Surprises and Anomalies in Heavy Quarkonium Production

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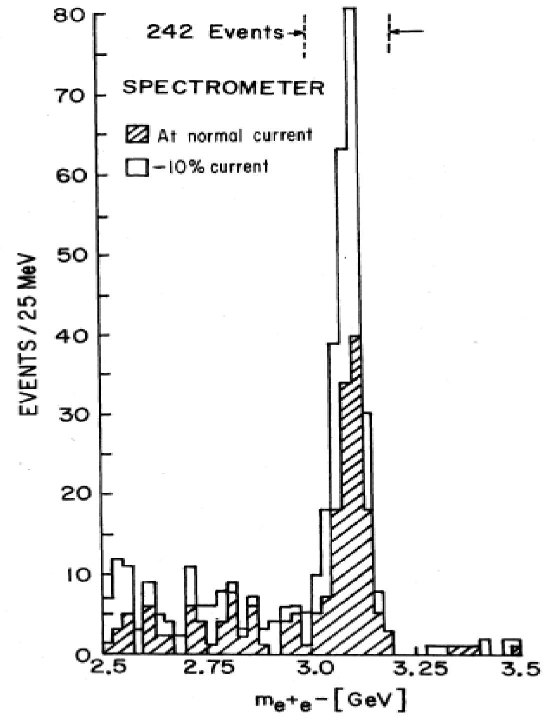
Based on work done with Kang, Nayak, Sterman, and …

INT Program (INT-11-3): Frontiers in QCD
Institute for Nuclear Theory, University of Washington
Seattle, WA, September 19 - November 18, 2011

November revolution (1974)



VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 19

Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu

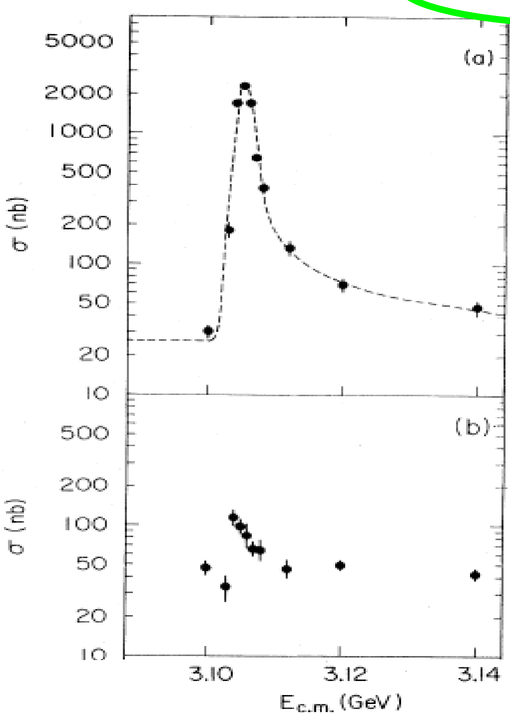
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee

Brookhaven National Laboratory, Upton, New York 11973

(Received 12 November 1974)



November, 1974

Discovery of a Narrow Resonance in e^+e^- Annihilation*

J.-E. Augustin,† A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie,† R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, and F. Vannucci‡

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre,§ G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse

Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

(Received 13 November 1974)

Particle physics since Nov 1974 (standard model)

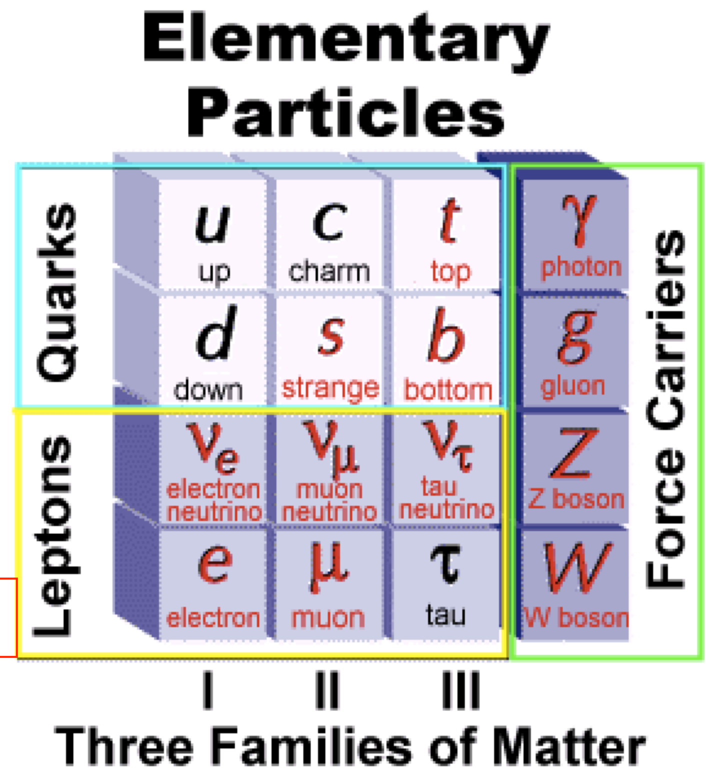
- Elementary particles – new periodic table:

Flavor	Mass
u	$1.5 - 4.5$ MeV
d	$5.0 - 8.5$ MeV
s	$80 - 155$ MeV
c	$1.0 - 1.4$ GeV
b	$4.0 - 4.5$ GeV
t	174.3 ± 5.1 GeV

Light quarks

$\Lambda_{\rm QCD}$

heavy quarks



- Quark masses span a wide kinematical range: $\frac{m_t}{m_u} \sim 10^4$?

- QCD can have bound states w/wo localized color charge!

Hadrons with localized color charge(s)

- **Heavy-light meson – "atom-like" system:**
  - **Charmed mesons:** $D^+ = c\bar{d},\ D^0 = c\bar{u},\ \bar{D}^0 = \bar{c}u,\ D^- = \bar{c}d,\ \ldots$
  - **Charmed, strange mesons:** $D_s^+ = c\bar{s},\ D_s^- = \bar{c}s,\ \ldots$
  - **Bottom mesons:** $B^+ = u\bar{b},\ B^0 = d\bar{b},\ \bar{B}^0 = \bar{d}b,\ B^- = \bar{u}b,\ \ldots$

  Heavy quark symmetry → HQET

- **Heavy-heavy meson/quarkonium – NR system**
  - **Bottom, charmed mesons:** $B_c^+ = c\bar{b},\ B_c^- = \bar{c}b,\ \ldots$
  - **$c\bar{c}$ mesons:** $J/\psi,\ \chi_c,\ \psi',\ \ldots$
  - **$b\bar{b}$ mesons:** $\Upsilon,\ \chi_b,\ \ldots$

  Heavy-heavy system: → NRQCD, pNRQCD

- **Recent review:**

  N. Brambilla et al. Eur. Phys. J. C71, 1534 (2011) [arXiv: 1010.5827]

No top quarkonium

- Charm and bottom quarks decay slowly and leave enough time to form charm and bottom mesons

m_c and m_b << M_W Charm and bottom decay via a virtual W into light $q\bar{q}$ or $\ell\bar{\nu}$

→ Semi-leptonic decay width: $\Gamma \sim 10^{-10}$ MeV

- Top quark decay very fast, before any meson can be formed

$m_t > M_W + m_b$

$t \to W^+ + b$

$\hookrightarrow \ell^+ + \nu$ (or $q + \bar{q}$)

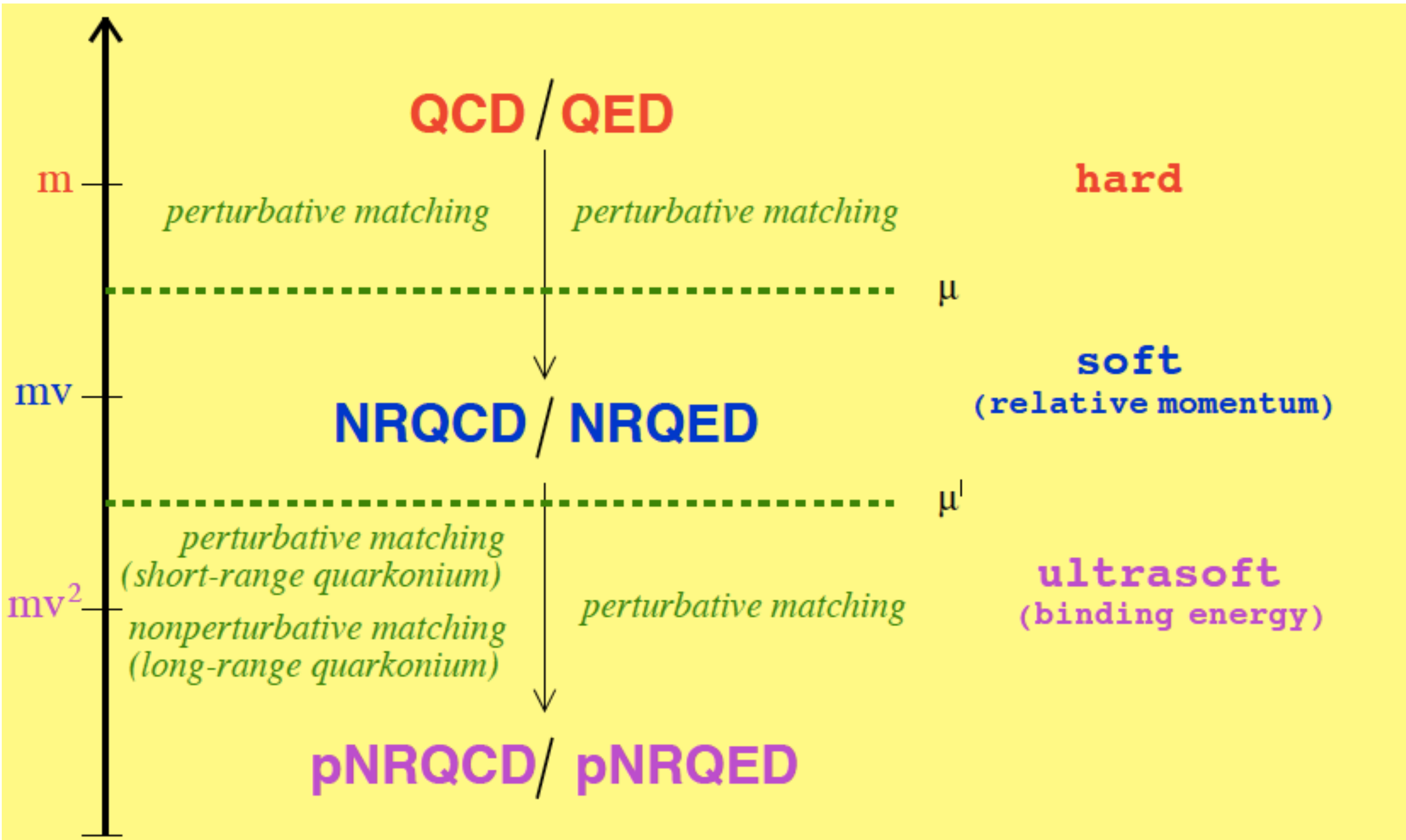
$$\Gamma(t \to W^+ b) \approx \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2$$

$$\approx 1.76 \text{ GeV} \left[\frac{m_t}{175\text{GeV}}\right]^3$$

→ Top quark should be a better candidate for studying heavy quark production, and heavy quark properties

Non-relativistic effective field theory

❑ **Quarkonium scales:**



Another relevant scale in QCD: Λ_{QCD}

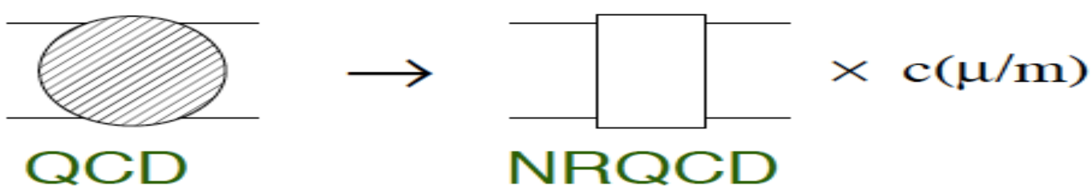
Non-relativistic QCD (NRQCD)

- **Perturbative expansion in the relative velocity:** $v \propto \frac{1}{m}$

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \ldots \right) \psi$$
$$+ \chi^\dagger \left(\qquad \ldots \qquad \right) \chi$$
$$+ \sum_K \frac{f}{m^2} \psi^\dagger\, K\, \chi \chi^\dagger\, K\, \psi + \cdots$$
$$- \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i \not{D}\, q + \ldots$$

Caswell, Lepage 86, Bodwin Braaten Lepage 95, Manohar 97

- **Integrate out the degrees of freedom that scales like “m”:**

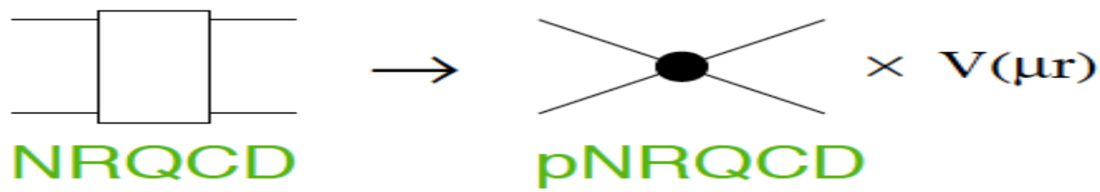


- **Works very well for calculating the decay rate!**

Potential non-relativistic QCD (pNRQCD)

Pineda, Soto 98, Brambilla, Pineda, Soto, Vairo, 2000, review 2005

- Integrate out the degrees of freedom scales like "mv" (>Λ_{QCD})



- Expansion in color states of heavy quark pairs and "r"

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S + O^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) O \right\}$$

LO in "r"

$$\theta(T)\, e^{-iTH_s} \qquad \theta(T)\, e^{-iTH_o} \left(e^{-i \int dt\, A^{\mathrm{adj}}} \right)$$

