Symbology and Scattering Amplitudes

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based on work with Goncharov, Spradlin and Vergu

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Scattering Amplitudes

- The last few years have seen a lot of progress in our understanding of the mathematical structure of gauge theories and in our ability to do computations both for theoretical and phenomenological purposes.

- Remarkable results range from precision predictions in QCD which are important for understanding the LHC data to the discovery of new miraculous structures in N=4 Yang-Mills and N=8 supergravity.

The goals of this research program are:

- to explore the rich hidden mathematical structure of amplitudes
- to exploit this structure as much as possible to make previously impossible computations trivial

Many have contributed to these recent developments including:

Alday, Arkani-Hamed, Bargheer, Beisert, Belitski, Bern, Berkovits, Boels, Bourjaily, Brandhuber, Brodel, Bjerrum-Bohr, Bullimore, Britto, Cachazo, Caron-Huot, Carrasco, Cheung, Damgaard, Del Duca, Duhr, Dixon, Dolan, Duhr, Drummons, Eden, Elvang, Fend, Ferro, Forde, Freedman, Gaiotto, Goddard, Goncharov, Green, Henn, Heslop, Hodges, Huang, Ita, Johnannson, Kallosh, Kaplan, Khoze, Kiermaier, Korchemsky, Kosower, Lipatov, Loebbert, Maitre, Mafra, Maldacena, Mason, Naculich, Nastase, Plefka, Prygarin, Roiban, Sabio Vera, Schnizer, Sever, Skinner, Smirnov, Sokatchev, Spence, Spradlin, Staudacher, Stelle, Steiberger, Svrcek, Taylor, Travaglini, Trnka, Tye, Vanhove, Vergu, Vieira, Wen, Witten

In My Talk

- I will present a new very powerful tool for analysis of multiloop amplitudes = symbol of an amplitude = which comes from modern mathematics called theory of motives.

- Symbol of an amplitude

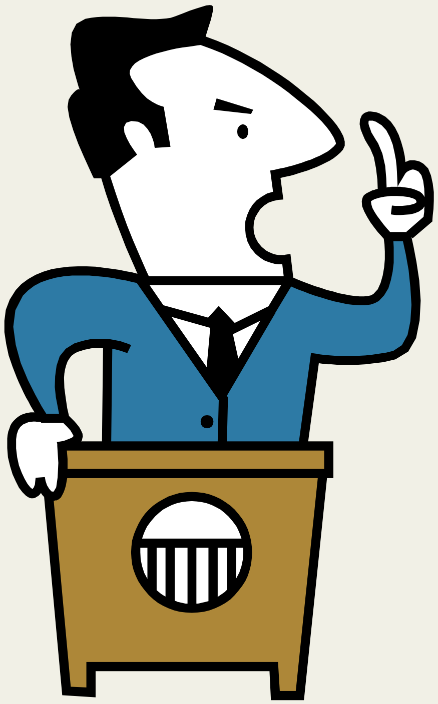
-- a very general powerful tool which leads to the compact expression for the amplitude

--captures the essential physical (and "motivic") content of the amplitude

Plan

- Introduction
- Definition of Symbol
- Why is Symbol very useful
- Results: 2-loop 6-points;

  1-loop 2m-gons;
- Outlook and Conclusions



N=4 Yang-Mills Amplitudeology

We have discovered a lot about N=4 amplitudes

1. Dual conformal and Yangian symmetries
   Beisert, Henn, Drummond, Korchemsky, McLoughlin, Plefka, Sokatchev, Smirnov
2. Amplitude/Wilson loop/Correlation funcs triality
   Adamo, Bullimore, Caron-Huot, Mason, Skinner
   Alday, Eden, Heslop, Henn, Drummond, Korchemsky, Maldacena, Sokachev,
3. Color-kinematic relations
   Bern, Carrasco, Johansson; Dennen, Huang, Kiermaier
4. Grassmanian & Integrand & Polytopes & Twistors
   Arkani-Hamed, Bourjaily, Cachazo, Cheung, Kaplan, Trnka;
   Mason, Hodges, Skinner; Spradlin, Wen, AV
5. Strong Coupling Y-system
   Alday, Gaiotto, Maldacena, Sever, Vieira

