Parton fragmentation within an identified jet

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Outline

- Single inclusive light hadron production: the fragmentation function $D_j^h(z)$
- Constraints on jet invariant mass s : the fragmenting jet function $\mathcal{G}_j^h(s,z)$
- Relations between $\mathcal{G}_j^h(s,z)$, the jet function $J_j(s)$ (pert.) and $D_j^h(z)$ (non-pert.)
- $e^+e^- \to X\pi^+$ in the dijet limit ($\tau^{\rm cut} \ll 1$) : $\mathrm{d}^2\sigma/\mathrm{d}\tau\,\mathrm{d}z$ up to NNLL accuracy
- Perp-momentum dependence : $\mathcal{G}_j^h(s,z,p_\perp^h)$ for $p_\perp^h \sim \sqrt{s}$ and beam functions

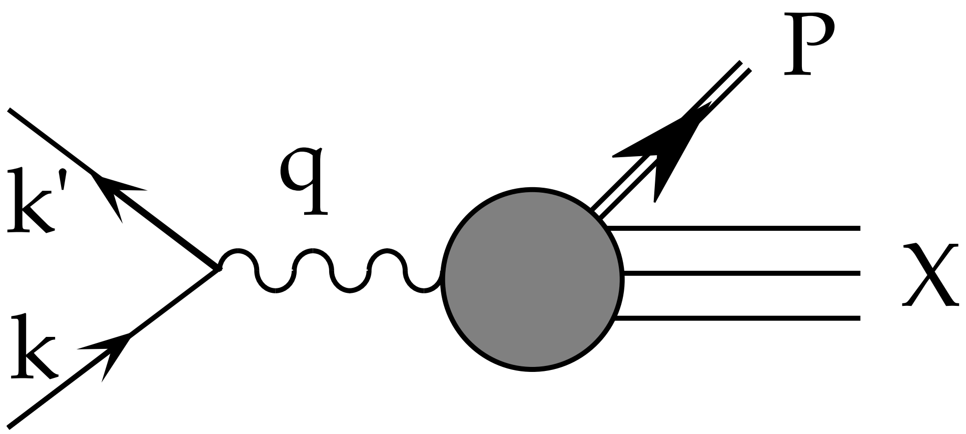
MP and I. W. Stewart, PRD 81 (2010)

A. Jain, MP and W. J. Waalewijn, JHEP 1105 and forthcoming

Single inclusive (SI) hadron production

e.g. : $e^+ e^- \to h\, X$

$q^2 > 0\,,\;\; \nu = P \cdot q$



$$d\sigma \sim L^{\alpha\beta} W_{\alpha\beta}\, \frac{d^3 P}{(2\pi)^3 2E}$$

$$W_{\alpha\beta} = \frac{1}{4\pi} \int d^4\xi\, e^{iq\cdot\xi} \sum_X \langle 0 | J_\alpha(\xi) | hX \rangle \langle hX | J_\beta(0) | 0 \rangle$$

vs. $$W^{\rm DIS}_{\alpha\beta} = \frac{1}{4\pi} \int d^4\xi\, e^{iq\cdot\xi} \sum_X \langle P | J_\alpha(\xi) | X \rangle \langle X | J_\beta(0) | P \rangle$$

Factorization in SI hadron production

- $e^+ e^- \to hX$ at high c.m. energy Q ($q^2, \nu \to \infty$) , to all orders in α_s , at leading power in $\Lambda_{\rm QCD}/Q$:

Collins, Soper, Sterman

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \sigma_0 \sum_{i=g,u,\bar{u},d,\ldots} \int_z^1 \frac{\mathrm{d}x}{x} \, C_i\left(Q, \frac{z}{x}, \mu\right) D_i^h(x,\mu)$$

pert. non-pert.

z is the fraction hadron/parton large light-cone momentum component:

$$z^{\rm exp} = 2\,\nu/q^2 = 2\,E_h^{\rm c.m.}/Q$$

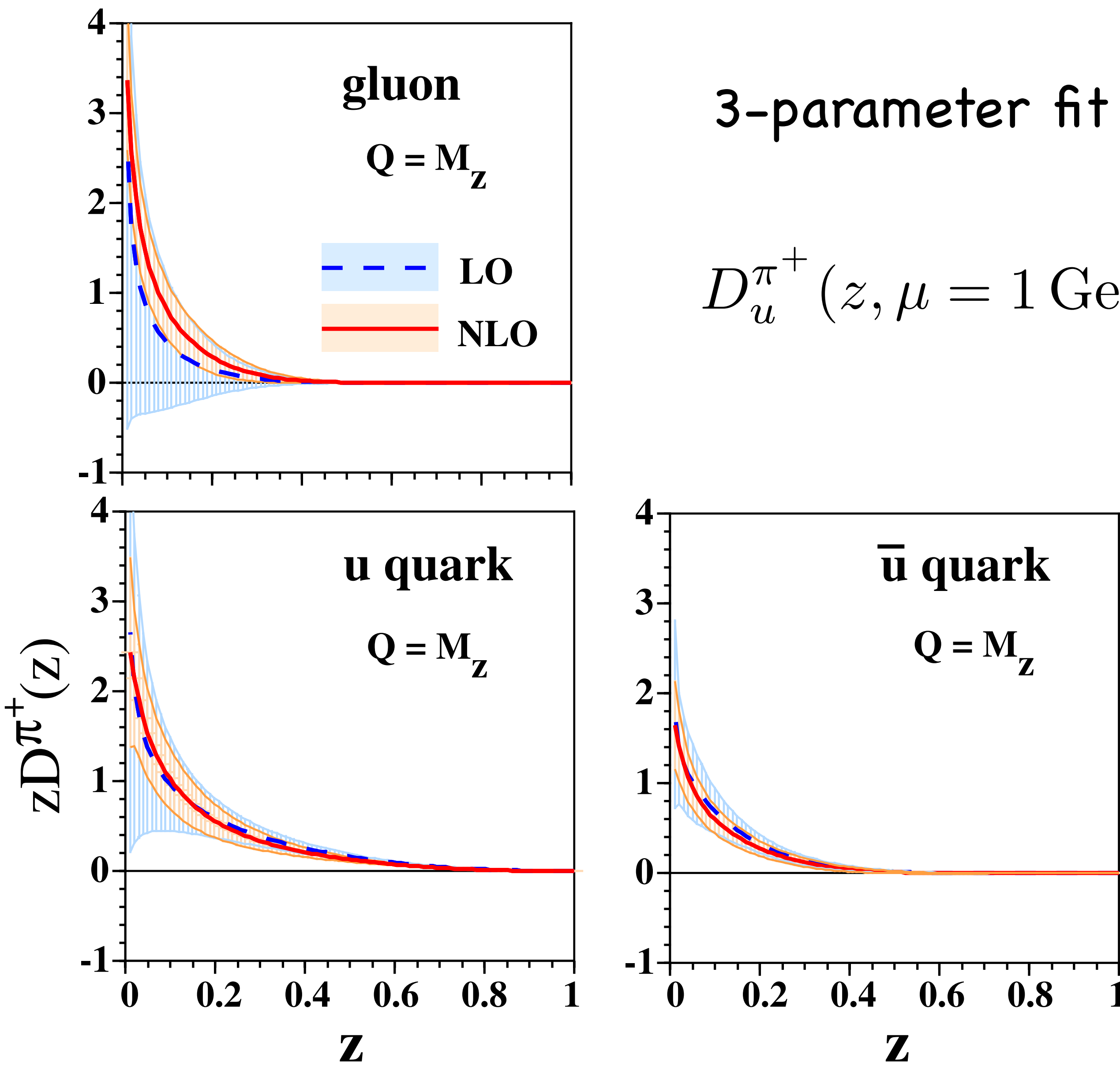
- The fragmentation function $D_i^h(z)$ is non-perturbative but universal

Collins and Soper (1982)

→ constraints on model parameters from phenomenology

Pion fragmentation from phenomenology

$$\frac{1}{\sigma_0}\frac{d\sigma}{dz}(e^+e^- \to \pi^+ X) = \sum_{i=u,\bar{u},d,g\ldots} \int_z^1 \frac{dx}{x}\, C_i\left(\frac{Q^2}{\mu^2}, \frac{z}{x}, \alpha_S(\mu)\right) D_i^{\pi^+}(x,\mu)$$



3-parameter fit Ansatz (CERN, DESY, KEK & SLAC data):

$$D_u^{\pi^+}(z,\mu = 1\,\mathrm{GeV}) = \frac{M_u^{\pi^+}}{B(\alpha_u^{\pi^+}+2, \beta_u^{\pi^+}+1)}\, z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}}$$

$$M_i^h = \int_0^1 \mathrm{d}z\, z\, D_i^h(z,\mu) < 1$$

Hirai et al. (2007)

input for our numerical analysis of $\mathrm{d}^2\sigma/\mathrm{d}\tau\,\mathrm{d}z$

The fragmentation function

Collins and Soper

$$n^\mu = (1,0,0,1), \quad \bar{n}^\mu = (1,0,0,-1), \quad p^+ = n\cdot p, \quad p^- = \bar{n}\cdot p \quad \text{(large)}$$

$$D_q^h(z) = z\int \frac{\mathrm{d}x^+}{4\pi}\, e^{ik^- x^+/2}\, \frac{1}{4N_c}\,\mathrm{Tr}\sum_X \langle 0|\bar{\not n}\,\Psi(x^+,0,0_\perp)|Xh\rangle\langle Xh|\bar{\Psi}(0)|0\rangle\Big|_{p_h^\perp=0}$$

- Gauge invariance: $\Psi(x^+,0,0_\perp)$ contains a Wilson line of gluon fields
- Boost invariance: D is a function of $z = p_h^-/k^-$

Spin-averaged fragmentation in SI jet-like processes where a light hadron fragments from a collimated jet whose invariant mass s is constrained:

$$\mathcal{G}(s,z)$$

Fragmentation in $e^+ e^- \rightarrow$ dijets

- Light quark fragmentation from B-factory data: restrict to dijet configurations

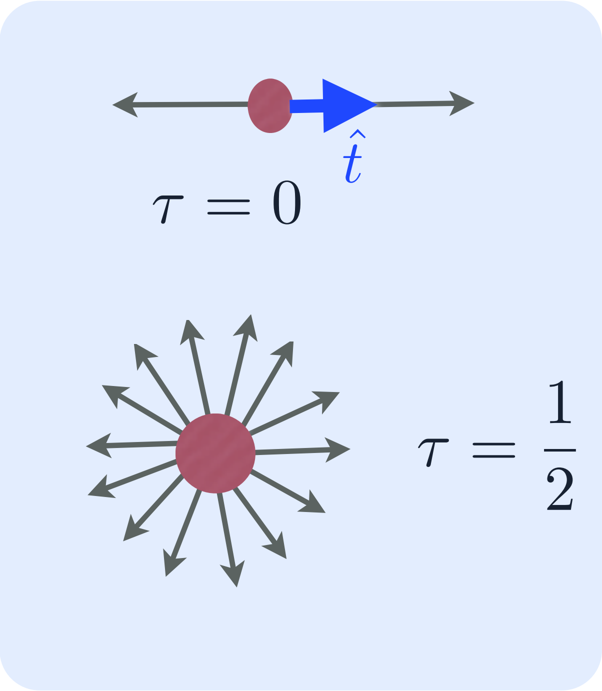
  Belle collaboration, Seidl et al. (2008)

  → very high statistical precision compared to previous e^+e^- analyses

- Dijets through a cut on the event shape variable thrust :

