Hadronic light-by-light contribution to the muon g-2 from lattice QCD+QED

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with

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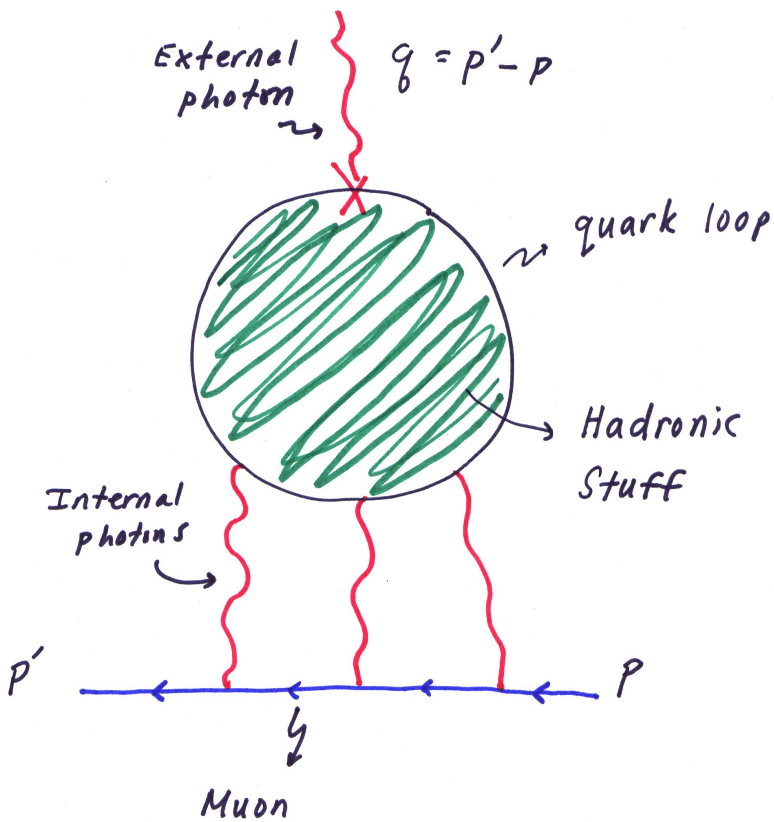
Outline of the talk

- Method
- Tests in pure QED
- Towards full QCD+QED result
- Summary

The hadronic light-by-light contribution ($O(\alpha^3)$)



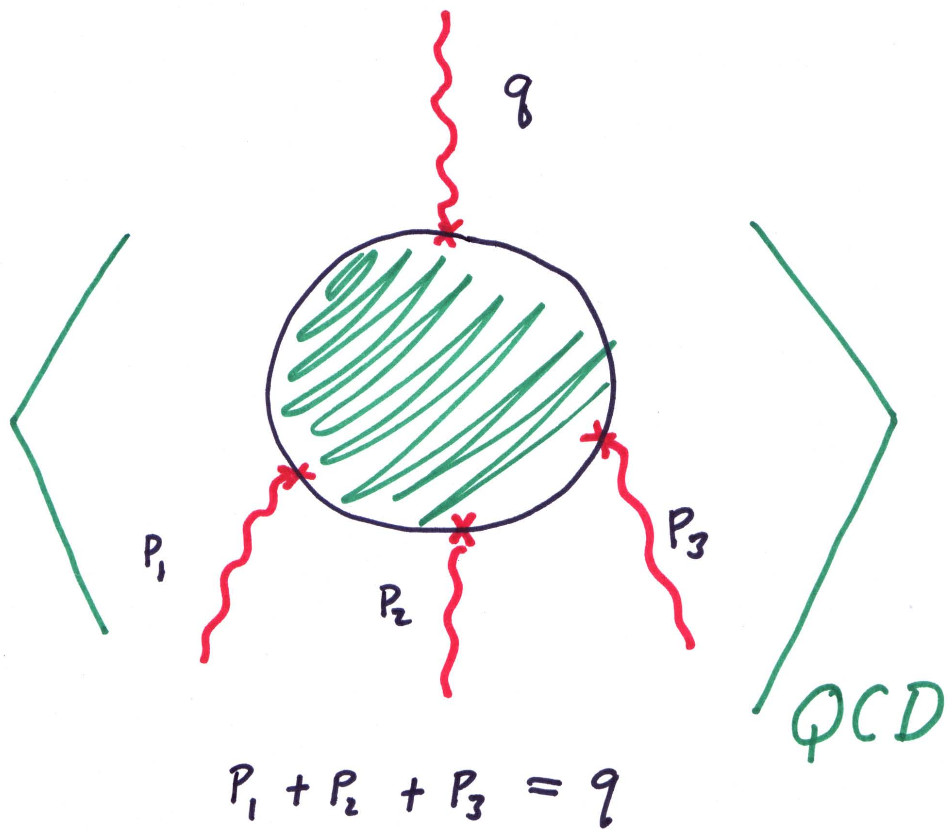
Model estimates put this $\mathcal{O}(\alpha^3)$ contribution at about $(10-12)\times 10^{-10}$ with a 25-40% uncertainty

Blob contains all possible hadronic states

No dispersion relation *a'la* vacuum polarization

Lattice regulator: model independent, approximations systematically improvable

Conventional approach (QCD only on the lattice)



Correlation of 4 currents

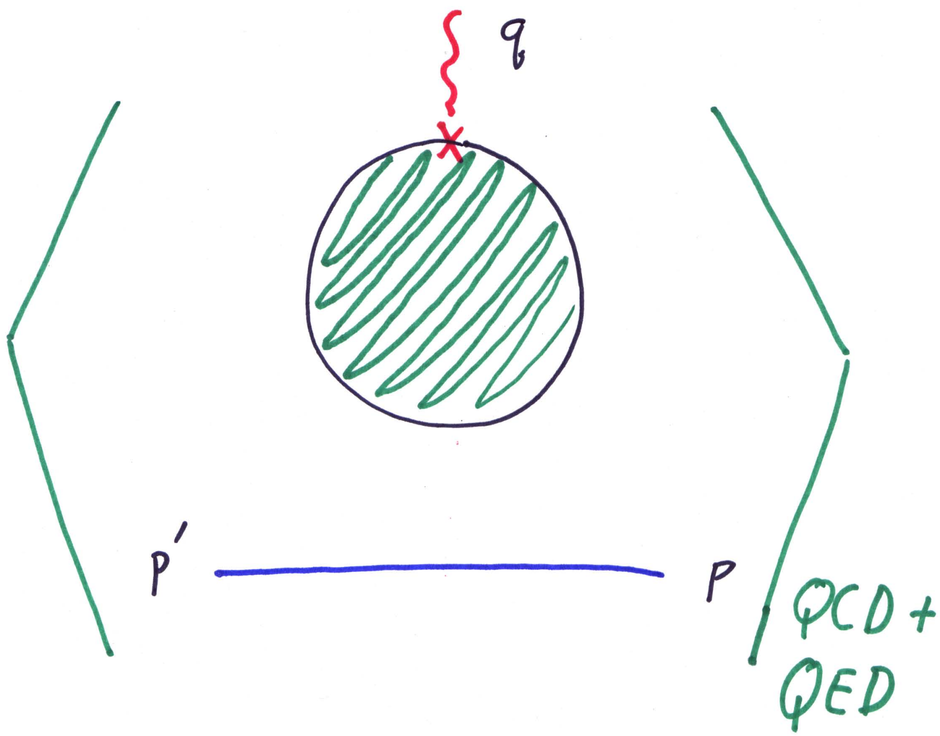
Two independent momenta
+external mom q

Compute for all possible values of p_1 and p_2,
four index tensor

several q, (extrap. $q \to 0$),
plug into perturbative QED two-loop integrals

Pursued by Rakow, *et al*,
(QCDSF collaboration)

New approach QCD and QED on the lattice



Average over combined gluon *and* photon gauge configurations

Quarks coupled to gluons and photons

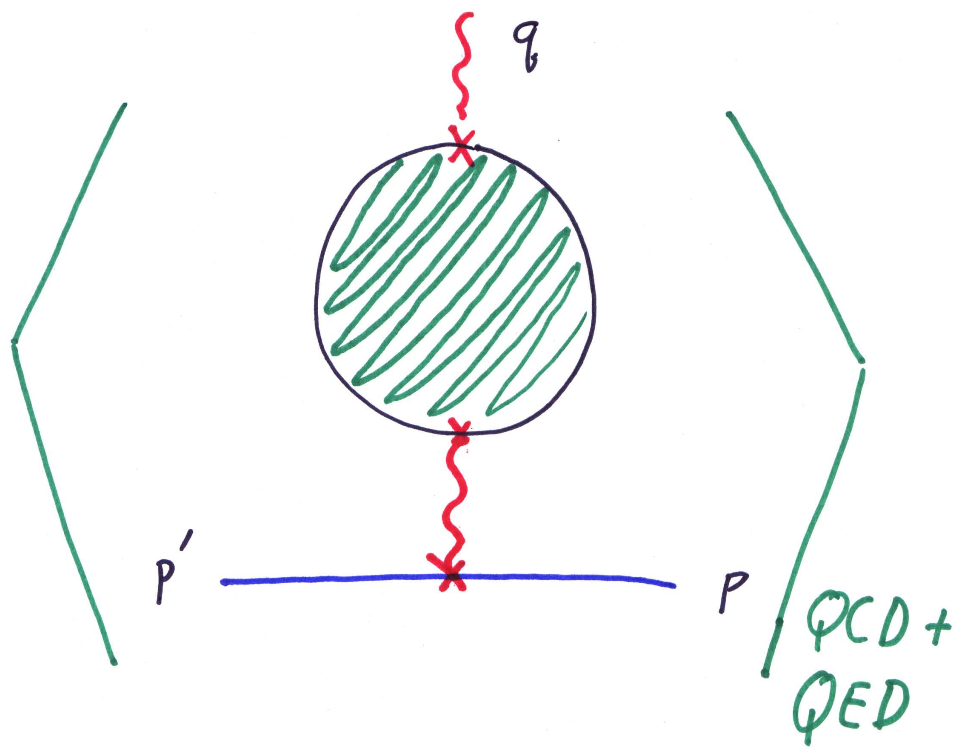
muon coupled to photons

[hep-lat/0509016;

Chowdhury *et al.* (2008);

Chowdhury Ph. D. thesis (2009)]

New approach...



