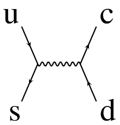
Employing Helicity Amplitudes for Resummation in SCET

Wouter Waalewijn

UCSD



INT workshop - Frontiers in QCD
September 27-29, 2011

In collaboration with Iain Stewart and Frank Tackmann

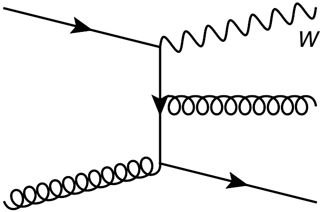
Outline

Introduction

Overview

- Powerful methods for calculating helicity amplitudes analytically and numerically [BlackHat (Zvi Bern's talk), Rocket, MadLoop, . . .]
- We discuss how to seamlessly incorporate helicity amplitudes in SCET: Helicity operator basis
- Illustrate ease of use with explicit LO and NLO results for:
  - $pp \to W/Z/\gamma + 0, 1, 2$ jets
  - $pp \to H + 0, 1, 2$ jets
  - $pp \to 2, 3$ jets

Example: $pp \to W + 2$ jets



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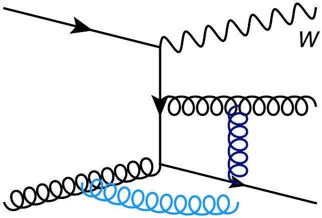

QCD

- Real and virtual corrections
- IR divergences cancel, but costly phase-space integration
- General observables

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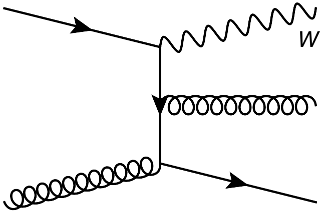

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SCET

- Match QCD onto SCET:
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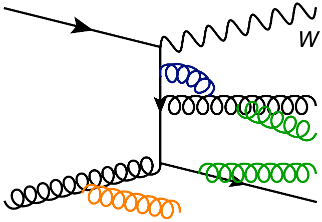

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- Real radiation described by collinear and soft degrees of freedom

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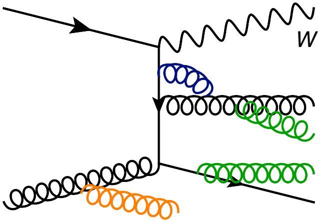

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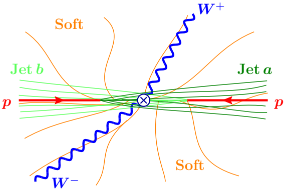
- Match QCD onto SCET:
  Partons correspond to energetic well-separated jets
- Only (IR finite part of) virtual QCD corrections
- Real radiation described by collinear and soft degrees of freedom
- Resummation for exclusive jet cross sections

Example: $gg \to H + 0$ jets



- Jet veto needed to remove large $t\bar{t}$ background: use beam thrust $\mathcal{T}_{\mathbf{cm}}$
- Factorization for $\mathcal{T}_{\mathbf{cm}} \ll m_H$ [Stewart, Tackmann, WW (2009)]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_{\mathbf{cm}}} = \underbrace{H_{gg}(\mu)}_{|\text{virtual QCD}|^2} \underbrace{B_g(\mu) \otimes B_g(\mu)}_{\text{collinear}} \otimes \underbrace{S^{gg}(\mu)}_{\text{soft}}$$

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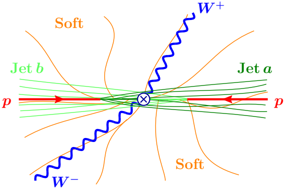

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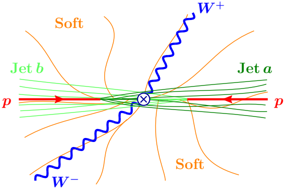

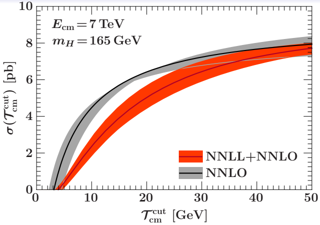

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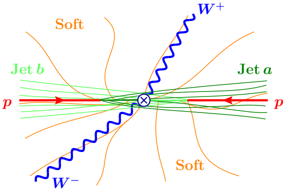

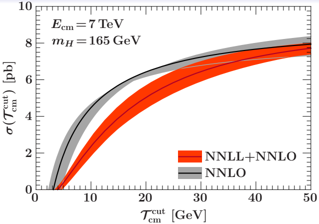

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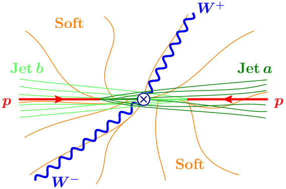

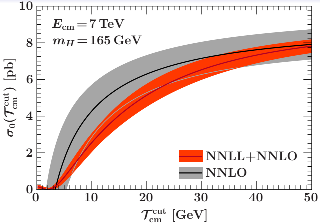

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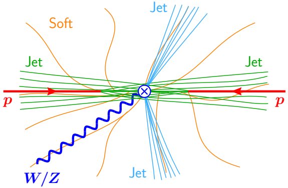
Factorization for N jets

$$\mathrm{d}\sigma_N = \underbrace{\int \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}\Phi_N}_{\text{phase space}} \underbrace{\sum_\kappa}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_N^\kappa \hat{S}_N^\kappa]}_{\text{color trace}} \otimes \underbrace{B_{\kappa_a} \otimes B_{\kappa_b}}_{\text{PDF}\otimes\text{ISR}} \otimes \underbrace{\prod_j J_{\kappa_j}}_{\text{FSR}}$$



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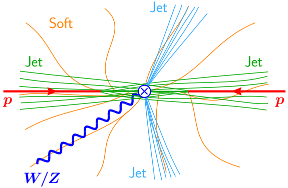
- Hard function H_N^κ depends on process (κ) but not observable/factorization theorem
    - Summed over spins
    - Matrix in color space