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{Q} \qquad \tau = 1 - T$$

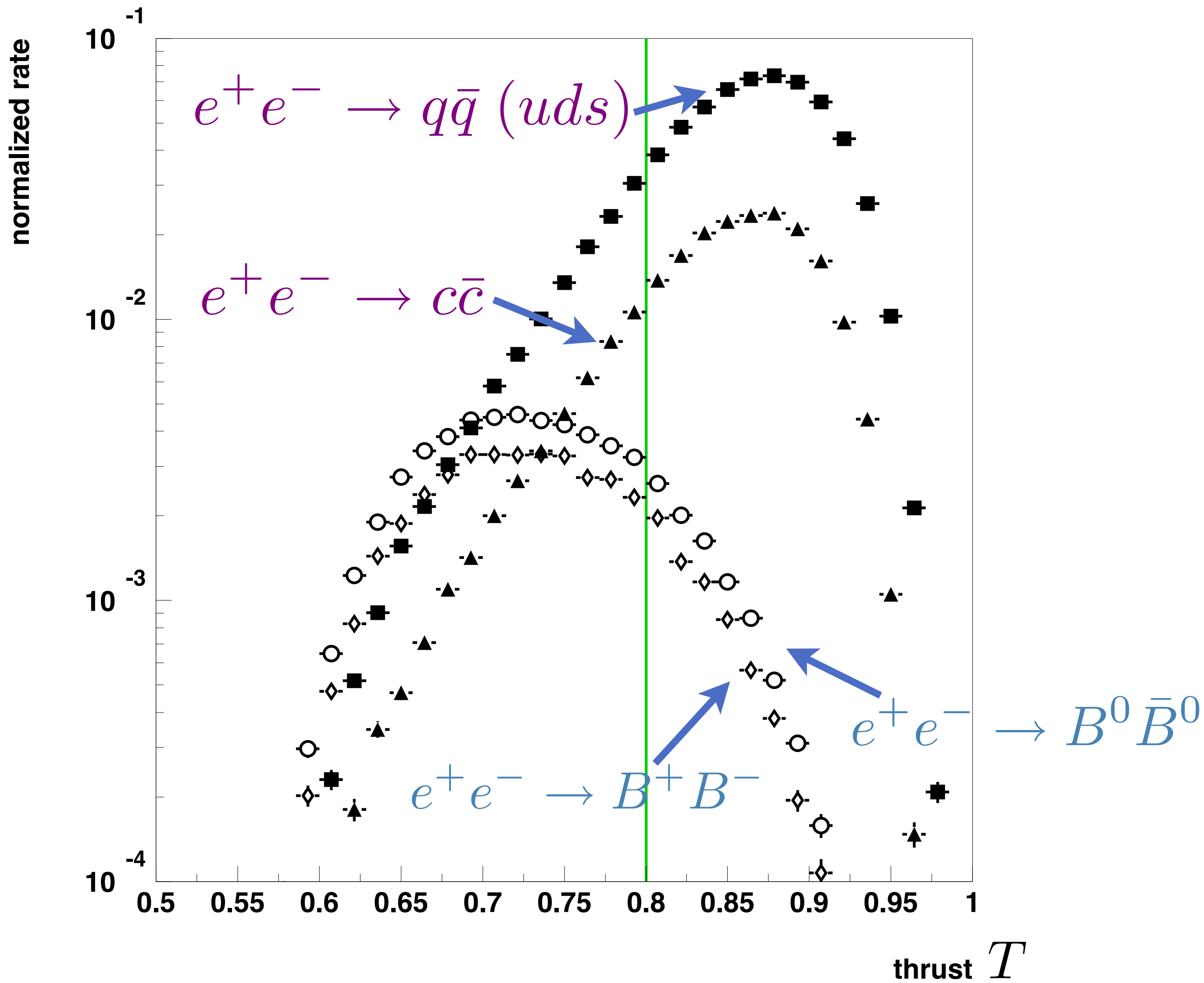
$\hat{t}$ defines the thrust axis



Fragmentation in $e^+e^- \to$ dijets

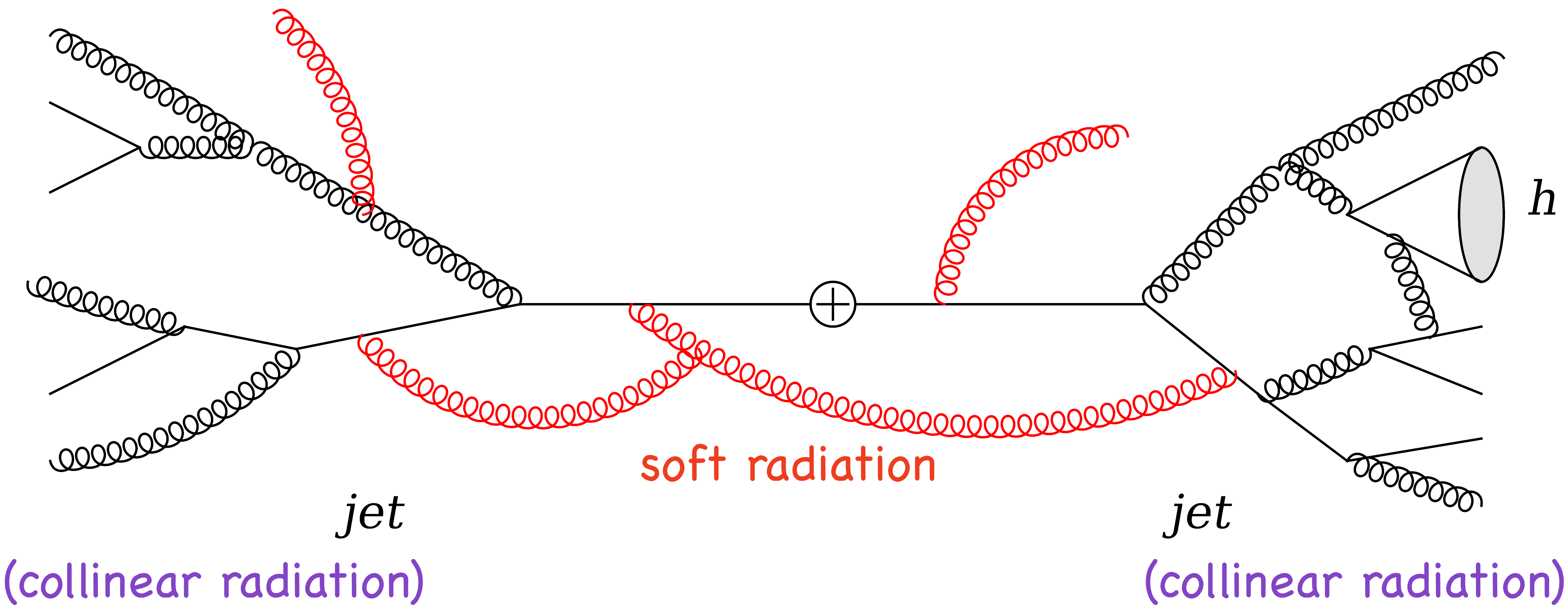
- Light quark fragmentation from B-factory data: restrict to dijet configurations

Simulated thrust distribution on the Υ(4S) resonance



- Cut on thrust $\tau < \tau^{\text{cut}} = 1 - T^{\text{cut}} = 0.2$ removes the b-quark contribution

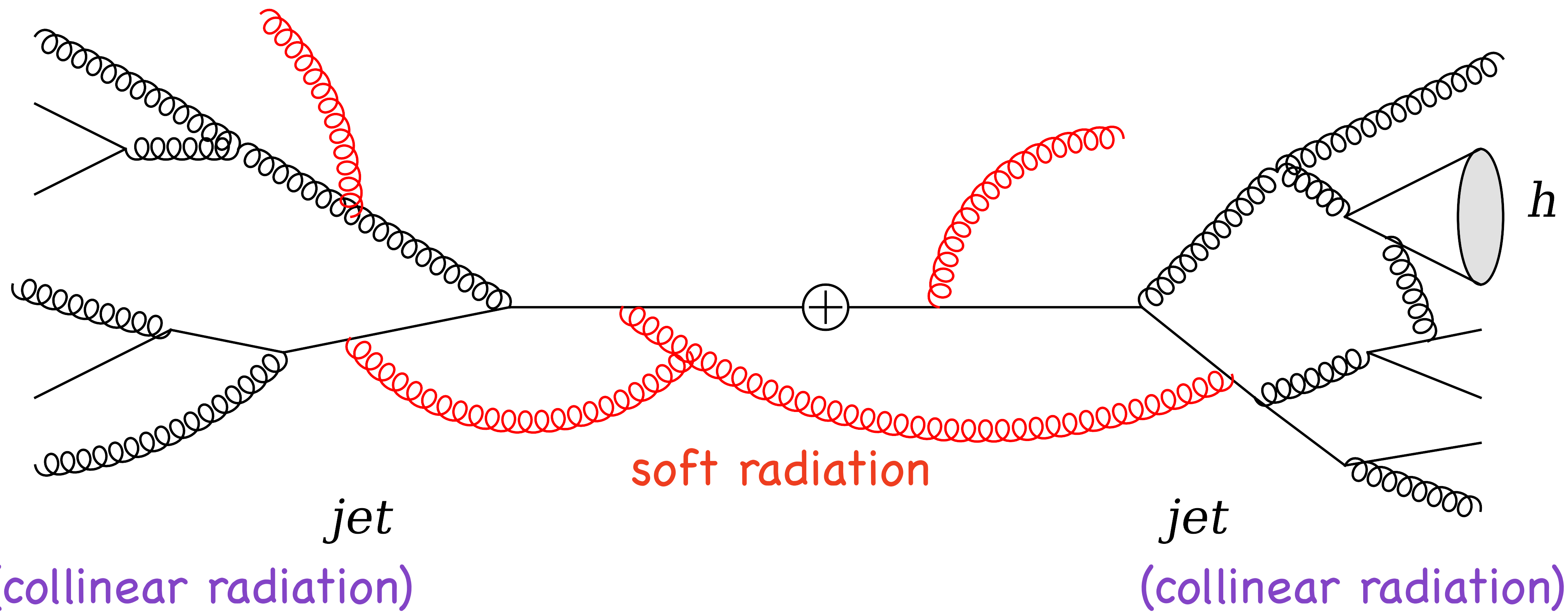
Fragmentation in e⁺ e⁻ → dijets



In the dijet limit (τ close to 0) : $\tau = \frac{s_{\mathrm{jet}_1} + s_{\mathrm{jet}_2}}{Q^2} + \frac{k_{\mathrm{soft}}}{Q}$

Additional restriction on τ introduces a new (jet) scale : $\Lambda_{\mathrm{QCD}} \ll \sqrt{\tau} Q \ll Q$

Fragmentation in e$^+$ e$^-$ → dijets



In the dijet limit (τ close to 0) : $\tau = \frac{s_{\text{jet}_1} + s_{\text{jet}_2}}{Q^2} + \frac{k_{\text{soft}}}{Q}$

Factorization theorem for $\mathrm{d}^2\sigma/\mathrm{d}\tau\,\mathrm{d}z$ involves $\mathcal{G}_i^h(s,z)$

Using EFT: resummation of large logs induced by the cut on τ

Fragmentation within a jet and SCET

- hard scale $E_{\rm jet}$, jet scale $\sqrt{s}$, soft scale $s/E_{\rm jet}$; $m_h \ll \sqrt{s}$

