Holographic Models of QCD and Muon $g-2$

Deog Ki Hong

Pusan National University

March 1, 2011

INT workshop on HLBL and $(g-2)_\mu$

What is holographic QCD?

- QCD is accepted as the microscopic theory of strong interactions.
- It is however difficult to describe strong interactions directly from QCD, because quarks and gluons are not relevant degrees of freedom at low energy.
- Holographic QCD is an attempt to describe strong interactions directly with hadrons, which are the right degrees of freedom at low energy, $E < \Lambda_{\rm QCD}$.

What is holographic QCD?

- QCD is accepted as the microscopic theory of strong interactions.
- It is however difficult to describe strong interactions directly from QCD, because quarks and gluons are not relevant degrees of freedom at low energy.
- Holographic QCD is an attempt to describe strong interactions directly with hadrons, which are the right degrees of freedom at low energy, $E < \Lambda_{\rm QCD}$.

What is holographic QCD?

- QCD is accepted as the microscopic theory of strong interactions.
- It is however difficult to describe strong interactions directly from QCD, because quarks and gluons are not relevant degrees of freedom at low energy.
- Holographic QCD is an attempt to describe strong interactions directly with hadrons, which are the right degrees of freedom at low energy, $E < \Lambda_{\mathrm{QCD}}$.

What is holographic QCD?

But, people have already tried with some success:

- χPT: theory of pions at $E < 4\pi f_\pi$.

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{2} \mathrm{Tr} \partial_\mu U \partial^\mu U^\dagger + \mathcal{O}(p^4)\,. \qquad (1)$$

where $U = e^{2i\pi/f_\pi} \mapsto g_L U g_R^\dagger$.

- What about vector mesons like $\rho, \omega, \cdots$? Hidden local symmetry (Bando et al '85):

$$U = \xi_L^\dagger \xi_R, \quad \xi_L \mapsto h(x)\xi_L(x)g_L^\dagger, \quad \xi_R \mapsto h(x)\xi_R(x)g_R^\dagger\,. \qquad (2)$$

What is holographic QCD?

But, people have already tried with some success:

- χPT: theory of pions at $E < 4\pi f_\pi$.

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{2}\mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger + \mathcal{O}(p^4)\,. \quad (1)$$

where $U = e^{2i\pi/f_\pi} \mapsto g_L U g_R^\dagger$.

- What about vector mesons like $\rho, \omega, \cdots$? Hidden local symmetry (Bando et al '85):

$$U = \xi_L^\dagger \xi_R, \quad \xi_L \mapsto h(x)\xi_L(x)g_L^\dagger, \quad \xi_R \mapsto h(x)\xi_R(x)g_R^\dagger\,. \quad (2)$$

- Introduce vector fields, $V_\mu \mapsto hV_\mu h^\dagger + ih\partial_\mu h^\dagger$ and write down, fixing a gauge $\xi_L^\dagger = \xi_R = \xi \,(= e^{i\pi/f_\pi})$,

$$\mathcal{L}_{\rm HLS} = \mathcal{L}_{\chi PT} + a\, f_\pi^2 {\rm Tr}\left\{ V_\mu - \frac{1}{2i}\left(\partial_\mu \xi \xi^\dagger + \xi \partial_\mu \xi^\dagger\right)\right\}^2 - \frac{1}{4g^2} V_{\mu\nu}^2 + \cdots$$

- If we identify the vector field as ρ meson, the unknown parameters a, g are related as, after rescaling $V \mapsto gV$,

$$m_\rho^2 = a\, g^2 f_\pi^2, \quad g_{\rho\pi\pi} = \frac{1}{2} a\, g \tag{3}$$

(When $a = 2$, KSRF holds: $a^{\rm exp} = 2.07 \pm 0.03$.)

- Introduce vector fields, $V_\mu \mapsto hV_\mu h^\dagger + ih\partial_\mu h^\dagger$ and write down, fixing a gauge $\xi_L^\dagger = \xi_R = \xi\,(= e^{i\pi/f_\pi})$,

$$\mathcal{L}_{\rm HLS} = \mathcal{L}_{\chi PT} + a f_\pi^2 {\rm Tr}\left\{ V_\mu - \frac{1}{2i}\left(\partial_\mu \xi \xi^\dagger + \xi \partial_\mu \xi^\dagger\right)\right\}^2 - \frac{1}{4g^2} V_{\mu\nu}^2 + \cdots$$

- If we identify the vector field as ρ meson, the unknown parameters a, g are related as, after rescaling $V \mapsto gV$,

$$m_\rho^2 = a\,g^2 f_\pi^2, \quad g_{\rho\pi\pi} = \frac{1}{2} a\,g \tag{3}$$

(When $a = 2$, KSRF holds: $a^{\rm exp} = 2.07 \pm 0.03$.)

What is holographic QCD?

- What about other vector mesons? (They are stable if $N_c \gg 1$)
- Open moose model (Son+Stephanov '04): We introduce K number of gauge theories

$$\mathcal{L} = \sum_{k=1}^{K+1} f_k^2 \mathrm{Tr} \left| D_\mu \Sigma^k \right|^2 - \sum_{k=1}^{K} \frac{1}{2} \mathrm{Tr} \left(F_{\mu\nu}^k \right)^2 . \quad (4)$$

where $D_\mu \Sigma^k = \partial_\mu \Sigma^k - i(gA_\mu)^{k-1}\Sigma^k + i\Sigma^k (gA_\mu)^k$.
$\Sigma^k \mapsto g_{k-1}(x)\Sigma^k g_k^\dagger(x)$ and $U = \Sigma^1\Sigma^2 \cdots \Sigma^{K+1} = e^{2i\pi/f_\pi}$.