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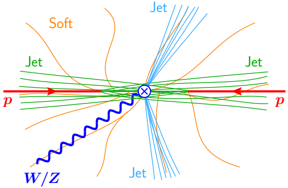


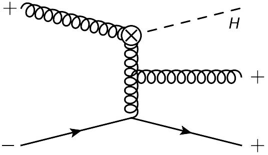
For N-Jettiness eventshape:

- Soft function $\hat{S}_N^\kappa$ known at NLO [Jouttenus, Stewart, Tackmann, WW (2011)]
- Beam functions B_{κ_a} known to NLO [Fleming, Leibovich, Mehen (2006); Stewart, Tackmann, WW (2010); Berger et. al. (2010)]
- Jet functions J_{κ_j} even known at NNLO [Becher, Neubert (2006); Becher, Bell (2010)]

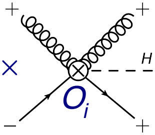
$\rightarrow$ Need to extract the (NLO) hard function = topic of this talk

Matching helicity amplitudes

$$\mathcal{A}(1^+2^+3_q^+4_{\bar{q}}^-5_H) =$$



$$= \sum_i C_i \times$$



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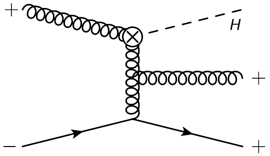
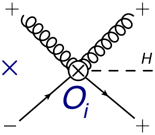
$\mathcal{A}(1^+ 2^+ 3_q^+ 4_{\bar{q}}^- 5_H) =$ 

$= \sum_i C_i \times$ 

- Building blocks of SCET operators: quark field χ_n, gluon field $\mathcal{B}^\mu_{n\perp}$
- Traditional approach to spin:

$$O_1 = \bar{\chi}_{n_3}\, \not{n}_2\, \chi_{n_4}\, \mathcal{B}_{n_1\perp} \cdot \mathcal{B}_{n_2\perp}\, H_5$$
$$O_2 = \bar{\chi}_{n_3}\, \not{\mathcal{B}}_{n_1\perp}\, \chi_{n_4}\, n_4 \cdot \mathcal{B}_{n_2\perp}\, H_5$$
$$O_3 = \bar{\chi}_{n_3}\, \not{n}_1 \not{n}_2\, \not{\mathcal{B}}_{n_1\perp}\, \chi_{n_4}\, n_4 \cdot \mathcal{B}^\perp_{n_2}\, H_5$$
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[Marcantonini, Stewart (2008)]

  - Counting operators is difficult

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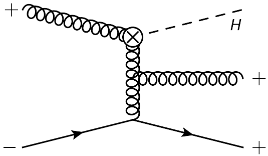
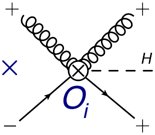
$\mathcal{A}(1^+ 2^+ 3_q^+ 4_{\bar{q}}^- 5_H) =$ 

$= \sum_i C_i \times$ 

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\end{aligned}
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[Marcantonini, Stewart (2008)]

  - Counting operators is difficult
- χ_n and $\mathcal{B}^\mu_{n\perp}$ only support two spins $\to$ introduce $\chi_{n\pm}$ and $\mathcal{B}_{n\pm}$
  e.g. $O_{+++-} \approx \mathcal{B}_{n_1+} \mathcal{B}_{n_2+}\, \bar{\chi}_{n_3+} \chi_{n_4-} H_5$

Spinor review

Helicity amplitudes written in spinors

Definition:

- $|p\pm\rangle = u_{\pm}(p) = v_{\mp}(p)$ (for antiparticles chirality and helicity opposite)
- $\langle pq\rangle = \langle p-|q+\rangle\,, \quad [pq] = \langle p+|q-\rangle$

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Some properties:

- $|\langle pq\rangle| = |[pq]| = \sqrt{|2p\cdot q|}$ (phase important too)
- $\langle pq\rangle[qp] = 2p\cdot q$
- $\langle pq\rangle = -\langle qp\rangle\,, \qquad [pq] = -[qp]$

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- $\langle pq\rangle = -\langle qp\rangle\,,\qquad [pq] = -[qp]$
- $\sum_i [ji]\langle ik\rangle = 0$ (momentum conservation)
  e.g. $[13]\langle 32\rangle = -[14]\langle 42\rangle$
- $\langle p+|\gamma_\mu|q+\rangle\langle k+|\gamma^\mu|l+\rangle = 2[pk]\langle lq\rangle$ (Fierz)

Goal

Helicity operators seamlessly connect helicity amplitudes to SCET

- Matching is independent of IR regulator
- Matching coefficient = IR finite part of virtual hel. amplitudes in dim. reg.
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Challenges:

- Organize spin and color
- Discrete symmetries
- Crossing symmetry
- Renormalization:
  $\mathbf{2}$ vs. $\mathbf{2 - 2\epsilon}$ gluon polarizations
- Intrinsic phases
  (for more than four massless particles)

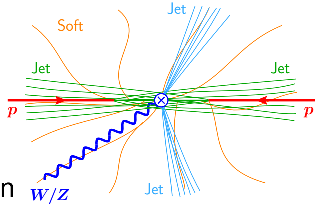
Helicity Operators

SCET basics

Modes:

- Collinear: energetic radiation collimated along a direction $\vec{n}$, decompose using $n^\mu = (1, \vec{n}), \bar{n}^\mu = (1, -\vec{n})$:

$$p^\mu = \bar{n}\cdot p\,\frac{n^\mu}{2} + n\cdot p\,\frac{\bar{n}^\mu}{2} + p^\mu_{n\perp}$$
$$\sim \quad 1 \quad + \quad \lambda^2 \quad + \quad \lambda\,.$$



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- Jets match onto collinear fields

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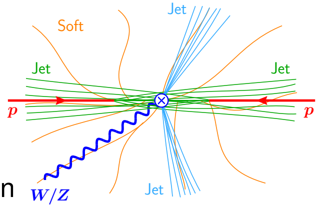

