Jet Quenching and Holographic Thermalization

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with Berndt Muller

Outline

- Energy loss in the holographic picture
- Thermalization and gravitational collapse :

  The falling mass shell in AdS-Vaidya spacetime
- Stopping diatance :

  Null geodesic of a massless particle moving in AdS-Vaidya spacetime
- Approximating jet quenching parameter :

  The moving string in AdS-Vaidya spacetime
- Summary

Energy loss of a heavy quark in N=4 SYM

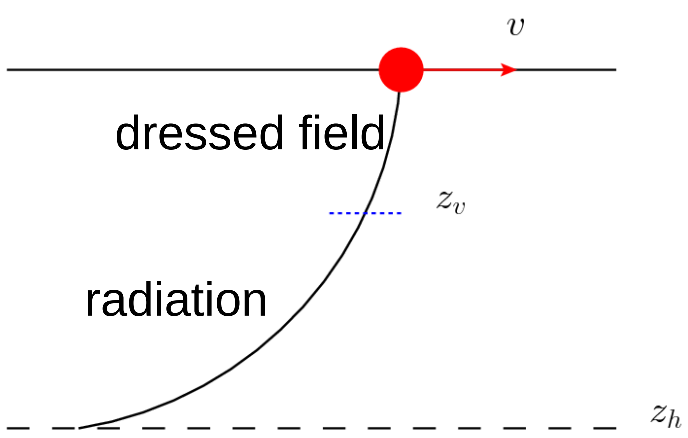
- Trailing string : drag force and diffusion Gubser, Herzog et.al.
- Lightcone Wilson loop : multiple scattering and momentum broadening Liu et.al.
- The string profile can be obtained by extremized the Nambu-Goto action in AdS-Schwarzschild spacetime .
- By using Langevin equation, the drag coefficient can be derived.

$$x^3(t,z) = vt + \xi(z)$$

$$\Rightarrow \frac{d\xi(z)}{dz} = -\frac{v}{1-(z/z_h)^4}\left(\frac{z}{z_h}\right)^2$$

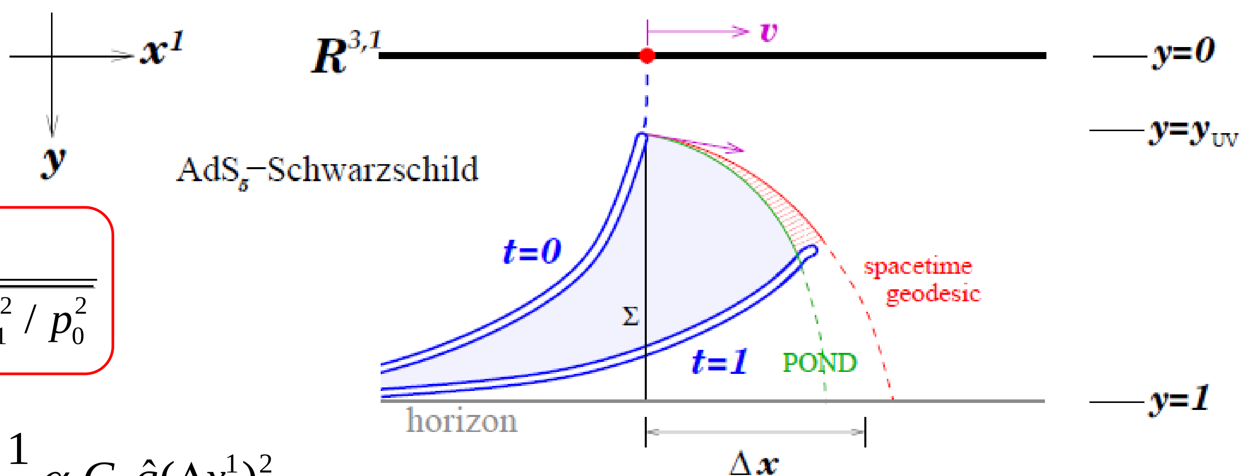
$$\Rightarrow \frac{dp_3}{dt} = -\frac{\pi\sqrt{g_{YM}^2 N_c}}{2} T^2 \frac{p_3}{m}$$

$z > z_v = z_h / \gamma$:causally disconnected



Falling string and gluon energy loss

- The string with two ends attached to the branes below horizon may describe a gluon traveling in the medium.
  - tip : null geodesic (trajectory of a massless particle)
  - lower part: trailing string
- The string will eventually fall to the horizon
- longitudinal displacement : maximum stopping distance



$$\Delta x^1 = -z_h \int_{y_{UV}}^{1} dy \frac{p_1 / p_0}{\sqrt{1-(1-y^4) p_1^2 / p_0^2}}$$

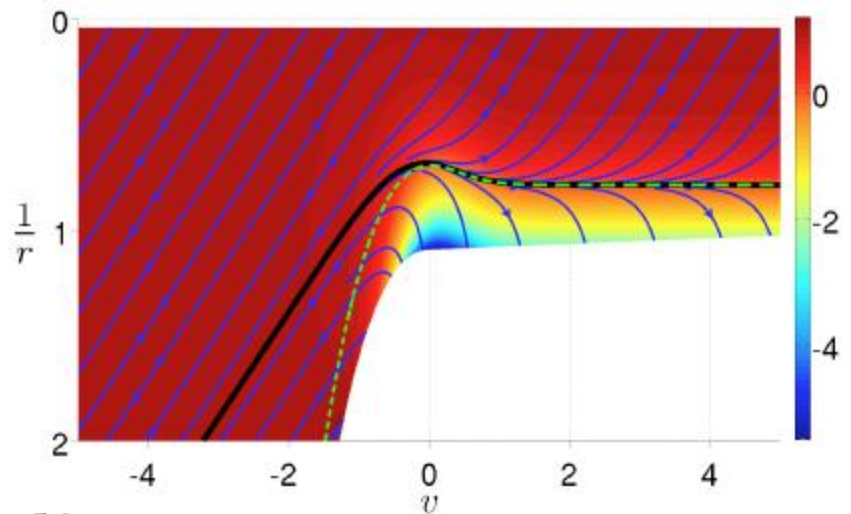
BDMPS formalism: $E = \frac{1}{4} \alpha_s C_R \hat{q} (\Delta x^1)^2$

Gubser et.al. JHEP0810:052,2008

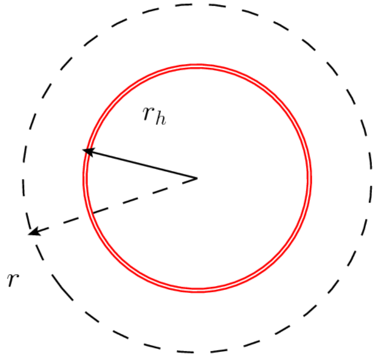
Gravitational collapse and the thermalization

- The stationary black hole can only describe the physics after the thermalization.
- Gravitation collapse : the formation of black hole corresponds to the thermalization of medium.
- Anisotropic and time-dependent metric on the boundary: radiation to the bulk and the formation of horizon.