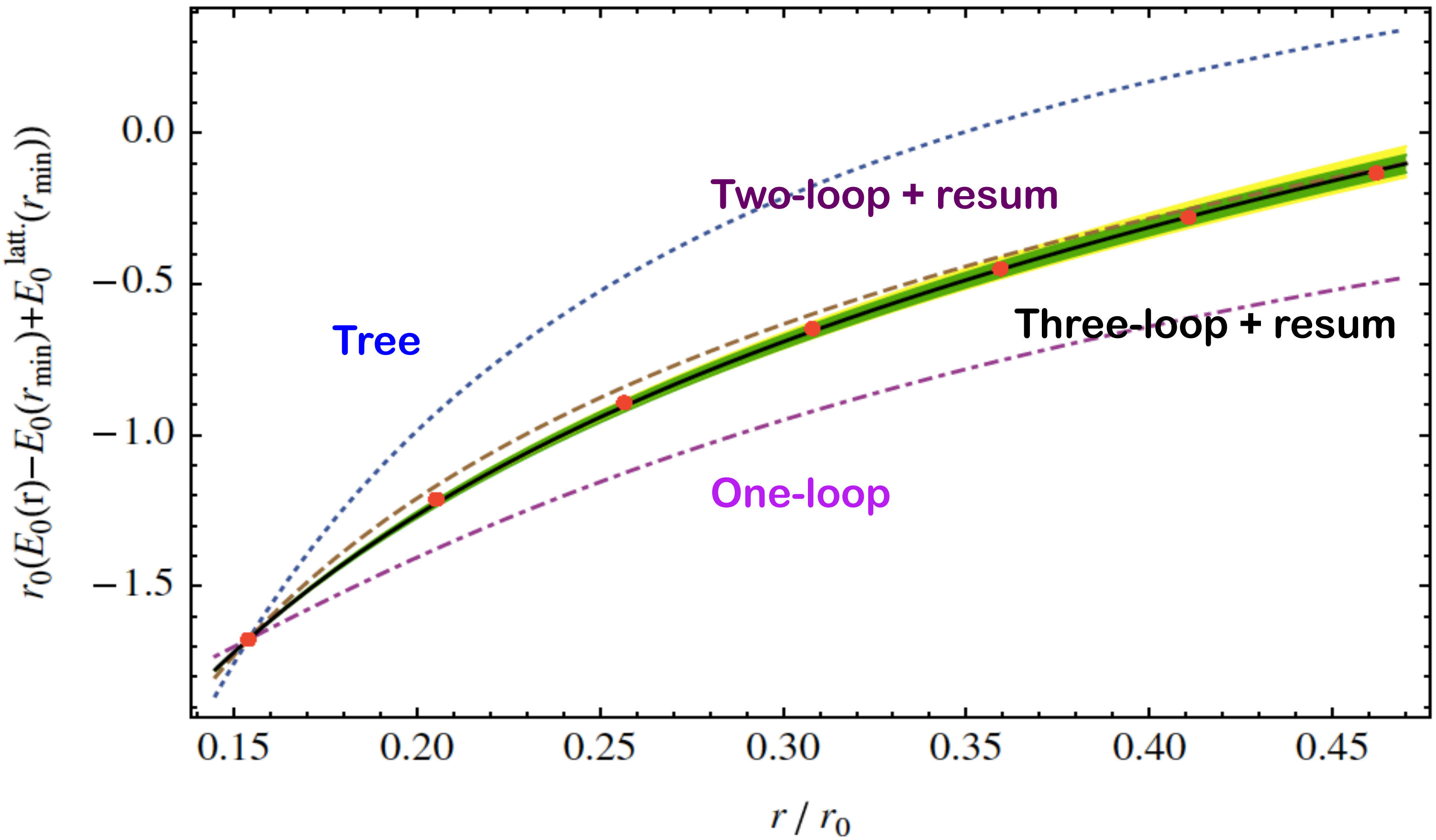
S: color singlet $Q\text{-}\bar{Q}$, O: color octet $Q\text{-}\bar{Q}$

- Systematic calculation of static potential when r << r_0 ~ 0.5 fm

$$V_s(r, \mu, \alpha_s(r))$$

Static potential energy vs lattice QCD

Brambilla, et al. PRL 2010



● **Lattice QCD data points** Necco and Sommer, 2002

With a few parameters, potential models extended to a larger r have done a good job in fitting the quarkonia spectra!

Questions

- **What about the production?**

  Can we predict the production rate?

- **In the vacuum:**

  Hadronization with localized color charge?

- **In a hot QCD medium:**

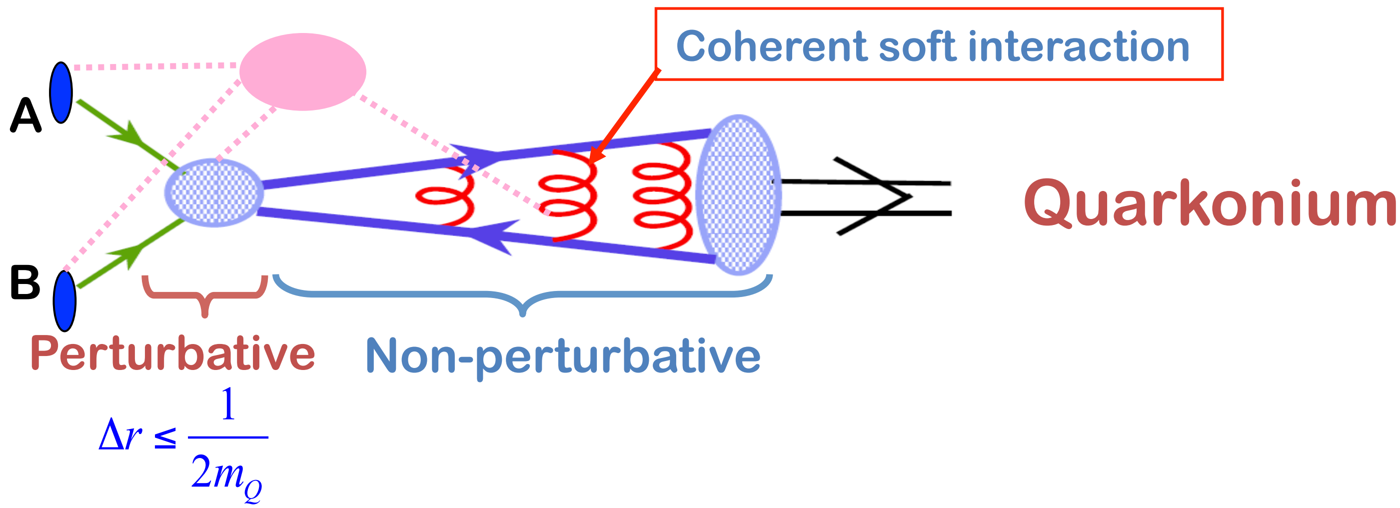
  Multiple momentum scales – effective probe of medium properties?

Outline of the rest of my talk

- Heavy quarkonium production models
- Surprises and anomalies
- High order in α_s is not necessarily leading in $1/p_T$
- Perturbative QCD factorization approach
- Connect pQCD factorization to NRQCD factorization
- Suppressions and puzzles in nuclear collisions
- Summary

Basic production mechanism

❑ **Production of an off-shell heavy quark pair:**



❑ **Approximation: on-shell pair + hadronization**

$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}}\right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

✧ **Different models ⇔ Different assumptions/treatments on how the heavy quark pair becomes a quarkonium?**

✧ **Factorization – No proof!**

A long history for the production

- Discovery of J/ψ – November revolution – 1974
- Color singlet model: 1975 –

  Einhorn, Ellis (1975),
  Chang (1980),
  Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- Color evaporation model: 1977 –

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- NRQCD model: 1986 –

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and α_s

- pQCD factorization approach: 2005 –

  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …

  $P_T \gg M_H$: M_H/P_T power expansion + α_s – expansion

  Universal fragmentation functions – evolution/resummation

Color singlet model (CSM)

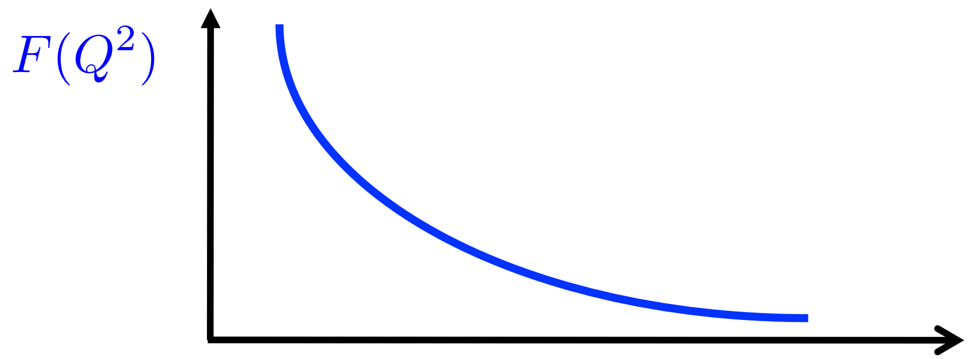
❑ Basic assumptions:

Einhorn and Ellis (1975), Chang (1980), Berger and Jone (1981), …

✧ **Only pairs with right quantum number can become quarkonia**

$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}}\right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

✧ **Steep falling:**



➡ $\hat{\sigma}(Q^2 \approx 4m_c^2)$

➡ $\int dQ^2 F(Q^2) \propto |\psi(0)|^2$ – **fixed by decay**

➡ $\sigma_{AB\to J/\psi} \propto \hat{\sigma}(Q^2 \approx 4m_c^2)\ |\psi(0)|^2$

No free parameter!

Color evaporation model (CEM)

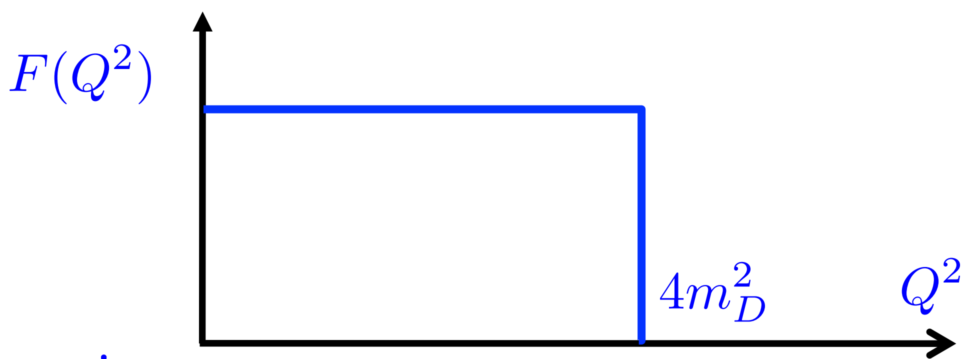
❑ Basic assumptions:

Fritsch (1977), Halzen (1977), …

✧ **All colored or color singlet pairs with invariant mass less then open charm threshold could become bound quarkonia**

$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}}\right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

✧ **Threshold:**



➡ $F(Q^2)$ – Constant!

➡ $$\sigma_{AB\to J/\psi} \approx f_{J/\psi} \int_{4m_c^2}^{4m_D^2} dQ^2 \left[\frac{d\sigma(Q^2)}{dQ^2}\right]$$

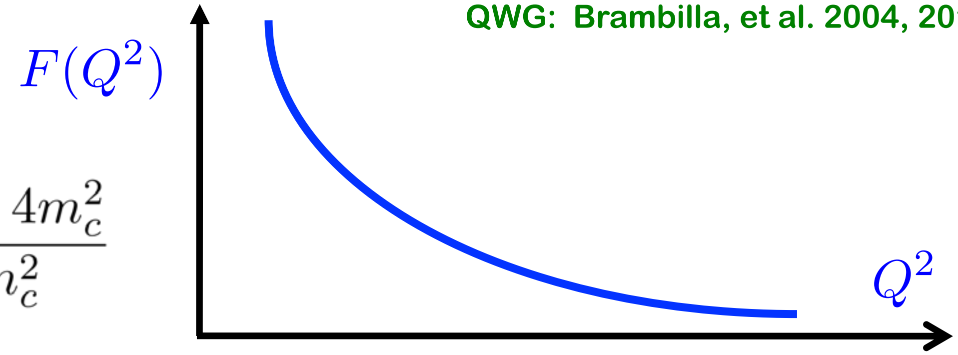
One constant per a quarkonium state

NRQCD model

Caswell and Lapage, 1986
Bodwin, Braaten, Lepage, PRD, 1995
QWG: Brambilla, et al. 2004, 2010

❑ Basic assumptions:

✧ **Steeply falling:**



✧ **v-expansion:** $v_{c\bar{c}}^2 \approx \frac{Q^2 - 4m_c^2}{4m_c^2}$

$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}}\right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

$$\Rightarrow \frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}} \approx \frac{d\hat{\sigma}(Q^2 = 4m_c^2)}{d\Gamma_{Q\bar{Q}}} + \frac{d\hat{\sigma}(Q^2 = 4m_c^2)}{dQ^2\, d\Gamma_{Q\bar{Q}}}(Q^2 - 4m_c^2) + \mathcal{O}\left[(Q^2 - 4m_c^2)^2\right]$$

$$\Rightarrow \sigma_{AB\to \mathrm{J}/\psi}(M_{\mathrm{J}/\psi}) = \sum_n \sigma_{[\mathcal{O}_n]}(4m_c^2 = M_{\mathrm{J}/\psi})\, \langle \mathcal{O}_n(0)\rangle$$

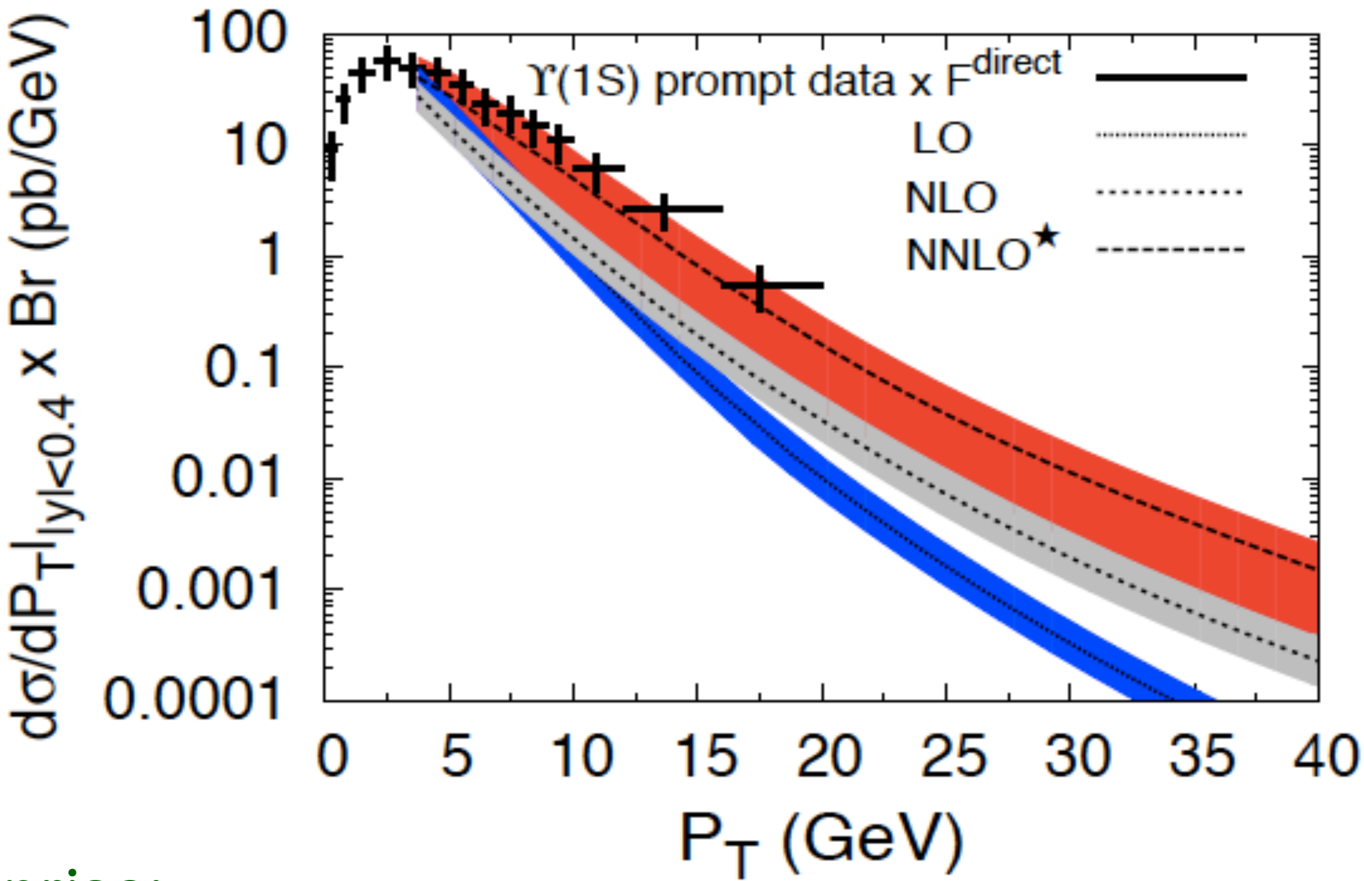
$$\langle \mathcal{O}(0)\rangle \propto \int dQ^2\; v_{c\bar{c}}^2\; F(Q^2)$$ ← **universal**

Infinite number of parameters!

