Hadronic Vacuum Polarization Contribution to g-2 from the Lattice

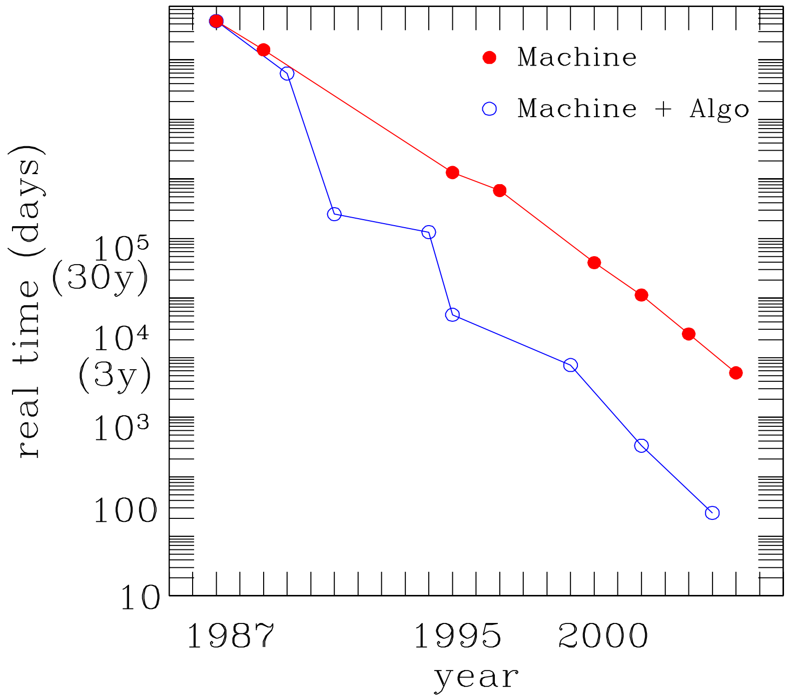
Karl Jansen

Xu Feng, Marcus Petschlies, Dru Renner

- **Understanding hadronic contributions to $g_\mu - 2$ from the lattice**
  (necessary first step)
  - systematics: finite volume, non-zero lattice spacing
  - dis-connected contribution
  - improved observables
- **Outlook: going further**
  - include strange and charm in simulations
  - attack light-by-light scattering

Computer and algorithm development over the years

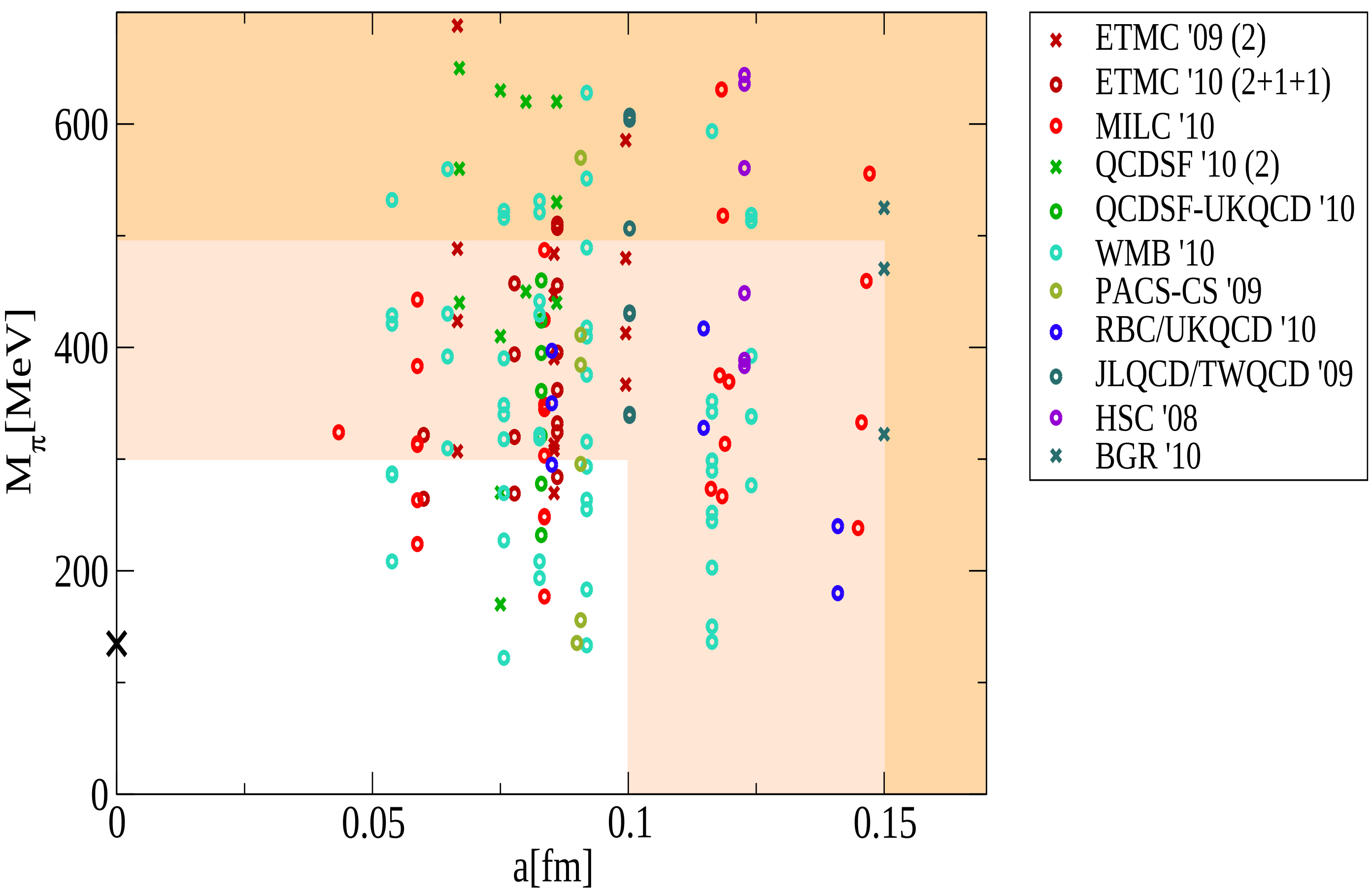
time estimates for simulating $32^3 \cdot 64$ lattice, 5000 configurations



