



# Geneva: Event Generation at NLO

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# Outline

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- Report on progress towards Event Generation at NLO
- What do event generators do - working parts
- Why merging NLO with Resummation is important and challenging
- Geneva Approach
  - SCET
  - First Results
  - Next Steps
- Conclusions

# What do Event Generators Do?

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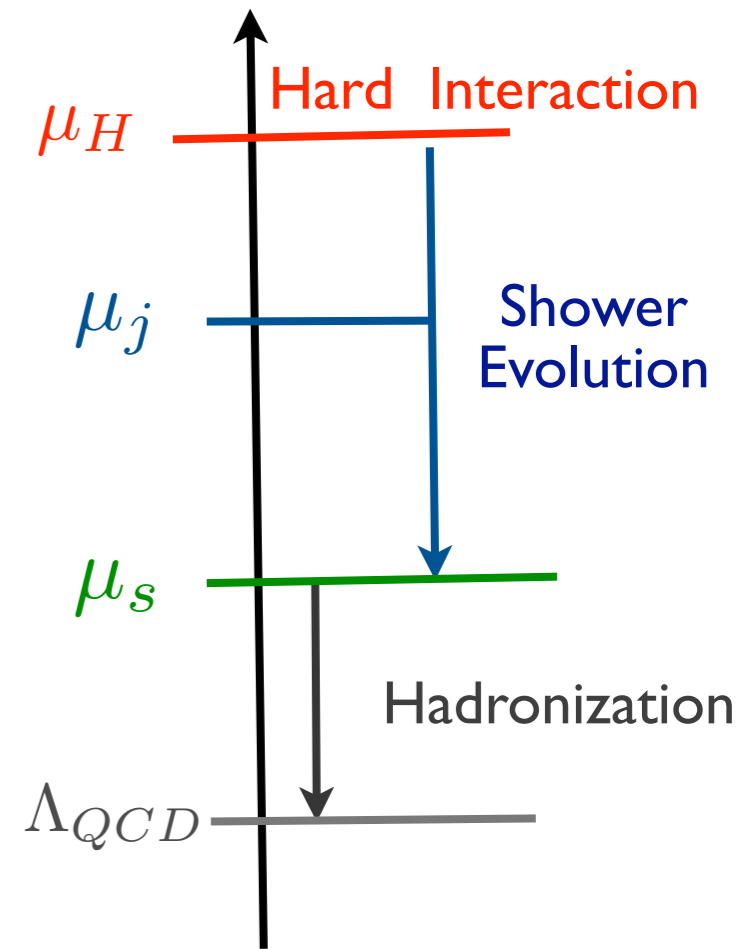
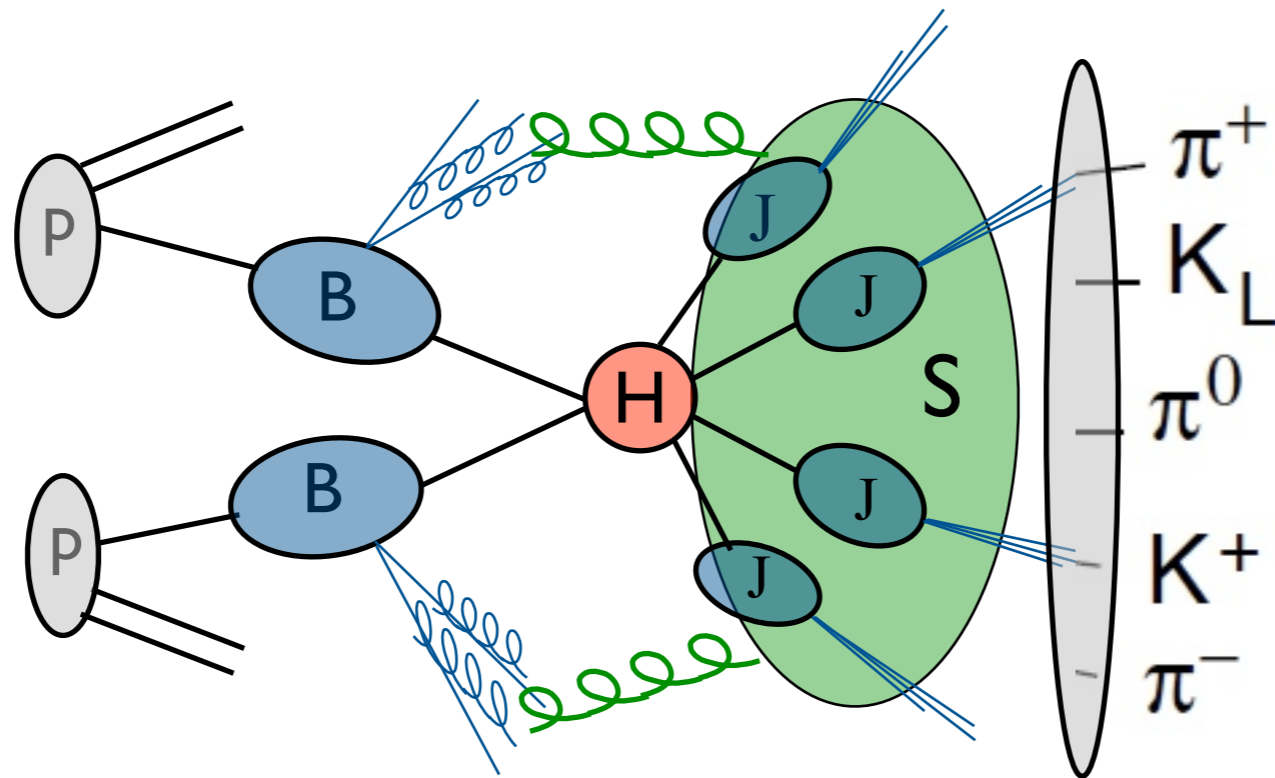
$$\frac{d\sigma}{d\mathcal{O}} = \int d\Phi_n \underbrace{\frac{d\sigma}{d\Phi_n}}_{\text{Basic role: return weight for each point in N-body phase space}} \delta \left( \mathcal{O} - \hat{\mathcal{O}}(\Phi_n) \right)$$

Basic role: return weight for each point in N-body phase space

- Address many of the challenges in connecting theoretical calculations to experimental searches:
  - High multiplicity final states
  - Complicated final state cuts  $\{\eta_{\text{cut}}, p_T^{\text{cut}}, R\}$
  - Hadronic final states
  - Tune models of underlying event and pile-up

# Parts of the Monte Carlo

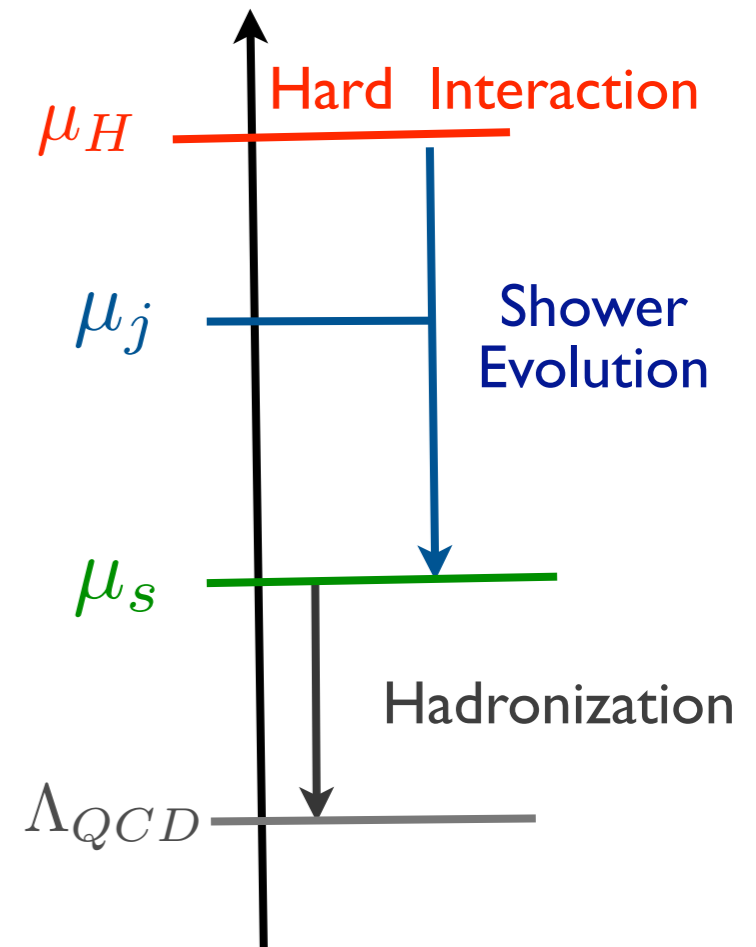
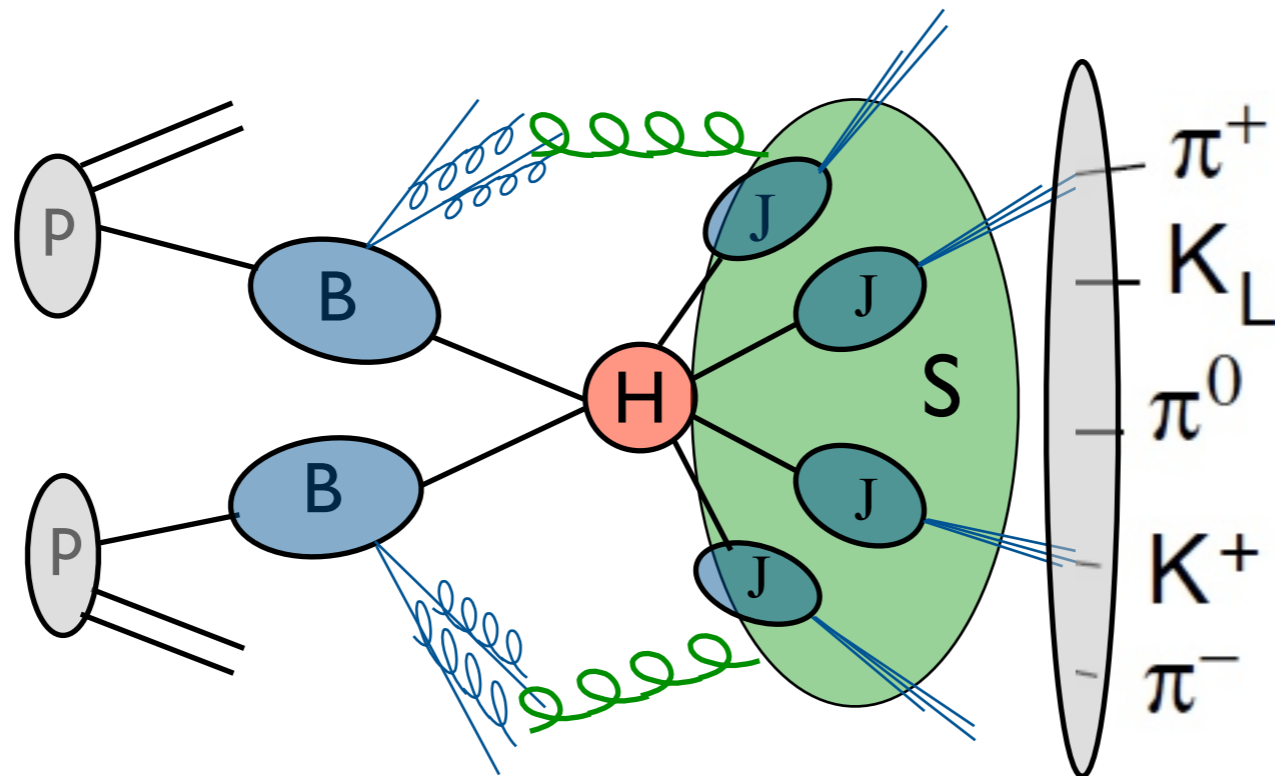
- Monte Carlo is built on the idea of factorization



$$d\sigma^{MC} = \text{Hard Interaction} \otimes \text{Collinear Evolution} \otimes \text{Soft Radiation} \otimes \text{PDFs} \otimes \text{Hadronization} \otimes \text{Underlying Event}$$

# Parts of the Monte Carlo

- Monte Carlo is built on the idea of factorization



$$d\sigma^{MC} = \text{Hard Interaction} \otimes \left( \text{Collinear Evolution} \otimes \text{Soft Radiation} \right) \otimes \left( \text{PDFs} \otimes \text{Hadronization} \otimes \text{Underlying Event} \right)$$

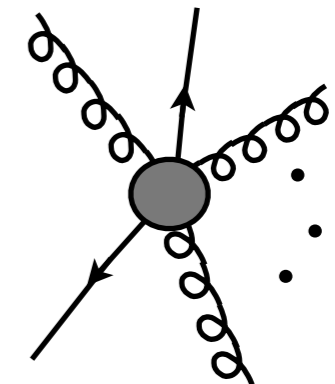
Parton Shower Evolution

# Parts of the Monte Carlo

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$$d\sigma^{MC} = \text{Hard Interaction} \otimes \text{Parton Shower} \otimes \text{Hadronization Underlying Event}$$

- **Hard Interaction:** Fixed order partonic matrix elements.
  - Challenge of perturbative corrections many final states
  - Many developments in calculation techniques



Automization of NLO has seen much progress:

Blackhat, Rocket, MadLoop, GOLEM  
and more...