  P. Chesler and L. Yaffe



- Different approach : Black hole formation as a shrinking shell?

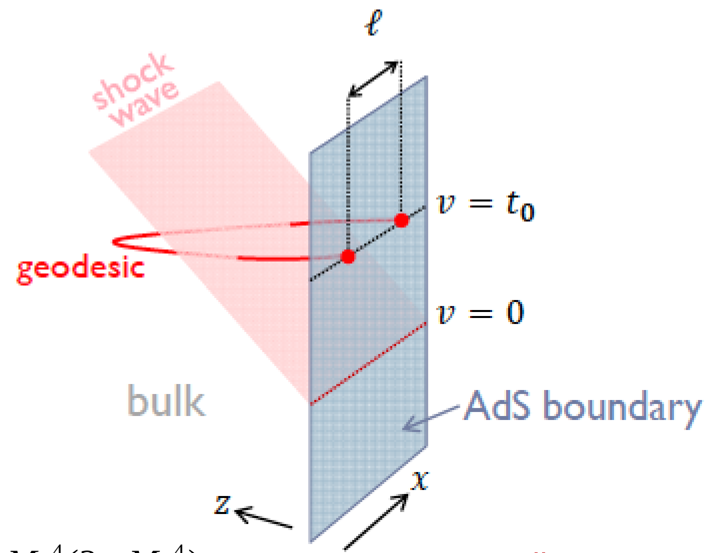


Falling mass shell and AdS-Vaidya spacetime

- A light-like falling mass shell (shock wave) finally forms a black hole. V. Balasubramanian et.al. Phys.Rev.D84:026010,2011
  - Outside : AdS-Schwarzschild (thermalized medium)
  - Inside : quasi-AdS (vacuum)
- AdS-Vaidya spacetime:

$$ds^2 = \frac{1}{z^2}\left(-(1-m(v)z^4)dv^2 - 2dvdz + dx^2\right)$$

- Thin shell limit: $m(v) = \frac{M}{2}\left(1+\tanh\left(\frac{v}{v_0}\right)\right)_{v_0\to 0}$



$$dv = \begin{cases} dt - \dfrac{1}{1-Mz^4}dz & \text{if} \quad v > 0 (z < z_0) \\ \underline{c(z_0)}dt - dz & \text{if} \quad v < 0 (z > z_0) \end{cases}$$

red shift

$$c(z_0) = \exp[-\frac{Mz_0^4(2-Mz_0^4)}{2(1-Mz_0^4)}] < 1$$

B. Muller QM2011

Derivation of the red shift factor

- Original metric: $ds^2 = -\frac{\dot{m}^2}{f^2}(1-\frac{m}{r})dt^2 + (1-\frac{m}{r})^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$

  $m(v) = m(t,r)$ $\quad f(m) = \frac{\partial m}{\partial r}(1-\frac{m}{r})$

- Generalization in AdS spacetime: $(R=1)$

$$ds^2 = -\frac{\dot{m}^2}{f^2}(1-\frac{m}{r^2}+r^2)dt^2 + (1-\frac{m}{r^2}+r^2)^{-1}dr^2 + r^2 d\Omega_3^2$$

By using the advanced null coordinate $dv = \frac{dm}{f(m)}$, $r^2 - m(v)/r^2 \gg 1$

$$ds^2 = \frac{1}{z^2}\left(-(1-mz^4)dv^2 - 2dvdz + dx^2\right) \qquad f(m) = -m'(1-mz^4)$$

$$dv = \frac{dm}{f(m)} = \frac{1}{f(m)}(m'dz + \dot{m}dt).$$

where $z=1/r$

- Finding m' and $\dot{m}$ for the specific shell.

Derivation of the red shift factor

- Specific function of the sell: $m(v) = \frac{M}{2}\left(1 + \tanh\left(\frac{v}{v_0}\right)\right)$

- We derive $m' = \frac{Mv'}{2v_0}\mathrm{sech}^2\left(\frac{v}{v_0}\right) = \frac{Mm'}{2v_0 f(m)}\mathrm{sech}^2\left(\frac{v}{v_0}\right)$ $f(m) = \frac{M}{2v_0}\mathrm{sech}^2\left(\frac{v}{v_0}\right) = \frac{2m}{v_0}\left(1 - \frac{m}{M}\right)$

we define $\dot{m} = f(m)g(t,z)$ and rewrite $dv = g(t,z)dt - (1 - mz^4)^{-1}dz$

- Spacetime metric requires $\frac{\partial^2 v}{\partial z \partial t} = \frac{\partial^2 v}{\partial t \partial z}$

$$g(t,z) = g_0(t)exp\left[\int_0^z dy \frac{-2m(v)y^4}{v_0(1 - my^4)^2}\left(1 - \frac{m(v)}{M}\right)\right]$$

- In the thin shell limit, by taking the expansion of zero of v

$$v \approx v'(y_0, t)(y - y_0)$$

- Red shift factor ⇨ $$g(t,z) = g_0(t)exp\left[-U(z - z_0)\frac{Mz_0^4(2 - Mz_0^4)}{2 - 2Mz_0^4}\right]$$