$\rightarrow$ O(few months) nowadays with a typical collaboration supercomputer contingent

Todays landscape of lattice simulations worldwide

(from C. Hoelbling, Lattice 2010)



Our fermion discretization: twisted mass fermions

$$D_{\mathrm{tm}} = m_q + i\mu\tau_3\gamma_5 + \frac{1}{2}\gamma_\mu\left[\nabla_\mu + \nabla_\mu^*\right] - a\frac{1}{2}\nabla_\mu^*\nabla_\mu$$

quark mass parameter m_q , twisted mass parameter μ

- $m_q = m_{\mathrm{crit}} \leftrightarrow m_{\mathrm{PCAC}} = 0 \to$ O(a) improvement for *hadron masses, matrix elements, form factors, decay constants, $\cdots$, physical quantities*

  $\star$ **based on symmetry arguments**

- no need for further operator specific improvement coefficients

- $\det[D_{\mathrm{tm}}] = \det[D^2_{\mathrm{Wilson}} + \mu^2]$
  $\Rightarrow$ protection against small eigenvalues

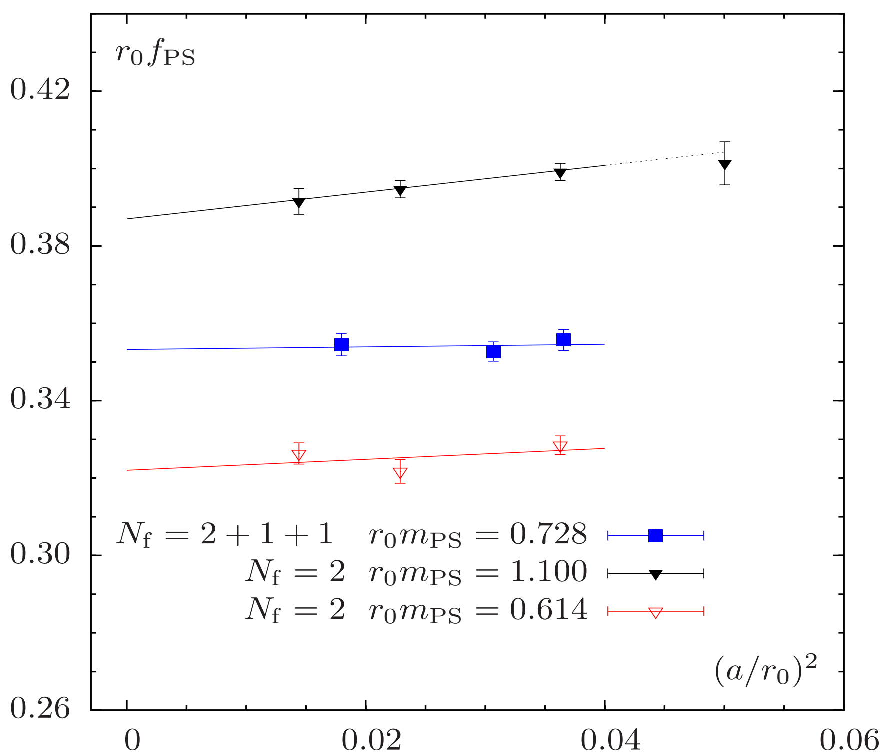
- computational cost comparable to Wilson

