Manifest power-counting, the UV properties of N=8 supergravity and the origin of simplicity in N=4 sYM theory

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Based on work with
Z. Bern, J.J. Carrasco, L. Dixon, H. Johansson and Q. Jin

The master plan:

1. $\mathcal{N}=8$ supergravity through 4 loops
   - why forms of amplitudes w/ manifest powercounting?
   - a curious relation between sYM and Sugra UV residues
   - 4-loops with manifest power counting
   - UV properties and more unexpected relations
2. Just how magical is nonplanar $\mathcal{N}=4$ sYM theory?
   - a close relative of $\mathcal{N}=4$ sYM theory though 2 loops
     with planar integrability & some tree-level numerator relations
   - how to calculate; find some all-loop prop's of 2-trace terms
   - explore 1 & 2-loop nonplanar amplitudes
3. Comments and conclusions

Why chase presentations of amplitudes with manifest powercounting?

- usually simpler than other presentations
- (much) easier to analyze
- expose the UV properties of the theory:

  $\mathcal{N}=8$ supergravity: $D_c = 4 + \frac{6}{L}$ though 4 loops

- (sufficiently many) symmetries are linearly realized
- may expose unexpected relations between theories and unexpected symmetries

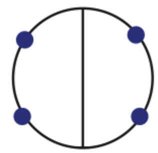
Should they exist?

- not clear; perhaps akin to freedom of gauge choice
- depends on the symmetries at work (linear vs. nonlinear)
- confusing when several symmetries have similar consequences

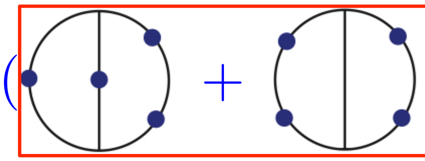
An example:

- 2-loop sYM vs. 2-loop supergravity $(d = 7)$

$$\mathcal{A}_4^{(2)}\Big|_{\mathrm{pole}}^{SU(N_c)} = -\, g^6\, \mathcal{K}\, \Big(N_c^2$$



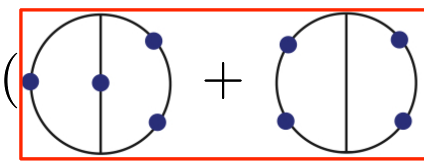
$$+\, 12\Big($$



$$\Big)\Big)$$

$$\times\Big(s\,(\mathrm{Tr}_{1324}+\mathrm{Tr}_{1423})+t\,(\mathrm{Tr}_{1243}+\mathrm{Tr}_{1342})+u\,(\mathrm{Tr}_{1234}+\mathrm{Tr}_{1432})\Big)+\ldots$$

$$\mathcal{M}_4^{(2)}\Big|_{\mathrm{pole}} = -2\left(\frac{\kappa}{2}\right)^6 stu(s^2+t^2+u^2)\, M_4^{\mathrm{tree}}\, \Big($$

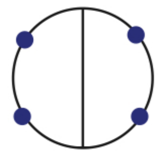


$$\Big)$$

$$= -2\left(\frac{\kappa}{2}\right)^6 stu(s^2+t^2+u^2)\, M_4^{\mathrm{tree}}\, \frac{\pi}{12(4\pi)^7\,\epsilon}$$

An example:

- 2-loop sYM vs. 2-loop supergravity $(d=7)$

$$\mathcal{A}_4^{(2)}\Big|_{\text{pole}}^{SU(N_c)} = -\,g^6\,\mathcal{K}\,\Big(N_c^2$$

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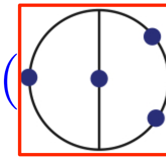
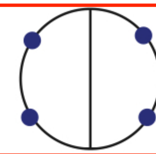
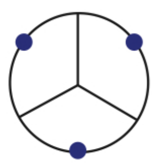
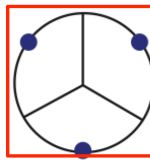
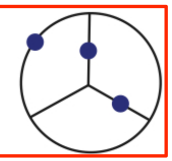
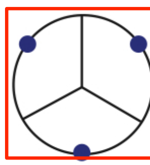

$$+$$

$$\Big)\Big)$$
$$\times\Big(s\,(\text{Tr}_{1324}+\text{Tr}_{1423})+t\,(\text{Tr}_{1243}+\text{Tr}_{1342})+u\,(\text{Tr}_{1234}+\text{Tr}_{1432})\Big)+\ldots$$

$$\mathcal{M}_4^{(2)}\Big|_{\text{pole}} = -2\left(\frac{\kappa}{2}\right)^6 stu(s^2+t^2+u^2)\,M_4^{\text{tree}}\,\boxed{\frac{\pi}{12(4\pi)^7\,\epsilon}}$$

- 3-loop sYM vs. 3-loop supergravity $(d=6)$

$$\mathcal{A}_4^{(3)}\Big|_{\text{pole}}^{SU(N_c)} = 2\,g^8\,\mathcal{K}\,\Big(N_c^3$$

$$+\,12\,N_c\,\Big($$

$$+\,3$$

$$\Big)\Big)$$
$$\times\Big(s\,(\text{Tr}_{1324}+\text{Tr}_{1423})+t\,(\text{Tr}_{1243}+\text{Tr}_{1342})+u\,(\text{Tr}_{1234}+\text{Tr}_{1432})\Big)$$

$$\mathcal{M}_4^{(3)}\Big|_{\text{pole}} = -\left(\frac{\kappa}{2}\right)^8 (stu)^2\,M_4^{\text{tree}}\,\Big(10\,\Big($$

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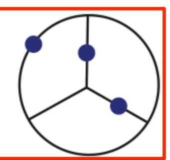
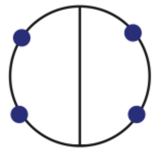

$$\Big)\Big)$$
$$= -\left(\frac{\kappa}{2}\right)^8 (stu)^2\,M_4^{\text{tree}}\left(10\times\frac{\zeta(3)}{2(4\pi)^9\,\epsilon}\right)$$

An example:

- 2-loop sYM vs. 2-loop supergravity $(d=7)$

$$\mathcal{A}_4^{(2)}\Big|_{\rm pole}^{SU(N_c)} = -\,g^6\,\mathcal{K}\Big(N_c^2$$

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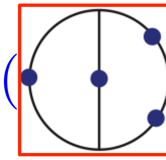
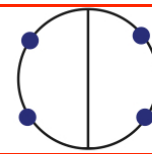
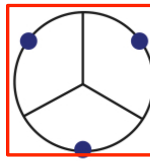
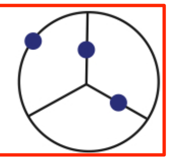

$$+$$

$$\Big)\Big)$$
$$\times\Big(s\,(\mathrm{Tr}_{1324}+\mathrm{Tr}_{1423})+t\,(\mathrm{Tr}_{1243}+\mathrm{Tr}_{1342})+u\,(\mathrm{Tr}_{1234}+\mathrm{Tr}_{1432})\Big)+\ldots$$

$$\mathcal{M}_4^{(2)}\Big|_{\rm pole} = -2\left(\frac{\kappa}{2}\right)^6 stu(s^2+t^2+u^2)\,M_4^{\rm tree}\,\boxed{\frac{\pi}{12(4\pi)^7\,\epsilon}}$$

- 3-loop sYM vs. 3-loop supergravity $(d=6)$

$$\mathcal{A}_4^{(3)}\Big|_{\rm pole}^{SU(N_c)} = 2\,g^8\,\mathcal{K}\Big(N_c^3$$

$$+\,12\,N_c\Big($$

$$+\,3$$

$$\Big)\Big)$$
$$\times\Big(s\,(\mathrm{Tr}_{1324}+\mathrm{Tr}_{1423})+t\,(\mathrm{Tr}_{1243}+\mathrm{Tr}_{1342})+u\,(\mathrm{Tr}_{1234}+\mathrm{Tr}_{1432})\Big)$$

$$\mathcal{M}_4^{(3)}\Big|_{\rm pole} = -\left(\frac{\kappa}{2}\right)^8 (stu)^2\,M_4^{\rm tree}\left(10\times\boxed{\frac{\zeta(3)}{2(4\pi)^9\,\epsilon}}\right)$$

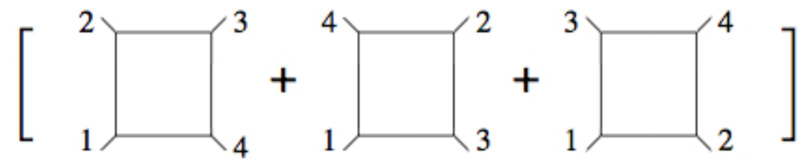
Does this continue?

Supergravity amplitudes with manifest power counting

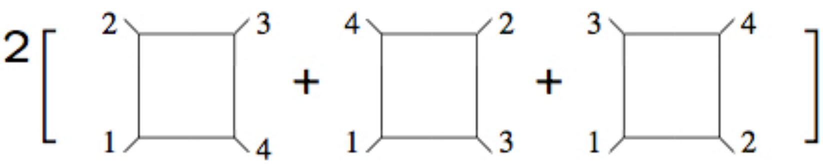
- 1-loop sYM vs. 1-loop supergravity

Green, Schwarz

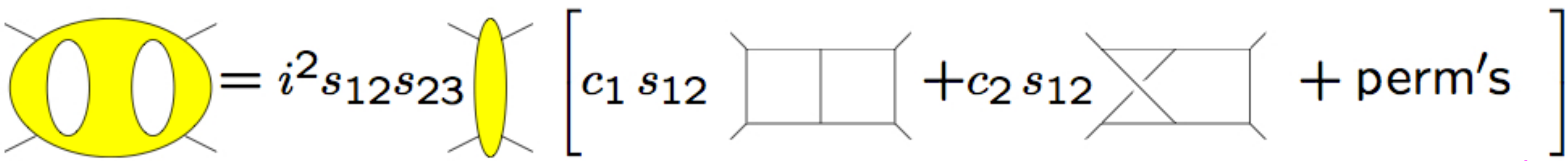
$$A_4^{1\,\text{loop}}(1,2,3,4) = s_{12}s_{23}A_4^{\text{tree}}(1,2,3,4)$$



$$M_4^{1\,\text{loop}}(1,2,3,4) = \left[s_{12}s_{23}A_4^{\text{tree}}(1,2,3,4)\right]^2$$

