

# From Dipoles to Quadrupoles

Bo-Wen Xiao

Penn State

- ▶ F. Dominguez, BX and F. Yuan, Phys.Rev.Lett.106:022301,2011.
- ▶ F.Dominguez, C.Marquet, BX and F. Yuan, Phys.Rev.D83:105005,2011.
- ▶ F. Dominguez, A. Mueller, S. Munier and BX, Phys.Lett. B705 (2011) 106-111.
- ▶ A. Stasto, BX and F. Yuan, arXiv:1109.1817 [hep-ph].
- ▶ F. Dominguez, J.W. Qiu, BX and F. Yuan, arXiv:1109.6293 [hep-ph].

INT Oct. 2011

## Introduction

## Normal Gluon Distributions

DIS dijet

$\gamma$ +Jet in  $pA$

Gluon+Jet in  $pA$

Dihadron correlations at RHIC

## The small- $x$ evolution of quadrupoles

## Linearly polarized gluon distributions

## Conclusion and Outlook

Introduction

Normal Gluon  
Distributions

DIS dijet

$\gamma$ +Jet in  $pA$

Gluon+Jet in  $pA$

Dihadron correlations at RHIC

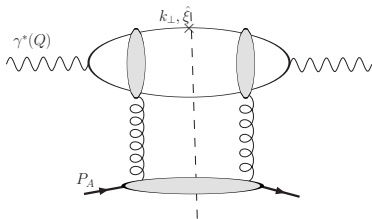
The small- $x$  evolution of  
quadrupoles

Linearly polarized gluon  
distributions

Conclusion and Outlook

Inclusive and Semi-inclusive DIS at small- $x$ 

The **Dipole Model** has become the common practice of QCD calculation at small- $x$ . Using QCD dipole model, one can write the cross section of SIDIS as



$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow qX}}{dz dk_\perp^2} = \frac{1}{4\pi} \int d^2\mathbf{x} d^2\mathbf{y} \Phi_{T,L}(z, \mathbf{x}, \mathbf{y}, Q^2) e^{-ik_\perp \cdot (\mathbf{x}-\mathbf{y})} \\ \times \int d^2\mathbf{b} [1 - S_{q\bar{q}}(\mathbf{x}, x) + S_{q\bar{q}}(\mathbf{y}, x) + S_{q\bar{q}}(\mathbf{x}-\mathbf{y}, x)]$$

- ▶ Integrating over  $z$  and  $k_\perp$  gives inclusive cross section.
- ▶ Convoluting with the fragmentation  $D_{h/q}(\frac{\hat{\xi}}{z})$  yields the single inclusive hadron spectrum.

## Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

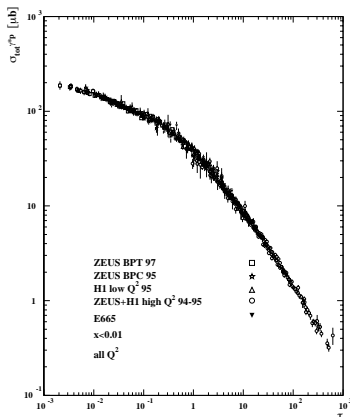
Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

## Conclusion and Outlook

# Golec-Biernat Wusthoff model and Geometrical Scaling

[Golec-Biernat, Wusthoff,; 98], [Golec-Biernat, Stasto, Kwiecinski; 01]



- ▶ The dipole amplitude in the GBW model

$$S_{q\bar{q}}(r_{\perp}) = \exp\left[-\frac{Q_s^2 r_{\perp}^2}{4}\right]$$

with  $Q_s^2(x) = Q_{s0}^2(x_0/x)^{\lambda}$  where  $Q_{s0} = 1\text{GeV}$ ,  $x = 3.04 \times 10^{-3}$  and  $\lambda = 0.288$ .

## Introduction

### Normal Gluon Distributions

- DIS dijet
- $\gamma$ +Jet in  $pA$
- Gluon+Jet in  $pA$
- Dihadron correlations at RHIC

### The small- $x$ evolution of quadrupoles

### Linearly polarized gluon distributions

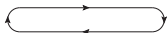
### Conclusion and Outlook

## Dipoles, Quadrupoles and higher point functions

Dipole

and

Quadrupole



(a)



(b)

$$S^{(2)} = \frac{1}{N_c} \text{Tr} [U(x_1) U^\dagger(x_2)]$$

$$S^{(4)} = \frac{1}{N_c} \text{Tr} [U(x_1) U^\dagger(x_2) U(x_3) U^\dagger(x_4)]$$