Processes like  $pp \rightarrow W + 4 \text{ jets}$  at NLO

[Berger, Bern, Dixon, Febres-Cordero, Gleisberg, Forde, Ita, Kosower, Maitre], [Diana, Ozeren, SH]

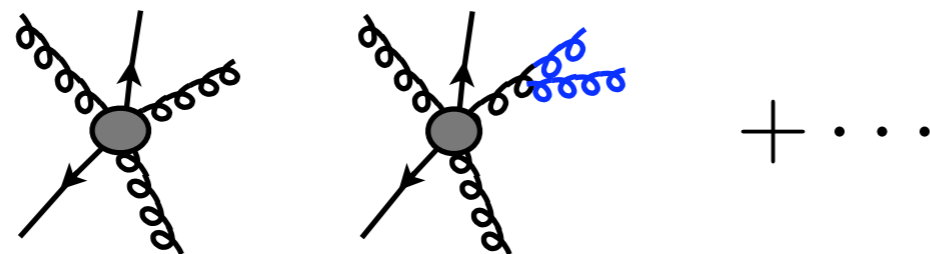
\* See Zvi Bern's talk

# Parts of the Monte Carlo

$$d\sigma^{MC} = \text{Hard Interaction} \otimes \text{Parton Shower} \otimes \text{Hadronization Underlying Event}$$

- **Parton Shower:** Collinear and soft splittings added to hard partons fills out phase space.

- Sums leading double logs  $\alpha_s \ln^2 \frac{\mu_S}{\mu_H}$



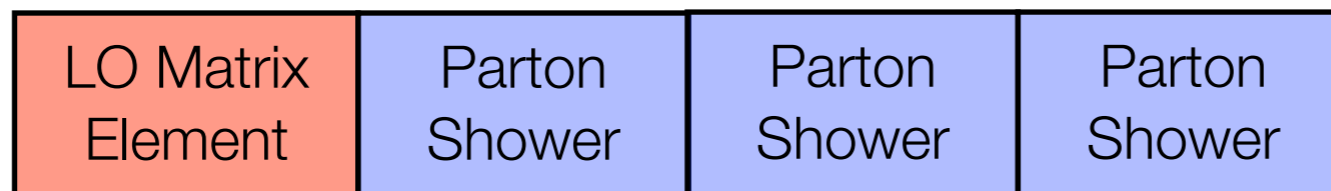
$d\Phi_n$

$d\Phi_{n+1}$

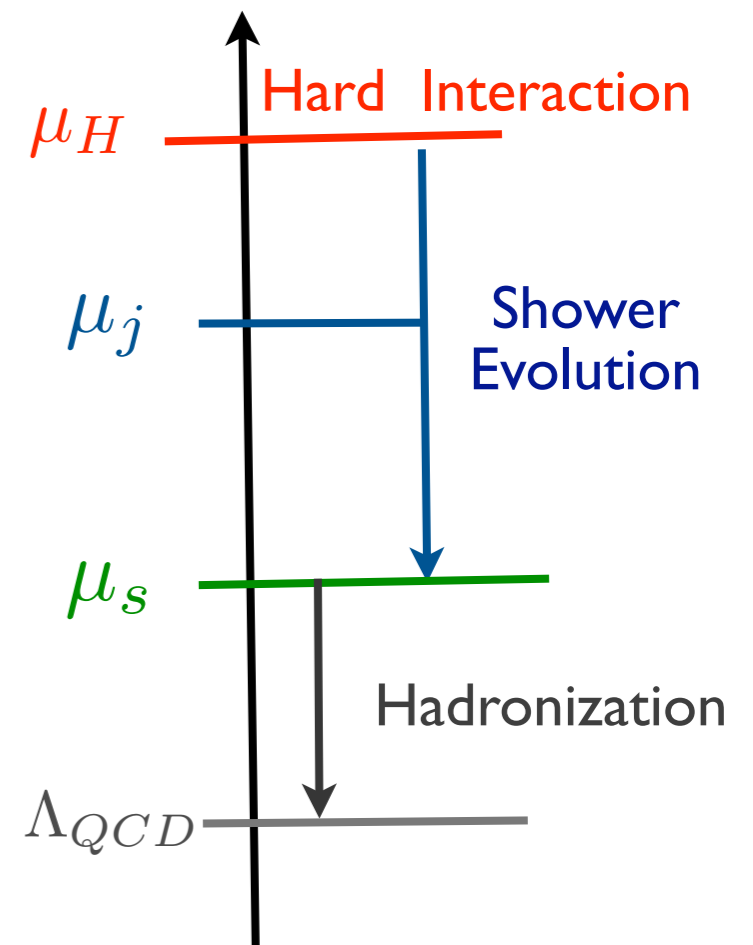
$d\Phi_{n+2}$

$d\Phi_{n+3}$

Sample:

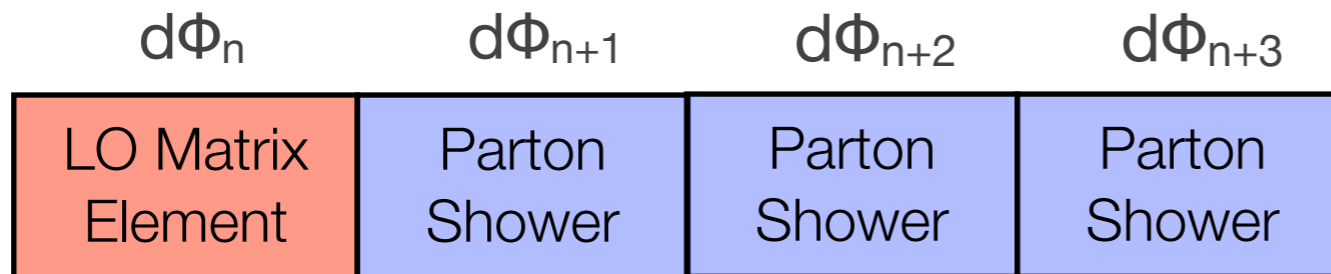


1 LO  $\otimes$  Parton Shower

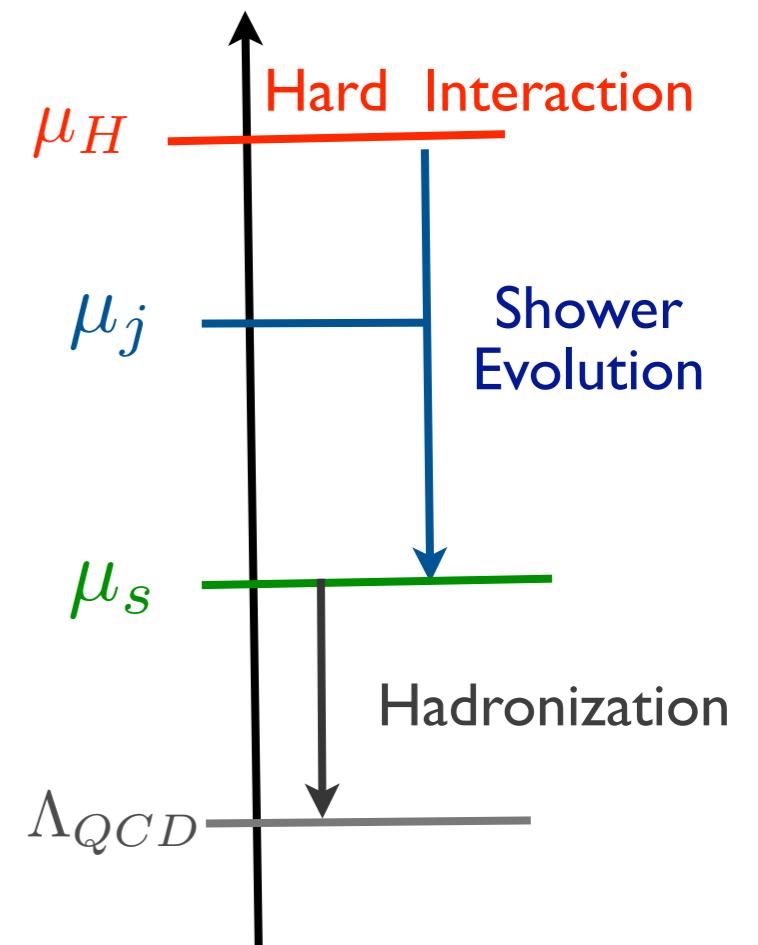


# Parts of the Monte Carlo

$$d\sigma^{MC} = \text{Hard Interaction} \otimes \text{Parton Shower} \otimes \text{Hadronization Underlying Event}$$



I will focus on **Fixed Order**  $\otimes$  **Parton Shower**





# Combining FO and Resummation is Important

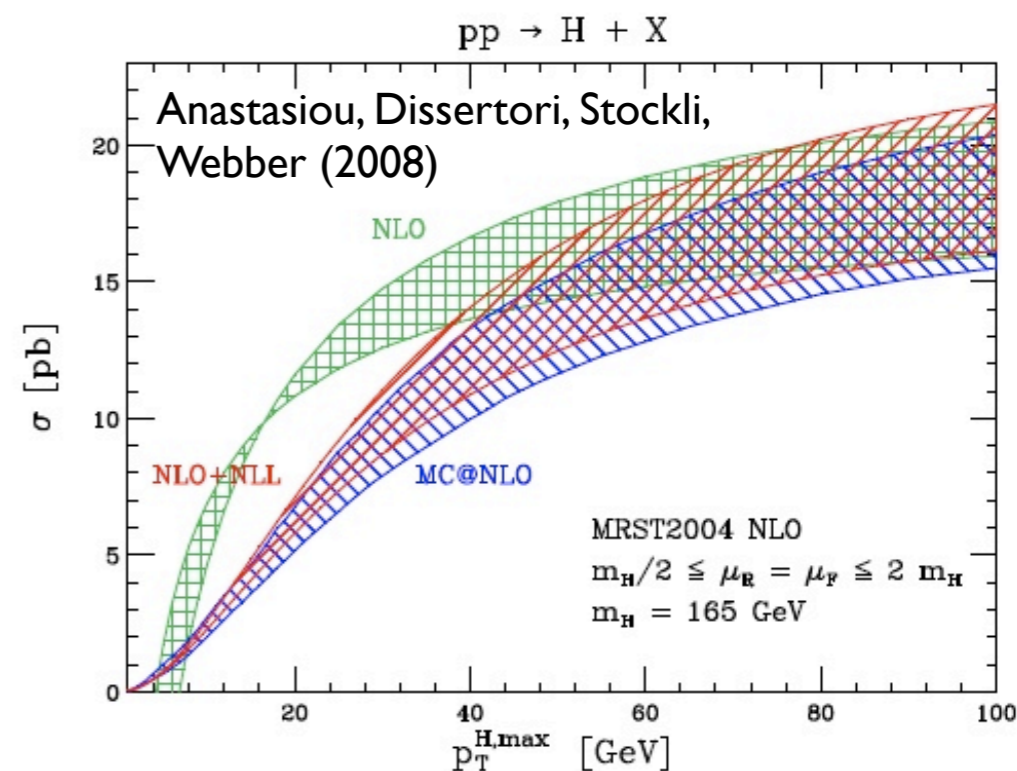
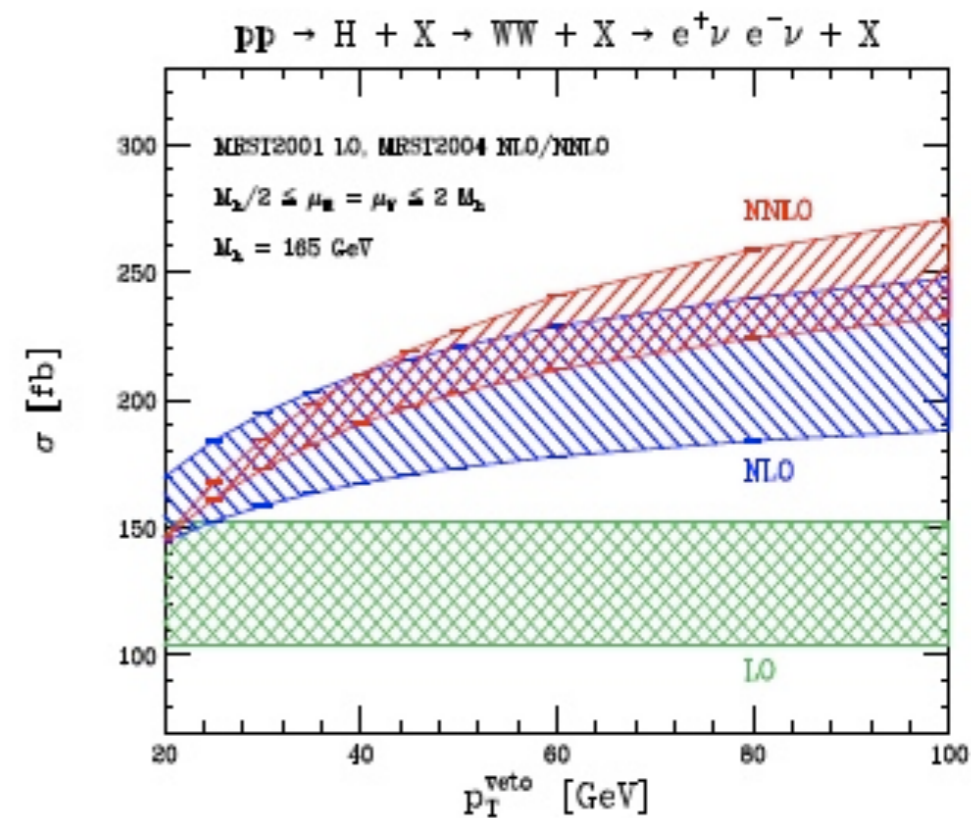
- Example  $pp \rightarrow H \rightarrow WW \rightarrow l\nu l\nu$  :
- **Going to NLO is important**  
Large fixed order corrections. Vary with  $p_T^{\text{cut}}$