The Position of Shell

- Spacetimes in two regions:

$$ds^2 = \begin{cases} \frac{1}{z^2}\left(-f dt^2 + \frac{dz^2}{f} + dx^2\right) & \text{if } v > 0 (z < z_0), \\ \frac{1}{z^2}\left(-c(z_0)^2 dt^2 + dz^2 + dx^2\right) & \text{if } v < 0 (z > z_0), \end{cases}$$

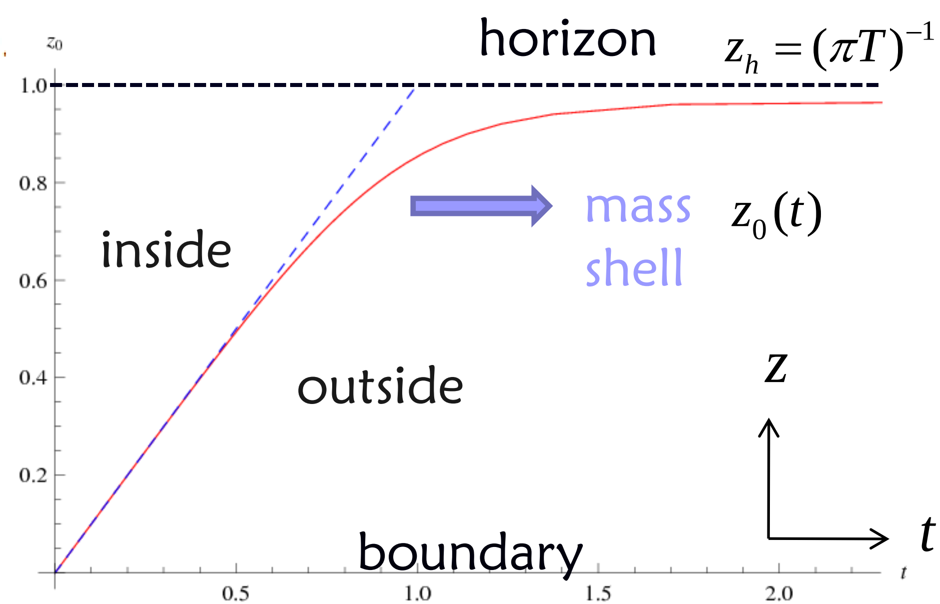
$f = 1-(z/z_h)^4$

- The position of shell can be found by using

$dv = g(t,z)dt - \frac{dz}{1-mz^4}$ and $v(z_0) = 0.$

- The position of shell is a function of t.
- The shell coincides with the horizon at large t.
- Linear approximation leads to $t_0 = 1/(T\pi)$

(e.g. T=300 MeV, $t_0 \approx 0.2\, fm$)



$$\frac{dz_0}{dt} = \left(1 - \frac{z_0^4}{2z_h^4}\right) g(z_0)$$

Null geodesic in AdS-Vaidya spacetime

- A massless particle traveling in AdS-Vaidya spacetime follows the null geodesic, P. Arnold and D. Vaman, JHEP 1104:027,2011

$$\frac{dx^\mu}{dz} = \sqrt{g_{zz}}\frac{g^{\mu\nu}q_\nu}{(-q_\alpha q_\beta g^{\alpha\beta})^{1/2}} \qquad q_\mu = g_{\mu\nu}dx^\nu/d\lambda \quad \text{4-momentum} \quad \mu = 0,1,2,3$$

- Ejecting a particle with $q_\mu = (-\omega, 0, 0, |\vec{q}|)$ $\quad \omega > |\vec{q}|$
- Outside the shell(AdS-Schwarzschild spacetime):

$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2\right) \Rightarrow \begin{cases} \dfrac{dx^3}{dz} = \dfrac{1}{\left(\frac{z^4}{z_h^4} - \frac{q^2}{|\vec{q}|^2}\right)^{1/2}}, \\ \dfrac{dt}{dz} = \dfrac{1}{\left(1-\frac{z^4}{z_h^4}\right)\left(1-\left(1-\frac{z^4}{z_h^4}\right)\frac{|\vec{q}|^2}{\omega^2}\right)^{1/2}}; \end{cases}$$

- Inside the shell(quasi-AdS spacetime):

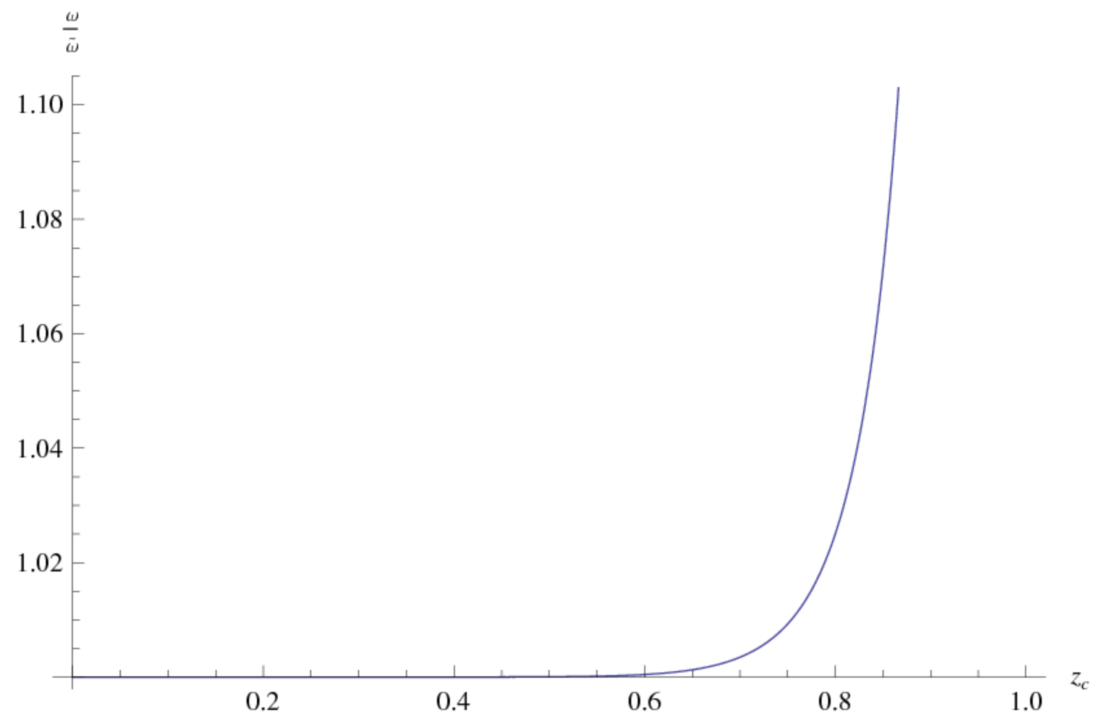
$$ds^2 = \frac{1}{z^2}\left(-c(z_0)^2 dt^2 + dz^2 + dx^2\right) \xrightarrow[\tilde{q}_\mu = (-\tilde{\omega}, 0, 0, |\vec{q}|)]{d\tilde{t} = c(z_0)dt} \begin{cases} \dfrac{dx^3}{dz} = \dfrac{1}{\left(\frac{\tilde{\omega}^2}{|\vec{q}|^2} - 1\right)^{1/2}}, \\ \dfrac{d\tilde{t}}{dz} = \dfrac{1}{\left(1-\frac{|\vec{q}|^2}{\tilde{\omega}^2}\right)^{1/2}}. \end{cases}$$

