Holographic QCD and the HLBL

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Why holography for HLBL?

- **Holographic principle**: Conjectured duality between strongly coupled gauge theories in d dimensions and gravitational weakly coupled theories in $(d+1)$ dimensions. AdS/CFT one of the most powerful examples.

- Lagrangian formulation: consistent treatment of the different channels (no double counting).

- (leading) short distances: automatically implemented due to the conformal invariance of AdS.

- Large-N_c: full realization of the large-N_c limit (∞ resonances with short distances).

- number of parameters: minimal.

Motivating questions

- Which are the parameters that mostly affect the uncertainty on the HLBL?

- Given
$$K(Q_1^2,Q_2^2)\simeq 1+\hat{\alpha}\,(Q_1^2+Q_2^2)+\hat{\beta}\,Q_1^2Q_2^2+\hat{\gamma}\,(Q_1^4+Q_2^4)$$
what is the impact of the low energy parameters α, β and γ? α determined by CLEO, but γ and β out of current experimental reach. Can one obtain reasonable predictions?

- Is vector meson dominance a good enough approximation?

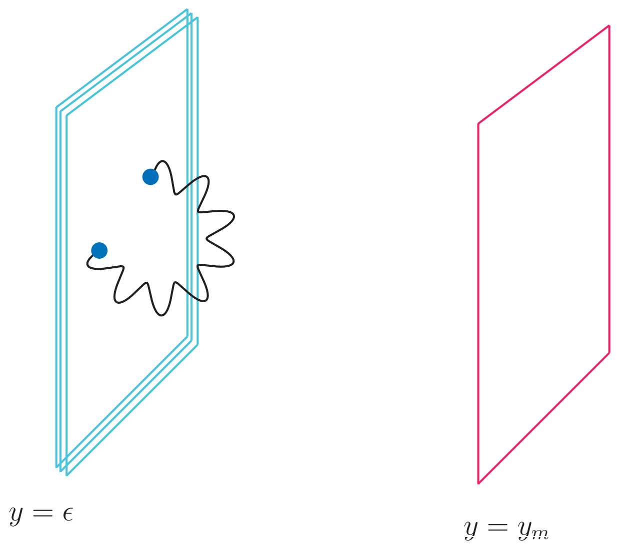
- How accurate is the pion-pole approximation?

The setting

- AdS$_5$ space with metric

$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{y^2}(-dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

with $\eta_{\mu\nu}$ mostly negative.



- Intuitively, UV boundary captures pQCD (leading term trivial: conformal limit reproduced by AdS metric). IR deals with non-perturbative physics (OPE and non-OPE, confinement scale, SχSB, spectrum patterns)

Holographic models

- There are different realizations of the holographic principle for QCD. Generically,

$$S_5 = S_{\rm YM} + S_X + S_{\rm CS}$$

where

$$\begin{aligned} S_{\rm YM}\left[\mathcal{B}_L, \mathcal{B}_R\right] &= {\rm tr} \int d^4x \int_0^{z_0} dz\ e^{-\Phi(z)} \frac{-1}{8g_5^2} w(z) \left[\mathcal{F}_{(L)}^{MN}\mathcal{F}_{(L)MN} + \mathcal{F}_{(R)}^{MN}\mathcal{F}_{(R)MN}\right] \\ S_X\left[X\right] &= {\rm tr} \int d^4x \int_0^{z_0} dz\ e^{-\Phi(z)} w(z)^3 \left[D^M X D_M X^\dagger + V(X^\dagger X)\right] \\ S_{\rm CS}\left[\mathcal{B}_L, \mathcal{B}_R\right] &= \frac{N_c}{24\pi^2} \int {\rm tr} \left[\mathcal{B}_L \mathcal{F}_L^2 - \frac{i}{2}\mathcal{B}_L^3\mathcal{F}_L - \frac{1}{10}\mathcal{B}_L^5 - (L \to R)\right] \end{aligned}$$

- Notice that chiral symmetry is incorporated.
- The models differ mostly in S_X (spontaneous symmetry breaking) and $(\Phi(z), z_0)$ (spectrum).
- Major drawback: plethora of models...
- Advantage: stability tests of phenomenological results.

