QCD sum rules concepts and pion's structure

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Outline:

- QCD sum rules (SR) approach
- Introducing Nonlocal Condensates in OPE
- Pion DA, moments, end-point behavior of pion DA
- Electromagnetic pion FF in space-like region
- Pion-photon transition FF in LC SR
- Pion DA from experiment
- Conclusions

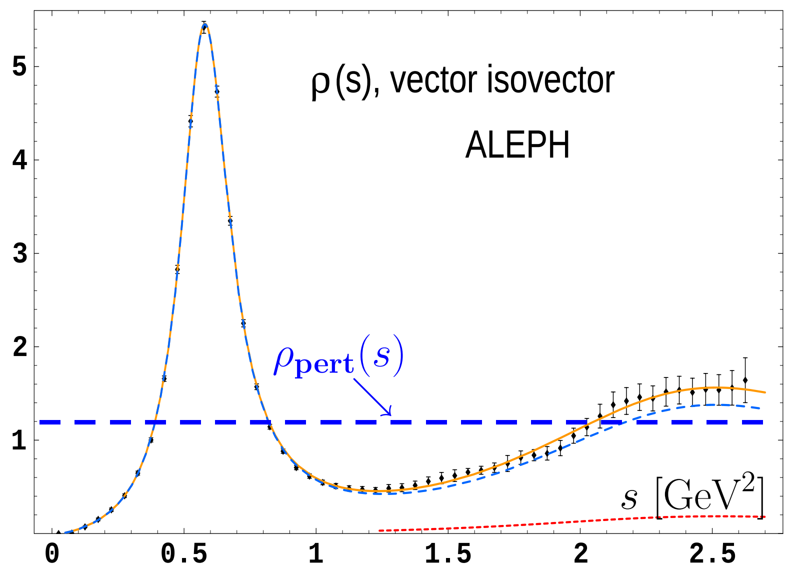
QCD SR Approach

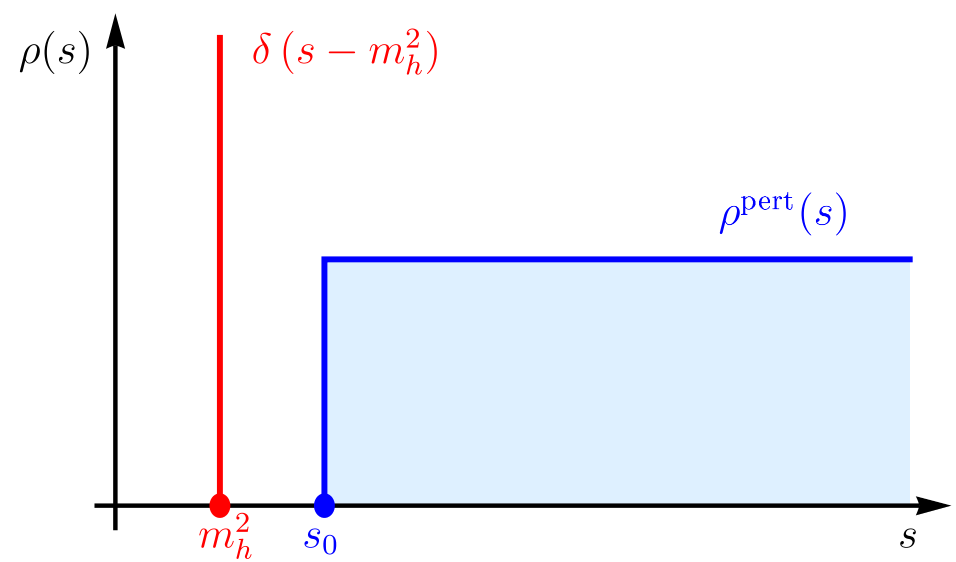
Determination of spectrum parameters from requirement of agreement between two ways for correlator $\Pi(Q^2)$ of currents:

- 1th way — Dispersion relation: decay constants f_h, masses m_h and others,

$$\Pi_{\text{had}}\left(Q^2\right) = \int_0^\infty \frac{\rho_{\text{had}}(s)\ ds}{s+Q^2} + \text{subtractions}\,.$$