Matching condition

- The energy defined in two spacetimes match at the collision point

$$\begin{cases} \left.\dfrac{dt}{d\lambda}\right|_{z_c} = z_c^2 f(z_c)^{-1}\omega \\ \left.\dfrac{d\tilde{t}}{d\lambda}\right|_{z_c} = z_c^2 \tilde{\omega} \\ d\tilde{t} = c(z_0)dt \end{cases}$$

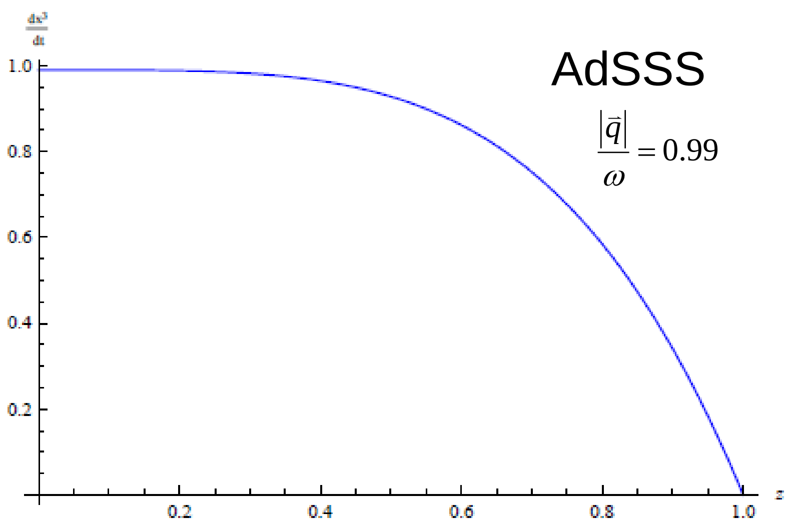
$$\frac{\omega}{\tilde{\omega}} = \frac{1-\frac{z_c^4}{z_h^4}}{c(z_c)}$$

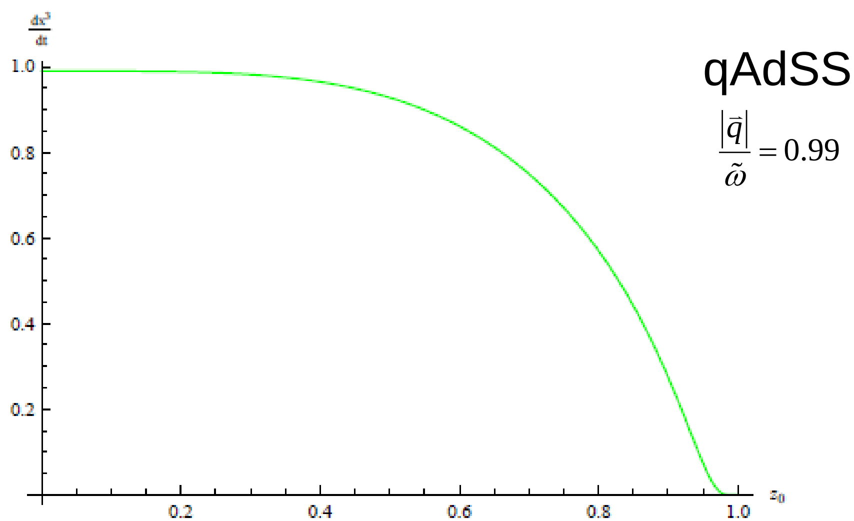


- $\tilde{\omega} \approx \omega$ when the collision point is not too close to the future horizon.

Longitudinal velocity of the almost onshell particle

- In AdSSS $$\frac{dx^3}{dt} = \left(1 - \frac{z^4}{z_h^4}\right)\left(\frac{|\vec{q}|}{\omega}\right)$$
- In qAdSS $$\frac{dx^3}{dt} = c(z_0)\left(\frac{|\vec{q}|}{\tilde{\omega}}\right)$$





- The particle ceases moving in both AdSSS and qAdSS, which leads to the maximum stopping distances.

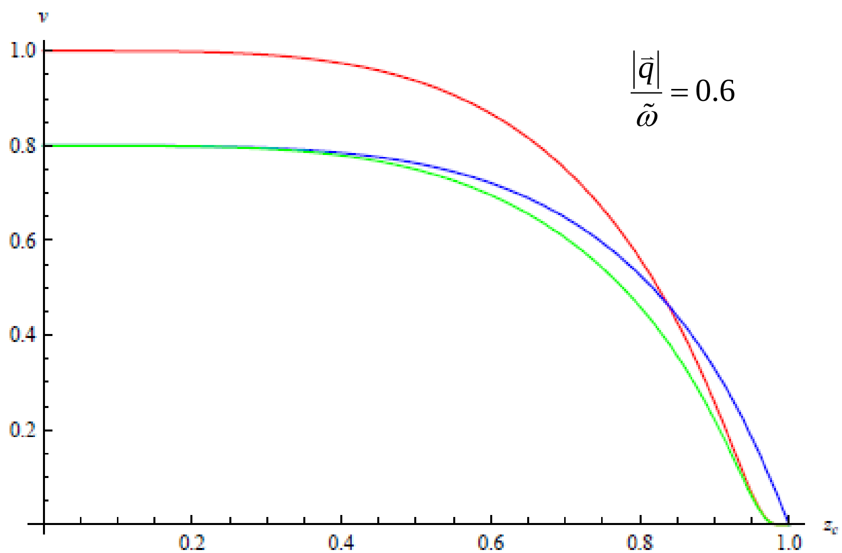
Transverse velocities in the vicinity of shell