- What about other vector mesons? (They are stable if $N_c \gg 1$)
- Open moose model (Son+Stephanov '04): We introduce K number of gauge theories

$$\mathcal{L} = \sum_{k=1}^{K+1} f_k^2 \mathrm{Tr} \left| D_\mu \Sigma^k \right|^2 - \sum_{k=1}^{K} \frac{1}{2} \mathrm{Tr} \left(F_{\mu\nu}^k \right)^2 . \qquad (4)$$

where $D_\mu \Sigma^k = \partial_\mu \Sigma^k - i(gA_\mu)^{k-1}\Sigma^k + i\Sigma^k (gA_\mu)^k$.
$\Sigma^k \mapsto g_{k-1}(x)\Sigma^k g_k^\dagger(x)$ and $U = \Sigma^1 \Sigma^2 \cdots \Sigma^{K+1} = e^{2i\pi/f_\pi}$.

What is holographic QCD?

- The gauge fields are coupled like a coupled harmonic oscillator. By diagonalizing them we get the physical spectrum (normal modes), which are identified as (excited) vector mesons $(\rho, a, \rho', a', \cdots)$
- When $K \to \infty$, the open Moose theory becomes a 5D gauge theory, $\Sigma^k \approx 1 + iaA_5(u)$:

$$S = -\mathrm{Tr}\int \mathrm{d}u\,\mathrm{d}^4x \left(-f^2(u)F_{5\mu}^2 + \frac{1}{2g^2(u)}F_{\mu\nu}^2\right), \quad (5)$$

which is nothing but a 5D gauge theory in a warped geometry, $ds^2 = -du^2 + f^2g^2\eta_{\mu\nu}dx^\mu dx^\nu$.

- The gauge fields are coupled like a coupled harmonic oscillator. By diagonalizing them we get the physical spectrum (normal modes), which are identified as (excited) vector mesons $(\rho, a, \rho', a', \cdots)$
- When $K \to \infty$, the open Moose theory becomes a 5D gauge theory, $\Sigma^k \approx 1 + iaA_5(u)$:

$$S = -\mathrm{Tr}\int \mathrm{d}u\,\mathrm{d}^4x\left(-f^2(u)F_{5\mu}^2 + \frac{1}{2g^2(u)}F_{\mu\nu}^2\right)\,, \quad (5)$$

which is nothing but a 5D gauge theory in a warped geometry, $ds^2 = -du^2 + f^2g^2\eta_{\mu\nu}dx^\mu dx^\nu$.

Holographic models of QCD

- Previous attempts, however, have no guiding principle on how to construct the 5D theory, unless we measure all m_n's, f_n's.
- Inspired by AdS/CFT duality found by Madacena ('98): $(\mathcal{N}=4)\,SYM \equiv$ *Type IIB superstring theory on* $AdS_5 \times S^5$.
- String theory construction of hQCD was made by Witten ('98) and Sakai-Sugimoto ('04), based on Gauge/gravity duality conjecture:
  There is a holographic dual of QCD in the large N_c limit

Holographic models of QCD

- Previous attempts, however, have no guiding principle on how to construct the 5D theory, unless we measure all m_n's, f_n's.
- Inspired by AdS/CFT duality found by Madacena ('98):
  $(\mathcal{N}=4)$ SYM $\equiv$ Type IIB superstring theory on $AdS_5 \times S^5$.
- String theory construction of hQCD was made by Witten ('98) and Sakai-Sugimoto ('04), based on Gauge/gravity duality conjecture:
  There is a holographic dual of QCD in the large N_c limit

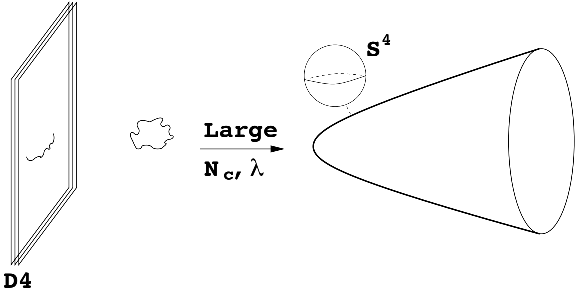
Holographic models of QCD

- Previous attempts, however, have no guiding principle on how to construct the 5D theory, unless we measure all m_n's, f_n's.
- Inspired by AdS/CFT duality found by Madacena ('98):
  $(\mathcal{N}=4)\,SYM \equiv$ Type IIB superstring theory on $AdS_5 \times S^5$.
- String theory construction of hQCD was made by Witten ('98) and Sakai-Sugimoto ('04), based on Gauge/gravity duality conjecture:

  There is a holographic dual of QCD in the large N_c limit

Witten model

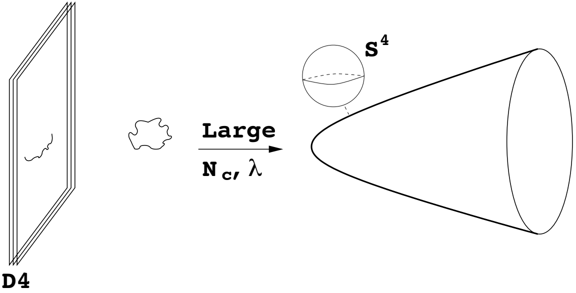
- A stack of N_c D4 branes describes $\mathcal{N}$=4 SYM in 5D.
- By circle-compactifying the extra dimension, $R = M_{KK}^{-1}$, we get 4D theory at $E < M_{KK}$.
- Imposing antiperiodic boundary conditions along S^1 to make gauginos massive, breaking SUSY, we get non-supersymmetric $SU(N_c)$ glue theory.



What is holographic QCD?

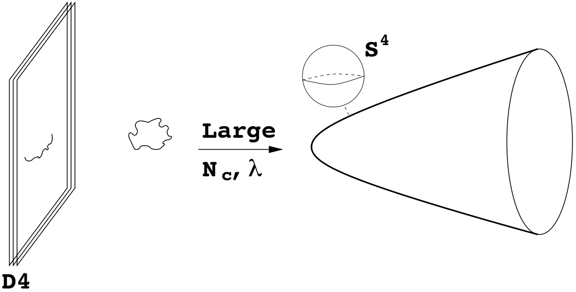
Witten model

- A stack of N_c D4 branes describes $\mathcal{N}$=4 SYM in 5D.
- By circle-compactifying the extra dimension, $R = M_{KK}^{-1}$, we get 4D theory at $E < M_{KK}$.
- Imposing antiperiodic boundary conditions along S^1 to make gauginos massive, breaking SUSY, we get non-supersymmetric $SU(N_c)$ glue theory.



Witten model

- A stack of N_c D4 branes describes $\mathcal{N}=4$ SYM in 5D.
- By circle-compactifying the extra dimension, $R = M_{KK}^{-1}$, we get 4D theory at $E < M_{KK}$.
- Imposing antiperiodic boundary conditions along S^1 to make gauginos massive, breaking SUSY, we get non-supersymmetric $SU(N_c)$ glue theory.



- Since the brane tension $\sim 1/g_s l_s^5$, we take $g_s \sim g_{YM}^2 \to 0$ to freeze the brane fluctuations.
- If we take $N_c \gg 1$, the backreaction of D4 brane is so large the spacetime forms horizon and D4 branes disappear:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$\left(R^3 = \pi g_s N_c l_s^3,\ f = 1 - \frac{U_{KK}^3}{U^3},\ e^\phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}},\ dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4\right)$$

- For weak gravity limit, we take $g_s N_c \sim g_{YM}^2 N_c \equiv \lambda \to \infty$.
- $SU(N_c)$ glue theory is now described by weak gravity when $N_c, \lambda \gg 1$: glueball mass $\left(\frac{M_{0^{-+}}}{M_{0^{++}}}\right) = 1.20$ for hQCD while 1.36 ± 0.032 in lattice.

- Since the brane tension $\sim 1/g_s l_s^5$, we take $g_s \sim g_{YM}^2 \to 0$ to freeze the brane fluctuations.
- If we take $N_c \gg 1$, the backreaction of D4 brane is so large the spacetime forms horizon and D4 branes disappear:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$\left(R^3 = \pi g_s N_c l_s^3,\ f = 1 - \frac{U_{KK}^3}{U^3},\ e^\phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}},\ dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4\right)$$

- For weak gravity limit, we take $g_s N_c \sim g_{YM}^2 N_c \equiv \lambda \to \infty$.
- $SU(N_c)$ glue theory is now described by weak gravity when $N_c, \lambda \gg 1$: glueball mass $\left(\frac{M_{0-+}}{M_{0++}}\right) = 1.20$ for hQCD while 1.36 ± 0.032 in lattice.

What is holographic QCD?

- Since the brane tension $\sim 1/g_s l_s^5$, we take $g_s \sim g_{YM}^2 \to 0$ to freeze the brane fluctuations.
- If we take $N_c \gg 1$, the backreaction of D4 brane is so large the spacetime forms horizon and D4 branes disappear:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$\left(R^3 = \pi g_s N_c l_s^3,\ f = 1 - \frac{U_{KK}^3}{U^3},\ e^\phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}},\ dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4\right)$$

- For weak gravity limit, we take $g_s N_c \sim g_{YM}^2 N_c \equiv \lambda \to \infty$.
- $SU(N_c)$ glue theory is now described by weak gravity when $N_c, \lambda \gg 1$: glueball mass $\left(\frac{M_{0^{-+}}}{M_{0^{++}}}\right) = 1.20$ for hQCD while 1.36 ± 0.032 in lattice.

- Since the brane tension $\sim 1/g_s l_s^5$, we take $g_s \sim g_{YM}^2 \to 0$ to freeze the brane fluctuations.
- If we take $N_c \gg 1$, the backreaction of D4 brane is so large the spacetime forms horizon and D4 branes disappear:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$\left(R^3 = \pi g_s N_c l_s^3,\ f = 1 - \frac{U_{KK}^3}{U^3},\ e^\phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}},\ dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4\right)$$

- For weak gravity limit, we take $g_s N_c \sim g_{YM}^2 N_c \equiv \lambda \to \infty$.
- $SU(N_c)$ glue theory is now described by weak gravity when $N_c, \lambda \gg 1$: glueball mass $\left(\frac{M_{0-+}}{M_{0++}}\right) = 1.20$ for hQCD while 1.36 ± 0.032 in lattice.

