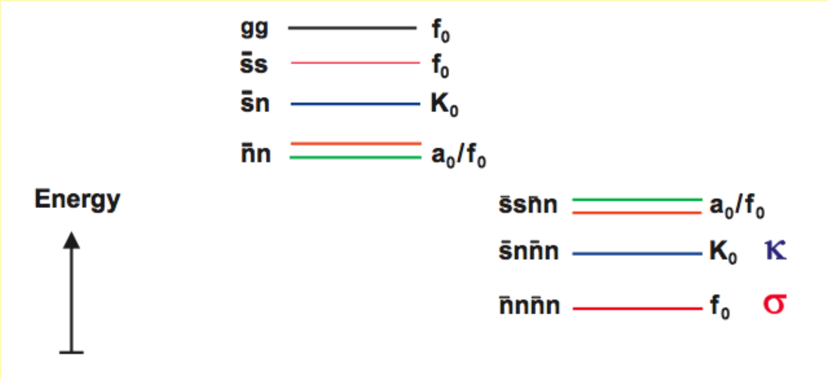
Sparse data so far, new measurements important; in particular momentum dependent $\Gamma(a_1 \to \gamma\gamma^*)$ etc.

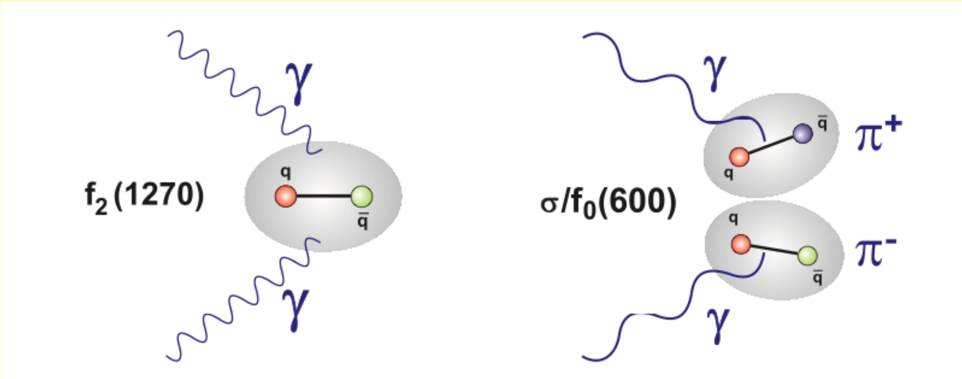
Expected contribution from axial mesons:

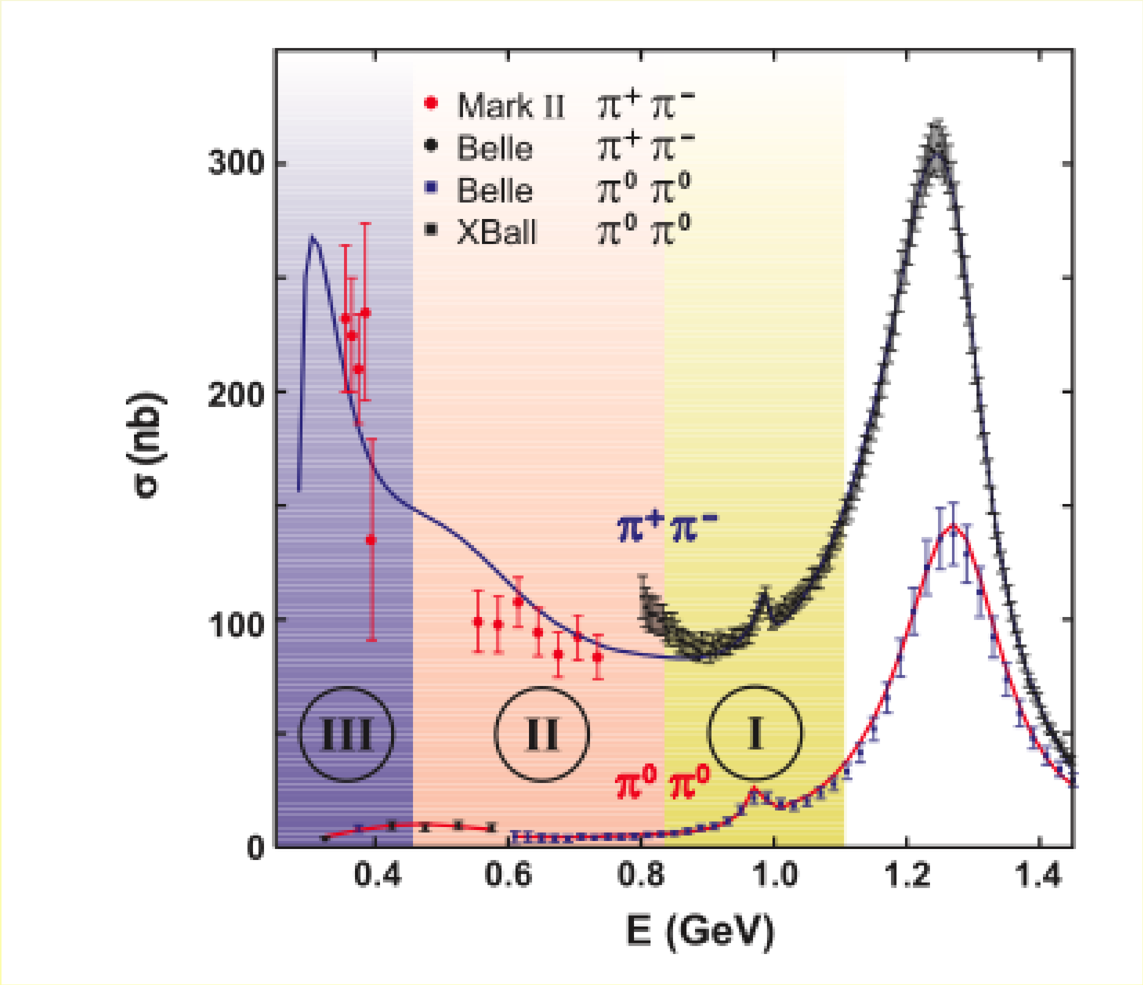
$$a_\mu[a_1, f_1', f_1] \sim (28.13 \pm 5.63) \times 10^{-11}$$

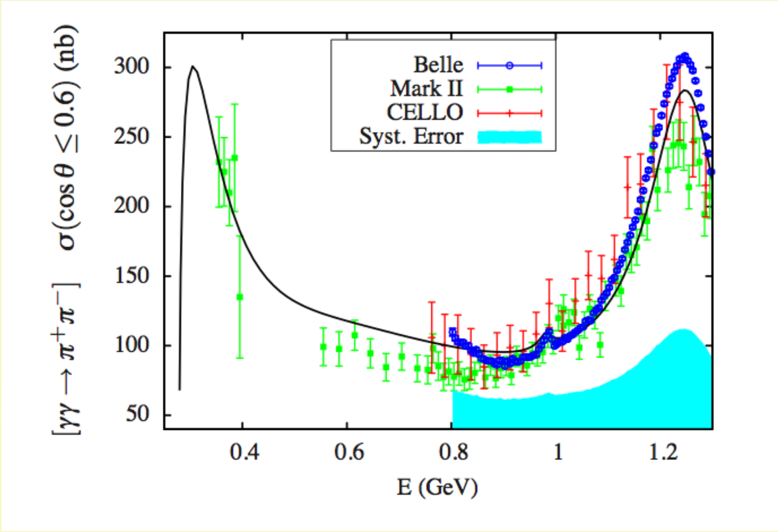
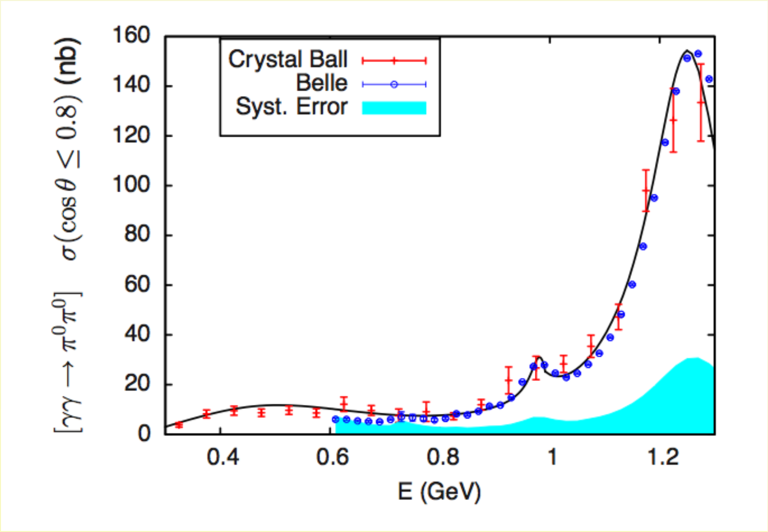
Scalar exchanges: $a_0, f_0', f_0, \cdots$