- hierarchy allows us to employ Soft-Collinear Effective Theory (SCET) :

hard dynamics integrated out by matching QCD onto SCET currents

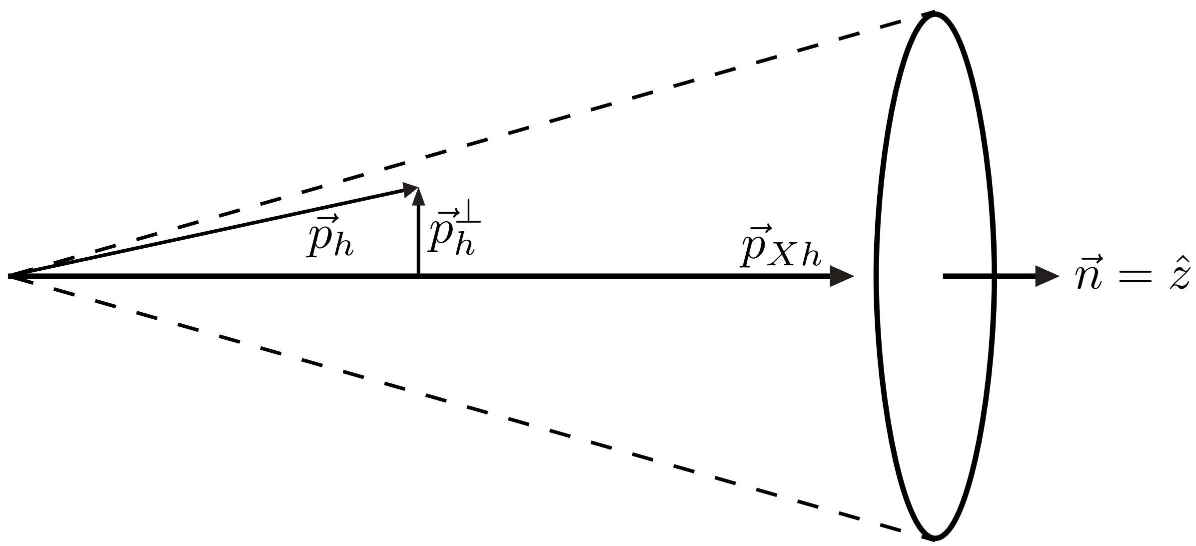
collinear d.o.f. (radiation inside jets): $p^\mu = (p^+, p^-, p_\perp^\mu) \sim p^-(\lambda^2, 1, \lambda)$

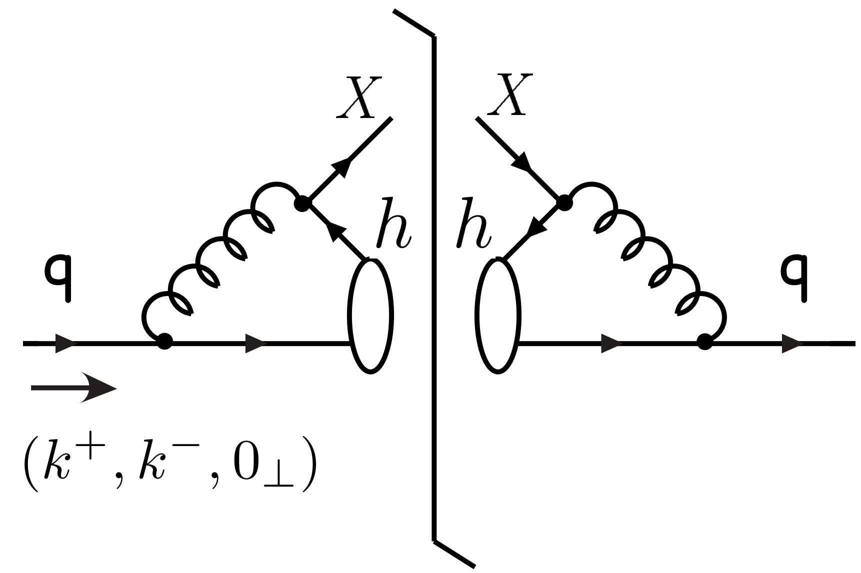
usoft d.o.f. (soft emissions between jets): $q^\mu = (q^+, q^-, q_\perp^\mu) \sim p^-(\lambda^2, \lambda^2, \lambda^2)$

$\lambda \sim m_{Xh}/E_{Xh} \ll 1$ is the SCET expansion parameter

Fragmenting jet functions

In a frame where the jet perpendicular momentum vanishes,

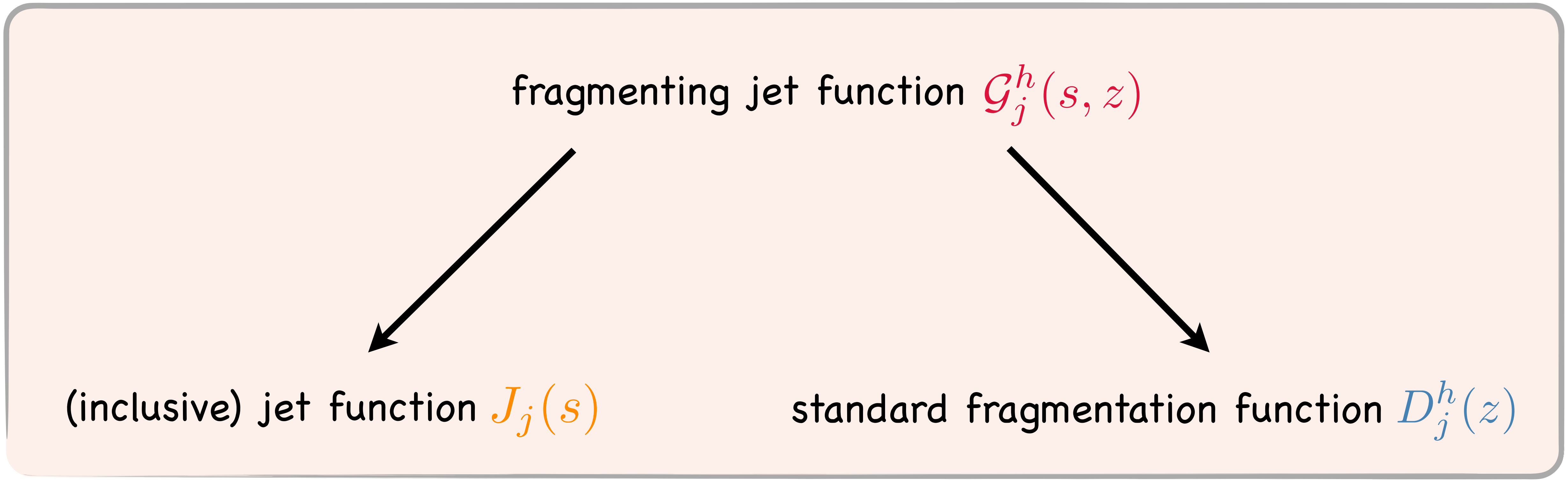




$s = k^- k^+$ $\qquad$ $z = p_h^- / k^-$

$$\mathcal{G}_q^h(s,z) = \frac{2(2\pi)^3}{2N_c\, z} \int d^2 p_h^\perp \sum_X \mathrm{tr}\Big[\frac{\bar{n}\!\!\!/}{2} \langle 0 | [\delta(k^- - \overline{\mathcal{P}}_n)\, \delta^2(\vec{\mathcal{P}}_{n\perp}) \chi_n(0)]\, \delta(s - k^- \hat{p}^+) | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle \Big]$$

$$\mathcal{G}_g^h(s,z) = -\frac{2(2\pi)^3\, k^-}{2(N_c^2 - 1) z} \int d^2 p_h^\perp \sum_X \langle 0 | [\delta(k^- - \overline{\mathcal{P}}_n)\, \delta^2(\vec{\mathcal{P}}_{n\perp}) \mathcal{B}_{n\perp}^{\mu,a}(0)]\, \delta(s - k^- \hat{p}^+) | Xh \rangle \langle Xh | \mathcal{B}_{n\perp,\mu}^{a}(0) | 0 \rangle$$



fragmenting jet function $\mathcal{G}_j^h(s,z)$
(inclusive) jet function $J_j(s)$
standard fragmentation function $D_j^h(z)$

Inclusive vs. semi-inclusive case

- Factorization for inclusive observables: convolution of jet- and soft-functions

$$\frac{1}{2N_c}\,\mathrm{Tr}\sum_{X_u}\langle 0|\frac{\bar{n}\!\!\!/}{2}\,\chi_n(x)|X_u\rangle\langle X_u|\bar{\chi}_{n,k^-,0_\perp}(0)|0\rangle = \delta(x^+)\,\delta^2(x_\perp)\;k^-\!\int dk^+\,e^{-ik^+x^-/2}\,J_q(\underbrace{k^+k^-}_{s})$$

- If a light hadron h fragments within an identified jet

MP and Stewart (2010)

$$J_i(s,\mu)\;\longrightarrow\;\frac{1}{2(2\pi)^3}\,\mathcal{G}_i^h(s,z,\mu)\,\mathrm{d}z$$

Example: $\bar{B}\to(X\pi)_u\ell\bar{\nu}$, at leading power

$$\frac{d^3\Gamma}{dp^+_{X\pi}\,dp^-_{X\pi}\,dz} = \Gamma_0\,H(m_B,p^-_{X\pi},p^+_{X\pi},\mu)\,p^-_{X\pi}\int_0^{p^+_{X\pi}}\mathrm{d}k^+\,\mathcal{G}^\pi_u(\underbrace{k^+p^-_{X\pi}}_{s},\,\overset{p^-_\pi/p^-_{X\pi}}{z},\mu)\,S(p^+_{X\pi}-k^+,\mu)$$

Fragmenting jet function vs. jet function

- Renormalization and RG evolution of $\mathcal{G}_i^h$ and J_i are the same:

$$\mathcal{G}_{i,\mathrm{bare}}^h(s,z) = \int_0^s \mathrm{d}s'\, Z_{\mathcal{G}}^i(s-s',\mu)\, \mathcal{G}_i^h(s',z,\mu)\,, \qquad Z_{\mathcal{G}}^i(s,\mu) = Z_J^i(s,\mu)$$

- Summing over all hadrons in the jet originated by parton i,

$$\int_0^1 \mathrm{d}z\, z \sum_{h\in\mathcal{H}_i} \sum_X |Xh(z)\rangle\langle Xh(z)| = \sum_{X_i} |X_i\rangle\langle X_i| = \mathbb{1}$$

Jain, MP and Waalewijn (2011)

$\rightarrow$

$$\sum_h \int_0^1 \mathrm{d}z\, z\, D_j^h(z,\mu) = 1$$

$$\sum_{h\in\mathcal{H}_i} \int_0^1 \mathrm{d}z\, z\, \mathcal{G}_i^h(s,z,\mu) = 2(2\pi)^3 J_i(s,\mu)$$

Relation with fragmentation function D

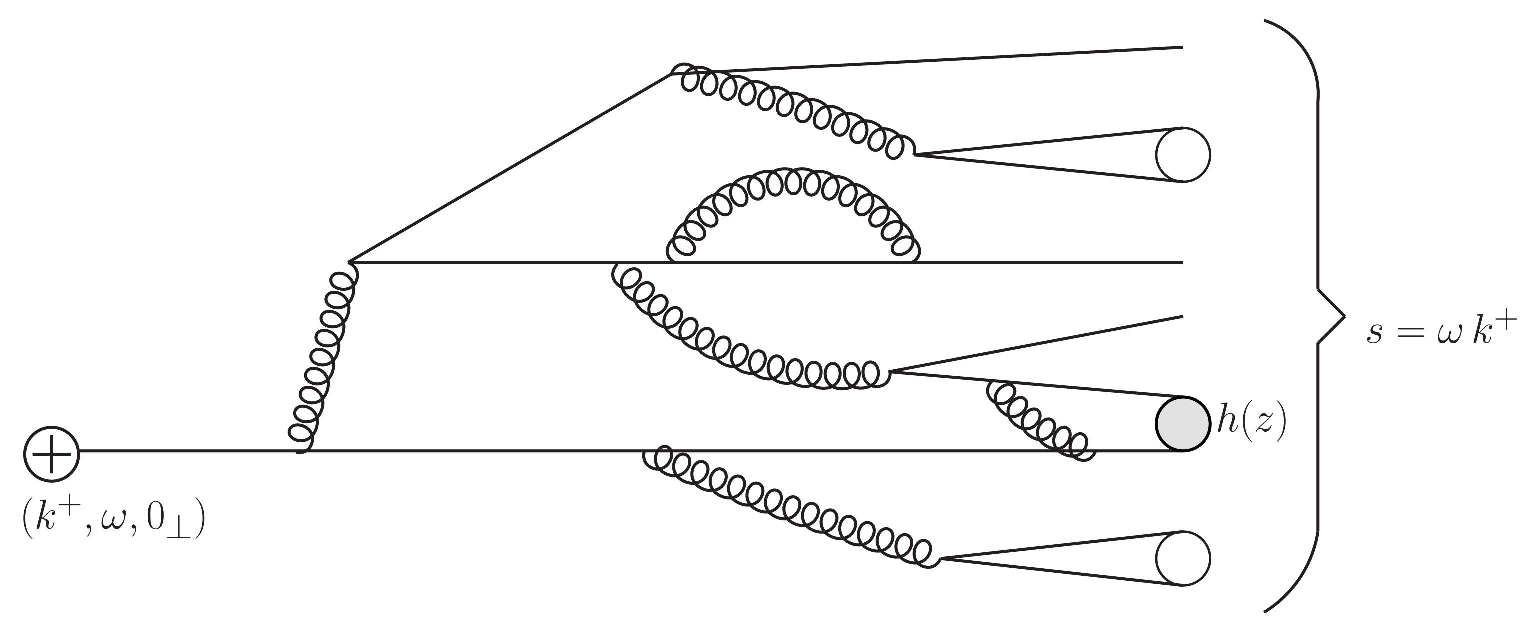
By performing an OPE, match $\mathcal{G}_i^h(s,z)$ onto $D_j^h(z)$ at the intermediate scale $\sqrt{s}$:

$$\mathcal{G}_i^h(s,z,\mu_J) = \sum_{j=g,u,\bar{u},d,\dots} \int_z^1 \frac{\mathrm{d}z'}{z'} \mathcal{J}_{ij}\Big(s, \frac{z}{z'}, \mu_J\Big) D_j^h(z',\mu_J)\Big[1 + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}^2}{s}\Big)\Big]$$