- ▶ Scattering amplitudes between different projectiles and the target.
- ▶ Semi-inclusive processes only involve **dipoles**.
- ▶ Dijet and more exclusive processes involve **dipoles**, **quadrupoles** and higher point functions.
- ▶ For instance, the **sextupole**  $S^{(6)} = \frac{1}{N_c} \text{Tr} [U(x_1) U^\dagger(x_2) U(x_3) U^\dagger(x_4) U(x_5) U^\dagger(x_6)]$  appears in the  $gg \rightarrow gg$  channel of dijet processes. Similar results can be obtained in *TMD* approach as well. [Bomhof, Mulders and Pijlman; 06]

## Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

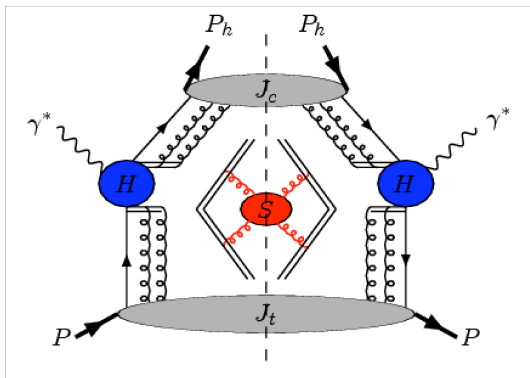
Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook

# Transverse Momentum Dependent (TMD) factorization

[Collins-Soper-Sterman,85], [Ji-Ma-Yuan, 04]



- ▶ (Mueller's dipole model)  $k_t$  factorization is widely used in small- $x$  physics. TMD factorization is widely used in spin physics (large  $x$ ).
- ▶  $k_t$  factorization = TMD factorization? **Yes!**
- ▶ **Universality** of the TMD parton distributions? **Yes and No!**

## Introduction

Normal Gluon  
Distributions

DIS dijet

$\gamma$ +Jet in  $pA$

Gluon+Jet in  $pA$

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupoles

Linearly polarized gluon  
distributions

Conclusion and Outlook

# The effective $k_t$ factorization

For  $pA$  (**dilute-dense system**) collisions, there is an effective  $k_t$  factorization.

$$\frac{d\sigma^{pA \rightarrow qfX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p, \mu^2) x f(x, q_\perp^2) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.$$

Remarks:

- ▶ For  $pp$ ,  $AA$  collisions, there is no  $k_t$  factorization [Rogers, Mulders; 10].
- ▶ **Penalty:**  $K_t$  dependent Parton distributions  $x f(x, q_\perp^2)$  are not universal.  $x f(x, q_\perp^2)$  here can be the quark or gluon distributions of the dense target. It contains the **anomalous terms** after resummation.
- ▶  $x_p q(x_p, \mu^2)$  is the Feynman parton distribution of the dilute projectile.
- ▶ Thanks to the nuclear enhancement, soft gluon exchange from the dilute proton can be neglected.
- ▶ Assuming small- $x$  limit, namely,  $s \rightarrow \infty$ , fixed  $Q^2$ ,  $x \rightarrow 0$ .
- ▶ For DIS, this  $k_t$  factorization works as well. The question is which gluon distributions one should use.

Introduction

Normal Gluon  
Distributions

DIS dijet

$\gamma$ +Jet in  $pA$

Gluon+Jet in  $pA$

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupoles

Linearly polarized gluon  
distributions

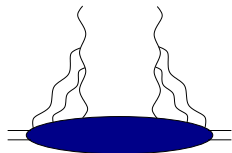
Conclusion and Outlook

## A Tale of Two Gluon Distributions

In small- $x$  physics, two gluon distributions are widely used:

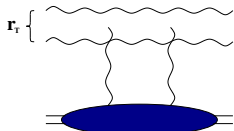
I. **Weizsäcker Williams** gluon distribution (**MV model**): [Mueller, Kovchegov, 98], [McLerran, Venugopalan, 99]

$$xG^{(1)} = \frac{S_{\perp} N_c^2 - 1}{\pi^2 \alpha_s N_c} \Leftrightarrow \int \frac{d^2 r_{\perp}}{(2\pi)^2} \frac{e^{-ik_{\perp} \cdot r_{\perp}}}{r_{\perp}^2} \left( 1 - e^{-\frac{r_{\perp}^2 Q_{sg}^2}{4}} \right)$$