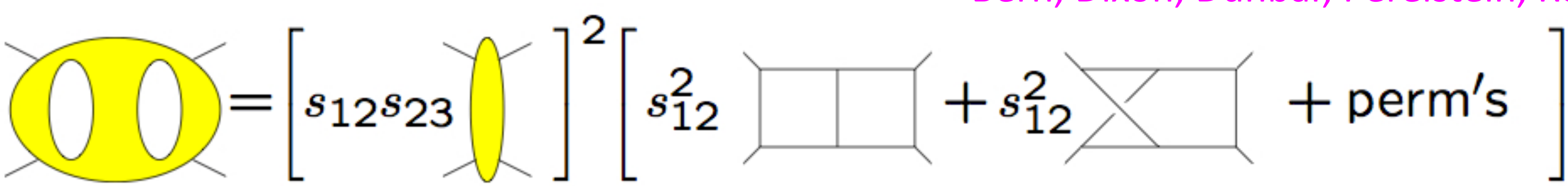


- 2-loop sYM vs. 2-loop supergravity



Bern, Rozowski, Yan

Bern, Dixon, Dunbar, Perelstein, Rozowski



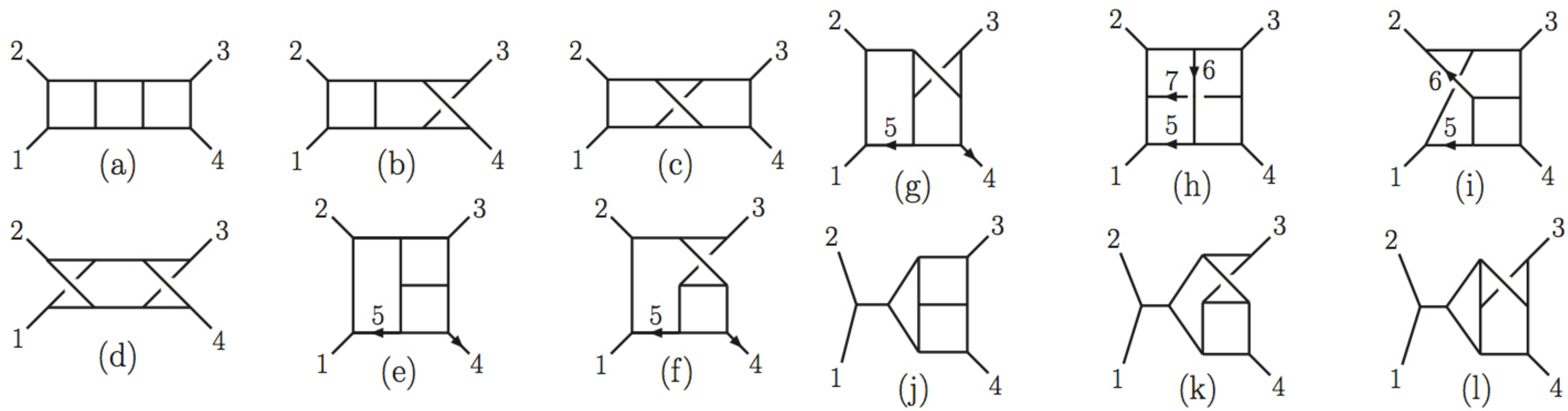
Bern, Dixon, Dunbar, Perelstein, Rozowski

- At 3 loops: use color/kinematics + squaring relation

• Color-kinematics duality provides a continuation of such squaring relations

Bern, Carrasco, Johansson

see JJ. Carrasco and H. Johansson's talks

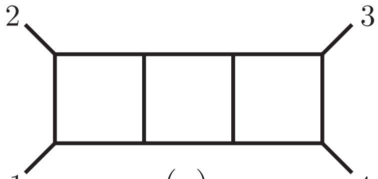


Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8}$ supergravity) numerator
(a)–(d)	s^2
(e)–(g)	$\big(s(-\tau_{35}+\tau_{45}+t)-t(\tau_{25}+\tau_{45})+u(\tau_{25}+\tau_{35})-s^2\big)/3$
(h)	$\big(s(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u)$ $+t(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17})+s^2\big)/3$
(i)	$\big(s(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t)$ $+t(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46})+u\,\tau_{25}+s^2\big)/3$
(j)-(l)	$s(t-u)/3$

• It has the same cuts as previously-derived expressions

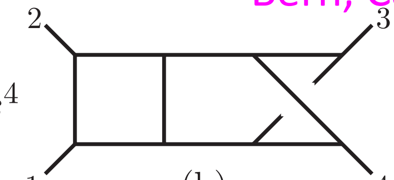
Bern, Carrasco, Dixon, Johansson, Kosower, RR
Bern, Carrasco, Dixon, Johansson, RR

s^4



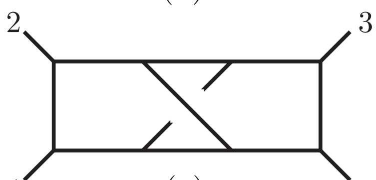
(a)

s^4



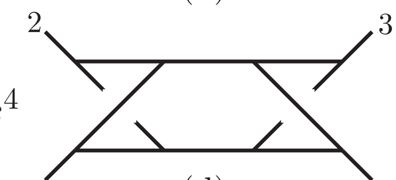
(b)

s^4



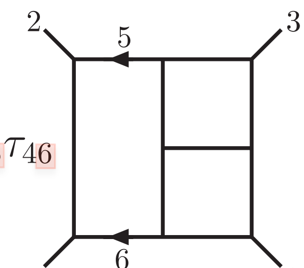
(c)

s^4



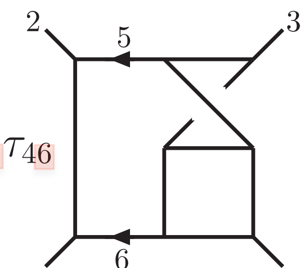
(d)

$s^2\tau_{35}\tau_{46}$



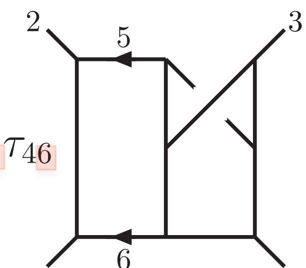
(e)

$s^2\tau_{35}\tau_{46}$



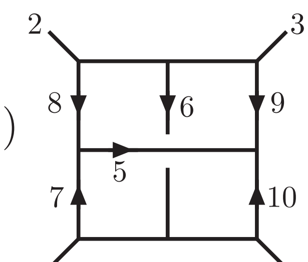
(f)

$s^2\tau_{35}\tau_{46}$



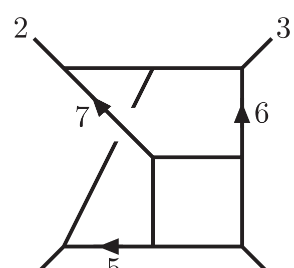
(g)

$$
\begin{aligned}
&(s(\tau_{26}+\tau_{36})+t(\tau_{15}+\tau_{25})+st)^2\\
&+(s^2(\tau_{26}+\tau_{36})-t^2(\tau_{15}+\tau_{25}))(\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10})\\
&+s^2(\tau_{17}\tau_{28}+\tau_{39}\tau_{4,10})+t^2(\tau_{28}\tau_{39}+\tau_{17}\tau_{4,10})\\
&+u^2(\tau_{17}\tau_{39}+\tau_{28}\tau_{4,10})
\end{aligned}
$$



(h)

$$
\begin{aligned}
&(s\tau_{45}-t\tau_{46})^2-\tau_{27}(s^2\tau_{45}+t^2\tau_{46})\\
&-\tau_{15}(s^2\tau_{47}+u^2\tau_{46})-\tau_{36}(t^2\tau_{47}+u^2\tau_{45})\\
&+l_5^2s^2t+l_6^2st^2-\tfrac{1}{3}l_7^2stu
\end{aligned}
$$



(i)

The 4-loop plan:

1. Construct 4-loop $\mathcal{N}=4$ sYM 4-point amplitude in BCJ form
see J.J. Carrasco's talk

2. Square numerators ⟶ candidate $\mathcal{N}=8$ amplitude
(address subtleties - certain sYM contributions "square" to zero: $\frac{0}{0}$ vs. $\frac{0^2}{0}$)

3. Check that it is indeed the $\mathcal{N}=8$ amplitude (it works out)

4. Analyze the result; extract UV divergences; etc

Some features:

Bern, Carrasco, Dixon, Johansson, RR (to appear)

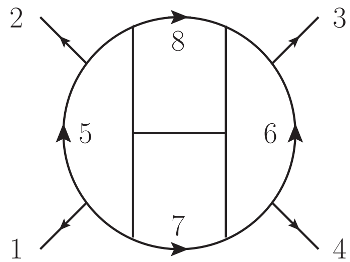
1. Expressed in terms of 82 integrals (85 in sYM; 3 "nilpotent" contrib's)
19 different numerators (up to signs)

2. Manifest power counting; finite in $d = 5$ by inspection

3. Similar to the 3-loop amplitude it contains 3-point sub-amplitudes

Three classes of terms; examples (a factor of $stuM_4^{(0)}$ is stripped off)

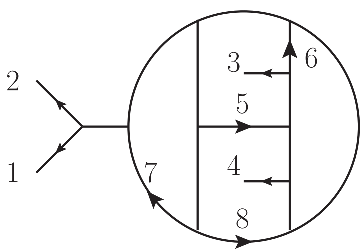
13-propagator integrals:



$$N_{12}^{\text{sYM}} = \tfrac{1}{2}s(s(\tau_{16} - \tau_{26} - \tau_{35} + \tau_{45} + 2\tau_{56} + 2t) - 2(4\tau_{16}\tau_{25} + 4\tau_{15}\tau_{26} + \tau_{45}(\tau_{36} - 3\tau_{46}) + \tau_{35}(\tau_{46} - 3\tau_{36})))$$

$$N_{12}^{\text{sugra}} = \Big[\tfrac{1}{2}s(s(\tau_{16} - \tau_{26} - \tau_{35} + \tau_{45} + 2\tau_{56} + 2t) - 2(4\tau_{16}\tau_{25} + 4\tau_{15}\tau_{26} + \tau_{45}(\tau_{36} - 3\tau_{46}) + \tau_{35}(\tau_{46} - 3\tau_{36})))\Big]^2$$