  - model spectral density: $\rho_{\text{had}}(s) = f_h^2\,\delta\left(s-m_h^2\right) + \rho_{\text{pert}}(s)\,\theta\left(s-s_0\right)\,.$



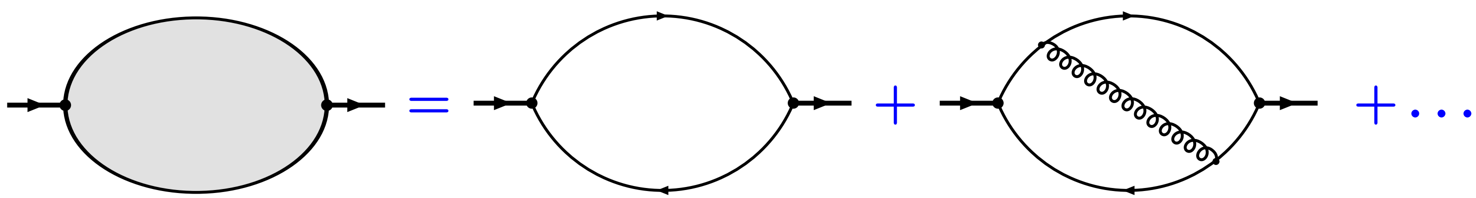


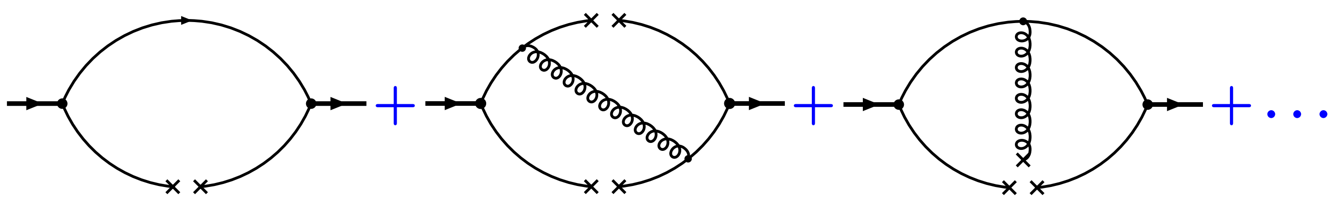
Theoretical part of QCD SR

- 2th way — Operator product expansion:

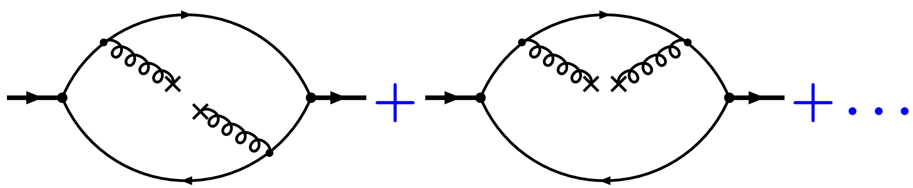
$$\Pi_{\text{OPE}}\left(Q^2\right) = \Pi_{\text{pert}}\left(Q^2\right) + \sum_n C_n \frac{\langle 0|:O_n:|0\rangle}{Q^{2n}}\,.$$

  - Condensates $\langle 0|:O_n:|0\rangle \equiv \langle O_n\rangle = ?$ (next slides).





$\sim \alpha_s \langle \bar{q}q\rangle^2/Q^4$



$\sim \langle \frac{\alpha_s}{\pi} GG\rangle/Q^2$

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- 2th way — Operator product expansion:

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  - Condensates $\langle 0|:O_n:|0\rangle \equiv \langle O_n\rangle = ?$ (next slides).

QCD SR reads:

$$\Pi_{\text{had}}\left(Q^2, m_h, f_h\right) = \Pi_{\text{OPE}}\left(Q^2\right)\,.$$

Borel Transform

$$\Phi(M^2) = \hat{B}_{Q^2 \to M^2}\left[\Pi(Q^2)\right] = \lim_{n\to\infty} \frac{(-Q^2)^n}{\Gamma(n)} \left[\frac{d^n}{dQ^{2n}}\Pi(Q^2)\right]_{Q^2=nM^2}.$$

$\Pi(Q^2)$	$C = \text{const}$	Q^{2n}	$1/Q^{2n}$	$1/\left(s+Q^2\right)$
$\Phi(M^2)$	0	0	$1/\left(\Gamma(n)\, M^{2n}\right)$	$e^{-s/M^2}/M^2$