- expected to simplify mixing problems for renormalization

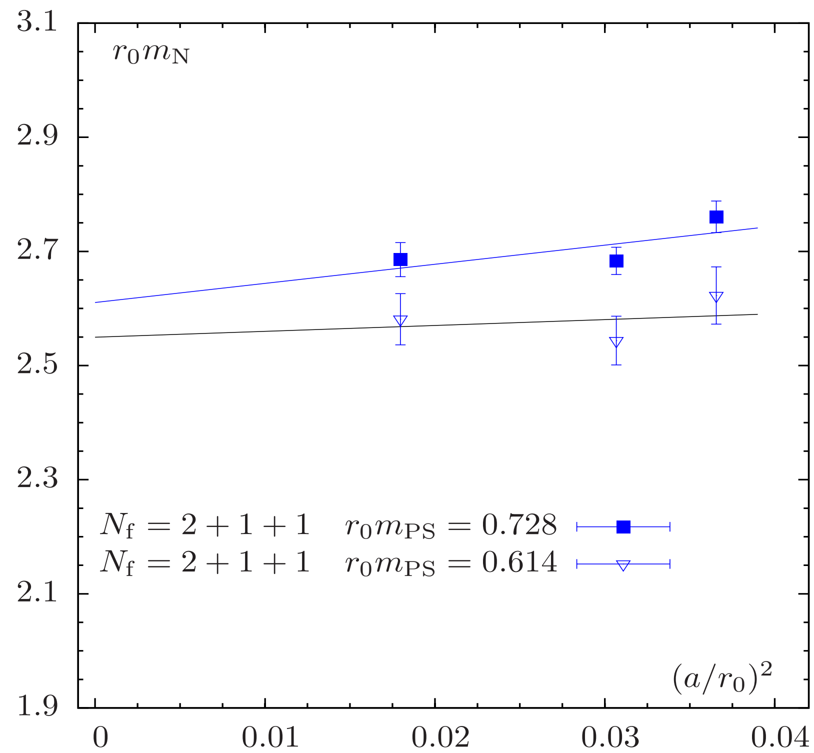
- drawback: explicit violation of isospin
  $\to$ seems to affect only neutral pion sector

- **Cyprus (Nicosia)**
  C. Alexandrou, M. Constantinou, T. Korzec, G. Koutsou
- **France (Orsay, Grenoble)**
  R. Baron, B. Bloissier, Ph. Boucaud, M. Brinet, J. Carbonell, V. Drach, P. Guichon, P.A. Harraud, Z. Liu, O. Pène
- **Italy (Rome I,II,III, Trento)**
  P. Dimopoulos, R. Frezzotti, V. Lubicz, G. Martinelli, G.C. Rossi, L. Scorzato, S. Simula, C. Tarantino
- **Netherlands (Groningen)**
  A. Deuzeman, E. Pallante, S. Reker
- **Poland (Poznan)**
  K. Cichy, A. Kujawa
- **Spain (Valencia)**
  V. Gimenez, D. Palao
- **Switzerland (Bern)**
  U. Wenger
- **United Kingdom (Glasgow, Liverpool)**
  G. McNeile, C. Michael, A. Shindler
- **Germany (Berlin/Zeuthen, Hamburg, Münster)**
  F. Farchioni, X. Feng, J. González López, G. Herdoiza, K. Jansen, I. Montvay, G. Münster, M. Petschlies, D. Renner, T. Sudmann, C. Urbach, M. Wagner

$N_f = 2+1+1$ light quark sector: scaling



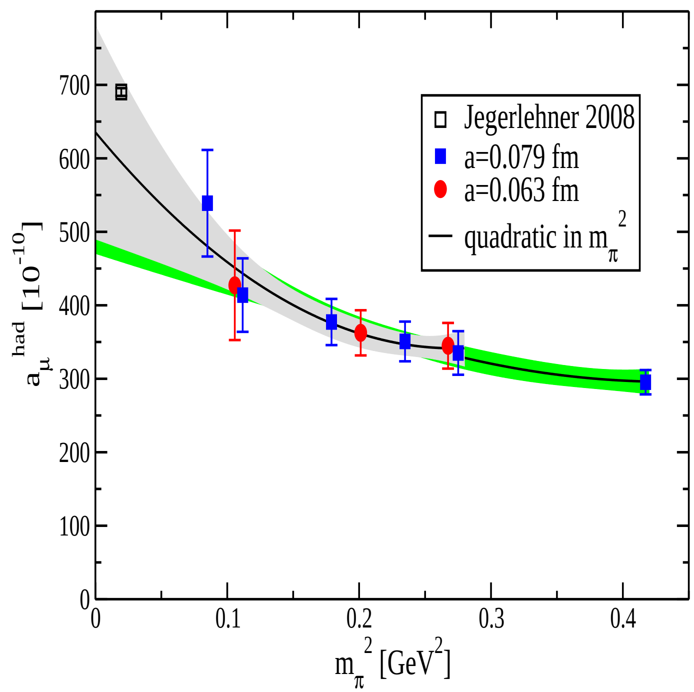
pseudoscalar decay constant f_{PS}



nucleon mass

Do we control hadronic vacuum polarisation?

(Xu Feng, Dru Renner, Marcus Petschlies, K.J.; Lattice 2010)



- experiment: $a_{\mu,N_f=2}^{\mathrm{hvp,exp}} = 5.66(05)10^{-8}$
- lattice: $a_{\mu,N_f=2}^{\mathrm{hvp,old}} = 2.95(45)10^{-8}$

