Holographic light-cone wavefunctions for the ρ meson

Frontiers in QCD

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November 1, 2011

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Outline

- Discuss recent work done with J. R. Forshaw (University of Manchester, UK) JHEP1011 (2010) 037 & JHEP10 (2011) 093
- Attempt to interpret results in the light of the AdS/QCD correspondance Work in progress

Diffractive ρ meson production at HERA

$$\gamma^* + p \to \rho + p$$

Current data from HERA

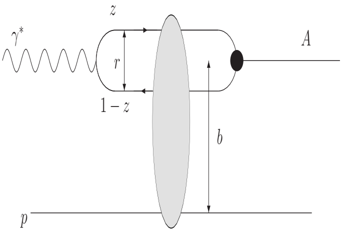
- $\sigma = \sigma_L + \sigma_T$
- $\mathrm{d}\sigma/\mathrm{d}t$
- σ_L/σ_T
- $Q^2 \in [0, 36]$ GeV^2 : includes photoproduction region

ZEUS Collaboration (2007) & H1 Collaboration (2010)

Our aim

Use data to extract information on the light-cone wavefunctions and Distribution Amplitudes of the ρ

Colour dipole model



- $A = \rho$
- r : transverse dipole size
- z : fraction of photon's light-cone momentum carried by quark

At high energy ($s \gg t, Q^2, M_\rho^2$), amplitude factorises

$$\Im\mathrm{m}\mathcal{A}_\lambda(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\mathrm{d}z \Psi^{\gamma^*,\lambda}_{h,\bar{h}} \Psi^{\rho,\lambda *}_{h,\bar{h}} e^{-iz\mathbf{r}.\boldsymbol{\Delta}} \mathcal{N}(x,\mathbf{r},\boldsymbol{\Delta})$$

Universal dipole cross-section

$$\hat{\sigma}(x,\mathbf{r}) = \mathcal{N}(x,\mathbf{r},\mathbf{0})/s$$

$\hat{\sigma}$ is well-constrained by very precise F_2 HERA data

Dipole models

Color Glass Condensate (CGC)-inspired

Marquet, Peschanski & Soyez (2007), Kowalski & Watt (2008)

$$\mathcal{N}(rQ_s, x, 0) = \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left[\gamma_s + \frac{\ln(2/rQ_s)}{\kappa\lambda \ln(1/x)}\right]} \quad \text{for} \quad rQ_s \leq 2$$

$$= \{1 - \exp[-a \ln^2(brQ_s)]\} \quad \text{for} \quad rQ_s > 2$$

Saturation scale $Q_s = (x_0/x)^{\lambda/2}$

- CGC[0.63] : anomalous dimension $\gamma_s = 0.63$ (fixed)
- CGC[0.74] : anomalous dimension $\gamma_s = 0.74$ (fitted)

Non forward extension of CGC[0.74] : t-CGC

$$Q_s \to Q_s(t) = (x_0/x)^{\lambda/2} \times (1 + c\sqrt{|t|})$$

Dipole models

Regge-inspired (FSSat)

Forshaw & Shaw (2004)

$r < r_0$: Hard Pomeron

$$\hat{\sigma}^{\text{hard}}(x, r) = A_H r^2 x^{-\lambda_H}$$

$r > r_1$: Soft Pomeron

$$\hat{\sigma}^{\text{soft}}(x, r) = A_S x^{-\lambda_S}$$

- r_0 varies with x $\rightarrow$ saturation radius
- Linear interpolation for intermediate $r_0 < r < r_1$

Fock expansion

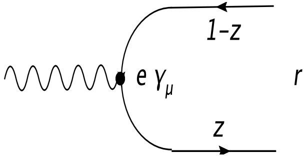
Hadronic fluctuation of photon

$$|\gamma(q,\lambda)\rangle_{\text{had.}} \propto \sum_{h,\bar{h}} \int \frac{\mathrm{d}k^+ \mathrm{d}^2\mathbf{k}}{\sqrt{k^+(q^+ - k^+)}} \Psi^{\rho,\lambda}_{h,\bar{h}}(k^+/q^+, \mathbf{k}) \hat{b}^\dagger_h \hat{d}^\dagger_{\bar{h}} |0\rangle$$