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Gauge-invariant collinear quark and gluon fields:

$$\chi_{n,\omega} = \delta(\omega - \bar{n}\cdot\mathcal{P}) \, W_n^\dagger \, \xi_n$$
$$\mathcal{B}^\mu_{n,\omega\perp} = \delta(\omega + \bar{n}\cdot\mathcal{P}) \, W_n^\dagger \, \big(\tfrac{1}{g}\mathcal{P}^\mu_{n\perp} + A^\mu_{n\perp}\big) W_n$$

- ξ_n and A_n^μ are collinear quark and gluon field

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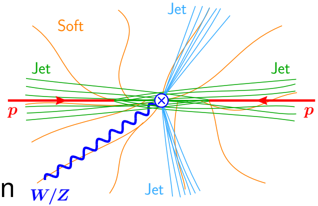

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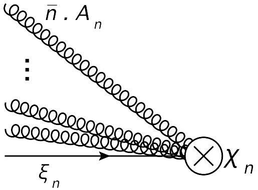

- ξ_n and A^μ_n are collinear quark and gluon field
- Wilson line W_n sums $\mathcal{O}(1)$ emissions of $\bar{n}\cdot A_n$ gluons

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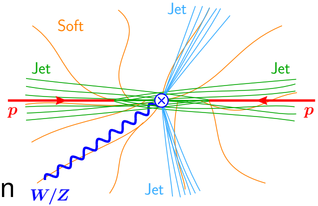

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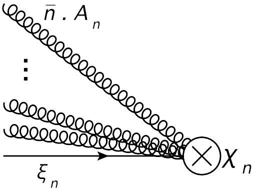

- ξ_n and A^μ_n are collinear quark and gluon field
- Wilson line W_n sums $\mathcal{O}(1)$ emissions of $\bar{n}\cdot A_n$ gluons
- $\delta(\omega \pm \bar{n}\cdot\mathcal{P})$ measures large momentum component

Helicity fields for gluons

- Polarization vectors for gluon with momentum p

$$\varepsilon_{+}^{\mu}(p,k) = \frac{\langle p+|\gamma^{\mu}|k+\rangle}{\sqrt{2}\langle kp\rangle}\,, \quad \varepsilon_{-} = \varepsilon_{+}^{*}\,,$$

satisfy the usual relations:

$$p\cdot\varepsilon_{\pm} = k\cdot\varepsilon_{\pm} = 0, \qquad \varepsilon_{\pm}\cdot\varepsilon_{\mp} = -1, \qquad \varepsilon_{\pm}\cdot\varepsilon_{\pm} = 0,$$

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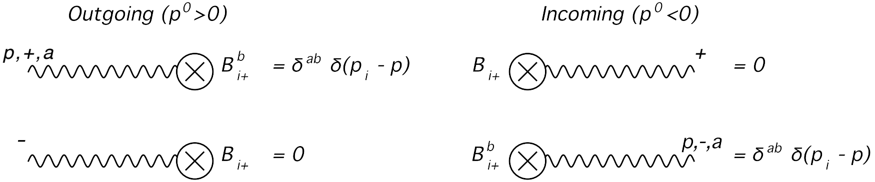
- Gluon helicity field

$$\mathcal{B}^a_{i\pm} = -\varepsilon_{\mp\mu}(n_i,\bar n_i)\,\mathcal{B}^{a\mu}_{n_i,\omega_i\perp_i}\,,$$

- Choice $k^\mu = \bar n^\mu$ gives simple Feynman rules:

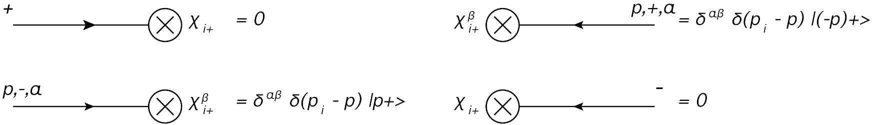


Helicity fields for quarks

$$\chi^{\alpha}_{i\pm} = \frac{1\pm\gamma_5}{2}\,\chi^{\alpha}_{n_i,-\omega_i}\,, \qquad \bar{\chi}^{\alpha}_{i\pm} = \bar{\chi}^{\alpha}_{n_i,\omega_i}\,\frac{1\mp\gamma_5}{2}$$

- Tree-level Feynman rules

Outgoing antiquark ($p^0>0$) *Incoming quark ($p^0<0$)*



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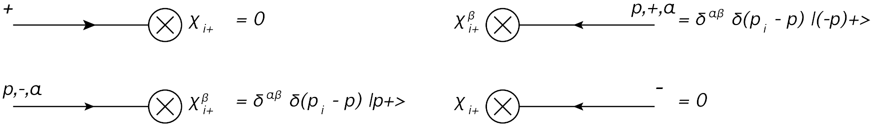
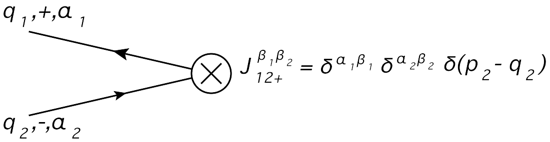


- Since chirality is conserved, we define a quark current

$$J^{\alpha\beta}_{ij\pm} = \mp\,\varepsilon^{\mu}_{\mp}(p_i,p_j)\,\frac{\bar{\chi}^{\alpha}_{i\pm}\,\gamma_\mu\chi^{\beta}_{j\pm}}{\sqrt{2}\langle p_j\mp|p_i\pm\rangle}$$

such that for example



Helicity operator basis

- Match onto the Lagrangian

$$\mathcal{L}_{\rm eff} = \int \prod_{i=1}^{n} \mathrm{d}p_i \, O_{+\cdot\cdot(\cdot\cdot-)}^{a_1\cdots\alpha_n}(p_1,\ldots,p_n)\, C_{+\cdot\cdot(\cdot\cdot-)}^{a_1\cdots\alpha_n}(p_1,\ldots,p_n)\,,$$

$$O_{\pm\pm\cdots(\pm\cdots\pm)}^{a_1 a_2\cdots\alpha_{i-1}\alpha_i\cdots\alpha_{n-1}\alpha_n} = S\, \mathcal{B}_{1\pm}^{a_1}\, \mathcal{B}_{2\pm}^{a_2}\cdots J_{q\,i-1,i\pm}^{\alpha_{i-1}\alpha_i}\cdots J_{q\,n-1,n\pm}^{\alpha_{n-1}\alpha_n}$$