Attach one photon by hand (see why in a minute)

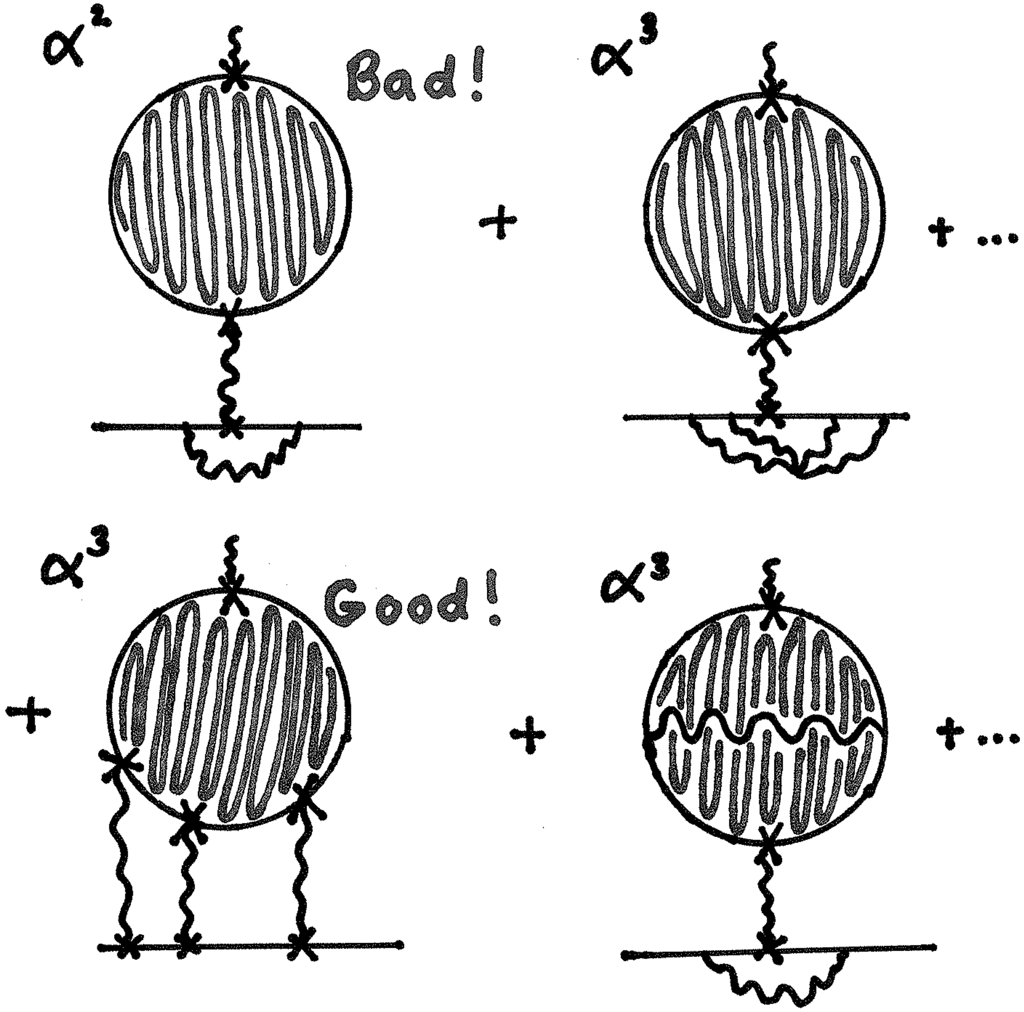
Correlation of hadronic loop and muon line

[hep-lat/0509016;

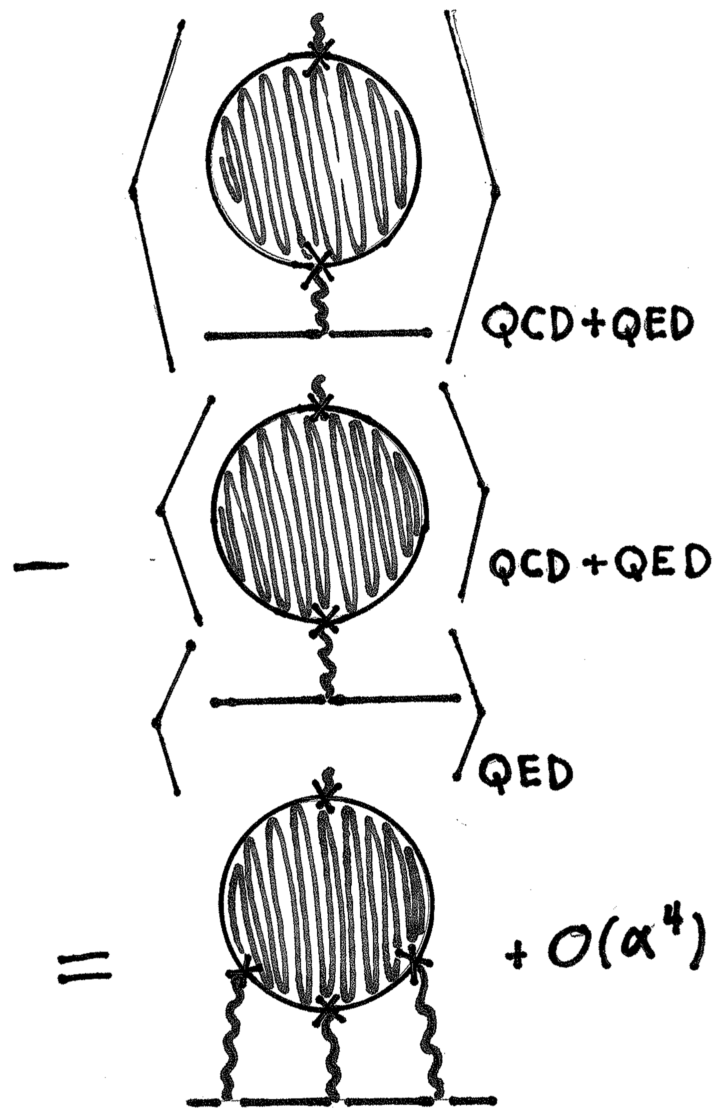
Chowdhury *et al.* (2008);

Chowdhury Ph. D. thesis (2009)]

The leading and next-leading contributions in α to magnetic part of correlation function come from



Subtraction of lowest order piece:

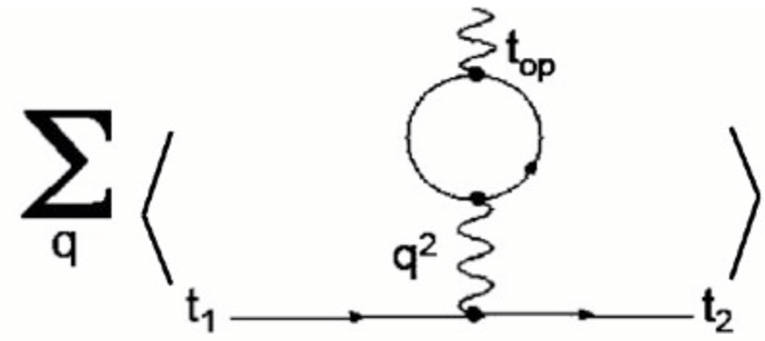


Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

Test calculation in pure QED [Chowdhury thesis]
(Compare to well known PT result)



- Incoming muon has $\vec{p} = 0$, outgoing $\vec{p}' = -\vec{p}_{op} = (1, 0, 0)$ (+ perms).
- external muons put on shell in usual way ($t_1 \ll t_{\rm op} \ll t_2$)
  $t_1 = 0$, $t_{\rm op} = 4, 6, 8$, $t_2 = 12$
- Single lattice size, $16^3 \times 32$
- (4d) Fourier transform "loop" and "line" separately
- to enhance signal, take $e = 1$, or $\alpha = 1/4\pi$
- Subtraction difficult in practice since need to average loop and line separately over gauge fields first, then sum over q^2.

more details

- Domain wall fermions (to match 2+1 QCD ensembles) ($L_s = 8$)
- loop/line masses degenerate: m_μ, $m_e = 0.4$ (heavy), and loop mass $= 0.01$
- Non-compact, quenched QED (easy to generate, fewer lattice artifacts)
- Fix to Feynman gauge (photon propagator is simple)
- O(100-1000) configurations (measurements)

Extracting the form factors

$$G^\mu(t',t) \;=\; \langle \psi(t',\vec{p}')\, J^\mu(t,q)\, \psi^\dagger(0,\vec{p})\rangle .$$

Insert two complete sets of states, take $t' \gg t \gg 0$,

$$\begin{aligned} G^\mu(t',t) &= \sum_{s,s'} \langle 0|\chi_N|p',s'\rangle\langle p',s'|J^\mu|p,s\rangle\langle p,s|\chi_N^\dagger|0\rangle \frac{1}{2E\,2E'} e^{-E'(t'-t)} e^{-Et} + \ldots \\ &= G^\mu(q^2) \times f(t,t',E,E') + \ldots, \end{aligned}$$

(like LHZ reduction, but in Euclidean space)

For example, $J^\mu = J^x$,

$$\begin{aligned} \mathrm{tr}\mathcal{P}^{xy}\, G^x(q^2) &= p_y\, m(F_1(q^2) + F_2(q^2)) \\ \mathcal{P}^{xy} &= \frac{i}{4}\frac{1+\gamma^t}{2}\gamma^y\gamma^x \end{aligned}$$

Similarly,

$$\begin{aligned} \mathrm{tr}\mathcal{P}^{t}\, G^t(q^2) &= m\,(E+m)\left(F_1(q^2) + \frac{q^2}{(2m)^2}F_2(q^2)\right), \\ \mathcal{P}^{t} &= \frac{1}{4}\frac{1+\gamma^t}{2}, \end{aligned}$$

Check Ward-Takahashi Identities

Use conserved (lattice) currents

WTI satisfied on each configuration

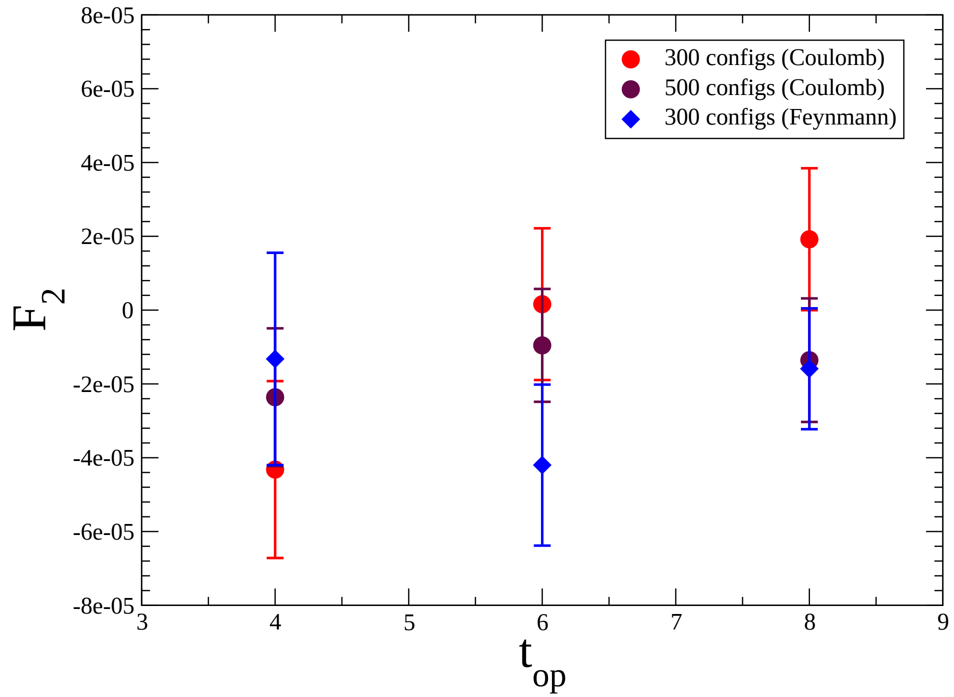
The loop is checked using $q^\mu \Pi^{\mu\nu} = 0$, same as for vacuum polarization