The $F_{\pi^0\gamma\gamma}$ form factor

- From holography $F_{\pi^0\gamma\gamma}$ can be extracted from the Chern-Simons 5-dimensional term: [Grigoryan et al'08,09]

$$S_{\mathrm{CS}}[\mathcal{B}] = \frac{N_c}{24\pi^2}\int \mathrm{tr}\left[\mathcal{B}_L\mathcal{F}_L^2 - \mathcal{B}_R\mathcal{F}_R^2\right] + \cdots$$
$$= \frac{N_c}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}\int_0^{z_0} dz\,(\partial_z\beta)\int d^4x\ \pi^a\left(\partial_\rho V_\mu^a\right)\left(\partial_\sigma \hat{V}_\nu\right)$$

- Applying the holographic recipe for correlators:

$$K(Q_1^2,Q_2^2) = -\int_0^{z_0}\mathcal{J}(Q_1,z)\mathcal{J}(Q_2,z)\,\partial_z\beta(z)\,dz$$

- For instance,

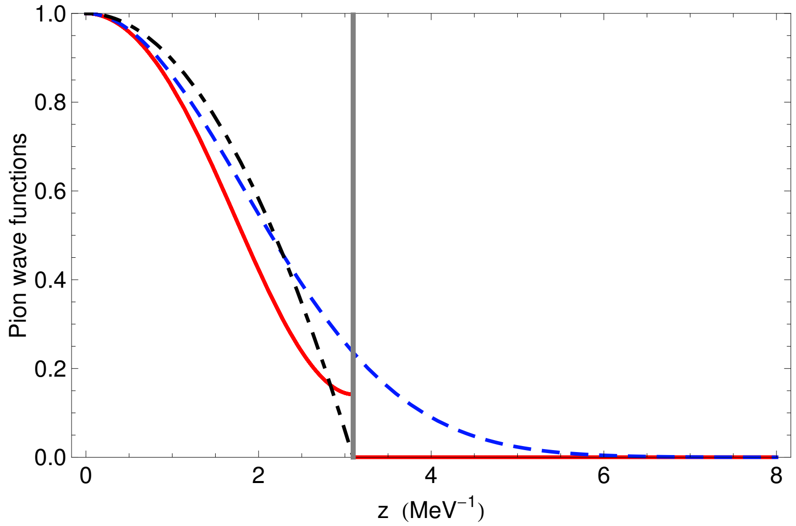
$$\mathcal{J}(Q,z) = Qz\left[K_1(Qz) + I_1(Qz)\frac{K_0(Qz_0)}{I_0(Qz_0)}\right]$$
$$\beta(z) = 1 - \frac{z^2}{z_0^2}$$

The $F_{\pi^0\gamma\gamma}$ form factor (high energies)

- Master formula for the different holographic models. It only depends on z_0, which for consistency (matching with the $\rho(770)$ mass) has to be

$$z_0 = \frac{2}{g_5^2 f_\pi} = \frac{N_c}{6\pi^2 f_\pi}$$

- In any model with (asymptotic) AdS metric, it can be shown that (at least) the leading short distance constraints from QCD are satisfied. **Not surprising**: the asymptotic behavior of $\mathcal{J}(Q,z)$ fixed by AdS, while the pion wave function is only relevant at the origin:



- ... but **non-trivial**. For instance, $m_\rho^2 = 8\pi^2 f_\pi^2$.

The $F_{\pi^0\gamma\gamma}$ form factor (low energies)

- Low energies will depend on the specific model. At small virtualities one can expand $K(Q_1^2, Q_2^2)$ in the form

$$K(Q_1^2,Q_2^2) \simeq 1 + \hat{\alpha}\,(Q_1^2+Q_2^2) + \hat{\beta}\,Q_1^2Q_2^2 + \hat{\gamma}\,(Q_1^4+Q_2^4)$$

- Experimentally, $\hat{\alpha} = -1.76 \pm 0.22$, $\hat{\gamma}$ waiting for better statistics while $\hat{\beta}$ challenging (2 virtual photons needed).
- Predictions from holographic models:

Model	$\hat{\alpha}$ (GeV^{-2})	$\hat{\beta}$ (GeV^{-4})	$\hat{\gamma}$ (GeV^{-4})
HW1	-1.60	3.01	2.63
HW2 (AdS)	-1.81	3.65	3.06
HW2 (Flat)	-1.37	2.25	2.25
SS	-2.04	4.56	3.55
SW	-1.66	3.56	2.76

- Compliance with experiment used as a holographic model filter. From the "acceptable" models we can then estimate the quartic terms to be