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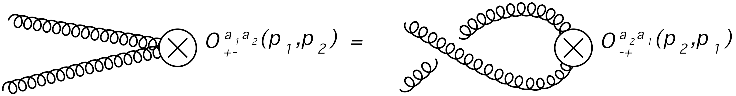
- Helicity labels are ordered

$$O_{+\cdot\cdot(\cdot\cdot-)} = O_{\underbrace{+\cdots+}_{n_g^+}\,\underbrace{-\cdots-}_{n_g^-}\,(\underbrace{+\cdots+}_{n_q^+}\,\underbrace{-\cdots-}_{n_q^-})}$$

Since



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- Match onto the Lagrangian

$$\mathcal{L}_{\rm eff} = \int \prod_{i=1}^{n} {\rm d}p_i \, O^{a_1\cdots\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(p_1,\ldots,p_n)\, C^{a_1\cdots\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(p_1,\ldots,p_n)\,,$$

$$O^{a_1 a_2\cdots\alpha_{i-1}\alpha_i\cdots\alpha_{n-1}\alpha_n}_{\pm\pm\cdots(\pm\cdots\pm)} = S\, \mathcal{B}^{a_1}_{1\pm}\, \mathcal{B}^{a_2}_{2\pm} \cdots J^{\alpha_{i-1}\alpha_i}_{q\, i-1,i\pm} \cdots J^{\alpha_{n-1}\alpha_n}_{q\, n-1,n\pm}$$

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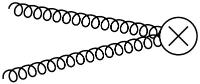
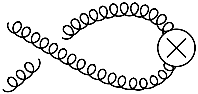
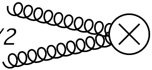
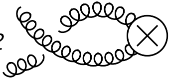
 $O^{a_1 a_2}_{+-}(p_1,p_2)$ =  $O^{a_2 a_1}_{-+}(p_2,p_1)$

- Symmetry factor S simplifies matching for identical particles, e.g.

$\langle g^{a_1}_{+}(q_1) g^{a_2}_{+}(q_2)| L_{\rm eff} |0\rangle = 1/2$  $+1/2$ 

$$= 1/2[C^{a_1 a_2}_{++}(q_1,q_2) + C^{a_2 a_1}_{++}(q_2,q_1)] = C^{a_1 a_2}_{++}(q_1,q_2)$$

General matching

- Very simple tree-level matching in our basis:

$$\mathcal{A}^{\text{tree}}_{\text{QCD}}(1^+2\cdots n^+_{\bar{q}}) = \langle g^{a_1}_+(p_1)g_2(p_2)\cdots q^{\alpha_{n-1}}_-(p_{n-1})\bar{q}^{\alpha_n}_+(p_n)\big|i\mathcal{L}_{\text{eff}}\big|0\rangle|_{\text{tree}}$$
$$= iC^{a_1 a_2\cdots\alpha_{n-1}\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(p_1, p_2, \ldots, p_{n-1}, p_n)$$

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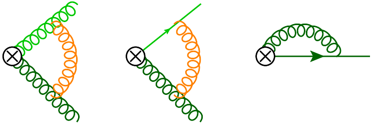
- In SCET only soft gluons connect different collinear directions, e.g.



- Loops are thus scaleless and vanish in pure dim. reg: $1/\epsilon_{\text{UV}} - 1/\epsilon_{\text{IR}} = 0$
- Renormalization removes $1/\epsilon_{\text{UV}}$, leaves $1/\epsilon_{\text{IR}}$:

$$\mathcal{A}_{\text{SCET}} = \int (\langle O \rangle^{\text{tree}} + \langle O \rangle^{\text{loop}}) \cdot \mathrm{i}C = \langle O \rangle^{\text{tree}} \cdot \big[1 + \delta_O(\epsilon_{\text{IR}})\big] \cdot \mathrm{i}C$$

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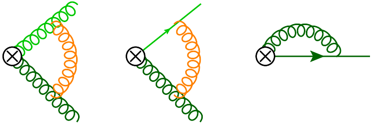

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The IR divergences cancel in the matching, leaving

$$\mathcal{A}^{\text{fin}}_{\text{QCD}}(1^+ 2 \cdots n^+_{\bar{q}}) = \mathrm{i}C^{a_1 \cdots \alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(p_1, \ldots, p_n)$$

Color structures

- Use a basis of color-singlet structures. E.g. for $ggq\bar{q}$

$$\vec{T}^{\dagger ab\,\alpha\beta} = \left(T^a T^b \quad T^b T^a \quad \mathrm{tr}[T^a T^b] 1\right)_{\alpha\beta} .$$

- Decompose operators and Wilson coefficients

$$\vec{O}^{\dagger} = O^{a_1 \cdots \alpha_n}\, \vec{T}^{\dagger a_1 \cdots \alpha_n} , \quad C^{a_1 \cdots \alpha_n} = \vec{T}^{\dagger a_1 \cdots \alpha_n} \cdot \vec{C} .$$

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$$\mathcal{L}_{\mathrm{eff}} = \int \prod_{i=1}^{n} \mathrm{d}p_i\, \vec{O}^{\dagger}_{+\cdot\cdot(\cdot\cdot-)}(p_1, \ldots, p_n) \vec{C}_{+\cdot\cdot(\cdot\cdot-)}(p_1, \ldots, p_n) ,$$