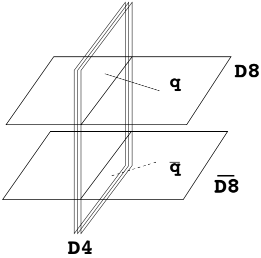
Sakai-Sugimoto model

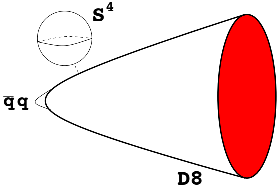
- To describe chiral quarks, introduce D8 branes.
- Large N_c QCD = open string theory on D8:

$$S_{D8} = -\kappa \int_{x,z} \mathrm{Tr}\left[\frac{1}{2}(1+z^2)^{-\frac{1}{3}} F_{\mu\nu}^2 + (1+z^2) F_{\mu z}^2\right] + \mathcal{O}(F^3) + S_{CS}\,.$$

- Vector mass:

$$\frac{m_{a_1}^2}{m_\rho^2} \simeq \frac{1.6}{0.67} = 2.4\,(\mathrm{th}); \qquad \frac{(1230\,\mathrm{MeV})^2}{(776\,\mathrm{MeV})^2} \simeq 2.51\,(\mathrm{exp})\,.$$





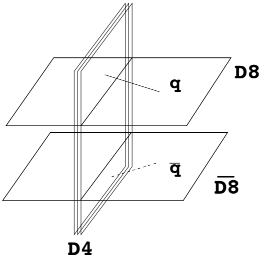
Sakai-Sugimoto model

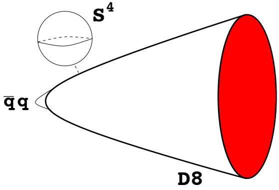
- To describe chiral quarks, introduce D8 branes.
- Large N_c QCD = open string theory on D8:

$$S_{D8} = -\kappa \int_{x,z} \mathrm{Tr}\left[\frac{1}{2}(1+z^2)^{-\frac{1}{3}} F_{\mu\nu}^2 + (1+z^2) F_{\mu z}^2\right] + \mathcal{O}(F^3) + S_{CS}\,.$$

- Vector mass:

$$\frac{m_{a_1}^2}{m_\rho^2} \simeq \frac{1.6}{0.67} = 2.4\,(\mathrm{th}); \qquad \frac{(1230\,\mathrm{MeV})^2}{(776\,\mathrm{MeV})^2} \simeq 2.51\,(\mathrm{exp})\,.$$





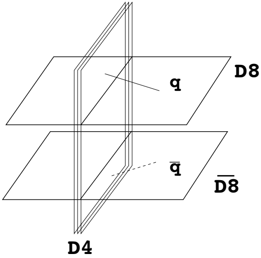
Sakai-Sugimoto model

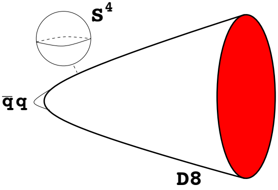
- To describe chiral quarks, introduce D8 branes.
- Large N_c QCD = open string theory on D8:

$$S_{D8} = -\kappa \int_{x,z} \mathrm{Tr}\left[\frac{1}{2}(1+z^2)^{-\frac{1}{3}} F_{\mu\nu}^2 + (1+z^2) F_{\mu z}^2\right] + \mathcal{O}(F^3) + S_{CS} .$$

- Vector mass:

$$\frac{m_{a_1}^2}{m_\rho^2} \simeq \frac{1.6}{0.67} = 2.4\,(\mathrm{th}); \quad \frac{(1230\,\mathrm{MeV})^2}{(776\,\mathrm{MeV})^2} \simeq 2.51\,(\mathrm{exp}) .$$





What is holographic QCD?

Gauge/gravity duality

- Gauge theory (or open string theory) is dual to closed string theory (or gravity) in the large N_c and large 't Hooft coupling ($\lambda = g_{YM}^2 N_c$) limit. (Maldacena 1997)

$$Z(J) = \left\langle e^{i\int J\mathcal{O}} \right\rangle = e^{iS_c(\phi(x,z))} \quad \text{with} \quad \phi(x,\epsilon) = J(x)$$

- hQCD is 5D flavor gauge theory and supposedly a gravity dual of QCD in the large N_c, λ limit, describing QCD with hadrons.
- hQCD allows us to calculate those hadronic corrections in the EW process such as (g-2)$_\mu$, K-$\bar{K}$ mixing, S-parameter, $\cdots$.

Gauge/gravity duality

- Gauge theory (or open string theory) is dual to closed string theory (or gravity) in the large N_c and large 't Hooft coupling $(\lambda = g_{YM}^2 N_c)$ limit. (Maldacena 1997)

$$Z(J) = \left\langle e^{i\int J\mathcal{O}} \right\rangle = e^{iS_c(\phi(x,z))} \quad \text{with} \quad \phi(x,\epsilon) = J(x)$$

- hQCD is 5D flavor gauge theory and supposedly a gravity dual of QCD in the large N_c, λ limit, describing QCD with hadrons.
- hQCD allows us to calculate those hadronic corrections in the EW process such as (g-2)$_\mu$, K-$\bar{K}$ mixing, S-parameter, $\cdots$.