The muon-line is a little more complicated

$$\begin{aligned} -iq_\rho \langle \psi(\vec{p_2}, t_2) J_\rho \bar{\psi}(\vec{p_1}, t_1) \rangle &= \\ e^{iq_4 t_2} \langle \psi(\vec{q} + \vec{p_2}, t_2) \bar{\psi}(\vec{p_1}, t_1) \rangle \delta(\vec{q} + \vec{p_2} - \vec{p_1}) & \\ -e^{iq_4 t_1} \langle \psi(\vec{p_2}, t_2) \bar{\psi}(\vec{q} - \vec{p_1}, t_1) \rangle \delta(\vec{p_2} - \vec{q} - \vec{p_1}) & \end{aligned}$$

Checked for free case and non-trivial gauge field

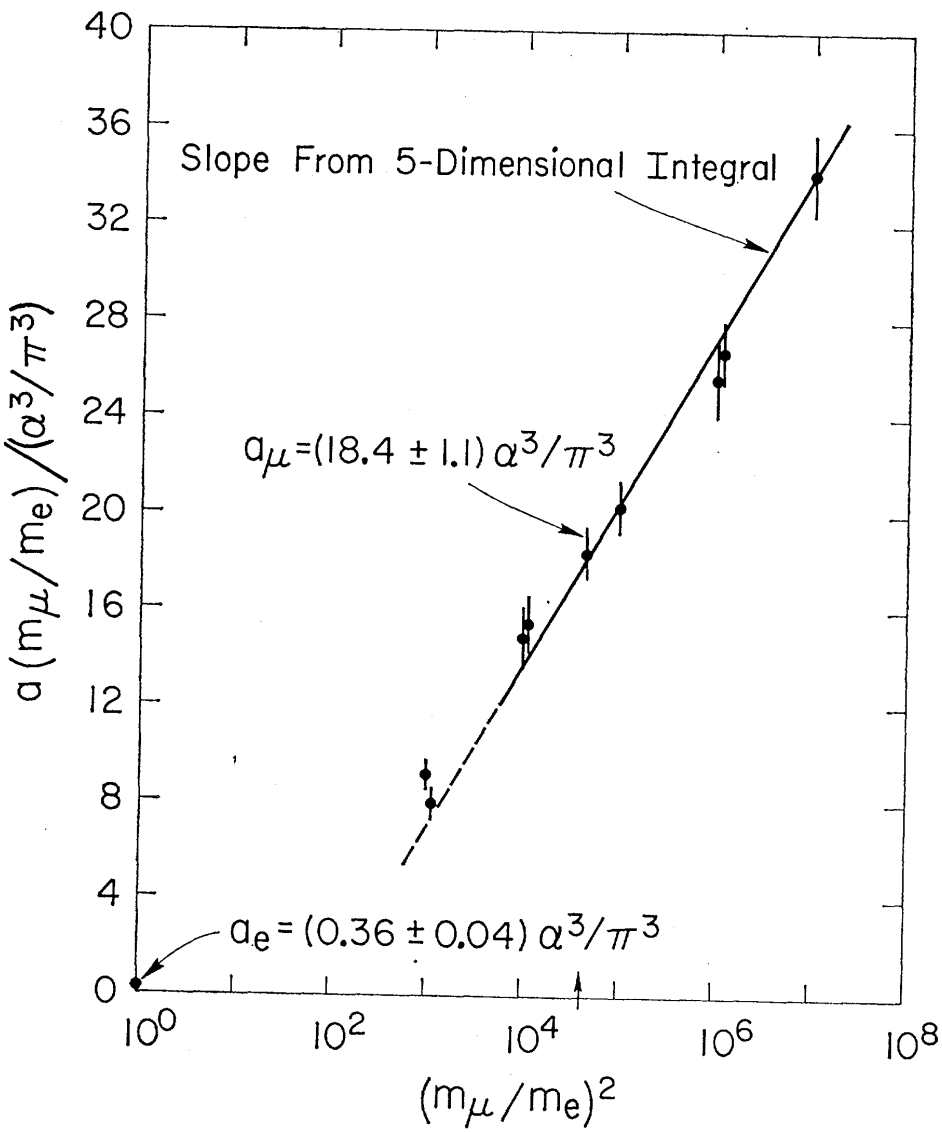
$F_2(0)$ (QED only, degenerate leptons)



$F_2 = (-0.50 \pm 0.37) \times 10^{-5}$ (lowest non-zero momentum, **stat error only**)

Continuum PT result: $0.36(\alpha/\pi)^3 = 0.585 \times 10^{-5}$ $(e = 1)$

Statistical error same order as PT result



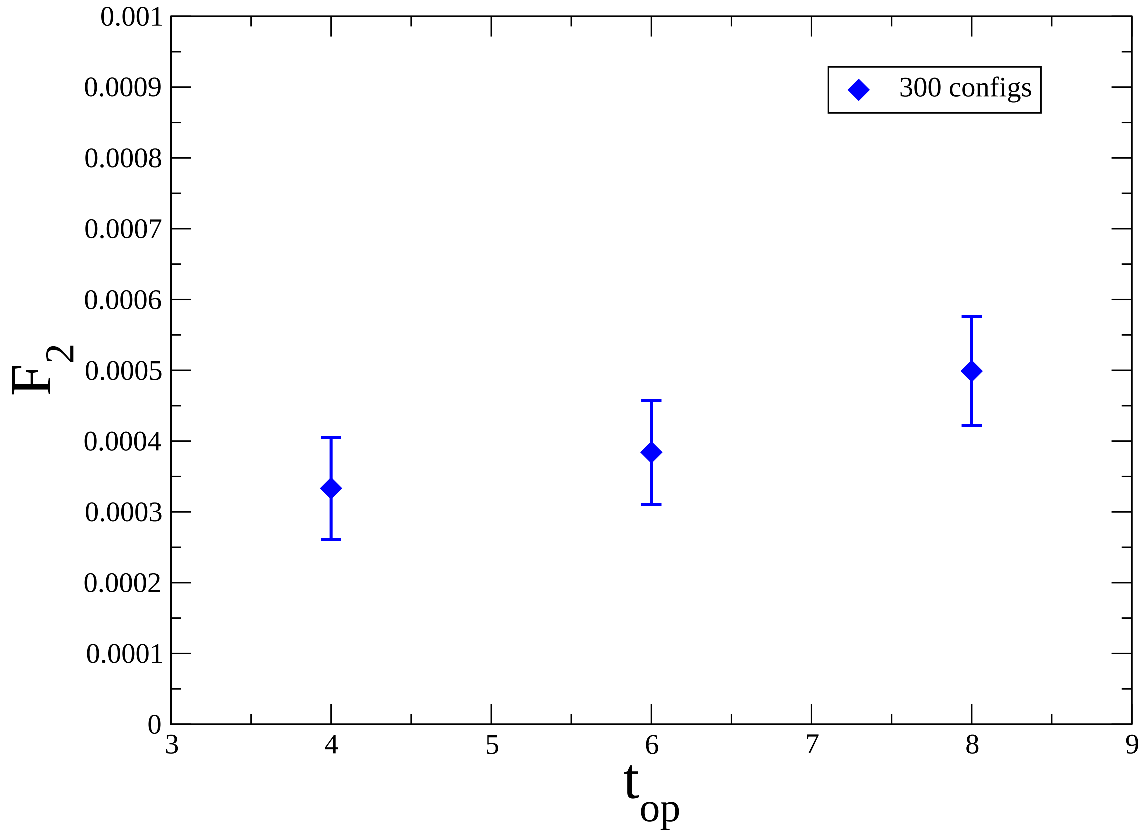
[Aldins, Brodsky, Duffner, Kinoshita (1970)]

Large m_μ/m_e enhancement seen in perturbation theory

Try $m = 0.01$ in the loop, or $m_\mu/m_e = 40$

Finite volume effects could be large

F_2 $m_\mu/m_e = 40$ (QED only)



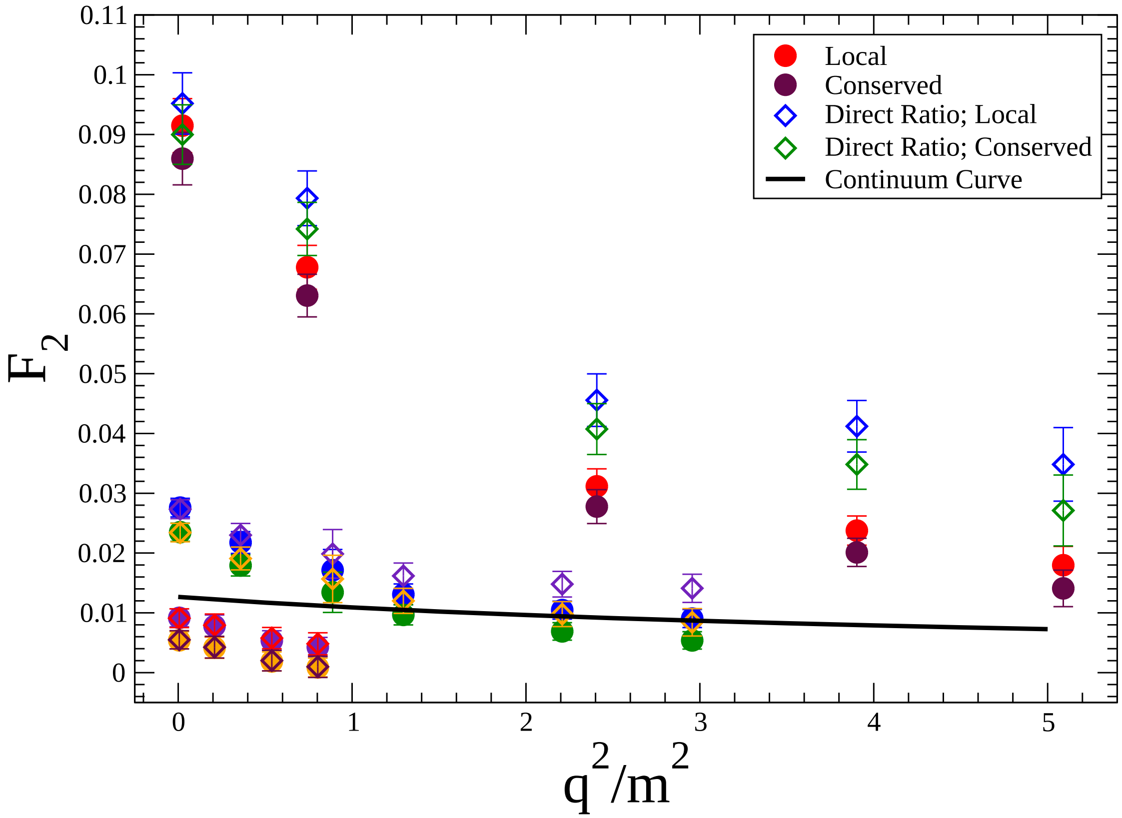
$F_2 = (1.32 \pm 0.13) \times 10^{-4}$ (lowest non-zero momentum, **stat error only**)

Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ $(e = 1)$

roughly consistent with PT result

Finite volume effects (QED only, **Schwinger** term)

Large finite volume effect in $O(\alpha)$ Schwinger term ($e = 1$, $m_\mu = 0.2$):



F_2 in 2+1 flavor QCD+QED

Include hadronic part in the loop only (same in subtraction)

2+1 flavors of DWF (RBC/UKQCD)

$a = 0.114$ fm, $16^3 \times 32\,(\times 16)$, $a^{-1} = 1.73$ GeV

$m_q \approx 0.013$ ($m_\pi \approx 400$ MeV)

~ 1000 configurations (one QED conf. for each QCD conf.)