Jain, MP and Waalewijn (2011)

$$\mathcal{G} = \mathcal{J} \otimes D$$

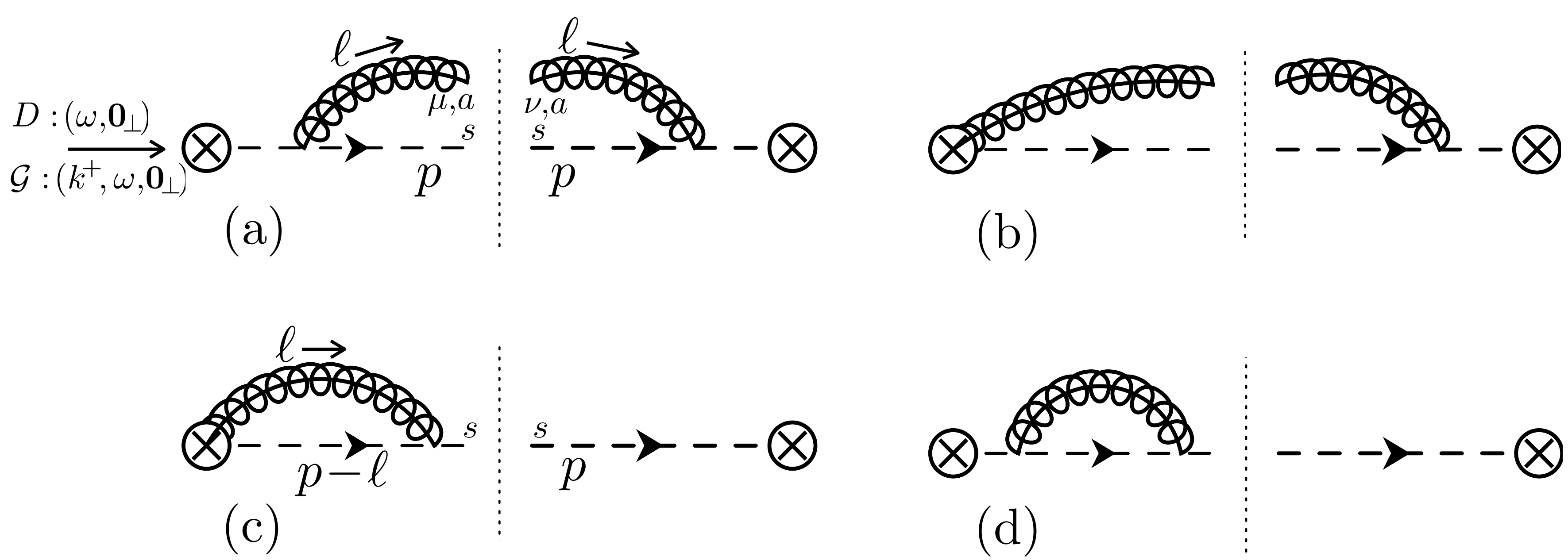
emissions with larger virtualities



$\mathcal{J}_{qj}$ matching coefficients to one loop

$$\mathcal{G}_i^{j(1)}(s,z,\mu_J) = 2(2\pi)^3\,\delta(s)\,D_i^{j(1)}(z,\mu_J) + \mathcal{J}_{ij}^{(1)}(s,z,\mu_J)$$

- parton i is a quark, non-vanishing diagrams in the Feynman gauge:



- gluon mass + δ-regulator for IR, dim. reg. only for UV, $\overline{\mathrm{MS}}$ scheme, to check:

$$\gamma_{qq}^D(z,\mu) = \frac{\alpha_s(\mu)}{\pi}\,\theta(1-z)\theta(z)P_{qq}(z)\,, \qquad \gamma_{qg}^D(z,\mu) = \frac{\alpha_s(\mu)}{\pi}\theta(1-z)\theta(z)P_{gq}(z)$$

$$\gamma_{\mathcal{G}}^i(\alpha_s) = \gamma_J^i(\alpha_s)\,,$$

the IR divergences cancel in the matching

$\mathcal{J}_{qj}$ matching coefficients to one loop

$$\frac{\mathcal{J}_{qq}^{(1)}(s,z,\mu_J)}{2(2\pi)^3} = \frac{\alpha_s(\mu_J)C_F}{2\pi}\,\theta(z)\left\{\frac{2}{\mu_J^2}\mathcal{L}_1\left(\frac{s}{\mu_J^2}\right)\delta(1-z)+\frac{1}{\mu_J^2}\mathcal{L}_0\left(\frac{s}{\mu_J^2}\right)(1+z^2)\mathcal{L}_0(1-z)\right.$$
$$\left. + \delta(s)\Big[(1+z^2)\mathcal{L}_1(1-z) + P_{qq}(z)\ln z + \theta(1-z)(1-z) - \frac{\pi^2}{6}\delta(1-z)\Big]\right\},$$
$$\frac{\mathcal{J}_{qg}^{(1)}(s,z,\mu_J)}{2(2\pi)^3} = \frac{\alpha_s(\mu_J)C_F}{2\pi}\,\theta(z)\left\{\Big[\frac{1}{\mu_J^2}\mathcal{L}_0\left(\frac{s}{\mu_J^2}\right) + \delta(s)\ln\left(z(1-z)\right)\Big]P_{gq}(z) + \delta(s)\,\theta(1-z)z\right\}$$

with $\mathcal{L}_n(x) \equiv \left[\dfrac{\theta(x)\ln^n x}{x}\right]_+$

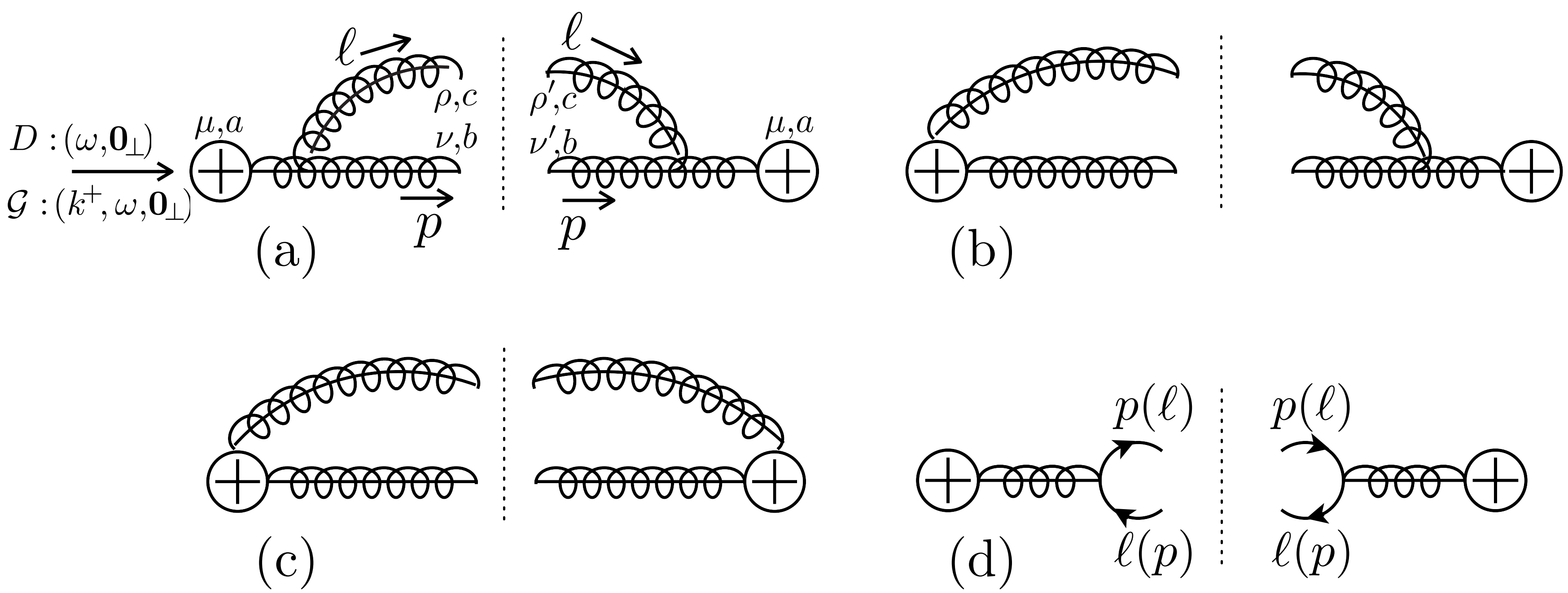
$\mu_J \simeq \sqrt{s}$ to avoid large logs

agrees with result in dim. reg. by X.Liu (2010)

$\mathcal{J}_{gj}$ matching coefficients to one loop

$$\mathcal{G}_i^{j(1)}(s,z,\mu_J) = 2(2\pi)^3\,\delta(s)\,D_i^{j(1)}(z,\mu_J) + \mathcal{J}_{ij}^{(1)}(s,z,\mu_J)$$

- parton i is a gluon:



- diagrams (b) and (c) vanish
- used dim. reg. both for UV and IR (no contribution from virtual emission)

$\mathcal{J}_{gj}$ matching coefficients to one loop

$$\frac{\mathcal{J}_{gg}^{(1)}(s,z,\mu_J)}{2(2\pi)^3} = \frac{\alpha_s(\mu_J)C_A}{2\pi}\theta(z)\Big\{\frac{2}{\mu_J^2}\mathcal{L}_1\Big(\frac{s}{\mu_J^2}\Big)\delta(1-z) + \frac{1}{\mu_J^2}\mathcal{L}_0\Big(\frac{s}{\mu_J^2}\Big)P_{gg}(z)$$
$$+\,\delta(s)\Big[\mathcal{L}_1(1-z)\frac{2(1-z+z^2)^2}{z} + P_{gg}(z)\ln z - \frac{\pi^2}{6}\delta(1-z)\Big]\Big\}\,,$$
$$\frac{\mathcal{J}_{gq}^{(1)}(s,z,\mu_J)}{2(2\pi)^3} = \frac{\alpha_s(\mu_J)T_F}{2\pi}\theta(z)\Big\{\Big[\frac{1}{\mu_J^2}\mathcal{L}_0\Big(\frac{s}{\mu_J^2}\Big)+\delta(s)\ln[z(1-z)]\Big]P_{qg}(z)+2\delta(s)\theta(1-z)z(1-z)\Big\}$$

with $\mathcal{L}_n(x) \equiv \Big[\frac{\theta(x)\ln^n x}{x}\Big]_+$

$\mu_J \simeq \sqrt{s}$ to avoid large logs

Fragmentation in a jet with measured invariant mass involves $\mathcal{G}_j^h(s, z)$