II. **Color Dipole** gluon distributions:

$$xG^{(2)} = \frac{S_{\perp} N_c}{2\pi^2 \alpha_s} \Leftrightarrow \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot r_{\perp}} \nabla_{r_{\perp}}^2 e^{-\frac{r_{\perp}^2 Q_{sq}(g)}{4}}$$



Remarks:

- ▶ The WW gluon distribution simply counts the number of gluons.
- ▶ The Color Dipole gluon distribution often appears in calculations.  $N(r_{\perp})$  is the color dipole amplitude. It is now in fundamental representation.
- ▶ Does this mean that gluon distributions are non-universal? **Yes** and **No!**
- ▶ These two distributions are used in  $R_{pA}$  calculation. [Kharzeev, Kovchegov, Tuchin; 03].

Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

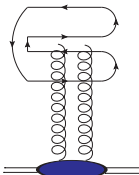
Conclusion and Outlook



# A Tale of Two Gluon Distributions

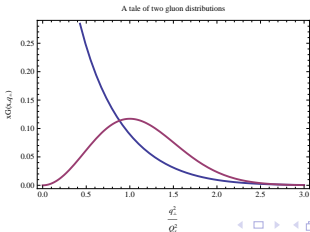
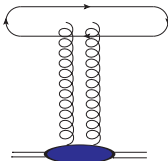
## I. Weizsäcker Williams gluon distribution (MV model):

$$xG^{(1)} = \frac{S_{\perp} N_c^2 - 1}{\pi^2 \alpha_s N_c} \Leftrightarrow \int \frac{d^2 r_{\perp}}{(2\pi)^2} \frac{e^{-ik_{\perp} \cdot r_{\perp}}}{r_{\perp}^2} \left( 1 - e^{-\frac{r_{\perp}^2 Q_{sg}^2}{4}} \right)$$



## II. Color Dipole gluon distributions:

$$xG^{(2)} = \frac{S_{\perp} N_c}{2\pi^2 \alpha_s} \Leftrightarrow \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot r_{\perp}} \nabla_{r_{\perp}}^2 e^{-\frac{r_{\perp}^2 Q_{sq}^2}{4}}$$



## A Tale of Two Gluon Distributions

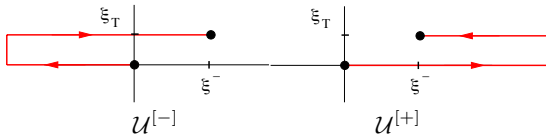
The operator definitions of these two gluon distributions: [Bomhof, Mulders and Pijlman; 06][F. Dominguez, BX and F. Yuan, 11]

I. **Weizsäcker Williams** gluon distribution:

$$xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[+] \dagger} F^{+i}(0) \mathcal{U}^{[+]} | P \rangle.$$

II. **Color Dipole** gluon distributions:

$$xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[-] \dagger} F^{+i}(0) \mathcal{U}^{[+]} | P \rangle.$$



Remarks:

- ▶ The WW gluon distribution is the **conventional gluon distributions**. In light-cone gauge, it is the **gluon density**. (**Only final state interactions.**)
- ▶ The dipole gluon distribution has no such interpretation. (**Initial and final state interactions.**)
- ▶ Both definitions are gauge invariant.
- ▶ Same after integrating over  $q_\perp$ .
- ▶ Same perturbative tail.

## A Tale of Two Gluon Distributions

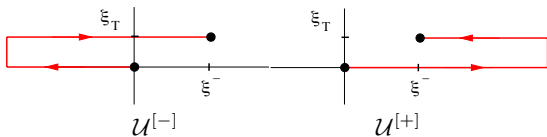
The operator definitions of these two gluon distributions: [Bomhof, Mulders and Pijlman; 06][F. Dominguez, BX and F. Yuan, 11]

I. **Weizsäcker Williams** gluon distribution (**Easy to evaluate in the MV model**):

$$xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[+]\dagger} F^{+i}(0) \mathcal{U}^{[+]} | P \rangle.$$

II. **Color Dipole** gluon distributions:

$$xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[-]\dagger} F^{+i}(0) \mathcal{U}^{[+]} | P \rangle.$$

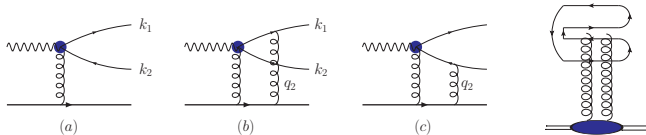


Questions:

- ▶ Can we distinguish these two gluon distributions in physical processes?
- ▶ How to measure  $xG^{(1)}$  directly? **DIS dijet.**
- ▶ How to measure  $xG^{(2)}$  directly? **Direct  $\gamma$ +Jet in  $pA$  collisions. Maybe single-inclusive particle production in  $pA$  (Subtle).**
- ▶ What happens in gluon+jet production in  $pA$  collisions? **It's complicated!**

## DIS dijet in the TMD and CGC approaches

Resummation of the gauge links in TMD approach.