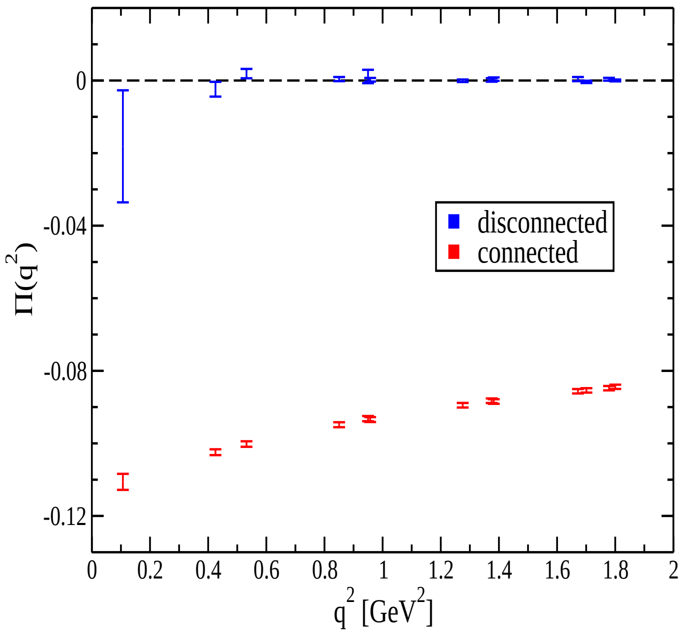
$\rightarrow$ misses the experimental value

$\rightarrow$ order of magnitude larger error

- have used different volumes
- have used different values of lattice spacing

Can it be the dis-connected contribution?

(Xu Feng, Dru Renner, Marcus Petschlies, K.J.)



- dedicated effort
- have included dis-connected contributions for first time
- smallness consistent with chiral perturbation theory (Della Morte, Jüttner)

Different extrapolation to the physical point

lattice: simulations at unphysical quark masses, demand only

$$\lim_{m_{\rm PS}\to m_\pi} a_l^{\rm hvp,latt} = a_l^{\rm hvp,phys}$$

$\Rightarrow$ flexibility to define $a_l^{\rm hvp,latt}$

standard definitions in the continuum

$$a_l^{\rm hvp} = \alpha^2 \int_0^\infty dQ^2 \frac{1}{Q^2}\omega(r)\Pi_R(Q^2)$$

$$\Pi_R(Q^2) = \Pi(Q^2) - \Pi(0)$$

$$\omega(r) = \frac{64}{r^2\left(1+\sqrt{1+4/r}\right)^4\sqrt{1+4/r}}$$

with $r = Q^2/m_l^2$

Redefinition of $a_l^{\mathrm{hvp,latt}}$

redefinition of r for lattice computations

$$r_{\mathrm{latt}} = Q^2 \cdot \frac{H^{\mathrm{phys}}}{H}$$

choices

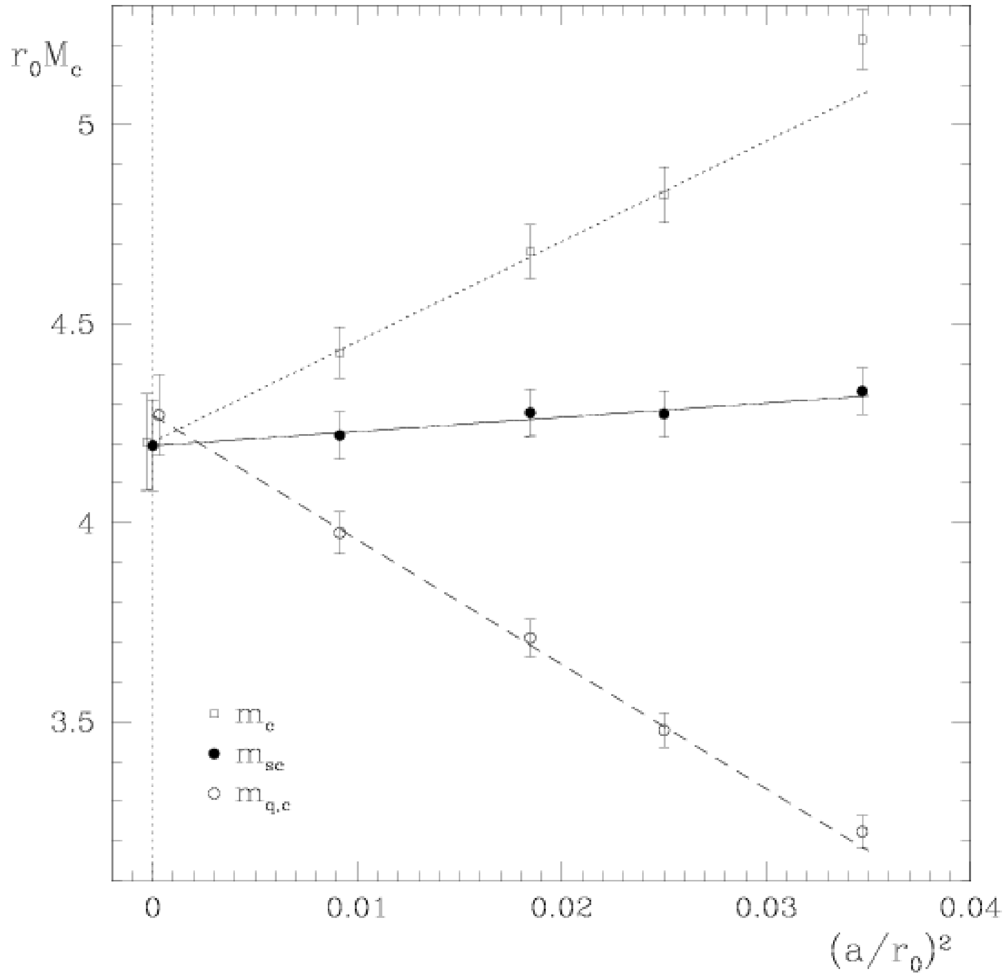
- r_1: $H = 1$; $H^{\mathrm{phys}} = 1/m_l^2$
- r_2: $H = m_V^2(m_{\mathrm{PS}})$; $H^{\mathrm{phys}} = m_\rho^2/m_l^2$
- r_3: $H = f_V^2(m_{\mathrm{PS}})$; $H^{\mathrm{phys}} = f_\rho^2/m_l^2$

