Oct. 2012
INT talk

Jet Physics from Static Charges in AdS

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Motivation:

Many Examples of Applications of AdS & CFT
to strong dynamics
{ strongly int. fluids, gluon scatt amplitudes, DIS / large S operators, ... }
(Son) (N=4, last week) (Korchemsky, Maldacena, ...)

Our Goal: Study Soft Interactions for Jets (anom. dim.)
(Make stronger connection to things we care about for phen.)

Outline: Intro Jets & Wilson Lines: $n_i^2 \neq 0$] eikonal, Sudakov, Fact. / Soft Fn
$W =$
Properties

Coordinates] $t, r =$, metrics, $\Gamma = iE$
conformal comments
$\Delta S = \beta_{ij}$

source A_τ

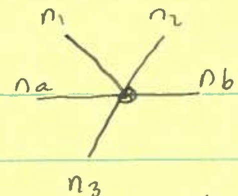
light like

Energies = Anom. Dim. (bndry. cond.)] $\nabla^2 A_\tau = \delta^3(x)$, AdS, S^3, phantom's
$\delta^3(x) - \frac{1}{2\pi^2}$, Coulomb, b. c., -field thy
light-like limit -A
- confined $\vec{E}$ lines

"Conformal Gauge" in d-dimensions] $D_{\mu\nu}$ trnsfm
• Def'n
$A_\mu =$
$D_{\mu\nu} =$ (good for $n_i^2 \neq 0$ calcs
no $1/\epsilon$ & radial gauge
light-like comments
Two-Loop Calc

Comments - Three Loops & Conf. Cross-ratio
- Witten Graphs ?

erase



$\equiv W$

eikonal

$$\int dm^2 \frac{d\sigma}{dm^2} \sim \exp\left(-\Gamma^{cusp} \ln^2 \frac{m^2}{Q^2}\right)$$

$$W_{d_1 \cdots d_N} = t_{c_1 \cdots c_N} \prod_{i=1}^{N} \left(P \exp\left(ig \int_0^\infty ds\; n_i \cdot A^a(s n_i)\, T_i^a\right)\right)_{d_i}^{c_i}$$

outgoing $n_i = (1, \hat{n})$

incoming $n_i = (-1, \hat{n})$

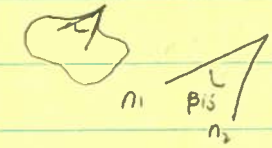
$$\frac{d\sigma}{dE_Z\, dm^2 \cdots} = H\, J \cdots J\, I\, I \otimes S\, f\, f$$

$$S(k) = \sum_{X'(k)} \langle 0 | W | X' \rangle \langle X' | W^\dagger | 0 \rangle$$

$$\mu \, d/d\mu \; W = \Gamma\, W$$

Properties $(n_i^2 \neq 0)$

1. $\Gamma = \Gamma(\{\beta_{ij}\})$



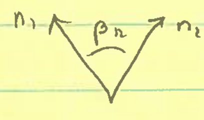
$$\cosh \beta_{ij} = \frac{n_i \cdot n_j}{|n_i||n_j|}$$

2. One-Loop

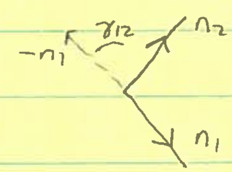
real

real

$$\Gamma = -\frac{\alpha_s}{\pi} \sum_{i<j} T_i \cdot T_j \left[(\beta_{ij} - i\pi) \coth \beta_{ij} - 1 \right] - \frac{\alpha_s}{\pi} \sum_{i<j} T_i \cdot T_j \left[\gamma_{ij} \coth \gamma_{ij} - 1 \right]$$



2-jets



$$\gamma_{ij} = \beta_{ij} - i\pi$$

(Exact for Abelian) (Exponentiation)

3. Linearity $\beta_{ij} \to \infty$ $(n_i^2 \to 0)$

$$\Gamma = -\sum_{i<j} \Gamma^{ij}(\alpha_s)\, \underbrace{\beta_{ij}}_{\ln\left(-n_i \cdot n_j \frac{\Lambda^2}{\mu^2}\right)} + \cdots$$

mention:

Brandt, Neri, Sato

Korchemsky, Radyushkin

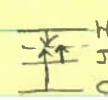
Manohar

Alday & Maldacena

Discuss Λ^2

Discuss Proof

$\Gamma_S + \Gamma_C = -\sum \Gamma_J$



Manny

Randy

Aneesh

Vogt, Moch, Vermoseren
– calc 2 lines to 3-loops
Aybat, Dixon, Sterman
– 2 loop pairwise
Gardi, Magnea, Dixon,
Becher, Neubert, ...
– arg's to 4-loops

Conjectures

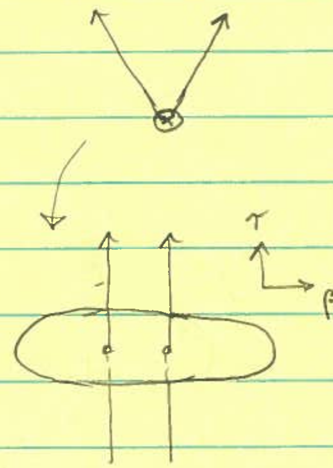
(A) Pairwise $\quad \Gamma^{ij} = \Gamma^{ij}(\alpha_s)\, T_i \cdot T_j$

(B) Casimir Scaling $\quad \Gamma^{ij}(\alpha_s) = \Gamma_{cusp}(\alpha_s)$

(C) Con~~formal~~ Cross Ratios $\quad (\beta_{ij})^0$ term: $\gamma(\alpha_s, \{n_i \cdot n_j\}) = \sum_i \gamma^i(\alpha_s)$

Known @ α_s^2

Coordinates



$n^\mu = (\cosh\beta, \sinh\beta\, \hat{n})$ $\qquad$ fixed β, θ, ϕ

$t = e^\tau \cosh\beta \qquad \tau = -\infty$ to $+\infty$

$r = e^\tau \sin\beta$

$$ds^2_{\mathbb{R}^{1,3}} = dt^2 - dr^2 - r^2 d\Omega_2^2$$
$$= e^{2\tau}\left[d\tau^2 - \underbrace{(d\beta^2 + \sinh^2\beta\, d\Omega_2^2)}_{H^3 = (\text{Eucl})\,AdS}\right]$$

often drops out

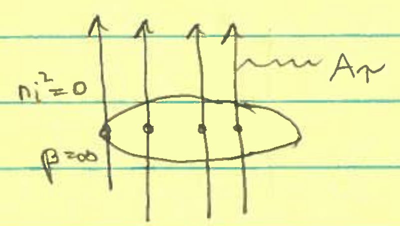
$\mathbb{R} \times AdS$

⊗ $\tau = -\infty$

[explain idea of coord change & conformal calc's]

radial quantization in CFT

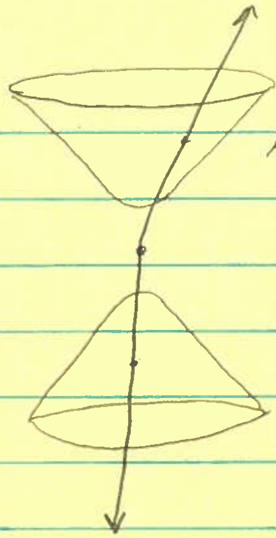
$$\mathcal{D} = x^\mu \partial_\mu = \partial_\tau = iH$$



$\Gamma = iE$

$\Delta s = \beta_{ij}$ $\quad$ • Energy only depends on proper dist btwn lines in homogeneous space (1)

• $x^\mu A_\mu(x) = \frac{dx^\mu}{d\tau} A_\mu(x) = A_\tau(x)$



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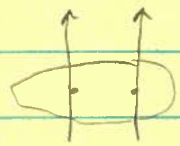
Energies



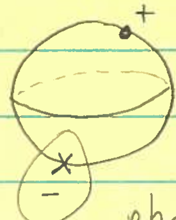
$\nabla^2 A_\tau = \delta^3(x)$ [use superposition]

$\left[\frac{1}{\sinh^2\beta} \partial_\beta \sinh^2\beta \, \partial_\beta A_\tau = 0 \right]$

$\rightarrow \quad A_\tau = C_1 + C_2 \underbrace{\coth\beta}_{\sim 1/\beta \checkmark}$? $A_\tau \xrightarrow[\beta\to\infty]{} \text{const}$