N=4 Yang-Mills Amplitudeology

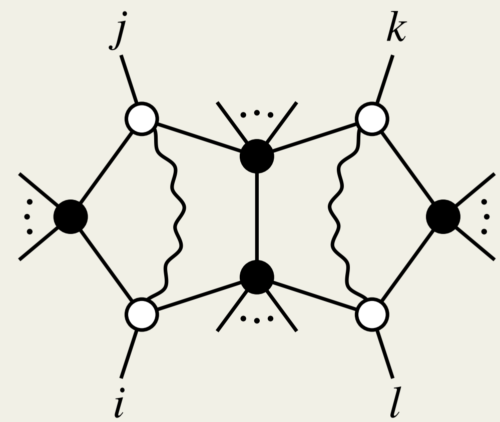
1. Dual conformal and Yangian symmetries
2. Amplitude/Wilson loop/Correlation funcs triality
3. Color-kinematic relations
4. Grassmanian & Integrand & Polytopes &Twistors

Important Lessons

All of these are completely invisible in the Lagrangian, and were discovered only after doing very hard calculations and then simplifying and analyzing the answers. Simplifications don't happen by accident. This is an experimental science, which requires collecting and analyzing lots of "data" to see the hidden structure.

Simplicity has to be believed to be seen!

Amplitudes and Integrand



- Standard Lore

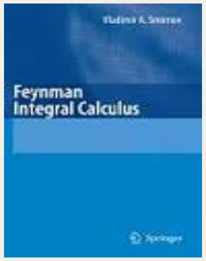
$$Amplitude = \int d(loop\ momenta)\ (Integrand)$$

- Integrand = solved for all loops and all legs in N=4

Bern, Dixon, et al, Arkani-Hamed et al

- Integral = still very hard to evaluate! This step requires a lot of blood and tears begging for some new technology!

e.g. Smirnov's book



- Symbol = "half way between integrand and amplitude"

Goncharov, Spradlin, Vergu, AV

MHV Remainder function

Simplest nontrivial multi-loop amplitude in N=4 Yang-Mills is the 2-loop 6-particle MHV remainder function = what is left after subtracting IR divergences, leaving a finite and dual conformal invariant quantity

$$R(u_1, u_2, u_2)$$

$$u_1 = \frac{s_{12}s_{45}}{s_{123}s_{345}} \quad u_2 = \frac{s_{23}s_{56}}{s_{234}s_{456}} \quad u_3 = \frac{s_{34}s_{61}}{s_{345}s_{234}} \qquad s_{123} = (k_1 + k_2 + k_3)^2$$

Numerically known since '08 Bern, Dixon, Kosower, Roiban, Vergu, Spradlin, AV
Henn,Drummond, Korchemsky, Sokatchev

In a heroic effort, Del Duca, Duhr, Smirnov '09 found a manageable way to evaluate the relevant diagrams and obtained an analytic formula for it:

DDS Formula

$$R^{(2)}_{6,WL}(u_1,u_2,u_3) = \qquad \text{(H.1)}$$

$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+$$

$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+$$

$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+$$

$$\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+$$

$$\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-$$

$$\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_3};1\right)+$$

$$G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-$$

$$\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_1};1\right)+$$

$$G\left(0,\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_3};1\right)+G\left(0,\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)-$$

$$\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$

$$\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+$$

$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)+$$

$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-$$

$$\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_1};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)+$$

$$G\left(0,\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$

$$\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+$$

$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)-$$

$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$

$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+$$

$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)+$$

$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3},0;1\right)+\frac{1}{2}G\left(\frac{1}{1-u_1},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1,0;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},1;1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_2};1\right)+$$

$$\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_3};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1},0;1\right)+$$

$$\frac{1}{2}G\left(\frac{1}{1-u_2},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,1;1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1,0;1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},1;1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_1};1\right)+$$

$$\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_2+u_3};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2},0;1\right)+$$

$$\frac{1}{2}G\left(\frac{1}{1-u_3},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,1;1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1,0;1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$