- Elimination of subtractions in dispersion relation
- Exponential suppression of higher states contribution
- Factorial suppression of condensate terms

$$f_h^2\, e^{-m_h^2/M^2} + \int_{s_0}^{\infty} \rho_{\text{pert}}(s)\, e^{-s/M^2} ds$$

$$= \int_0^{\infty} \rho_{\text{pert}}(s)\, e^{-s/M^2} ds + \frac{c_G}{M^2} \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \rangle + \frac{c_{\bar{q}q}}{M^4}\, \alpha_s \langle \bar{q}q \rangle^2 .$$

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$$f_h^2\, e^{-m_h^2/M^2} = \int_0^{s_0} \rho_{\text{pert}}(s)\, e^{-s/M^2} ds + \frac{c_G}{M^2}\,\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}\rangle + \frac{c_{\bar{q}q}}{M^4}\,\alpha_s\langle\bar{q}q\rangle^2 .$$

Introducing condensates in QCD calculations

$$\langle 0|T\left(\bar{q}_B(0)q_A(x)\right)|0\rangle = \boxed{\langle 0| : \bar{q}_B(0)q_A(x) : |0\rangle} - i\hat{S}_{AB}(x)$$

QCD PT

$$\langle \bar{q}q \rangle \stackrel{\text{def}}{=} 0$$

QCD SR

$$\langle \bar{q}_A(0)q_A(x)\rangle = \langle \bar{q}q\rangle$$

CONST $\neq 0$

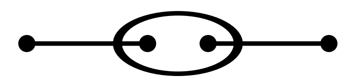
[SVZ'79]