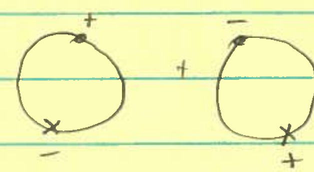
Analog on S^3

$\beta = i\alpha$



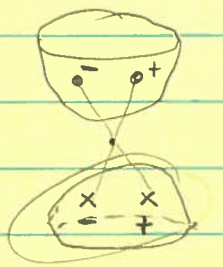
phantom charge

$A_\tau(\alpha) = C_1 + C_2 \cot\alpha$ ← poles $\alpha = 0$, $\alpha = \pi$



no good

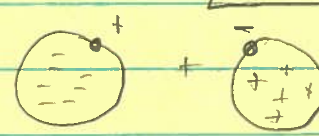
AdS:



$\beta^{ph} = \beta - i\pi$

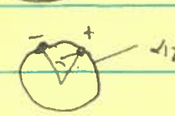
$\int (\text{Electric Field})^2$ gives one charges ptnl evaluated at other's location ↓

Trick $\quad \nabla^2 A_\tau = \delta^3(x) - \frac{1}{2\pi^2}$



✓

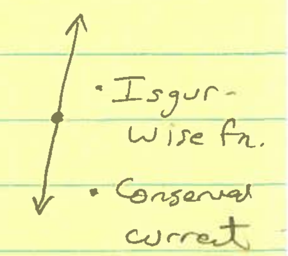
S^3: $\quad E_{pair} = \frac{q_1 q_2}{4\pi^2}\left[(\pi - \alpha_{12}) \cot\alpha_{12} + C \right] =$



AdS: $\quad E_{pair} = \frac{q_1 q_2}{4\pi^2}\left[\underbrace{(\pi + i\beta_{12}) \coth\beta_{12}}_{} + C \right]$

$\beta \to 0 \sim \frac{1}{4\pi\beta} \qquad \overset{\beta\to\infty}{\sim} \beta$

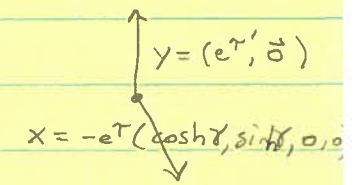
$$E_{pair} = i\,\frac{q_1 q_2}{4\pi^2}\left(\gamma_{12}\coth\gamma_{12} - i\,\underbrace{C}_{1}\right)$$



$$\Gamma = iE = -\frac{\alpha_s}{\pi}\sum_{i<j} T_i\cdot T_j\left[(\beta_{ij} - i\pi)\coth\beta_{ij} - 1\right] \quad \checkmark$$

One-Loop

$$D^F_{\mu\nu}(x,y) = \frac{1}{4\pi^2}\,\frac{g^{\mu\nu}}{(x-y)^2}$$



$$D^F_{\tau\tau} = \frac{x\cdot y}{4\pi^2(x-y)^2} = \frac{-\cosh\gamma}{8\pi^2\left[\cosh(\tau-\tau') + \cosh\gamma\right]}$$

$$A_\tau = -i\int d^4y\, D^F_{\tau\tau}(x,y)\, J^\tau(y)$$

$$= -i\int d\tau'\, D^F_{\tau\tau} = \frac{i}{4\pi^2}\,\gamma\coth\gamma$$

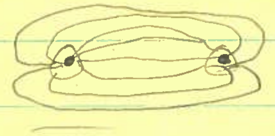
or

$$\int_\infty^0 ds\int_0^\infty dt\,\frac{n_1\cdot n_2}{(sn_1 - tn_2)^2} = \frac{1}{2}\underbrace{\int_0^\infty\frac{ds}{s}}_{1/\epsilon_{UV}}\int_{-\infty}^\infty d\tau\,\frac{\cosh\gamma}{\cosh\tau + \cosh\gamma}$$

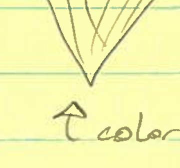
Comment (• −1 from self energy)

- Abelian Exponentiation
- γ-case → only radiation, no "potential"

  β-case → "potential" $\neq 0$, no trivial limit
- linearity, $\alpha i\beta$ is const. decay rate / unit scale