$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{79\pi^4}{360}+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)-G\left(\frac{1}{u_3},0,0,\frac{1}{u_1};1\right)-$$

$$G\left(\frac{1}{u_3},0,0,\frac{1}{u_2};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_1+u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{132};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{123},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{132},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{132},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{213},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{213},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{312},\frac{1}{1-u_3};1\right)+\mathcal{G}\left(0,0,v_{321},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{132};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{231};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{321};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1},0;1\right)-$$

$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{123},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{123};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{123};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{132};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},1;1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{231};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,\frac{1}{1-u_2};1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{321};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},0;1\right)-$$

$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},0,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},0,\frac{1}{1-u_1},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,0,\frac{1}{1-u_1};1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},1,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},0,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,1,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1,1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,1,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},1,0,\frac{1}{1-u_2};1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},1,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},1,0,\frac{1}{1-u_3};1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},1,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,1,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1,1;1\right)-\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0;1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_1\right)-$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H\left(0;u_1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_1\right)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0;1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1;1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0;1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H\left(0;u_2\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_2\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H\left(0;u_2\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_2\right)-$$

$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_2\right)-\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+\frac{5}{24}\pi^2H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0;1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_3\right)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_3\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H\left(0;u_3\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_3\right)+$$

$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H\left(0;u_1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)H\left(0;u_3\right)+$$
$$\frac{5}{24}\pi^2H\left(0;u_1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H\left(0;u_2\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)H\left(0;u_3\right)+\frac{5}{24}\pi^2H\left(0;u_2\right)H\left(0;u_3\right)+$$
$$3H\left(0;u_2\right)H\left(0,0;u_1\right)H\left(0;u_3\right)+3H\left(0;u_1\right)H\left(0,0;u_2\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H\left(0;u_3\right)+\frac{1}{2}H\left(0;u_1\right)H\left(0,1;\left(u_1+u_3\right)\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H\left(0;u_3\right)+\frac{1}{2}H\left(0;u_2\right)H\left(0,1;\left(u_2+u_3\right)\right)H\left(0;u_3\right)+$$
$$\frac{3}{4}H\left(0;u_2\right)H\left(1,0;u_1\right)H\left(0;u_3\right)+\frac{3}{4}H\left(0;u_1\right)H\left(1,0;u_2\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H\left(0,0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H\left(0,0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0,0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0,0;u_1\right)-\frac{23}{24}\pi^2H\left(0,0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H\left(0,0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H\left(0,0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0,0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0,0;u_2\right)-$$
$$\frac{25}{4}H\left(0,0;u_1\right)H\left(0,0;u_2\right)-\frac{23}{24}\pi^2H\left(0,0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H\left(0,0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H\left(0,0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H\left(0,0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H\left(0,0;u_3\right)+3H\left(0;u_1\right)H\left(0;u_2\right)H\left(0,0;u_3\right)-$$
$$\frac{25}{4}H\left(0,0;u_1\right)H\left(0,0;u_3\right)-\frac{25}{4}H\left(0,0;u_2\right)H\left(0,0;u_3\right)-\frac{23}{24}\pi^2H\left(0,0;u_3\right)+\frac{1}{12}\pi^2H\left(0,1;u_1\right)+$$
$$\frac{1}{12}\pi^2H\left(0,1;u_2\right)-\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H\left(0;u_1\right)H\left(0;u_2\right)H\left(0,1;\left(u_1+u_2\right)\right)+$$