- **Resummation is important**

- FO  $\alpha_s$  expansion describe large  $p_T^{\text{cut}}$  region  
Unreliable at small  $p_T^{\text{cut}}$

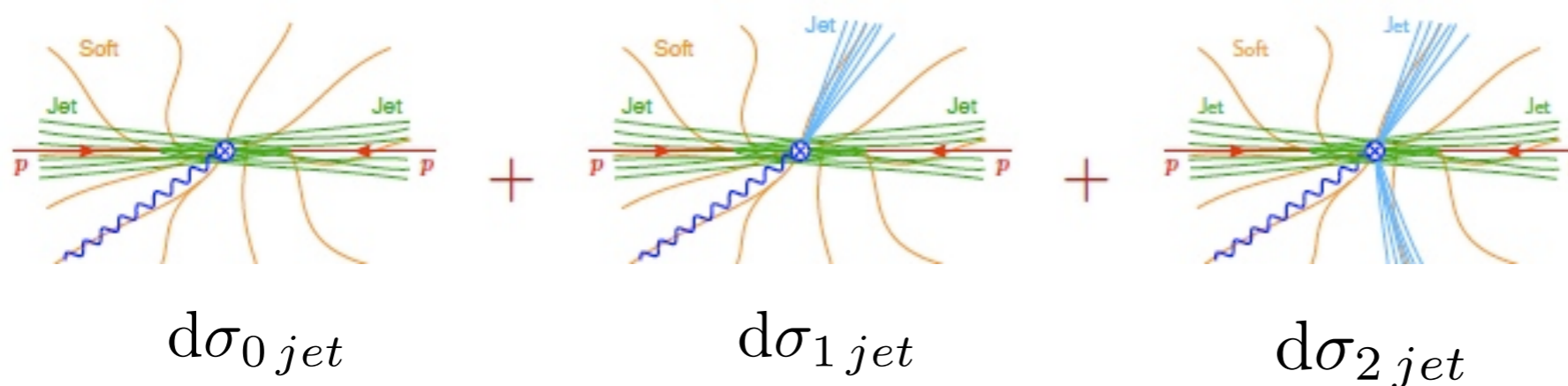
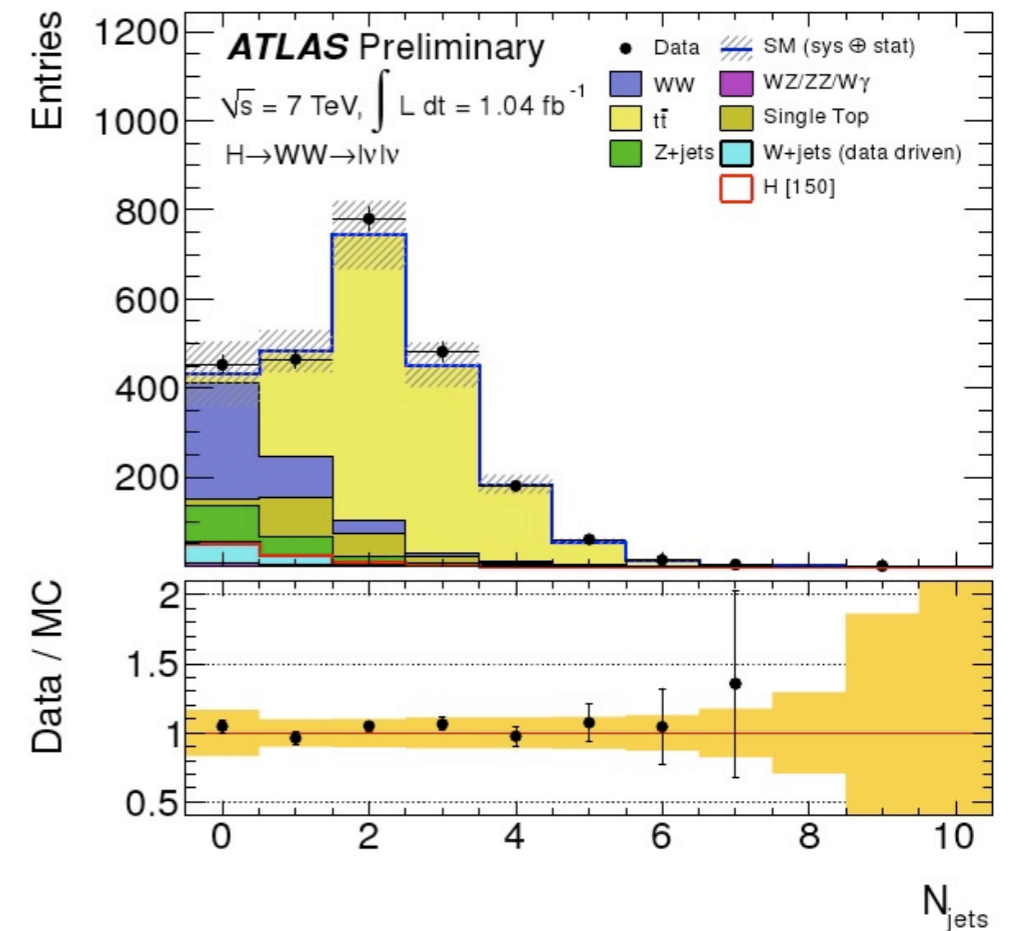
Large logs of  $\alpha_s \ln \frac{p_T^{\text{cut}}}{m_H}$

Anastasiou, Dissertori, Stockli (2007)



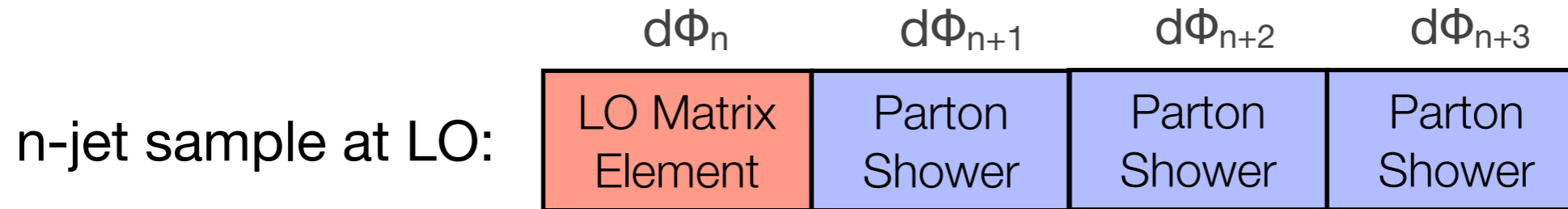
# Combining FO and Resummation is Important

- Often interested in exclusive jet samples
- For example  $pp \rightarrow H + 0, 1, 2 \text{ jets}$   
Backgrounds depend on jet multiplicity.
- Want exclusive jet multiplicities all at NLO with resummation

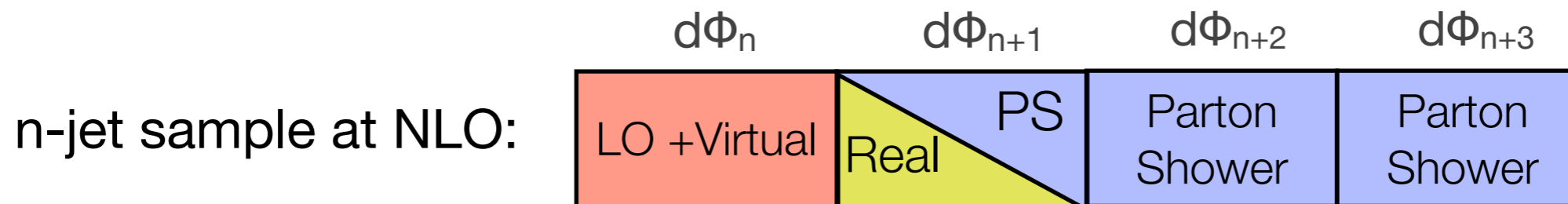


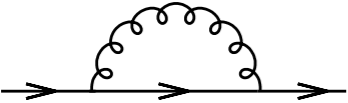
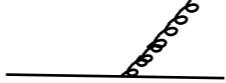
\*Image from Frank Tackmann

# Challenge : Fixed Order and Parton Shower



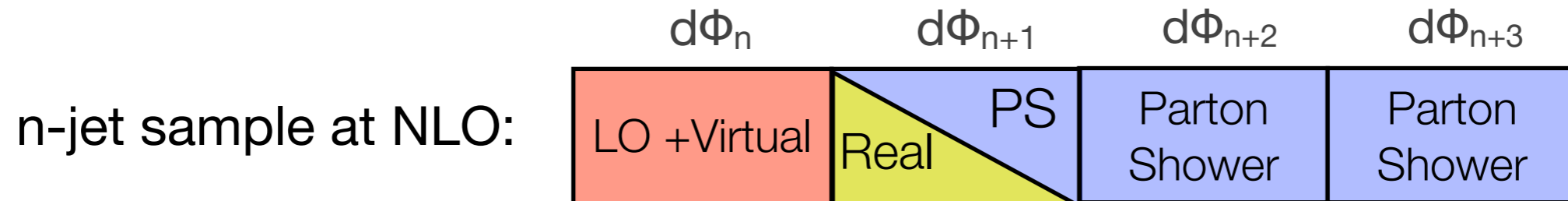
- Beyond LO: N-parton Phase Space  $\neq$  N-body Phase Space



-   $d\Phi_n$  +   $d\Phi_{n+1}$  = IR finite NLO

- FO ME requires cancellation in singular limit. Collinear/soft limit described by parton shower

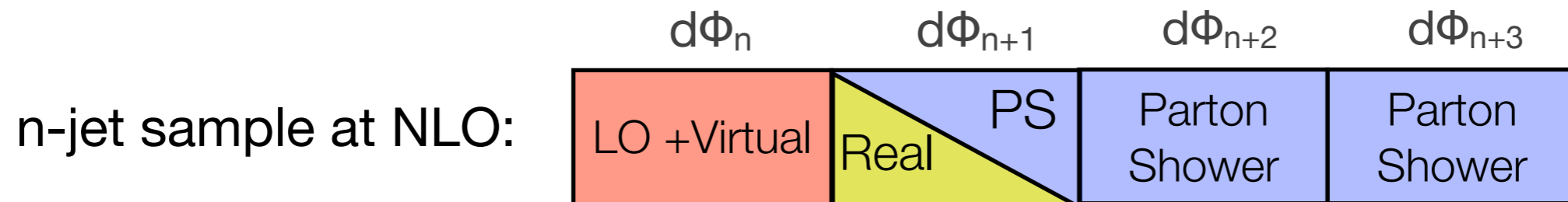
# Challenge : Fixed Order and Parton Shower



- Challenge:
  - ➔ Make each  $d\Phi_n$  weight well-defined
  - ➔ Avoid double-counting
- Broadly speaking, two approaches on the market:

- Subtraction
 
$$d\Phi_n \left[ V_n + \int d\Phi_{n+1|n} S \right] + d\Phi_{n+1} [R_{n+1} - S]$$