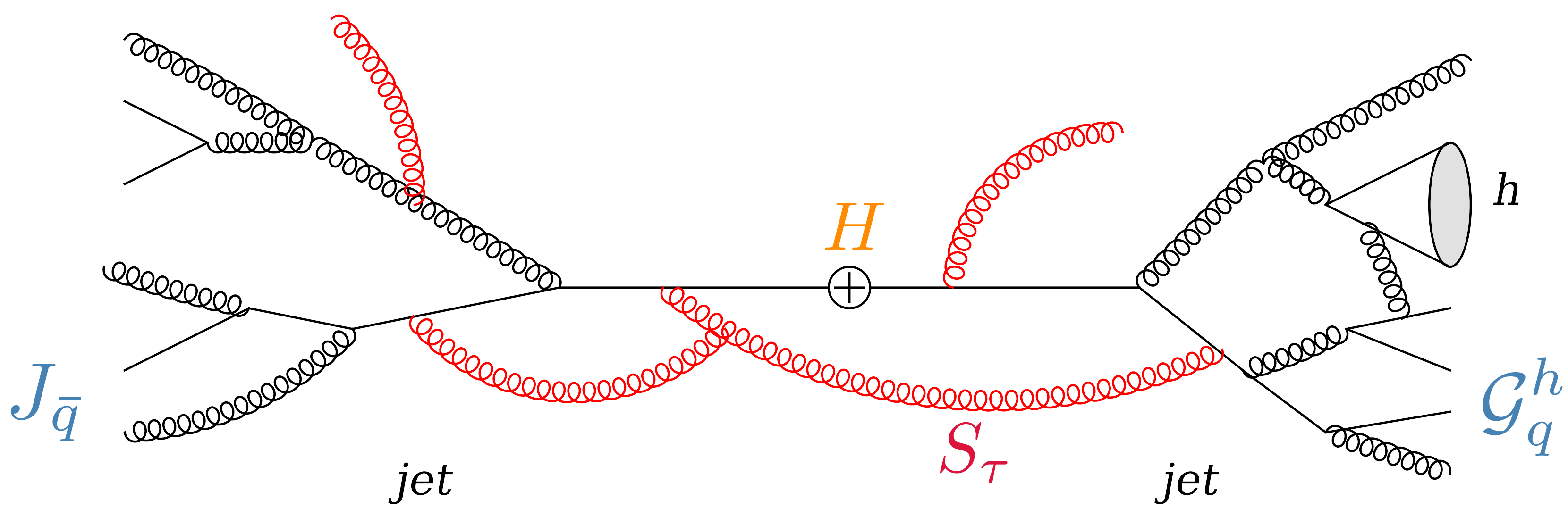
Since $\mathcal{G} = \mathcal{J} \otimes D$, factorization formulae are related with $D_j^h(z)$

Factorization theorem e⁺ e⁻ → dijet + h

Leading-order factorization formula for the singular part of the thrust distribution :

$\sim (\ln^k \tau)/\tau$ for $\tau \ll 1$

dominates in the dijet limit



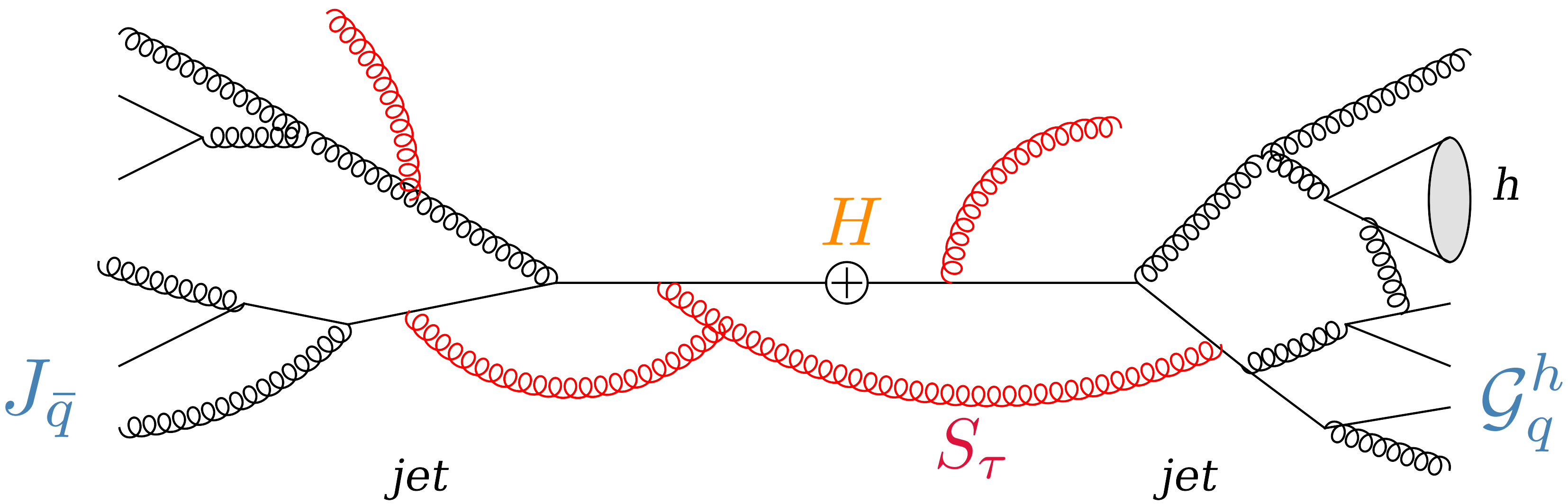
$$\frac{d^2\sigma}{d\tau\, dz} = \sum_q \frac{\sigma_0^q}{2(2\pi)^3} H(Q^2,\mu) \int ds_a\, ds_b\, dk \left[\mathcal{G}_q^h(s_a,z,\mu)\, J_{\bar{q}}(s_b,\mu) + J_q(s_a,\mu)\, \mathcal{G}_{\bar{q}}^h(s_b,z,\mu)\right]$$

$$\times\, S_\tau(k,\mu)\, \delta\Big(\tau - \frac{s_a+s_b}{Q^2} - \frac{k}{Q}\Big) \left[1 + \mathcal{O}(\tau)\right]$$

from Catani et al. (1993), Korchemsky and Sterman (1999), Fleming et al. (2008), Schwartz (2008)

Factorization theorem $e^+ e^- \to$ dijet + h

Leading-order factorization formula for the singular part of the thrust distribution :



$$\frac{d^2\sigma}{d\tau\, dz} = \sum_q \frac{\sigma_0^q}{2(2\pi)^3} H(Q^2,\mu) \int ds_a\, ds_b\, dk \left[\mathcal{G}_q^h(s_a,z,\mu)\, J_{\bar{q}}(s_b,\mu) + J_q(s_a,\mu)\, \mathcal{G}_{\bar{q}}^h(s_b,z,\mu)\right]$$

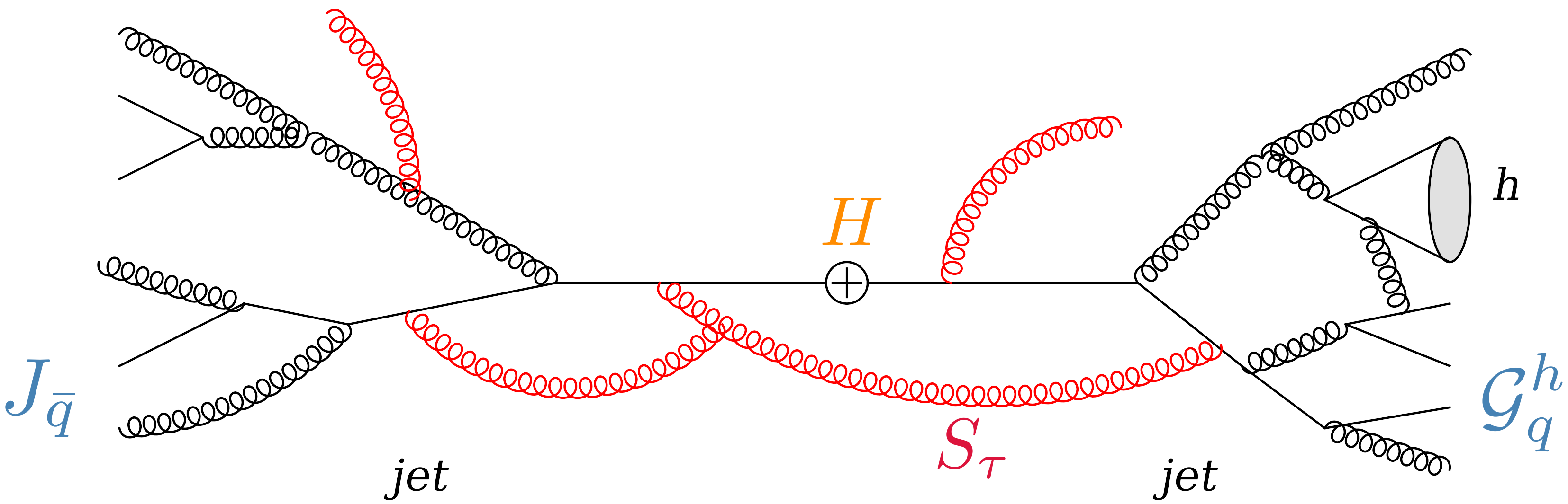
$$\times S_\tau(k,\mu)\, \delta\Big(\tau - \frac{s_a+s_b}{Q^2} - \frac{k}{Q}\Big)\Big[1+\mathcal{O}(\tau)\Big]$$

$$= \sum_{q,j} \frac{\sigma_0^q}{2(2\pi)^3} H(Q^2,\mu) \int ds_a\, ds_b\, \frac{dx}{x} \left[\mathcal{J}_{qj}\Big(s_a,\frac{z}{x},\mu\Big) J_{\bar{q}}(s_b,\mu) + J_q(s_a,\mu)\, \mathcal{J}_{\bar{q}j}\Big(s_b,\frac{z}{x},\mu\Big)\right]$$

$$\times D_j^h(x,\mu)\, Q\, S_\tau\Big(Q\tau - \frac{s_a+s_b}{Q},\mu\Big)\left[1+\mathcal{O}\Big(\tau,\frac{\Lambda_{\rm QCD}^2}{\tau Q^2}\Big)\right]$$

Factorization theorem $e^+ e^- \to$ dijet + h

Leading-order factorization formula for the singular part of the thrust distribution :



large logs as $\tau \to 0$ need to be summed for reliable predictions and uncertainty

(y = Fourier conjugate variable of τ)

$$\ln \frac{d\sigma}{dy} \sim \left[\ln y \sum_{k=1}^{\infty} (\alpha_s \ln y)^k\right]_{\mathrm{LL}} + \left[\sum_{k=1}^{\infty} (\alpha_s \ln y)^k\right]_{\mathrm{NLL}} + \left[\alpha_s \sum_{k=1}^{\infty} (\alpha_s \ln y)^k\right]_{\mathrm{NNLL}} + \ldots$$

EFT approach: H, $\mathcal{G}_i^h$, J_j , S_τ at their natural scales (no large logs) and then use RGEs

$e^+ e^- \to X\pi^+$ for small τ up to NNLL

$$\frac{d^2\sigma_{\rm sing}}{d\tau\, dz} = H(Q^2,\mu_H)\, U_H(Q^2,\mu_H,\mu) \sum_{q=\{u,\bar u,d,\bar d,s,\bar s\}} \frac{\sigma_0^q}{2(2\pi)^3} \int ds_a\, ds_b$$
$$\times \int ds'_a\, \mathcal{G}_q^{\pi^+}(s_a - s'_a, z, \mu_J)\, U_{\mathcal{G}}^q(s'_a,\mu_J,\mu) \int ds'_b\, J_{\bar q}(s_b - s'_b,\mu_J)\, U_J^q(s'_b,\mu_J,\mu)$$
$$\times \int dk\, Q\, S_\tau\Big(Q\,\tau - \frac{s_a+s_b}{Q} - k, \mu_S\Big)\, U_S(k,\mu_S,\mu)$$