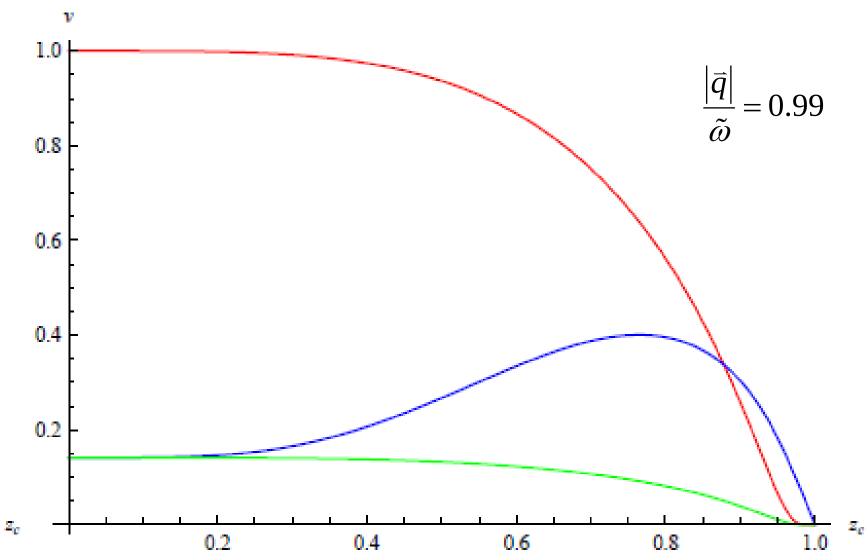
- What happen at the collision point z_c ?
- Comparing the transverse velocities dz/dt:

$$v_{shell} = \left(1 - \frac{z_c^4}{2z_h^4}\right) g(z_c),$$

$$v_{AdS-SS} = \frac{dz}{dt}\Big|_{z=z_c^-} = \left(1 - \frac{z_c^4}{z_h^4}\right) \left(1 - \left(1 - \frac{z_c^4}{z_h^4}\right) \frac{|\vec{q}|^2}{\omega^2}\right)^{1/2},$$

$$v_{qAdS} = \frac{dz}{dt}\Big|_{z=z_c^+} = c(z_c) \left(1 - \frac{|\vec{q}|^2}{\tilde{\omega}^2}\right)^{1/2},$$

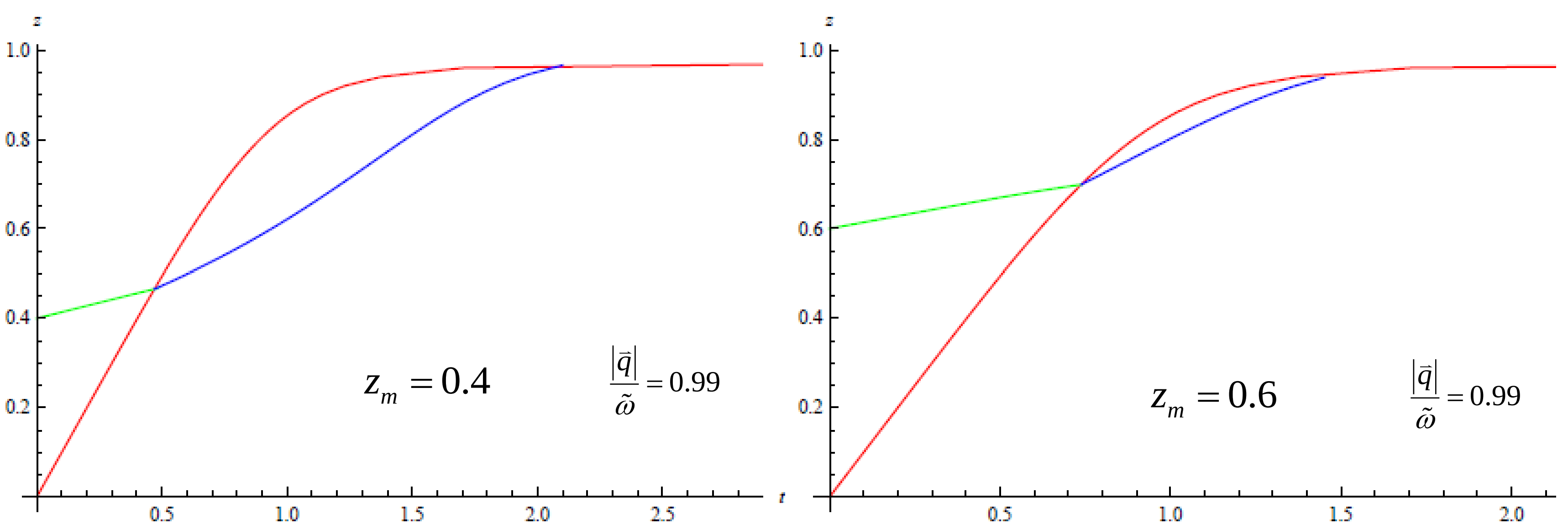




Trajectory of the particle

- Assuming no interaction
  - 1st collision: outpaced by the shell
  - 2nd collision: accrete to the shell
- For the particle ejected near the boundary, the result will be the same as that in pure AdSSS.