ρ meson

$$|\rho(P,\lambda)\rangle \propto \sum_{h,\bar{h}} \int \frac{\mathrm{d}k^+ \mathrm{d}^2\mathbf{k}}{\sqrt{k^+(P^+ - k^+)}} \Psi^{\rho,\lambda}_{h,\bar{h}}(k^+/P^+, \mathbf{k}) \hat{b}^\dagger_h \hat{d}^\dagger_{\bar{h}} |0\rangle$$

Light cone wavefunctions



Photon

γ^μ

QED

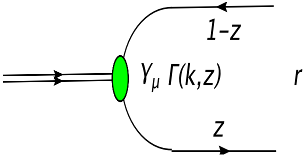
Spinor $\times$ Scalar

$$\Psi^{\gamma\{\lambda\}}_{h,\bar{h}}(\mathbf{k}, z; Q^2) \propto S^{\gamma,\lambda}_{h,\bar{h}}(\mathbf{k}, z) \times \phi_\gamma(\mathbf{k}, z; Q^2)$$

$$S^{\gamma,\lambda}_{h,\bar{h}}(\mathbf{k}, z) = \frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}}\gamma^\mu \cdot \varepsilon^\lambda_\mu \frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}$$

Well known (high Q^2)

Light cone wavefunctions



Meson
$\gamma^\mu \Gamma(\mathbf{k}, z)$
QCD

Vector meson : Spinor $\times$ Scalar

$$\Psi^{\lambda}_{h,\bar{h}}(\mathbf{k}, z) \propto S^{\lambda}_{h,\bar{h}}(\mathbf{k}, z) \times \phi_\lambda(\mathbf{k}, z)$$

$$S^{\lambda}_{h,\bar{h}}(\mathbf{k}, z) = \frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}} \gamma^\mu \cdot e^{\lambda}_{\mu} \frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}$$

Unknown scalar wavefunction $\triangleright$ models

Spinor light-cone wavefunctions

Brodsky & Lepage (1980)

Longitudinal

$$S^{\rho,L}_{h,\bar{h}}(z,\mathbf{k}) = -\frac{1}{M_\rho z(1-z)}\left[z(1-z)M_\rho^2 + m_f^2 + \mathbf{k}^2\right]\delta_{h,-\bar{h}} \; .$$

Transverse

$$S^{\rho,T(\pm)}_{h,\bar{h}}(z,\mathbf{k}) = \pm\frac{\sqrt{2}}{z(1-z)}\{[z\delta_{h\pm,\bar{h}\mp} - (1-z)]\delta_{h\mp,\bar{h}\pm} k e^{\pm i\theta_k} + m_f \delta_{h\pm,\bar{h}\pm}\}$$

Contraints on meson wavefunction

Normalisation

$$1 = \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\mathrm{d}z |\Psi^{\rho,\lambda}_{h,\bar{h}}(r,z)|^2 \equiv \int \mathrm{d}^2\mathbf{r}\mathrm{d}z |\Psi^{\rho,\lambda}(r,z)|^2$$

Leptonic decay width

$$f_\rho M_\rho = \frac{N_c}{\pi} e_f \int_0^1 \frac{\mathrm{d}z}{z(1-z)} [z(1-z)M_\rho^2 + m_f^2 - \nabla_r^2]\phi_L(r,z)\Big|_{r=0}$$

Models for scalar wavefunction

CM frame to the light-cone

$$\phi^{\mathrm{RF}}\left(\vec{k}^2 \to \frac{\mathbf{k}^2 + m_f^2}{4z(1-z)} - m_f^2\right) = \tilde{\phi}_\lambda^{\mathrm{BG}}(\mathbf{k}, z)$$

Equate invariant mass of $q\bar{q}$ pair in CM and LC frame

Brodsky, Huang & Lepage (1980)

Boosted Gaussian

$$\phi_\lambda^{\mathrm{BG}}(r,z) = \mathcal{N}_\lambda z\bar{z} \exp\left(-\frac{m_f^2 R_\lambda^2}{8z\bar{z}}\right) \exp\left(-\frac{2z(1-z)r^2}{R_\lambda^2}\right)$$