$$\frac{1}{12}\pi^2H\left(0,1;\left(u_1+u_2\right)\right)+\frac{1}{12}\pi^2H\left(0,1;u_3\right)+\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)+\frac{1}{12}\pi^2H\left(0,1;\left(u_1+u_3\right)\right)-\frac{1}{24}\pi^2H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{12}\pi^2H\left(0,1;\left(u_2+u_3\right)\right)-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H\left(1,0;u_1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H\left(1,0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(1,0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H\left(1,0;u_1\right)-\frac{3}{4}H\left(0,0;u_2\right)H\left(1,0;u_1\right)-\frac{3}{4}H\left(0,0;u_3\right)H\left(1,0;u_1\right)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)H\left(1,0;u_1\right)-\frac{1}{3}\pi^2H\left(1,0;u_1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H\left(1,0;u_2\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H\left(1,0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(1,0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H\left(1,0;u_2\right)-\frac{3}{4}H\left(0,0;u_1\right)H\left(1,0;u_2\right)-\frac{3}{4}H\left(0,0;u_3\right)H\left(1,0;u_2\right)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H\left(1,0;u_2\right)-\frac{1}{4}H\left(1,0;u_1\right)H\left(1,0;u_2\right)-\frac{1}{3}\pi^2H\left(1,0;u_2\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H\left(1,0;u_3\right)-\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_3\right)-\frac{1}{3}\pi^2H\left(1,0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H\left(1,0;u_3\right)+$$
$$\frac{3}{4}H\left(0;u_1\right)H\left(0;u_2\right)H\left(1,0;u_3\right)-\frac{3}{4}H\left(0,0;u_1\right)H\left(1,0;u_3\right)-\frac{3}{4}H\left(0,0;u_2\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H\left(1,0;u_3\right)-\frac{1}{4}H\left(1,0;u_1\right)H\left(1,0;u_3\right)-\frac{1}{4}H\left(1,0;u_2\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{24}\pi^2H\left(1,1;u_1\right)+\frac{1}{24}\pi^2H\left(1,1;u_2\right)+\frac{1}{24}\pi^2H\left(1,1;u_3\right)+\frac{1}{2}H\left(0;u_2\right)H\left(0,0,0;u_1\right)+$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,0;u_2\right)+\frac{1}{2}H\left(0;u_1\right)H\left(0,0,0;u_3\right)-\frac{1}{2}H\left(0;u_2\right)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-H\left(0;u_1\right)H\left(0,0,1;\left(u_1+u_2\right)\right)-$$
$$H\left(0;u_2\right)H\left(0,0,1;\left(u_1+u_2\right)\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$

$$\frac{1}{2}H\left(0;u_2\right)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-H\left(0;u_1\right)H\left(0,0,1;\left(u_1+u_3\right)\right)-$$
$$H\left(0;u_3\right)H\left(0,0,1;\left(u_1+u_3\right)\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-H\left(0;u_2\right)H\left(0,0,1;\left(u_2+u_3\right)\right)-$$
$$H\left(0;u_3\right)H\left(0,0,1;\left(u_2+u_3\right)\right)-\frac{1}{2}H\left(0;u_2\right)H\left(0,1,0;u_1\right)-\frac{1}{2}H\left(0;u_3\right)H\left(0,1,0;u_2\right)-$$
$$\frac{1}{2}H\left(0;u_1\right)H\left(0,1,0;u_3\right)+\frac{1}{4}H\left(0;u_2\right)H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(0;u_1\right)H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{4}H\left(0;u_3\right)H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H\left(0;u_2\right)H\left(1,0,0;u_1\right)-\frac{1}{2}H\left(0;u_3\right)H\left(1,0,0;u_1\right)-$$
$$\frac{1}{2}H\left(0;u_1\right)H\left(1,0,0;u_2\right)+\frac{1}{2}H\left(0;u_3\right)H\left(1,0,0;u_2\right)+\frac{1}{2}H\left(0;u_1\right)H\left(1,0,0;u_3\right)-$$
$$\frac{1}{2}H\left(0;u_2\right)H\left(1,0,0;u_3\right)-\frac{1}{4}H\left(0;u_3\right)H\left(1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$7H\left(0,0,0,0;u_1\right)-7H\left(0,0,0,0;u_2\right)-7H\left(0,0,0,0;u_3\right)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+$$
$$3H\left(0,0,0,1;\left(u_1+u_2\right)\right)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+3H\left(0,0,0,1;\left(u_1+u_3\right)\right)+$$
$$\frac{3}{2}H\left(0,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+3H\left(0,0,0,1;\left(u_2+u_3\right)\right)+\frac{9}{4}H\left(0,0,1,0;u_1\right)+$$
$$\frac{9}{4}H\left(0,0,1,0;u_2\right)+\frac{9}{4}H\left(0,0,1,0;u_3\right)-\frac{1}{2}H\left(0,1,0,0;u_1\right)-\frac{1}{2}H\left(0,1,0,0;u_2\right)-$$
$$\frac{1}{2}H\left(0,1,0,0;u_3\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{2}H\left(0,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(0,1,1,0;u_1\right)+H\left(0,1,1,0;u_2\right)+H\left(0,1,1,0;u_3\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(1,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+H\left(1,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$H\left(1,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+2H\left(1,0,1,0;u_1\right)+2H\left(1,0,1,0;u_2\right)+2H\left(1,0,1,0;u_3\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H\left(1,1,1,0;u_1\right)+\frac{1}{2}H\left(1,1,1,0;u_2\right)+\frac{1}{2}H\left(1,1,1,0;u_3\right)-$$
$$\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{u_{123}}\right)-\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{u_{231}}\right)-\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{8}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{123}}\right)-\frac{1}{8}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-$$