each definition of r will show a different dependence on m_{PS} but agree *by construction* at the physical point

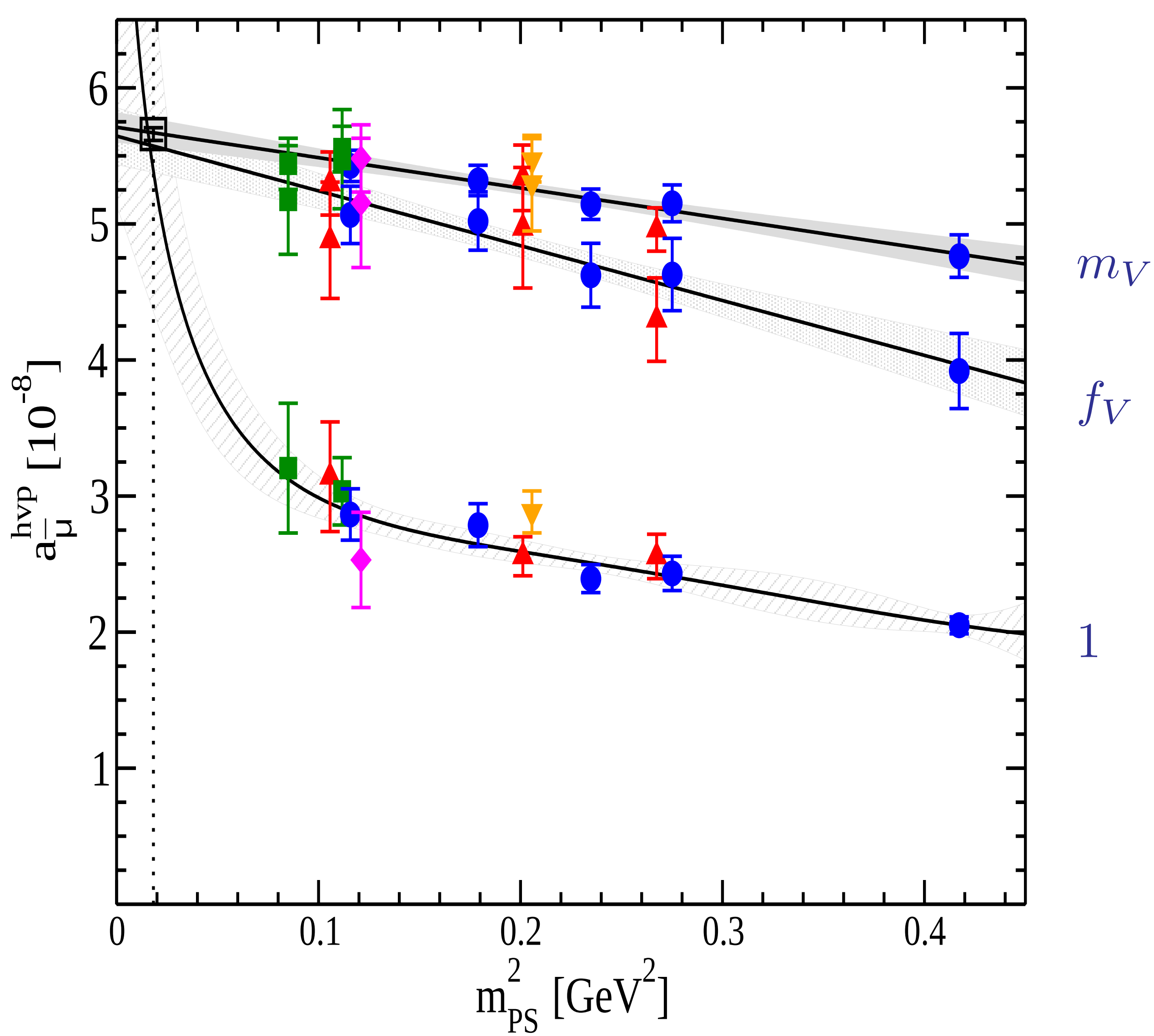
remark: strategy often used in continuum limit extrapolations, e.g. charm quark mass determination