- $\zeta = \sqrt{z(1-z)}r$ is called the impact variable
- R_λ and $\mathcal{N}_\lambda$ fixed using normalisation and decay width constraints
- Cannot fit current data with most dipole models

Models for scalar wavefunction

BG inspired wavefunction

$$\phi_\lambda^{\mathrm{BG}}(r,z) = \mathcal{N}_\lambda [z\bar{z}]^{b_\lambda} \exp\left(-\frac{m_f^2 R_\lambda^2}{8[z\bar{z}]^{b_\lambda}}\right) \exp\left(-\frac{2[z\bar{z}]^{b_\lambda} r^2}{R_\lambda^2}\right)$$

- Allow b_λ to vary freely
- This enhances end-point contribution

Additional enhancement

$$\phi_\lambda(r,z) = \phi_\lambda^{\mathrm{BG}}(r,z) \times [1 + c_\lambda \xi^2 + d_\lambda \xi^4]$$

$$\xi \equiv 2z - 1$$

BG fits

Data points

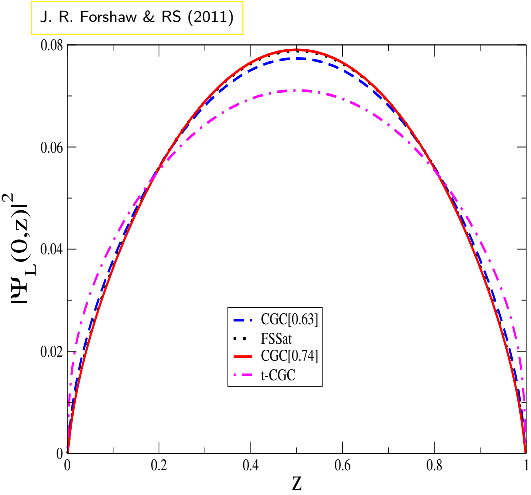
- σ_{tot} : 59
- $\sigma_{\text{L}}/\sigma_{\text{T}}$: 16
- $\mathrm{d}\sigma/\mathrm{d}t$: 46
- f_ρ : 1

Best fit parameters

J. R. Forshaw & RS (2011)

Model	R_L^2	R_T^2	b_L	b_T	c_T	d_T	χ^2/d.o.f
FSSat	26.8	27.5	0.57	0.75	0.33	1.31	68/70
CGC[0.63]	27.3	31.9	0.55	0.73	1.70	2.15	67/70
CGC[0.74]	26.7	21.3	0.57	0.79	0	0	64/72
t-CGC	29.6	21.6	0.50	0.74	0	0	114/116
t-CGC (alt.)	29.7	21.0	0.50	0.73	−0.16	−0.17	112/114

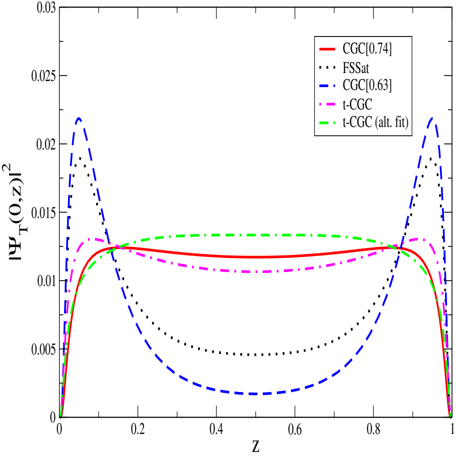
Extracted light-cone wavefunctions



- Longitudinal polarisation
- Not much model dependent

Extracted light-cone wavefunctions

J. R. Forshaw & RS (2011)



- Transverse polarisation
- More model dependent

Distribution Amplitudes

Meson-to-vacuum matrix elements on the light-cone

Ball, Braun & Lenz (2007)

$$\langle 0|\bar{q}(0)[0,x]\gamma_\mu q(x)|\rho(P,\lambda)\rangle \propto \{ \frac{e^{(\lambda)}\cdot x}{P\cdot x} P_\mu \int_0^1 du\, e^{-iuP\cdot x} \phi_\parallel(u,\mu) + \left(e_\mu^{(\lambda)} - P_\mu \frac{e^{(\lambda)}\cdot x}{P\cdot x} \right) \int_0^1 du\, e^{-iuP\cdot x} g_\perp(u,\mu) \}$$