Condensate

Decay constants, masses of hadrons

NLC QCD SR

$$\langle \bar{q}(0)q(x)\rangle = F_S(x^2) + \hat{x}F_V(x^2)$$



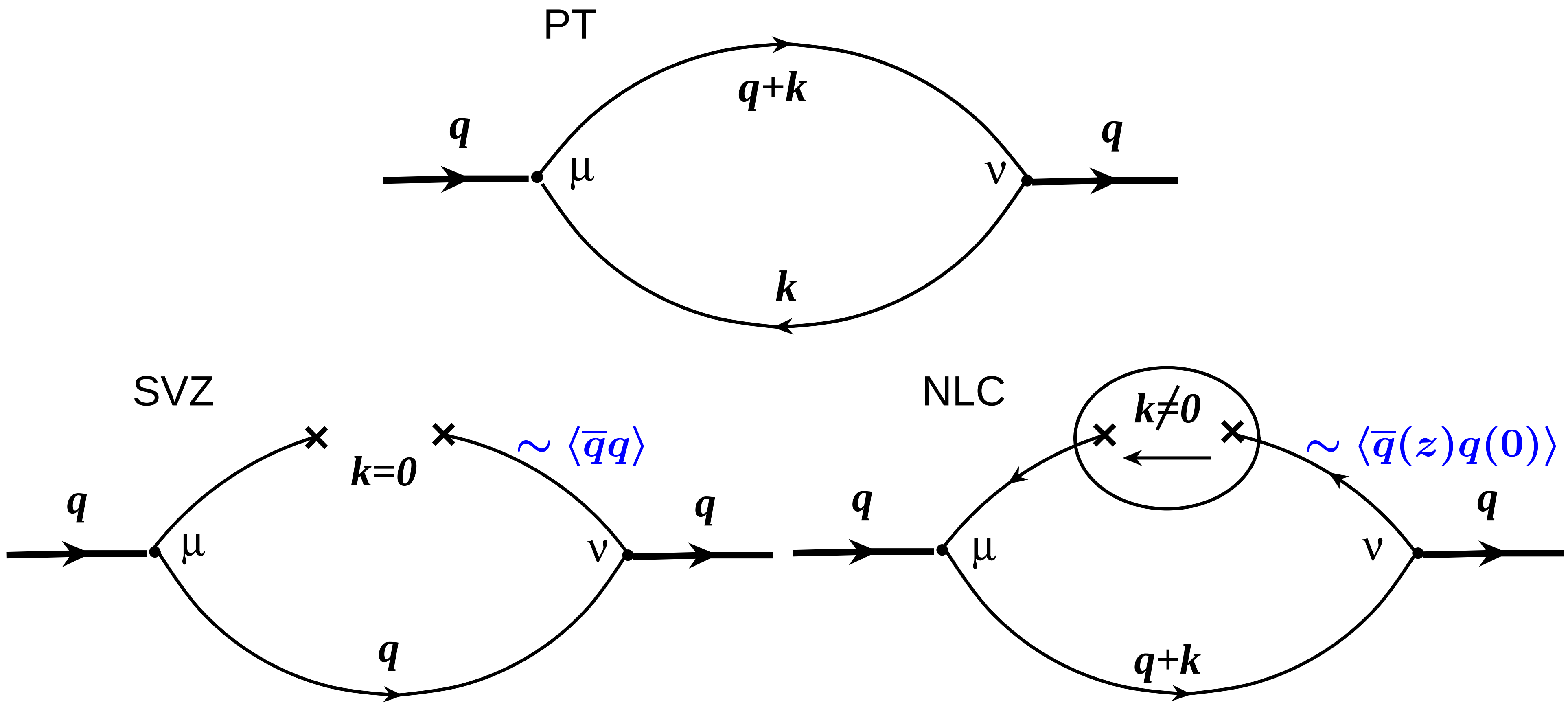
M&R '86

Nonlocal condensate

Distribution Amplitudes, Form Factors

$$\langle \bar{q}_B(0)\, q_A(x)\rangle = \frac{\delta_{AB}}{4}\left[\langle \bar{q}q\rangle + \frac{x^2}{4}\frac{\langle \bar{q}D^2q\rangle}{2} + \ldots\right] + i\frac{\widehat{x}_{AB}}{4}\frac{x^2}{4}\left[\frac{2\alpha_s\pi\langle \bar{q}q\rangle^2}{81} + \ldots\right].$$

Diagrams for $\langle T\,(J_\nu(z)J_\mu(0))\rangle$



- Quarks run through vacuum with nonzero momentum $k \neq 0$:

$$2\langle k^2\rangle = \frac{\langle\bar{q}D^2q\rangle}{\langle\bar{q}q\rangle} = \lambda_q^2 = 0.40(5)\,\text{GeV}^2$$