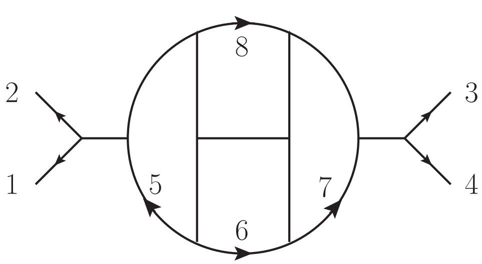
12-propagator integrals:



$$N_{66}^{\text{sYM}} = s(4t(\tau_{35} - 2\tau_{36}) + 2u(\tau_{35} + 3\tau_{45} - 4\tau_{46}) - s(6u + \tau_{15} - 6t + 5\tau_{25} - 8\tau_{26}))$$

$$N_{66}^{\text{sugra}} = \Big[s(4t(\tau_{35} - 2\tau_{36}) + 2u(\tau_{35} + 3\tau_{45} - 4\tau_{46}) - s(6u + \tau_{15} - 6t + 5\tau_{25} - 8\tau_{26}))\Big]^2$$

11-propagator integrals:



$$N_{80}^{\text{sYM}} = 16s^2(u - t)$$

$$N_{80}^{\text{sugra}} = \Big[16s^2(u - t)\Big]^2$$

- Note mechanism for manifest powercounting
- in sYM: only 11-propagator integrals are divergent in d=11/2

In supergravity, most integrals are divergent in d=11/2

- leaves open the possibility of additional magic

Extract UV divergences in d=11/2 (in general in dimensions with log divergences)

- expand at small external momenta
- use Lorentz-invariance to reorganize tensor integrals

2-tensors

$$l_i^{\mu_i} l_j^{\mu_j} \mapsto \frac{1}{d} \eta^{\mu_i \mu_j} l_i \cdot l_j$$

4-tensors

$$l_i^{\mu_i} l_j^{\mu_j} l_k^{\mu_k} l_l^{\mu_l} \mapsto \frac{1}{(d-1)d(d+2)} \left(A \eta^{\mu_i \mu_j} \eta^{\mu_k \mu_l} + B \eta^{\mu_i \mu_k} \eta^{\mu_j \mu_l} + C \eta^{\mu_i \mu_l} \eta^{\mu_j \mu_k} \right)$$

$$A = (d+1) l_i \cdot l_j \, l_k \cdot l_l - l_i \cdot l_k \, l_j \cdot l_l - l_i \cdot l_l \, l_j \cdot l_k$$

$$B = -l_i \cdot l_j \, l_k \cdot l_l + (d+1) l_i \cdot l_k \, l_j \cdot l_l - l_i \cdot l_l \, l_j \cdot l_k$$

$$C = -l_i \cdot l_j \, l_k \cdot l_l - l_i \cdot l_k \, l_j \cdot l_l + (d+1) l_i \cdot l_l \, l_j \cdot l_k$$

In supergravity, most integrals are divergent in d=11/2

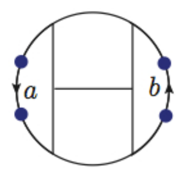
- leaves open the possibility of additional magic

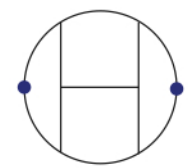
Extract UV divergences in d=11/2 (in general in dimension with 1st log divergence)

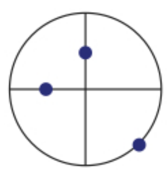
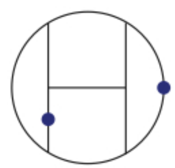
- expand at small external momenta
- use Lorentz-invariance to reorganize tensor integrals
- do the permutation sum
  - . external momentum dependence factorizes as

$$M_4^{(4)} \mapsto stu M_4^{(0)} (s^2 + t^2 + u^2)^2 (\text{vacuum integrals})$$

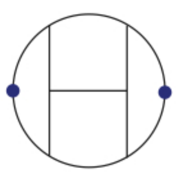
The examples:

$I_{12}^{\text{sugra}} \mapsto 4\,(s^2+t^2+u^2)^2 \left(\frac{d^3 - 19d^2 + 146d - 96}{(d-1)d(d+2)} \tau_{ab}^2 \right.$ 

$+\, 16 \frac{17d - 25}{(d-1)d(d+2)}$  $\left. \right)$

$I_{66}^{\text{sugra}} \mapsto \frac{16}{d} (s^2 + t^2 + u^2)^2\, (17$  $- 64$  $)$

$$\tau_{ab} = 2k_a \cdot k_b$$

$I_{80}^{\text{sugra}} \mapsto 1024\,(s^2 + t^2 + u^2)^2$ 

In supergravity, most integrals are divergent in d=11/2

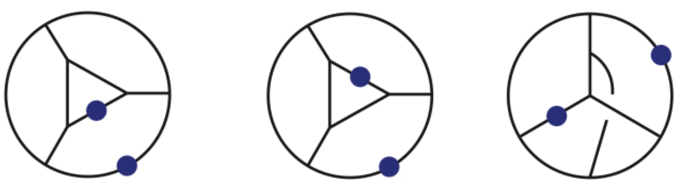
- leaves open the possibility of additional magic

Extract UV divergences in d=11/2 (in general in dimensions with log divergences)

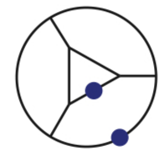
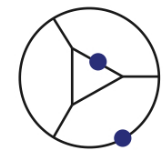
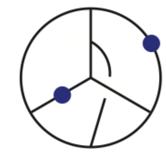
- expand at small external momenta
- use Lorentz-invariance to reorganize tensor integrals
- do the permutation sum
  - . external momentum dependence factorizes as

$$M_4^{(4)} \mapsto stu M_4^{(0)} (s^2 + t^2 + u^2)^2 (\text{vacuum integrals})$$

  - . some statistics: 69 vacuum graph topologies
    32 scalar integrals
    24 2-tensor integrals (τ_{ab} numerator)
    13 4-tensor integrals (τ_{ab}^2 numerator)
  - . Reducible (though Laporta algorithm) to only 3 vacuum scalar integrals



So the leading UV pole in d=11/2 is

$$\mathcal{M}_4^{(4)}\Big|_{\rm pole} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2\, M_4^{\rm tree}\,\Big($$  $$+\,2$$  $$+$$  $$\Big)$$

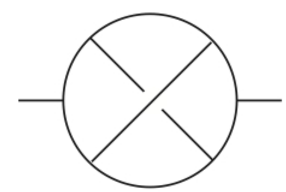
$$-256 + \frac{2025}{8}$$

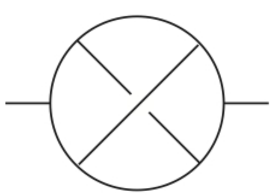
-256: 11-propagator integrals; same as in sYM

$\frac{2025}{8}$: 12- and 13-propagator integrals

and it evaluates to

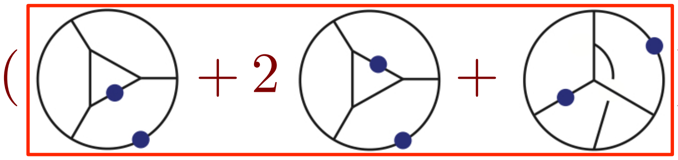
$$\mathcal{M}_4^{(4)}\Big|_{\rm pole} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2\, M_4^{\rm tree}$$

$$\times\Big(\frac{30208}{2625}\,\Gamma^4(\tfrac{3}{4}) - \frac{1024}{525}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4}) + \frac{4}{21\,\Gamma(\tfrac{3}{4})}$$  $$\Big)$$

 $$= -6.198399226750(2)$$

So the leading UV pole in d=11/2 is

$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2\, M_4^{\text{tree}}\,\Big($$



$$\Big)$$

$$-256 + \frac{2025}{8}$$

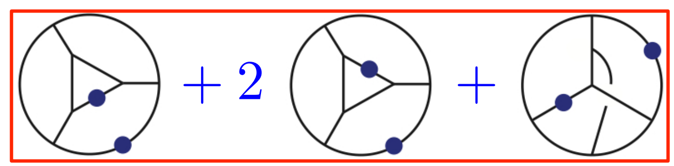
12- and 13-propagator integrals

11-propagator integrals; same as in sYM

As for comparison with the single-trace subleading color sYM

$$\mathcal{A}_4^{(4)}\Big|_{\text{pole}}^{SU(N_c)} = -6\, g^{10}\, \mathcal{K}\, N_c^2 \Big(N_c^2$$

$$+\,12\,\Big($$



$$\Big)\Big)$$

$$\times \Big(s\,(\text{Tr}_{1324}+\text{Tr}_{1423}) + t\,(\text{Tr}_{1243}+\text{Tr}_{1342}) + u\,(\text{Tr}_{1234}+\text{Tr}_{1432}) \Big)$$

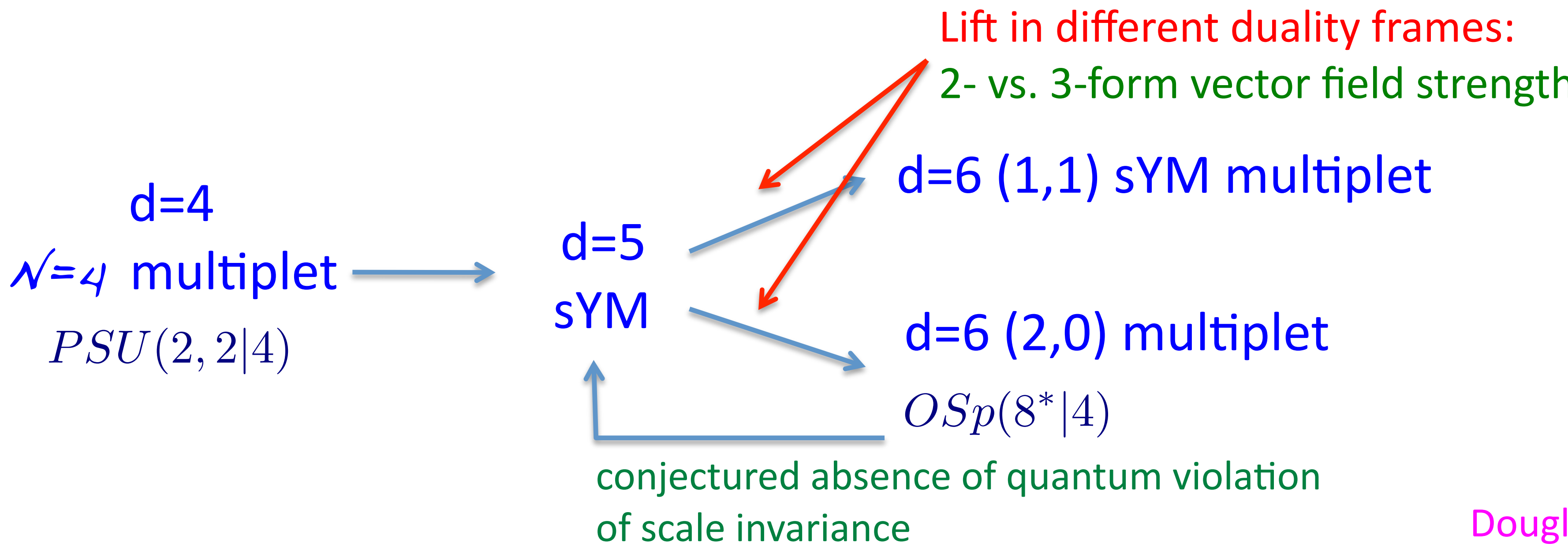
It seems unlikely that relation is a coincidence; its origin and implications however are not clear; may continue at higher loops