Predictive power: truncation of the *v*-expansion

Color singlet model – huge HO contribution

Campbell, Maltoni, Tramontano (2007), Artoisenet, Lansburg, Maltoni (2007)
Artoisenet, Campbell, Lansburg, Maltoni, Tramontano (2008)

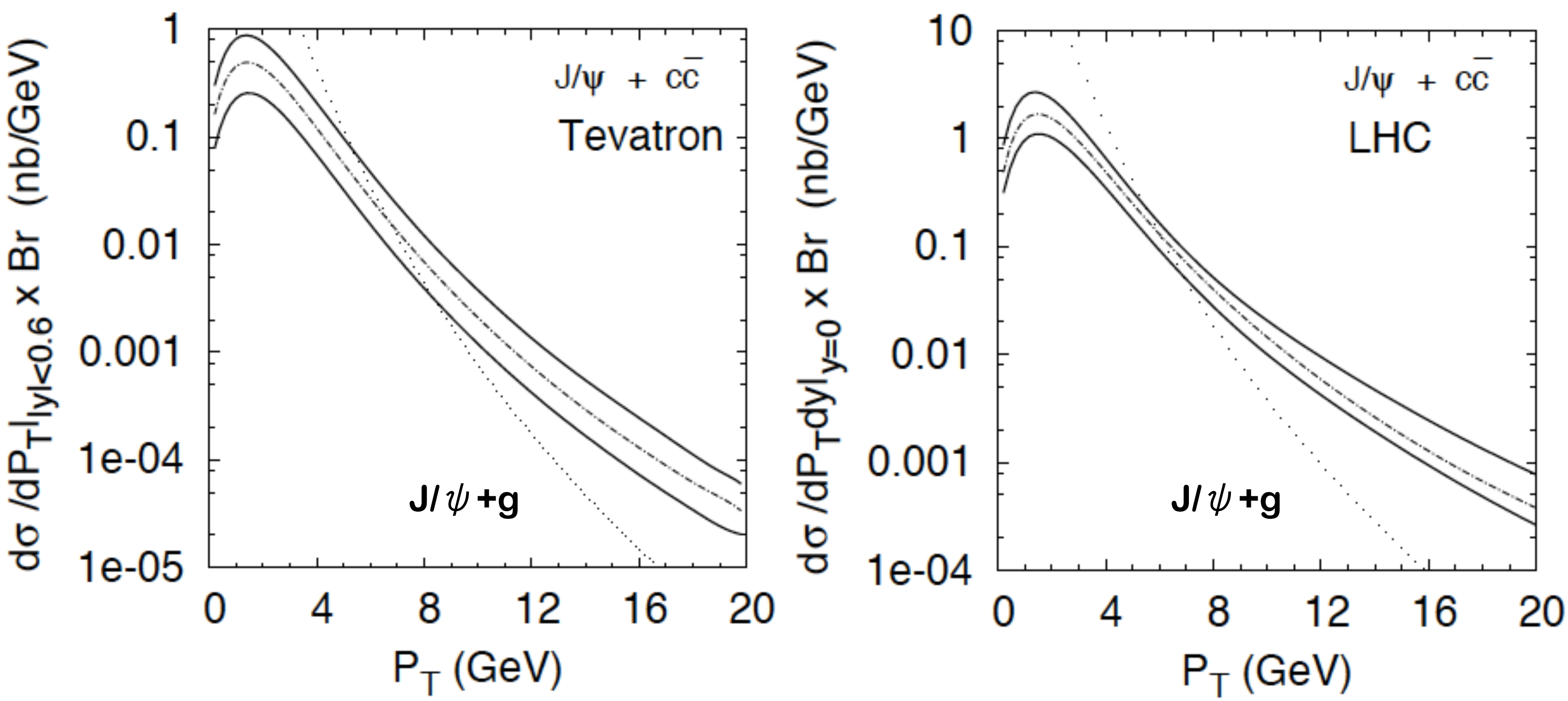


- **Surprise:**

Order of magnitude enhancement from high orders?

Color singlet model – huge associate production

Artoisenet, Lansburg, Maltoni (2007)



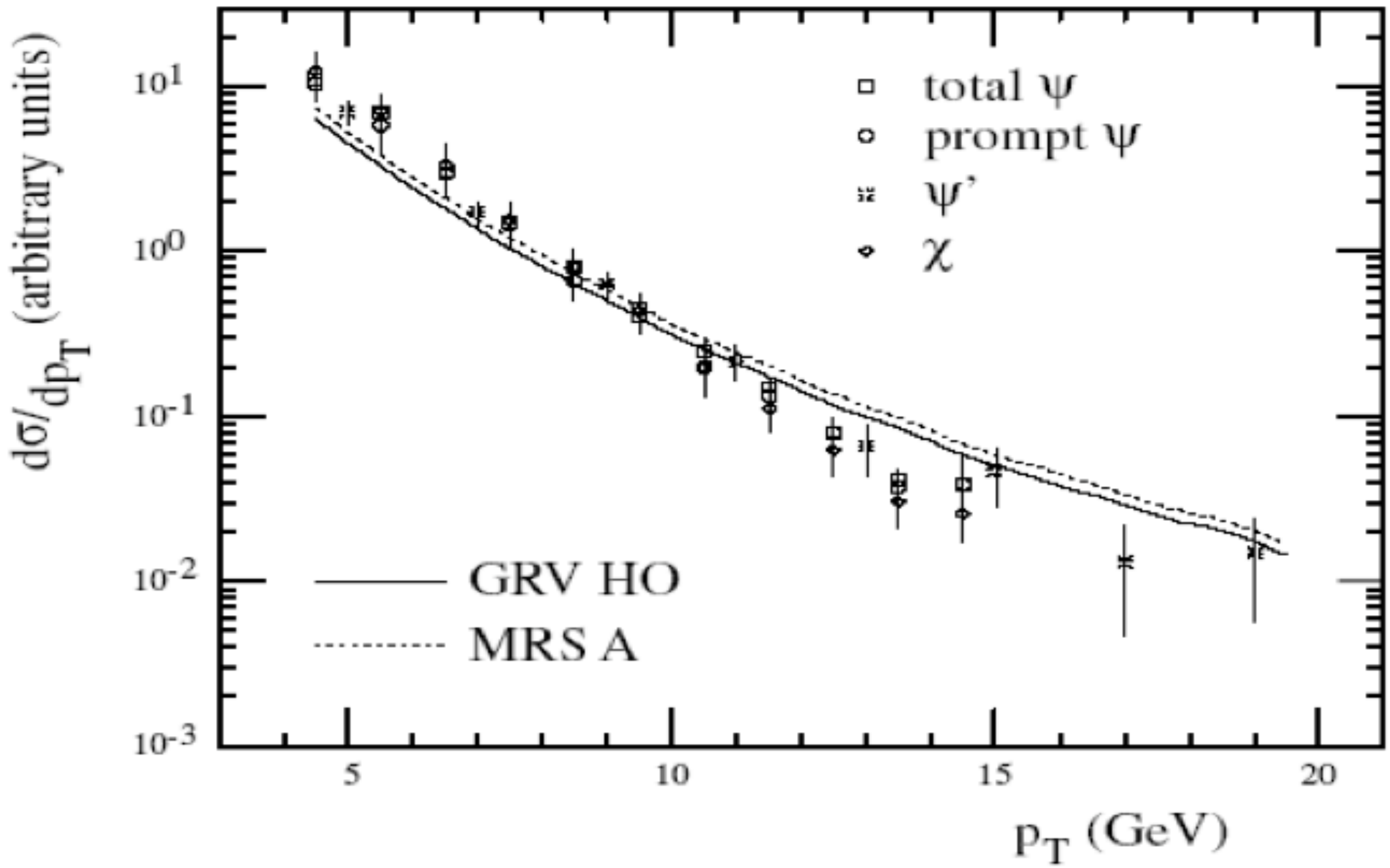
- ❑ **More surprises and question:**

  Wrong shape and strong collision energy dependence?

  How reliable is the perturbative expansion?

Color evaporation model

❑ **Good for total cross section, ok for p_T distribution:**



Amundson et al, PLB 1997

❑ **Question:**

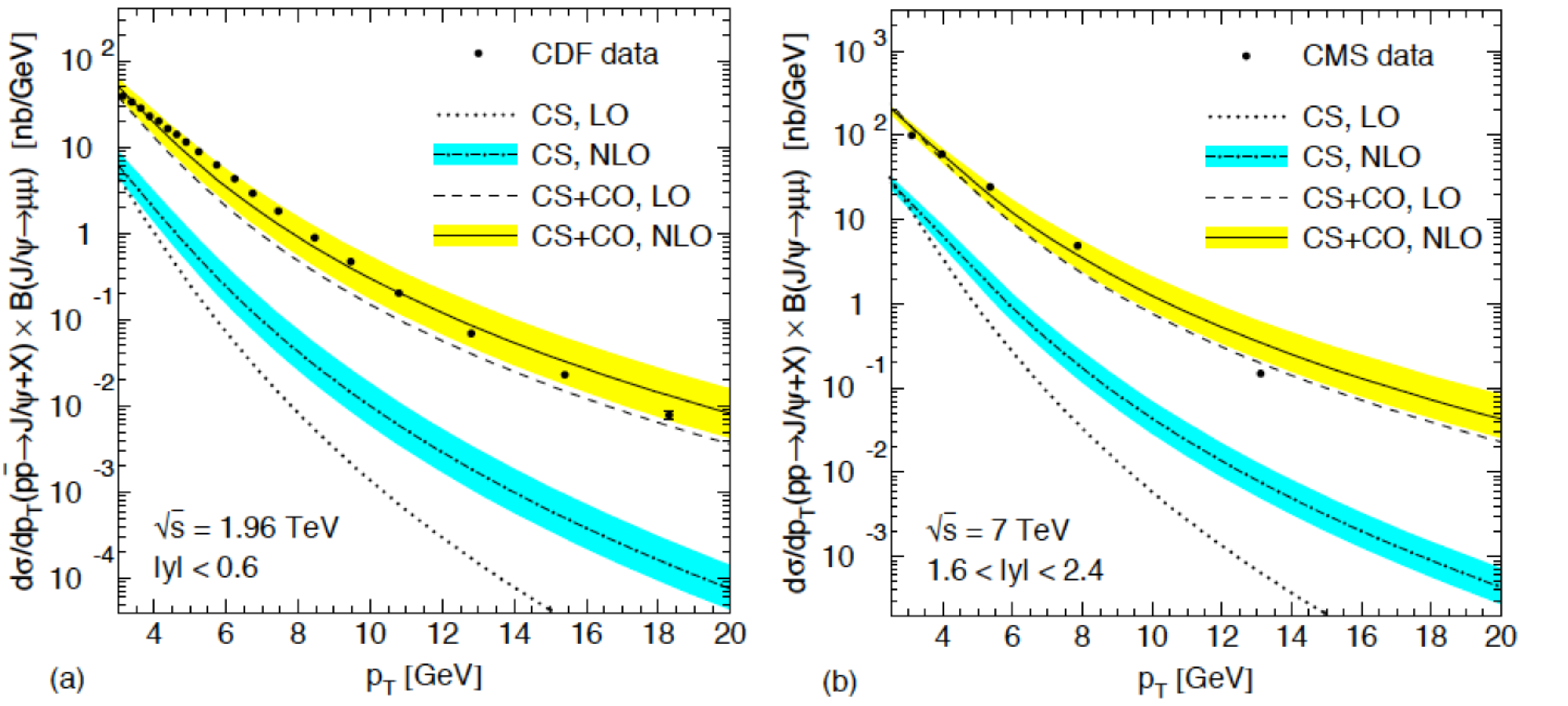
Better p_T distribution – the shape?

NRQCD – most successful so far

- NLO color octet contributions – becoming available:

  Most hard calculations were done in China and Germany!