↑ concentrated energy density



↑ color coherence

Conformal Gauge

Fenyman Gauge $\quad \overset{\tau}{\sim\!\!\sim}\!\times\!\overset{i}{\sim\!\!\sim}$ mix $\qquad \hat{x}^\mu = \frac{x^\mu}{|x|}$

$$g^{\mu\nu} = \underbrace{\left[g^{\mu\nu} - \hat{x}^\mu \hat{x}^\nu - \hat{y}^\mu \hat{y}^\nu + \hat{x}\cdot\hat{y}\, \hat{x}^\mu \hat{y}^\nu \right]}_{ij} + \underbrace{\left[\hat{x}\cdot\hat{y}\, x^\mu y^\nu \right]}_{\tau\tau}$$

$$+ \underbrace{\left[\hat{x}^\mu \hat{x}^\nu + \hat{y}^\mu \hat{y}^\nu - 2(\hat{x}\cdot\hat{y})\, \hat{x}^\mu \hat{y}^\nu \right]}_{\tau i + i\tau}$$

$\boxed{D_{\tau i} = D_{i\tau} = 0}$

Conf. Gauge : when $\underbrace{y_\mu}_{\text{no } \tau \text{ component}} A^\mu(y) = 0$ then $\underbrace{x^\mu}_{\text{no } \tau \text{ component after // transport}} D_{\mu\nu}(x,y) A^\nu(y) = 0$

loose translation inv. of $D^{\mu\nu}$ $\qquad$ 0 special

Consider $\quad D_{\mu\nu}(x,y) = \underbrace{D^F_{\mu\nu}}_{\frac{-g_{\mu\nu} K_d}{[-(x-y)^2]^{d/2-1}}} + \frac{\partial}{\partial y^\nu} \underbrace{\Lambda_\mu(y,x)}_{= \frac{K_d}{(|x||y|)^{d/2-1}} \left[x_\mu g_1(\alpha,\beta) + y_\mu g_2(\alpha,\beta) \right]} + \frac{\partial}{\partial x^\mu} \Lambda_\nu(x,y)$

$$\alpha = \frac{x\cdot y}{|x||y|}, \quad \beta = \frac{|y|}{|x|}$$

solve diff eqtn's for g_i

$$K_d = \frac{\Gamma(d/2 - 1)}{4\pi^{d/2}}$$

eg 1 $\quad g_2 = 0 \qquad d/d\alpha\, g_1 = \cdots$

$$D_{\mu\nu}(x,y) = \frac{-K_d}{[-(x-y)^2]^{d/2-1}} \quad |x||y|\, \partial^x_\mu \partial^y_\nu \left(\frac{x\cdot y}{|x||y|} \right)$$

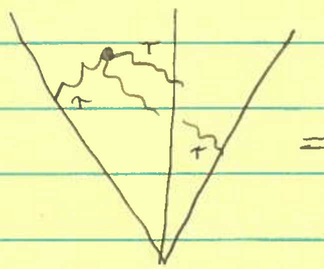
$$- K_d \frac{x_\mu x_\nu}{x^2 y^2} \left[\frac{x\cdot y}{(-(x-y)^2)^{d/2-1}} - (-(x-y)^2)^{2-d/2} + \chi(|x|,|y|)^{4-d} \right]$$

eg 2 $\quad g_1 = -\alpha\beta\, g_2$

Comment on Radial (Fock-Schwinger) Gauge

$x^\mu A_\mu(x) = 0$ ie $A^\tau = 0$

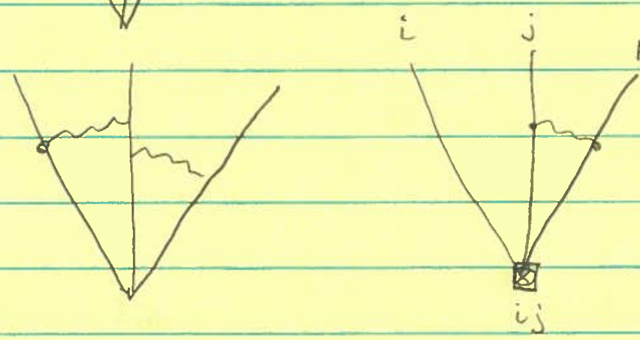
eg.