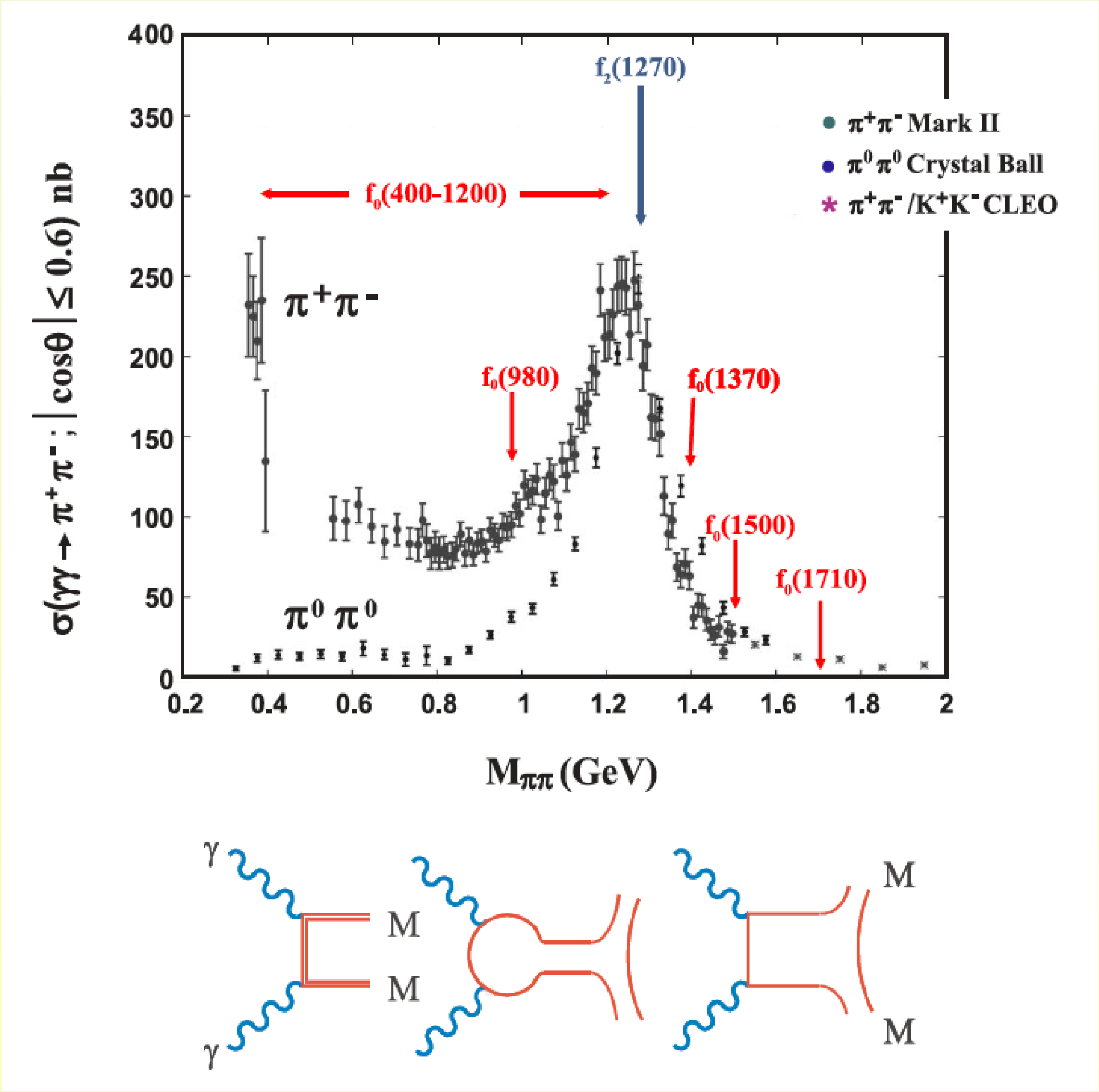
Mesons: $M(q\bar{q})$, $M(qq\bar{q}\bar{q})$, glueballs mixing
Experimental: Crystal Ball, Mark II, Belle!
Theory: Mennessier, Pennington et al., Mousallam et al., Achasov et al., ...