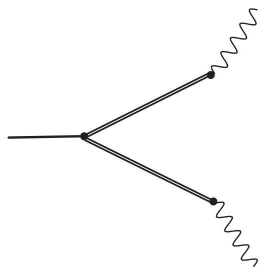
$$\hat{\beta} = 3.33 \pm 0.32$$
$$\hat{\gamma} = 2.84 \pm 0.21$$

Vector meson dominance revisited

- Spectral decomposition:

$$\mathcal{J}(z,Q)=\sum_{n=1}^{\infty}\frac{f_n}{Q^2+m_n^2}\psi_n(z); \qquad K(Q_1^2,Q_2^2)\equiv\sum_{k,l=1}^{\infty}\frac{B_{kl}}{(Q_1^2+m_k^2)(Q_2^2+m_l^2)}$$

- Graphically:



- Each resonance contribution can be evaluated

Model	$\hat{\alpha}_n/\hat{\alpha}$			$\hat{\beta}_n/\hat{\beta}$			$\hat{\gamma}_n/\hat{\gamma}$		
HW1	1.20	-0.18	-0.04	1.10	-0.06	0.01	1.20	-0.22	0.06
HW2 (AdS)	1.30	-0.37	0.06	1.10	-0.11	0.01	1.30	-0.37	0.08
HW2 (flat)	0.99	0.01	0.00	1.00	0.00	0.00	1.00	0.00	0.00
SS	1.70	-1.10	0.49	1.30	-0.34	0.07	1.60	-1.10	0.54
SW	0.75	0.14	0.05	0.87	0.09	0.02	0.88	0.09	0.02

- Lowest meson dominance qualitatively holds. Enough for the accuracy of HLBL?

Estimation of the π^0-HLBL

- Strategy: encode the short and long distance information in an interpolator (DIP):

$$K(q_1^2,q_2^2)=1+\lambda\left(\frac{q_1^2}{q_1^2-m_V{}^2}+\frac{q_2^2}{q_2^2-m_V{}^2}\right)+\eta\frac{q_1^2q_2^2}{(q_1^2-m_V{}^2)(q_2^2-m_V{}^2)}$$

- Used before as low energy parameterization for kaon decays [D'Ambrosio et al'97]
- Consistent with the anomaly and Bose symmetry.
- Parameters easily interpreted in terms of $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$.
- Interesting feature: it can be cast in the general form

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)=-\frac{N_c}{12\pi^2 f_\pi}\left[f(q_1^2)-\sum_i\frac{1}{q_2^2-m_i^2}g_i(q_1^2)\right]$$

with

$$\begin{aligned} f(q^2) &= 1+\lambda+(\lambda+\eta)\frac{q^2}{q^2-m_V^2} \\ g(q^2) &= -m_V^2\left[\lambda+\eta\frac{q^2}{q^2-m_V^2}\right] \end{aligned}$$

which allows to easily perform the two-loop integrals as [Knecht et al'01]

$$a_\mu^{\pi^0}=\left(\frac{\alpha_{em}}{\pi}\right)^3\int_0^\infty dQ_1\int_0^\infty dQ_2\Big[w_1G_1+w_2(m_V)G_2+w_3(m_V)G_3+w_3(m_\pi)G_4\Big]$$

Numerical analysis

- Our parameterization has 3 free parameters, λ, η and m_V, to be fixed by

$$\begin{aligned} 1+2\lambda+\eta &= 0\,, && \text{(OPE)} \\ \lambda+\eta &= -\frac{4\pi^2 f_\pi^2}{3m_V^2}\,, && \text{(OPE)} \\ \frac{\lambda}{m_V^2} &= -1.76\pm 0.22\ \ (=\hat{\alpha}) && \text{(exp.)} \end{aligned}$$

- This yields

$$\begin{aligned} \lambda &= -0.73\pm 0.05\,, \\ \eta &= 0.46^{+0.10}_{-0.13}\,, \\ m_V &= (0.64^{+0.07}_{-0.06})\,\text{GeV}\,, \\ \chi_0 &= \frac{N_c}{4\pi^2 f_\pi^2}(1+\lambda) = (2.42\pm 0.17)\,\text{GeV}^{-2} \end{aligned}$$

Several comments:

- $\eta = m_V^4\hat{\beta}$, fixed by short distance constraints, agrees very well with holographic predictions. Recall that $\lambda = m_V^2\hat{\alpha}$ also does.
- $m_V = 0.64^{+0.07}_{-0.06}$ GeV determined dynamically. In LMD models one assumes $m_V \equiv m_\rho$. In our case m_V comes from using the experimental slope of $F_{\pi^0\gamma\gamma}$. Effective resummation of ∞ resonances.