Twist classification

$$\text{Twist-2} : \phi_\parallel(z,\mu) \propto \int \mathrm{d}x^- \, e^{izP^+x^-} \langle 0|\bar{q}(0)\gamma^+ q(x^-)|\rho(P,L)\rangle$$

$$\text{Twist-3} : g_\perp(z,\mu) \propto P^+ \int \mathrm{d}x^- \, e^{izP^+x^-} \langle 0|\bar{q}(0) e^{T*}.\gamma q(x^-)|\rho(P,T)\rangle$$

Distribution Amplitudes

Explicit forms

Twist-2 :

$$\phi_{\parallel}(z,\mu) = 6z(1-z)\left[1 + a_2^{\parallel}(\mu)\frac{3}{2}(5\xi^2 - 1)\right]$$

Twist-3 :

$$\begin{aligned} g_{\perp}(z,\mu) &= \frac{3}{4}(1+\xi^2) + \left(\frac{3}{7}a_2^{\parallel}(\mu) + 5\zeta_3(\mu)\right)(3\xi^2 - 1) \\ &+ \left[\frac{9}{112}a_2^{\parallel}(\mu) + \frac{15}{64}\zeta_3(\mu)\left(3\omega_3^V(\mu) - \omega_3^A(\mu)\right)\right] \\ &\quad \times \left(3 - 30\xi^2 + 35\xi^4\right) \end{aligned}$$

$$\xi \equiv 2z - 1$$

Distribution Amplitudes

QCD Sum Rules and evolution

- QCD Sum Rules to estimate parameters at $\mu = 1$ GeV
- pQCD evolution
- Parameters vanish as $\mu \to \infty$

Asymptotic DAs

Twist-2 :

$$\phi_{||}(z,\infty) = 6z(1-z)$$

Twist-3 :

$$g_{\perp}(z,\infty) = \frac{3}{4}(1+\xi^2)$$

Can predict Distribution Amplitudes

J. R. Forshaw & RS (2011)

Meson to vacuum matrix elements

$$\langle 0|\bar{q}(0)\gamma^\mu q(x^-)|\rho(P,\lambda)\rangle \propto \sum_{h,\bar{h}} \int \left[\frac{\mathrm{d}k^+ \mathrm{d}^2\mathbf{k}\,\Theta(|\mathbf{k}|<\mu)}{16\pi^3\sqrt{k^+(P^+-k^+)}}\right]$$
$$\times\ \Psi^{\rho,\lambda}_{h,\bar{h}}(k^+/P^+,\mathbf{k})$$
$$\times\ \bar{v}_{\bar{h}}(P^+-k^+,-\mathbf{k})\gamma^\mu u_h(k^+,\mathbf{k})e^{-ik^+x^-}$$

Can predict Distribution Amplitudes

J. R. Forshaw & RS (2011)

Twist-2 DA sensitive to longitudinal light-cone wavefunction

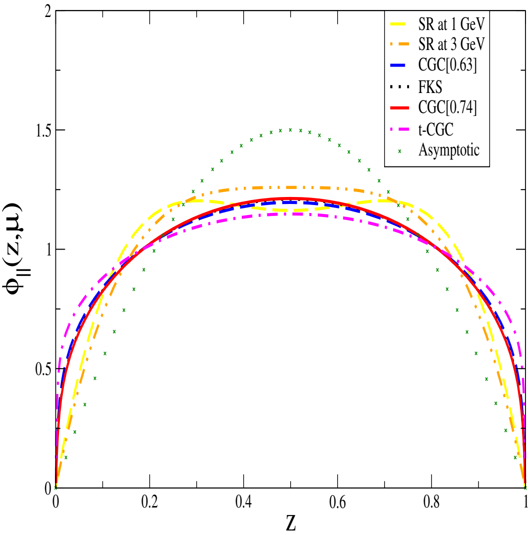
$$\phi_{\parallel}(z,\mu) = \frac{N_c}{\pi\sqrt{2}f_\rho M_\rho}\int \mathrm{d}r\mu J_1(\mu r)[M_\rho^2 z\bar{z} + m_f^2 - \nabla_r^2]\frac{\phi_L(r,z)}{z\bar{z}}$$