Example of continuum limit of renormalized charm quark mass

(Rolf, Sint)



comparison using r_1, r_2, r_3



Some preliminary numbers

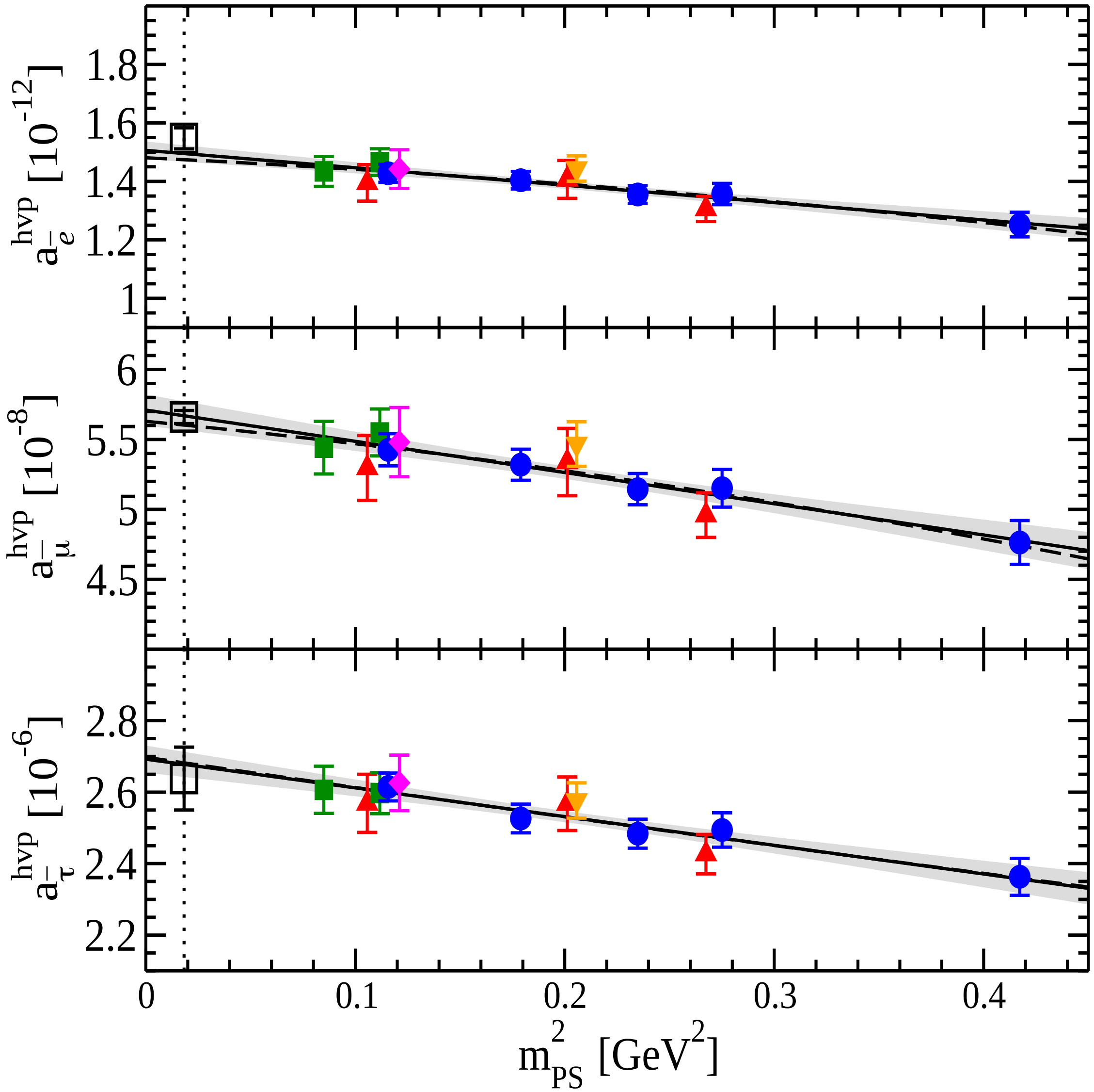
- experimental value: $a_{\mu,N_f=2}^{\mathrm{hvp,exp}} = 5.66(05)10^{-8}$
- from our old analysis: $a_{\mu,N_f=2}^{\mathrm{hvp,old}} = 2.95(45)10^{-8}$

$\rightarrow$ misses the experimental value
$\rightarrow$ order of magnitude larger error