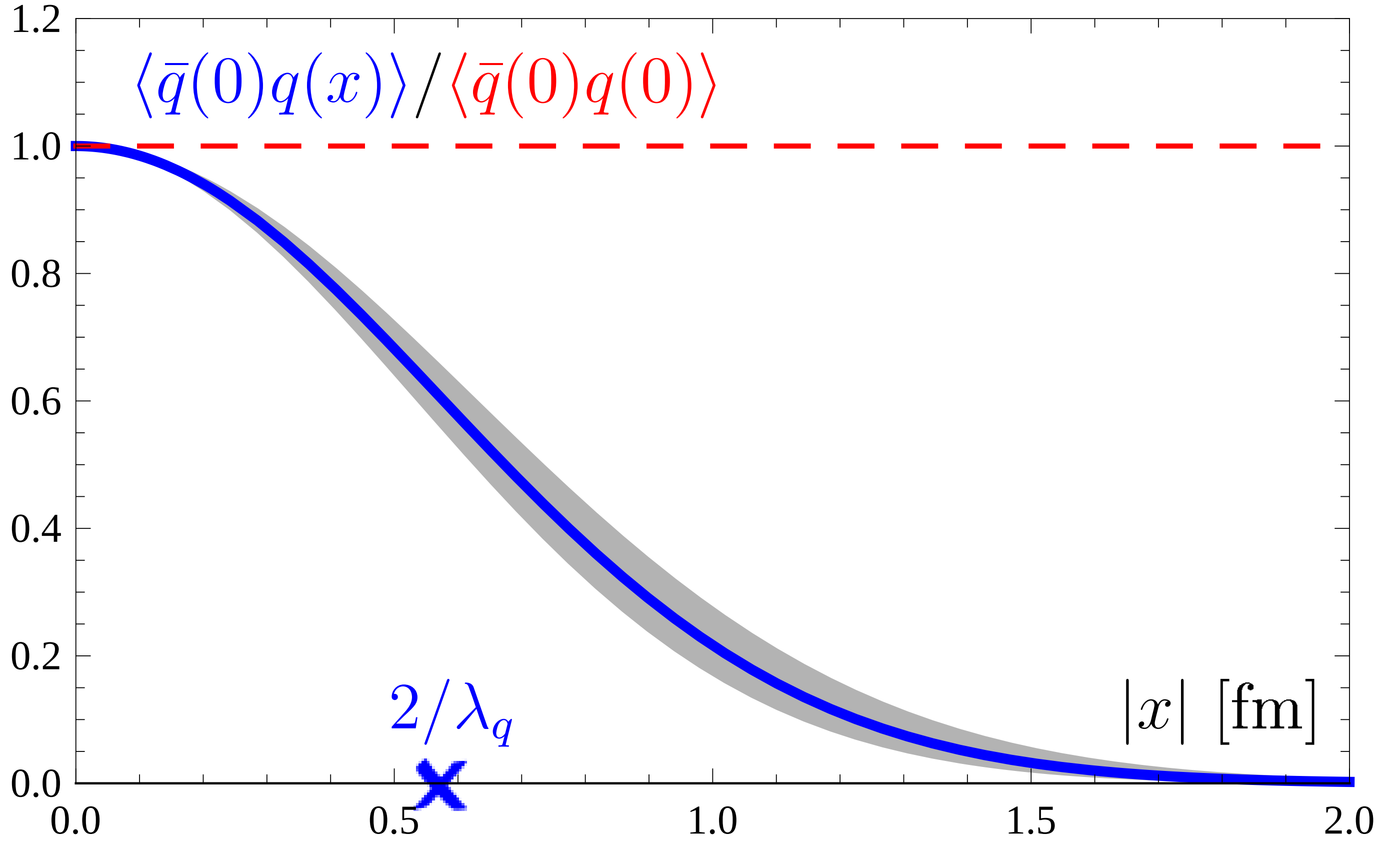
Coordinate dependence of condensates

Parameterization for scalar condensate was suggested in works of Bakulev, Mikhailov and Radyushkin:

$$\langle :\bar{q}_A(0) q_A(x): \rangle = \langle \bar{q} q \rangle \int_0^\infty \boxed{f_S(\alpha)} \, e^{\alpha x^2/4} \, d\alpha \,, \text{ where } x^2 < 0.$$

- First approximation which takes into account finite width of quark distribution in vacuum: $f_S(\alpha) = \delta\left(\alpha - \lambda_q^2/2\right)$, $\lambda_q^2 = \langle \bar{q} D^2 q \rangle / \langle \bar{q} q \rangle$.
- Such representation corresponds to Gaussian form $\sim \mathbf{exp}\left(\lambda_q^2 x^2/8\right)$ of NLC in coordinate representation.
- The heavy-quark effective theory **(Radyushkin 91)** tells us that the scalar condensate decreases exponentially at large distances.
- The **smooth model** $f_S(\alpha) \sim \alpha^{n-1} \mathbf{exp}\left(-\Lambda^2/\alpha - \sigma^2 \alpha\right)$ has a sensible asymptotic form $\langle \bar{q}(0) q(x) \rangle \Big|_{x^2 \to \infty} \sim \mathbf{exp}\left(-\Lambda x\right)$ in x-representation.

Lattice data of Pisa group

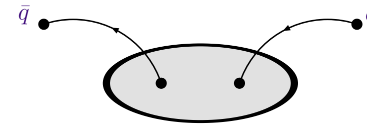


Nonlocality of quark condensates $\lambda_q^2 = 0.42(8)$ GeV^2 from lattice data of Pisa group in comparison with **local limit**.

- Even at $|z| \simeq 0.5$ fm nonlocality is quite important!