- focus on "tail region" $\left(\frac{2\Lambda_{\rm QCD}}{Q} \ll \tau \lesssim 1/3\right)$, $\mu_H \simeq -iQ\,, \quad \mu_J \simeq \sqrt{\tau} Q\,, \quad \mu_S \simeq \tau Q$

Belle cut to remove b-quark contribution is $\tau < 0.2$

large π^2-terms in the hard function get summed :

$$H(Q^2,\mu_H) = \Big|1 + \frac{\alpha_s(\mu_H)\, C_F}{4\pi}\Big[-\ln^2\Big(\frac{-Q^2 - i0}{\mu_H^2}\Big) + \dots\Big]\Big|^2$$

$\sqrt{\tau}Q$: typical momentum transverse to the thrust axis within each hemisphere

$e^+ e^- \to X\pi^+$ for small τ up to NNLL

$$\frac{d^2\sigma_{\rm sing}}{d\tau\, dz} = H(Q^2,\mu_H)\, U_H(Q^2,\mu_H,\mu) \sum_{q=\{u,\bar u,d,\bar d,s,\bar s\}} \frac{\sigma_0^q}{2(2\pi)^3} \int ds_a\, ds_b$$
$$\times \int ds_a'\, \mathcal{G}_q^{\pi^+}(s_a - s_a', z, \mu_J)\, U_{\mathcal{G}}^q(s_a',\mu_J,\mu) \int ds_b'\, J_{\bar q}(s_b - s_b',\mu_J)\, U_J^q(s_b',\mu_J,\mu)$$
$$\times \int dk\, Q\, S_\tau\Big(Q\,\tau - \frac{s_a+s_b}{Q} - k, \mu_S\Big)\, U_S(k,\mu_S,\mu)$$

- focus on "tail region" $\left(\frac{2\Lambda_{\rm QCD}}{Q} \ll \tau \lesssim 1/3\right)$, $\mu_H \simeq -\mathrm{i}Q\,, \quad \mu_J \simeq \sqrt{\tau}Q\,, \quad \mu_S \simeq \tau Q$

- input for fixed-order and resummed results:

$$\mathcal{G}_i^{\pi^+}(s,z,\mu) = \sum_{j=u,d,g,\bar u\ldots} \int_z^1 \frac{dx}{x}\, \mathcal{J}_{ij}\Big(s,\frac{z}{x},\mu\Big)\, D_j^{\pi^+}(x,\mu)$$

at NLO from Hirai et al. (2007), with $\alpha_s(M_Z)$=0.125

	matching	γ_x	$\Gamma_{\rm cusp}$	β
LO	0-loop	-	-	1-loop
NLO	1-loop	-	-	2-loop
LL	0-loop	-	1-loop	2-loop
NLL	0-loop	1-loop	2-loop	2-loop
NNLL	1-loop	2-loop	3-loop	3-loop

RG counting

$e^+ e^- \to X\pi^+$ for small τ up to NNLL

$$\frac{d^2\sigma_{\text{sing}}}{d\tau\, dz} = H(Q^2,\mu_H)\, U_H(Q^2,\mu_H,\mu) \sum_{q=\{u,\bar{u},d,\bar{d},s,\bar{s}\}} \frac{\sigma_0^q}{2(2\pi)^3} \int ds_a\, ds_b$$
$$\times \int ds'_a\, \mathcal{G}_q^{\pi^+}(s_a - s'_a, z, \mu_J)\, U_{\mathcal{G}}^q(s'_a,\mu_J,\mu) \int ds'_b\, J_{\bar{q}}(s_b - s'_b,\mu_J)\, U_J^q(s'_b,\mu_J,\mu)$$
$$\times \int dk\, Q\, S_\tau\Big(Q\,\tau - \frac{s_a+s_b}{Q} - k, \mu_S\Big)\, U_S(k,\mu_S,\mu)$$

- Distinct kinematic regions, different importance of summation of large (double) logs of τ and non-perturbative corrections to S_τ:

$$\begin{aligned} \text{peak:} &\quad \mu_H \simeq -\mathrm{i}Q\,, \qquad \mu_J \simeq \sqrt{\Lambda_{\text{QCD}} Q}\,, \qquad \mu_S = \Lambda_{\text{QCD}} \\ \text{tail:} &\quad \mu_H \simeq -\mathrm{i}Q\,, \qquad \mu_J \simeq \sqrt{\tau}Q\,, \qquad \mu_S \simeq \tau Q \\ \text{far tail:} &\quad \mathrm{i}\mu_H = \mu_J = \mu_S \simeq Q \end{aligned}$$

→ profile functions: τ-dependent scales which smoothly connect 3 regions

cf. Ligeti et al. (2008), Abbate et al. (2010), Berger et al. (2010)

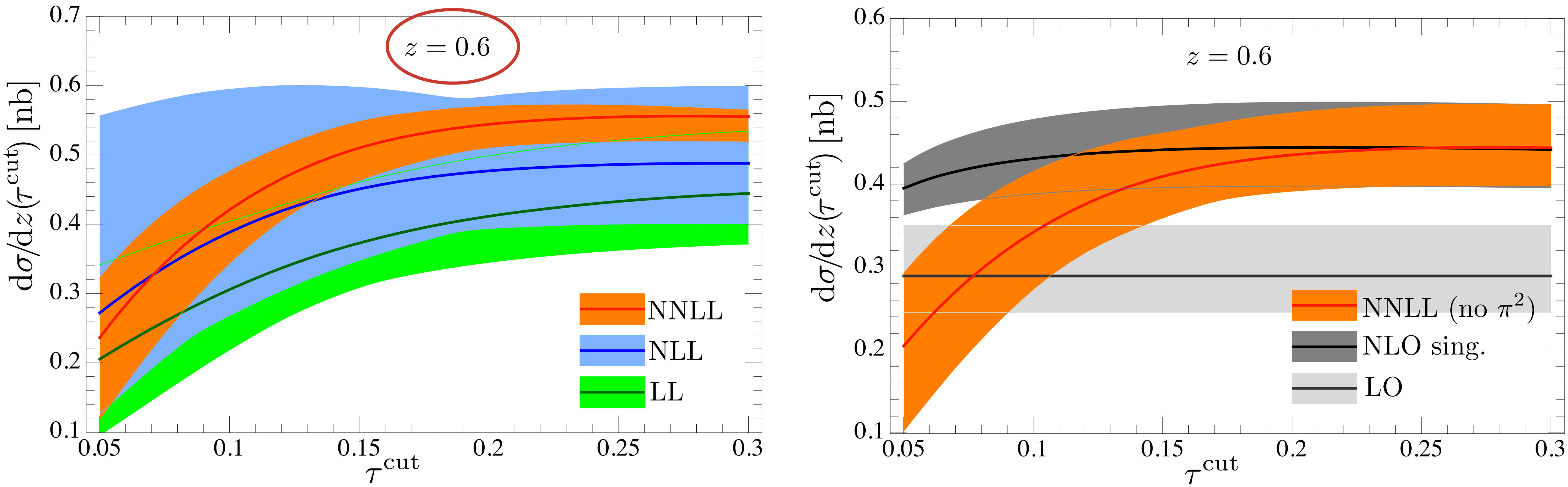
- Non-perturbative effects in S_τ in the tail region not included here

The $e^+ e^- \to X\pi^+$ cross section up to τ^{cut}

$$\frac{d\sigma}{dz}(\tau^{cut}) = \int_0^{\tau^{cut}} d\tau \, \frac{d^2\sigma}{d\tau\, dz}$$

$Q = 10.6\,\mathrm{GeV}$

$\tau^{cut} \ll 1$ selects contribution from light quark flavors (u d s)



Uncertainty estimate
- at fixed order: $Q/2 \le \mu \le 2Q$
- RG-improved: independent variations of μ_H , μ_J , μ_S

The $e^+ e^- \to X\pi^+$ cross section up to τ^{cut}

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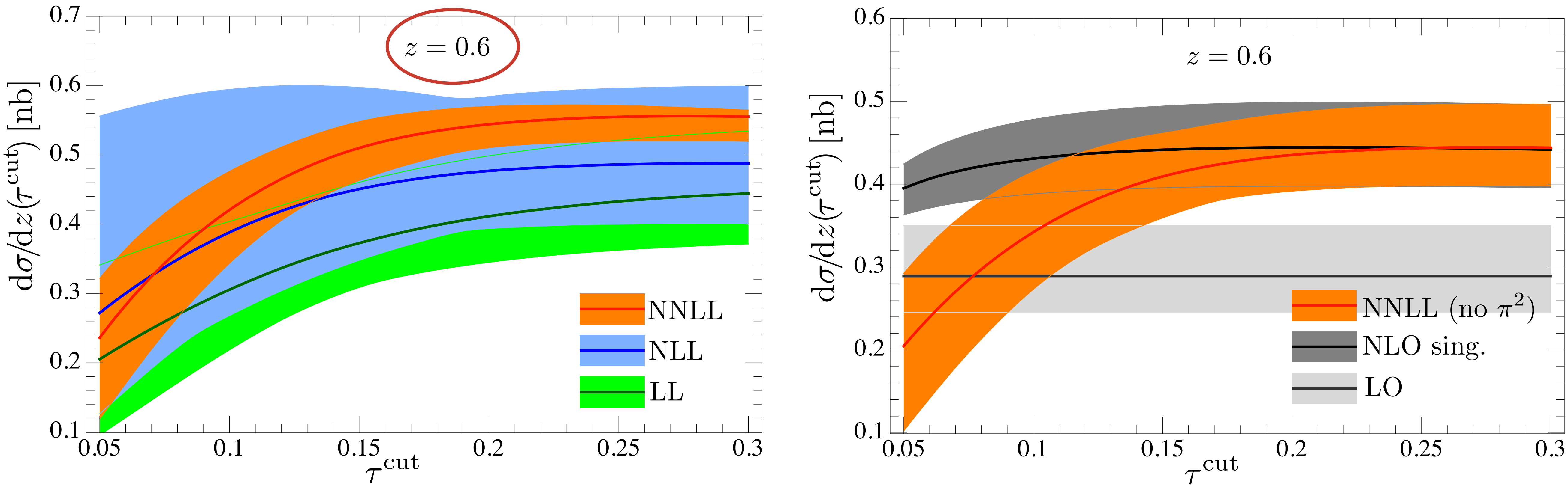

clear convergence pattern in resummed perturbation theory

The e⁺ e⁻ → Xπ⁺ cross section up to τ^{cut}

$$\frac{d\sigma}{dz}(\tau^{cut}) = \int_0^{\tau^{cut}} d\tau \, \frac{d^2\sigma}{d\tau\, dz}$$