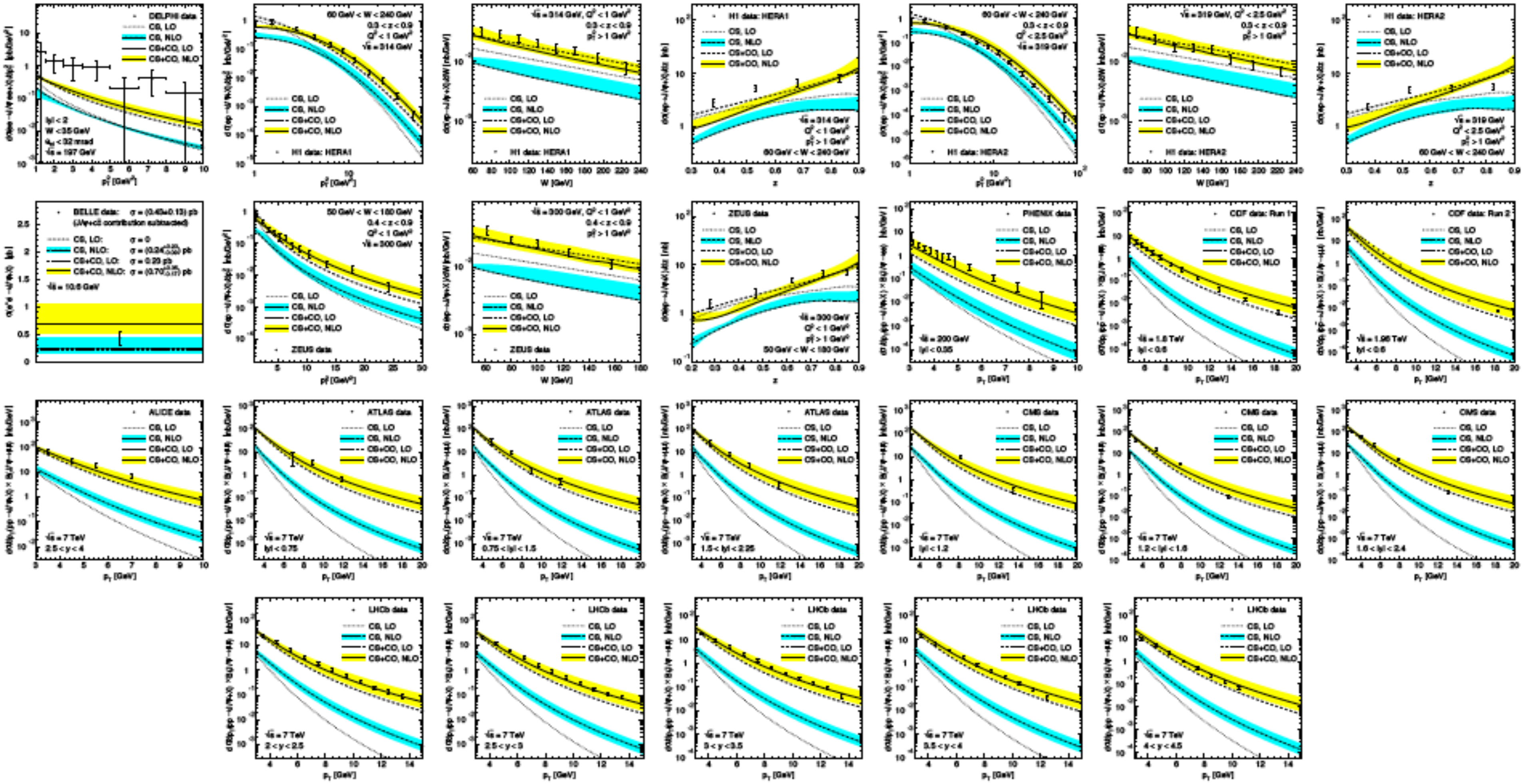
- Phenomenology:



- Fine details – shape?

PRL 106, 022003 (2011)

NRQCD – global analysis



194 data points from 10 experiments, fix singlet $\langle O[^3S_1^{[1]}]\rangle = 1.32\ \mathrm{GeV}^3$

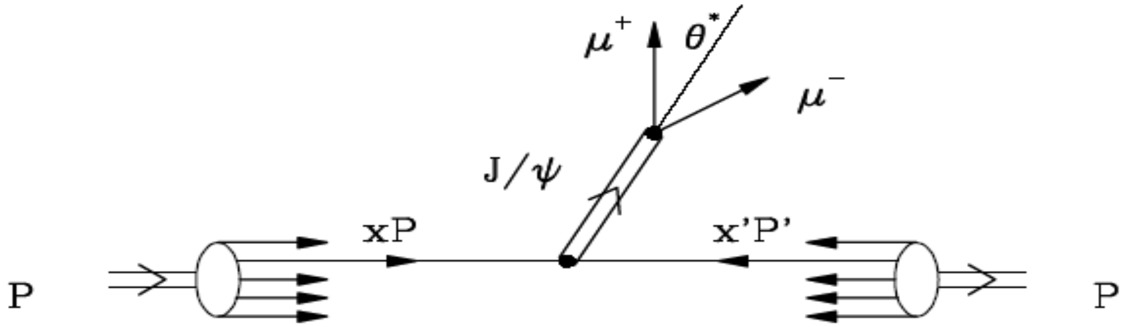
→ $\langle O[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\langle O[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

Butenschoen and Kniehl, arXiv: 1105.0820

Heavy quarkonium polarization

- Measure angular distribution of $\mu^+\mu^-$ in J/ψ decay



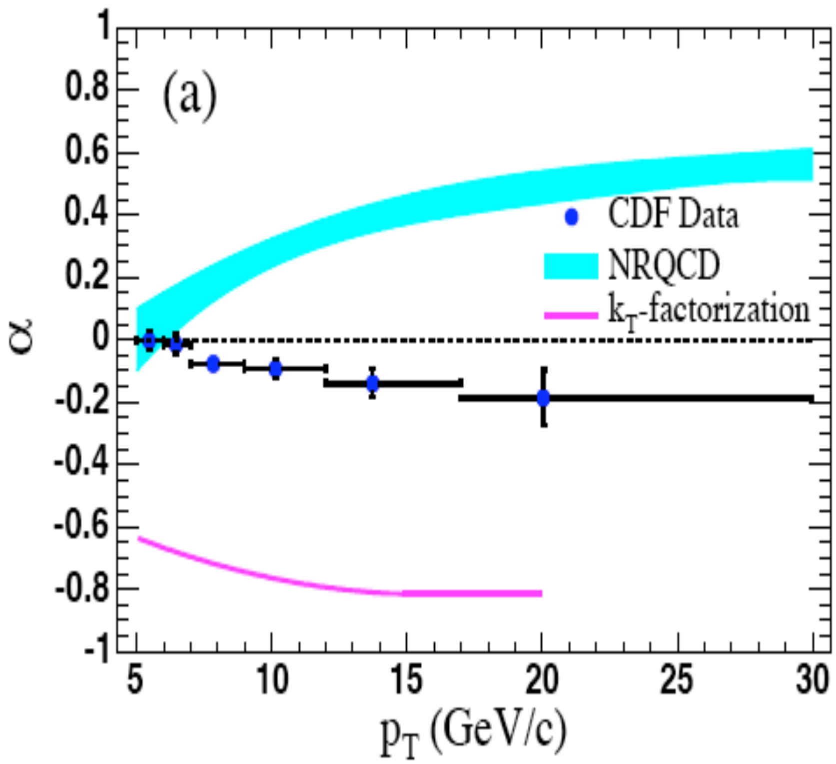
- Normalized distribution:

$$I(\cos\theta^*) = \frac{3}{2(\alpha+3)}\left(1+\alpha\cos^2\theta^*\right)$$

$$\alpha = \begin{cases} +1 & \text{fully transverse} \\ 0 & \text{unpolarized} \\ -1 & \text{fully longitudinal} \end{cases}$$

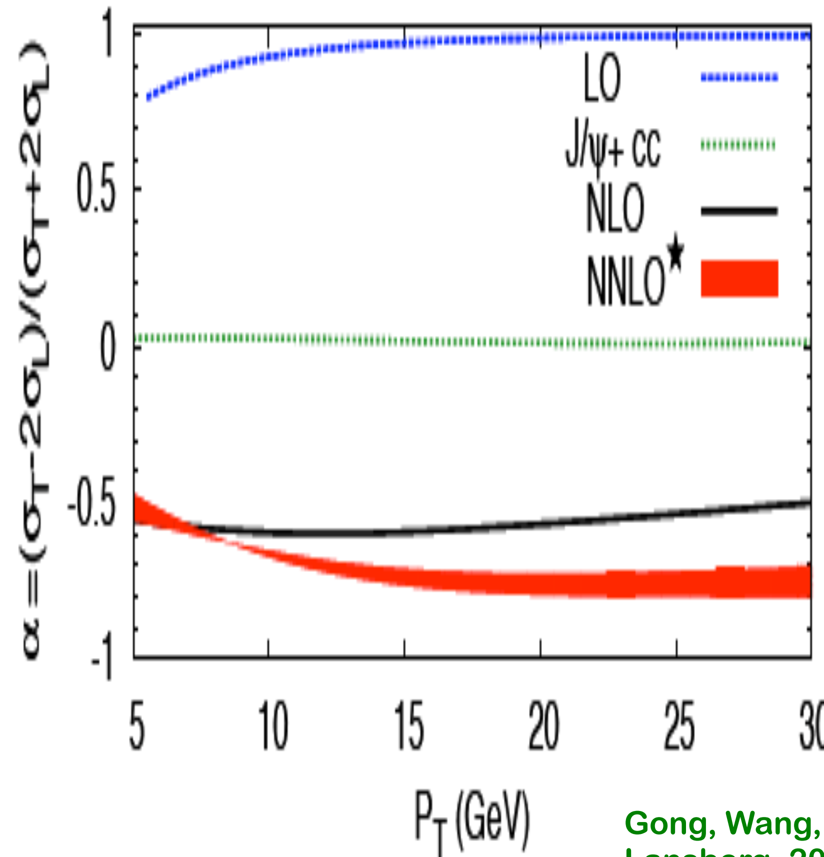
Anomalies from J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

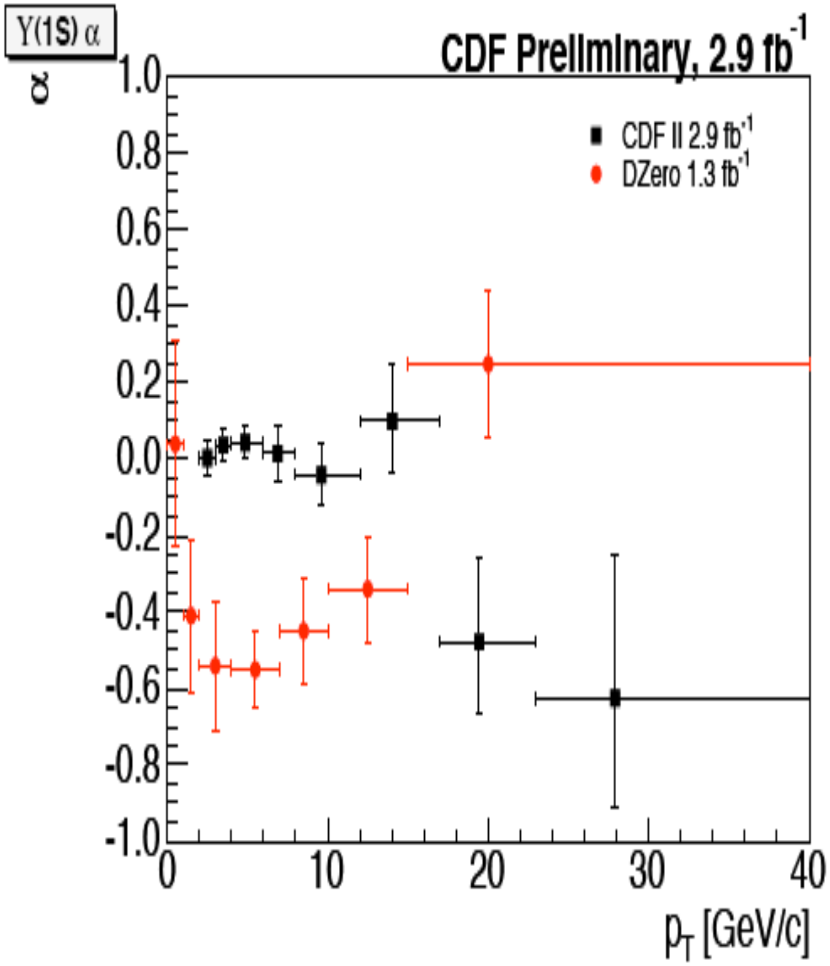
CSM

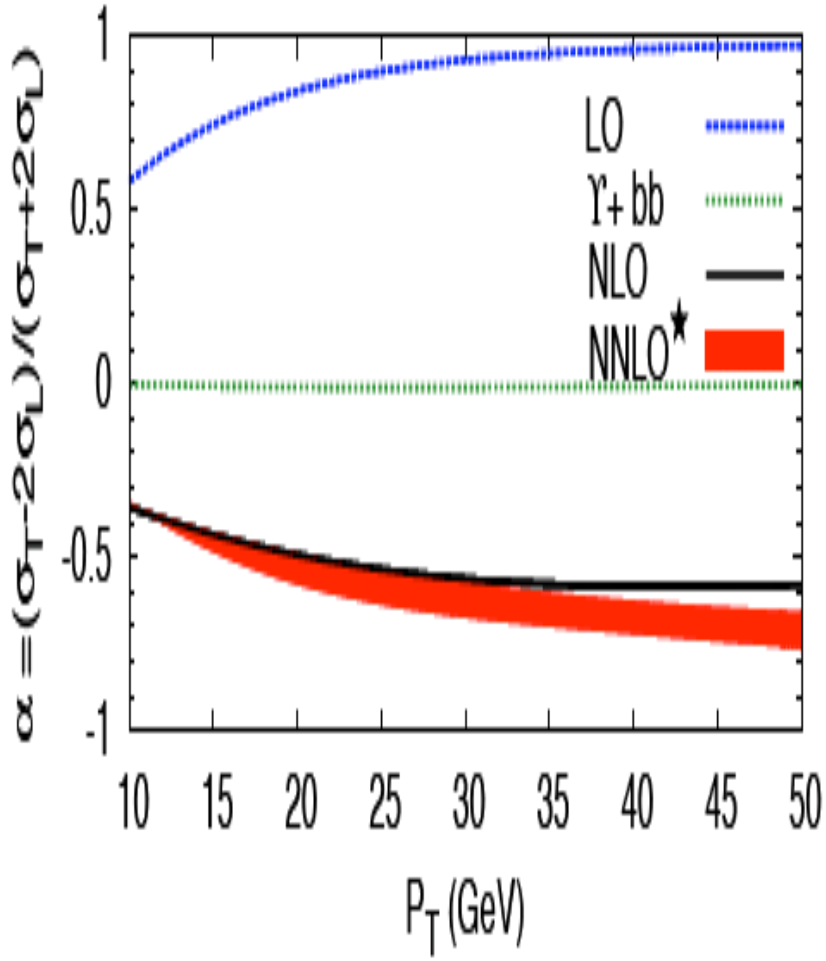


Gong, Wang, 2008
Lansberg, 2009

- ✧ **NRQCD:** Dominated by color octet – NLO is not a huge effect
- ✧ **CSM:** Huge NLO – change of polarization?

Confusions from Upsilon polarization





Resolution between CDF and D0?

Gong, Wang, 2008

Artoisenet, et al. 2008

Lansberg, 2009

Heavy quarkonium associate production

- **Inclusive J/ψ + charm production:**

$$\sigma(e^+e^- \to J/\psi c\bar{c})$$

Belle: $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb

NRQCD-LO: $:\ 0.07$ pb

Kiselev, et al 1994,
Cho, Leibovich, 1996
Yuan, Qiao, Chao, 1997
...
Zhang, Chao, 2007 (NLO)

- **Ratio to light flavors:**

$$\sigma(e^+e^- \to J/\psi c\bar{c})/\sigma(e^+e^- \to J/\psi X)$$

Belle: $0.59^{+0.15}_{-0.13} \pm 0.12$

- **Message:**

Production rate of $e^+e^- \to J/\psi c\bar{c}$ is larger than

all these channels: $e^+e^- \to J/\psi gg$, $e^+e^- \to J/\psi q\bar{q}$, ...

combined ?