What is holographic QCD?

Gauge/gravity duality

- Gauge theory (or open string theory) is dual to closed string theory (or gravity) in the large N_c and large 't Hooft coupling $(\lambda = g_{YM}^2 N_c)$ limit. (Maldacena 1997)

$$Z(J) = \left\langle e^{i\int J\mathcal{O}} \right\rangle = e^{iS_c(\phi(x,z))} \quad \text{with} \quad \phi(x,\epsilon) = J(x)$$

- hQCD is 5D flavor gauge theory and supposedly a gravity dual of QCD in the large N_c, λ limit, describing QCD with hadrons.
- hQCD allows us to calculate those hadronic corrections in the EW process such as $(\text{g-2})_\mu$, K-$\bar{K}$ mixing, S-parameter, $\cdots$.

EKSS model (bottom-up approach)

- Construct $U(3)_L \times U(3)_R$ flavor gauge theory in a slice of AdS_5 ($\epsilon \leq z \leq z_m = (0.323)^{-1}$) as a model for hQCD:

4D	$\bar{q}_L \gamma^\mu t^a q_L$	$\bar{q}_R \gamma^\mu t^q q_R$	$\bar{q}_L^\alpha q_R^\beta$
5D	$A^a_{L\mu}$	$A_{R\mu}$	$\frac{2}{z} X^{\alpha\beta}$

$$S = \int d^5x \sqrt{g}\ \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}({F_L}^2 + {F_R}^2) \right\} + S_Y + S_{CS},$$

- Flavor-singlet bulk scalar, Y, dual to F^2 $(F\tilde{F})$, described by

$$S_Y = \int d^5x \sqrt{g} \left[\frac{1}{2}|DY|^2 - \frac{\kappa}{2}(Y^{N_f} \det(X) + \mathrm{h.c.}) \right].$$

- Finally we introduce CS term for QCD flavor anomaly:

$$S_{CS} = \frac{N_c}{24\pi^2} \int [\omega_5(A_L) - \omega_5(A_R)],$$

where $d\,\omega_5(A) = \mathrm{Tr} F^3$. (N.B. We need either bulk fermions or a counter term in IR to recover the gauge invariance at IR.)

EKSS model (bottom-up approach)

- Construct $U(3)_L \times U(3)_R$ flavor gauge theory in a slice of AdS_5 ($\epsilon \leq z \leq z_m = (0.323)^{-1}$) as a model for hQCD:

4D	$\bar{q}_L\gamma^\mu t^a q_L$	$\bar{q}_R\gamma^\mu t^q q_R$	$\bar{q}_L^\alpha q_R^\beta$
5D	$A^a_{L\mu}$	$A_{R\mu}$	$\frac{2}{z}X^{\alpha\beta}$

$$S = \int d^5x\sqrt{g}\ \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}({F_L}^2 + {F_R}^2)\right\} + S_Y + S_{CS},$$

- Flavor-singlet bulk scalar, Y, dual to F^2 ($F\tilde{F}$), described by

$$S_Y = \int d^5x\sqrt{g}\left[\frac{1}{2}|DY|^2 - \frac{\kappa}{2}(Y^{N_f}\det(X) + \mathrm{h.c.})\right].$$

- Finally we introduce CS term for QCD flavor anomaly:

$$S_{CS} = \frac{N_c}{24\pi^2}\int\left[\omega_5(A_L) - \omega_5(A_R)\right],$$

where $d\,\omega_5(A) = \mathrm{Tr}F^3$. (N.B. We need either bulk fermions or a counter term in IR to recover the gauge invariance at IR.)

EKSS model (bottom-up approach)

- Construct $U(3)_L \times U(3)_R$ flavor gauge theory in a slice of AdS_5 ($\epsilon \leq z \leq z_m = (0.323)^{-1}$) as a model for hQCD:

4D	$\bar{q}_L \gamma^\mu t^a q_L$	$\bar{q}_R \gamma^\mu t^q q_R$	$\bar{q}^\alpha_L q^\beta_R$
5D	$A^a_{L\mu}$	$A_{R\mu}$	$\frac{2}{z} X^{\alpha\beta}$

$$S = \int d^5x \sqrt{g}\ \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}({F_L}^2 + {F_R}^2) \right\} + S_Y + S_{CS},$$

- Flavor-singlet bulk scalar, Y, dual to F^2 ($F\tilde{F}$), described by

$$S_Y = \int d^5x \sqrt{g} \left[\frac{1}{2}|DY|^2 - \frac{\kappa}{2}(Y^{N_f} \det(X) + \text{h.c.}) \right] .$$

- Finally we introduce CS term for QCD flavor anomaly:

$$S_{CS} = \frac{N_c}{24\pi^2} \int [\omega_5(A_L) - \omega_5(A_R)] ,$$

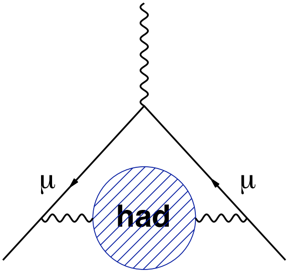
where $d\,\omega_5(A) = \mathrm{Tr} F^3$. (N.B. We need either bulk fermions or a counter term in IR to recover the gauge invariance at IR.)