$= 0$ conf.

no $(A^0)^3$ or $(A^\tau)^3$ vertex

[$\neq 0$ Feyn & non-light like
$= 0$ Feyn like like from explicit computation]



History:
Aybat, Dixon, Sterman
Mitov, Sterman, Sung
Ferroglia, Neubert, Pecjak, Yang

$$A_\tau(\gamma_{ij}) = \int ds\, n_i^\mu D_{\mu\nu}(sn_i, tn_j)\, n_j^\nu$$
$$\propto E_F^{(0)} + \epsilon E_F^{(1)} + \epsilon E_C^{(1)}$$

$$\int_0^\infty \frac{dt_1}{t_1^{1-2\epsilon}} \left[E_F^{(0)} + \epsilon E_F^{(1)} + \epsilon E_C^{(1)}\right](\gamma_{ij}) \left\{ -\frac{1}{\epsilon} E_F^{(0)}(\gamma_{jk}) + \int_0^{t_1} \frac{dt_2}{t_2^{1-2\epsilon}} \left[E_F^{(0)} + \epsilon E_F^{(1)} + \epsilon E_C^{(1)}\right](\gamma_{jk}) \right\}$$

+ perms

$$\propto \frac{1}{\epsilon} E_F^{(0)}(\gamma_{ij}) \left[E_F^{(1)}(\gamma_{jk}) + E_C^{(1)}(\gamma_{jk})\right]$$

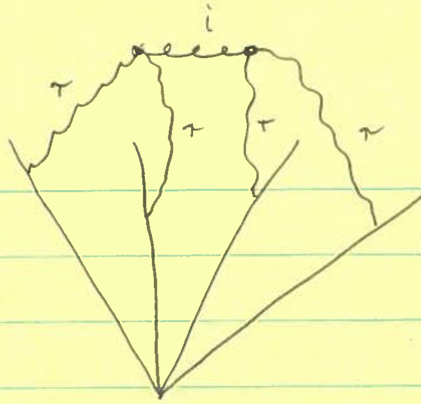
$$= \frac{1}{\epsilon}(\gamma_{ij}\coth\gamma_{ij}) \left[\coth\gamma_{jk}\left\{\gamma_{jk}^2 + 2\gamma_{jk}\ln(1-e^{-2\gamma_{jk}}) - \mathrm{Li}_2(e^{-2\gamma_{jk}}) + \frac{\pi^2}{6}\right\} - \gamma_{jk}^2\right] + (\text{anti-symm})$$

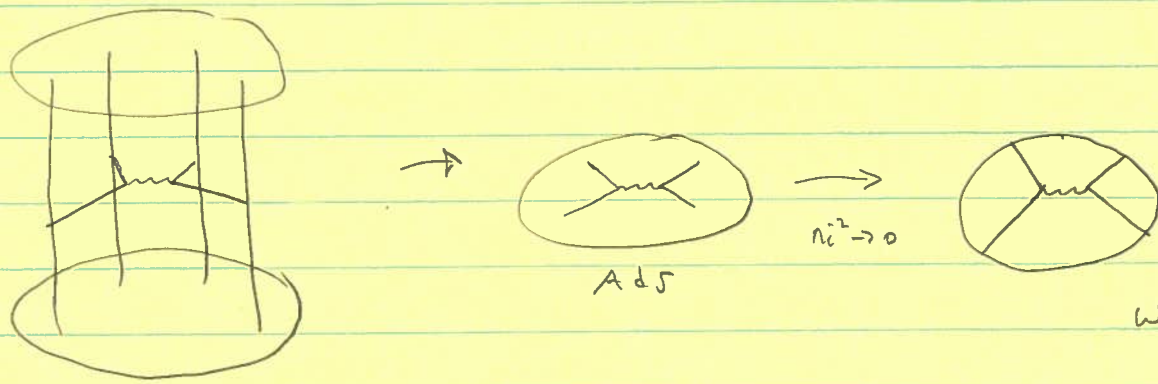
$* \; f^{abc}\, T_1^a T_2^b T_3^c$

disuss $\gamma_{jk} \to \infty$

Discuss: Light-Like Calc directly

3-loops





$\mathbb{R} \times AdS$

Witten Graphs

- Mention difference for propagator
- mention conformal cross ratio dependence