Questions

- ❑ Why the high order correction in CSM is so large?
  How many orders should we calculate?

- ❑ Why the CSM predicts the longitudinally polarized
  J/psi?

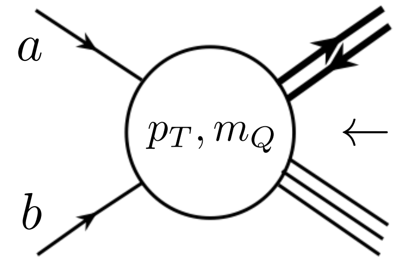
- ❑ Color singlet model is a special case of NRQCD

- ❑ Why NRQCD model predicts the transverse polarization
  and "wrong rate" for associate production?

Why high orders in CSM are so large?

- **Hard part in CSM (or NRQCD):**

  Expansion in power of α_s



$\leftarrow \quad \Delta r \sim \frac{1}{m_Q}$

$$\hat{\sigma}_{ab\to Q\bar{Q}}(p_T, m_Q, \mu, \alpha_s(\mu)) = \sum_n \hat{\sigma}^{(n)}_{ab\to Q\bar{Q}}(p_T, m_Q, \mu) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- **Complication with more than one hard scale:**

  ✧ **IF $p_T^2 \gg m_Q^2$, high order in α_s is NOT necessary smaller!**
  Different power in p_T^2

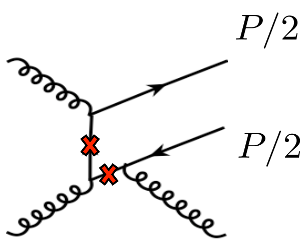
  ✧ **p_T^2-dependence is sensitive to where the pair is produced!**

  ✧ **The size of the hard coefficients also depends on where the pair was produced!**

α_s-expansion vs 1/p_T-expansion

- **LO in α_s but higher power in 1/p_T:**

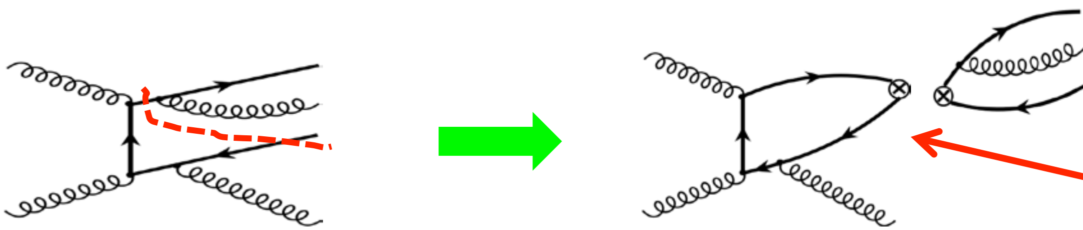
LO in α_s:



$$\hat{\sigma}^{\rm LO} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

NNLP in 1/p_T!

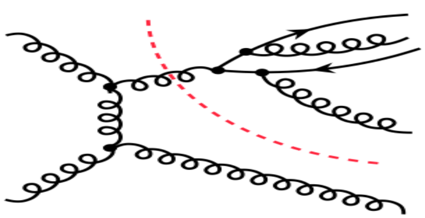
- **NLO in α_s but lower power in 1/p_T:**



Leading power Projection "Octet pair"

$$\hat{\sigma}^{\rm NLO} \rightarrow \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)$$

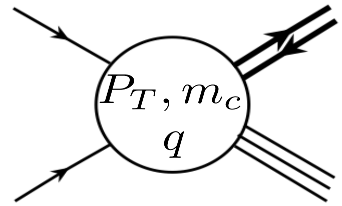
- **NNLO in α_s but leading power in 1/p_T:**



$$\hat{\sigma}^{\rm NNLO} \rightarrow \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^2(\mu)\log^m(\mu^2/\mu_0^2)$$

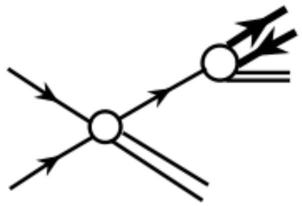
Where heavy quark pairs produced?

❑ IF $p_T >> m_Q$:



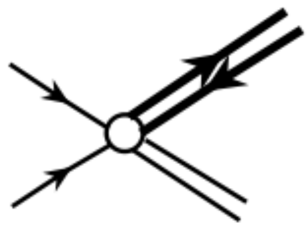
$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

✧ at $1/m_Q$:



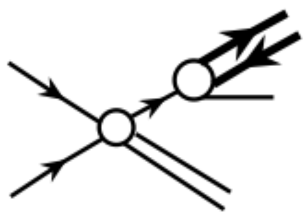
Only final-state fragmentation – large P_T

✧ at $1/P_T$:



Short-distance production - perturbative

✧ between: [$1/m_Q$, $1/P_T$]



Parton evolution + short-distance

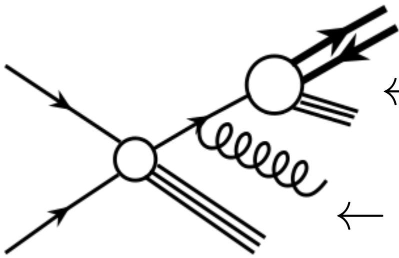
❑ Role of color:

✧ Color can be perturbatively resolved between m_Q and P_T

✧ Option to factorize into colored partonic states: singlet vs octet

PQCD power counting

- **Single parton produced at high P_T:**

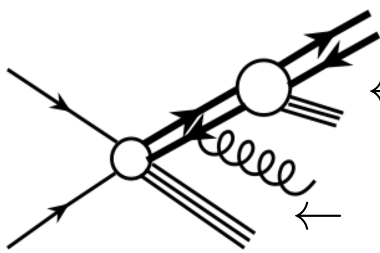


$$\leftarrow \frac{1}{m_Q}$$

$$\leftarrow \Delta r \sim \frac{1}{p_T}$$

$$\Rightarrow \quad \frac{1}{p_T^4}\left[\log(\frac{p_T^2}{\mu_0^2})\right]^n \left[\frac{m_Q^2}{\mu_0^2}, \ldots\right] \qquad \mu_0 \sim 2m_Q$$

- **Two-parton produced at high P_T:**

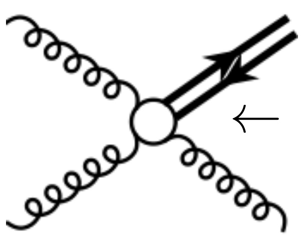


$$\leftarrow \frac{1}{m_Q}$$

$$\leftarrow \Delta r \sim \frac{1}{p_T}$$

$$\Rightarrow \quad \frac{1}{p_T^6}\left[\log(\frac{p_T^2}{\mu_0^2})\right]^n \left[m_Q^2 \left(\log \frac{\mu_0^2}{m_Q^2}, \frac{m_Q^2}{\mu_0^2}, \ldots\right)\right]$$

- **Color constraint can suppress the leading behavior:**



$$\leftarrow \Delta r \sim \frac{1}{p_T}$$

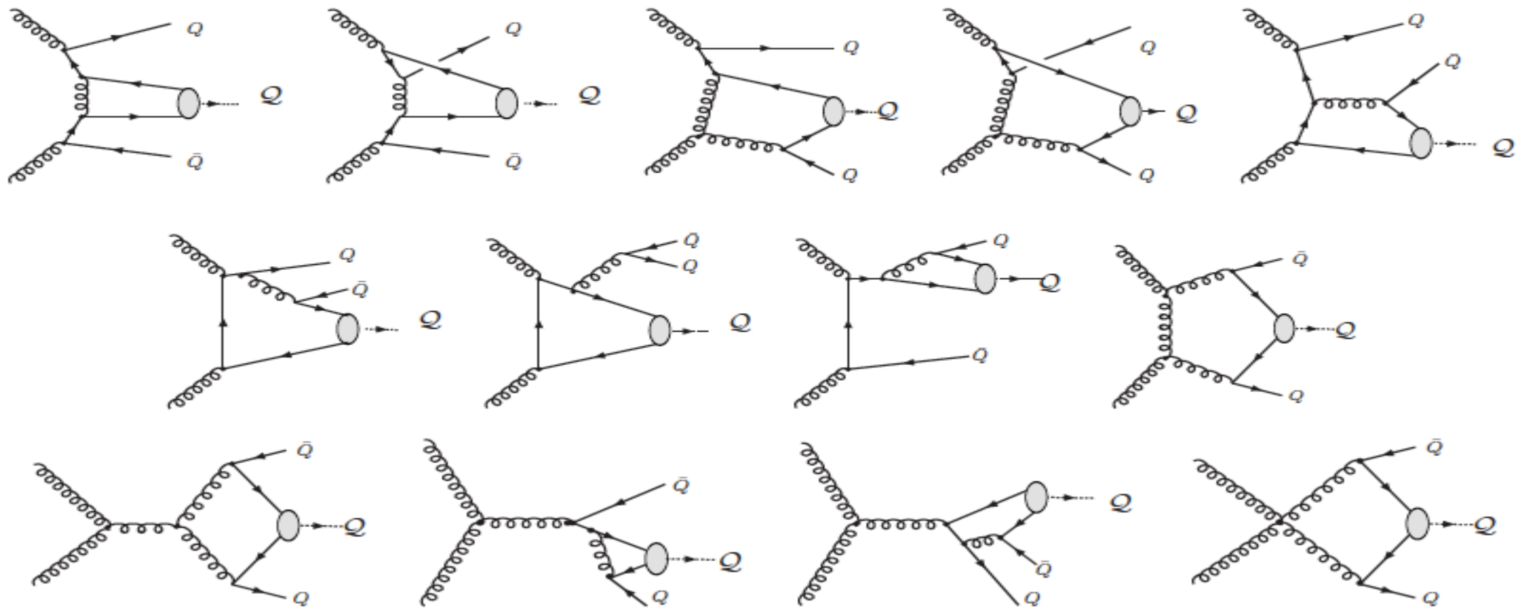
$$\Rightarrow \quad \frac{1}{p_T^4}\left[\frac{m_Q^4}{p_T^4}\right]$$ **LO singlet**

$$\frac{1}{p_T^4}\left[\frac{m_Q^2}{p_T^2}\right]$$ **LO octet**

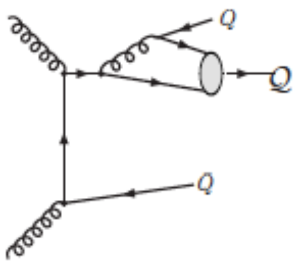
Associate production as an example

- **Complete set of diagrams:**

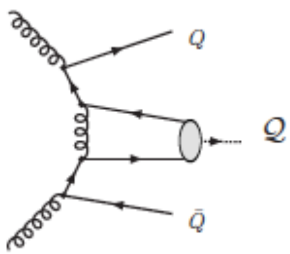
Artoisenet, Lansburg, Maltoni (2007)



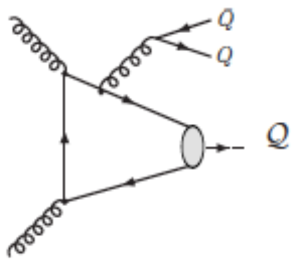
- **Physical observables:** inclusive J/ψ, $J/\psi+D$, $J/\psi+D+\bar{D}$, …



Q-fragmentation



Logs in PDF



Need interference diagrams

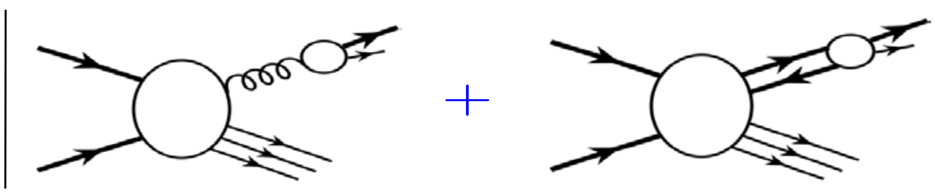
Perturbative factorization approach

Nayak, Qiu, and Sterman, 2005
Kang, Qiu and Sterman, 2010

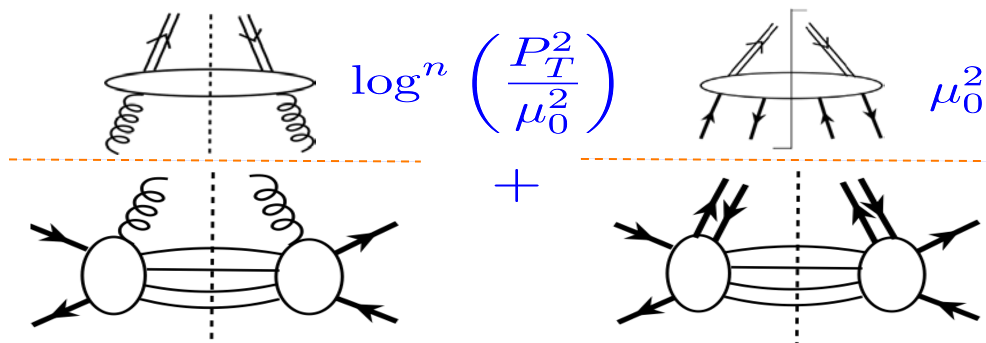
- **Ideas:**
  - ✧ Expand cross section in powers of μ_0^2/p_T^2 with $\mu_0 \gtrsim 2m_Q$
  - ✧ Resum logarithmic contribution into "fragmentation functions"
  - ✧ Apply NRQCD to input fragmentation functions at $\mu_0 \sim 2m_Q$

- **Factorization – all orders in α_s:**

$E\frac{d\sigma_{J/\psi}}{d^3P}$:



$\Big| \quad + \quad + \ldots \Big|^2$



$\log^n\left(\frac{P_T^2}{\mu_0^2}\right)$ $\quad + \quad$ $\mu_0^2 \log^n\left(\frac{P_T^2}{\mu_0^2}\right)$

$\mu_0 \sim 2m_Q$

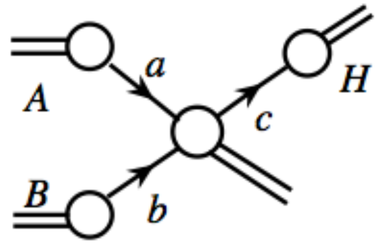
$\mathcal{O}\left(\frac{1}{P_T^4}\right)$ $\quad$ $\mathcal{O}\left(\frac{1}{P_T^6}\right)$

Power series in α_s without large logarithms

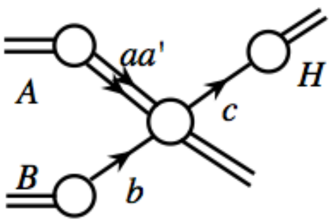
Why such power correction important?