Holographic Calculation of Hadronic Leading Contribution

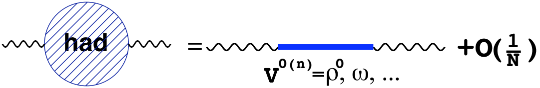
The HLO contribution is given as (Blum '03)

$$a_\mu^{\rm HLO} = 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\, \bar{\Pi}_{\rm em}^{\rm had}(Q^2)\,,$$

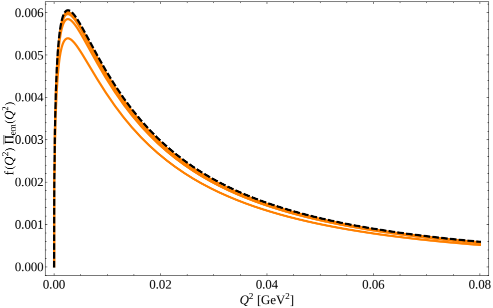
$$f(Q^2) = \frac{m_\mu^2 Q^2 Z^3 (1 - Q^2 Z)}{1 + m_\mu^2 Q^2 Z^2} \quad \text{and} \quad Z = -\frac{Q^2 - \sqrt{Q^4 + 4 m_\mu^2 Q^2}}{2 m_\mu^2 Q^2}\,.$$



Holographic Calculation of Hadronic Leading Contribution



We have $\bar{\Pi}_V(Q^2) \simeq \sum_{n=1}^{4} \frac{Q^2 F_{V_n}^2}{(Q^2+M_{V_n}^2)M_{V_n}^4} + \mathcal{O}(Q^2/(M_{V_5}^2))$



Holographic Calculation of Hadronic Leading Contribution

We obtain (arXiv:0911.0560, done with D. Kim and S. Matsuzaki)

$$a_\mu^{\rm HLO}|^{N_f=2}_{\rm AdS/QCD} = 470.5 \times 10^{-10}, \quad (6)$$

which agrees, within 10% errors, with the currently updated value (BaBar 2009)

$$a_\mu^{\rm HLO}[\pi\pi]|_{\rm BABAR} = (514.1 \pm 3.8) \times 10^{-10}\,. \quad (7)$$

We expect that the discrepancy may be due to the $1/N_c$ corrections together with the isospin-breaking corrections.

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- For the hadronic LBL we need to calculate 4-point functions of flavor currents:

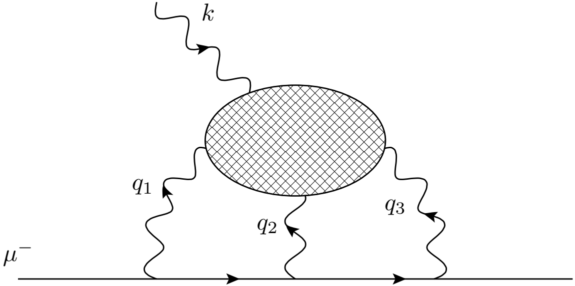


Figure: Hadronic Light-by-light corrections to muon $g-2$.

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- Since there is no quartic term for $A_{Q_{\rm em}}$ ($Q_{\rm em} = 1/2 + I_3$), there is no 1PI 4-point function for the EM currents in hQCD:

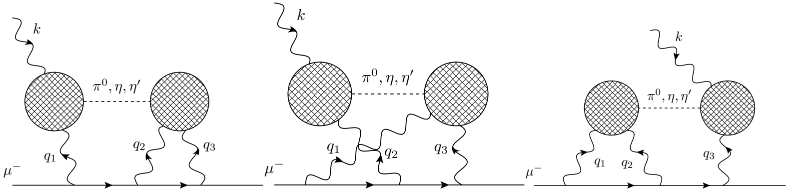


Figure: Light-by-light correction is dominated by the pseudo scalar mesons (and also axial vectors) exchange.

- Higher order terms like F^4 or F^2X^2 terms are α' suppressed.

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- Since there is no quartic term for $A_{Q_{\rm em}}$ ($Q_{\rm em} = 1/2 + I_3$), there is no 1PI 4-point function for the EM currents in hQCD:

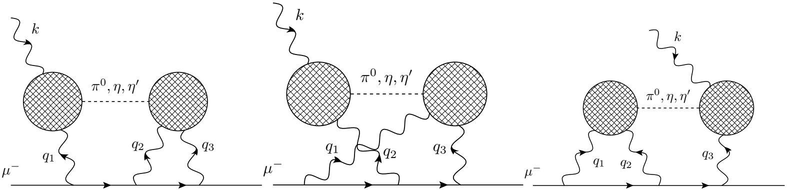


Figure: Light-by-light correction is dominated by the pseudo scalar mesons (and also axial vectors) exchange.

- Higher order terms like F^4 or F^2X^2 terms are α' suppressed.