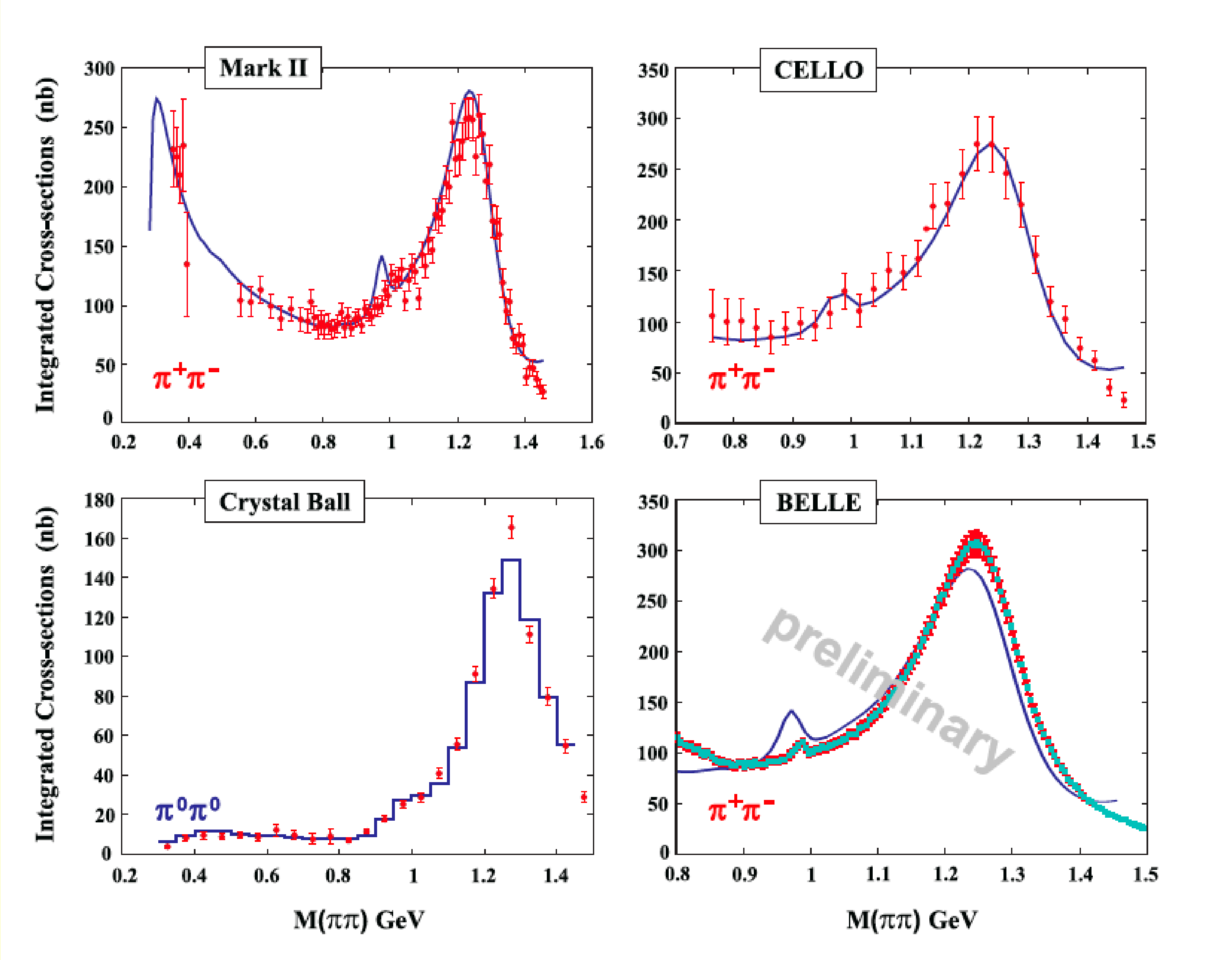



Mark II π+ π-
Belle π+ π-
Belle π0 π0
XBall π0 π0
300
200
100
0
σ (nb)
0.4
0.6
0.8
1.0
1.2
1.4
E (GeV)
π+π-
π0 π0
III
II
I


Belle
Mark II
CELLO
Syst. Error
$[\gamma\gamma \to \pi^+\pi^-]$ $\sigma(\cos\theta \leq 0.6)$ (nb)
E (GeV)

Crystal Ball
Belle
Syst. Error
$[\gamma\gamma \to \pi^0\pi^0]$ $\sigma(\cos\theta \leq 0.8)$ (nb)
E (GeV)



f₂(1270)
f₀(400-1200)
f₀(980)
f₀(1370)
f₀(1500)
f₀(1710)
π⁺π⁻ Mark II
π⁰π⁰ Crystal Ball
π⁺π⁻/K⁺K⁻ CLEO
π⁺π⁻
π⁰ π⁰
σ(γγ → π⁺π⁻; |cosθ| ≤ 0.6) nb
Mππ (GeV)
γ
γ
M
M
M
M


Mark II
CELLO
Crystal Ball
BELLE
Integrated Cross-sections (nb)
$\pi^+\pi^-$
$\pi^0\pi^0$
preliminary
M(ππ) GeV

Strong tensor meson resonance in $\pi\pi$ channel $f_2(1270)$

So: expect usual pion-loop in HLbL plays role like pion-loop in VP. i.e. like missing the ρ.
➠ Need to explicitly include tensor mesons

The di-pion amplitude $M_{\mathrm{res}}^{\mathrm{direct}}(\gamma\gamma \to \pi^+\pi^-; s)$ gets contribution caused by mixed $\sigma(600)$ and $f_0(980)$ resonances with the direct coupling constants of the $\sigma(600)$ and $f_0(980)$ to photons, $g^{(0)}_{\sigma\gamma\gamma}$ and $g^{(0)}_{f_0\gamma\gamma}$,

$$M_{\mathrm{res}}^{\mathrm{direct}}(\gamma\gamma \to \pi^+\pi^-; s) = s\, e^{i\delta_B^{\pi\pi}(s)}$$