$F_2 = (-5.3 \pm 6.0) \times 10^{-5}$ (lowest non-zero momentum, $e = 1$)

Magnitude of error is about 13$\times$ model estimates

model calculations (physical mass and charge) about 200 times smaller than QED light-by-light contribution.

Signal has disappeared, but statistical error stayed about the same ($m_\mu/m_\pi \approx 2$)

Low mode average

Need large improvement in statistics

Current method uses point source at external vertex

Gauge (ensemble) average enforces momentum conservation

Volume average (fourier transform) would improve statistics by $O(V)$
(project onto $\vec{q} = \vec{p}' - \vec{p}$)

V point source propagators (D^{-1}'s) too expensive

Low-mode average instead

Low mode average

Just usual spectral decomposition of the (Hermitian) Dirac operator

$$
\begin{aligned}
\not{D}_H &= \gamma_5 \not{D} \\
\not{D}_H \psi_\lambda &= (\lambda + m)\psi_\lambda \\
\not{D}^{-1} &= \not{D}_H^{-1} \gamma_5 = \sum_\lambda \frac{\psi_\lambda \psi_\lambda^\dagger \gamma_5}{\lambda + m}
\end{aligned}
$$

Still too expensive to calculate all the eigenmodes. Compute the low-modes, and treat the high(er) ones in the usual way with a stochastic source(s)

generate "all-to-all" propagator

Low mode average

Test calculation underway, uses same parameters as before

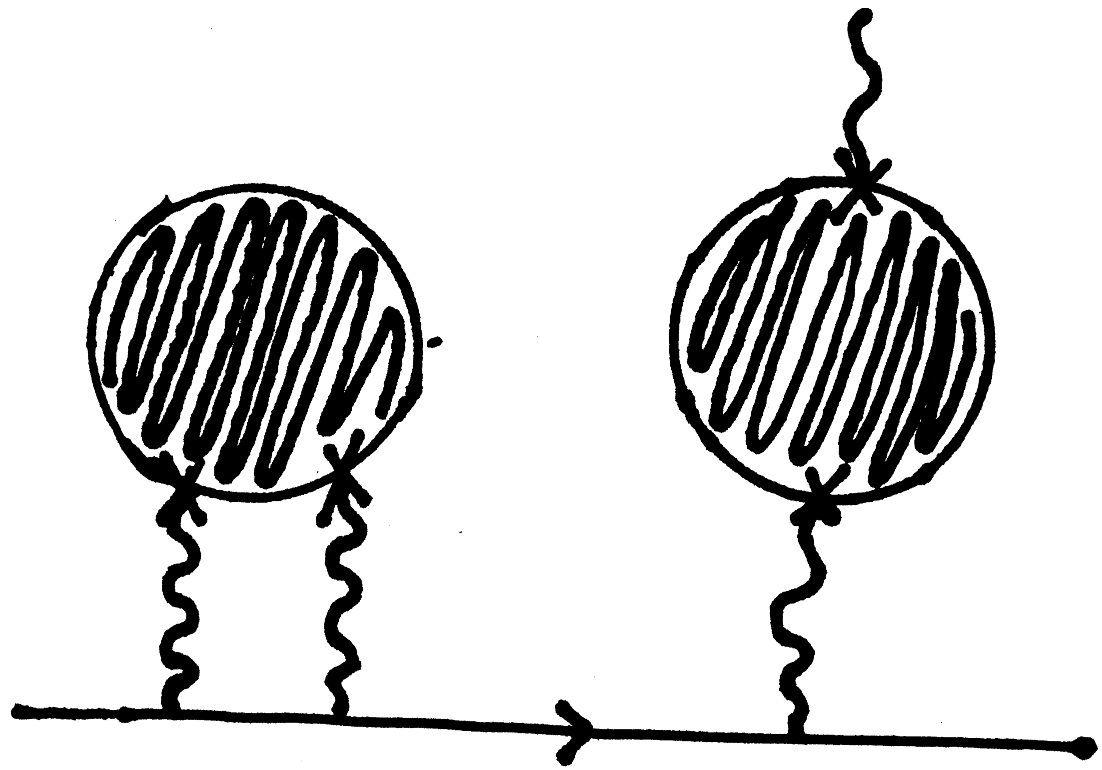
Low-modes only (O(100)), high-mode (stochastic) part later

Have O(150) low modes on larger 24^3 lattices, $m_\pi \approx 300$ MeV, QCD only

Use them to compute QCD four-point function (momentum sums)

Directly compare the two methods

“Disconnected” diagram(s) (connected by gluons)



Haven’t tried these yet...

Summary/Outlook: light-by-light contributions ($O(\alpha^3)$)

- Pure QED calculation on the lattice roughly reproduces the perturbative result. Encouraging.
- Full hadronic contribution is $O(10^2)$ times smaller, still swamped by the statistical noise
- Small volumes, poor statistics. Try
  - Volume (low-mode) averaging for the loop
  - Larger volumes
  - More statistics, i.e. more QED configurations per QCD configuration
  - conventional calculation using "all-to-all" propagator
- multi-quark loops not yet attempted

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The order α^2 hadronic contribution to g-2 Fits

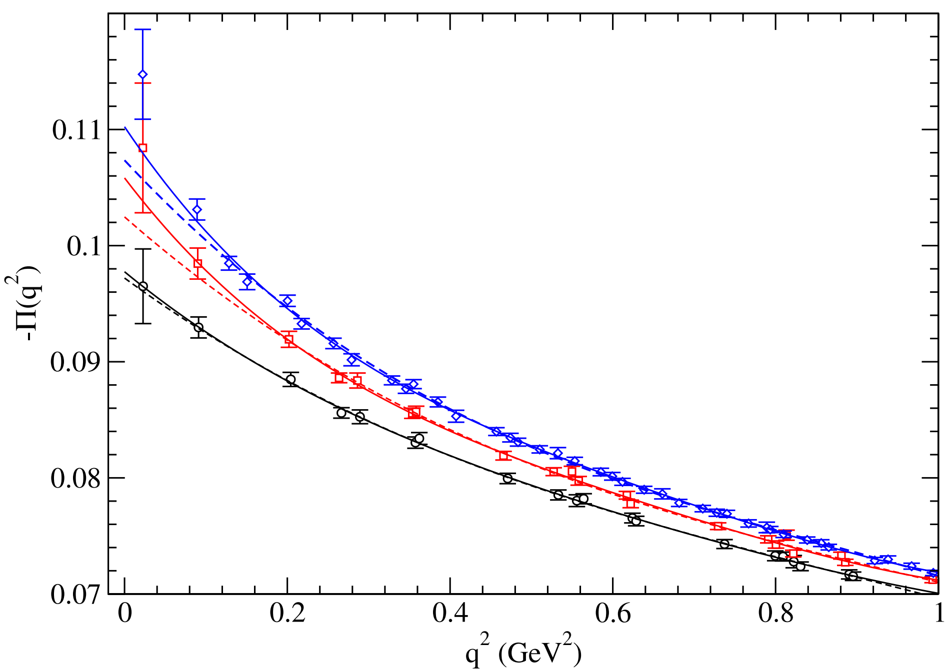


FIG. 12 (color online). Cubic (dashed line) and quartic (solid line) fits to $-\Pi(\hat{q}^2)$ for $am_l = 0.0031$ (diamonds), 0.0062 (squares), and 0.0124 (circles). The strange quark mass is fixed to 0.031 in each case.

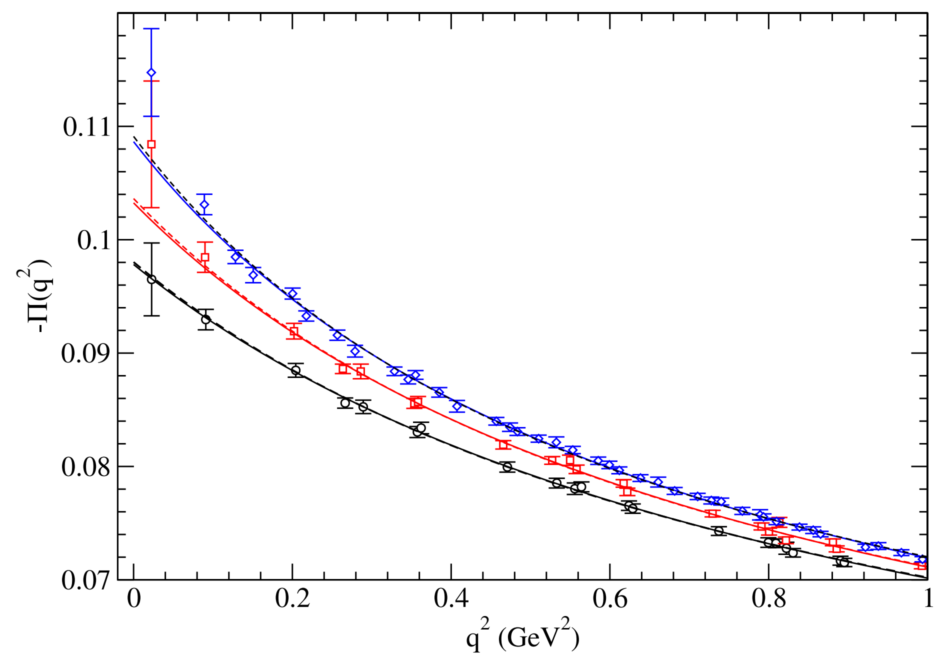
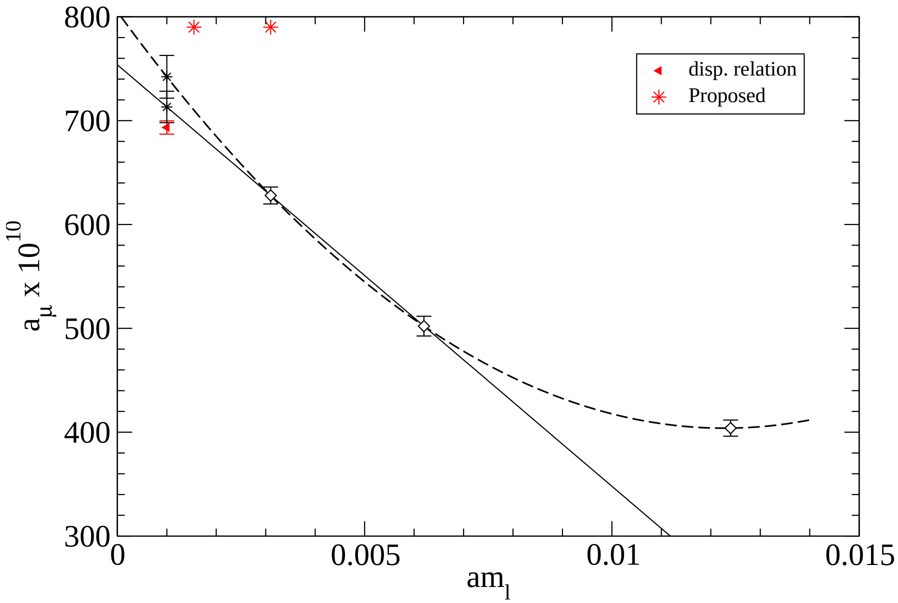