# Challenge : Fixed Order and Parton Shower

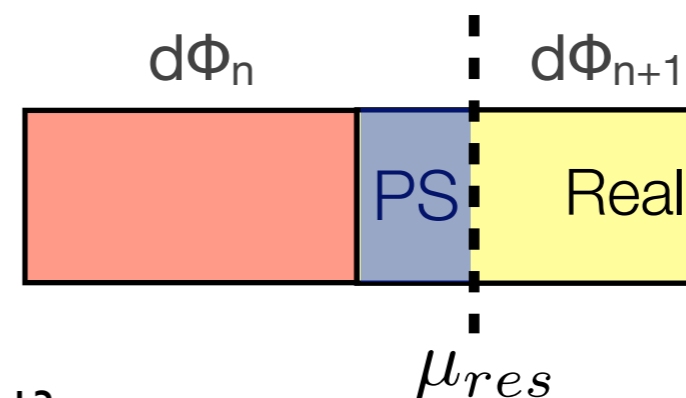


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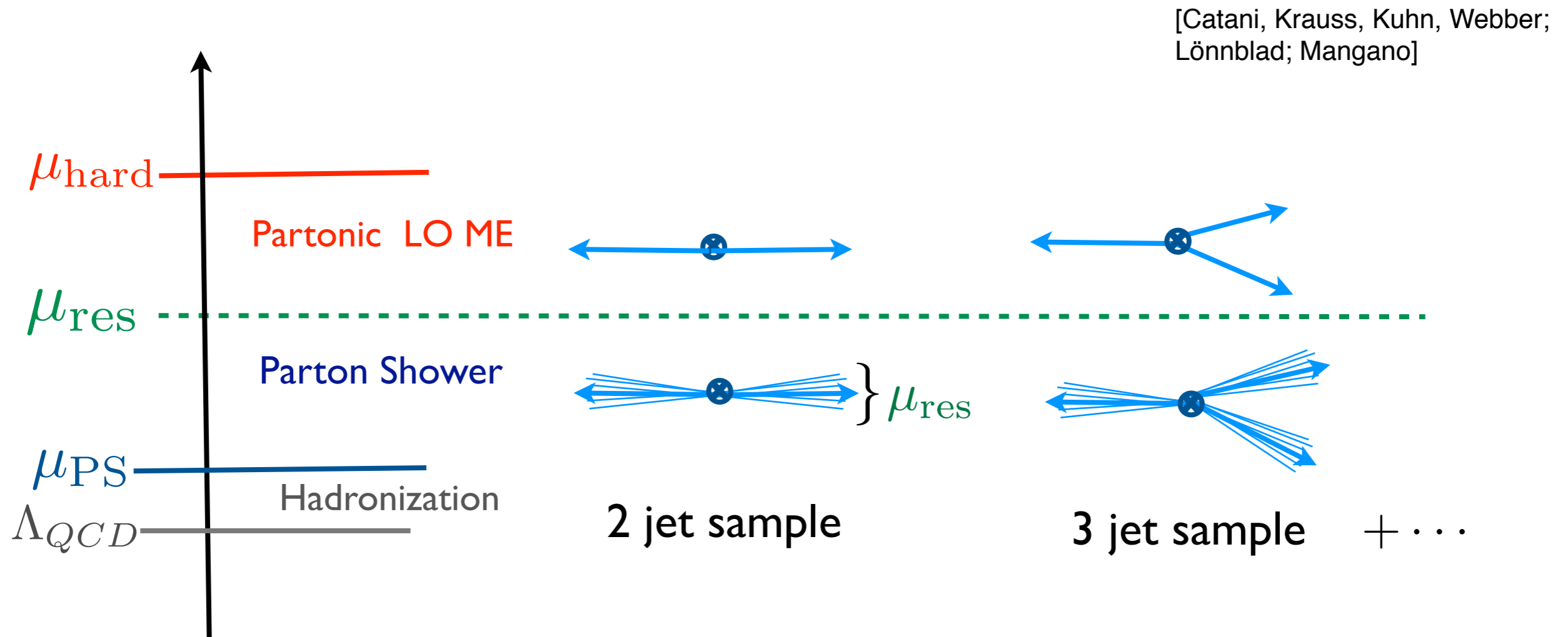
- Subtraction
 
$$d\Phi_n \left[ V_n + \int d\Phi_{n+1|n} S \right] + d\Phi_{n+1} [R_{n+1} - S]$$

- Phase space separation



# Current Approaches: Fixed Order $\otimes$ Parton Shower

- Leading Order ME for all jet multiplicities  $\otimes$  Parton Shower



- Hard matrix element combined with Sudakov to cancel  $\mu_{\text{res}}$  dependence from parton shower.

# Current Approaches: Fixed Order $\otimes$ Parton Shower

- NLO for single jet multiplicity  $\otimes$  LL Parton Shower MC@NLO/ POWHEG

[Frixione, Webber; Nason; Frixione, Nason, Oleari]

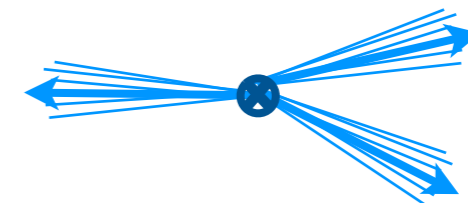
→ Divergences in  $d\Phi_n$  at NLO:

Define Subtraction function  $S(\Phi_n)$   $\int d^d k S(k) = \frac{A_{QCD}}{\epsilon^2} + \frac{B_{QCD}}{\epsilon} + C'$



$$d\Phi_2 \left[ V_n + \int d\Phi_{3|2} S \right] + d\Phi_3 [R_3 - S]$$

2 jet sample at NLO



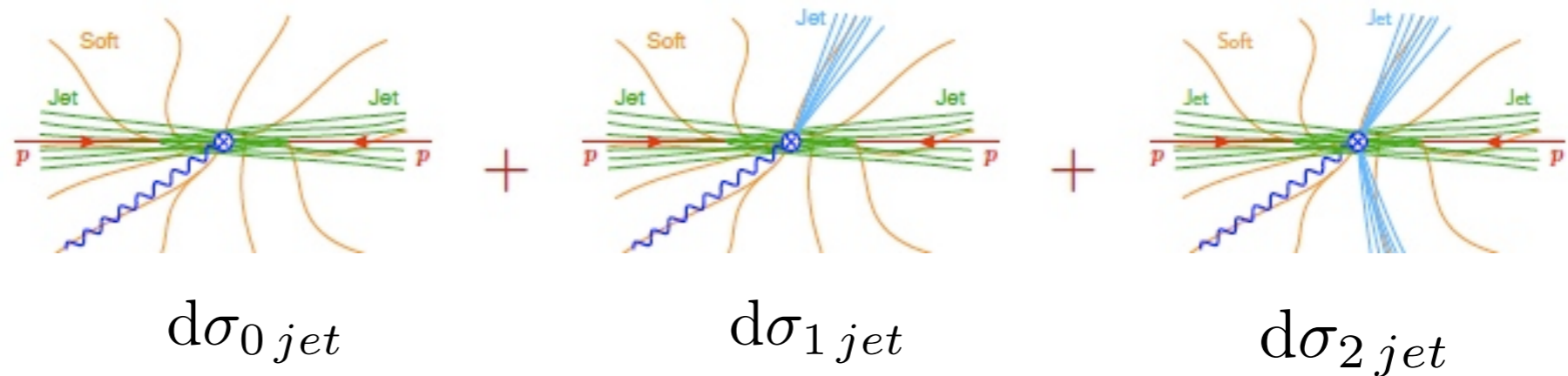
$$d\Phi_3 R_3$$

3 jet sample at LO

- Maintain map from N-Parton Phase Space to N-Body Phase Space
- Avoid Double Counting: Modify 1st emission of parton shower.  
Inclusive jet observable at NLO

# Current Approaches: Fixed Order $\otimes$ Parton Shower

\*Image from Frank Tackmann



- CKKW/MLM

LOxPS

LOxPS

LOxPS

- POWHEG  
MC@NLO

NLOxPS

LOxPS

- MENLOPS  
Genva 0.1

NLOxPS

LOxPS

LOxPS

[Bauer, Tackmann, Thaler;  
Hamilton, Nason;  
Hoche, Krauss,  
Schonherr, Siegert]

- **Goal Geneva**

NLOxPS

NLOxPS

NLOxPS



# The Geneva Approach

- Goal: Exclusive jet multiplicities all at NLO + resummation

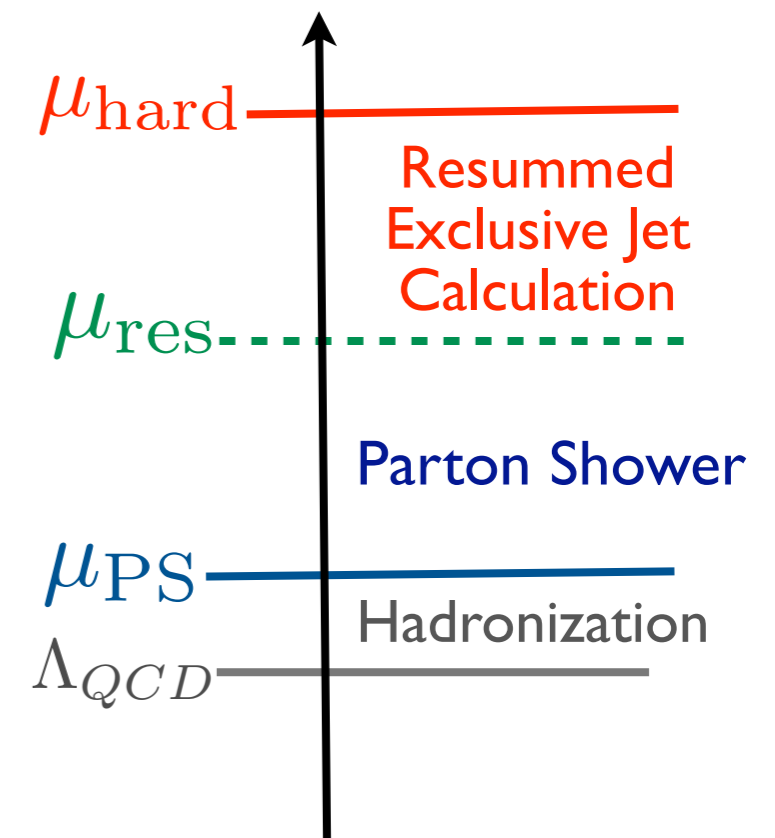
$pp \rightarrow H/W + 0, 1, 2$  jets.      Start with  $e^+e^- \rightarrow 2, 3, 4$  jets.

- Divergences in  $d\Phi_n$  at NLO:  
MC theory input: Exclusive jet cross-sections  
Map N-jet Phase Space to N-body Phase Space

- Avoid Double Counting with Parton Shower:  
Phase space separation  $\mu_{res}$

Resummed calculation to cancel dependence

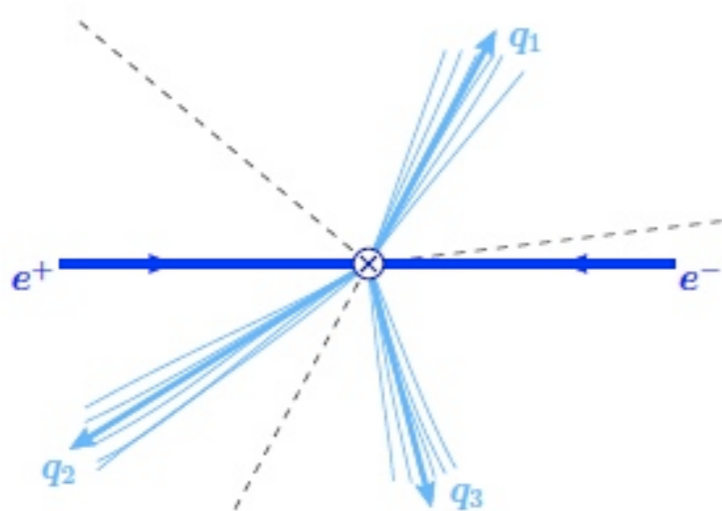
- Divide Phase Space in to N-jet samples using resolution variable, N-jettiness.



# N-jettiness

- Use N-jettiness to divide into exclusive-jet regions.
- Quantifies distance of particles from jet directions  $q_i$

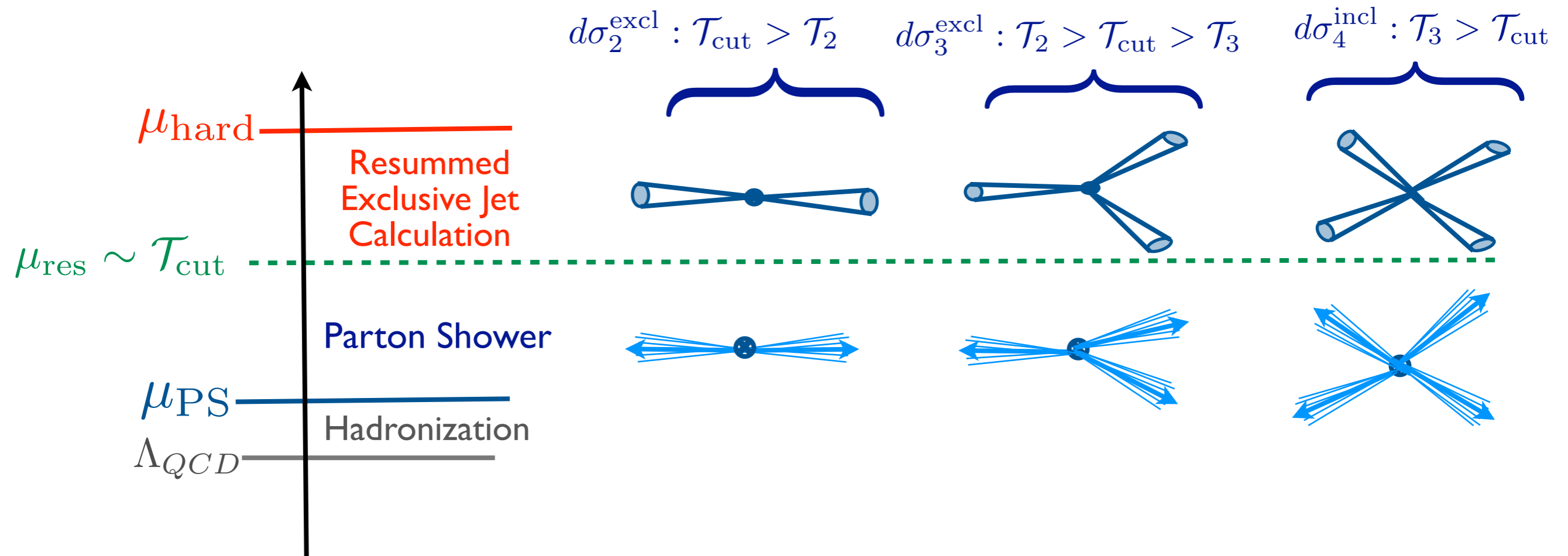
[Stewart, Tackmann, Waalewijn; Jouttenus, Stewart, Tackmann, Waalewijn]



$$\mathcal{T}_N = \sum_k \min_i \{ 2\hat{q}_i \cdot p_k \}$$

- Each final state particle is assigned to a region.
- $\mathcal{T}_N \rightarrow 0$  : N pencil-like jets.  $\mathcal{T}_N \rightarrow Q$  : More than N-jets
- Vetoes  $> N$  jets. Well defined for any number of partonic final states.