$$\times \frac{g^{(0)}_{\sigma\gamma\gamma}[D_{f_0}(s) g_{\sigma\pi^+\pi^-} + \Pi_{f_0\sigma}(s) g_{f_0\pi^+\pi^-}] + g^{(0)}_{f_0\gamma\gamma}[D_\sigma(s) g_{f_0\pi^+\pi^-} + \Pi_{f_0\sigma}(s) g_{\sigma\pi^+\pi^-}]}{D_\sigma(s) D_{f_0}(s) - \Pi^2_{f_0\sigma}(s)}\,.$$

For $\sqrt{s} < 2m_K$, the phase coincides with the I=0, S wave $\pi\pi$ phase shift $\delta_0^0(s) = \delta_B^{\pi\pi}(s) + \delta_{\mathrm{res}}(s)$.

Scalars everywhere. Many scalars many small contributions may sum up to substantial effect!

Expected contribution from $q\bar{q}$ scalars:

$$a_\mu[a_0, f_0', f_0] \sim (-5.98 \pm 1.20) \times 10^{-11}$$

So far nobody has evaluated $qq\bar{q}\bar{q}$ in $SU(3)$ sector [u, d, s] many possible states, which individually are expected rather small

Present & Future

Details given by following talks:

Dario Morriciani	KLOE small angle tagger (low energy $\pi^0\gamma\gamma$)
Achim Denig	BaBar and BES results and plans
Simeon Eidelman	Belle and KEDR results and plans
Henryk Czyż	EKHARA a Monte Carlo for $\gamma^*\gamma^*$ physics

- $\gamma\gamma, \gamma^*\gamma$ and $\gamma^*\gamma^*$ physics a mandatory input for constraining hadronic LbL amplitudes.