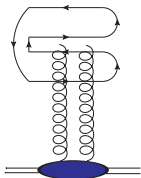
TMD factorization approach:

$$\frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}+X}}{dy_1 dy_2 d^2P_\perp d^2q_\perp} = \delta(x_{\gamma^*} - 1) x_g G^{(1)}(x_g, q_\perp) H_{\gamma_T^* g \rightarrow q\bar{q}}.$$

CGC approach:[Jalilian-Marian, Gelis, 02]

$$\begin{aligned} \frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}+X}}{d\mathcal{P} \cdot \mathcal{S}} &\propto N_c \alpha_{em} e_q^2 \int \frac{d^2x}{(2\pi)^2} \frac{d^2x'}{(2\pi)^2} \frac{d^2b}{(2\pi)^2} \frac{d^2b'}{(2\pi)^2} e^{-ik_{1\perp} \cdot (x-x')} \\ &\times e^{-ik_{2\perp} \cdot (b-b')} \sum \psi_T^*(x-b) \psi_T(x'-b') \\ &\left[ 1 + S_{x_g}^{(4)}(x, b; b', x') - S_{x_g}^{(2)}(x, b) - S_{x_g}^{(2)}(b', x') \right], \end{aligned}$$

Two independent calculations agree perfectly in the correlation limit (Large  $P_\perp$  and Small  $q_\perp$ ).[F. Dominguez, BX and F. Yuan, 11]



In the dijet correlation limit, where  $u = x - b$  and  $v = zx + (1 - z)b$

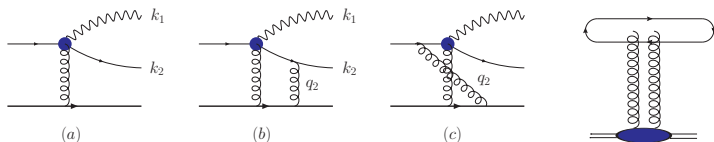
$$\begin{aligned} & \left[ 1 + S_{x_g}^{(4)}(x, b; b', x') - S_{x_g}^{(2)}(x, b) - S_{x_g}^{(2)}(b', x') \right] \\ & \simeq -u_i u'_j \frac{1}{N_c} \langle \text{Tr} [\partial^i U(v)] U^\dagger(v') [\partial^j U(v')] U^\dagger(v) \rangle_{x_g} \\ & = u_i u'_j \frac{g^2}{N_c} \int_{-\infty}^{\infty} dv^+ dv'^+ \left\langle \text{Tr} \left[ F^{i-}(v) \mathcal{U}^{[+] \dagger} F^{j-}(v') \mathcal{U}^{[+]} \right] \right\rangle_{x_g} \end{aligned}$$

Remarks:

- ▶ Dijet in DIS is the **only physical** process which can directly measure **Weizsäcker Williams** gluon distributions.
- ▶ **Golden measurement** for the **Weizsäcker Williams** gluon distributions of nuclei at small-x.
- ▶ **EIC** will provide us a **perfect machine** to study the strong gluon fields in nuclei.

$\gamma$ +Jet in  $pA$  collisions

The direct photon + jet production in  $pA$  collisions. (Drell-Yan Process follows the same factorization.)



TMD factorization approach:

$$\frac{d\sigma^{(pA \rightarrow \gamma q + X)}}{d\mathcal{P}.S.} = \sum_f x_1 q(x_1, \mu^2) x_g G^{(2)}(x_g, q_\perp) H_{qg \rightarrow \gamma q}.$$

Remarks:

- ▶ Independent CGC calculation gives the identical result in the correlation limit. [Jalilian-Marian, Gelis, 02],[Dominguez, Marquet, BX, Yuan, 11]
- ▶ This process can be calculated exactly for all range of azimuthal angles.
- ▶ Direct measurement of the **Color Dipole** gluon distribution.
- ▶ The RHIC and future LHC experiments shall provide us some information on this.

Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

Dihadron correlations at RHIC

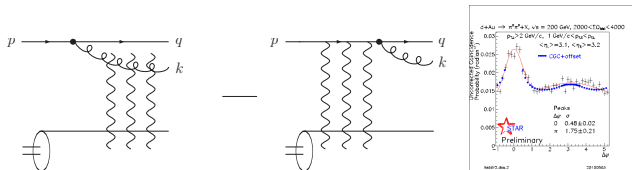
The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook

# Existing calculations on dijet production

Let us first look back, and re-examine the existing calculations on dijet productions.

**Quark+Gluon channel** [Marquet, 07] and [Albacete, Marquet, 10]



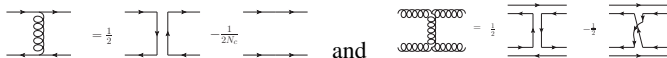
- ▶ Prediction of saturation physics.
- ▶ All the framework is correct, but over-simplified 4-point function.
- ▶ Improvement [F. Dominguez, C. Marquet, BX and F. Yuan, 11.]

$$S_{x_g}^{(4)}(x_1, x_2; x'_2, x'_1) \simeq e^{-\frac{CF}{2} [\Gamma(x_1 - x_2) + \Gamma(x'_2 - x'_1)]}$$

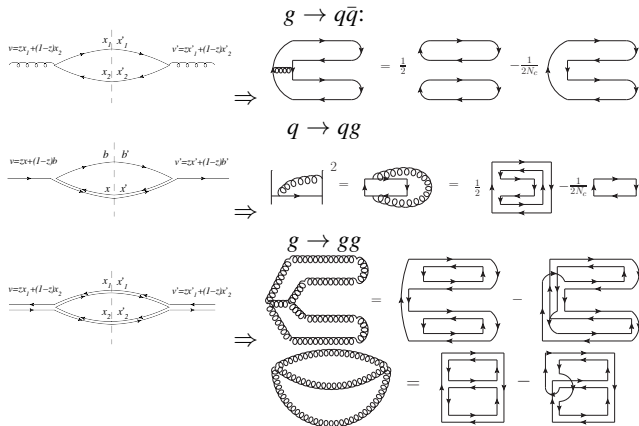
$$- \frac{F(x_1, x_2; x'_2, x'_1)}{F(x_1, x'_2; x_2, x'_1)} \left( e^{-\frac{CF}{2} [\Gamma(x_1 - x_2) + \Gamma(x'_2 - x'_1)]} - e^{-\frac{CF}{2} [\Gamma(x_1 - x'_1) + \Gamma(x'_2 - x_2)]} \right)$$

Dijet processes in the large  $N_c$  limit

The Fierz identity:



Graphical representation of dijet processes

The **Octupole** and the **Sextupole** are suppressed.

Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook



## Gluon+quark jets correlation

Including all the  $qg \rightarrow qg$ ,  $gg \rightarrow gg$  and  $gg \rightarrow q\bar{q}$  channels, a lengthy calculation gives

$$\begin{aligned} \frac{d\sigma^{(pA \rightarrow \text{Dijet}+X)}}{d\mathcal{P} \cdot \mathcal{S}} &= \sum_q x_1 q(x_1, \mu^2) \frac{\alpha_s^2}{\hat{s}^2} \left[ \mathcal{F}_{qg}^{(1)} H_{qg}^{(1)} + \mathcal{F}_{qg}^{(2)} H_{qg}^{(2)} \right] \\ &+ x_1 g(x_1, \mu^2) \frac{\alpha_s^2}{\hat{s}^2} \left[ \mathcal{F}_{gg}^{(1)} \left( H_{gg \rightarrow q\bar{q}}^{(1)} + \frac{1}{2} H_{gg \rightarrow gg}^{(1)} \right) \right. \\ &\left. + \mathcal{F}_{gg}^{(2)} \left( H_{gg \rightarrow q\bar{q}}^{(2)} + \frac{1}{2} H_{gg \rightarrow gg}^{(2)} \right) + \mathcal{F}_{gg}^{(3)} \frac{1}{2} H_{gg \rightarrow gg}^{(3)} \right], \end{aligned}$$

with the various gluon distributions defined as

$$\begin{aligned} \mathcal{F}_{qg}^{(1)} &= xG^{(2)}(x, q_\perp), \quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)} \otimes F, \\ \mathcal{F}_{gg}^{(1)} &= \int xG^{(2)} \otimes F, \quad \mathcal{F}_{gg}^{(2)} = - \int \frac{q_{1\perp} \cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)} \otimes F, \\ \mathcal{F}_{gg}^{(3)} &= \int xG^{(1)}(q_1) \otimes F \otimes F, \end{aligned}$$

where  $F = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} \frac{1}{N_c} \langle \text{Tr} U(r_\perp) U^\dagger(0) \rangle_{x_g}$ .