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Example: tree-level matching for n gluons

- QCD color decomposition

$$\mathcal{A}_n^{\mathrm{tree}}(1\cdots n) = \mathrm{i}g_s^{n-2}\sum_{\sigma\in S_n/Z_n}\mathrm{tr}[T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}}]A_n^{(0)}[\sigma(1),\ldots,\sigma(n)] ,$$

Using the color-basis $T_k^{\dagger a_1\cdots a_n} = \mathrm{tr}[T^{a_{\sigma_k(1)}}\cdots T^{a_{\sigma_k(n)}}]$, we find

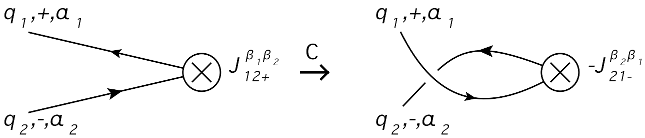
$$C^k(p_1,\ldots,p_n) = g_s^{n-2}A_n^{(0)}[\sigma_k(1),\ldots,\sigma_k(n)]$$

C and P

- C and P for helicity fields

$$\mathrm{P}\,\mathcal{B}^{a}_{i\pm}(p_i)\,\mathrm{P} = \mathcal{B}^{a}_{i\mp}(p_i^{\mathrm{P}})\,, \qquad \mathrm{C}\,\mathcal{B}^{a}_{i\pm}\,T^{a}_{\alpha\beta}\,\mathrm{C} = -\mathcal{B}^{a}_{i\pm}\,T^{a}_{\beta\alpha}$$

$$\mathrm{P}\,J^{\alpha\beta}_{ij\pm}(p_i,p_j)\,\mathrm{P} = J^{\alpha\beta}_{ij\mp}(p_i^{\mathrm{P}},p_j^{\mathrm{P}})\,, \qquad \mathrm{C}\,J^{\alpha\beta}_{ij\pm}\,\mathrm{C} = -J^{\beta\alpha}_{ji\mp}\,,$$



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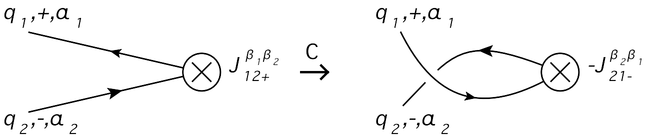

- C and P of operators easily determined

- For example for $O^{ab\,\alpha\beta}_{++(+)} = \frac{1}{2}\,\mathcal{B}^{a}_{1+}\,\mathcal{B}^{b}_{2+}\,J^{\alpha\beta}_{34+}$
  - Parity: $C_{++(+)} = C_{--(-)}$ up to a phase
    since Lorentz invariants $s_{ij} = (p_i + p_j)^2 = (p_i^{\mathrm{P}} + p_j^{\mathrm{P}})^2$

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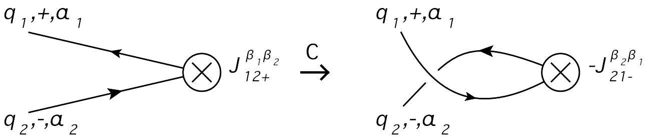

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  - Charge conjugation relates $C_{++(+)}$ and $C_{++(-)}$:

$$\mathrm{C}\,O^{ab\,\alpha\beta}_{++(+)}(p_1,p_2;p_3,p_4)\,\mathrm{C} = -O^{ba\,\alpha\beta}_{++(-)}(p_1,p_2;p_4,p_3)$$

Example: $ggq\bar{q}$

$gg q\bar{q}$: Basis and matching

- Six helicity operators:

$$O_{++(+)}^{ab\,\alpha\beta} = \frac{1}{2}\,\mathcal{B}_{1+}^{a}\,\mathcal{B}_{2+}^{b}\,J_{34+}^{\alpha\beta}\,,$$
$$O_{+-(+)}^{ab\,\alpha\beta} = \mathcal{B}_{1+}^{a}\,\mathcal{B}_{2-}^{b}\,J_{34+}^{\alpha\beta}$$
$$\ldots$$

- Color structures:

$$\vec{T}^{\dagger ab\,\alpha\beta} = \big(T^aT^b \quad T^bT^a \quad \mathrm{tr}[T^aT^b]1\big)_{\alpha\beta}$$

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$$\dots$$

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$$\mathcal{A}(1 2\, 3_q^+ 4_{\bar{q}}^-) = \mathrm{i}\sum_{\sigma\in S_2}\big[T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}\big]_{\alpha_3\alpha_4} A(\sigma(1),\sigma(2); 3_q^+, 4_{\bar{q}}^-)$$

$$+\, \mathrm{i}\,\mathrm{tr}[T^{a_1}T^{a_2}]\,\delta_{\alpha_3\alpha_4}\, B(1,2;3_q^+,4_{\bar{q}}^-)$$

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- Matching coefficients:

$$\vec{C}_{+-(+)}(p_1,p_2;p_3,p_4) = \begin{pmatrix} A_{\mathrm{fin}}(1^+,2^-;3^+_q,4^-_{\bar{q}}) \\ A_{\mathrm{fin}}(2^-,1^+;3^+_q,4^-_{\bar{q}}) \\ B_{\mathrm{fin}}(1^+,2^-;3^+_q,4^-_{\bar{q}}) \end{pmatrix}$$

and similarly for $\vec{C}_{++(+)}$

$gg q\bar{q}$: Matching results

- Nonvanishing tree-level helicity amplitudes

$$A^{(0)}(1^{+},2^{-};3_{q}^{+},4_{\bar{q}}^{-})=-2g^{2}\,\frac{\langle 23\rangle\langle 24\rangle^{3}}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle}$$

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$$s_{ij} = (p_i + p_j)^2 = 2p_i\cdot p_j\,, \qquad e^{i\Phi_{+-}} = \frac{\langle 24\rangle}{[24]}\frac{[13][14]}{\sqrt{|s_{13}\, s_{14}|}}$$