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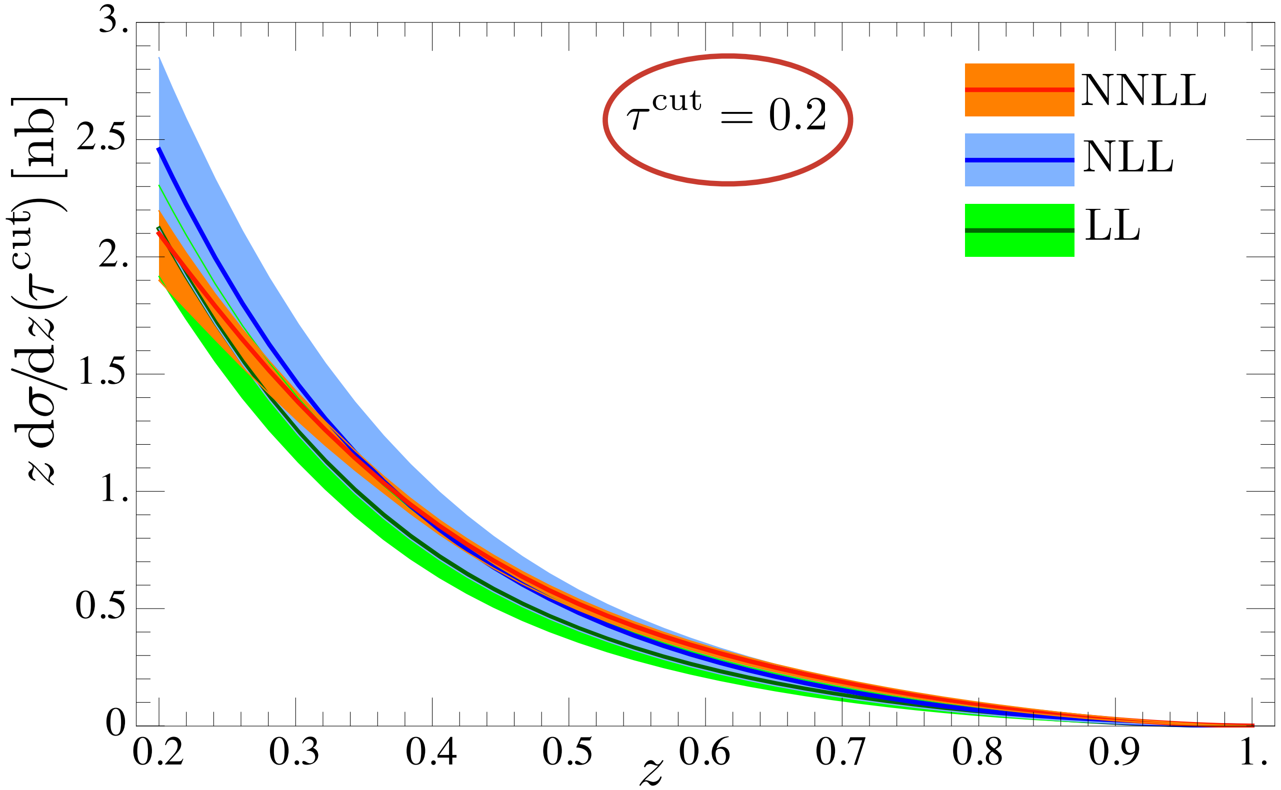

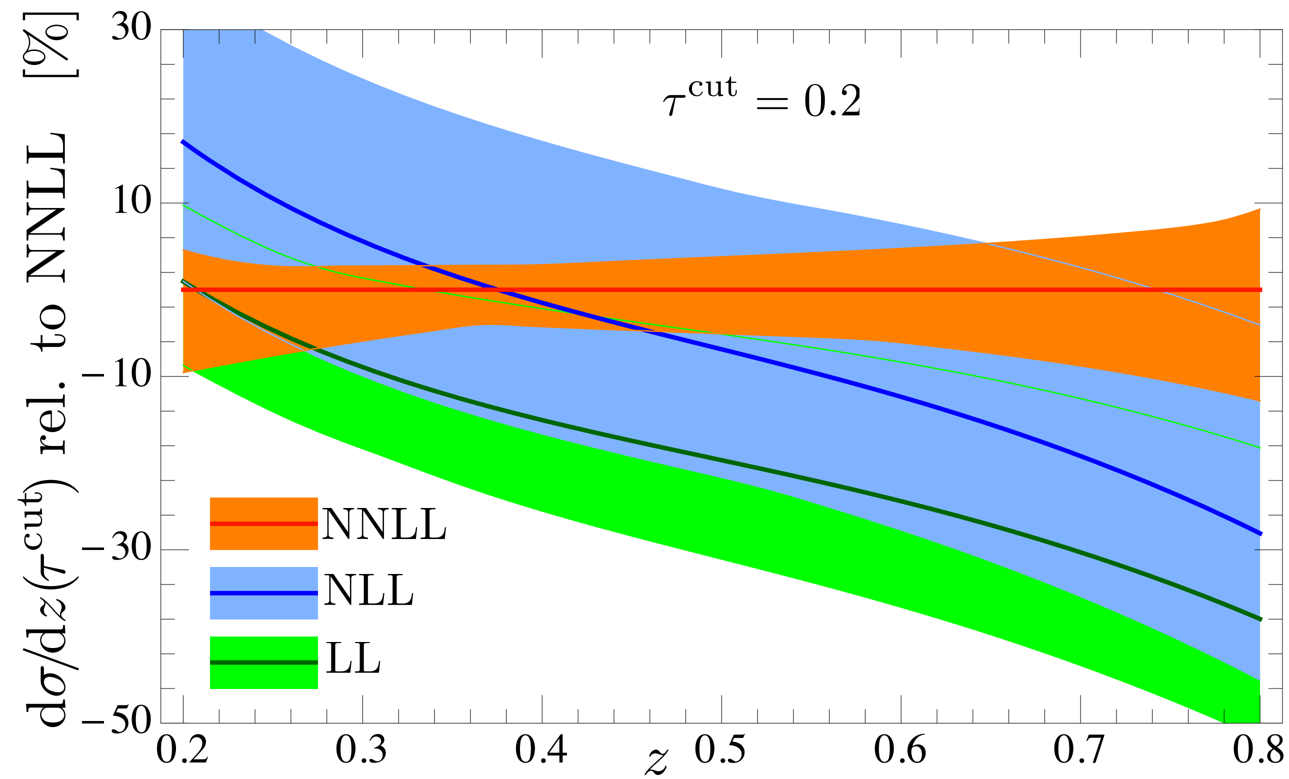

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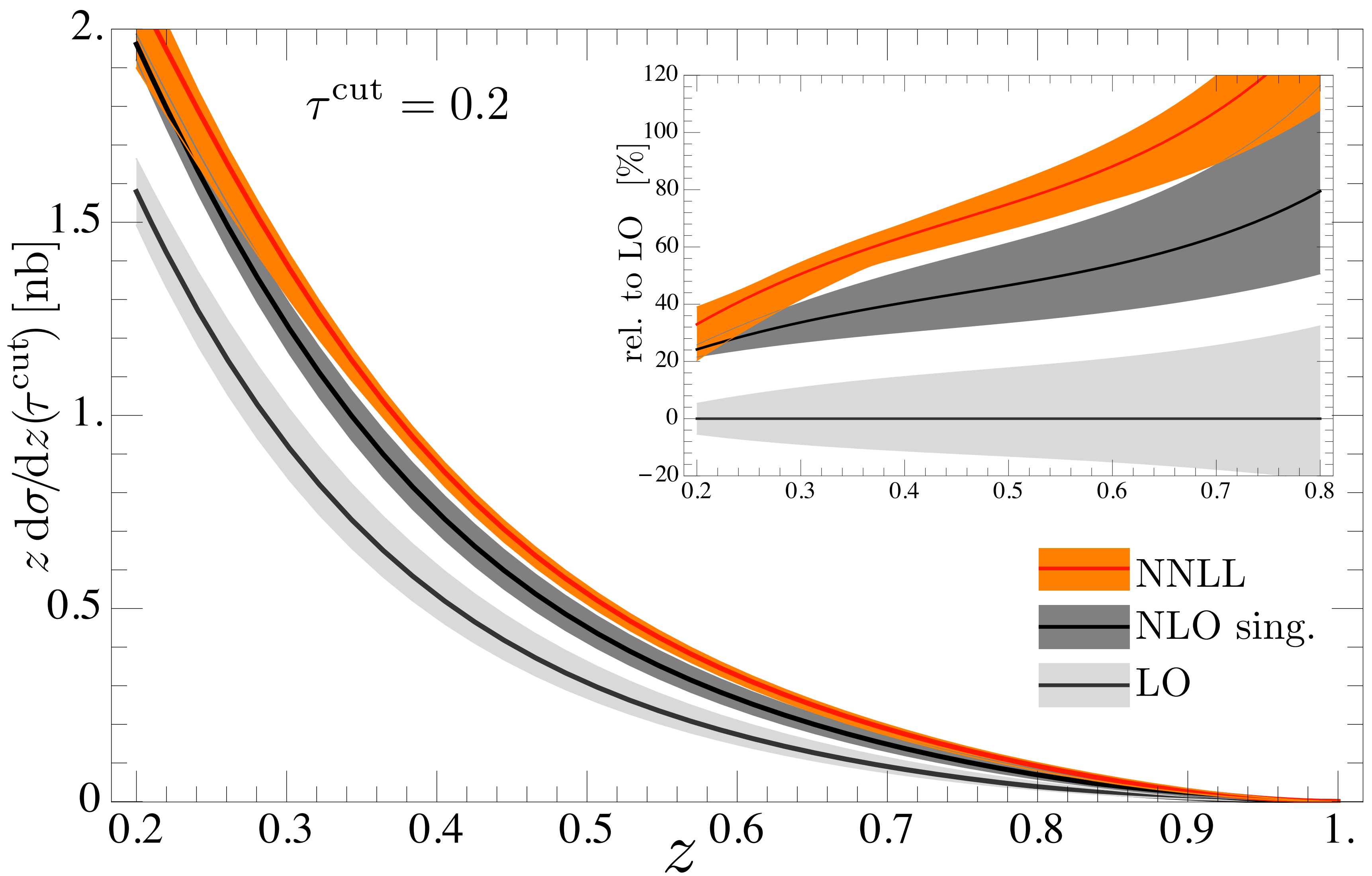
The $e^+ e^- \to X\pi^+$ cross section up to τ^{cut}



Fitting to NNLL result with LO cross-section $\mathrm{d}\sigma^u_{\rm LO}/\mathrm{d}z = \sigma^u_0\, D^{\pi^+}_u(z,\mu=Q)$ using

$$D^{\pi^+}_u(z,\mu=1\,\mathrm{GeV}) = \frac{M^{\pi^+}_u}{B(\alpha^{\pi^+}_u+2,\beta^{\pi^+}_u+1)}\, z^{\alpha^{\pi^+}_u}(1-z)^{\beta^{\pi^+}_u}$$

→ $\alpha^{\pi^+}_u$ and $\beta^{\pi^+}_u$ change by about 30%, $M^{\pi^+}_u$ by about 70% !

Generalized fragmenting jet functions

- Depend on the transverse momentum of the hadron w.r.t. the jet axis:

$$\mathcal{G}_q^h(s,z,\vec{p}_{h\perp}^{\,2}) = \frac{2(2\pi)^3}{2N_c\,z}\sum_X \mathrm{tr}\Big[\frac{\bar{n}\!\!\!/}{2}\,\langle 0|[\delta(k^- - \overline{\mathcal{P}}_n)\,\delta^{d-2}(\vec{\mathcal{P}}_{n\perp})\chi_n(0)]\,\delta(s-k^-\hat{p}^+)\big|Xh(p_h)\rangle\langle Xh(p_h)\big|\bar{\chi}_n(0)|0\rangle\Big]$$

- We consider $p_{h\perp}\sim\sqrt{s}$ (perturbative) : matching onto standard FFs

$$\mathcal{G}_i^h(s,z,\vec{p}_{h\perp}^{\,2},\mu_J) = \sum_j\int_z^1\frac{\mathrm{d}z'}{z'}\tilde{\mathcal{J}}_{ij}\Big(s,\frac{z}{z'},\vec{p}_{h\perp}^{\,2},\mu_J\Big)D_j^h(z',\mu_J)\Big[1+\mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{s},\frac{\Lambda_{\mathrm{QCD}}^2}{\vec{p}_{h\perp}^{\,2}}\Big)\Big]$$