shell
pt in qAdS
pt in AdSSS



Maximum stopping distance

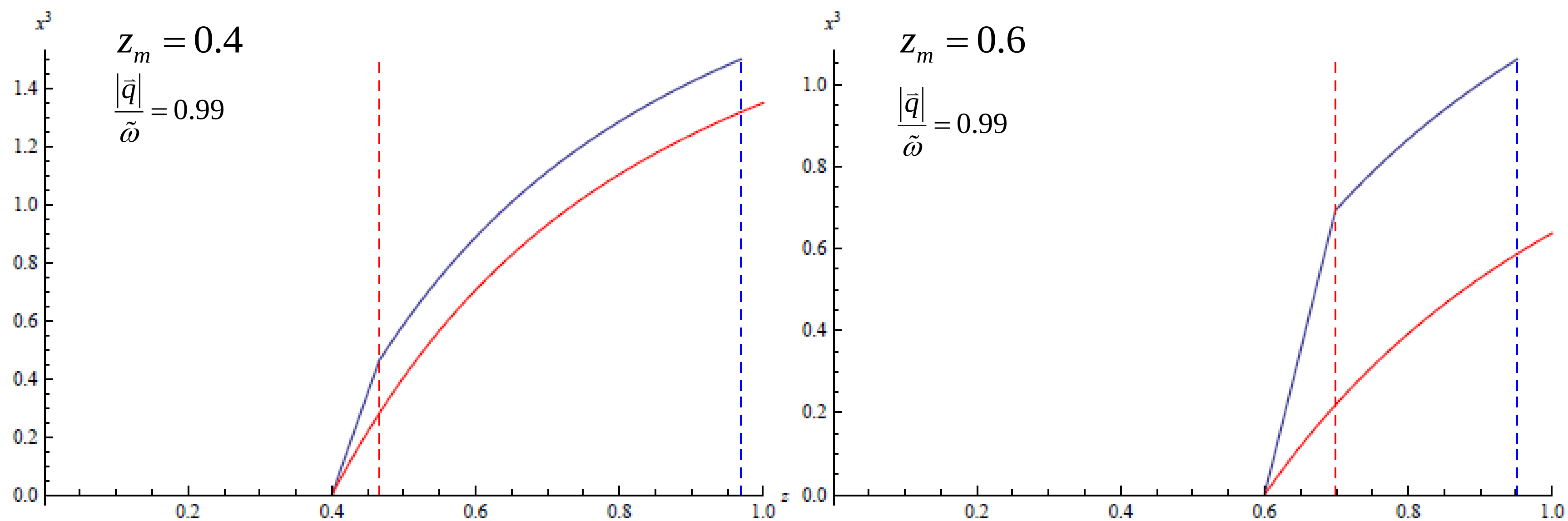
- The maximum stopping distance : the longitudinal displacement from the starting point to the accretion point.

$$x_s^3 = \int_{z_m}^{z_c} \frac{dz}{\left(\frac{\tilde{\omega}^2}{|\vec{q}|^2} - 1\right)^{1/2}} + \int_{z_c}^{z_b} \frac{dz}{\left(\frac{z^4}{z_h^4} - \frac{q^2}{|\vec{q}|^2}\right)^{1/2}}.$$

pt in AdS-Vaidya

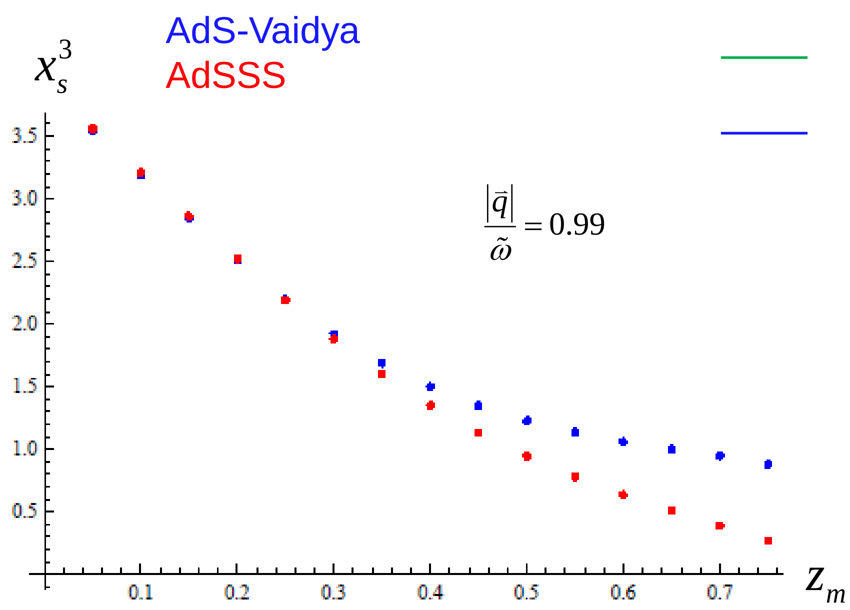
pt in AdSSS

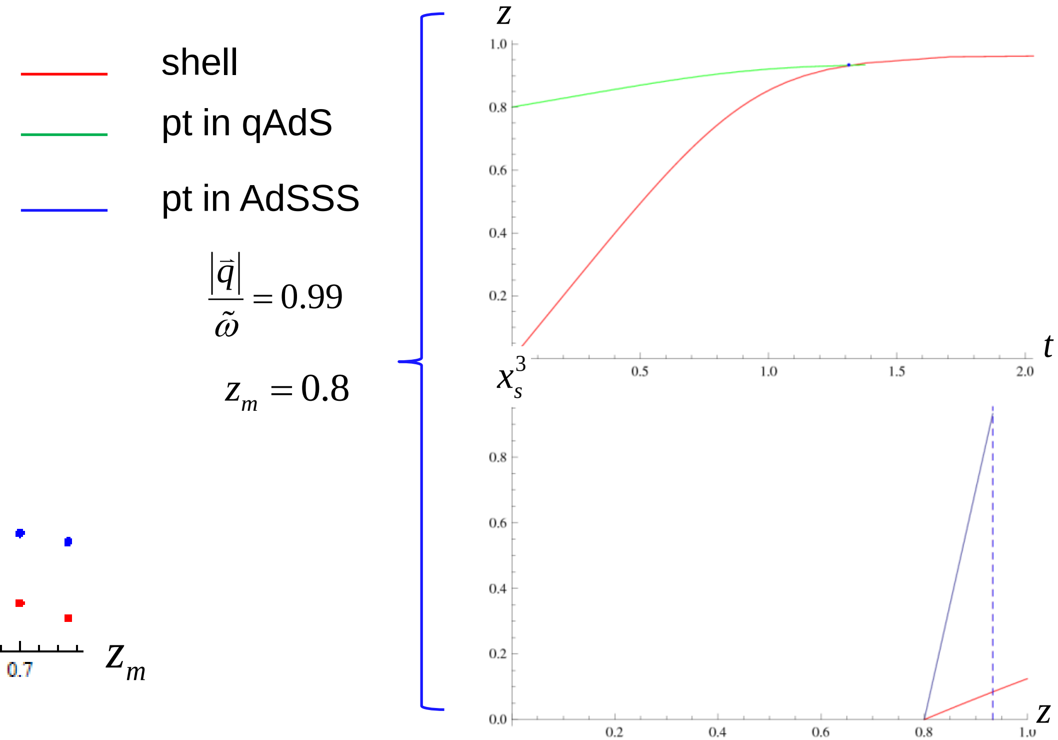
- In qAdSS: $x_s^3\big|_{qAdS} \approx (z_c - z_m)\left(\frac{\tilde{\omega}}{\delta\tilde{\omega}}\right) \approx (EL)^{1/2}$ $\quad L^{-1} = \delta\omega \;\&\; \tilde{\omega} \approx \omega$
- In pure AdSSS: $x_s^3\big|_{AdSSS} \approx (EL)^{1/4}$ P. Arnold and D. Vaman, JHEP 1104:027,2011