  - Pull out convention-dependent overall phase

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  - Pull out convention-dependent overall phase

- One-loop helicity amplitudes were calculated in [Kunszt, Signer, Trocsanyi (1994)]

$$A^{(1)}_{\text{div}}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = A^{(0)}\frac{\alpha_s}{4\pi}\left[-\frac{2}{\epsilon^2}(C_A + C_F) + \frac{1}{\epsilon}(2C_F\, L_{12} + 2C_A\, L_{13} - 3C_F - \beta_0)\right],$$

$$A^{(1)}_{\text{fin}}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = A^{(0)}\frac{\alpha_s}{4\pi}\left\{C_A\left(-L_{13}^2 + L_{12/13}^2 + 1 + \frac{7\pi^2}{6}\right) + C_F\left(-L_{12}^2 + 3L_{12} - 8 + \frac{\pi^2}{6}\right)\right.$$
$$\left. + (C_A - C_F)\frac{s_{12}}{s_{14}}\left(L_{12/13}^2 + \pi^2\right)\right\}$$

$$L_{ij} = \ln\left(-\frac{s_{ij}}{\mu^2} - i0\right) \qquad L_{ij/kl} = L_{ij} - L_{kl}$$

$gg q\bar{q}$: Matching results

- Nonvanishing tree-level helicity amplitudes

$$A^{(0)}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = -2g^2 \frac{\langle 23\rangle\langle 24\rangle^3}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle} = 2g^2 \frac{\sqrt{|s_{13}\, s_{14}|}}{s_{12}}\, e^{i\Phi_{+-}}$$

$$A^{(0)}(2^-,1^+;3_q^+,4_{\bar{q}}^-) = -2g^2 \frac{\langle 23\rangle\langle 24\rangle^3}{\langle 21\rangle\langle 14\rangle\langle 43\rangle\langle 32\rangle} = 2g^2 \frac{s_{13}\sqrt{|s_{13}\, s_{14}|}}{s_{12}\, s_{14}}\, e^{i\Phi_{+-}}$$

$$s_{ij} = (p_i + p_j)^2 = 2p_i\cdot p_j\,, \qquad e^{i\Phi_{+-}} = \frac{\langle 24\rangle}{[24]} \frac{[13][14]}{\sqrt{|s_{13}\, s_{14}|}}$$

  - Pull out convention-dependent overall phase
- One-loop helicity amplitudes were calculated in [Kunszt, Signer, Trocsanyi (1994)]

$$A^{(1)}_{\rm div}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = A^{(0)} \frac{\alpha_s}{4\pi}\Big[-\frac{2}{\epsilon^2}(C_A + C_F) + \frac{1}{\epsilon}(2C_F\, L_{12} + 2C_A\, L_{13} - 3C_F - \beta_0)\Big]\,,$$

$$A^{(1)}_{\rm fin}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = A^{(0)} \frac{\alpha_s}{4\pi}\Big\{C_A\Big(-L_{13}^2 + L_{12/13}^2 + 1 + \frac{7\pi^2}{6}\Big) + C_F\Big(-L_{12}^2 + 3L_{12} - 8 + \frac{\pi^2}{6}\Big) + (C_A - C_F)\frac{s_{12}}{s_{14}}(L_{12/13}^2 + \pi^2)\Big\}$$

$$L_{ij} = \ln\Big(-\frac{s_{ij}}{\mu^2} - i0\Big) \qquad L_{ij/kl} = L_{ij} - L_{kl}$$

  - Cross check: IR divergences with anomalous dimension in SCET

$$\frac{\alpha_s}{4\pi}\Big[-\frac{2}{\epsilon^2}(C_A + C_F) + \frac{1}{\epsilon}\Big(-\beta_0 - 3C_F + 2\hat{\Delta}_{gg\,q\bar{q}}(\mu^2)\Big)\Big]\vec{C}^{(0)}_{+-(+)} = \begin{pmatrix} A^{(1)}_{\rm div}(1^+,2^-;3_q^+,4_{\bar{q}}^-) \\ A^{(1)}_{\rm div}(2^-,1^+;3_q^+,4_{\bar{q}}^-) \\ B^{(1)}_{\rm div}(1^+,2^-;3_q^+,4_{\bar{q}}^-) \end{pmatrix}$$

$\hat{\Delta}_{gg\,q\bar{q}}$ = anomalous dim. mixing matrix

$gg q\bar{q}$: Hard function

$$d\sigma_N = \int dx_a dx_b \int d\Phi_N \underbrace{\sum_{\kappa}}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_N^{\kappa} \hat{S}_N^{\kappa}]}_{\text{color trace}} \otimes B_{\kappa_a} \otimes B_{\kappa_b} \otimes \prod_j J_{\kappa_j}$$

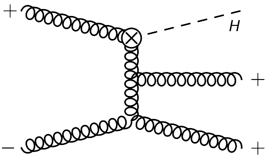
$$\hat{H}_4^{gg q\bar{q}} = \sum_{\lambda_1,\lambda_2,\lambda_3} \vec{C}_{\lambda_1\lambda_2(\lambda_3)} \vec{C}^{\dagger}_{\lambda_1\lambda_2(\lambda_3)}$$

- Calculate: $\vec{C}_{++(+)}$ and $\vec{C}_{+-(+)}$
- Identical particles: $\vec{C}_{-+(+)}(p_1, p_2, p_3, p_4) = \hat{V}\vec{C}_{+-(+)}(p_2, p_1, p_3, p_4)$
- Charge conj: $\vec{C}_{++(-)}(p_1, p_2, p_3, p_4) = -\hat{V}\vec{C}_{++(+)}(p_1, p_2, p_4, p_3)$
- Parity gives remaining $\vec{C}_{--(+)} = \vec{C}_{++(-)}$ etc.

$$\hat{V} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{(interchanging colors)}$$