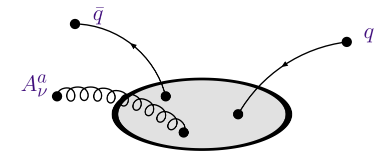
Basis condensates

- Bilocal quark-anti quark condensate ($A_0 = 2\alpha_s\pi\langle\bar{q}q\rangle^2/81$):



$$\langle\bar{q}_A(0)q_B(x)\rangle = \frac{1}{4}\int_0^\infty \left\{\delta_{BA}\langle\bar{q}q\rangle \boxed{f_S(\alpha)} - iA_0\,\widehat{x}_{BA}\boxed{f_V(\alpha)}\right\} e^{\alpha x^2/4}\,d\alpha\,.$$

- Quark-gluon condensate in fixed-point gauge $x^\mu A_\mu(x) = 0$:



$$\langle\bar{q}_B(0)(-gA_\nu^a(y)\,t^a)q_A(x)\rangle = A_0\sum_{i=1}^{3}\Gamma_\nu^i(x,y)_{AB}\times$$

$$\times\int_0^\infty\!\!\int_0^\infty\!\!\int_0^\infty d\alpha_1\,d\alpha_2\,d\alpha_3\boxed{f_i(\alpha_1,\alpha_2,\alpha_3)}e^{(\alpha_1x^2+\alpha_2y^2+\alpha_3(x-y)^2)/4}\,,$$

where

$$\Gamma_\nu^1(x,y) = -\frac{3}{2}\left(\hat{y}x_\nu - \gamma_\nu(xy)\right)\,;$$

$$\Gamma_\nu^2(x,y) = 2\left(\hat{y}y_\nu - \gamma_\nu y^2\right)\,;$$

$$\Gamma_\nu^3(x,y) = i\frac{3}{2}\varepsilon_{\nu\sigma yx}\gamma_5\gamma^\sigma\,.$$

Minimal Gaussian Model

Bakulev, Mikhailov, Radyushkin, and Stefanis use the minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\left(\alpha - \lambda_q^2/2\right)\,, \quad f_V(\alpha) = \delta'\left(\alpha - \lambda_q^2/2\right)\,,$$

$$f_i\left(\alpha_1, \alpha_2, \alpha_3\right) = \delta\left(\alpha_1 - \lambda_q^2/2\right)\,\delta\left(\alpha_2 - \lambda_q^2/2\right)\,\delta\left(\alpha_3 - \lambda_q^2/2\right)$$

- There is one parameter $\lambda_q^2 = 0.4 - 0.5\,\text{GeV}^2$.
- The transition to local condensate case is $\lambda_q^2 \to 0$.
- This model provides the DAs and FFs of light mesons in good agreement with experimental data.

Problems:

- **QCD equations of motion are violated**
- **Vector current correlator is not transverse**
  $\Rightarrow$ **gauge invariance is broken**

QCD equation of motion for condensates

From Dirac equation for massless quark ($\hat{A}_\mu(x) \equiv A^a_\mu(x)\, t^a$):

$$(\partial_\mu - ig\hat{A}_\mu(x))\gamma_\mu q(x) = 0\,,$$

one cans obtain QCD equation of motion for splitted vector quark current

$$(\partial_\mu - ig\hat{A}_\mu(x))\bar{q}(0)\gamma_\mu q(x) = 0$$

- If we average it over physical QCD vacuum, then we obtain the equation for condensates:

$$\partial_\mu \langle \bar{q}(0)\gamma^\mu q(x)\rangle = i\langle \bar{q}(0) g\hat{A}_\mu(x)\gamma^\mu q(x)\rangle\,.$$

- Minimal Gaussian ansatz does not satisfy this equation.

Improved Gaussian model

We modify functions f_i by introducing new parameters:

$$f_S(\alpha) = \delta\left(\alpha - \Lambda_{\rm S}\right) \,, \quad f_V(\alpha) = \delta'(\alpha - \Lambda_{\rm V}) \,,$$

$$f_i^{\rm imp}(\alpha_1, \alpha_2, \alpha_3) = \left(1 + X_i\partial_x + Y_i\partial_y + Y_i\partial_z\right) \delta\left(\alpha_1 - x\Lambda_{\rm V}\right) \delta\left(\alpha_2 - y\Lambda_{\rm V}\right) \delta\left(\alpha_3 - z\Lambda_{\rm V}\right) .$$

What does it give?:

- If these conditions $12\,(X_2 + Y_2) - 9\,(X_1 + Y_1) = 1\,,\ x + y = 1\,,$ are fulfilled than QCD equations of motion are satisfied;
- We minimize nontransversity of polarization operator by special choice of model parameters;
- Using improved model causes changing results (pion DA, pion em. FF) but on values that are smaller than theoretical errors.