Supergravity in the UV: the status

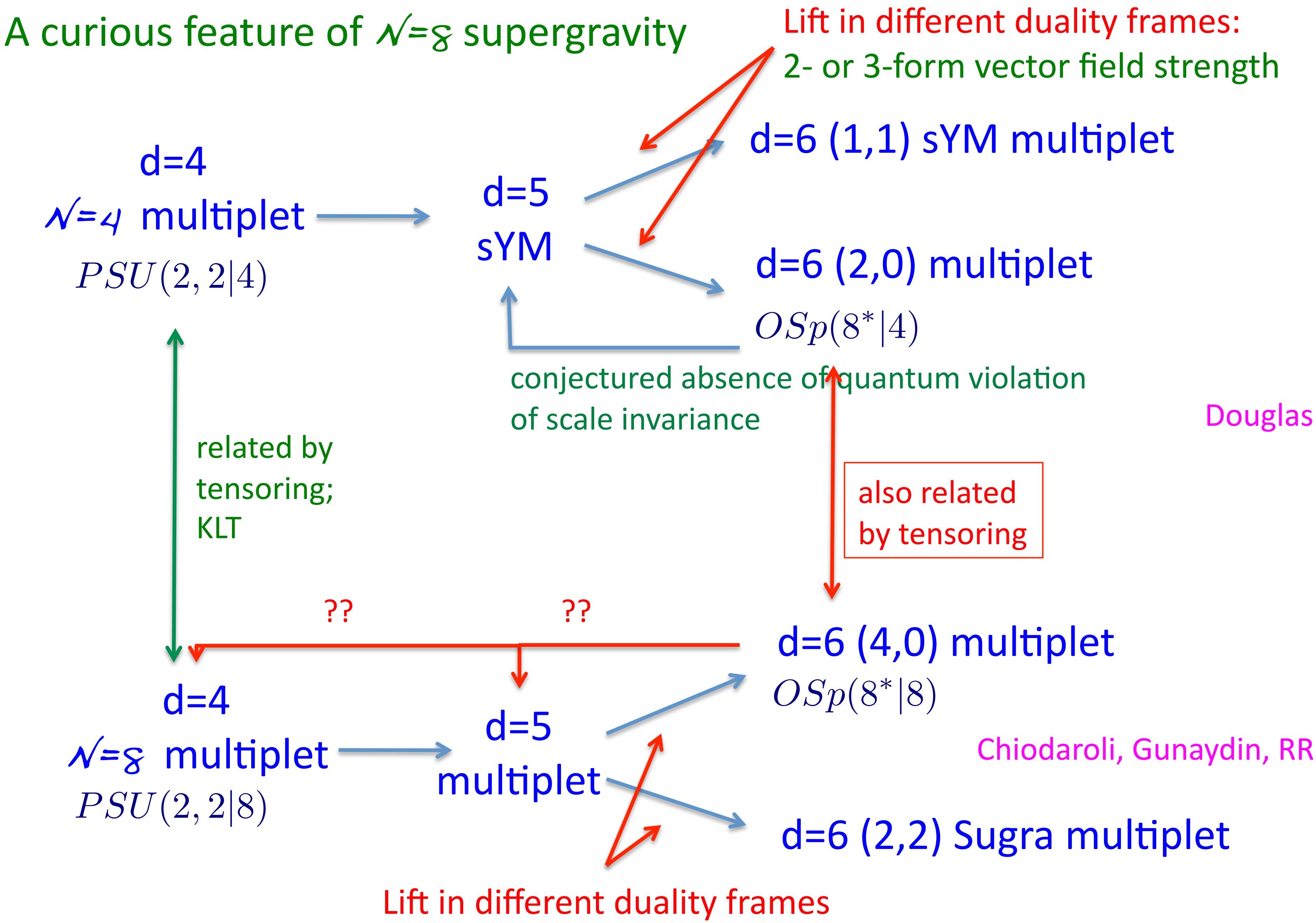
- Explicit calculations: $D_c = 4 + \frac{6}{L}$ for L=2,3,4
- Arguments based on fixed-order calculations for all-loop cancellations of certain dangerous terms — Bern, Dixon, RR
- String dualities reduced to d=4 suggest finite eff. act. — Green, Russo, Vanhove
  - Certain kind of string-inspired superspace
  - susy Ward identities + duality symmetry

  → potential L=7 d=4 divergence — Bjornson, Green; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger
- Power counting: L=7, d=4 divergence shows up in higher dimensions at lower loops: L=5 in d=5-1/5. A 5-loop calculation is needed to settle this issue — in progress: Bern, Carrasco, Dixon, Johansson, RR

A curious feature of $\mathcal{N}$=8 supergravity



Douglas


A curious feature of $\mathcal{N}$=8 supergravity
Lift in different duality frames:
2- or 3-form vector field strength
d=4
$\mathcal{N}$=4 multiplet
$PSU(2,2|4)$
d=5
sYM
d=6 (1,1) sYM multiplet
d=6 (2,0) multiplet
$OSp(8^*|4)$
conjectured absence of quantum violation
of scale invariance
Douglas
related by
tensoring;
KLT
also related
by tensoring
??
??
d=6 (4,0) multiplet
$OSp(8^*|8)$
d=4
$\mathcal{N}$=8 multiplet
$PSU(2,2|8)$
d=5
multiplet
Chiodaroli, Gunaydin, RR
d=6 (2,2) Sugra multiplet
Lift in different duality frames

Chiodaroli, Gunaydin, RR

	$h_{\hat{\gamma}\hat{\delta}}$	$\lambda^{\breve{a}}_{\hat{\gamma}}$	$\phi^{[\breve{a}\breve{b}]\vert}$
$h_{\hat{\alpha}\hat{\beta}}$	$R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}} \oplus \partial_{\hat{\alpha}(\hat{\gamma}} h_{\hat{\delta})\hat{\beta}} \oplus \partial_{\hat{\alpha}(\hat{\gamma}}\partial_{\hat{\delta})\hat{\beta}}\phi^0$	$\psi^{\breve{a}}_{\hat{\alpha}\hat{\beta}\hat{\gamma}} \oplus \partial_{\hat{\gamma}(\hat{\alpha}}\lambda^{\breve{a}}_{\hat{\beta})}$	$h^{[\breve{a}\breve{b}]\vert}_{\hat{\alpha}\hat{\beta}}$
$\lambda^a_{\hat{\alpha}}$	$\psi^a_{\hat{\alpha}\hat{\gamma}\hat{\delta}} \oplus \partial_{\hat{\alpha}(\hat{\gamma}}\lambda^a_{\hat{\delta})}$	$h^{a\breve{a}}_{\hat{\alpha}\hat{\gamma}} \oplus \partial_{\hat{\alpha}\hat{\gamma}}\phi^{a\breve{a}}$	$\lambda^{a[\breve{a}\breve{b}]\vert}_{\hat{\alpha}}$
$\phi^{[ab]\vert}$	$h^{[ab]\vert}_{\hat{\gamma}\hat{\delta}}$	$\lambda^{\breve{a}[ab]\vert}_{\hat{\gamma}}$	$\phi^{[ab]\vert[\breve{a}\breve{b}]\vert}$

Irreducible, positive energy representation of $OSp(8^*|8)$

$6D$ Field	$SU^*(4)_D$	$USp(8)$	$4D$ Decomposition
$\phi^{[ABCD]\vert}$	(0,0,0)	42	$\phi^{[ABCD]\vert}$
$\lambda^{[ABC]\vert}_{\hat{\alpha}}$	(1,0,0)	48	$\lambda^{[ABC]\vert}_{\alpha} \oplus \lambda^{[ABC]\vert}_{\dot{\alpha}}$
$h^{[AB]\vert}_{\hat{\alpha}\hat{\beta}}$	(2,0,0)	27	$h^{[AB]\vert}_{\alpha\beta} \oplus h^{[AB]\vert}_{\dot{\alpha}\dot{\beta}} \oplus \partial_{\alpha\dot{\beta}}\phi^{[AB]\vert}$
$\psi^A_{(\hat{\alpha}\hat{\beta}\hat{\gamma})}$	(3,0,0)	8	$\psi^A_{(\alpha\beta\gamma)} \oplus \psi^A_{(\dot{\alpha}\dot{\beta}\dot{\gamma})} \oplus \partial_{\dot{\gamma}(\alpha}\lambda^A_{\beta)} \oplus \partial_{\alpha(\dot{\beta}}\lambda^A_{\dot{\gamma})}$
$R_{(\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta})}$	(4,0,0)	1	$R_{(\alpha\beta\gamma\delta)} \oplus R_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})} \oplus \partial_{\dot{\delta}(\gamma}h^0_{\alpha\beta)} \oplus \partial_{\delta(\dot{\gamma}}h^0_{\dot{\alpha}\dot{\beta})} \oplus \partial_{\alpha(\dot{\gamma}}\partial_{\dot{\delta})\beta}\phi^0$

Comments

- Discussed advantages of manifest power-counting presentations of scattering amplitudes

- extracted the pole in d=11/2; confirmed critical dimension pattern

- result has unexpected features
  - transcendental part of residue is the same as the residue of the $\mathcal{N}=4$ $1/N^2$-suppressed single-trace terms

- pointed out a curious kinematic similarity between the lift of the $\mathcal{N}=4$ vector multiplet to d=6 and that of the $\mathcal{N}=8$ multiplet and the potential importance of the duality frame.
  Interesting to explore the existence of an interacting theory

see Y.-t Huang's talk

What makes the amplitudes of $\mathcal{N}=4$ sYM "simple"?