FIG. 13 (color online). SχPT fits to $\Pi(\hat{q}^2)$ for the three light masses, $am_l = 0.0031$ (diamonds), 0.0062 (squares), and 0.0124 (circles). The strange quark mass is fixed to 0.031 in each case. The solid lines correspond to fit B, and the dashed lines to Fit A, as described in the text.

The order α^2 hadronic contribution to g-2



Extrapolate $m_l \to m_{u,d}$

Simple linear and quadratic chiral extrapolations consistent with $e^+e^- \to$ hadrons result

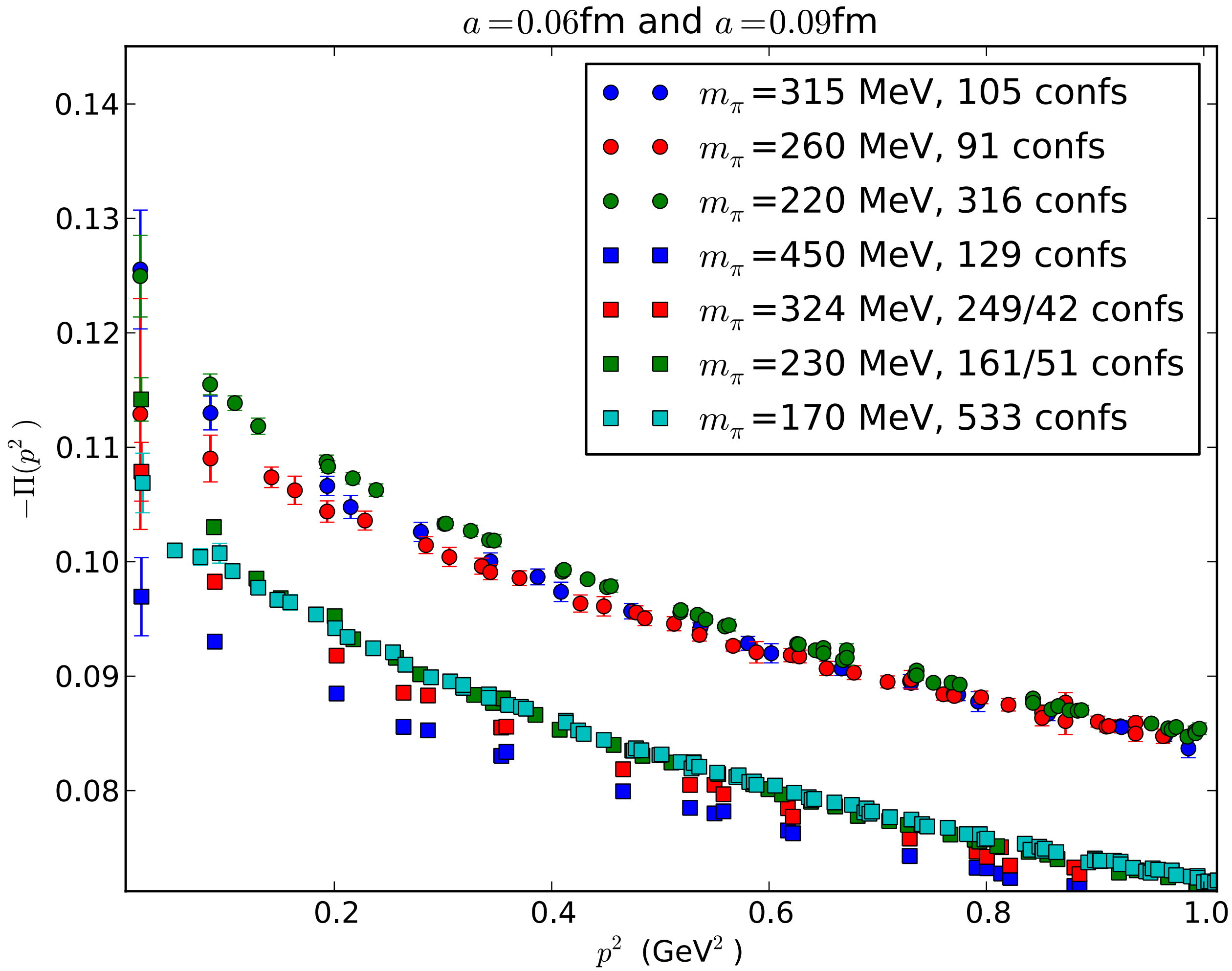
$a_\mu^{HLO} = (713 \pm 15) \times 10^{-10}$ (linear)
$a_\mu^{HLO} = (742 \pm 21) \times 10^{-10}$ (quad)
(statistical errors only).

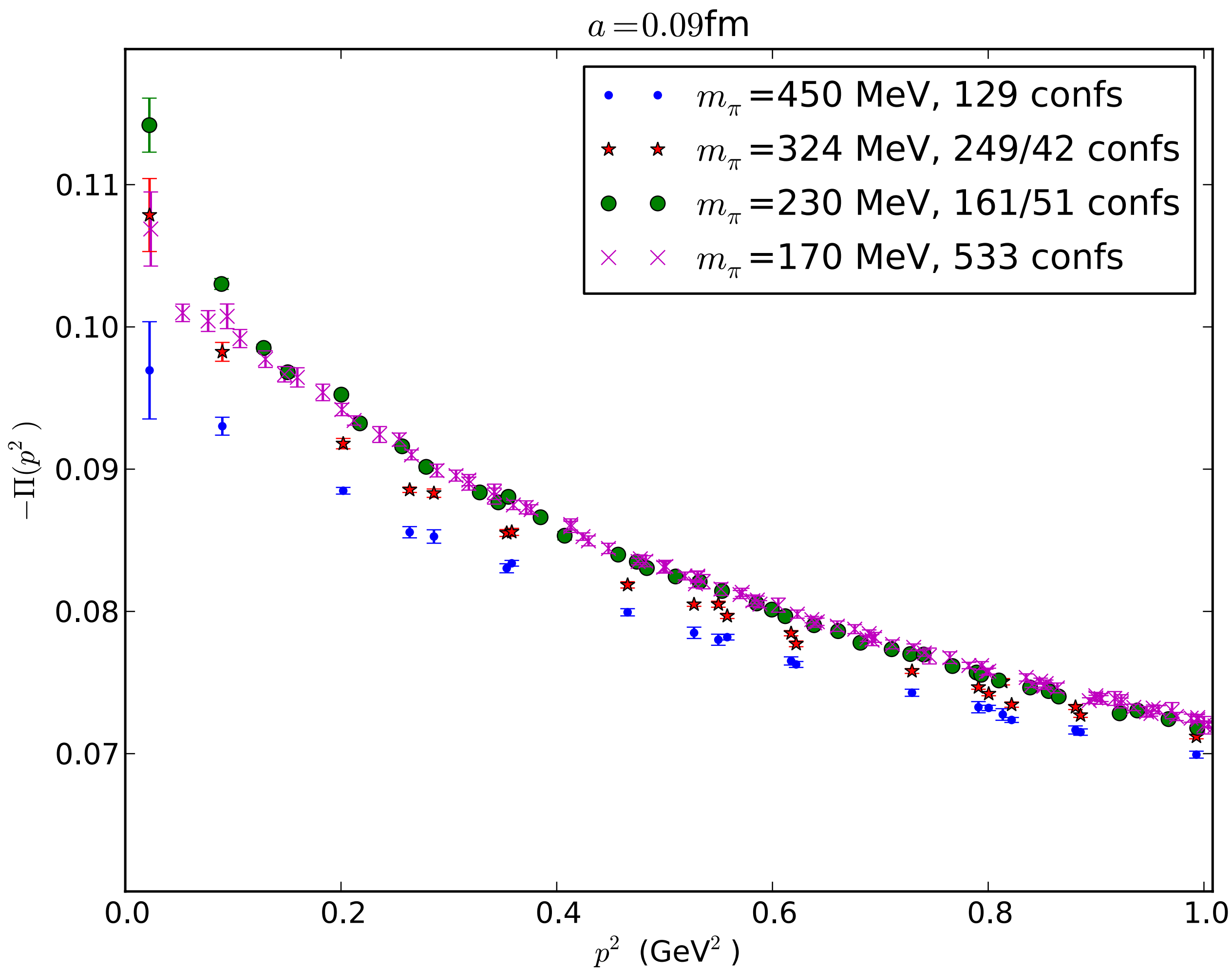
[Aubin, Blum, Phys. Rev. D, 2006]

Fit	quenched	$am_l = 0.0124$	$am_l = 0.0062$	$am_l = 0.0031$
Poly 3	381 (63)	370(49)	445(43)	542(24)
Poly 4	588 (142)	410(91)	639(123)	729(59)
A	366.6 (7.0)	412.3 (7.8)	516.0 (9.5)	646.9 (8.1)
B		403.9 (7.8)	502.1 (9.5)	628.0 (8.1)
C		403.9 (7.8)	502.1 (9.5)	628.0 (8.1)