Pion distribution amplitude $\varphi_\pi(x, \mu^2)$

- The pion DA parameterizes this matrix element:

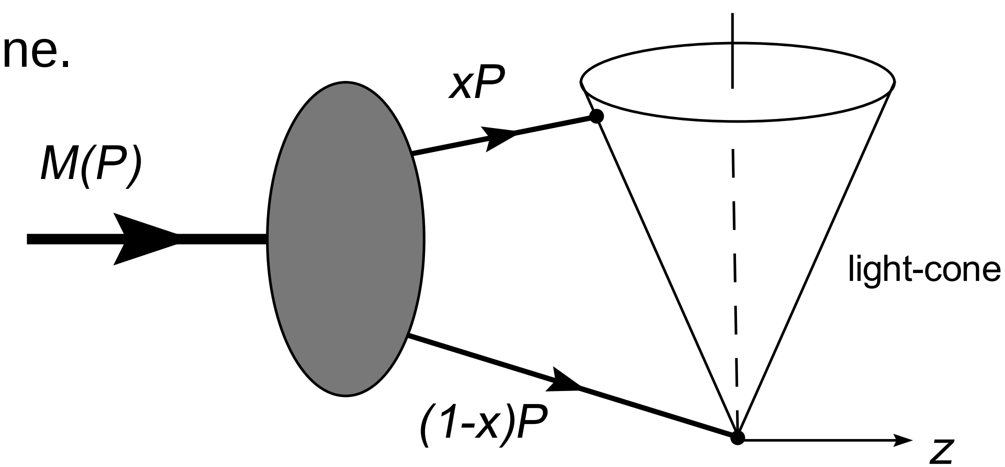
$$\langle 0|\,\bar{d}(z)\gamma_\nu\gamma_5[z,0]u(0)\,|\,\pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\nu \int_0^1 dx\, e^{ix(zP)} \varphi_\pi(x,\mu^2)\,.$$

where the path-ordered exponential

$$[z,0] = \mathcal{P}\, exp\left[ig \int_0^z t^a A_\mu^a(y) dy^\mu\right],$$

i.e., the light-like gauge link, ensures the gauge invariance.

- Pion DA describes the transition of a physical pion into two valence quarks, separated at light cone.



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Distribution amplitudes are **nonperturbative** quantities to be derived from

- QCD SR **[CZ 1984]**,
  NLC QCD SR **[M&Radyushkin 1988-91,B&Mikhailov&S 1998,2001–04]**
- instanton-vacuum approaches, e.g.
  [Dorokhov *et al.* 2000; Polyakov *et al.* 1998, 2009]
- Lattice QCD, **[Braun *et al.* 2006; Donnellan *et al.* 2007]**
- from experimental data **[Schmedding&Yakovlev 2000, BMS 2003–2006]**

DA evolves with μ_F^2 according to **ERBL equation in pQCD**.

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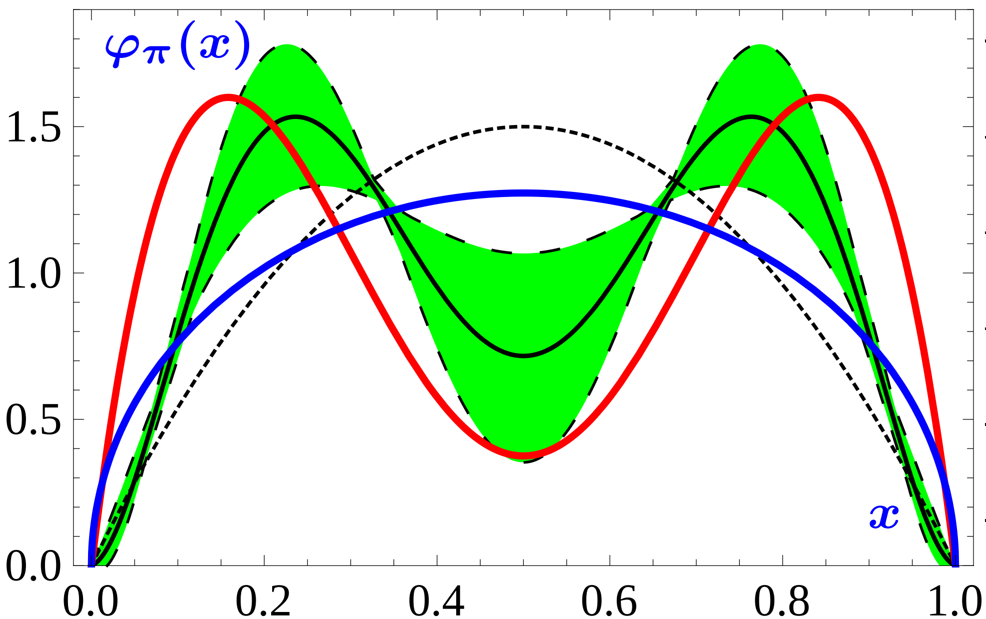