Comparing stopping distances

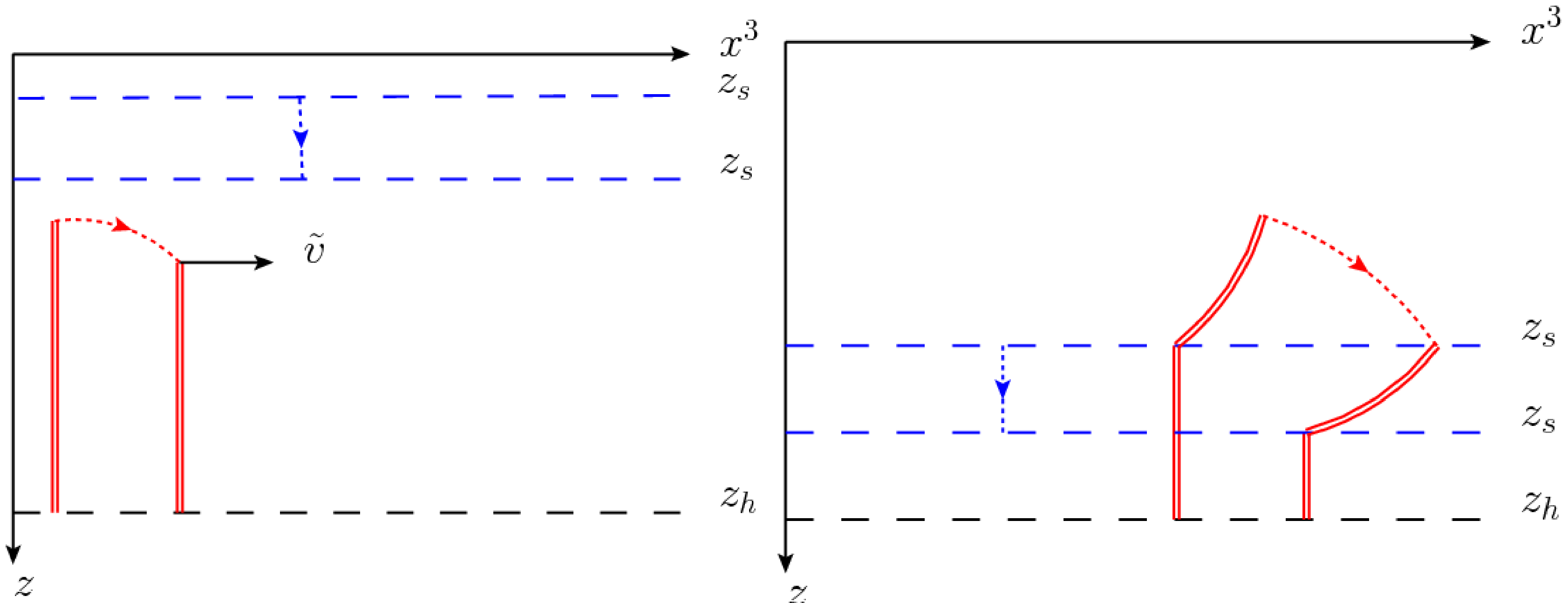
- The particle will travel further in AdS-Vaidya than in AdSSS.
- When the starting point is close to the boundary, two results coincide.
- When the starting point is close to the future horizon, the particle will travel all the way in qAdSS.





Falling String in AdS-Vaidya spacetime

- All pieces of the straight string may move parallely along the null geodesic in qAdSS.
- When the string penetrates the shell, the tip of the string may still move along the null geodesic.
- The rest part outside the shell will be trailed, while the part inside the shell will remain straight.



Set up the initial condition of string

- The string move with a constant velocity along x^3 in qAdSS.
- Setting $x^3(\tilde{t}, z) = \tilde{v}\tilde{t}$ ⇒ $S = \frac{-1}{2\pi\alpha'} \int d\tilde{t}dz \frac{\sqrt{1-\tilde{v}^2}}{z^2}$
- Momentum density:

$$\pi^0_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = (\pi^0_{x^3}, \pi^0_z, \pi^0_{\tilde{t}}) = \frac{-1}{\pi\alpha' z^2\sqrt{1-\tilde{v}^2}}(-\tilde{v}, 0, 1)$$

⇒

$$\tilde{E} = -P_0 = \frac{1}{\pi\alpha'\sqrt{1-\tilde{v}^2} z_m},$$
$$P_3 = \tilde{v}\tilde{E}.$$