$$\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{8}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{8}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)-\frac{1}{4}H\left(1,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H\left(1,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-\frac{1}{4}H\left(1,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{2}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{1}{2}H\left(0,0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)-\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{213}}\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{1}{2}H\left(0,0;u_1\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_1\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{1}{2}H\left(0,0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{321}}\right)+\frac{1}{2}H\left(0;u_2\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+\frac{1}{2}H\left(0;u_3\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{2}H\left(0;u_1\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H\left(0;u_3\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H\left(0;u_1\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{2}H\left(0;u_2\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{231}}\right)+$$

$$
\begin{aligned}
&\frac{1}{4}H(0;u_2)\mathcal{H}\left(0,1,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H(0;u_2)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H(0;u_3)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-\\
&\frac{1}{4}H(0;u_2)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H(0;u_3)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H(0;u_1)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-\\
&\frac{1}{4}H(0;u_3)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H(0;u_1)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H(0;u_3)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+\\
&\frac{1}{4}H(0;u_1)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H(0;u_2)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H(0;u_1)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+\\
&\frac{1}{4}H(0;u_2)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H(0;u_3)\mathcal{H}\left(1,0,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H(0;u_1)\mathcal{H}\left(1,0,1;\frac{1}{u_{231}}\right)+\\
&\frac{1}{4}H(0;u_2)\mathcal{H}\left(1,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H(0;u_2)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H(0;u_3)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-\\
&\frac{1}{4}H(0;u_2)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H(0;u_3)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H(0;u_1)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-\\
&\frac{1}{4}H(0;u_3)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H(0;u_1)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H(0;u_3)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+\\
&\frac{1}{4}H(0;u_1)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H(0;u_2)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H(0;u_1)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+\\
&\frac{1}{4}H(0;u_2)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+H(0;u_2)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-H(0;u_3)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-\\
&H(0;u_1)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H(0;u_3)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H(0;u_1)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-\\
&H(0;u_2)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{123}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{231}}\right)-\\
&\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{312}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{132}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{213}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{321}}\right)-\\
&\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{312}}\right)-\\
&\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{312}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{132}}\right)+\zeta_3H(0;u_1)+\zeta_3H(0;u_2)+\zeta_3H(0;u_3)+\\
&\frac{5}{2}\zeta_3H(1;u_1)+\frac{5}{2}\zeta_3H(1;u_2)+\frac{5}{2}\zeta_3H(1;u_3)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{123}}\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{231}}\right)+\\
&\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{312}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{312}}\right)+\\
&\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{132}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+\\
&\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{312}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{213}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{312}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{123}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{312}}\right)+
\end{aligned}
$$

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There should be a better presentation!