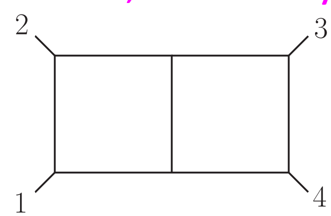
Non-planar amplitudes are simpler than what they could have been and, to some extent, related to their planar counterparts:

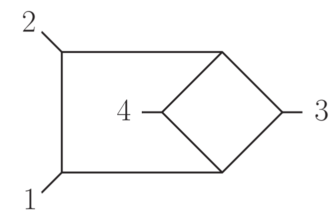
U(1) decoupling: • 1-loop sub-leading color i.t.o. leading color

→ combination of box integrals — Bern, Kosower

• parts of 2-loop 2-trace related to leading color

Bern, Rozowsky, Yan; Bern, de Freitas, Dixon

→ combination of 

and 

Higher loops: • 3 & 4 loops: 2-trace better in UV than rest

$D_c = 4 + \frac{6}{L}$ vs $D_c = 4 + \frac{8}{L}$ — Bern, Carrasco, Dixon, Johansson, RR

Color-kinematic duality; $c_i + c_j + c_k = 0 \leftrightarrow n_i + n_j + n_k = 0$

Bern, Carrasco, Johansson

→ (potential) all-order relations between l. and sub-l. color

→ simple and structured expressions

To have a glimpse at the origin of some of these properties…

Analyze QFT-s which share most of the properties of $\mathcal{N}=4$ sYM

→ Deform it in a controlled way

1. orbifolds

Inheritance principle: Bershadsky, Johansen
Bershadsky, Kakushadze, Vafa

$$\varphi_i^I = R^I{}_J\, g^{-1} \varphi_i^J\, g \quad R \in SU(4) \quad g \in SU(4) \subset SU(N)$$

2. the h deformation

Leigh, Strassler

$$W = \mathrm{Tr}[\Phi_1[\Phi_2,\Phi_3]] \longrightarrow f(h,N)(\mathrm{Tr}[\Phi_1[\Phi_2,\Phi_3]] + h(\mathrm{Tr}[\Phi_1^3] + \mathrm{Tr}[\Phi_2^3] + \mathrm{Tr}[\Phi_3^3]))$$

3. the β deformation

Leigh, Strassler

$$W = \mathrm{Tr}[\Phi_1[\Phi_2,\Phi_3]] \longrightarrow f(\beta,N)\mathrm{Tr}[\Phi_1(e^{i\beta}\Phi_2\Phi_3 - e^{-i\beta}\Phi_3\Phi_2)]$$

	super-conf.	dual super-conf.	planar integrable	Amp/W.L.
1.	yes; N=2, 1, 0	yes; inherited	yes	quite likely
2.	yes; N=1	not (well) known	sometimes	not clear
3.	yes; N=1, 0	yes	yes	yes

The supersymmetric β -deformed $\mathcal{N}=4$ super-Yang-Mills theory

- the same field content as $\mathcal{N}=4$ sYM
- real β : the same planar properties except for supersymmetry
- a pattern for the deformation:

Lunin, Maldacena

noncommutative deformation: $\varphi_I \varphi_J \;\mapsto\; e^{i\hat{\beta}_{ij} q_I^i q_J^j} \varphi_I \varphi_J$

$\mathcal{N}=4$ R-charge vectors

$$\hat{\beta}_{ij} = -\hat{\beta}_{ji} \qquad \hat{\beta}_{12} = \hat{\beta}_{23} = \hat{\beta}_{31} = \beta$$

	ϕ^{14}	ϕ^{24}	ϕ^{34}	A_μ	ψ^1	ψ^2	ψ^3	ψ^4	Q^1	Q^2	Q^3	Q^4
J_{12}	1	0	0	0	1/2	-1/2	-1/2	1/2	-1/2	1/2	1/2	-1/2
J_{34}	0	1	0	0	-1/2	1/2	-1/2	1/2	1/2	-1/2	1/2	-1/2
J_{56}	0	0	1	0	-1/2	-1/2	1/2	1/2	1/2	1/2	-1/2	-1/2

Some consequences:

- most non-commutative results survive; planar amplitudes are inherited in dimensional regularization

Filk (space-time noncommutativity); Khoze; ...

- vector U(1) factors decouple; chiral superfield U(1) factors are coupled
- both f_{abc} and d_{abc} couplings

• non-vanishing tree-level double-trace amplitudes

$$\mathcal{L}_{2\text{tr}} = \frac{1}{2N}|f(\beta,N)^2|\epsilon_{ijk}\epsilon^{ilm}\text{Tr}[[\phi^j,\,\phi^k]_\beta]\text{Tr}[[\bar\phi_l,\,\bar\phi_m]_\beta]$$

→ crucial for finiteness; also $|f(\beta,N)|^2 = \dfrac{g^2_{YM}}{1-\frac{4}{N^2}\sin^2\beta}$

With the same planar properties, differences are creeping in at subleading color level in dimensional regularization

• Single-trace amplitudes:

$$A^{(0)} = \sum_{\rho\in S_n/Z_n} \text{Tr}[T^{a_{\rho(1)}}\ldots T^{a_{\rho(n)}}]A^{(0)}(k_{\rho(1)}\ldots k_{\rho(n)})$$

⬇

$$A^{(0)}(k_1\ldots k_n) \mapsto e^{i\Theta(1,\ldots,n)}A^{(0)}(k_1\ldots k_n);\ \Theta(1,\ldots,n) = \sum_{1\le i<j\le n} q_i\cdot\hat\beta\cdot q_j$$

- Account for the $\mathcal{O}(1/N^2)$ deformation of the coefficient of the superpotential

Here: focus on double-trace terms; ignore $\mathcal{O}(1/N^2)$ corrections

- Modified color-kinematic-like (BCJ) duality:

$$\mathcal{A}_4^{\beta,(0)}(1g^+, 2\phi^{23}, 3f^{134}, 4f^{124}) = \frac{n_{12}}{s_{12}} f^{12a} f_\beta{}^{34}{}_a + \frac{n_{23}}{s_{23}} f_\beta{}^{23a} f^{14}{}_a + \frac{n_{13}}{s_{13}} f^{31a} f_\beta{}^{24}{}_a$$

$$\mathcal{A}_4^{\beta,(0)}(1\phi^{23}, 2\phi^{14}, 3\phi^{13}, 4\phi^{24}) = \frac{n_{12}}{s_{12}} f^{12a} f^{34}{}_a + \frac{n_{23}}{s_{23}} f^{23a} f^{14}{}_a + \frac{n_{13}}{s_{13}} f_\beta{}^{31a} f_\beta{}^{24}{}_a$$

$$f_\beta^{abc} = \mathrm{Tr}[T^a[T^b, T^c]_\beta] = e^{i\Phi(a,b,c)}\mathrm{Tr}[T^aT^bT^c] - e^{i\Phi(a,c,b)}\mathrm{Tr}[T^aT^cT^b]$$

Numerator factors -- same as in $\mathcal{N}=4$ sYM:

$$n_{12} + n_{23} + n_{13} = 0$$

Color factors – different; generically no Jacobi identity:

$$f^{[12}{}_a f^{3]4a} = 0 \qquad f^{[12}{}_a d^{3]4a} = 0$$

but no Jacobi-like identity involving only d-structure constants

Some explicit examples 4-point loop amplitudes: 1 loop

Jin, RR

- Construct using generalized unitarity
  - use color-dressed cuts
  - supersums: use pictorial rules (Bern, Carrasco, Ita, Johansson, RR)
    dressed with the extra phase factors (structure hints at hidden susy)
  - focus on 3 terms: $\mathrm{Tr}[T^{a_1}T^{a_i}]\mathrm{Tr}[T^{a_j}T^{a_k}] \quad i,j,k=2,3,4$

- Classify following the number of vector multiplets
  - 4 vector multiplets: same as in $\mathcal{N}=4$ sYM
  - 3 vector multiplets + 1 chiral multiplet: vanish identically
  - 2 vector multiplets + 2 chiral multiplets: $A(1_{g^+}, 2_{g^-}, 3_{\phi^{34}}, 4_{\phi^{12}})$

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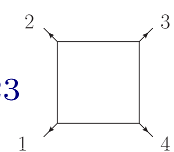
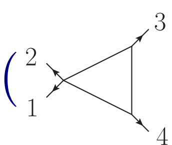
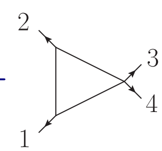
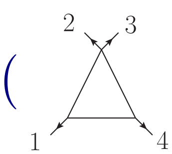
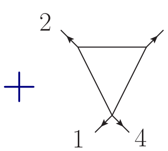
$$A(1234)_4^{(1)\beta} = A(1234)_4^{(1)\mathcal{N}=4} - 8\sin^2\beta\,(\mathrm{Tr}_{13}\mathrm{Tr}_{24} + \mathrm{Tr}_{14}\mathrm{Tr}_{23})A(1234)_{4;3}^{(1)\mathrm{extra}}$$

$$A(1234)_{4;3}^{(1)\mathrm{extra}} = \frac{\langle 23\rangle^2}{\langle 13\rangle^2}\Big[-s_{12}s_{23}$$


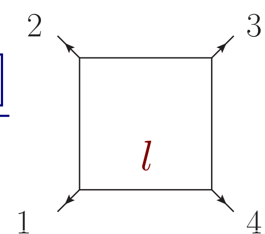
$$+ s_{12}\Big($$


$$+$$


$$\Big) + s_{23}\Big($$


$$+$$


$$\Big)\Big]$$

$$= -\frac{\langle 23\rangle^2}{\langle 13\rangle^2}\,\frac{G[l,1,2,3]}{s_{12}s_{23}}$$


- IR finite
- in d=4, expressible in terms of the d=6 box integral
- UV divergent in 6 dimensions; standard expectation for a conformal $\mathcal{N}=1$ theory