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- In hQCD the LBL diagram is dominated by VVA or VVP diagrams, which come from the CS term:

$$F_{\gamma^*\gamma^* P(A)}(q_1,q_2) = \frac{\delta^3}{\delta V(q_1)\delta V(q_2)\delta A(-q_1-q_2)} S_{5D\text{eff}} \quad (8)$$

where the gauge fields satisfy in the axial gauge, $V_5 = 0 = A_5$,

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^{\hat{a}}(q,z)\right) + \frac{q^2}{z} V_\mu^{\hat{a}}(q,z)\right]_\perp = 0\,, \quad (9)$$

$$\left[\partial_z\left(\frac{1}{z}\partial_z A_\mu^{\hat{a}}\right) + \frac{q^2}{z} A_\mu^{\hat{a}} - \frac{g_5^2 v^2}{z^3} A_\mu^{\hat{a}}\right]_\perp = 0\,, \quad (10)$$

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- For two flavors the longitudinal components, $A^a_{\mu\|} = \partial_\mu \phi^a$, and the phase of bulk scalar X are related by EOM as

$$\partial_z\left(\frac{1}{z}\partial_z\phi^a\right) + \frac{g_5^2 v^2}{z^3}(\pi^a - \phi^a) = 0\,, \qquad (11)$$

$$-q^2\partial_z\phi^a + \frac{g_5^2 v^2}{z^2}\partial_z\pi^a = 0\,. \qquad (12)$$

Anomalous pion form factors

- The anomalous FF is with $\psi^a(z) = \phi^a - \pi^a$ and $J_q = V(iq, z)$

$$F_{\pi\gamma^*\gamma^*} = \frac{N_c}{12\pi^2}\left[\psi(z_m)J(Q_1, z_m)J(Q_2, z_m) - \int_z \partial_z\psi J_{Q_1} J_{Q_2}\right].$$

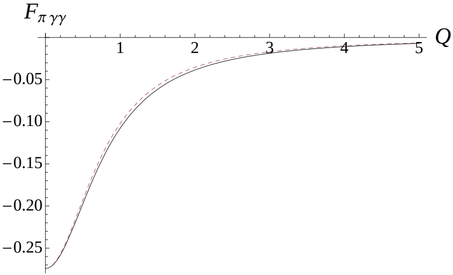


Figure: Anomalous pion form factor $F_{\pi\gamma^*\gamma^*}(Q^2, 0)$: dashed (VMD) and solid (AdS/QCD)

Anomalous pion form factors

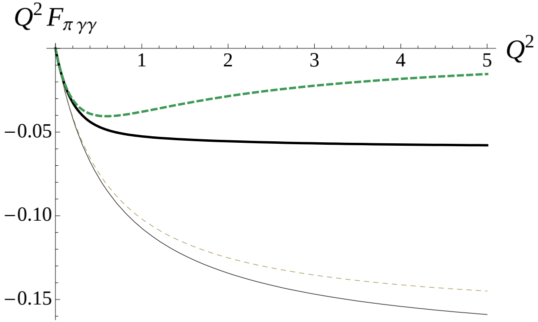
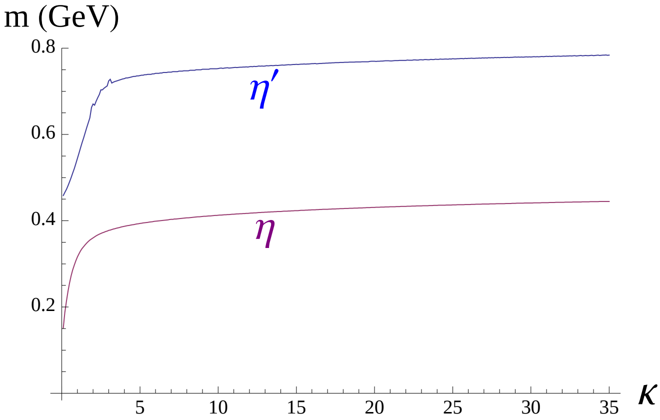


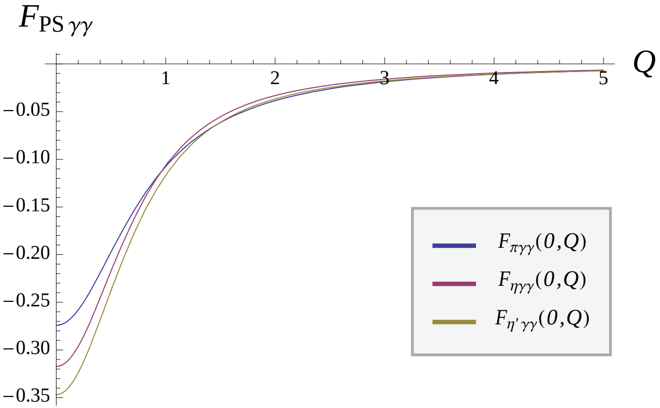
Figure: $F_{\pi\gamma^*\gamma}(Q^2, 0)$ for lower part; $F_{\pi\gamma^*\gamma^*}(Q^2, Q^2)$ for upper part (Brodsky-Lepage): solid line (AdS/QCD) and dashed line (VMD)

Anomalous form factors

- For η and η' we scan the parameter κ because of mixing ($m_q = 0.0022, m_s = 0.04$):



Anomalous form factors



Hadronic LBL in hQCD (DKH+D.Kim, PLB '09)

- To calculate the hadronic LBL contribution to a_μ we expand the photon line as

$$J(-iQ,z) = V(q,z) = \sum_{\rho} \frac{-g_5 f_\rho \psi_\rho(z)}{q^2 - m_\rho^2 + i\epsilon}$$

Table: Muon $g-2$ results from the AdS/QCD in unit of 10^{-10}.