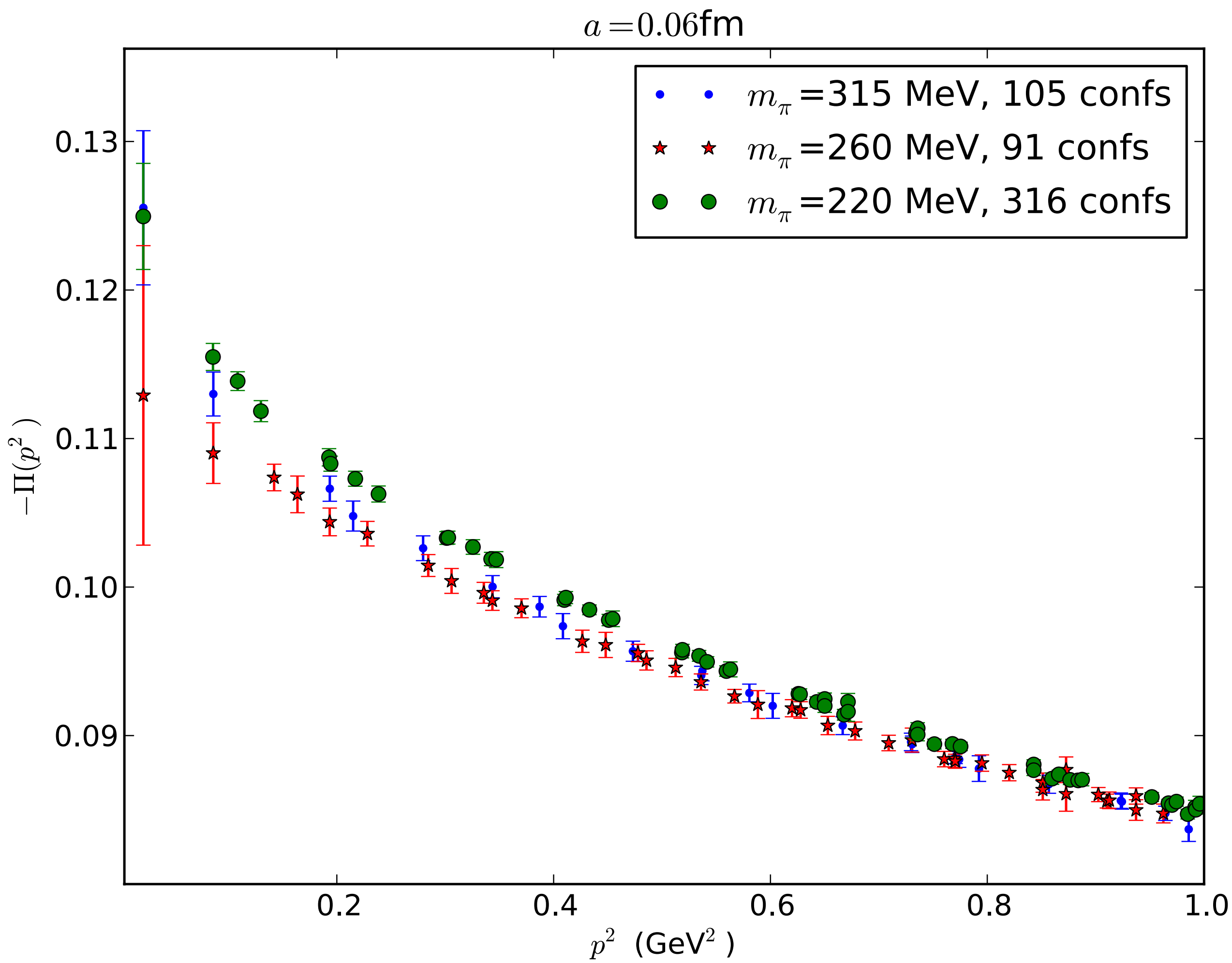
New results for vacuum polarization

$a = 0.09$ and 0.06 fm, m_π down to 170 MeV, volume $\lesssim (6\,\mathrm{fm})^3$





$a = 0.09$fm
m_π =450 MeV, 129 confs
m_π =324 MeV, 249/42 confs
m_π =230 MeV, 161/51 confs
m_π =170 MeV, 533 confs
$-\Pi(p^2)$
p^2 (GeV^2)
0.11
0.10
0.09
0.08
0.07
0.0
0.2
0.4
0.6
0.8
1.0



$a = 0.06$fm
m_π =315 MeV, 105 confs
m_π =260 MeV, 91 confs
m_π =220 MeV, 316 confs
0.13
0.12
0.11
0.10
0.09
$-\Pi(p^2)$
0.2
0.4
0.6
0.8
1.0
p^2 (GeV^2)

Backup Slides

Introduction to muon g-2

Classical interaction of particle with static magnetic field

$$V(\vec{x}) \;=\; -\vec{\mu} \cdot \vec{B}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin

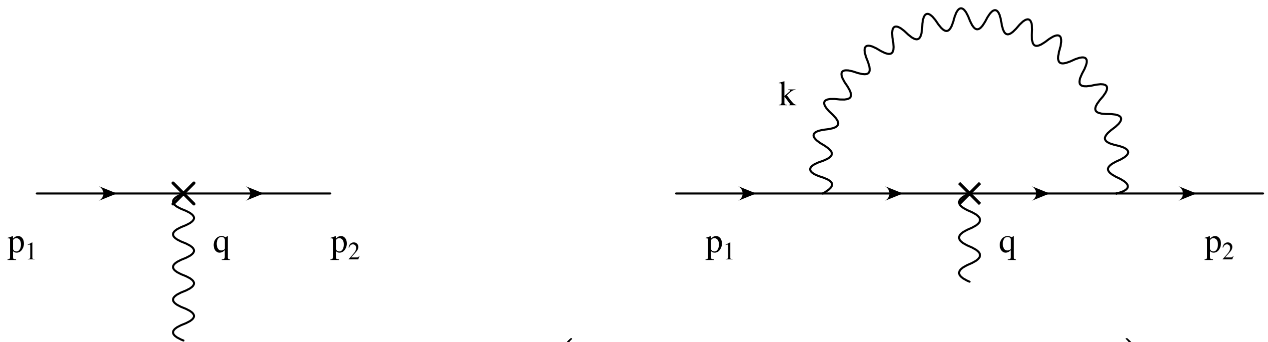
$$\vec{\mu} \;=\; g \left(\frac{e}{2m}\right) \vec{S}$$

The Landé g-factor is predicted from the free Dirac eq. to be

$$g \;=\; 2$$

for elementary fermions

In the quantum (field) theory g receives radiative corrections



$$\gamma_\mu \quad \rightarrow \quad \Gamma^\mu(q) = \left(\gamma^\mu \, F_1(q^2) + \frac{i \, \sigma^{\mu\nu} \, q_\nu}{2m} \, F_2(q^2) \right)$$

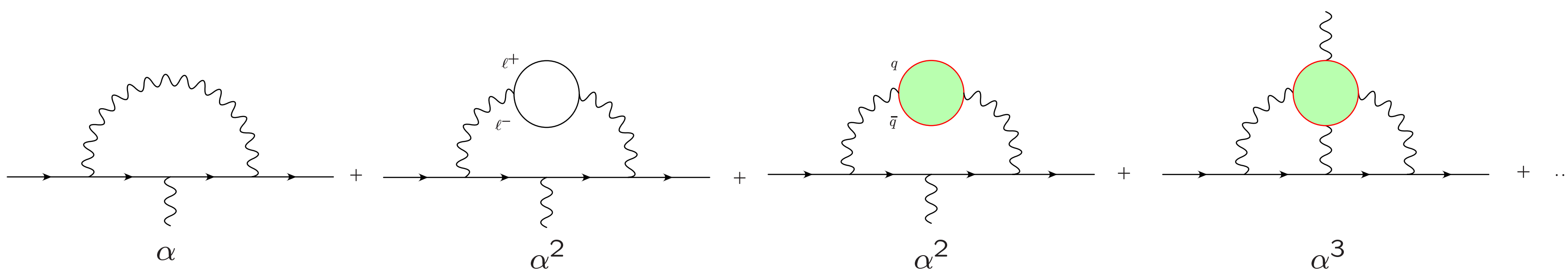
which results from Lorentz invariance and current-conservation (Ward-Takahashi identity) when the muon is <u>on-mass-shell</u>.

$$F_2(0) \quad = \quad \frac{g-2}{2} \equiv a_\mu$$

(the anomalous magnetic moment)

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

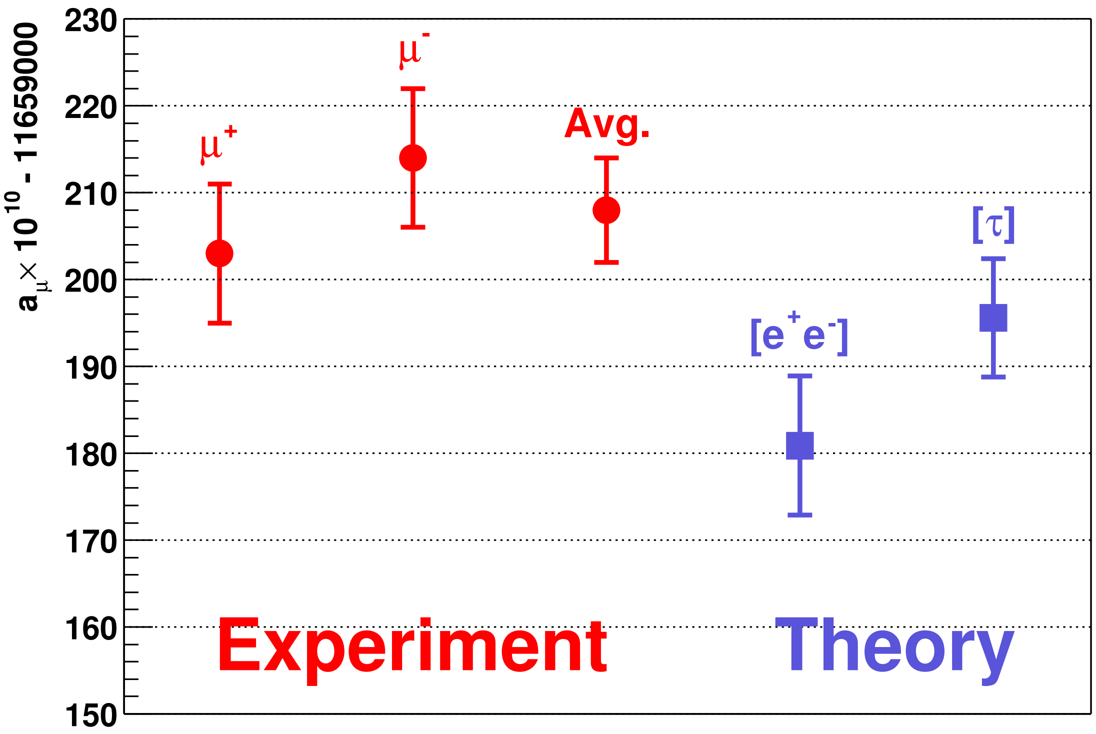
$$\alpha \;=\; \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$



Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$

<u>Hadronic contribution</u> $\sim 6 \times 10^{-5}$ times smaller

Status of the experimental measurement (Muon $(g-2)$ Collaboration, BNL-E821) of a_μ.