- DIP naturally gives $\chi_0 \neq 0$. Therefore, it provides one way to estimate the 'off-shellness' through a short distance constraint.
- Result for $HLBL$:

Model	$w_1 G_1$	$w_2 G_2$	$w_3 G_3$	$w_4 G_4$	a_μ
LMD	$+0.015$	$+0.042$	$+0.0016$	-0.0002	$7.3 \cdot 10^{-10}$
DIP$_{\hat{\alpha}}$	$+0.018(3)$	$+0.034(4)$	$+0.0016$	-0.0002	$6.7(3) \cdot 10^{-10}$
DIP$_{m_\rho}$	$+0.015$	$+0.043$	$+0.0016$	-0.0002	$7.35 \cdot 10^{-10}$

- Difference with LMD only due to the vector mass scale. With long distance constraints, no need to take $m_V = m_\rho$.
- Intriguing: parameterization dependence (DIP vs VMD) seems to be extremely small.
- Contribution dominated by $w_1 G_1, w_2 G_2$, peaked at (very) low energies.

Refinements and sanity checks

- We would like to use holographic input ($\hat{\beta}$, $\hat{\gamma}$) and check for stability.
- Simplest generalised version of the interpolator:

$$K(q_1,q_2)=1+\sum_i^2\lambda_i\left(\frac{q_1^2}{q_1^2-m_i^2}+\frac{q_2^2}{q_2^2-m_i^2}\right)+\sum_i^2\eta_i\frac{q_1^2q_2^2}{(q_1^2-m_i^2)(q_2^2-m_i^2)}$$

- It still satisfies

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)=-\frac{N_c}{12\pi^2 f_\pi}\left[f(q_1^2)-\sum_i\frac{1}{q_2^2-m_i^2}g_i(q_1^2)\right]$$

- Two candidates:

$$\begin{aligned}K_{(1)}(q_1,q_2) &= 1+\lambda\left(\frac{q_1^2}{q_1^2-m_1^2}+\frac{q_2^2}{q_2^2-m_1^2}\right)+\sum_i^2\eta_i\frac{q_1^2q_2^2}{(q_1^2-m_i^2)(q_2^2-m_i^2)}\\ K_{(2)}(q_1,q_2) &= 1+\sum_i^2\lambda_i\left(\frac{q_1^2}{q_1^2-m_i^2}+\frac{q_2^2}{q_2^2-m_i^2}\right)+\eta\frac{q_1^2q_2^2}{(q_1^2-m_1^2)(q_2^2-m_1^2)}\end{aligned}$$

Constraints

- The constraints we will impose now are the following:

  (a) Long distance constraints:

$$\hat{\alpha} = \sum_i^2 \frac{\lambda_i}{m_i^2} \quad = \quad -1.76 \pm 0.22 \ , \ \ \text{(exp.)}$$

$$\hat{\beta} = \sum_i^2 \frac{\eta_i}{m_i^4} \quad = \quad 3.33 \pm 0.32 \ , \ \ \text{(holography)}$$

$$\hat{\gamma} = -\sum_i^2 \frac{\lambda_i}{m_i^4} \quad = \quad 2.84 \pm 0.21 \ , \ \ \text{(holography)}$$

  (b) Short distance constraints:

$$1 + 2\sum_i^2 \lambda_i + \sum_i^2 \eta_i \quad = \quad 0 \ ,$$

$$\sum_i^2 m_i^2(\lambda_i + \eta_i) \quad = \quad -\frac{4\pi^2 f_\pi^2}{3} \ ,$$

$$1 + \sum_i^2 \lambda_i \quad = \quad -\frac{4\pi^2 f_\pi^2}{3}\chi_0 \ .$$

- Slightly over-constrained.