Otherwise we should abandon N=4 Yang Mills!

`FullSimplify[]` is not enough, we need a serious weapon to battle this 17-page

How do we proceed?

Classical Polylogs

$$Li_k(z) = \int_0^z Li_{k-1}(t) d\log t \qquad Li_1(z) = -\log(1-z)$$

Satisfy various identities

$$-Li_2(1-\frac{1}{x}) = Li_2(1-x) + \frac{1}{2}\log(x)^2$$

Goncharov polylogs satisfy huge number of identities

$$G(a_k, a_{k-1}, \ldots; z) = \int_0^z G(a_{k-1}, \ldots; t)\frac{dt}{t-a_k}, \qquad G(z) \equiv 1$$

How can we bring the identities under control?

How do we proceed?

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How can we bring the identities under control?

We will use motivic high tech!!!

Symbol of Transcendental Function

Goncharov

$$T_k \to S(T_k) = R_1 \otimes \cdots \otimes R_k$$

Symbol is an element of the k-fold tensor product of the multiplicative group of rational functions, it can be defined recursively

$$dT_k = \sum_i T^i_{k-1} d\log R_i \to S(T_k) = \sum_i S(T^i_{k-1}) \otimes R_i$$

Properties

$$R_1 \otimes (R_2 R_3) = R_1 \otimes R_2 + R_1 \otimes R_3$$

$$R_1 \otimes (cR_2) = R_1 \otimes R_2 \quad \mathrm{c} = \mathrm{const}$$

Examples

Function	Differential	Symbol
$\log R$	$d\log R$	R
$\log R_1 \log R_2$	$\log R_1 d\log R_2 + \log R_2 d\log R_1$	$R_1 \otimes R_2 + R_2 \otimes R_1$
$Li_2(R)$	$-\log(1-R) d\log R$	$-(1-R) \otimes R$
$Li_k(R)$	$Li_{k-1}(R) d\log R$	$-(1-R) \otimes R \cdots \otimes R$

Why is the Symbol Very Useful?

Symbol converts polylog functional equations into rational function identities

- $$Li_2(x) + Li_2(-x) = \frac{1}{2} Li_2(x^2)$$

$$-(1-x)\otimes x - (1+x)\otimes(-x) = -(1-x^2)\otimes x = -\frac{1}{2}(1-x^2)\otimes x^2$$

- $$Li_2(z) + Li_2(1-z) + \log(z)\log(1-z) = \frac{\pi^2}{6}$$

$$-z\otimes(1-z) - (1-z)\otimes z + z\otimes(1-z) + (1-z)\otimes z = 0$$

Symbol fixes only the leading functional transcendentality piece

Goncharov, Spradlin, Vergu, AV

Our Strategy

- Compute symbol of DDS = very simple! It has many nice properties and symmetries

Goncharov

$$A \wedge A = 0 \rightarrow \text{classical Li's only}$$

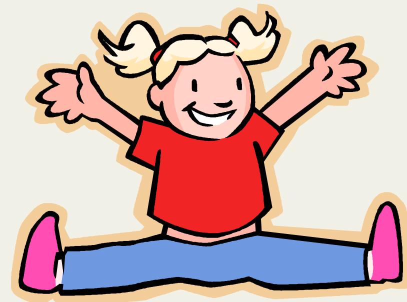
- Construct the function which has the same symbol, its nontrivial, need luck/guessing

$$(1 + x + x^2) \otimes x = \frac{1-x^3}{1-x} \otimes x = \frac{1}{3}(1 - x^3) \otimes x^3 - (1 - x) \otimes x$$