Example: $ggggH$

$gggg H$: Basis



- Five helicity operators:

$$O^{abcd}_{++++} = \frac{1}{4!}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}\mathcal{B}^c_{3+}\mathcal{B}^d_{4+}H_5$$

$$O^{abcd}_{+++-} = \frac{1}{3!}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}\mathcal{B}^c_{3+}\mathcal{B}^d_{4-}H_5$$

$$\dots$$

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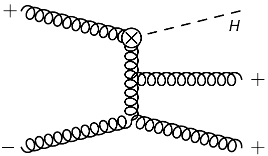

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$$\dots$$

- Six color structures:

$$\vec{T}^{\dagger abcd} = \Big(\tfrac{1}{2}(\mathrm{tr}[abcd] + \mathrm{tr}[dcba]), \dots \quad \mathrm{tr}[ab]\,\mathrm{tr}[cd], \;\dots\Big)$$

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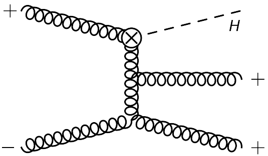

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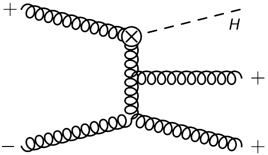
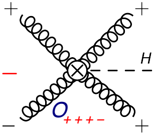
- Parity: $\vec{C}_{++++} = \vec{C}_{----}$ and $\vec{C}_{+++-} = \vec{C}_{+---}$ up to a phase
- Under charge conjugation

$$\mathrm{C}\, O^{abcd}_{\lambda_1\lambda_2\lambda_3\lambda_4}\, \vec{T}^{abcd}\, \mathrm{C} = O^{abcd}_{\lambda_1\lambda_2\lambda_3\lambda_4}\, \vec{T}^{dcba}$$

$\mathrm{tr}[abcd] - \mathrm{tr}[dcba]$ is thus not allowed

$ggggH$: Basis

$\mathcal{A}(1^+2^+3^+4^-5_H) =$ 

$+ \;= iC_{+++-}$ 

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$gggg H$: Intrinsic phases

- QCD partial amplitudes

$$\mathcal{A}(1234) = i \sum_{\sigma \in S_4/Z_4} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)} a_{\sigma(3)} a_{\sigma(4)}] A\big(\sigma(1), \sigma(2), \sigma(3), \sigma(4)\big)$$
$$+ i \sum_{\sigma \in S_4/Z_2^3} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)}] \mathrm{tr}[a_{\sigma(3)} a_{\sigma(4)}] B\big(\sigma(1), \sigma(2), \sigma(3), \sigma(4)\big)$$

- Matching coefficients are

$$\vec{C}_{++--}(p_1, p_2, p_3, p_4) = \begin{pmatrix} 2A_{\text{fin}}(1^+, 2^+, 3^-, 4^-) \\ \vdots \\ B_{\text{fin}}(1^+, 2^+, 3^-, 4^-) \\ \vdots \end{pmatrix}$$

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- Tree-level helicity amplitudes calculated by [Kauffmann, Desai, Risal (1997)]

$$A^{(0)}(1^+,2^+,3^-,4^-;5_H) = -2\left[\frac{[12]^4}{[12][23][34][41]} + \frac{\langle 34\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\right]$$

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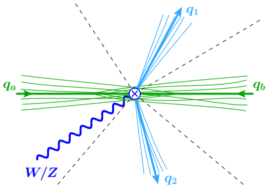
$$A^{(0)}(1^+,2^+,3^-,4^-;5_H)=2\left[\frac{s_{12}^2}{\sqrt{|s_{12}s_{23}s_{34}s_{14}|}}+e^{-2\mathrm{i}\phi_2}\frac{s_{34}^2}{\sqrt{|s_{12}s_{23}s_{34}s_{14}|}}\right]e^{\mathrm{i}\Phi}$$

- Two independent intrinsic phases:

$$e^{\mathrm{i}\phi_1}=\frac{\langle 13\rangle\langle 24\rangle}{\langle 12\rangle\langle 34\rangle}\frac{\sqrt{|s_{12}s_{34}|}}{\sqrt{|s_{13}s_{24}|}},\quad e^{\mathrm{i}\phi_2}=\frac{\langle 14\rangle\langle 23\rangle}{\langle 12\rangle\langle 34\rangle}\frac{\sqrt{|s_{12}s_{34}|}}{\sqrt{|s_{14}s_{23}|}}.$$

  - Independent of phase conventions
  - $e^{\mathrm{i}\phi_i}=\pm 1$ for four massless particles
  - ϕ_i can be written in terms of s_{ij} and $\epsilon_{\mu\nu\rho\sigma}p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$

N-Jettiness



$$\mathcal{T}_N = \frac{2}{Q} \sum_k \min\left\{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k \right\}$$

[Stewart, Tackmann, WW (2010)]

- $q_i = q_i^0(1, \hat{n}_i)$ are massless reference momenta for beams and jets

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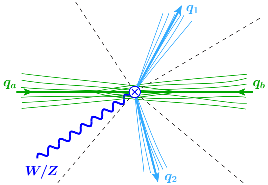

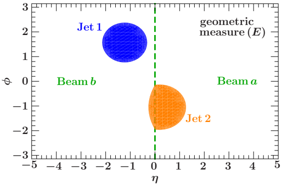

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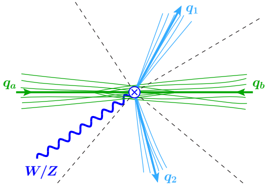

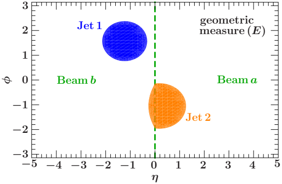

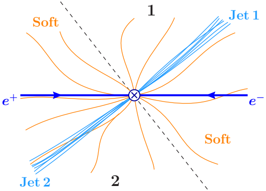

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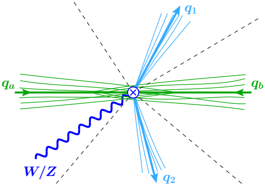