Vector modes	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	a_μ^{PS}
4	7.5	2.1	1.0	10.6
6	7.1	2.5	0.9	10.5
8	6.9	2.7	1.1	10.7

- In the LMD+V model (Nyffeler '09)

$$a_\mu^{\mathrm{PS}} = 9.9(1.6) \times 10^{-10}$$

Hadronic LBL in hQCD (DKH+D.Kim, PLB '09)

- To calculate the hadronic LBL contribution to a_μ we expand the photon line as

$$J(-iQ,z) = V(q,z) = \sum_\rho \frac{-g_5 f_\rho \psi_\rho(z)}{q^2 - m_\rho^2 + i\epsilon}$$

Table: Muon $g-2$ results from the AdS/QCD in unit of 10^{-10}.

Vector modes	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	$a_\mu^{\rm PS}$
4	7.5	2.1	1.0	10.6
6	7.1	2.5	0.9	10.5
8	6.9	2.7	1.1	10.7

- In the LMD+V model (Nyffeler '09)

$$a_\mu^{\rm PS} = 9.9(1.6) \times 10^{-10}$$

Hadronic LBL in hQCD (DKH+D.Kim, PLB '09)

- To calculate the hadronic LBL contribution to a_μ we expand the photon line as

$$J(-iQ,z) = V(q,z) = \sum_\rho \frac{-g_5 f_\rho \psi_\rho(z)}{q^2 - m_\rho^2 + i\epsilon}$$

Table: Muon $g-2$ results from the AdS/QCD in unit of 10^{-10}.

Vector modes	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	a_μ^{PS}
4	7.5	2.1	1.0	10.6
6	7.1	2.5	0.9	10.5
8	6.9	2.7	1.1	10.7

- In the LMD+V model (Nyffeler '09)

$$a_\mu^{\mathrm{PS}} = 9.9(1.6) \times 10^{-10}$$

Conclusion

- Recent advance in gauge/gravity duality provides a possible dual of QCD, with which we obtain nonperturbative results.
- The hadronic leading correction is found to be

$$a_\mu^{\mathrm{HLO}}\Big|_{\mathrm{AdS/QCD}}^{N_f=2} = 470.5\times 10^{-10}, \quad Exp : 514.13.8\times 10^{-10}$$

- hQCD naturally explains the PS (and axial vector) dominance in the hadronic LBL contribution to muon anomalous magnetic moment.
- Consistently with previous results, we find in hQCD

$$a_\mu^{\mathrm{PS}} = 10.7\times 10^{-10}$$

upto $1/N$ and $1/\lambda$ corrections.
- Currently the axial vector contributions are being calculated.
- In the future experiment new physics might be clearly visible.

Conclusion

- Recent advance in gauge/gravity duality provides a possible dual of QCD, with which we obtain nonperturbative results.
- The hadronic leading correction is found to be

$$a_\mu^{\rm HLO}|_{\rm AdS/QCD}^{N_f=2} = 470.5 \times 10^{-10}, \quad Exp: 514.13.8 \times 10^{-10}$$

- hQCD naturally explains the PS (and axial vector) dominance in the hadronic LBL contribution to muon anomalous magnetic moment.
- Consistently with previous results, we find in hQCD

$$a_\mu^{\rm PS} = 10.7 \times 10^{-10}$$

upto $1/N$ and $1/\lambda$ corrections.
- Currently the axial vector contributions are being calculated.
- In the future experiment new physics might be clearly visible.

Conclusion

- Recent advance in gauge/gravity duality provides a possible dual of QCD, with which we obtain nonperturbative results.
- The hadronic leading correction is found to be

$$a_\mu^{\rm HLO}\big|^{N_f=2}_{\rm AdS/QCD} = 470.5 \times 10^{-10}, \quad Exp : 514.13.8 \times 10^{-10}$$

- hQCD naturally explains the PS (and axial vector) dominance in the hadronic LBL contribution to muon anomalous magnetic moment.
- Consistently with previous results, we find in hQCD

$$a_\mu^{\rm PS} = 10.7 \times 10^{-10}$$

upto $1/N$ and $1/\lambda$ corrections.
- Currently the axial vector contributions are being calculated.
- In the future experiment new physics might be clearly visible.

Conclusion

- Recent advance in gauge/gravity duality provides a possible dual of QCD, with which we obtain nonperturbative results.
- The hadronic leading correction is found to be

$$a_\mu^{\rm HLO}|_{\rm AdS/QCD}^{N_f=2} = 470.5 \times 10^{-10}, \quad Exp : 514.13.8 \times 10^{-10}$$

- hQCD naturally explains the PS (and axial vector) dominance in the hadronic LBL contribution to muon anomalous magnetic moment.
- Consistently with previous results, we find in hQCD

$$a_\mu^{\rm PS} = 10.7 \times 10^{-10}$$

upto $1/N$ and $1/\lambda$ corrections.

- Currently the axial vector contributions are being calculated.
- In the future experiment new physics might be clearly visible.