Remarks:

- ▶ Only the term in NavyBlue color was known before.
- ▶ This can help us understand the **dihadron correlation data**.

Introduction

Normal Gluon  
Distributions

DIS dijet

$\gamma$ +Jet in  $pA$

**Gluon+Jet in  $pA$**

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupoles

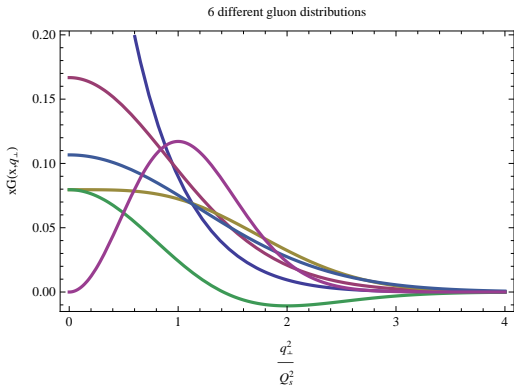
Linearly polarized gluon  
distributions

Conclusion and Outlook

# Illustration of gluon distributions

The various gluon distributions:

$$\begin{aligned}
 & xG_{\text{WW}}^{(1)}(x, q_{\perp}), \quad \mathcal{F}_{qg}^{(1)} = xG^{(2)}(x, q_{\perp}), \\
 \mathcal{F}_{gg}^{(1)} &= \int xG^{(2)} \otimes F, \quad \mathcal{F}_{gg}^{(2)} = - \int \frac{q_{1\perp} \cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)} \otimes F, \\
 \mathcal{F}_{gg}^{(3)} &= \int xG^{(1)}(q_1) \otimes F \otimes F, \quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)} \otimes F
 \end{aligned}$$



Introduction

Normal Gluon  
Distributions

DIS dijet

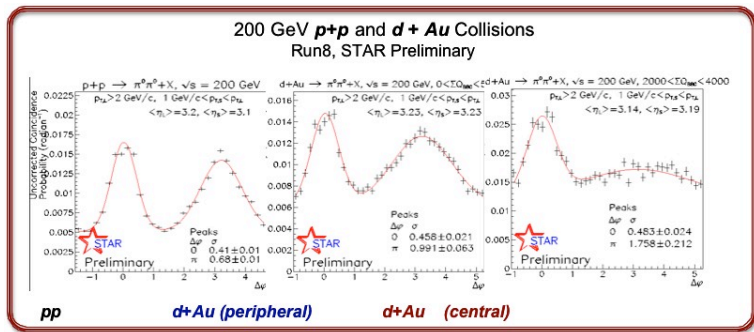
 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook

# STAR measurement on di-hadron correlation in $dA$ collisions

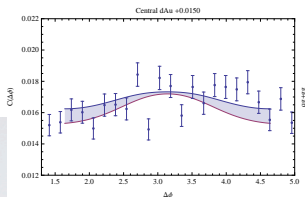
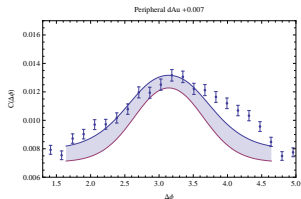


- ▶ There is no sign of suppression in the  $p + p$  and  $d + Au$  peripheral data.
- ▶ The suppression and broadening of the away side jet in  $d + Au$  central collisions is due to the multiple interactions between partons and dense nuclear matter (**CGC**).
- ▶ Probably the best evidence for saturation.
- ▶ Dissect the data into three features:  
Width  $\sigma$  of peaks, Pedestal  $P$  and Peak suppression.

# Comparing to STAR data

[A. Stasto, BX, F. Yuan, 11]

For away side peak in both **peripheral** and **central**  $dAu$  collisions



- ▶ The framework does not work for pp since the saturation scale  $Q_s$  is too low at  $\sqrt{s} = 200\text{GeV}$ .
- ▶ Use Golec-Biernat Wusthoff model for the saturation momentum,  $Q_s^2(x) = Q_{s0}^2(x_0/x)^\lambda$ .
- ▶ Adding the nuclear and impact factor dependence:  $Q_{sA}^2 = c(b)A^{1/3}Q_s^2(x)$ .
- ▶ Peripheral  $b = 6.8 \pm 1.7\text{fm}$  with  $c(b) = 0.45$  and width  $\sigma \simeq 0.99$ ;
- ▶ Central  $b = 2.7 \pm 1.3\text{fm}$  with  $c(b) = 0.85$  and width  $\sigma \simeq 1.6$ .