Twist-3 DA sensitive to transverse light-cone wavefunction

$$g_{\perp}(z,\mu) = \frac{N_c}{2\pi\sqrt{2}f_\rho M_\rho}\int \mathrm{d}r\mu J_1(\mu r)\left[(m_f^2 - (z^2+\bar{z}^2)\nabla_r^2\right]\frac{\phi_T(r,z)}{(z\bar{z})^2}$$

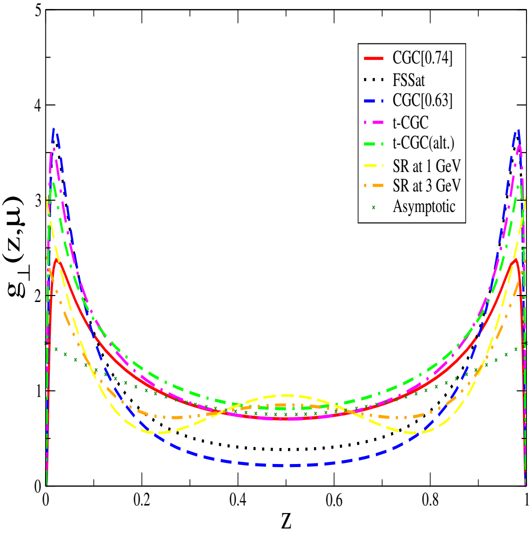
$$\bar{z} \equiv 1 - z$$

Comparison with Sum Rules predictions



- Twist-2 DAs at $\mu = 1$ GeV
- Consistent with Sum Rules
- Asymptotic : $\mu \to \infty$

Comparison with Sum Rules predictions



- Twist-3 DAs at $\mu = 1$ GeV
- Consistent with Sum Rules
- Asymptotic : $\mu \to \infty$

Moment of the twist-2 DA

Lowest moment

$$\langle \xi^2 \rangle_\mu = \int_0^1 \mathrm{d}z \xi^2 \varphi(z,\mu)$$

Approach	Scale μ	$\langle \xi^2 \rangle_\mu$
Old BG prediction	~ 1 GeV	0.181
CGC[0.74]	~ 1 GeV	0.266
CGC[0.63]	~ 1 GeV	0.271
t-CGC	~ 1 GeV	0.286
FSSat	~ 1 GeV	0.267
Sum Rules	1 GeV	0.254
Sum Rules	3 GeV	0.237
Lattice	2 GeV	0.24(4)
$6z(1-z)$	∞	0.2

Insights from AdS/QCD

Pion form factor in light-cone formalism

$$F_\pi(Q^2) = 2\pi \int_0^1 \frac{\mathrm{d}z}{z(1-z)} \int_0^\infty \mathrm{d}\zeta \zeta J_0 \left(\zeta Q \sqrt{\frac{1-z}{z}}\right) |\phi(z,\zeta)|^2$$

Pion form factor in soft-wall AdS at large $Q^2 \gg 4\kappa^2$

$$F_\pi(Q^2) = \int_0^\infty \mathrm{d}z_5 \int_0^1 \mathrm{d}z J_0 \left(z_5 Q \sqrt{\frac{1-z}{z}}\right) |\Phi_\kappa(z_5)|^2$$

Dilaton background : $\varphi(z_5) = \kappa^2 z_5^2$ Karch, Katz, Son & Stephanov (2006)

Brodsky and de Teramond mapping

$\zeta \Leftrightarrow z_5$

$$|\phi(z,\zeta)|^2 = z(1-z)\frac{|\Phi_\kappa(\zeta)|^2}{2\pi\zeta}$$

Insights from AdS/QCD

Schroedinger equation for string modes in AdS

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\Phi_\kappa(\zeta) = M^2\Phi_\kappa(\zeta)$$