❒ Need improved $Hadron \to \gamma\gamma$ measurements for π^0, η, η' as well for axial and scalar mesons

❒ Single tag form factors very much improvement to come for pseudoscalars

❒ Double tag form factors: experimentally not simple, requires very high luminosity

❒ $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0, K^+K^-, K_LK_S, \cdots$ important input for scalar sector and pion and kaon loops

- Challenge for theory: radiative corrections needed

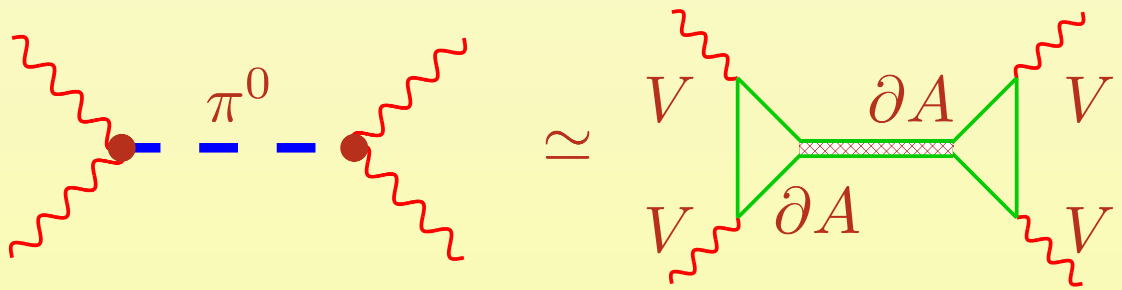
- Question of asymptotic behavior seen by BaBar, will likely be settled by Belle

- Can we check controversial dressed/undressed (i.e damping or not?) at external vertex? Can Primakoff-effect plus DR help?

Will learn more from the experts now.

Be damped or not to be

A counter example to Arcady's non-dressing theorem on external HLbL $\pi^0\gamma\gamma$ vertex.:
Forget about short distance expansion and look at physics. Assume a one pion exchange [experimental evidence] and look at valence quark structure:



Can use bare PCAC $\partial A(x) = \mathrm{i}\, m_0 \pi^0(x)$. In real world quarks carry mass. $\langle m_0 \pi^0 A A \rangle$ exhibits correct WZW effective behavior [pQCD as well as lattice QCD (Hashimoto)].

❒ ABJ anomaly seen as IR effect, while in axial current it appears as an UV effect (is conformal i.e. at any scale object).

❒ non-commuting singular limit

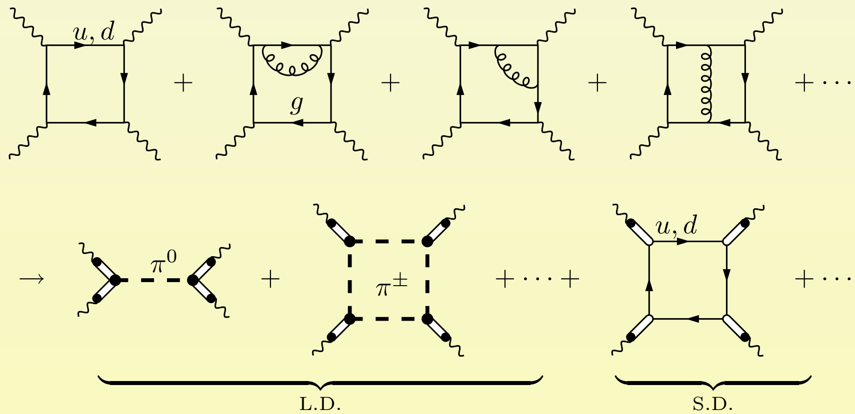
CQM behavior:

$$F^{\rm CQM}_{\pi^0\gamma^*\gamma}(0,p_1^2,0) \sim r\;\ln^2 r\;,\quad F^{\rm CQM}_{\pi^0\gamma^*\gamma^*}(0,p_1^2,p_1^2) \sim 2\,r\;\ln r$$

where $r=\frac{m_q^2}{-p_1^2}$ (and permutations).