The small- $x$  evolution equation of quadrupoles

- ▶ The **Balitsky-Kovchegov** equation for **dipoles**

$$\frac{\partial}{\partial Y} S = \int P_{d \rightarrow dd} (SS - S),$$

$$\text{with } P_{d \rightarrow dd} = \frac{\alpha_s N_c}{2\pi^2} \frac{d^2 z_{\perp} (x_{\perp} - y_{\perp})^2}{(x_{\perp} - z_{\perp})^2 (z_{\perp} - y_{\perp})^2}.$$

- ▶ The **quadrupole** evolution equation: Large  $N_c$  [Jalilian-Marian and Kovchegov, 04], Finite  $N_c$  [Dominguez, Mueller, Munier, BX, 11], [Iancu, Triantafyllopoulos, 11]

$$\frac{\partial}{\partial Y} Q = \int P_{q \rightarrow qd} (QS - Q) + \int P_{q \rightarrow dd} (SS - Q).$$

$$\text{with } P_{q \rightarrow qd} + P_{q \rightarrow dd} > 0.$$

- ▶ Dipoles and quadrupoles are very alike in terms of evolution.
- ▶ Follow BFKL in the dilute limit. Both have geometrical scaling. Confirmed by the numerical study [Dumitru, Jalilian-Marian, Lappi, Schenke and Venugopalan, 11]
- ▶ Saturate to a stable fixed point in the dense limit.

Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in pA

Gluon+Jet in pA

Dihadron correlations at RHIC

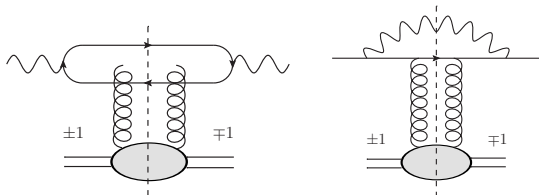
The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook

## The Linearly polarized gluon distribution

The **linearly polarized gluon distribution** effectively measures an averaged quantum **interference** between a scattering amplitude with an active gluon polarized along the **x(or y)**-axis and a complex conjugate amplitude with an active gluon polarized along the **y(or x)**-axis inside an unpolarized hadron.

[Mulders and Rodrigues, 01], [Boer, Brodsky, Mulders, Pisano, 10], [Metz and Zhou, 10], [Dominguez, Qiu, BX and Yuan, 10]

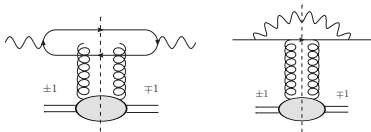


$$\frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}+X}}{dy_1 dy_2 d^2\tilde{P}_\perp d^2q_\perp} = \delta(x_{\gamma^*} - 1) H_{\gamma_T^* g \rightarrow q\bar{q}} \times \left[ x_g G^{(1)}(x_g, q_\perp) - \frac{2\epsilon_f^2 \tilde{P}_\perp^2}{\epsilon_f^4 + \tilde{P}_\perp^4} \cos(2\Delta\phi) x h_\perp^{(1)}(x, q_\perp) \right],$$

where  $\Delta\phi = \phi_{\tilde{P}_\perp} - \phi_{q_\perp}$  and  $\epsilon_f^2 = z(1-z)Q^2 \neq 0$ . There is similar cross section for DY-type dijet.

# The Linearly polarized gluon distribution

[Metz and Zhou,10], [Dominguez, Qiu, BX and Yuan, 10]



For DIS dijet: W.W. gluon distribution

$$2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[+] \dagger} F^{+j}(0) \mathcal{U}^{[+]} | P \rangle_{x_g}$$

$$= \frac{1}{2} \delta^{ij} xG^{(1)}(x, q_\perp) + \frac{1}{2} \left( \frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij} \right) xh_\perp^{(1)}(x, q_\perp).$$

For DY-type dijet processes: Dipole gluon distribution

$$2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - iq_\perp \cdot \xi_\perp} \langle P | \text{Tr} \left[ F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[-] \dagger} F^{+j}(0) \mathcal{U}^{[+]} \right] | P \rangle,$$

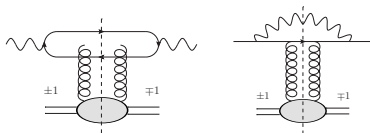
$$= \frac{1}{2} \delta^{ij} xG^{(2)}(x, q_\perp) + \frac{1}{2} \left( \frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij} \right) xh_\perp^{(2)}(x, q_\perp).$$

► Diagonal and off-diagonal part of the dipole and quadrupole amplitudes.