# Geneva Approach



- Choose  $\mathcal{T}_{\text{cut}}$  to be small. Narrow jets.
- Parton Shower fills up region below  $\mathcal{T}_{\text{cut}}$
- Integrate up to get other observables to LL

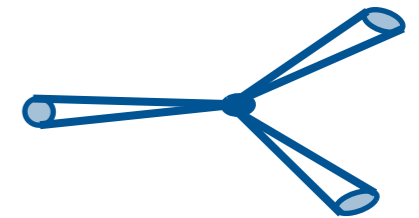
# Resummed Exclusive Jet Cross-Section

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- Use Soft Collinear Effective Theory to calculate

[Bauer, Fleming, Luke, Pirjol, Stewart]

$$\frac{d\sigma}{d\mathcal{T}_N}$$



- In the limit of small  $\mathcal{T}_N$  SCET provides framework to calculate resummed QCD distributions
- Systematically include:  $\alpha_s^n$  matching and resum renormalization group  $\alpha_s^n \ln^m(\mathcal{T}_{cut}/Q)$

# Soft Collinear Effective Theory

Bauer, Fleming, Luke, 2000

Bauer, Fleming, Pirjol, Stewart  
2001

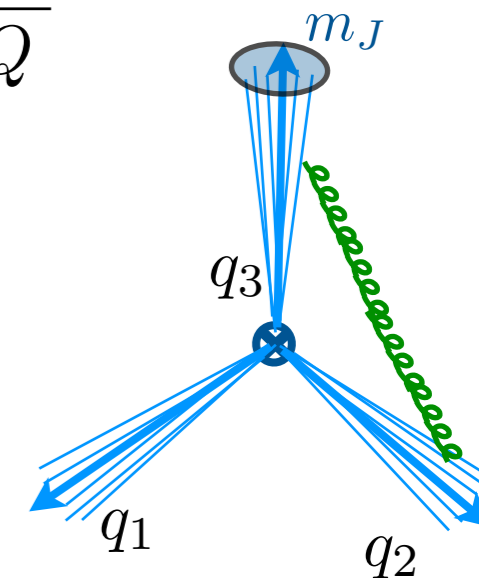
- Construct expansion: large energy, small invariant mass

$$\lambda \sim \sqrt{\frac{\mathcal{T}_N}{Q}}$$

$$q^\mu = q^+ \frac{\bar{n}^\mu}{2} + q^- \frac{n^\mu}{2} + q_\perp^\mu$$

$$n = (1, \vec{n}) \quad \bar{n} = (1, -\vec{n})$$

$$(\bar{n} \cdot q, n \cdot q, q_\perp)$$

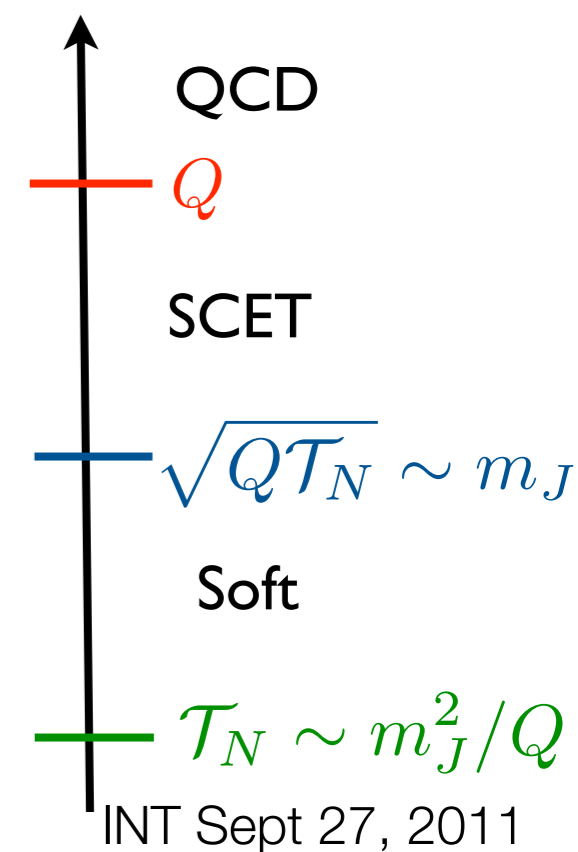


- Collinears in each jet direction.

$$p_c \sim Q(1, \lambda^2, \lambda) \quad p_c^2 \sim \lambda^2 Q^2 \sim \mathcal{T}_N Q$$

- Soft radiation between jets, without changing their virtuality

$$p_s \sim Q(\lambda^2, \lambda^2, \lambda^2) \quad p_s^2 \sim \lambda^4 Q^2 \sim \frac{2}{N}$$

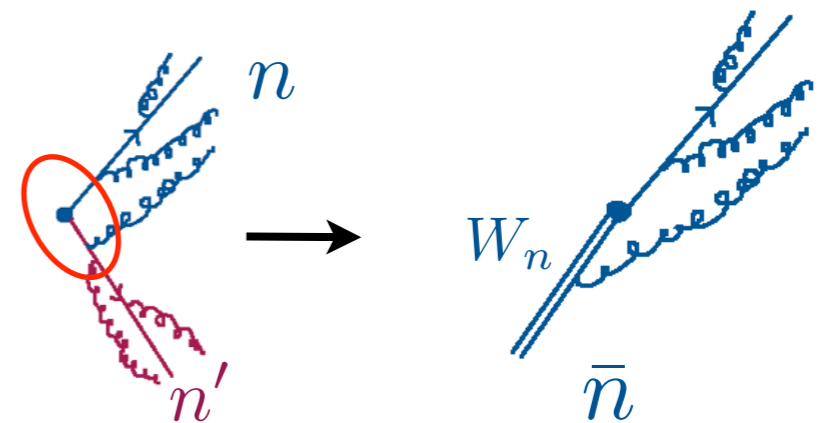
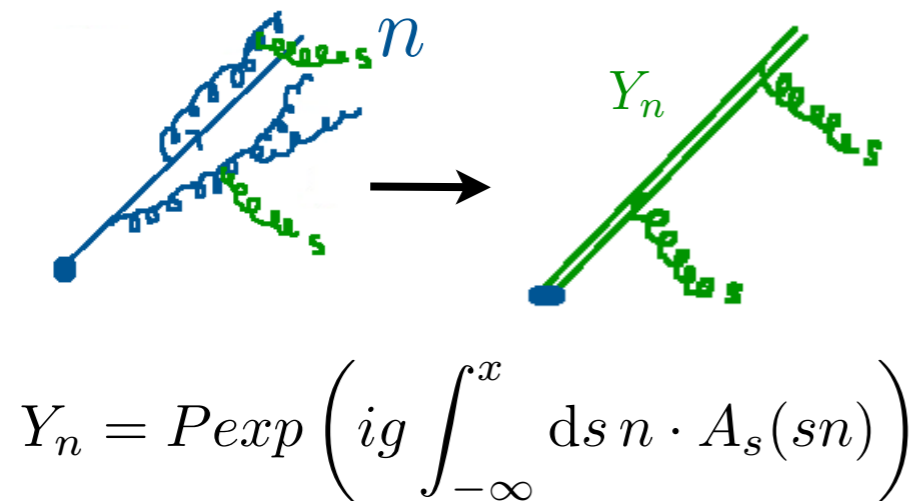


# SCET: An Overview

- Modes with different scaling are different fields in SCET.
- Interactions are simple, consequence of scaling.

➔ Soft only sees color and direction

➔ Collinears see other jet directions  
colour source moving in  $\bar{n}$ -direction.



- BPS Field redefinition  $\mathcal{L}_{SCET} = \mathcal{L}_s + \sum_i \mathcal{L}_{c_i}$  (Bauer, Pirjol, Stewart)

- Operators built from Jet field  $\chi_n = W_n^\dagger \xi_n$  and couples to soft through  $Y_n$

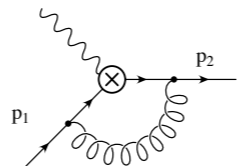
# Calculating in SCET

- Factorization shown for N-jettiness for general N

[Stewart, Tackmann, Waalewijn]

$$\frac{d\sigma_N}{d\mathcal{T}_N} = H_N(Q) \prod_i J_i(\sqrt{Q\mathcal{T}_N}) \otimes S_N(\mathcal{T}_N) \quad \mathcal{T}_N \ll 1$$

- Separately calculate  $\alpha_s^n$  corrections to each piece.

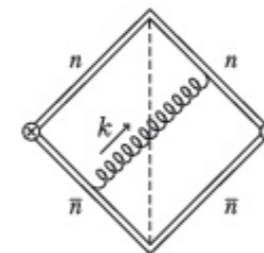
• **Hard Function:** QCD   $\longrightarrow C_N \langle \mathcal{O}_N \rangle$

Virtual correction of QCD. Independent of observable Use known NLO result

• **Jet Function**  $J_n(\mathcal{T}_N) \sim \langle \chi_n \widehat{\mathcal{M}}(\mathcal{T}_N^c) \bar{\chi}_n \rangle$



• **Soft Function:**  $S(\mathcal{T}_N) \sim \langle Y_j^\dagger Y_i \widehat{\mathcal{M}}(\mathcal{T}_N) Y_i^\dagger Y_j \rangle$



$\mathcal{M}$

observable  
dependent

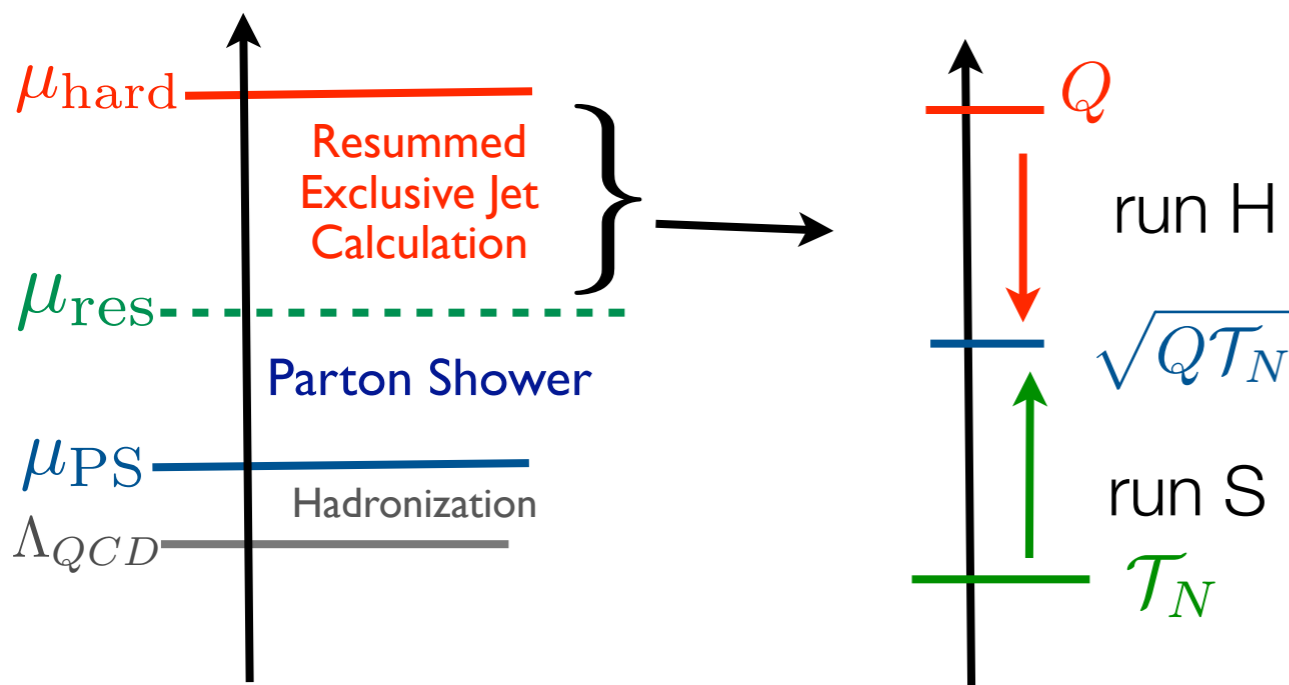
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$$\frac{d\sigma_N}{d\mathcal{T}_N} = H_N(Q) \prod_i J_i(\sqrt{Q\mathcal{T}_N}) \otimes S_N(\mathcal{T}_N) \quad \mathcal{T}_N \ll 1$$