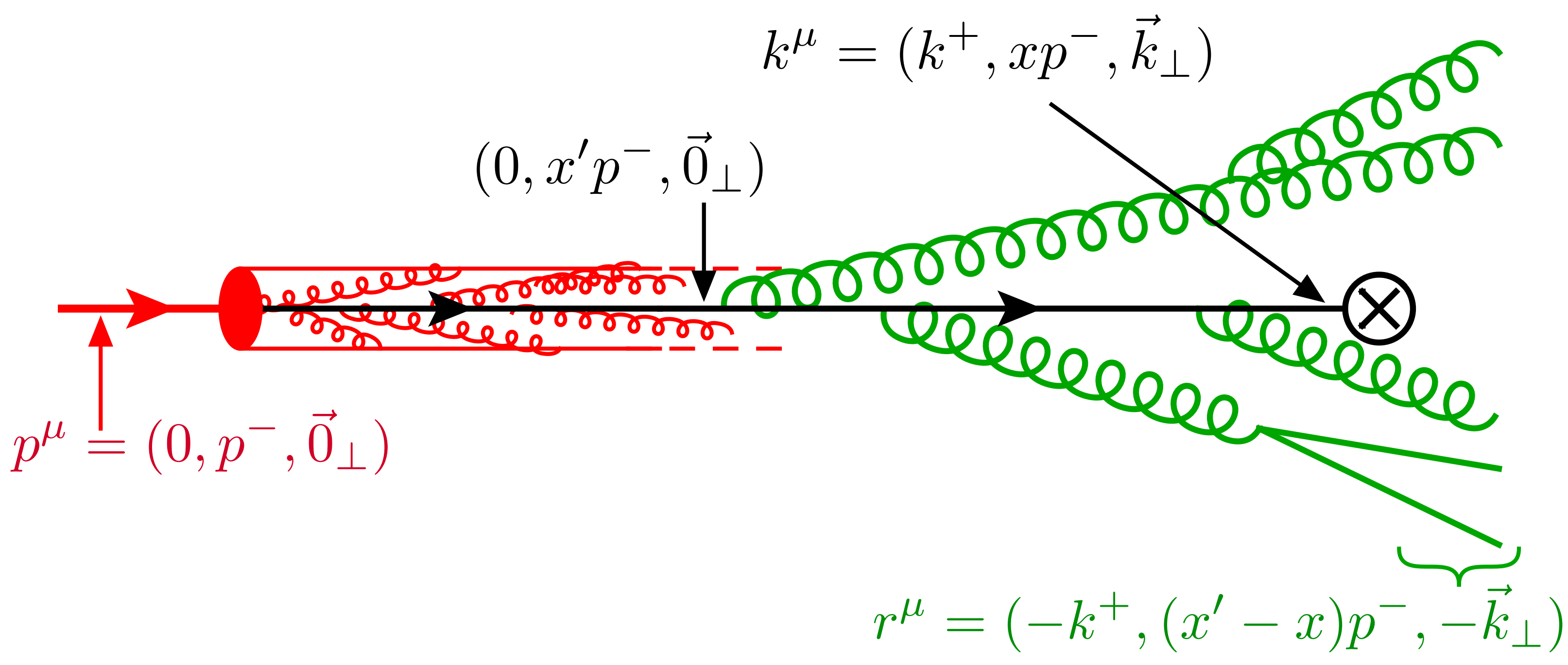
At one loop $\vec{p}_{h\perp}^{\,2} = z(1-z)s$:

$$\tilde{\mathcal{J}}_{ij}\Big(s,\frac{z}{z'},\vec{p}_{h\perp}^{\,2},\mu_J\Big) = \frac{1}{\pi}\,\delta\big(\vec{p}_{h\perp}^{\,2}-z(1-z)s\big)\,\mathcal{J}_{ij}\Big(s,\frac{z}{z'},\mu_J\Big)$$

Generalized beam functions

- Depend on all components of the four-momentum k^μ of a parton entering the hard subprocess and therefore describe the ISR

  FUPDFs [Watt et al. (2003), Collins et al. ('05, '07)] in SCET, $\Lambda_{\rm QCD}^2 \ll \{t, \vec{k}_\perp^{\,2}\} \ll Q^2$



- Unintegrated BFs $B_i(t, x, \vec{k}_\perp, \mu)$: $t = -k^+k^- \geq \dfrac{x}{1-x}\vec{k}_\perp^{\,2}$

  Mantry and Petriello ('09, '10): Higgs pT and rapidity distributions in gg $\longrightarrow$ H, pT and rapidity distributions for EW gauge boson production in Drell-Yan

Generalized beam functions

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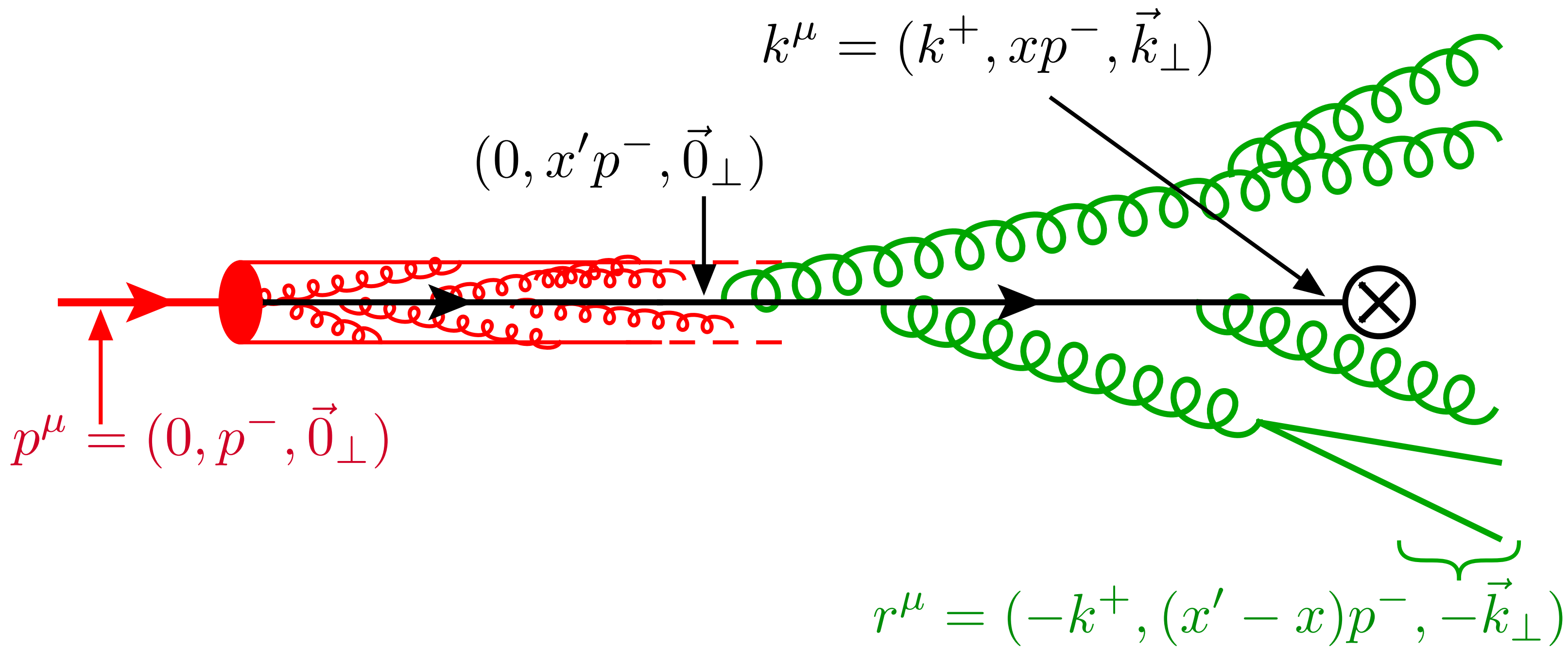

- $$\int \mathrm{d}\vec{k}_\perp \, B_i(t, x, \vec{k}_\perp, \mu) = B_i(t, x, \mu)$$

standard BF : Fleming, Leibovich and Mehen (2006), Stewart, Tackmann and Waalewijn ('09, '10)

Generalized beam functions

Jain, MP, Waalewijn

$$B_q^{\text{bare}}(t, x, \vec{k}_\perp^{\,2}) =$$
$$\theta(k^-)\,\langle p_n(p^-)\big|\bar{\chi}_n(0)\,\delta(t - k^-\hat{p}^+)\frac{\bar{n}\!\!\!/}{2}\big[\delta(k^- - \overline{\mathcal{P}}_n)\,\frac{1}{\pi}\delta(\vec{k}_\perp^{\,2} - \vec{\mathcal{P}}_{n\perp}^{\,2})\,\chi_n(0)\big]\,\big|p_n(p^-)\rangle$$

$$B_g^{\mu\nu,\,\text{bare}}(t, x, \vec{k}_\perp) =$$
$$-\,k^-\,\theta(k^-)\langle p_n(p^-)\big|\mathcal{B}_{n\perp}^{\mu c}(0)\,\delta(t - k^-\hat{p}^+)\big[\delta(k^- - \overline{\mathcal{P}}_n)\delta^2(\vec{k}_\perp - \vec{\mathcal{P}}_{n\perp})\,\mathcal{B}_{n\perp}^{\nu c}(0)\big]\big|p_n(p^-)\rangle$$

- In the gluon BF, the measurement of $\vec{k}_\perp$ introduces a new Lorentz structure:

$$B_g^{\mu\nu}(t, x, \vec{k}_\perp) = B_1(t, x, \vec{k}_\perp^{\,2})\,L_1^{\mu\nu} + B_2(t, x, \vec{k}_\perp^{\,2})\,L_2^{\mu\nu}(\vec{k}_\perp)$$

$L_1^{\mu\nu} \sim g_\perp^{\mu\nu}$, $\quad L_2^{\mu\nu}(\vec{k}_\perp) \sim k_\perp^\mu k_\perp^\nu$

- Renormalizable both in momentum and in impact parameter space

momentum: in dim. reg. : $\vec{k}_\perp$ in 2 not (d-2) dimensions

Matching onto standard PDFs

- We consider $k_\perp \sim \sqrt{t}$ (perturbative) : matching onto standard PDFs

$$B_i(s,x,\vec{k}_\perp^{\,2},\mu_B) = \sum_j \int_x^1 \frac{\mathrm{d}x'}{x'}\, \tilde{\mathcal{I}}_{ij}\Big(t,\frac{x}{x'},\vec{k}_\perp^{\,2},\mu_B\Big)\, f_j(x',\mu_B)\Big[1+\mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{t},\frac{\Lambda_{\mathrm{QCD}}^2}{\vec{k}_\perp^{\,2}}\Big)\Big]$$

At one loop $\vec{k}_\perp^{\,2} = \frac{(1-x)}{x}\, t$:

$$\tilde{\mathcal{I}}_{ij}\Big(t,\frac{x}{x'},\vec{k}_\perp^{\,2},\mu_B\Big) = \frac{1}{\pi}\,\delta\Big(\vec{k}_\perp^{\,2} - \frac{t(1-x)}{x}\Big)\,\mathcal{I}_{ij}\Big(t,\frac{x}{x'},\mu_B\Big)$$

↑ matching coefficients of standard BFs onto PDFs
Stewart et al. (2010)

- We correct a mistake by Mantry and Petriello in $\tilde{\mathcal{I}}_{gq}$ and $\tilde{\mathcal{I}}_{qg}$

Conclusions and outlook

- SCET factorization theorems describing fragmentation of a light hadron within a jet with constrained invariant mass involve fragmenting jet functions

- The calculation of matching coefficients $\mathcal{J}_{ij}(s,z,\mu)$ enables us to relate these factorization formulae with the standard fragmentation functions

- $e^+e^- \to X\pi^+$ on the $\Upsilon(4S)$ resonance in the dijet limit by imposing a cut on thrust (Belle collaboration): $\mathrm{d}^2\sigma_{\rm sing}/\mathrm{d}\tau\,\mathrm{d}z$ up to NNLL accuracy. Convergence of RG-improved perturbation theory better then fixed order. Using LO instead of NNLL or NLO has a sizable impact on the extracted numerical values for $D_i^{\pi^+}$ parameters

- Unintegrated FJFs and beam functions for perturbative transverse momenta: matching onto standard FFs and PDFs

- Future work: inclusion of NLO non-singular terms in the thrust distribution, non-perturbative corrections, effects of uncertainties on fragmentation functions and α_s, $e^+e^- \to h_1 h_2 X$ ...