Soft wall potential

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Mass spectrum

$$M^2 = 4\kappa^2(n+L+S/2)$$

ρ meson : $n = L = 0,\ S = 1$

$$M_\rho^2 = 2\kappa^2 \qquad \Phi_\kappa(\zeta) = \sqrt{\zeta}\exp\left(-\frac{\kappa^2\zeta^2}{2}\right)$$

AdS light-cone wavefunction

Apply BT mapping to obtain scalar part of ρ wavefunction

$$\phi_\lambda^{\mathrm{AdS}}(z,\zeta) = \mathcal{N}_\lambda \sqrt{z\bar{z}} \exp\left(-\frac{m_f^2}{2\kappa_\lambda^2 z\bar{z}}\right) \exp\left(-\frac{\kappa_\lambda^2\zeta^2}{2}\right)$$

Compare to old Boosted Gaussian

$$\phi_\lambda^{\mathrm{BG}}(r,z) = \mathcal{N}_\lambda z\bar{z} \exp\left(-\frac{m_f^2 R_\lambda^2}{8z\bar{z}}\right) \exp\left(-\frac{2z\bar{z}r^2}{R_\lambda^2}\right)$$

AdS broader than BG

$$R_\lambda^2 \equiv \frac{4}{\kappa_\lambda^2}$$

$$\sqrt{z\bar{z}}\phi_\lambda^{\mathrm{AdS}}(z,\zeta) = \phi_\lambda^{\mathrm{BG}}(z,r)$$

AdS fits

Fits (Preliminary)

Model	κ_L^2	κ_T^2	χ^2/d.o.f
FSSat	0.32	0.30	74/74
CGC[0.74]	0.32	0.29	67/74
CGC[0.63]	0.32	0.29	112/74

Fitted value of κ is consistent with Regge slope

$$\kappa = 0.54 \text{ GeV} - 0.57 \text{ GeV}$$

Better than Boosted Gaussian

AdS wavefunction able to fit with two dipole models

AdS inspired light-cone wavefunction

AdS inspired wavefunction

$$\phi_\lambda^{\mathrm{AdS}}(z,\zeta) = \mathcal{N}_\lambda [z\bar{z}]^{b_\lambda} \exp\left(-\frac{m_f^2}{2\kappa_\lambda^2 z\bar{z}}\right) \exp\left(-\frac{\kappa_\lambda^2 \zeta^2}{2}\right)$$

Compare to BG inspired wavefunction

$$\phi_\lambda^{\mathrm{BG}}(z,r) = \mathcal{N}_\lambda [z\bar{z}]^{b_\lambda} \exp\left(-\frac{m_f^2 R_\lambda^2}{8[z\bar{z}]^{b_\lambda}}\right) \exp\left(-\frac{2[z\bar{z}]^{b_\lambda} r^2}{R_\lambda^2}\right)$$

- $b_\lambda = 0.5$ gives AdS wavefunction
- $b_\lambda = 1$ gives old BG wavefunction
- Value of b_λ controls degree of end-point enhancement
- Allow b_λ to vary freely to fit data

AdS inspired fits

Fits (Preliminary)

Model	κ_L^2	κ_T^2	b_L	b_T	χ^2/d.o.f
FSSat	0.32	0.29	0.36	0.26	61/72
CGC[0.74]	0.32	0.29	0.39	0.47	64/72
CGC[0.63]	0.34	0.27	0.23	0.10	58/72