- Use renormalization group to run each component to a common scale  
RG arranges terms to all orders in log counting. Makes series more convergent

$$F(\mu) = \exp\left(\int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'} \gamma_F(\mu')\right) F(\mu_F) \equiv \Pi_F(\mu_F, \mu) F(\mu_F)$$

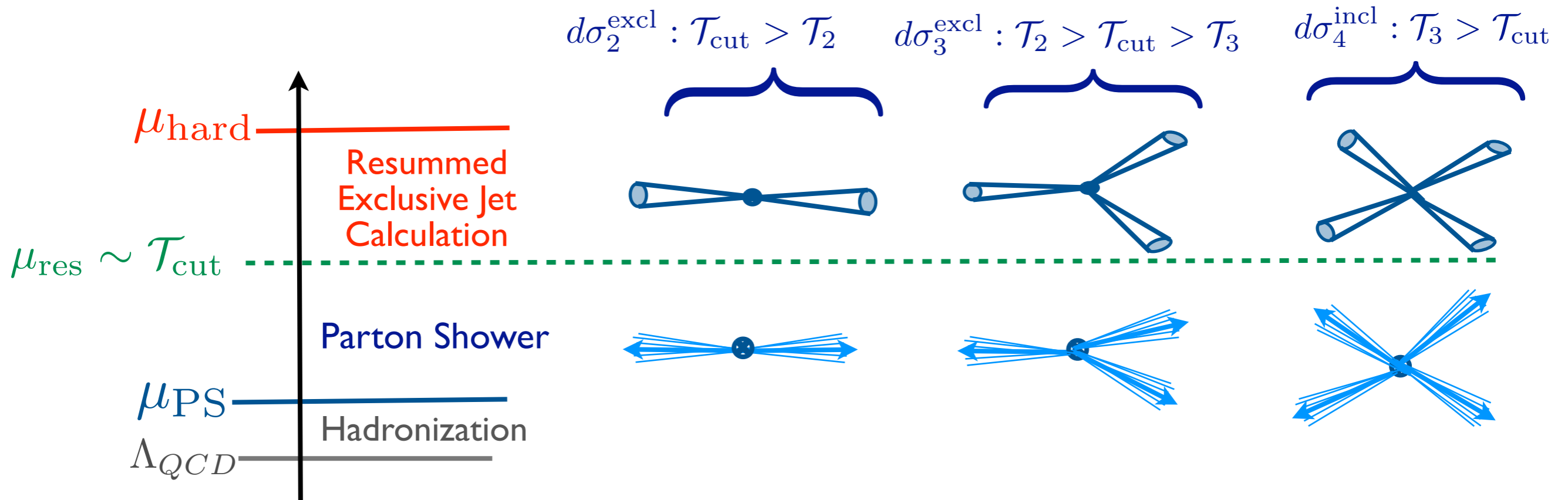


$$\sigma_2(\mathcal{T}_{\text{cut}}) = \int_0^{\mathcal{T}_{\text{cut}}} d\mathcal{T}_2 \frac{d\sigma}{d\mathcal{T}_2}$$

→ Resummed to NNLL



# Recall what we are after....



- Focus on exclusive 2 jet and inclusive 3 jets for now. Divide samples with cut on  $\mathcal{T}_2$ .
- Generate 2 body PS weights  $d\sigma_2(\mathcal{T}_{\text{cut}})$  at NLO  $\mathcal{O}(\alpha_s)$  and resummed.  
3 body PS weights  $d\sigma_{\geq 3}(\mathcal{T}_2 > \mathcal{T}_{\text{cut}})$  at NLO  $\mathcal{O}(\alpha^2)$ . Combine to  $d\sigma_2$  inclusive NLO.
- Naively 3 jets NLO would require 2 jet to N<sup>2</sup>LO to be IR finite.  
Geneva: Relevant pieces obtained from resummation

# Perturbative Accuracy

- For resummed 2-jet exclusive rate combine  $\mathcal{O}(\alpha_s)$  and leading log (LL)

$$\alpha L^2 \sim 1 \qquad L \sim \ln \frac{\mathcal{T}_{cut}}{Q}$$

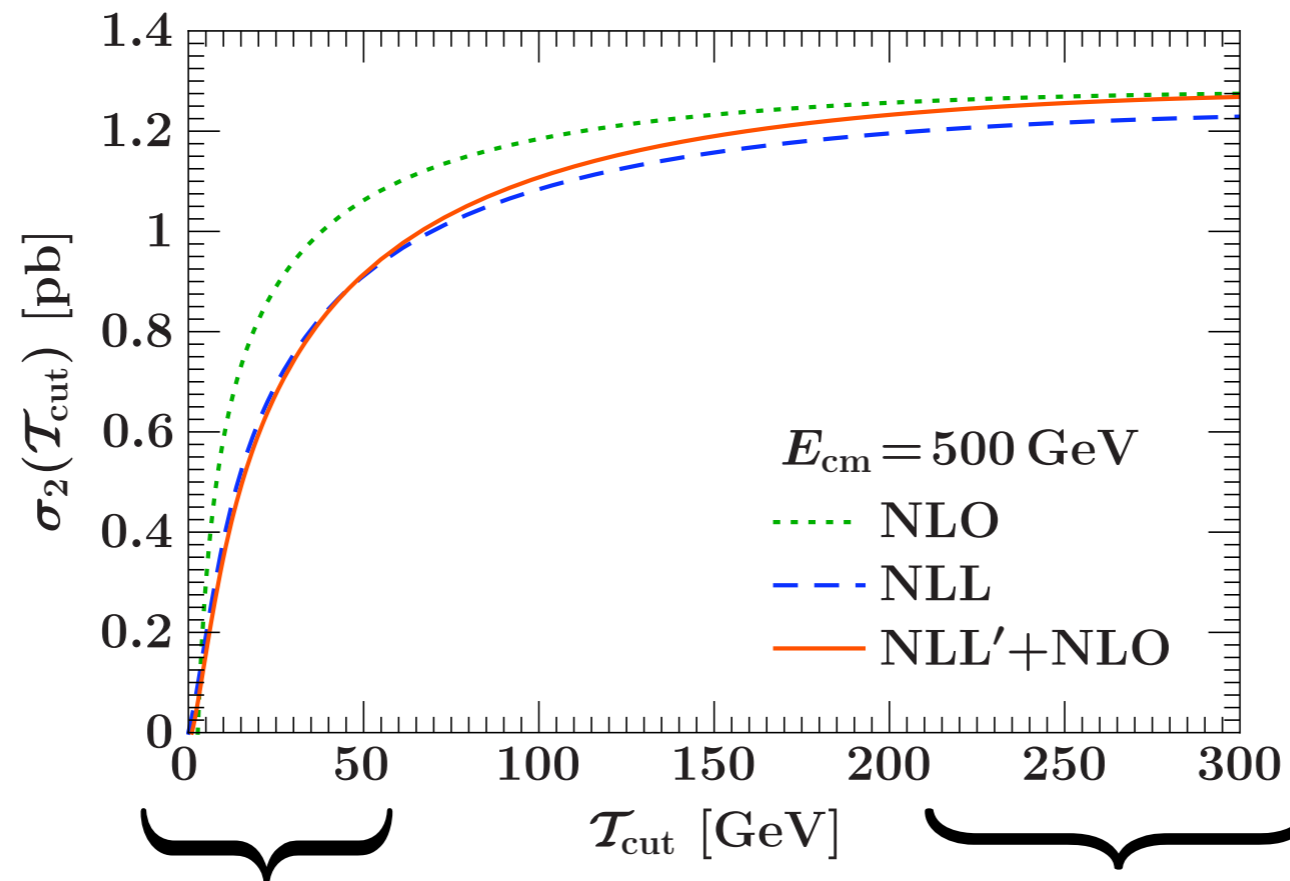
$$\sigma \sim \begin{array}{l} 1 \\ \text{NLO 2-jet} + \alpha_s L^2 + \alpha_s L + \alpha_s \\ \text{NNLO 2-jet/} + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 \\ \text{NLO 3-jet} \end{array}$$

$LL$ 
 $NLL$

- Consistent counting requires all  $\alpha_s^n L^{2n-2} \sim \alpha_s$  to be resummed. NLL'D

# Combining Jet Multiplicities

- Combine on 2-jet NLL'+NLO, 3 jet NLO
- Need to properly interpolate between  $d\sigma_2(\mathcal{T}_{cut})$  at NLO and resummed 3 body PS weights  $d\sigma_{\geq 3}(\mathcal{T}_2 > \mathcal{T}_{cut})$  at NLO

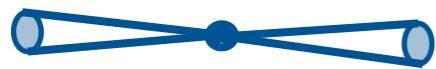


2-jet ResummedSCET  
 $NLL' = NLO \text{ FO} + \alpha_s^n L^{2n-2}$

More than 2-jets region.  
 Non-singular NLO

# Combining Jet Multiplicities

- Combine on 2-jet NLL'+NLO, 3 jet NLO
- Distribute events according to:



2-body events

$$\frac{d\sigma_2}{d\Phi_2}(\mathcal{T}_{\text{cut}}) = \int_0^{\mathcal{T}_{\text{cut}}} d\mathcal{T}_2 \frac{d\sigma_2}{d\Omega d\mathcal{T}_2} + \frac{d\sigma_{\geq 3}}{d\Phi_3} = \left( \frac{d\sigma_2}{d\Omega_2 d\mathcal{T}_2} / \frac{d\sigma_2}{d\Omega_2 d\mathcal{T}_2} \Big|_{\text{exp}} \right) \frac{d\sigma_{\geq 3}^{\text{FO}}}{d\Phi_3} \theta(\mathcal{T}_2 > \mathcal{T}_{\text{cut}})$$

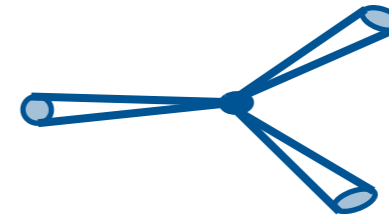
Constant  $\mathcal{T}_2$   
dependence for  
2-body events

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2-body events



3-body events

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Has full  $\Phi_3$   
dependence

Resummed to  
 $\alpha_s^n L^{2n-2}$

Expanded to  
 $\mathcal{O}(\alpha_s^2)$

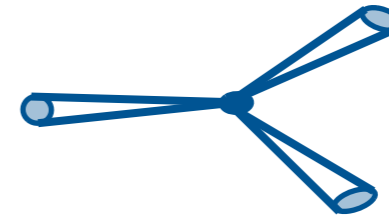
Full FO  
contribution  
at NLO

# Combining Jet Multiplicities

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- Distribute events according to:



2-body events



3-body events

$$\frac{d\sigma_2}{d\Phi_2}(\mathcal{T}_{\text{cut}}) = \int_0^{\mathcal{T}_{\text{cut}}} d\mathcal{T}_2 \frac{d\sigma_2}{d\Omega d\mathcal{T}_2} + \frac{d\sigma_{\geq 3}}{d\Phi_3} = \left( \frac{d\sigma_2}{d\Omega_2 d\mathcal{T}_2} / \frac{d\sigma_2}{d\Omega_2 d\mathcal{T}_2} \Big|_{\text{exp}} \right) \frac{d\sigma_{\geq 3}^{\text{FO}}}{d\Phi_3} \theta(\mathcal{T}_2 > \mathcal{T}_{\text{cut}})$$

Large  $\mathcal{T}_2$  :

Resummation starts to turn off.