- **Leading power in hadronic collisions:**

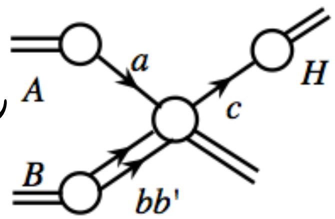
$$d\sigma_{AB\to H} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to cX} \otimes D_{c\to H}$$



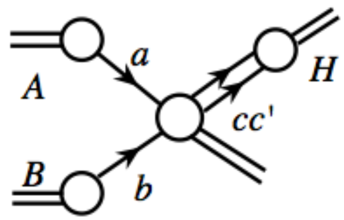
- **1st power corrections in hadronic collisions:**



$\sim$

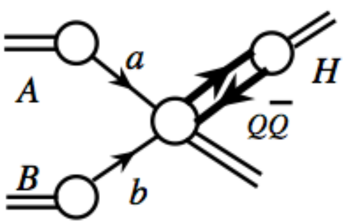


$\sim$



$$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$$

- **Dominated 1st power corrections:**



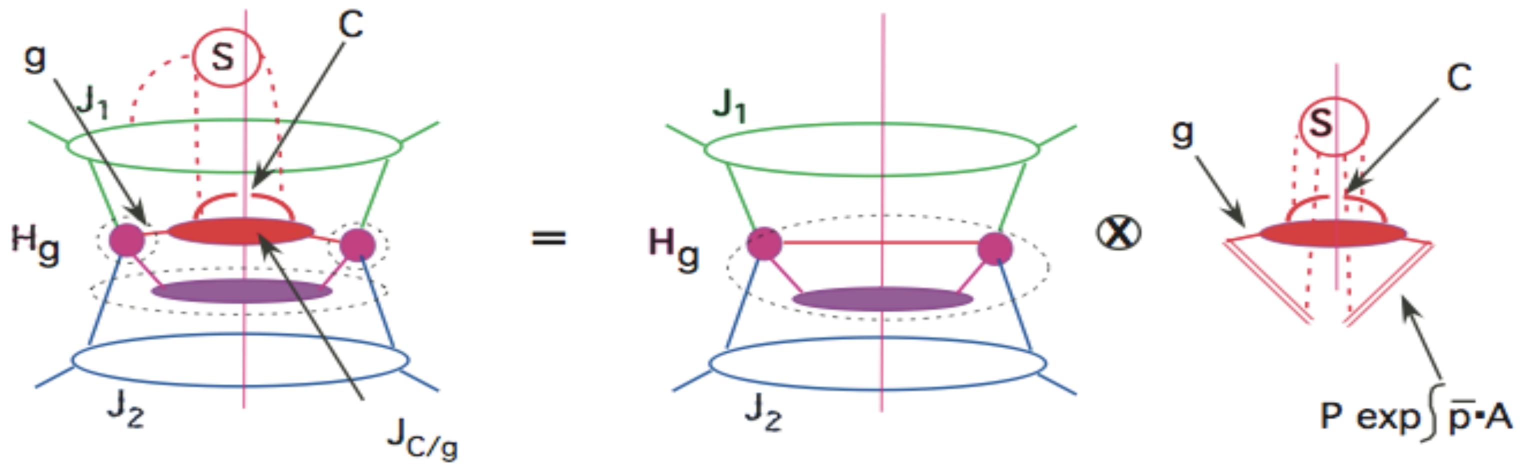
$$\sim \mathcal{O}\left(\frac{(2m_Q)^2}{P_T^2}\right) \otimes D^{(2)}_{[Q\bar{Q}]\to H}$$

Key: competition between $P_T^2 \gg (2m_Q)^2$ **and** $D^{(2)}_{[Q\bar{Q}]\to H} \gg D_{c\to H}$

pQCD Factorization

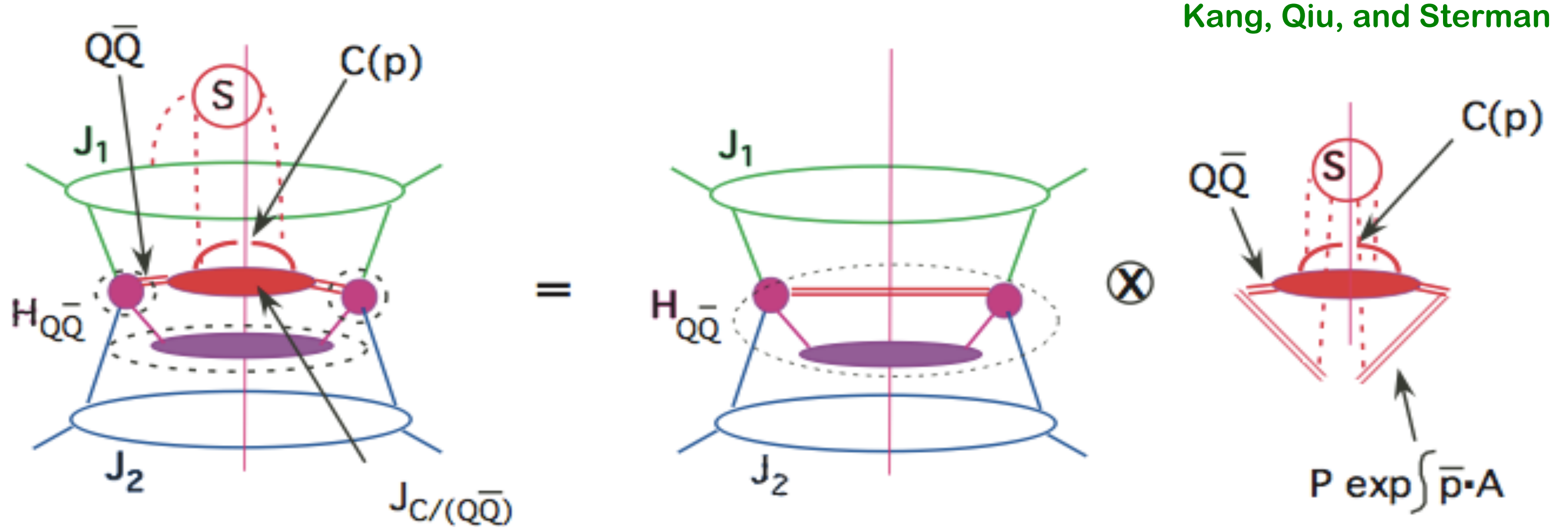
Nayak, Qiu, and Sterman, 2005

❑ Leading power – single hadron production



❑ Next-to-leading power – $Q\bar{Q}$ channel:

Qiu, Sterman, 1991
Kang, Qiu, and Sterman, 2010



Formalism and production of the pairs

❑ Factorization formalism:

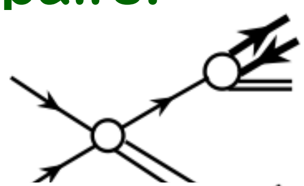
Kang, Qiu and Sterman, 2010

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$
$$+ \mathcal{O}(m_Q^4/p_T^4)$$

$$\hat{p}_Q = \frac{1+\zeta}{2z}\,\hat{p}\,, \quad \hat{p}_{\bar{Q}} = \frac{1-\zeta}{2z}\,\hat{p}$$

❑ Production of the pairs:

✧ at $1/m_Q$:

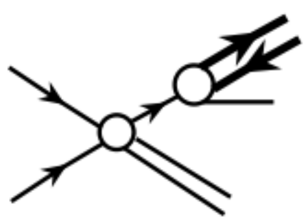


$$D_{i\to H}(z, m_Q, \mu_0)$$

✧ at $1/P_T$:

$$d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]}(\kappa), \mu)$$

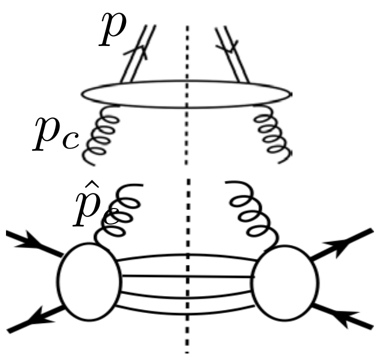
✧ between: $[\,1/m_Q\,,\,1/P_T\,]$



$$\frac{d}{d\ln(\mu)} D_{i\to H}(z, m_Q, \mu) = \ldots$$
$$+ \frac{m_Q^2}{\mu^2}\,\Gamma(z) \otimes D_{[Q\bar{Q}(\kappa)]\to H}(\{z_i\}, m_Q, \mu)$$

Cut vertices and projection operators

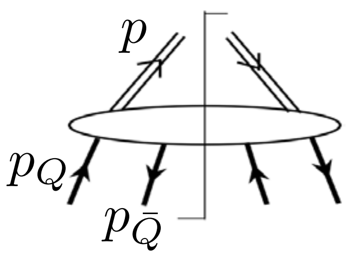
❑ Leading power:



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$\mathcal{P}_{\mu\nu}(p) = -g_{\mu\nu} + \bar{n}_\mu n_\nu + n_\mu \bar{n}_\nu \equiv d_{\mu\nu}$$

❑ Next-to-leading power – mass dependence:



$$\widetilde{\mathcal{P}}_v^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n$$

$$\widetilde{\mathcal{P}}_a^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma^5$$

$$\widetilde{\mathcal{P}}_t^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma_\perp^\alpha$$

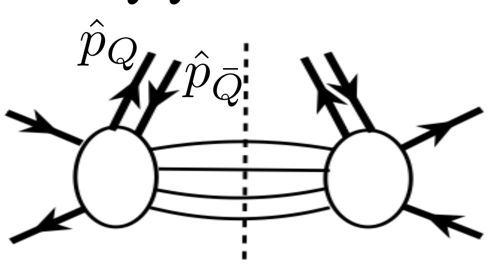
PQCD – relativistic:

Upper components

NRQCD – nonrelativistic:

Lower components

For a $Q\bar{Q}$ pair:



$$\mathcal{P}_v^L(\hat{p}_Q, \hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p} = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$

$$\mathcal{P}_a^L(\hat{p}_Q, \hat{p}_{\bar{Q}}) = \gamma_5\gamma\cdot\hat{p} = \gamma_5\gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$

$$\mathcal{P}_t^L(\hat{p}_Q, \hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p}\gamma_\perp^\alpha = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})\gamma_\perp^\alpha$$

Hard part is insensitive to the difference in quarkonium states!

Short-distance hard parts

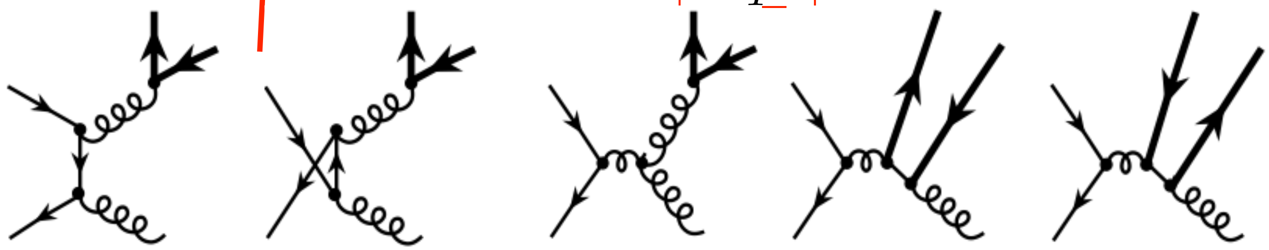
- Even tree-level needs subtraction:

$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

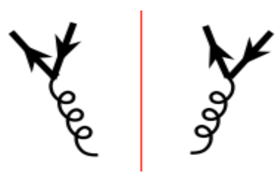
$$\Rightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$\frac{\alpha_s^3(\mu)}{p_T^6}$ $\quad$ $\frac{\alpha_s^2(\mu)}{p_T^4}$ $\quad$ $\frac{\alpha_s(2m_Q)}{(2m_Q)^2}$

$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$:



$D^{(1)}_{g\to[Q\bar{Q}]}$:



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$H^{(3)}_{q\bar{q}\to[Q\bar{Q}(a8)]g} = \frac{8\pi\alpha_s}{\hat{s}}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\frac{1}{(1-\zeta^2)(1-\zeta'^2)}\frac{N^2-1}{N}\left[1+\zeta\zeta' - \frac{4}{N^2}\right]$$

Normalized to 2 → 2 amplitude square

Predictive power

- **Calculation of short-distance hard parts in pQCD:**

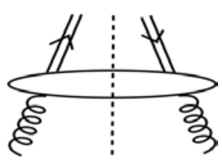
  Power series in α_s, without large logarithms

- **Calculation of evolution kernels in pQCD:**

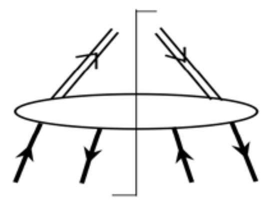
  Power series in α_s, scheme in choosing factorization scale μ

  Could affect the term with mixing powers

- **Universality of input fragmentation functions at μ_0:**



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

- **Physics of $\mu_0 \sim 2m_Q$ – a parameter:**

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

  Different quarkonium states require different input distributions!

NRQCD for input distributions

- Input distributions are universal, non-perturbative:

  Should, in principle, be extracted from experimental data

- NRQCD – single parton distributions – valid to 2-loop:

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

Nayak, Qiu and Sterman, 2005

Dominated by transverse polarization

- NRQCD – heavy quark pair:

$$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu) \to \sum_c d_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu)\langle O^H_{[Q\bar{Q}(c)]}\rangle$$

Kang, Qiu and Sterman, 2011

Dominated by longitudinal polarization

- No proof of such factorization yet!

  Single parton case was verified to two-loops (with gauge links)!