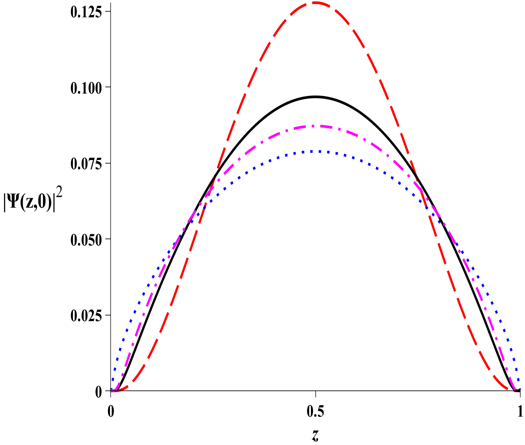
Fitted value of κ is consistent with Regge slope

$$\kappa = 0.52 \text{ GeV} - 0.58 \text{ GeV}$$

Compare to BG fits

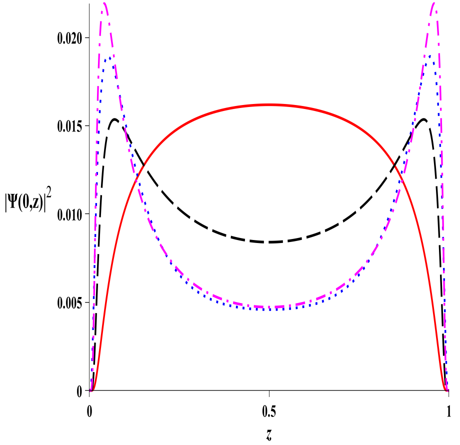
- 2 less free parameters for FSSat and CGC[0.63]
- Lower χ^2/d.o.f for all three dipole models

AdS light-cone wavefunctions



- Longitudinal polarisation
- FSSat dipole
- Red : BG
- Blue : BG fit
- Black :AdS fit
- Magenta : AdS inspired fit

AdS light-cone wavefunctions



- Transverse polarisation
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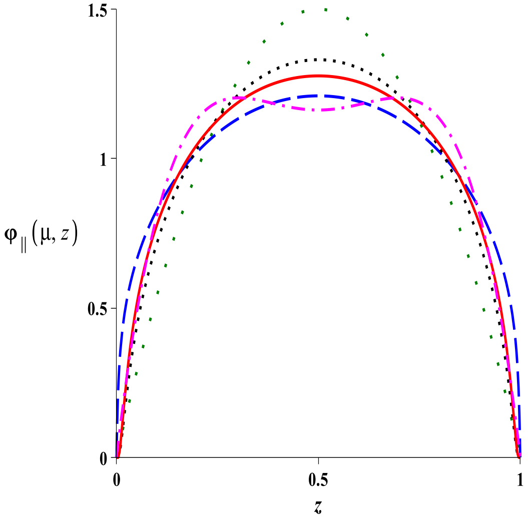
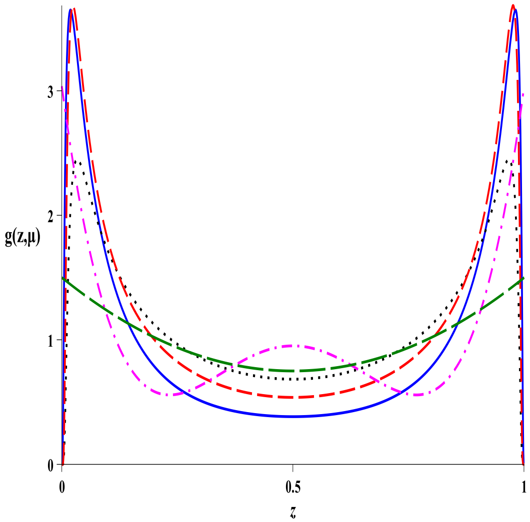
AdS Distribution Amplitudes



- Green : asymptotic
- FSSat dipole
- Blue : BG fit
- Black : AdS fit
- Red : AdS inspired fit
- Magenta : Sum Rules

AdS Distribution Amplitudes



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AdS inspired fit	~ 1 GeV	0.241
Lattice	2 GeV	0.24(4)
Sum Rules	1 GeV	0.254
Sum Rules	3 GeV	0.237
$6z(1-z)$	∞	0.2

RBC Collaboration, P. A. Boyle et. al.PoS LATTICE2008 (2008) 165, [arXiv :0810.1669]

Conclusions

- Extracted light-cone wavefunctions show end-point enhancement
- Extracted DAs consistent with Sum Rules and lattice predictions
- All extracted DAs broader than asymptotic distribution
- AdS/QCD inspired light-cone wavefunctions look promising