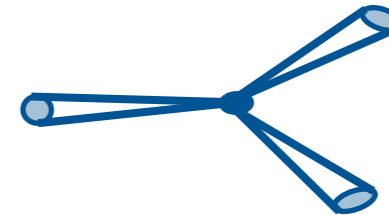
Ratio starts at  $\mathcal{O}(\alpha_s^3)$

# Combining Jet Multiplicities

- Combine on 2-jet NLL'+NLO, 3 jet NLO
- Distribute events according to:



2-body events



3-body events

$$\frac{d\sigma_2}{d\Phi_2}(\mathcal{T}_{\text{cut}}) = \int_0^{\mathcal{T}_{\text{cut}}} d\mathcal{T}_2 \frac{d\sigma_2}{d\Omega d\mathcal{T}_2} + \frac{d\sigma_{\geq 3}}{d\Phi_3} = \left( \frac{d\sigma_2}{d\Omega_2 d\mathcal{T}_2} / \frac{d\sigma_2}{d\Omega_2 d\mathcal{T}_2} \Big|_{\text{exp}} \right) \frac{d\sigma_{\geq 3}^{\text{FO}}}{d\Phi_3} \theta(\mathcal{T}_2 > \mathcal{T}_{\text{cut}})$$

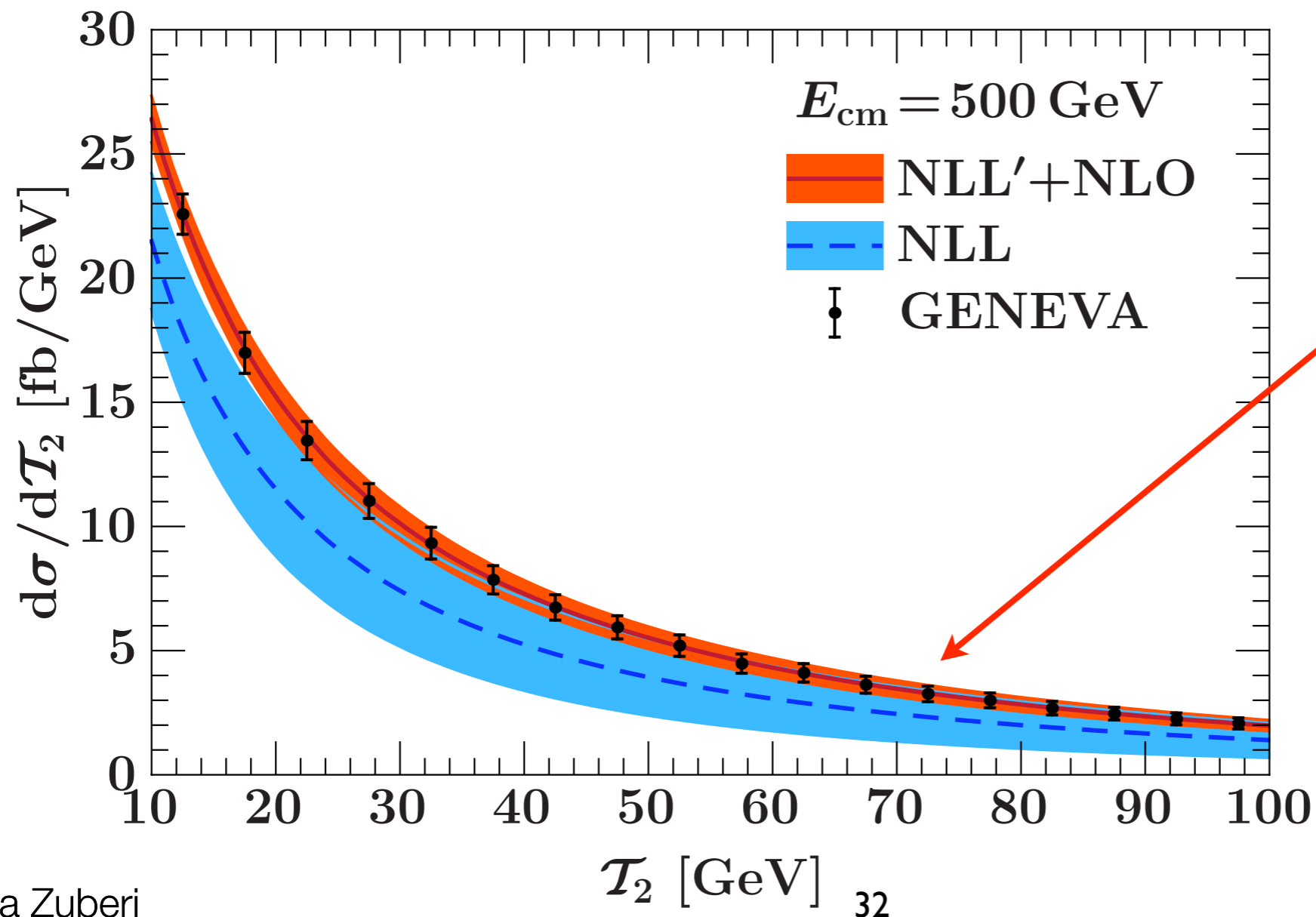
Small  $\mathcal{T}_2$  :

Resummation important

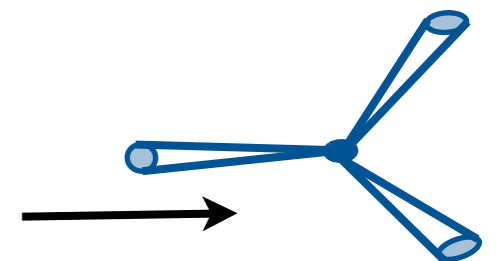
Ratio starts at  $\mathcal{O}(\alpha_s^2 L)$  NNLL

# First Results: 2 and 3 jets at NLO

- Previously: 2 jets at NLL+NLO and 3 jets at LO
- Now with Geneva have 2 jets at NLL+NLO and 3 jets at **NLO**  
**Systematically extendable**



- Monte Carlo with theory (scale) uncertainties not MC statistics!

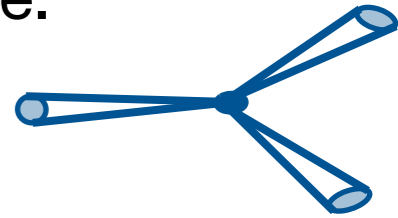




# First Results: 2 and 3 jets at NLO

---

- $\mathcal{T}_2$  was the spectrum we used to distinguish 2 and 3 body Phase Space.  
Consider variable sensitive to  $\Phi_3$  angular dependence.

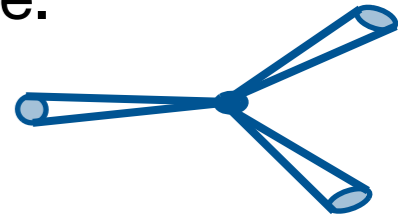


- Should reproduce shape.  $d\Phi_3 = d\Phi_2 dz d\mathcal{T}_2 d\phi$   
Definition of variables consistent for 3-jets (at any order).

$$z = \frac{E_1}{E_1 + E_2}$$

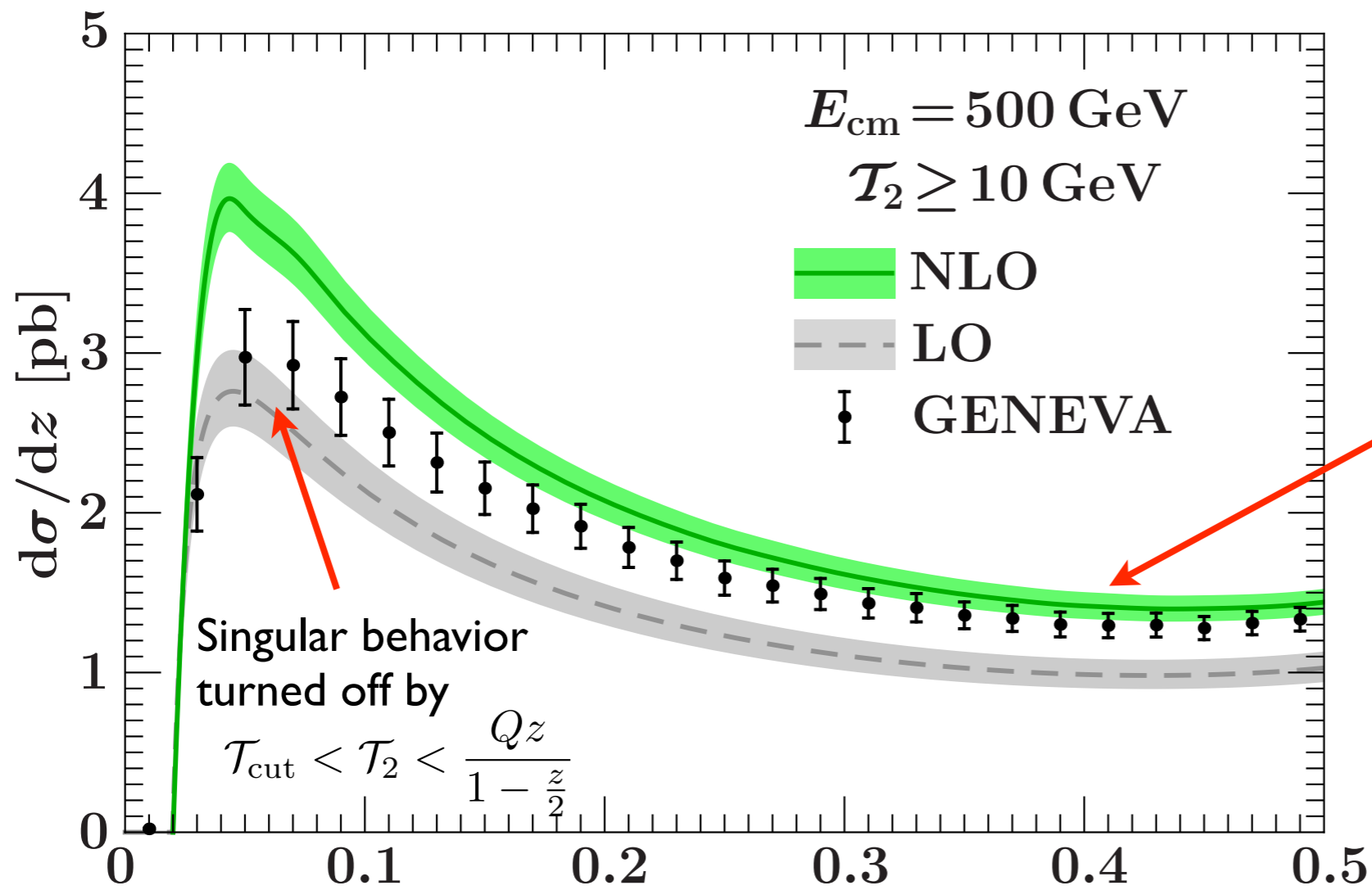
# Another variable $z$

- $\mathcal{T}_2$  was the spectrum we used to distinguish 2 and 3 body Phase Space.  
Consider variable sensitive to  $\Phi_3$  angular dependence.



$$z = \frac{E_1}{E_1 + E_2}$$

- Should reproduce shape.  $d\Phi_3 = d\Phi_2 dz d\mathcal{T}_2 d\phi$   
Definition of variables consistent for 3-jets (at any order).



- **2 + 3 jets at NLO**  
3 jet NLO shift  $\sim \alpha_s^2$
- **Theory scale uncertainties**

# Other Variables: Angularities

---

- A non-trivial check: Integrate up events distributed according to N-jettiness to get other observables to LL.

Check cancellation of cut dependence when combined with parton shower

$$\tau_a(X) = \frac{1}{Q} \sum_i |p_i^T| e^{-\eta_i(1-a)} \quad (\text{Berger, Kucs, Sterman})$$

2 jet like for  $\tau_a \rightarrow 0$   
Thrust  $a=0$

Consider  $a = -1$

# Other Variables: Angularities

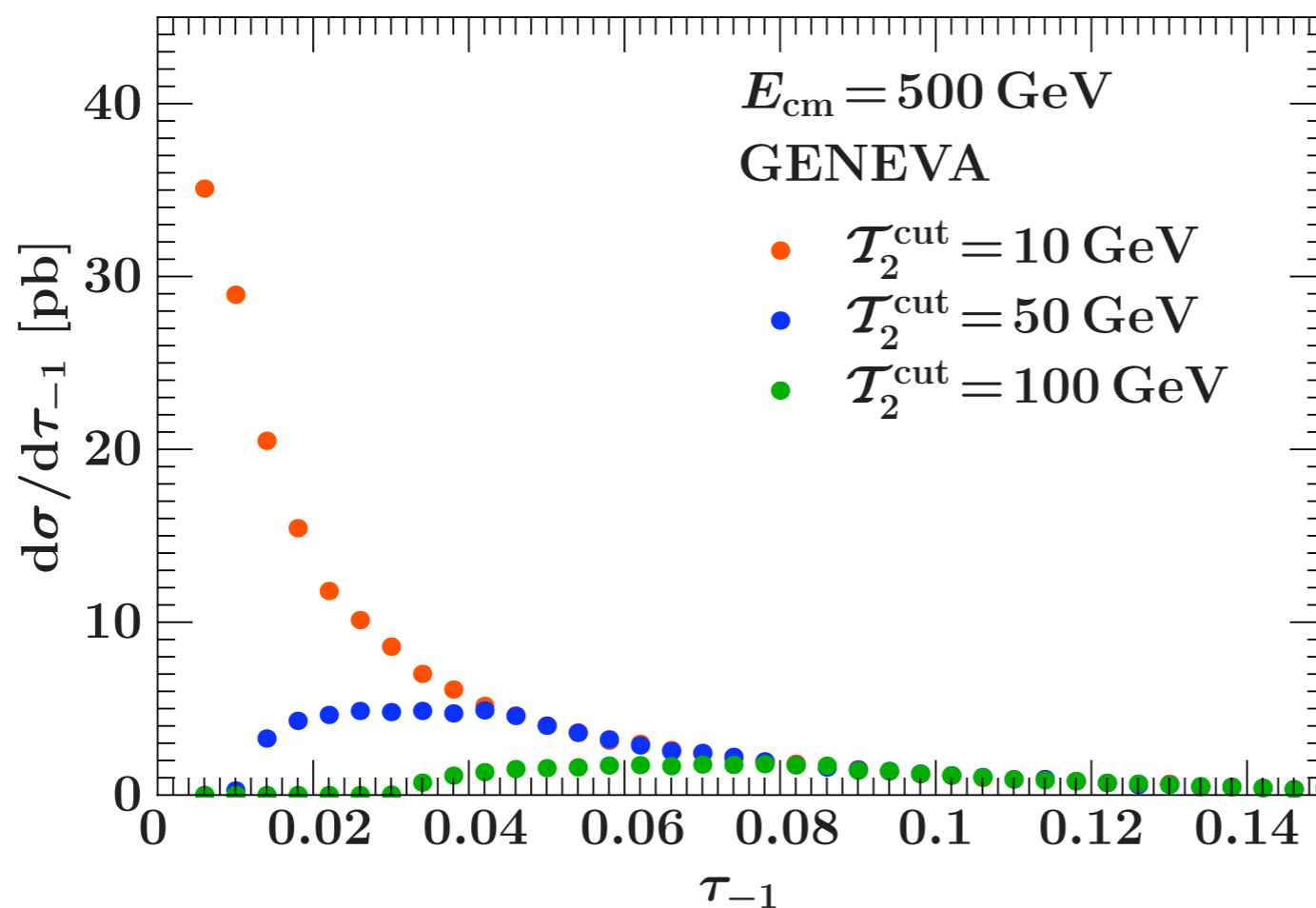
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# Other Variables: Angularities