►  $xG^{(1)}(x, q_\perp) \geq xh_\perp^{(1)}(x, q_\perp)$  and  $xG^{(2)}(x, q_\perp) = xh_\perp^{(2)}(x, q_\perp)$

# The Linearly polarized gluon distribution

[Dominguez, Qiu, BX and Yuan, 10]



## A few more comments

- ▶ The linearly polarized gluon distributions come from **quantum interference** and have **no probability interpretation**. They are different from the usual polarized gluon distribution ( $\frac{1}{2} [\varepsilon_+^{*i} \varepsilon_+^j - \varepsilon_-^{*i} \varepsilon_-^j]$ ).
- ▶ Need non-vanishing virtuality to have non-vanishing contribution in the dilute-dense factorization.
- ▶ The small- $x$  evolution of the W.W. type linearly polarized gluon distribution ( $\langle \text{Tr} [\partial^i U(v)] U^\dagger(v') [\partial^j U(v')] U^\dagger(v) \rangle_y$ ) is related to the quadrupole evolution equation, while the dipole type linearly polarized gluon distribution follows BK equation.
- ▶ They follow BFKL evolution in the dilute regime, and receive exponential growth in terms of rapidity. They also have geometrical scaling.
- ▶ They also saturate in the dense regime.

Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in  $pA$ Gluon+Jet in  $pA$ 

Dihadron correlations at RHIC

The small- $x$  evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook



# Conclusion

- ▶ The effective factorization for collisions between a dilute projectile and a dense target in the large  $N_c$  limit.
- ▶ DIS dijet provides **direct information** of the WW gluon distributions.
- ▶ **Modified Universality** for Gluon Distributions:

	Inclusive	Single Inc	DIS dijet	$\gamma$ +jet	g+jet
$xG^{(1)}$	×	×	√	×	√
$xG^{(2)}, F$	√	√	×	√	√

×  $\Rightarrow$  Do Not Appear.      √  $\Rightarrow$  Appear.

- ▶ **Two fundamental gluon distributions** which are related to the **quadrupole and dipole** amplitudes, respectively. Other gluon distributions are just different **combinations and convolutions** of these two.
- ▶ Dihadron correlation calculation and comparison with the STAR data.
- ▶ The small-x evolution of the quadrupole and the WW gluon distribution, a different equation from Balitsky-Kovchegov equation.
- ▶ Linearly polarized gluon distributions provide us the off-diagonal information of scattering amplitudes.

Introduction

Normal Gluon  
Distributions

DIS dijet

 $\gamma$ +Jet in pA

Gluon+Jet in pA

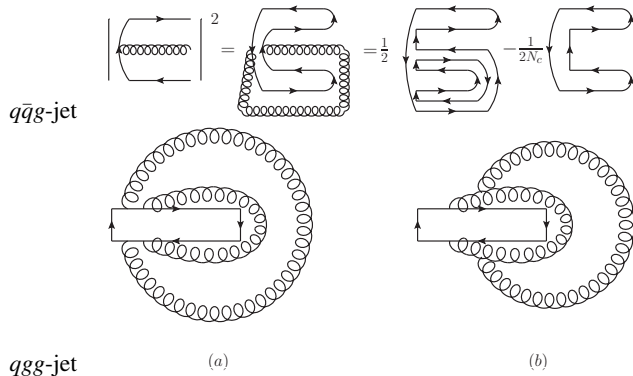
Dihadron correlations at RHIC

The small-x evolution of  
quadrupolesLinearly polarized gluon  
distributions

Conclusion and Outlook

[Dominguez, Marquet, Stasto, BX and Yuan, in preparation]

- ▶ The three-jet production processes in the large  $N_c$  limit:



- ▶ Conjecture: In the large  $N_c$  limit at small- $x$ , the **dipole** and **quadrupole** amplitudes are the **only two fundamental objects** in the cross section of multiple-jet production processes up to **all order**.
- ▶ Other higher point functions, such as **sextupoles**, **octupoles**, **decapoles** and **duodecapoles**, etc. are suppressed by factors of  $\frac{1}{N_c^2}$ .