- from our new analysis: $a_{\mu,N_f=2}^{\mathrm{hvp,new}} = 5.66(11)10^{-8}$

$\rightarrow$ error (including systematics) almost matching experiment

Anomalous magnetic moments, a check



$a^{hvp}_{\bar{e}}$

$a^{hvp}_{\bar{\mu}}$

$a^{hvp}_{\bar{\tau}}$

Why it works: fitting the Q^2 dependence

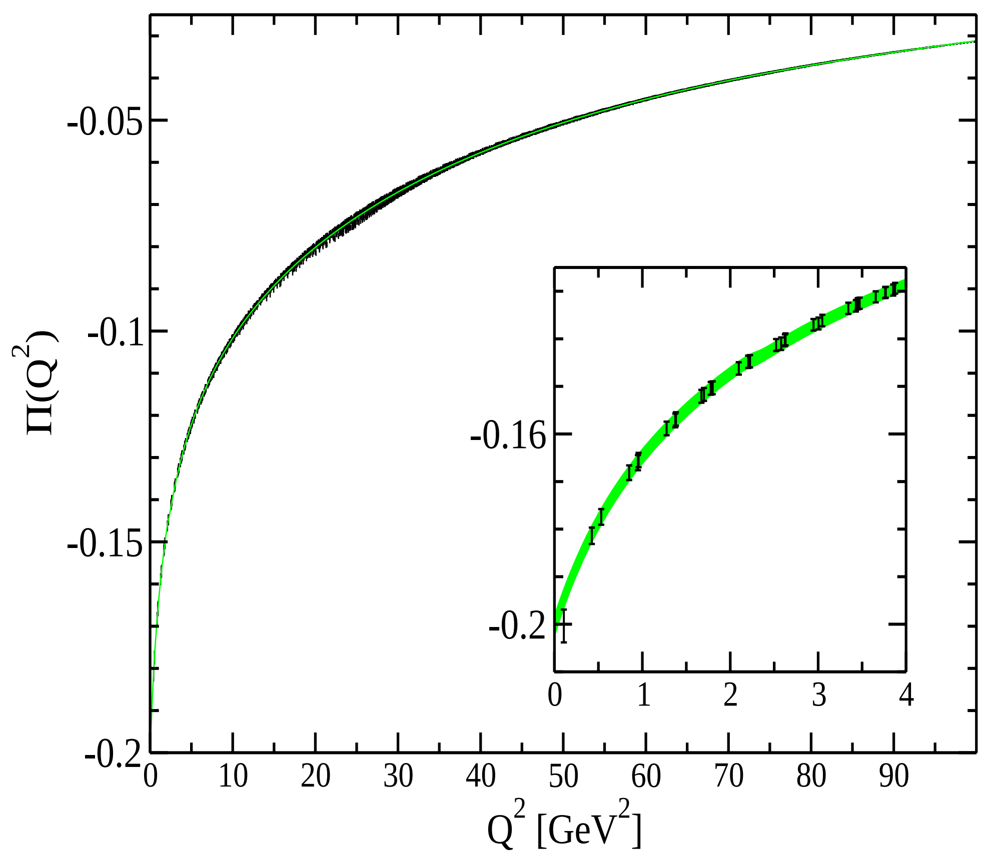
Fit function

$$\Pi_{M,N}(Q^2) = -\frac{5}{9}\sum_{i=1}^{M} \frac{m_i^2}{Q^2+m_i^2} + \sum_{n=0}^{N} a_n (Q^2)^n$$

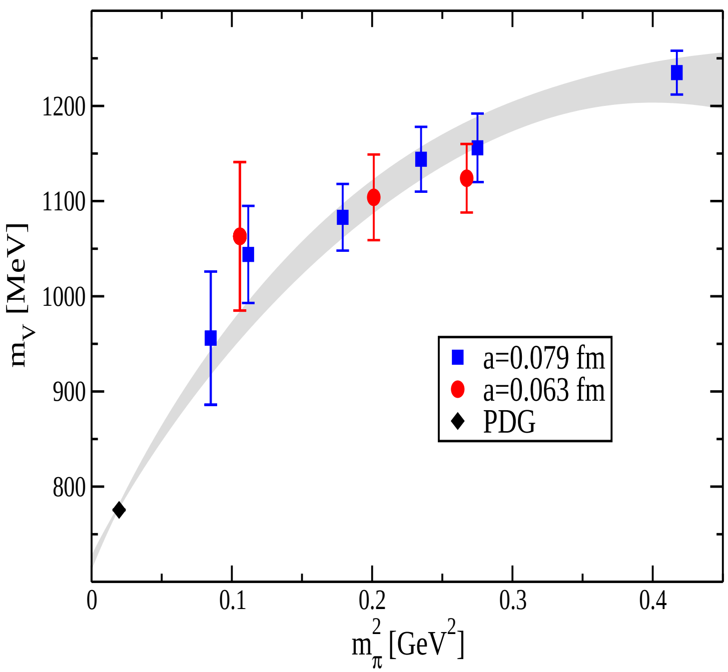
$i = 1$: ρ-meson $\rightarrow$ dominant contribution $\propto 5.010^{-8}$

$i = 2$: ω-meson $\propto 3.710^{-9}$

$i = 3$: ϕ-meson $\propto 3.410^{-9}$



Why it works



- m_V consistent with resonance analysis (Feng, Renner, K.J.)
- strong dependence on $m_{\rm PS}$

Next steps

Fit function

$$\Pi_{M,N}(Q^2) = -\frac{5}{9}\sum_{i=1}^{M}\frac{m_i^2}{Q^2+m_i^2} + \sum_{n=0}^{N} a_n (Q^2)^n$$

add $i = 4$: J/Ψ, $i = 5 \ldots$

- **ETMC** is performing simulations wit dynamical
  up, down, strange <u>and</u> charm quarks
  $\rightarrow$ unique opportunity

- generalized boundary conditions: $\Psi(L + a\hat{\mu}) = e^{i\theta}\Psi(x)$
  $\rightarrow$ θ *continuous* momentum
  $\rightarrow$ allows to realize arbitrary momenta on the lattice