$a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10}$ (accurate to about 0.5 ppm)

Theory calculation

(Summary from D. W. Hertzog [E821 Collaboration], hep-ex/0501053.)

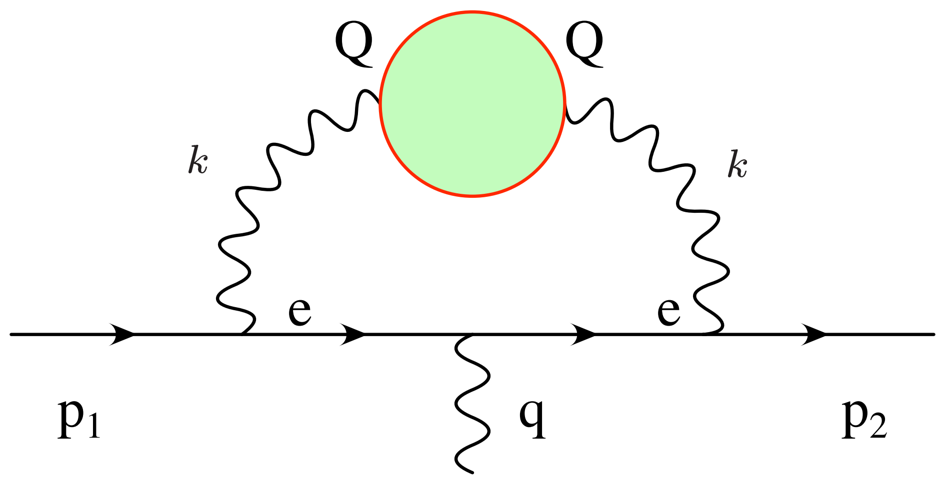
Table 1: Comparison of a_μ(SM) with a_μ(Exp)

	$a_\mu \times 10^{10}$	$\Delta a_\mu \times 10^{10}$
QED	11 658 471.94	0.14
QCD	695.4	7.3
Weak	15.4	0.22
Theory	11 659 182.7	7.4
Experiment	11 659 208	6
a_μ(EXP) $-$ a_μ(SM)	25.3	9.5

Table 2: QCD contribution to the muon $g-2$

	$a_\mu \times 10^{10}$	$\Delta a_\mu \times 10^{10}$
hadronic vacuum polarization ($\mathcal{O}(\alpha_{\mathrm{em}}^2)$)	693.4	6.4
hadronic vacuum polarization ($\mathcal{O}(\alpha_{\mathrm{em}}^3)$)	-10.0	0.6
hadronic light-by-light ($\mathcal{O}(\alpha_{\mathrm{em}}^3)$)	12.0	3.5
Total QCD	695.4	7.3

$\mathcal{O}(\alpha^2)$ hadronic contribution hadronic vacuum polarization: $(\Pi(k^2))$



The blob, which represents all possible intermediate hadronic states, is not calculable in perturbation theory, but can be calculated from

dispersion relation + experimental cross-section for $e^+e^- \to$ hadrons

first principles using lattice QCD

Dispersive method for the vacuum polarization contribution

[Bouchiat and Michel (1961); Durand (1962); ...]

The vacuum polarization is an *analytic* function.

$$\begin{aligned}\Pi(q^2) &= \frac{1}{\pi}\int_0^\infty \mathrm{d}s \frac{\Im\Pi(s)}{(s-q^2)} \\ \sigma_{\text{total}}(e^+e^- \to \text{hadrons}) &= \frac{4\pi^2\alpha}{s}\frac{1}{\pi}\Im\Pi(s)\end{aligned}$$

(by the optical theorem) which leads to

$$a_\mu^{\text{had}(2)} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^\infty \mathrm{d}s K(s)\sigma_{\text{total}}(s)$$

where where $K(s)$ is a known function

$K(s)$ is strongly weighted to low energy region: roughly 91% from $\sqrt{s} \lesssim$ 1.8 GeV, 73% from two pion final state which is dominated by the $\rho(770)$ resonance.

Can get part of σ_{total} from $\tau \to \pi^\pm\pi^0\nu$ decay (needs isospin correction)

Recent theory updates

hadronic vacuum polarization ($a_\mu^{\text{exp}} - a_\mu^{\text{theory}}$)

June 2009 new τ-based evaluation incl. Belle data: 1.8 σ (Davier, *et al.*)

Aug 2009 new e^+e^--based, incl. Babar ISR data: 3.1 σ (Davier, *et al.*)

Oct 2009 new e^+e^--based (Phipsi conf., Beijing): 4.0 σ (Hagiwara, *et al.*)

hadronic light-by-light

2009 $(10.5 \pm 2.6) \times 10^{-10}$ (Prades-de Rafael-Vainshtein, arXiv:0901.0306)

2009 $(11.6 \pm 4.0) \times 10^{-10}$ (Nyffeler, arXiv:0901.1172)

The precise measurement at Brookhaven (E821) allows a precision test of the standard model

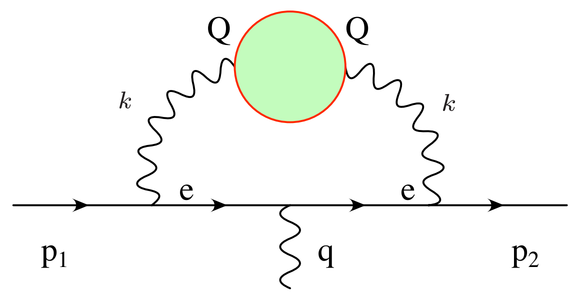
- $\mathcal{O}(\alpha^2)$ hadronic contribution dominates "theory" error: Dispersion relation for $\Pi(q^2)$ + optical theorem + exp. $e^+e^- \to$ hadrons cross section

- e^+e^-: 2.7 σ deviation with experiment

- τ decay: 1.4 σ deviation with experiment

- Lattice calculation (purely theoretical) provides an important check of the dispersive calculation, but required precision poses a great challenge.

- Lattice calculation of hadronic light-by-light contribution important since only model calculations exist



Inserting the quark loop (blob) into the one-loop diagram is easy since only the photon propagator is modified (*c.f.*, charge renormalization in QED),

$$\frac{1}{k^2 - i\,\epsilon} \quad \rightarrow \quad \frac{1}{k^2 - i\,\epsilon} \times (1 + \Pi(k^2) + \ldots)$$

The diagram boils down to [B.E.Lautrup and E. de Rafael (1969), Blum (2002)]

$$a_\mu^{(2)\mathrm{had}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2\, f(K^2)\, \Pi(K^2)$$

where K^2 is Euclidean (space-like) momentum-squared

Note: kernel $f(K^2)$ diverges as $K^2 \to 0$

Ok, since renormalized $\Pi(K^2)$ vanishes at $K^2 = 0$

But it means the integral is dominated by the low momentum region.

Lattice calculation of $\Pi(q^2)$

The continuum vacuum polarization is defined as

$$\begin{aligned}\Pi^{\mu\nu}(q) &= \int \mathrm{d}^4x\, \mathrm{e}^{i\,q\,(x-y)}\,\langle J^\mu(x) J^\nu(y)\rangle \qquad\qquad (J^\mu(x) = \bar{\psi}\gamma_\mu\psi(x)) \\ &= (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)\end{aligned}$$

and satisfies the Ward-Takahashi identity (charge conservation)

$$q_\mu\,\Pi^{\mu\nu}(q) = 0 \qquad\qquad\qquad (\partial_\mu\, J^\mu = 0)$$

<u>On the lattice</u> this also holds, provided <u>conserved current</u> is used

$$\begin{aligned}J^\mu(x) &= \frac{1}{2}\left(\bar{\psi}(x+\widehat{\mu})U^\dagger(x)(1+\gamma^\mu)\psi(x) - \bar{\psi}(x)U(x)(1-\gamma^\mu)\psi(x+\widehat{\mu})\right) \\ \Delta^\mu J^\mu(x) &= \sum_\mu \frac{J^\mu(x) - J^\mu(x-a\widehat{\mu})}{a}\end{aligned}$$

Fourier transformation of the two point function yields the continuum form of W-T identity

$$\begin{aligned}
\widehat{q}^{\mu}\Pi^{\mu\nu}(\widehat{q}^{2}) &= 0 \\
\widehat{q}^{\mu} &= \frac{2}{a}\sin\left(\frac{aq^{\mu}}{2}\right) \\
q^{\mu} &= \frac{2\pi \mathsf{n}_{\mu}}{a\,L_{\mu}}, \quad \mathsf{n}_{\mu} = 0, \pm 1, \pm 2, \ldots, \pm(L_{\mu}-1) \\
\Pi^{\mu\nu}(\widehat{q}) &= (\widehat{q}^{\mu}\widehat{q}^{\nu} - \widehat{q}^{2}\delta^{\mu\nu})\Pi(\widehat{q}^{2})
\end{aligned}$$

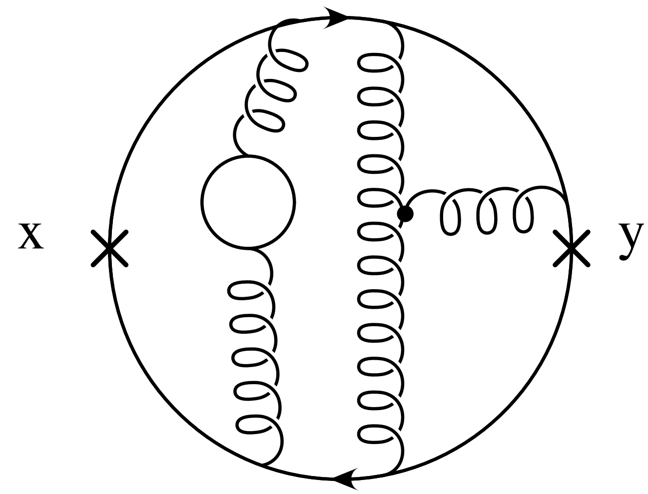
from which we compute $\Pi(\widehat{q}^{2})$

The W-T Identity provides a strong check on the calculation since it must be satisfied *exactly* (up to machine precision) in the numerical calculation

Anatomy of a lattice calculation

In practice, calculate the two-point correlation function of the electromagnetic current in coordinate space on a discrete Euclidean space-time lattice,

$\langle\bar{\psi}(y)\gamma_\mu\psi(y)\bar{\psi}(x)\gamma_\nu\psi(x)\rangle$



Wick contract quark fields into propagators (M^{-1}_{xy})
(M_{xy} is lattice Dirac op, large sparse matrix)

$$C^{\mu\nu}(x,y) \;=\; \mathrm{tr} M^{-1}_{xy}\,\gamma^\mu\, M^{-1}_{yx}\,\gamma^\nu$$