Some explicit examples 4-point loop amplitudes: 1 loop

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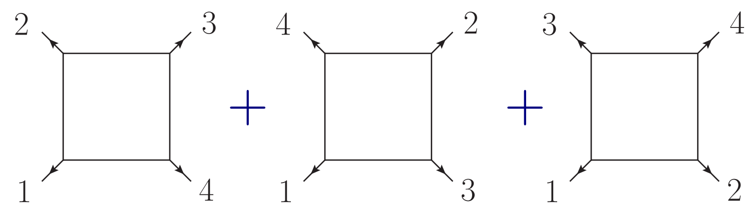
  - 2 vector multiplets + 2 chiral multiplets: $A(1_{g^+}, 2_{g^-}, 3_{\phi^{34}}, 4_{\phi^{12}})$
  - 1 vector multiplet + 3 chiral multiplets: $A(1_{g^-}, 2_{\phi^{34}}, 3_{\psi^1}, 4_{\psi^2})$

$$A^{(1)2\mathrm{tr}}(1234) = \cos\beta\, A^{(1)2\mathrm{tr}}_{\mathcal{N}=4}(1234)\big(\mathrm{Tr}_{12}\mathrm{Tr}_{34}+\mathrm{Tr}_{13}\mathrm{Tr}_{24}+\mathrm{Tr}_{14}\mathrm{Tr}_{23}\big)$$

$$A^{(1)2\mathrm{tr}}_{\mathcal{N}=4} = -2\, s_{12}s_{23}\,\frac{[23][34]}{[12][13]}\,\Big($$



$$\Big)$$

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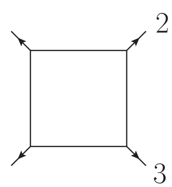
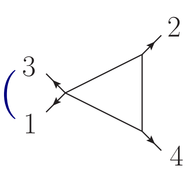
  - 1 vector multiplet + 3 chiral multiplets: $A(1_{g^-},2_{\phi^{34}},3_{\psi^1},4_{\psi^2})$
  - 4 chiral multiplets: $A(1_{\psi^1},2_{\psi^1},3_{\psi_1},4_{\psi_1})$, $A(1_{\psi^1},2_{\psi^2},3_{\psi_1},4_{\psi_2})$

$$A(1_{\psi^1},2_{\psi^1},3_{\psi_1},4_{\psi_1})_4^{(1)\beta} = A(1_{\psi^1},2_{\psi^1},3_{\psi_1},4_{\psi_1})_4^{(1)\mathcal{N}=4} - 8\sin^2\beta\,\mathrm{Tr}_{12}\mathrm{Tr}_{34}A(1234)^{(1)\mathrm{extra}}$$

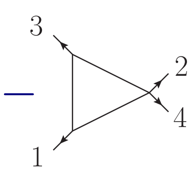
$$\frac{A(1234)^{(1)\mathrm{extra}}}{\cos^2\beta} = \frac{\langle 34\rangle}{\langle 12\rangle}\Bigg[s_{13}s_{14}$$



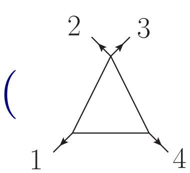
$$- s_{13}\Big($$



$$-$$



$$\Big) - s_{14}\Big($$



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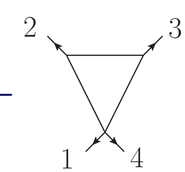
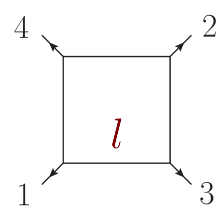


$$\Big)\Bigg]$$

$$= -\frac{\langle 34\rangle}{\langle 12\rangle}\,\frac{G[l,1,4,2]}{s_{13}s_{14}}$$



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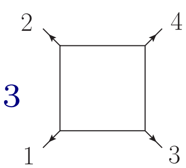
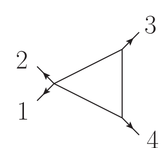
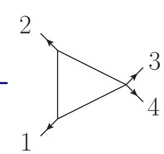
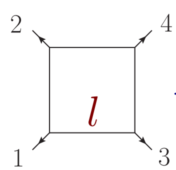
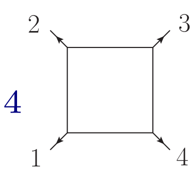
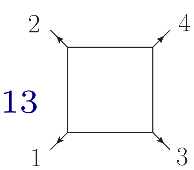
  - 1 vector multiplet + 3 chiral multiplets: $A(1_{g^-}, 2_{\phi^{34}}, 3_{\psi^1}, 4_{\psi^2})$
  - 4 chiral multiplets: $A(1_{\psi^1}, 2_{\psi^1}, 3_{\psi_1}, 4_{\psi_1})$, $A(1_{\psi^1}, 2_{\psi^2}, 3_{\psi_1}, 4_{\psi_2})$

$$A(1_{\psi^1}, 2_{\psi^2}, 3_{\psi_1}, 4_{\psi_2})_4^{(1)\beta} = A_4^{(1)\mathcal{N}=4} - 8\sin^2\beta\, \mathrm{Tr}_{12}\mathrm{Tr}_{34} A(1234)^{(1)\mathrm{extra}}_{12;34} - 8\sin^2\beta\, \mathrm{Tr}_{14}\mathrm{Tr}_{23} A(1234)^{(1)\mathrm{extra}}_{14;23}$$

$$A(1234)^{(1)\mathrm{extra}}_{12;34} = \cos^2\beta \frac{\langle 34\rangle}{\langle 12\rangle}\left(\frac{1}{2}s_{12}s_{13}\right.$$

$$- s_{12}\Big($$

$$-$$

$$\left.\Big)\right)$$

$$A(1234)^{(1)\mathrm{extra}}_{14;23} = \frac{\langle 34\rangle}{\langle 12\rangle}\left[\cos^2\beta \frac{G[l,1,2,4]}{2s_{12}s_{13}}\right.$$

$$+ s_{12}s_{14}$$

$$- s_{12}s_{13}$$

$$\left.\right]$$

Some comments:

- results consistent with expected structure of IR divergences
  - -- most corrections are in fact IR-finite; consistent with structure of IR div's
  - -- only small changes in the soft anomalous dimension matrix
- no real improvement over a finite "garden variety" $\mathcal{N}=1$ theory
  - -- except perhaps absence of incomplete cancellations (of bubbles)
- some details are as if there were more than $\mathcal{N}=1$ susy
  - -- supersums are perfect squares, characteristic to $\mathcal{N}=2$
  - -- persists at higher loops
- no immediate manifestation of modified color-kinematics relations

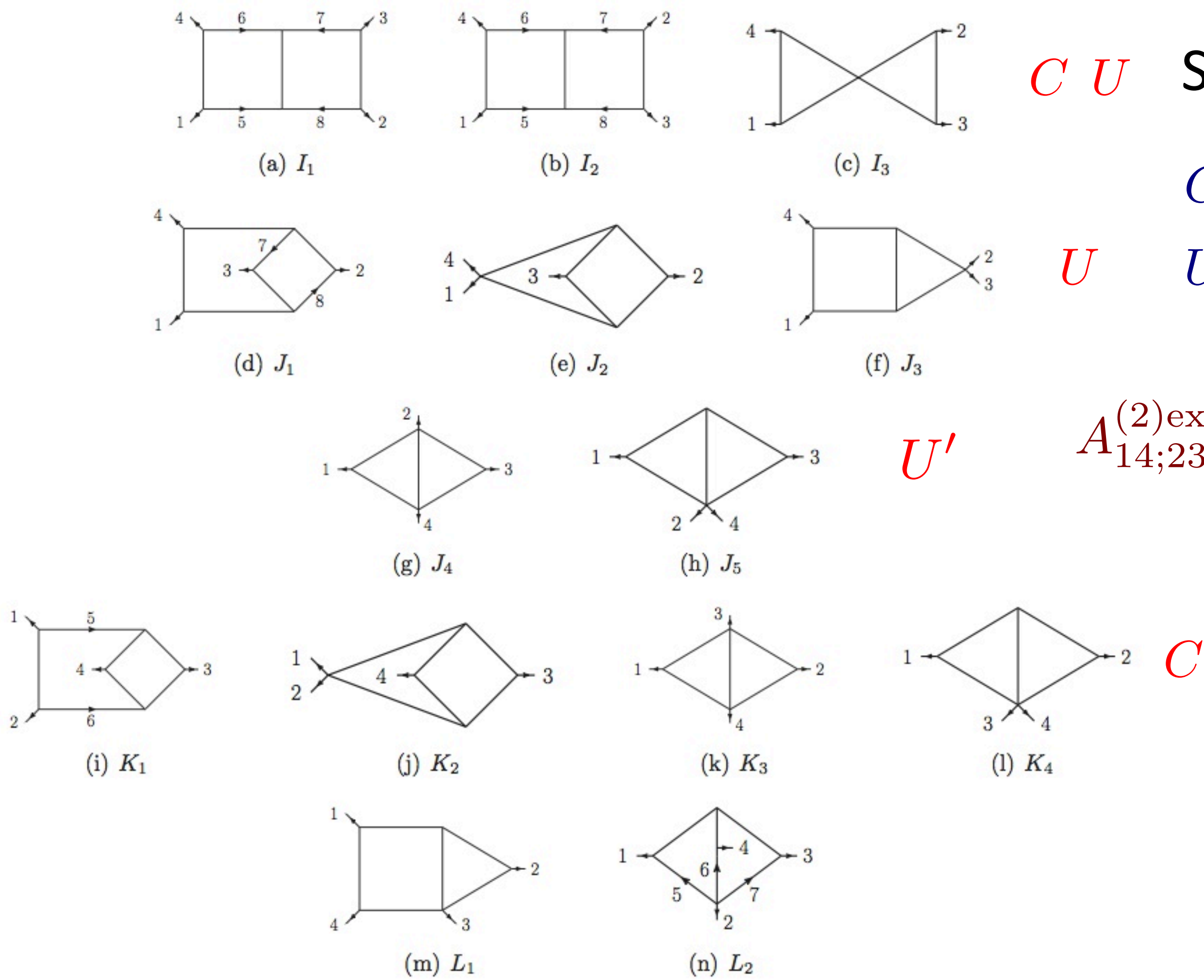
More explicit examples 4-point loop amplitudes: 2 loops

Jin, RR

Same classification:

- 2 vector multiplets + 2 chiral multiplets: $A(1_{g^+}, 2_{g^-}, 3_{\phi^{34}}, 4_{\phi^{12}})$

$$A(1234)^{(2)\beta}_{4;\,2\mathrm{tr}} = A(1234)^{(2)\mathcal{N}=4}_{4;\,2\mathrm{tr}} - 8\sin^2\beta\,\mathrm{Tr}_{13}\mathrm{Tr}_{24}A^{(2)\mathrm{extra}}_{13;24} - 8\sin^2\beta\,\mathrm{Tr}_{14}\mathrm{Tr}_{23}A^{(2)\mathrm{extra}}_{14;23}$$