Results

Model	$w_1 G_1$	$w_2 G_2$	a_μ
DIP$_{\hat{\alpha}}$	$+0.018(3)$	$+0.034(4)$	$6.7(3) \cdot 10^{-10}$
DIP$^{(1)}_{\hat{\beta};m_2=m_\rho}$	$+0.014$	$+0.037$	$6.52(15)(10) \cdot 10^{-10}$
DIP$^{(2)}_{\hat{\beta};m_2=m_\rho}$	$+0.014$	$+0.037$	$6.55(21)(6) \cdot 10^{-10}$
DIP$^{(1)}_{\hat{\beta};0<\chi_0<8.9}$	$[+0.003; +0.047]$	$[+0.043; +0.022]$	$[5.9; 8.9] \cdot 10^{-10}$
DIP$^{(2)}_{\hat{\beta};0<\chi_0<4.4}$	$[+0.002; +0.027]$	$[+0.044; +0.025]$	$[6.0; 6.7] \cdot 10^{-10}$

- $\hat{\alpha}$ drives the error over $\hat{\beta}$ and $\hat{\gamma}$.
- If I don't look at χ_0:

$$a_\mu = 6.54(25) \cdot 10^{-10}$$

  compatible with previous estimates. The uncertainty is driven by the experimental situation on $\hat{\alpha}$, whereas (holographic) model dependences turn out to be negligible.
- Notice stability: with the simplest interpolator, $a_\mu = 6.7(3) \cdot 10^{-10}$.

Recent determinations

- Most recent estimates for the π^0 and pseudoscalar exchange:

Model for $\mathcal{F}_{P^{(*)}\gamma^*\gamma^*}$	$a_\mu^{\mathrm{LbyL};\pi^0} 10^{-11}$	$a_\mu^{\mathrm{LbyL;PS}} 10^{-11}$
modified ENJL [BPP]	59(9)	85(13)
VMD / HLS [HKS, HK]	57(4)	83(6)
nonlocal χQM [DB]	65(2)	-
AdS/QCD [HoK]	69	107
LMD [KN]	73	-
LMD+V [KN1]	58(10)	83(12)
LMD+V [KN2]	63(10)	88(12)
LMD+V [MV]	77(7)	114(10)
LMD+V [N]	72(12)	99(16)
DIP	65(2)	-

The status of χ_0

- Showing up in the (off-shell) short distance constraint:

$$\lim_{Q^2\to\infty} F_{\pi^0\gamma\gamma}(Q^2,Q^2,0) = \frac{f_\pi}{3}\chi_0 + \cdots$$

- Determinations range from $0 \sim \chi_0 \sim 9$ GeV^{-2}.

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	χ_0
DIP$_{\hat{\alpha}}$	-1.76^*	2.67	4.25	2.42
DIP$_{m_\rho}$	-1.35	1.73	2.25	1.66
DIP$^{(1)}_{\hat{\beta}}$	-1.76^*	3.33^*	3.78	1.61
DIP$^{(2)}_{\hat{\beta}}$	-1.76^*	3.33^*	3.88	1.69
DIP$^{(1)}_{\hat{\beta},\chi_0}$	-1.76^*	3.33^*	$[3.10; -5\cdot 10^5]$	$[0; 8.9]^*$
DIP$^{(2)}_{\hat{\beta},\chi_0}$	-1.76^*	3.33^*	$[3.19; -3.18]$	$[0; 4.4]^*$

- Our analysis favors lower values of χ_0. If so, mild effect and pion-pole approximation very successful. However, χ_0 9 GeV^{-2} might amount to $10-15\%$ systematic deviations in Hlbl.

First results and Outlook

- Holographic models of QCD aim at providing a full description of hadronic processes. Extremely useful for the HLBL (all-channel, all-state approach) where so far each contribution is evaluated in a fragmentary way (interpolators for different channels). In contrast, with holographic models one starts from one Lagrangian (predictability, unitarity, Green functions highly constrained).

- This is not (yet) an holographic determination of HLBL, rather a 'classic' approach with new ingredients: DIP interpolator, low energy parameters determined from holography. Full calculation underway.

- We provided: (a) predicted values for $\hat{\beta}$ and $\hat{\gamma}$ from holographic models (consistent with short distance QCD constraints and the experimental slope of the pion form factor), soon to be checked at KLOE-2; (b) assessed the quantitative validity of LMD in the pion-exchange diagram; (c) estimated the impact of 'pion off-shellness' through χ_0.

- No free lunch: holographic models are not free from assumptions, but they open interesting avenues to study the hadronic contributions to $(g-2)_\mu$.