Then compute the Fourier transform

$$\Pi^{\mu\nu}(q) \;=\; \sum_x e^{i\,q(x-y)}\, C^{\mu\nu}(x,y),$$

and average over many gauge-field configurations (do the path intergral)

2+1 flavor QCD configuration summary

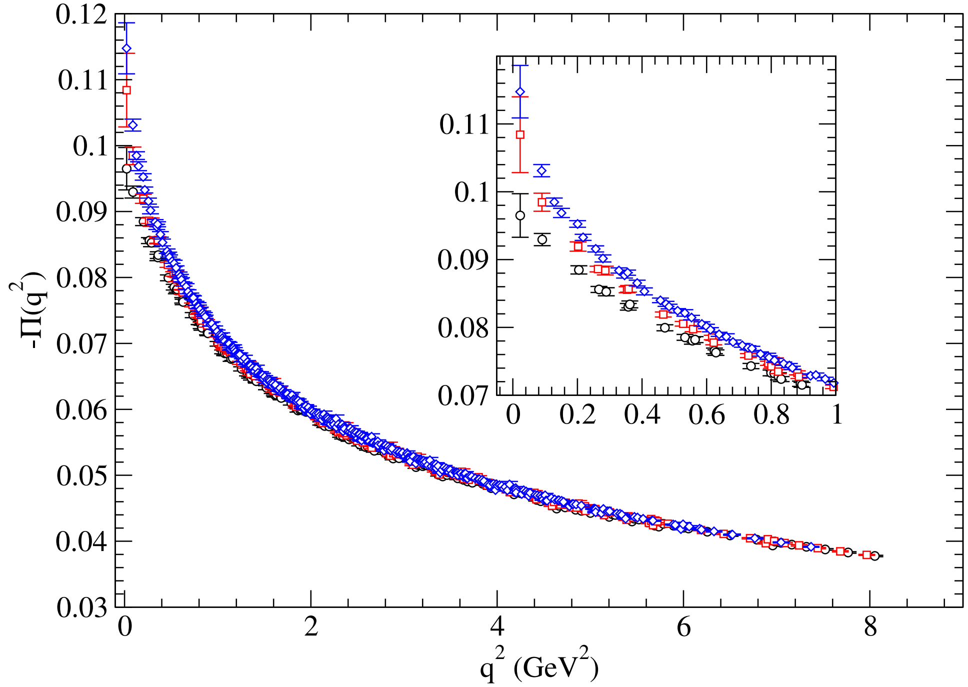
a (fm)	am_l	am_s	β	size	$m_\pi L$	# lats. avail.
$\approx$ 0.09*	0.0124	0.031	7.11	$28^3 \times 96$	5.78	531
$\approx$ 0.09*	0.0062	0.031	7.09	$28^3 \times 96$	4.14	591
$\approx$ 0.09	0.00465	0.031	7.085	$32^3 \times 96$	4.10	480
$\approx$ 0.09*	0.0031	0.031	7.08	$40^3 \times 96$	4.22	945
$\approx$ **0.09**	**0.00155**	**0.031**	**7.075**	$\mathbf{64^3 \times 96}$	**4.80**	**491**
$\approx$ 0.06	0.0036	0.018	7.47	$48^3 \times 144$	4.50	751
$\approx$ 0.06	0.0025	0.018	7.465	$56^3 \times 144$	4.38	768
$\approx$ **0.06**	**0.0018**	**0.018**	**7.46**	$\mathbf{64^3 \times 144}$	**4.27**	**826**
$\approx$ 0.045	0.0028	0.014	7.81	$64^3 \times 192$	4.56	801

MILC Asqtad ensembles

lattices labeled with * have been used in [Aubin and Blum (2006)]

New MILC ensembles: lightest mass on the fine ($a \approx$ 0.09 fm) ensemble and superfine ($a \approx$ 0.06 fm) ensemble, marked in bold.

Hadronic vacuum polarization: 2+1 flavors of quarks



large lattice, $L \sim 3.5$ fm
Small q^2: $q = \sin(2\pi n/L)$

Quark masses
$\sim (0.1, 0.2, 1.0) \times m_s$

slope increases as $m_{u,d} \to 0$

single spacing $a = 0.086$ fm

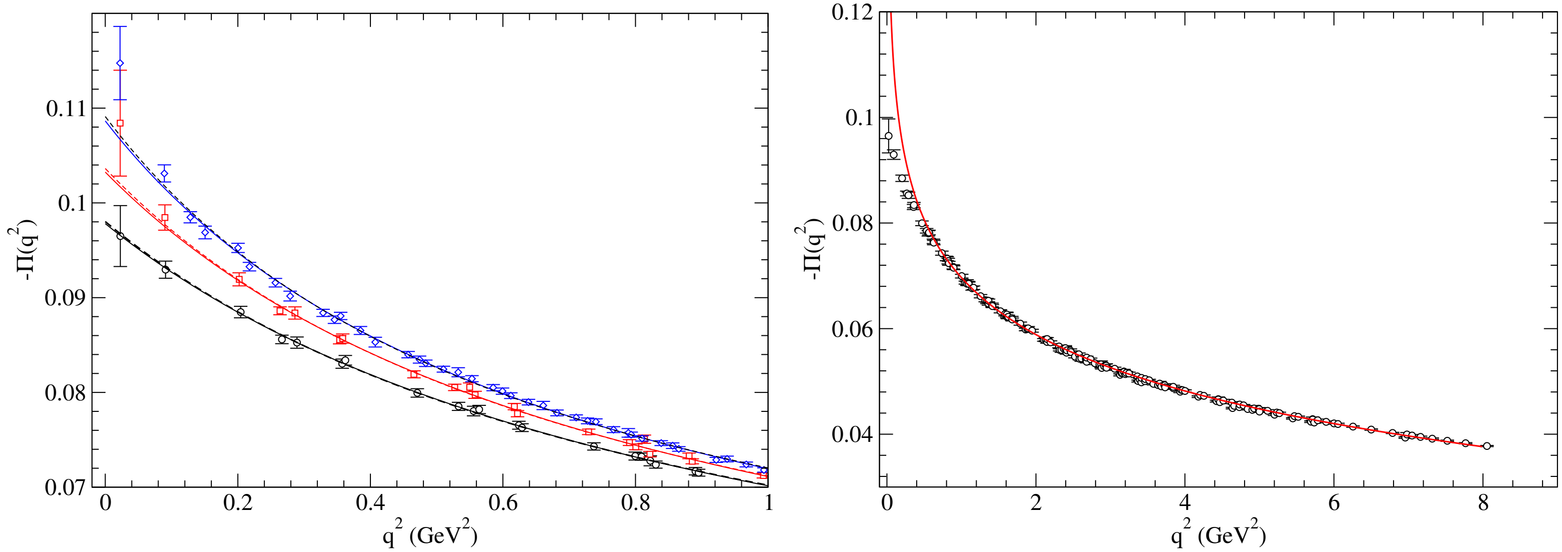
[Aubin, Blum, Phys. Rev. D, 2006]

The order α^2 hadronic contribution to g-2

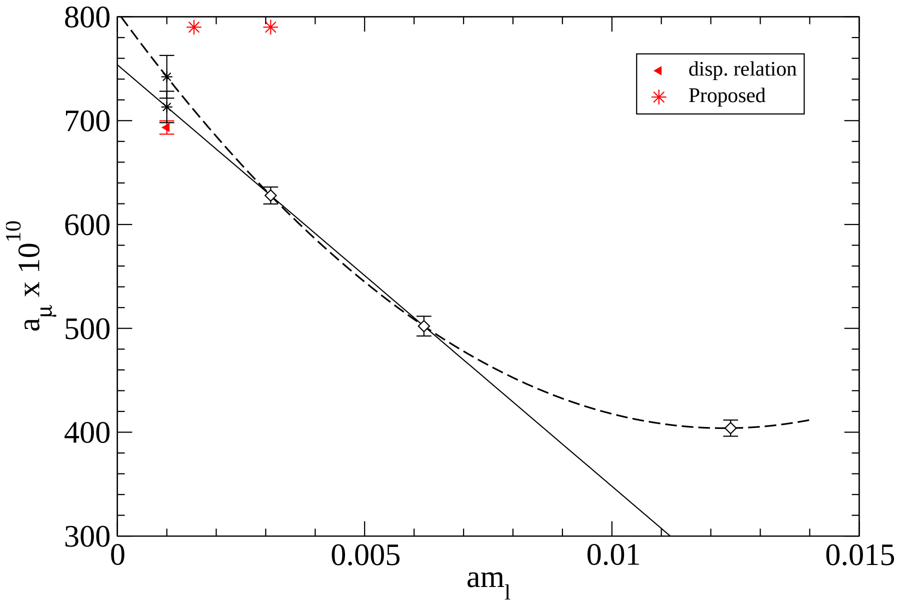
Fit $\Pi(q^2)$ to obtain smooth function of q^2, plug into a_μ formula

$$a_\mu^{(2)\mathrm{had}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2\, f(K^2)\, \Pi(K^2)$$

- χPT + vector meson (resonance χPT) [Aubin and Blum (2006)]
- High momentum part of integral done with 3-loop continuum PT [Chetyrkin, *et al.* (1996)] (small contribution $\sim 1-2\%$)



The order α^2 hadronic contribution to g-2



Extrapolate $m_l \to m_{u,d}$

Simple linear and quadratic chiral extrapolations consistent with $e^+e^- \to$ hadrons result

$a_\mu^{HLO} = (713 \pm 15) \times 10^{-10}$ (linear)
$a_\mu^{HLO} = (742 \pm 21) \times 10^{-10}$ (quad)
(statistical errors only).

[Aubin, Blum, Phys. Rev. D, 2006]

Fit	quenched	$am_l = 0.0124$	$am_l = 0.0062$	$am_l = 0.0031$
Poly 3	381 (63)	370(49)	445(43)	542(24)
Poly 4	588 (142)	410(91)	639(123)	729(59)
A	366.6 (7.0)	412.3 (7.8)	516.0 (9.5)	646.9 (8.1)
B		403.9 (7.8)	502.1 (9.5)	628.0 (8.1)
C		403.9 (7.8)	502.1 (9.5)	628.0 (8.1)

Summary/Outlook: vacuum polarization contribution($O(\alpha^2)$)

- Previous results (2006) consistent with e^+e^-/τ decay results, but need to reduce *systematic* errors (momentum and quark mass extrapolations)

- new configurations available from MILC (staggered) and RBC/UKQCD (DWF)

- New results for lighter quark mass and bigger volume soon (ρ above $\pi\pi$ threshold)

- Finer lattice spacing soon

- Still need *disconnected* quark loop diagrams, (vanish in the $SU(3)$ limit)

- Take continuum, infinite volume limits

- Several groups (RBC/UKQCD, LSD, DESY-Zeuthen, ...) now calculating vacuum polarization to calculate S-parameter and $g-2$.