Simulation setup for $N_f = 2+1+1$
Configurations available through ILDG

β	a[fm]	L^3T/a^4	m_π[MeV]	status
1.9	≈ 0.085	$24^3 48$	300 – 500	ready
1.95	≈ 0.075	$32^3 64$	300 –500	ready
2.0	≈ 0.065	$32^3 64$	300	ready
2.1	≈ 0.055	$48^3 96$	300 – 500	running/ready
		$64^3 128$	230	thermalizing
		$64^3 128$	200	queued
		$96^3 192$	160	planned

- trajectory length always one
- 1000 trajectores for thermalization
- $\geq$ 5000 trajectores for measurements

Momentum sources

(Alexandrou, Constantinou, Korzec, Panagopoulos, Stylianou)

$\leftarrow$ following Göckeler et.al.

for renormalization: need Green function in momentum space

$$G(p) = \frac{a^{12}}{V} \sum_{x,y,z,z'} e^{-ip(x-y)} \langle u(x)\overline{u}(z)\mathcal{J}(z,z')d(z')\overline{d}(y)\rangle$$

e.g. $\mathcal{J}(z,z')=\delta_{z,z'}\gamma_\mu$ corresponds to local vector current

sources:

$$b^a_\alpha(x) = e^{ipx}\delta_{\alpha\beta}\delta_{ab}$$

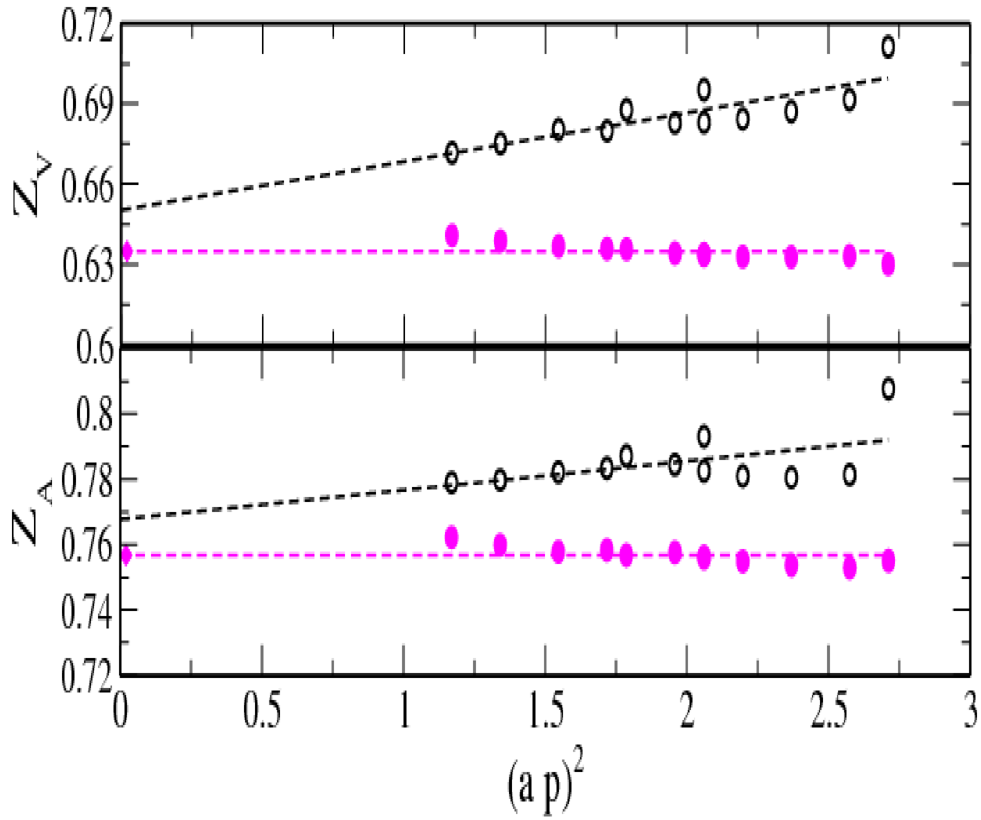
solve for

$$D_{\rm latt}G(p) = b$$

Advantage: very high, sub-percent precision data (only moderate statistics)

Disadvantage: need inversion for each momentum separately

Illustration of precision



with $O(a^2)$ effects

$O(a^2)$ effects pert. subtracted

with $O(a^2)$ effects

$O(a^2)$ effects pert. subtracted

use the momentum source method to attack the 4-point function
as needed for light-by-light scattering (P. Rakow et.al., lattice'08)

Summary

To tackle $g_\mu - 2$ on the lattice, need both

- **precise calculation of hadronic vacuum polarization**
- **quantitative calculation of light-by-light contribution**

European Twisted Mass Collaboration:

- automatic $O(a)$-improvement
  $\Rightarrow$ discretization effects start automatically at $O(a^2)$
- simulations with up, down, strange and charm
- improved observables
- techniques to compute dis-connected contributions
- generalized boundary conditions
- momentum source method

- **prospects to match experimental precision for a_μ^{hvp}**
- **prospects to obtain first quantitative value for a_μ^{lbl}**

A discussion point

complete light-by-light is very demanding

- **are there intermediate steps?**
  - chiral magnetic susceptibility χ
  - $\pi \to \gamma\gamma$ form factors
  - other suggestions?
- **discriminate between models**