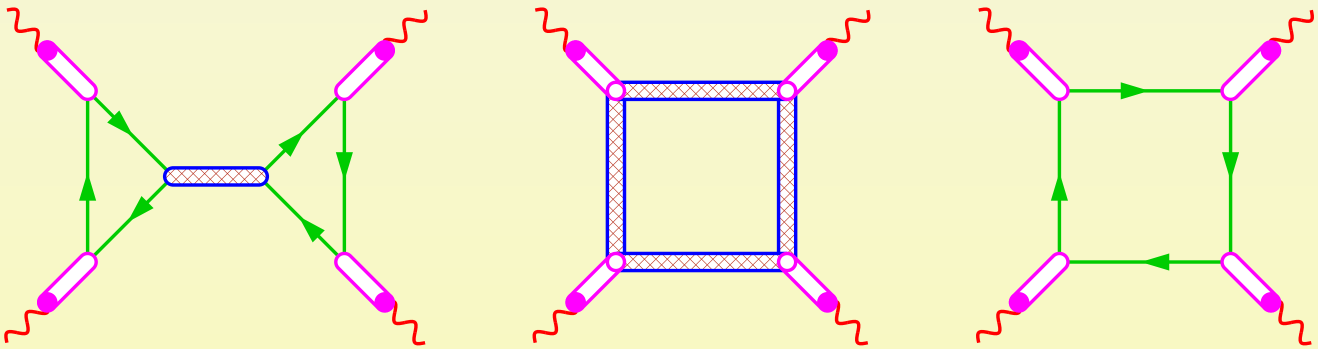
Note: not $\sim 1+O(m_q^2/Q^2)$ but $\sim O(m_q^2/Q^2)$, beyond pQCD $m_q \to M_{\rm eff}$ screening the anomaly! (same as Brodsky-Lepage derived in QCD via OPE) [BL $\Leftrightarrow$ CQM: $M_q^2=24\pi^2\,f_\pi^2/N_c$]

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}$$



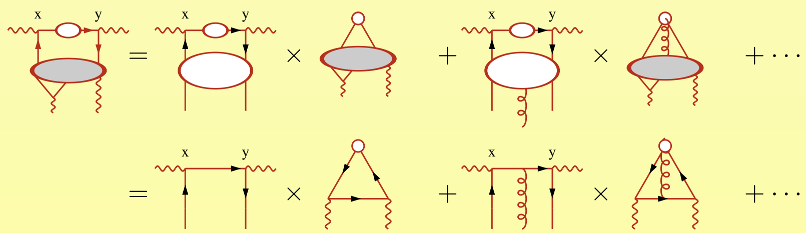
$\langle VVVV \rangle$ has non-trivial radiative corrections

Hint ENJL model:



$\langle AAAA\rangle$ exhibits radiative correction and even in strong coupling regime (non-perturbative). One cannot get rid of these by performing OPE in some place.

Look at OPE:



Adler-Bardeen theorem does **not** imply that there are no higher order corrections!

Also Adler-Bardeen theorem holds for renormalized axial current, e.g.

$$(J^5_\rho)_r = Z_5 Z_{\overline{\mathrm{MS}}}\,(J^5_\rho)_0 .$$

At two-loops

$$Z_5 = 1 - 4C_2(R)\,\frac{\alpha_s}{4\pi}\;,$$

where α_s is the QCD coupling and $C_2(R) = 4/3$ for QCD.

In OPE $C_W\,(J^5_\rho)_0 \to C_W Z_5^{-1}\,(J^5_\rho)_r$, what matters is renormalized Wilson coefficient $C_W \to C_W + \text{ direct correction } + C_W\,(Z_5^{-1} - 1)$.

A virtual photon attached to a quark line cannot know that it should dress or not depending on whether the quark line belongs to an axial triangle (i.e. somewhere else). Note that a VMD dressing is a multiplicative factor multiplying the bare loop. Such multiplicative factors are not excluded by the Adler-Bardeen theorem.

Corrections must be there! Anyway try to check by data and/or lattice QCD!