$$\Rightarrow \quad -\frac{1}{3} Li_2(x^3) + Li_2(x)$$

- Fix the lower transcendentality terms by numerical fit or collinear limits

Final Result



Goncharov, Spradlin, Vergu, AV ‘10

Two-loop six-point MHV remainder function is

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}Li_4(1-1/u_i)\right) - \frac{1}{8}\left(\sum_{i=1}^{3} Li_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}.$$

$$L_4(x^+,x^-) = \sum_{m=0}^{3} \frac{(-1)^m}{(2m)!!}\log(x^+x^-)^m(\ell_{4-m}(x^+) + \ell_{4-m}(x^-)) + \frac{1}{8!!}\log(x^+x^-)^4$$

$$\ell_n(x) = \frac{1}{2}\left(Li_n(x) - (-1)^n Li_n(1/x)\right)$$

$$J = \sum_{i=1}^{3}(\ell_1(x_i^+) - \ell_1(x_i^-))$$

$$x_i^\pm = u_i x^\pm$$

$$x^\pm = \frac{u_1+u_2+u_3-1\pm\sqrt{\Delta}}{2u_1u_2u_3}$$

$$\Delta = (u_1+u_2+u_3-1)^2 - 4u_1u_2u_3$$

Bartels, Prygarin, Lipatov ‘10: analyzed Regge limit

Gaiotto, Maldacena, Sever, Veira ‘11: OPE approach

What’s next?

- Our formula provides hope to the idea that we might be able to really unlock the secrets of multi-loop SYM amplitudes, and connect to strong coupling **Alday, Gaiotto, Maldacena, Sever, Veira**
- To do: higher points

  Gaiotto, Maldacena, Sever, Veiera OPE approach: one integral

  Caron-Huot: Symbol for all n, need to integrate.

  non-MHV ratio function

  higher loops **Bartels, Lipatov, Prygarin**

  Dixon, Drummond, Henn: 3-loops 6-pts symbol
- It would be nice to have a derivation from integrals: work on two-loops is in progress, but solved for one-loop higher dimensional polygons

More Motivic Magic

Spradlin, AV 05/11

- Consider one-loop 2m-gon in 2m-dimensions:

  appear in dim regulated one-loop MHV & can also be related to higher-loop four-dimensional integrals

- One-loop hexagon in six dimensions have been evaluated for massless & three-mass case:

  surprisingly the same L_3 function appears.....

Dixon, Drummond, Henn; Del Duca, Duhr, Smirnov 05/11

- It turns out its symbol for any m can be read of from theorem by Goncharov'96 on mixed Tate motives

Recursions for Symbols

- After Feynman parameterization, any one-loop 2m-gon integral in D=2m takes the form

$$F(Q) = \int_{\mathbb{CP}^{2m-1}} \frac{D^{2m-1}W}{(W \cdot Q \cdot W)^m} \qquad Q_{ij} = (p_i + \cdots + p_j)^2$$

in terms of a quadric Q in $\mathbb{CP}^{2m-1}$

- The symbol of the integral is given recursively by application of a result on mixed Tate motives Goncharov 96

$$S_m(Q) = \sum_{i<j} S_{m-1}(\overline{Q}_{ij}) \otimes \frac{Q_{ij}^{-1} - \sqrt{(Q_{ij}^{-1})^2 - Q_{ii}^{-1}Q_{jj}^{-1}}}{Q_{ij}^{-1} + \sqrt{(Q_{ij}^{-1})^2 - Q_{ii}^{-1}Q_{jj}^{-1}}}$$

$(\overline{Q} = Q \text{ with rows and columns } i, j \text{ deleted})$ Spradlin, AV 05/11

Conclusions

- If a problem is too hard, look for something simpler. The symbol is a useful stepping stone half way between an integrand and its integral.
- We need a technology for writing down symbols without first evaluating integrals at all. in progress
- Feynman integrals are natural projective objects, loop amplitude should also have a direct relation to the Grassmannian formulation for N=4. in progress
- Symbols are not the ultimate goal, but serve the simplest incarnation of a deeper motivic structure of amplitudes which we are only beginning to see.

A LOT OF WORK TO DO!!!