- All pieces move along the null geodesic: $|\vec{q}|/\tilde{\omega} = P_3/\tilde{E} = \tilde{v}$
- For an observer at the boundary, the energy varies with time.
- Same results can be obtained by taking the trailing string profile and setting $z_h \to \infty$ and substituting t with $\tilde{t}$.
- Compared with trailing string profile in AdSSS:

$$\frac{P_3}{P_0} = v\frac{\sqrt{1-v^2} - y_m^2}{v^2 y_m^2 - (1-y_m^4)\sqrt{1-v^2}} \qquad E = -P_0 = \tilde{E}$$

Gubser et.al. JHEP0810:052,2008

Computing the jet quenching parameter

- BDMPS formalism: $\hat{q} = \dfrac{4\tilde{E}}{\alpha_s C_R (x^3_{max})^2} = \dfrac{4\sqrt{g^2_{YM}N}\,y_m^{-1}}{\alpha_s C_R (\hat{x}^3_{max})^2}\dfrac{\pi^2 T^3}{\sqrt{1-\tilde{v}^2}}$

- $\hat{q}$ in AdS-Vaidya and in pure AdSSS coincide at the UV limit.
  - Obvious scheme: $T_{QCD} = T_{SYM}, \quad g^2_{YM}N = 6\pi$
  - Alternative scheme: $T_{QCD} = 3^{1/4}T_{SYM}, \quad g^2_{YM}N = 5.5$

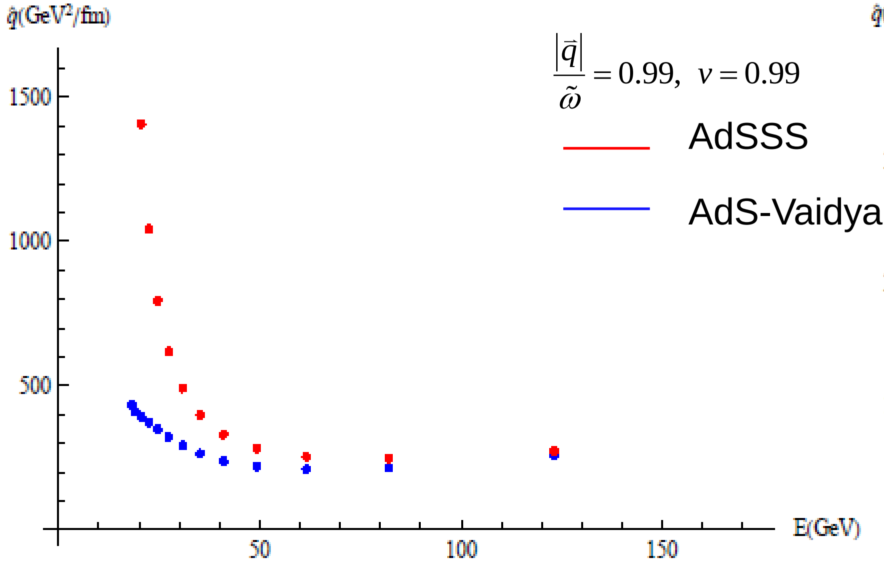
(S. Gubser, Phys.Rev.D76:126003,2007)

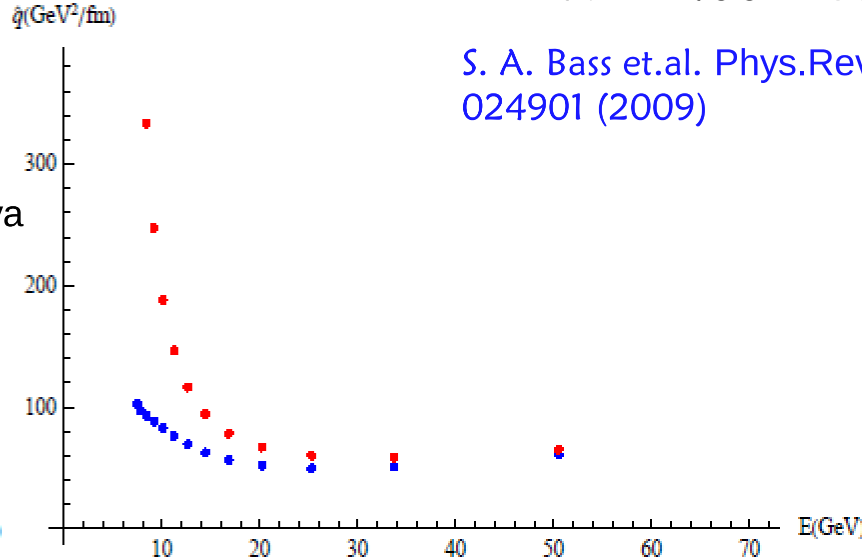
RHIC data:

$\hat{q} = 2 \sim 10 (GeV^2/fm)$

at T=400 MeV

S. A. Bass et.al. Phys.Rev.C 79:
024901 (2009)





Summary

- We have written down the Vaidya metric in the thin shell limit and approximated the thermal equilibration time.
- A massless particle moving along the null geodesics results in larger stopping distance in AdS-Vaidya spacetime.
- We found a distinct scenario of a falling string in the thermalization process.
- By applying BDMPS formalism, we obtained smaller $\hat{q}$.
- Caveats :
  - N=4 SYM is distinct from QCD.
  - The correspondence of non-local objects and physics in gauge theory side may be obscure.
- However, we are probably in the ballpark!

Thank you!

Dangling string and wave velocity

- Nambu-Goto action in qAdSS:

$$S=\frac{-1}{2\pi\alpha'}\int d\tilde{t}dz\frac{1}{z^2}\sqrt{1-\left(\frac{dx^3}{d\tilde{t}}\right)^2+\left(\frac{dx^3}{dz}\right)^2}$$

- Taking $x^3(\tilde{t},z)=\tilde{v}\tilde{t}+\delta x^3(\tilde{t},z)$ and finding the EOM of δL

$$\frac{d^2\delta x^3(\tilde{t},z)}{dz^2}-2z^{-1}\frac{d\delta x^3(\tilde{t},z)}{dz}=(1-\tilde{v}^2)^{-1}\frac{d^2\delta x^3(\tilde{t},z)}{d\tilde{t}^2}\Rightarrow \boxed{v_w=\sqrt{1-\tilde{v}^2}c(z_0)}$$