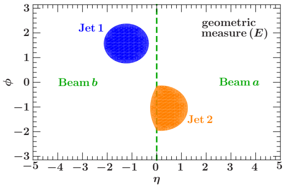

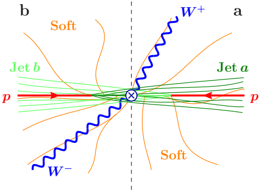

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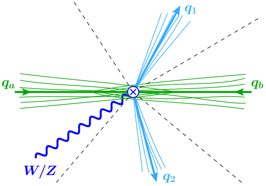

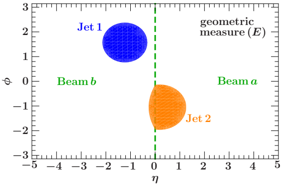

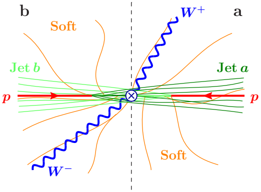

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- $pp \to 0$ jets: for $q^\mu_{a,b} = Q/2\,(1, \pm\hat{z})$ we have $\mathcal{T}_0$ = beam thrust
- Can use jet algorithm to get jet directions: $\mathcal{T}_N^{\text{alg.1}} = \mathcal{T}_N^{\text{alg.2}} + \mathcal{O}(\mathcal{T}_N^2/Q)$
- N-jettiness adapted to study substructure [Kim (2010), Thaler, Van Tilburg (2010)]

Further Discussion

Crossing symmetry

Crossing symmetry requires proper branch cuts

- For logarithms:

$$\ln\left(-\frac{s_{ij}}{\mu^2} - \mathrm{i}0\right) = \begin{cases} \ln\frac{s_{ij}}{\mu^2} - \mathrm{i}\pi & s_{ij} > 0 \\ \ln(-\frac{s_{ij}}{\mu^2}) & s_{ij} < 0 \end{cases}$$

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- For spinors:
  - We define conjugate spinors as

$$\langle p\pm| = \mathrm{sgn}(p^0)\,\overline{|p\pm\rangle}$$

  - For $p^0 < 0$

$$|p\pm\rangle = \mathrm{i}|(-p)\pm\rangle$$
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    so spinors and conjugate spinors have same branch cut
  - Spinor identities valid for $p^0 > 0$ and $p^0 < 0$
  - Additional signs only appear in relations with explicit complex conj.

$$\langle p-|q+\rangle^* = \mathrm{sgn}(p^0 q^0)\,\langle q+|p-\rangle$$

Renormalization Schemes

Gluon polarizations treated differently:

	CDR	HV	FDH
observed	d	4	4
unobserved	d	d	4

unobserved = virtual emissions or real emissions in the collinear/soft limit

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- Helicity operators require observed pol. in 4 dim. $\rightarrow$ use HV
- Partons are energetic and well-separated in matching, so HV = CDR:
  - Evanescent operators do not mix under renormalization
  - Jet function $J^{\mathrm{HV}} = J^{\mathrm{CDR}}/(1-\epsilon)$ $\rightarrow$ essentially unchanged

Soft Function

$$d\sigma_N = \int dx_a dx_b \int d\Phi_N \underbrace{\sum_{\kappa}}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_N^{\kappa} \hat{S}_N^{\kappa}]}_{\text{color trace}} \otimes B_{\kappa_a} \otimes B_{\kappa_b} \otimes \prod_j J_{\kappa_j}$$

- Soft function $\hat{S}_N^{\kappa}$ is matrix in color space
- At tree-level, soft function has no emissions,

$$\hat{S}_N{}^{b_1 \cdots \beta_N a_1 \cdots \alpha_N} \propto 1 = \delta^{b_1 a_1} \cdots \delta^{\beta_N \alpha_N} = \sum_{a_1, \ldots, \alpha_N} \vec{T}^{a_1 \cdots \alpha_N} \vec{T}^{\dagger a_1 \cdots \alpha_N}$$

Soft Function

$$d\sigma_N = \int dx_a dx_b \int d\Phi_N \underbrace{\sum_\kappa}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_N^\kappa \hat{S}_N^\kappa]}_{\text{color trace}} \otimes B_{\kappa_a} \otimes B_{\kappa_b} \otimes \prod_j J_{\kappa_j}$$

- Soft function $\hat{S}_N^\kappa$ is matrix in color space
- At tree-level, soft function has no emissions,

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- We work out the tree-level soft functions up to 5 partons. E.g. for $ggq\bar{q}$

$$1_{gg\,q\bar{q}} = \frac{C_A C_F}{2} \begin{pmatrix} 2C_F & 2C_F - C_A & 1 \\ 2C_F - C_A & 2C_F & 1 \\ 1 & 1 & C_A \end{pmatrix}$$

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- The N-jet soft function at NLO:
  - Jet angularities of cone-jets [Ellis, Hornig, Lee, Vermilion, Walsh (2010)]
  - N-Jettiness [Jouttenus, Stewart, Tackmann, WW (2011)]
  - Subtraction method for N-jet soft functions [Bauer, Dunn, Hornig (2011)]

Conclusions

We introduce a helicity operator basis such that

$$\mathcal{A}^{\text{fin}}_{\text{QCD}}(1^+2\cdots n^+_{\bar{q}}) = \text{i}C^{a_1\cdots\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(p_1,\ldots,p_n)$$

- C and P are easy
- Helicity operators require HV
  but HV and CDR essentially the same here
- Proper branch cuts ensure crossing symmetry
- Easy to use: we give explicit matching for V+jets, H+jets, $pp \to 2,3$ jets

Virtual helicity amplitudes can directly be used
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...it would be great if BlackHat, Rocket etc. would make this publicly available!

Thank you