Nayak, Qiu and Sterman, 2005

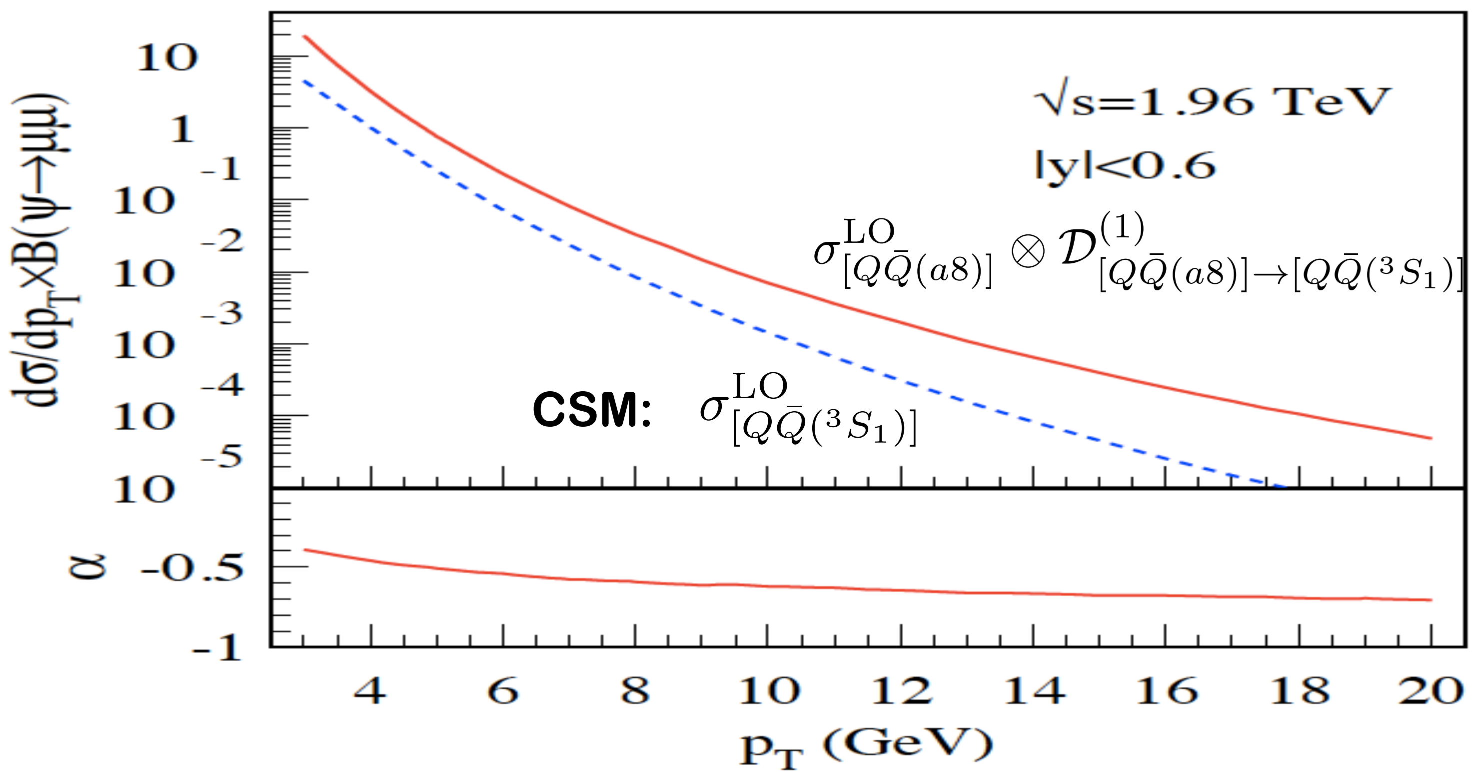
Polarization and production rate

Kang, Qiu and Sterman, 2011

- Fragmentation functions determine the polarization

  Short-distance dynamics at r ~ 1/pT is insensitive to the details taken place at the scale of hadron wave function ~ 1 fm

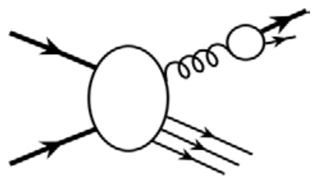
- LO fragmentation contribution reproduces the bulk of NLO color singlet contribution, and the polarization



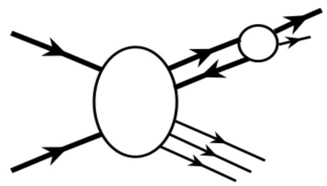
Polarization from different powers

- **Competition between LP and NLP:**

$$E\frac{d\sigma_{J/\psi}}{d^3P}:$$



$+$



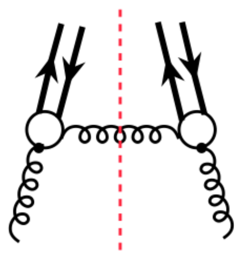
$+\ldots$

Transverse polarization (first diagram) **Longitudinal polarization** (second diagram)

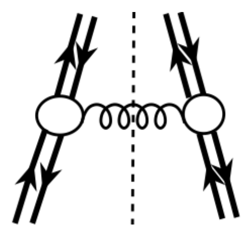
$$\frac{\langle\mathcal{O}_{\rm octet}\rangle}{p_T^4}\ \text{vs.}\ \frac{\langle\mathcal{O}_{\rm singlet}\rangle}{p_T^6}$$

- **Contribution of high spin states:**

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



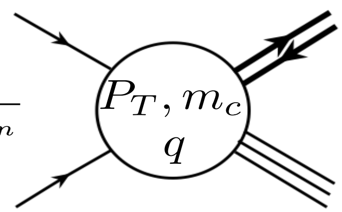
$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

instead of,

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

Universal and process independent, if NRQCD factorization is valid

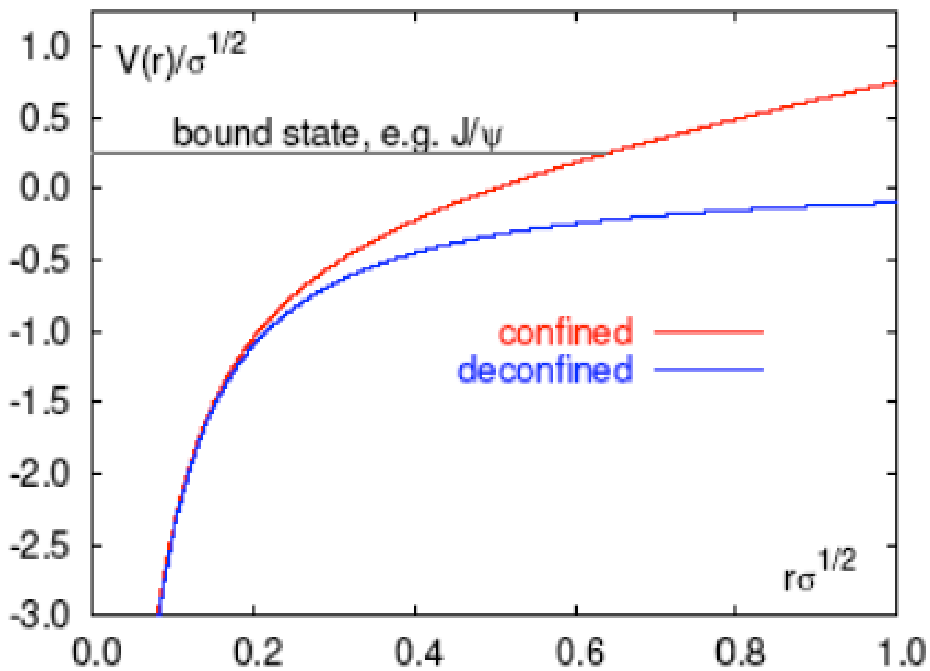
Quarkonium production at a finite T

❑ **Quark-antiquark color-screened potential:**

$$V_{Q\bar{Q}}(r,T) = -\frac{\alpha_{\mathrm{eff}}}{r} e^{-r/r_D(T)} + \sigma r_D(T)\left[1 - e^{-r/r_D(T)}\right]$$

Screening radius/length: $r_D(T) \to 0$ as $T \to \infty$

❑ **No heavy quarkonium in a deconfined medium:**

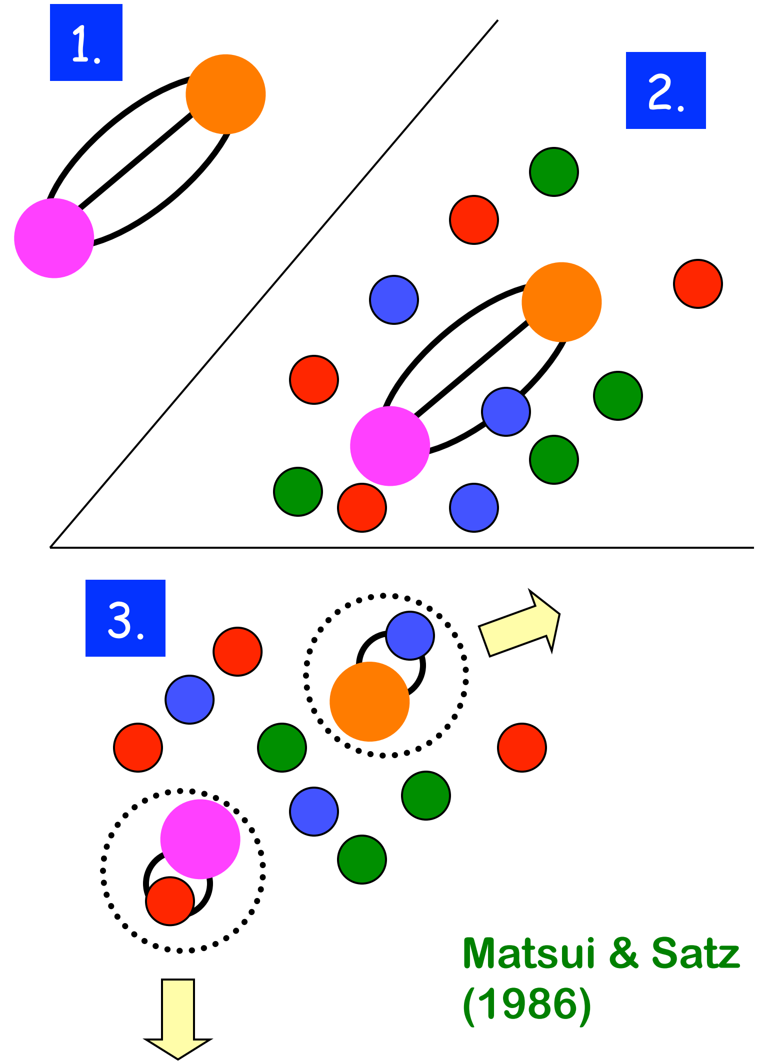


Matsui-Satz argument: (1986)

- ✧ **Deconfined QGP**
- ✧ **Color screen**
- ✧ **No quarkonium in QGP**

Melting a quarkonium in QGP

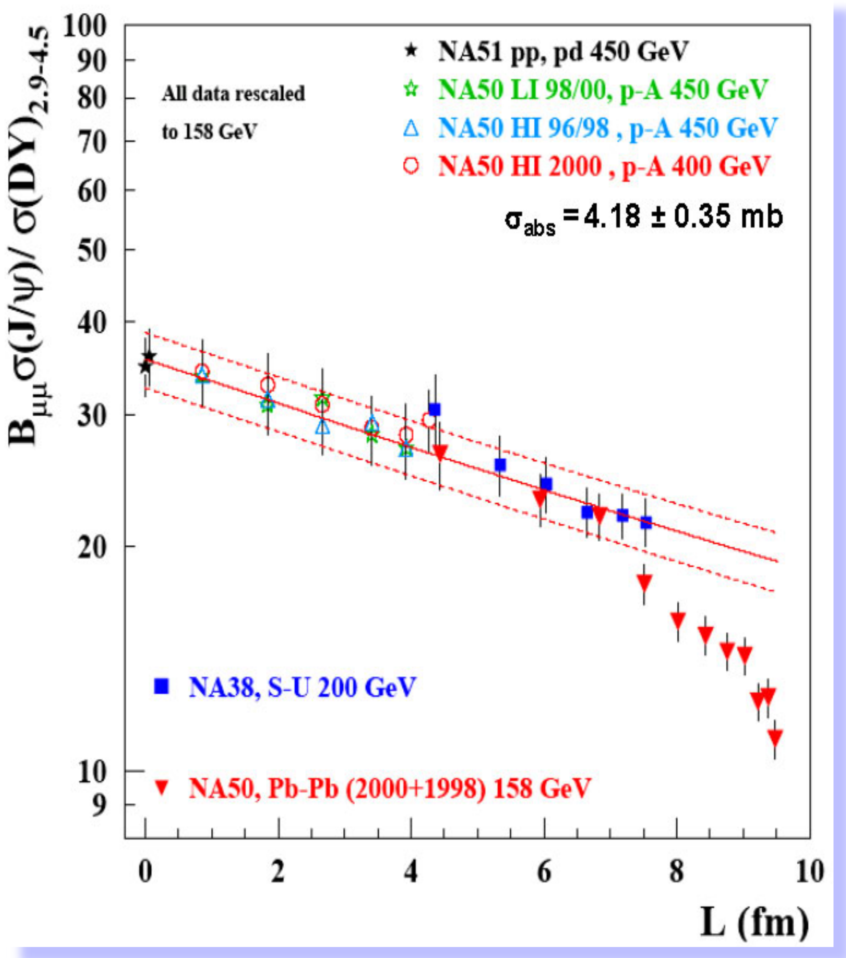
- Start with a J/ψ
  - ✧ This works with other charmonium states as well
  - ✧ The J/ψ is easiest to observe
- Put it in a sea of color charges
- The color lines attach themselves to other quarks
  - – This forms a pair of charmed mesons
- These charmed mesons “wander off” from each other
- When the system cools, the charmed particles are too far apart to recombine
  - – Essentially, the J/ψ has melted

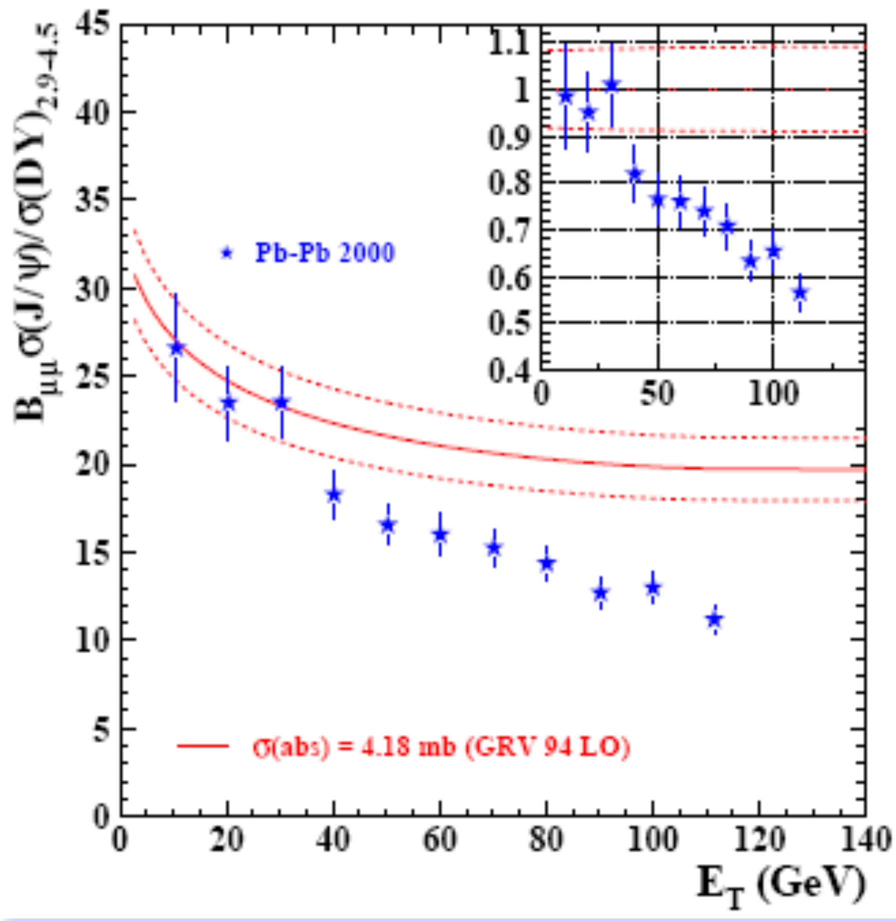


Anomalous suppression in pA

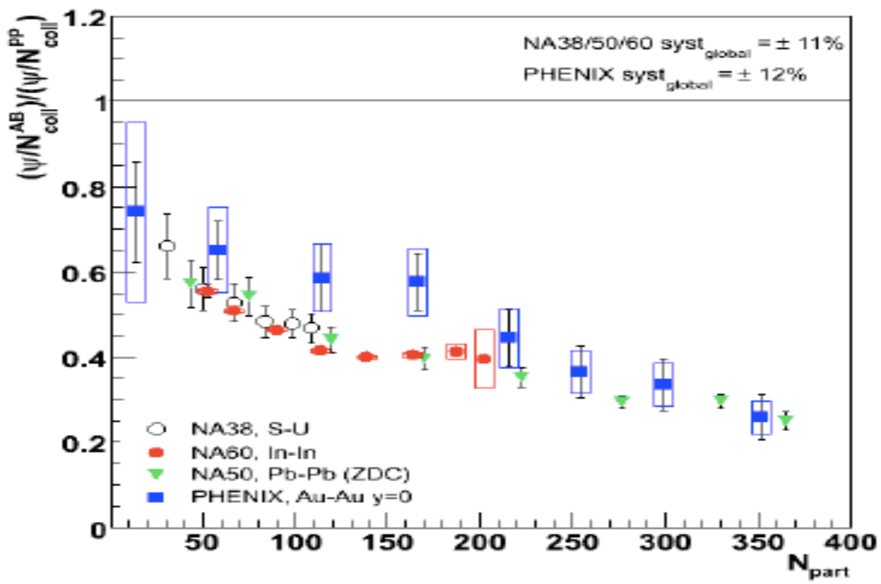
- Anomalous suppression:

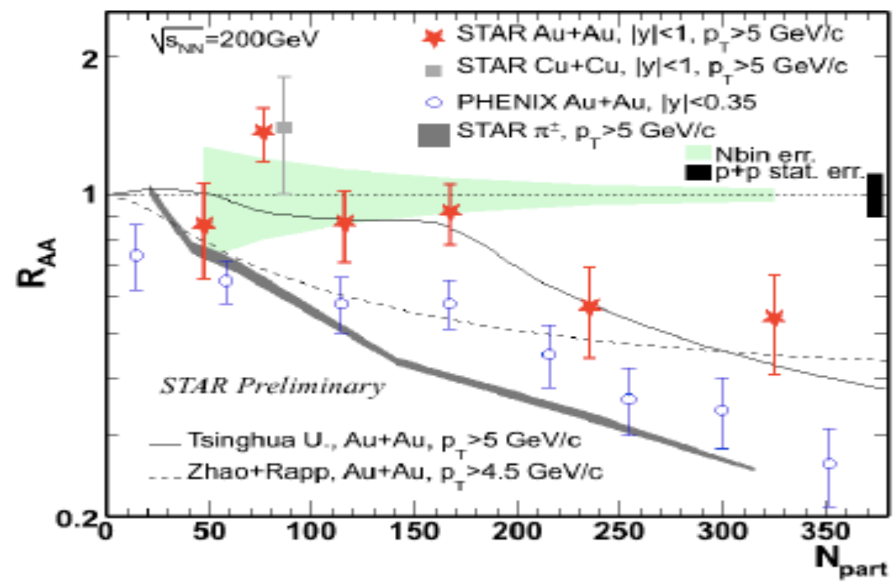
  Not a straight line on the semi-log plots – additional suppression!

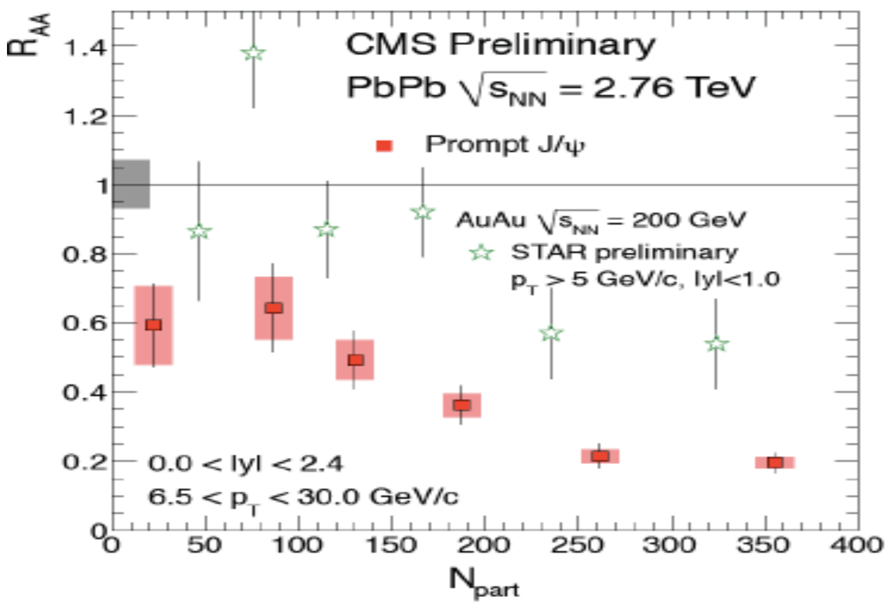


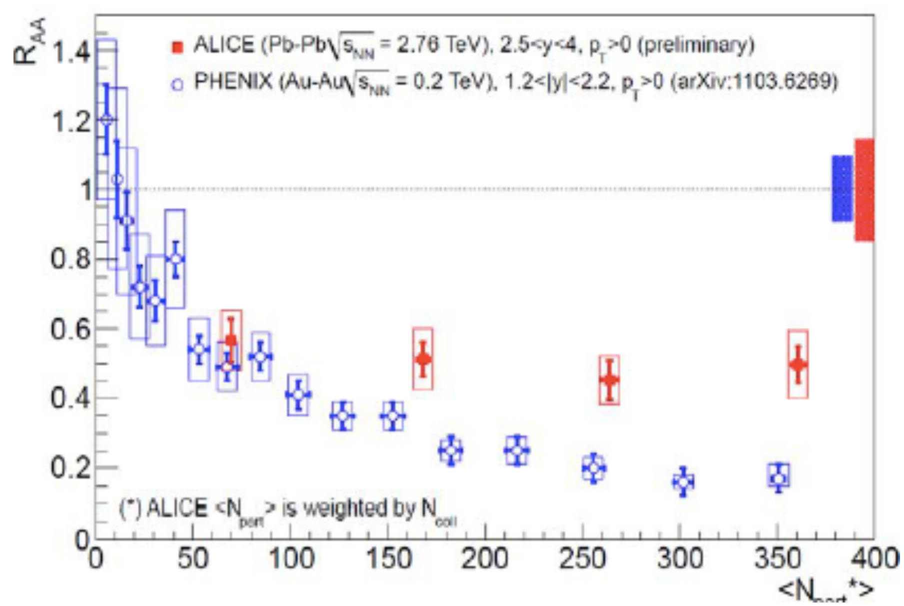


Confusion from data on AA



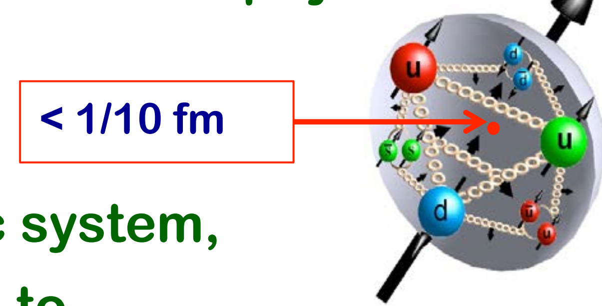






Summary

- QCD is a very successful theory for strong interaction physics.
  We have only learned a very little of it



- Heavy quarkonium provides a non-relativistic system, and could offer some important perspectives to the formation of QCD bound states

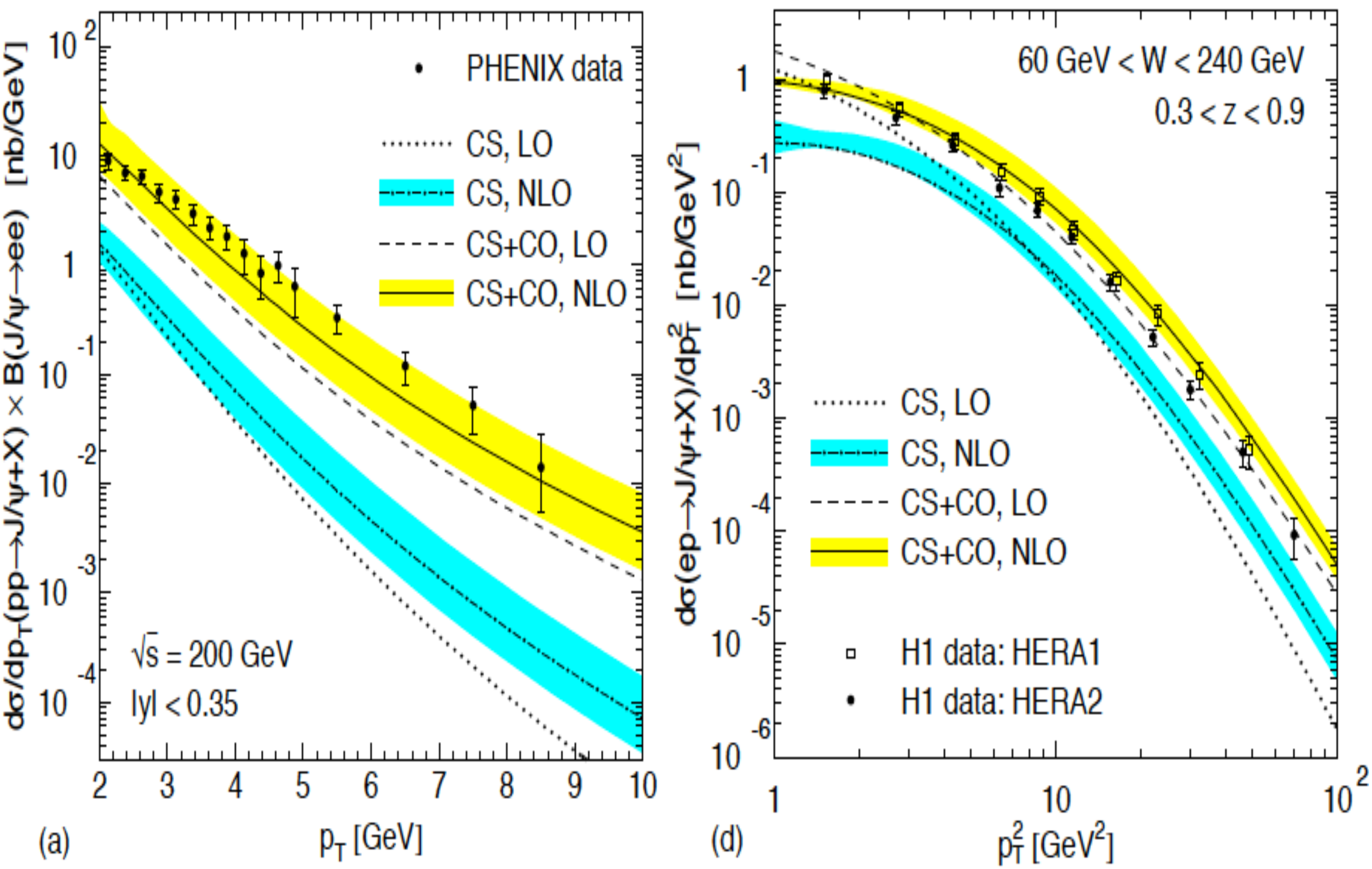
- After more than 35 years, since the discovery of J/ψ, theorists still have not been able to fully understand the production mechanism of heavy quarkonia

- RHIC/LHC are offering an excellent opportunity to learn and exam the formation of QCD bound states
  – in a vacuum, as well as at a finite temperature

Thank you!

Backup slices

Success of NRQCD model (II)



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Evolution of fragmentation functions

- **Independence of the factorization scale:**

Kang, Qiu and Sterman, 2011

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$$

- ✧ **at Leading power in $1/P_T$:**

DGALP evolution

$$\frac{d}{d\ln\mu^2}D_{i\to H}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{i\to j}(z)\otimes D_{f\to H}(z,m_Q,\mu)$$

- ✧ **next-to-leading power in $1/P_T$:**

$$\frac{d}{d\ln\mu^2}D_{i\to H}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{i\to j}(z)\otimes D_{f\to H}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{i\to[Q\bar{Q}(\kappa)]}(\{z_i\})\otimes D_{[Q\bar{Q}(\kappa)\to H}(\{z_i\},m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}D_{[Q\bar{Q}]\to H}(\{z_i\},m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}]\to[Q\bar{Q}](\kappa)]}(\{z_i\})\otimes D_{[Q\bar{Q}(\kappa)]\to H}(\{z_1\},m_Q,\mu)$$

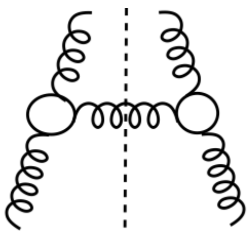
- **Evolution kernels are perturbative:**

- ✧ **Set mass:** $m_Q \to 0$ **with a caution**

Evolution kernels

Kang, Qiu and Sterman, 2011

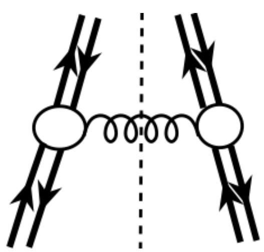
- **Single parton:**



Same as normal DGLAP
Difference from input distribution

$$D_{g\to H}(z,\mu_0,m_Q)$$

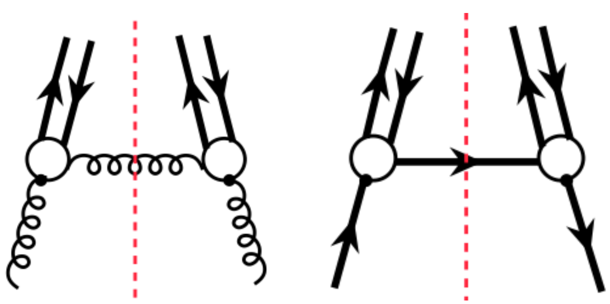
- **Heavy quark pair:**



Differ from DGLAP – still logarithmic
Spin-color sensitive
Infrared safe evolution

$$D_{[Q\bar{Q}(\kappa)]\to H}(z,\mu_0,m_Q)$$

- **Mixing:**



Non-logarithmic contribution
to the evolution
Needed to remove CO divergence

$$\sigma^{(4)}_{q\bar{q}\to Q\bar{Q}gg} \qquad \text{NLO to } \hat{\sigma}_{q\bar{q}\to Q\bar{Q}}$$