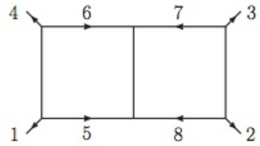


Symmetries of $A^{(2)\mathrm{extra}}_{14;23}$

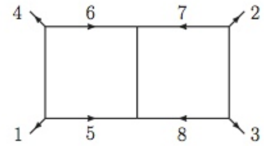
$C : (1 \leftrightarrow 2, 3 \leftrightarrow 4)$

$U : (1 \leftrightarrow 4, 2 \leftrightarrow 3)$

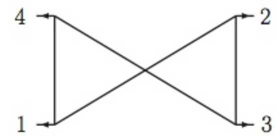
$$A^{(2)\mathrm{extra}}_{14;23} = \sum_i \alpha_i I_i + (1+C)\sum_i \beta_i J_i + (1+U)\sum_i \gamma_i K_i + (1+U)(1+C)\sum_i \delta_i L_i$$



(a) I_1



(b) I_2

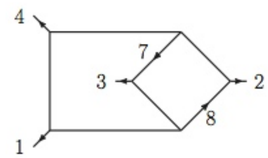


(c) I_3

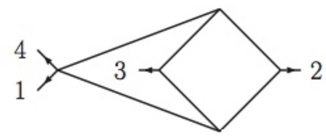
$$\alpha_1 = \tau_{1,4}\left(\tau_{1,8}^2 + \tau_{2,5}^2 + \tau_{4,7}^2 + \tau_{3,6}^2 + \tau_{1,2}(\tau_{1,8} + \tau_{2,5} + \tau_{4,7} + \tau_{3,6} - 2\tau_{1,4}) - \tau_{1,8}\tau_{2,5} - \tau_{4,7}\tau_{3,6}\right)$$

$$\alpha_2 = \tau_{1,4}\left(\tau_{1,8}^2 + \tau_{2,6}^2 + \tau_{4,7}^2 + \tau_{3,5}^2 + \tau_{1,2}(\tau_{1,8} + \tau_{2,6} + \tau_{4,7} + \tau_{3,5}) + \tau_{1,8}\tau_{2,6} + \tau_{4,7}\tau_{3,5}\right)$$

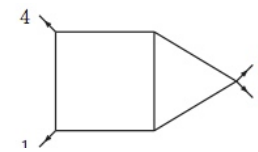
$$\alpha_3 = -4\tau_{1,3}\tau_{1,4}$$



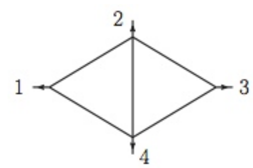
(d) J_1



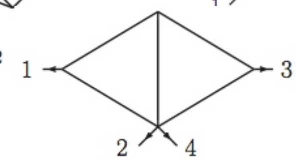
(e) J_2



(f) J_3



(g) J_4

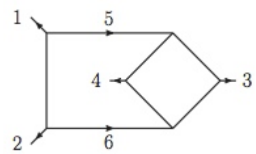


(h) J_5

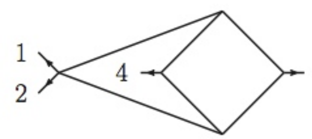
$$\beta_1 = 2\tau_{1,4}\left(\tau_{1,8}^2 + \tau_{4,7}^2 + \tau_{1,3}(\tau_{1,8} + \tau_{4,7})\right)$$

$$\beta_2 = -2\tau_{1,4}^2 \qquad \beta_3 = 2\tau_{1,4}^2$$

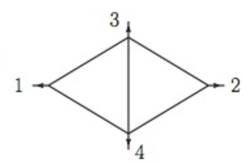
$$\beta_4 = 2\tau_{1,2} \qquad \beta_5 = (\tau_{1,4} - 2\tau_{1,2})$$



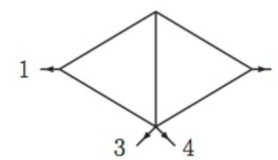
(i) K_1



(j) K_2



(k) K_3

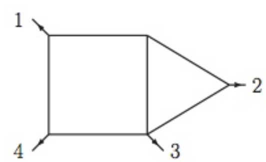


(l) K_4

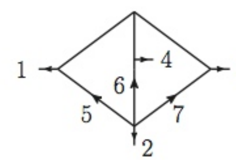
$$\gamma_1 = \tau_{1,2}\left(\tau_{1,5}^2 + \tau_{2,6}^2 + \tau_{1,2}(\tau_{1,5} + \tau_{2,6})\right)$$

$$\gamma_2 = 2\tau_{1,2}^2 \qquad \gamma_3 = 2\tau_{1,2} \qquad \gamma_4 = (\tau_{1,3} - \tau_{1,2})$$

$$\delta_1 = \tau_{1,4}^2 \qquad \delta_2 = -\tau_{1,4}\tau_{2,5} + \frac{1}{2}\tau_{12}(3\tau_{1,3} + \tau_{2,5} - 2\tau_{3,5} + 2\tau_{2,7})$$



(m) L_1

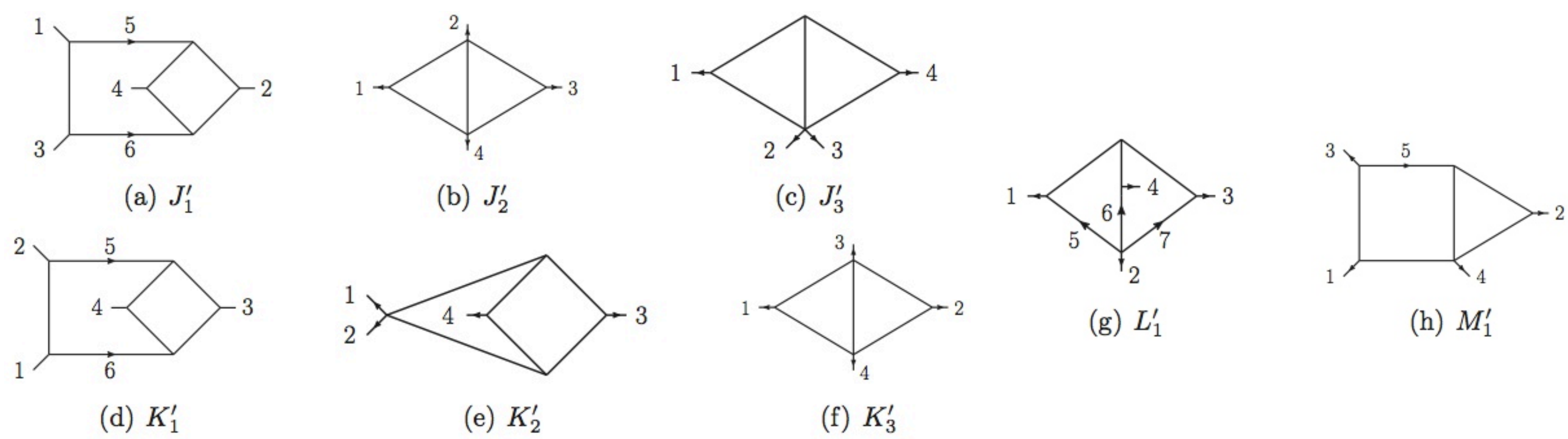


(n) L_2

- The other trace structure $A_{13;24}^{(2)\text{extra}}$: similar structure with $A_{14;23}^{(2)\text{extra}}$ with a few twists
  - planar double-boxes are absent
  - additional symmetries:

$$C : (1 \leftrightarrow 2, 3 \leftrightarrow 4)$$
$$U : (1 \leftrightarrow 3, 2 \leftrightarrow 4)$$
$$E : (1 \leftrightarrow 3)$$

$$A_{13;24}^{(2)\text{extra}} = (1+C)\sum_i \beta_i J_i' + (1+U)\sum_i \gamma_i K_i' + (1+U)(1+C)\sum_i \delta_i L_i' + (1+U)(1+C)(1+E)\sum_i \epsilon_i M_i'$$



More comments

- Despite extensive planar similarity with $\mathcal{N}=4$ sYM and tree-level numerator relations, the β-deformed theory does not show much simplicity at the non-planar level

- β-deformed theory seems to exhibit all the features of a typical finite $\mathcal{N}=1$ theory

- Calculations suggest that details of the theory are crucial for transferring planar info to non-planar level

- Disentangle effects of reduced supersymmetry and d_{abc}?
  (d_{abc} is also an obstruction for constructing a gravity theory via KLT relations)

Recap

- constructed one for $\mathcal{N}=8$
- extracted the pole in d=11/2; divergence is indeed present there
- result has unexpected features
    - transcendental part of residue is the same as the residue of the $\mathcal{N}=4$ $1/N^2$ -suppressed single-trace terms

- a 5-loop calculation will test these observations as well as the 7-loop UV behavior of $\mathcal{N}=8$ supergravity in d=4

- pointed out a curious kinematic similarity between the lift of the $\mathcal{N}=4$ vector multiplet to d=6 and that of the $\mathcal{N}=8$ multiplet and the potential importance of the duality frame

- In theories with several multiplets, the details of the interactions between multiplets are crucial for transfering planar simplicity to non-planar level via color/kinematics duality

- a better picture: disentangle effects of d_{abc}

Extra slides

Expect similar features at 4 loops

The plan:

1. Construct 4-loop N=4 sYM 4-point amplitude in BCJ form

   stay tuned for J.J. Carrasco's talk

2. Square numerators ⟶ candidate N=8 amplitude (address subtleties)

3. Check that it is indeed the N=8 amplitude (it works out)

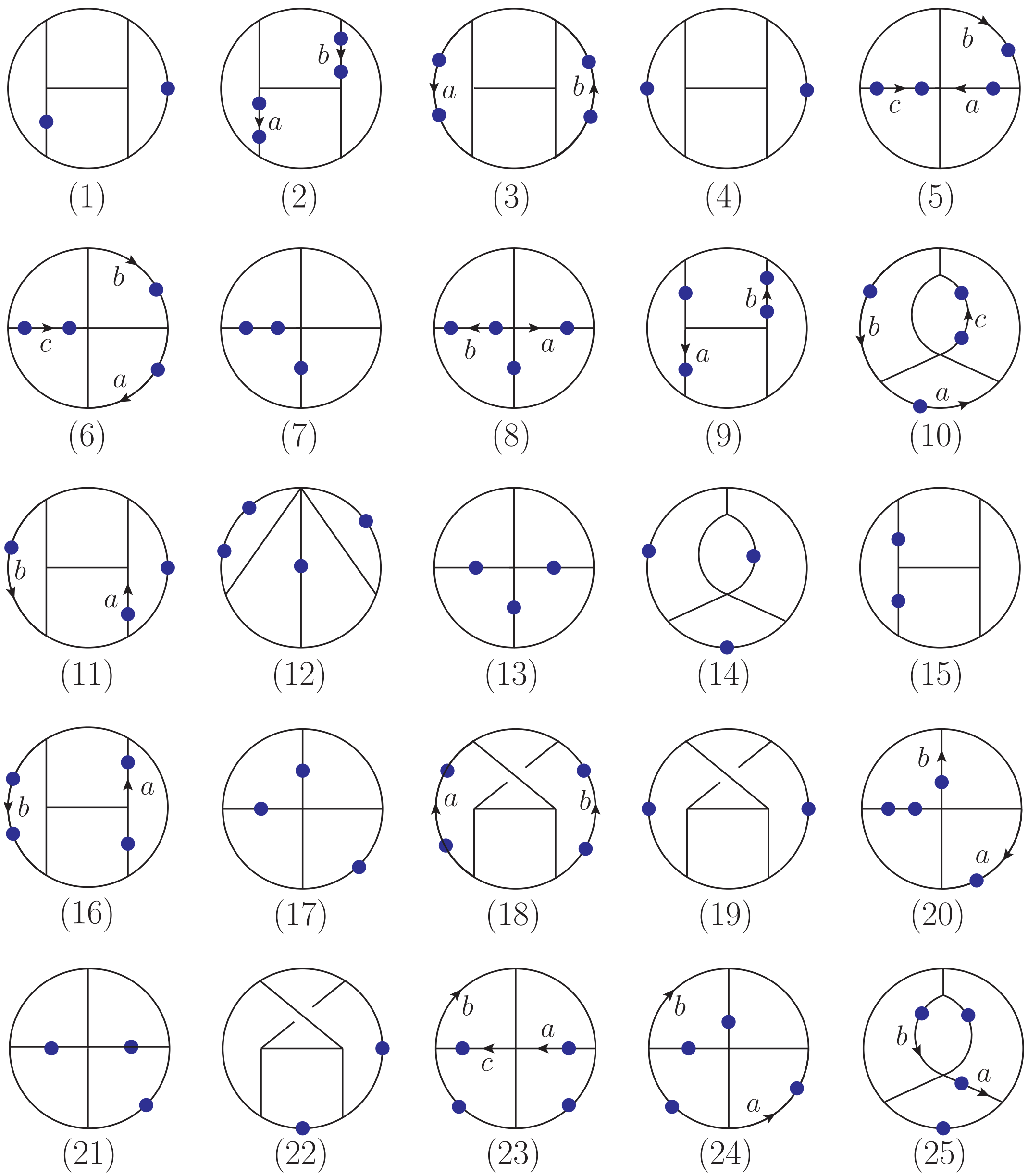
4. Analyze the result; extract UV divergences; etc

To anticipate:

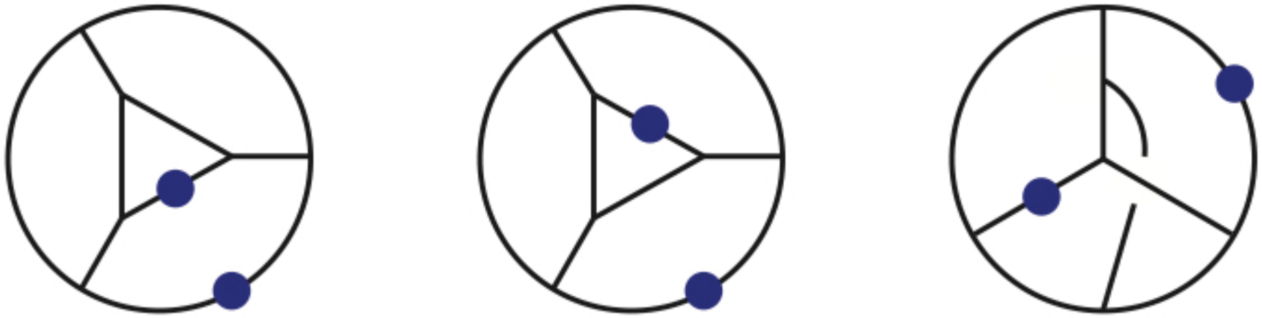
1. extensive cancellations between individual diagrams despite apparent perfect square integrands (Minkowski signature is important)

2. approx. 69 vac. int's irreducible through momentum conservation

3. 3 master integrals



I^v	Effective numerator	V_1	V_2	V_8
I_1^v	$-\frac{117674}{1485}$	0	$-\frac{117674}{1485}$	0
I_2^v	$\frac{19112}{1485}\ \tau_{a,b}^2$	$\frac{8798687}{5346000}$	$\frac{212621}{27000}$	0
I_3^v	$\frac{9556}{1485}\ \tau_{a,b}^2$	$\frac{15937019}{1782000}$	$-\frac{140951}{33000}$	0
I_4^v	$-\frac{16427}{495}$	$-\frac{16427}{495}$	0	0
I_5^v	$\frac{19112}{1485}\ \tau_{a,c}-\frac{19112}{1485}\ \tau_{b,c}$	$-\frac{2389}{2970}$	$-\frac{2389}{1485}$	0
I_6^v	$-\frac{4778}{495}\ \tau_{a,c}+\frac{4778}{1485}\ \tau_{b,c}$	$\frac{16723}{2970}$	$-\frac{4778}{1485}$	0
I_7^v	$-\frac{9556}{1485}$	$\frac{109894}{7425}$	$\frac{90782}{2475}$	0
I_8^v	$\frac{38224}{1485}\ \tau_{a,b}$	$-\frac{2389}{675}$	$-\frac{19112}{2475}$	0
I_9^v	$\frac{38224}{1485}\ \tau_{a,b}^2$	$-\frac{1617353}{148500}$	$\frac{2606399}{74250}$	0
I_{10}^v	$-\frac{19112}{1485}\ \tau_{a,c}-\frac{19112}{1485}\ \tau_{b,c}$	$-\frac{2389}{2970}$	$-\frac{2389}{1485}$	0
I_{11}^v	$-\frac{38224}{1485}\ \tau_{a,b}$	$\frac{90782}{22275}$	$-\frac{9556}{825}$	0
I_{12}^v	$-\frac{19112}{1485}$	$\frac{31057}{990}$	$\frac{38224}{495}$	0
I_{13}^v	$\frac{10048}{99}$	$\frac{2512}{99}$	$\frac{10048}{99}$	0
I_{14}^v	$-\frac{19112}{1485}$	$-\frac{4778}{275}$	$-\frac{324904}{7425}$	0
I_{15}^v	$\frac{19112}{1485}$	$\frac{66892}{4455}$	$\frac{19112}{495}$	0
I_{16}^v	$\frac{19112}{1485}\ \tau_{a,b}^2$	$\frac{977101}{267300}$	$\frac{88393}{14850}$	0
I_{17}^v	$\frac{39676}{1485}$	$\frac{9919}{495}$	$\frac{19838}{1485}$	0
I_{18}^v	$\frac{9556}{1485}\ \tau_{a,b}^2$	$-\frac{1478791}{297000}$	$\frac{661753}{148500}$	$\frac{2389}{396}$
I_{19}^v	$-\frac{64441}{1485}$	0	0	$-\frac{64441}{1485}$
I_{20}^v	$\frac{38224}{1485}\ \tau_{a,b}$	$-\frac{102727}{14850}$	$-\frac{74059}{7425}$	0
I_{21}^v	$\frac{5284}{1485}$	$\frac{18494}{7425}$	$\frac{34346}{7425}$	0
I_{22}^v	$\frac{934}{165}$	$\frac{467}{165}$	$\frac{1868}{165}$	$-\frac{934}{165}$
I_{23}^v	$\frac{526}{135}\ \tau_{a,b}-\frac{91}{1485}\ \tau_{a,c}$	$\frac{279199}{297000}$	$\frac{72052}{37125}$	0
I_{24}^v	$\frac{3736}{495}\ \tau_{a,b}$	$\frac{26152}{12375}$	$-\frac{91532}{12375}$	0
I_{25}^v	$-\frac{9556}{1485}\ \tau_{a,b}$	$-\frac{2389}{2475}$	$-\frac{16723}{7425}$	0



V_1 V_2 V_8

$$V_1 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[\frac{512}{5}\,\Gamma^4(\tfrac{3}{4}) - \frac{2048}{105}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4})\right] + \mathcal{O}(1)$$

$$V_2 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[-\frac{4352}{105}\,\Gamma^4(\tfrac{3}{4}) + \frac{832}{105}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4})\right] + \mathcal{O}(1)$$

$$V_8 = \frac{1}{(4\pi)^{11}}\frac{4}{21}\frac{1}{\Gamma(\frac{3}{4})}\frac{1}{\epsilon}\left[-\frac{5248}{125}\,\Gamma^5(\tfrac{3}{4}) + \frac{224}{25}\,\Gamma^4(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4}) + 2\,\mathrm{NO}_m\right] + \mathcal{O}(1)$$

Nonplanar amplitudes of $\mathcal{N} = 4$ super-Yang-Mills theory are not directly constrained properties of planar amplitudes:

- dual super-conformal invariance
- integrablity of planar dilatation operator
- amplitudes / Wilson loops relation
- amplitudes / correlation function relation

$$\beta_1 = 2\tau_{1,3}\left(\tau_{2,5}^2 + \tau_{4,6}^2 - \tau_{1,3}(\tau_{2,5} + \tau_{4,6})\right)$$
$$\beta_2 = -2\tau_{1,2}$$
$$\beta_3 = 2\tau_{1,4} - 2\tau_{1,3}$$

$$\gamma_1 = \tau_{1,2}\left(\tau_{4,5}^2 + \tau_{3,6}^2 + \tau_{1,3}(\tau_{4,5} + \tau_{3,6})\right)$$
$$\gamma_2 = \tau_{1,2}^2$$
$$\gamma_3 = 2\tau_{1,3}$$

$$\delta_1 = \frac{1}{2}\tau_{1,4}(2\tau_{1,7} + \tau_{2,7} + \tau_{1,4}) + \frac{1}{2}\tau_{1,2}(4\tau_{1,4} - 3\tau_{1,2} + 2\tau_{4,7} + \tau_{2,5})$$

$$\epsilon_1 = 2\tau_{1,3}\tau_{2,5}$$