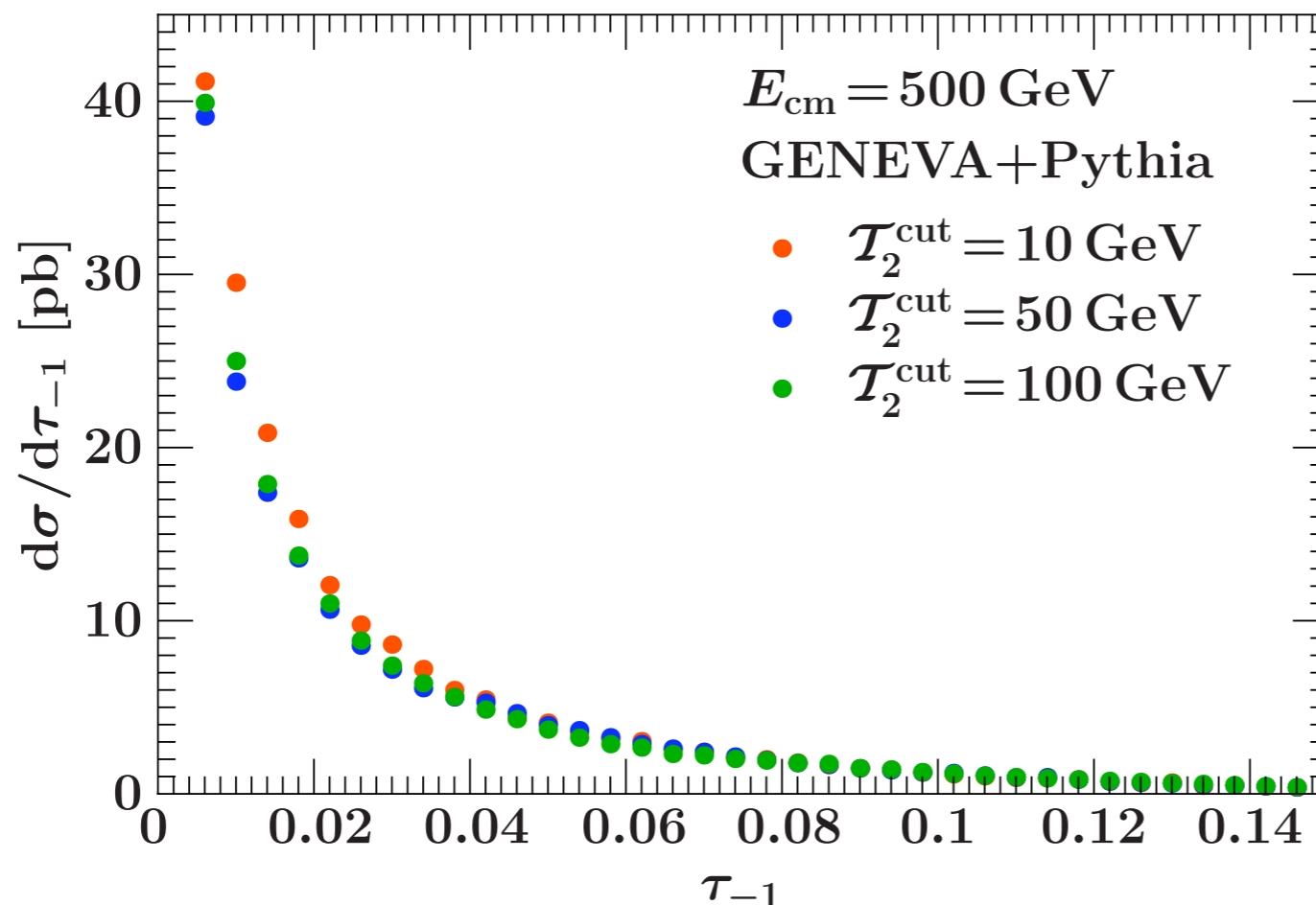
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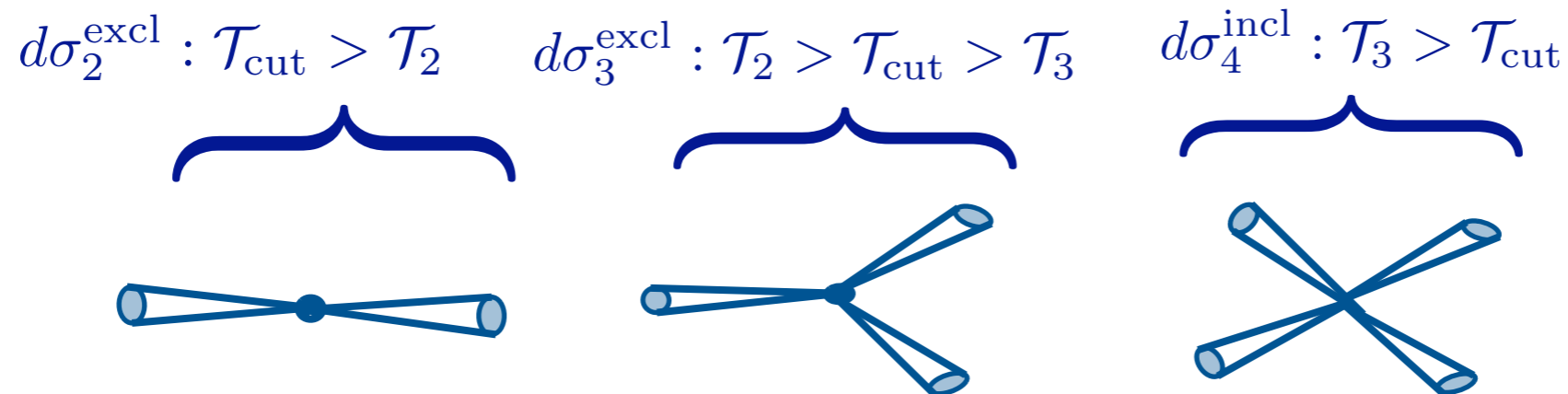
2 jet like for  $\tau_a \rightarrow 0$   
Thrust  $a=0$

Consider  $a = -1$



# Next Steps for Geneva: Theory Challenges

- **Combine higher jet multiplicities at NLO :**



This involves summing logs of wider range of kinematic configurations with additional scales  $\mathcal{T}_i$  .

(Bauer, Tackmann, Walsh, SZ)

\* See Jon Walsh's talk

- **Soft subtractions:**  $S(\mathcal{T}_N) \sim \langle Y_j^\dagger Y_i \widehat{\mathcal{M}}(\mathcal{T}_N) Y_i^\dagger Y_j \rangle = \frac{A_{SCET}}{\epsilon^2} + \frac{B_{SCET}}{\epsilon} + C$

Subtraction function:  $\int d^d k \text{Sub}(k) = \frac{A_{SCET}}{\epsilon^2} + \frac{B_{SCET}}{\epsilon} + C'$

Algorithm dependence!

(Bauer, Dunn, Hornig; Dunn Hornig)

# Conclusions

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- Want event generators with best possible accuracy to connect theory and experiment
- Goal of Geneva: Combining several jet multiplicities at NLO with resummation/parton shower
- Method: Use resummed exclusive cross-sections from SCET
- Status: 2 jet NLL'+ NLO and 3 jet NLO
- Coming soon at NLO  $pp \rightarrow H + 0, 1 \text{ jets}$ ,  $pp \rightarrow W + 0, 1 \text{ jets}$  !

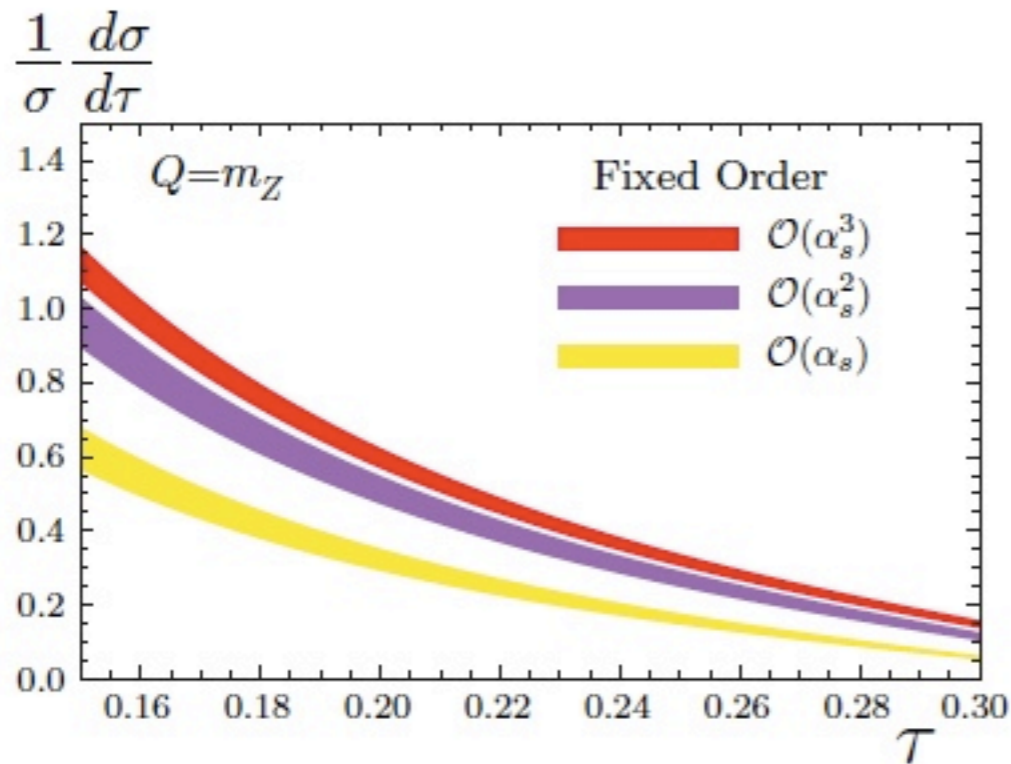
**Thank You**



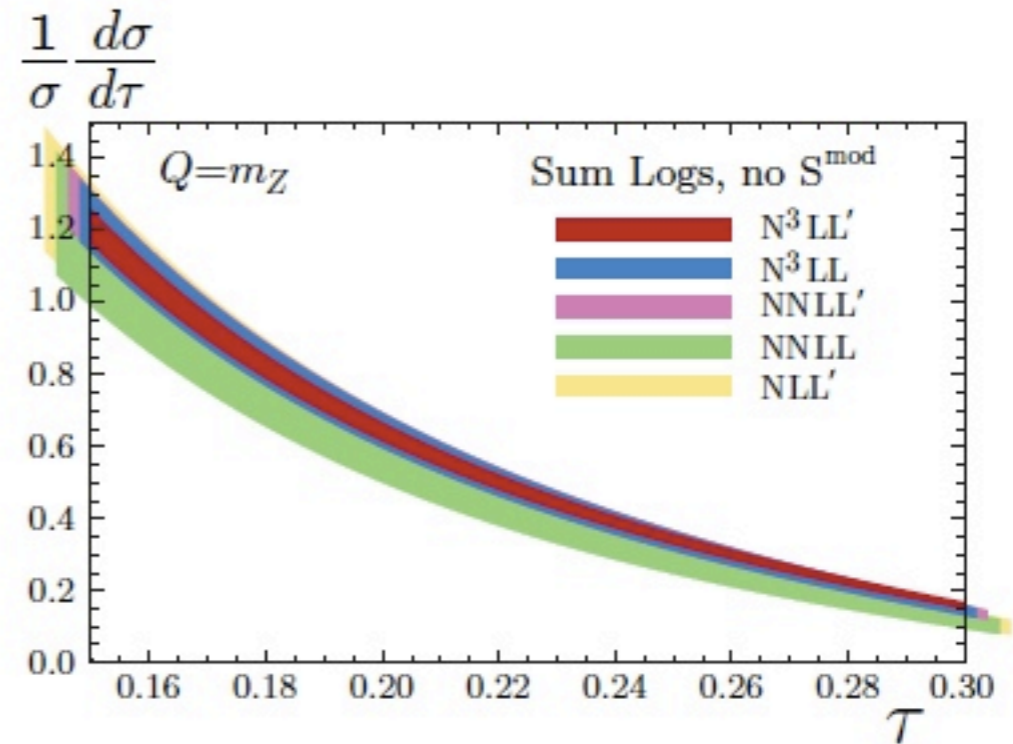
# Back Up Slides

# $\alpha_s^2$ corrections are Large

- NLL' much larger than  $\alpha_s$  contribution



(a)



(b)

[Abbate, Fickinger, Hoang, Mateu, Stewart]