Curve	Approach
black dashed line	Asymptotic
green band with black solid and dashed lines	BMS from NLC QCD SR
red line	CZ from QCD SR
blue line	AdS/QCD result

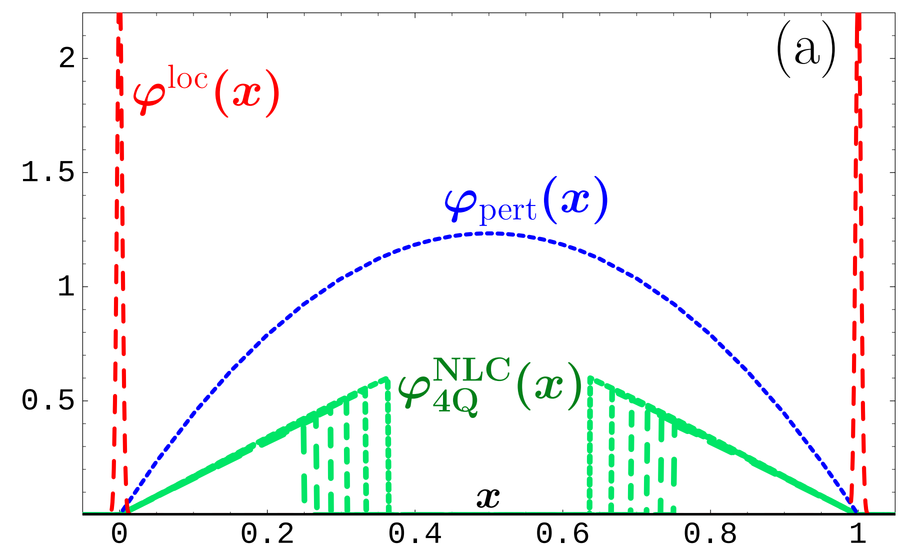
There are numbers of models for pion DA on a market. We could qualitatively collect them in two groups by their behavior at the end-point region $x = 0$:
end-point suppressed and **end-point enhanced** pion DAs.

QCD SR for pion DA

QCD SR technique for correlator of two axial current leads to SR for π-DA $\varphi_\pi(x)$:

$$f_\pi^2\,\varphi_\pi(x) + f_{A_1}^2\,\varphi_{A_1}(x)\,e^{-m_{A_1}^2/M^2} = \int_0^{s_0} \rho_{\text{pert}}(s,x)\,e^{-s/M^2} ds + \Phi_{\text{npert}}(x,M^2)\,,$$

where $\Phi_{\text{npert}} = \Phi_{\text{4Q}} + \Phi_{\text{T}} + \Phi_{\text{V}} + \Phi_{\text{G}}$,
M^2 – Borel parameter,
ρ_{pert} – pert. spec. density.
The largest nonperturbative term:
$\Phi_{\text{4Q}} \sim x\theta(\Delta - x) \xrightarrow{\text{loc. lim}} \Phi_{\text{4Q}}^{\text{loc}} \sim \delta(x)$,
is defined by scalar quark condensate,
where $\Delta = \lambda_q^2/M^2 \in [0.01, 0.3]$.



- Since nonperturbative contribution has **singularities** ($x\delta'(\Delta - x)$, $\delta(\Delta - x)$), we should study **integral characteristics** of π-DA in order to take into account all condensates and reduce model dependence.
- Exception is end-point region where only 4-quark condensate Φ_{4Q} contributes without any singularities.

Integral characteristics of pion DA

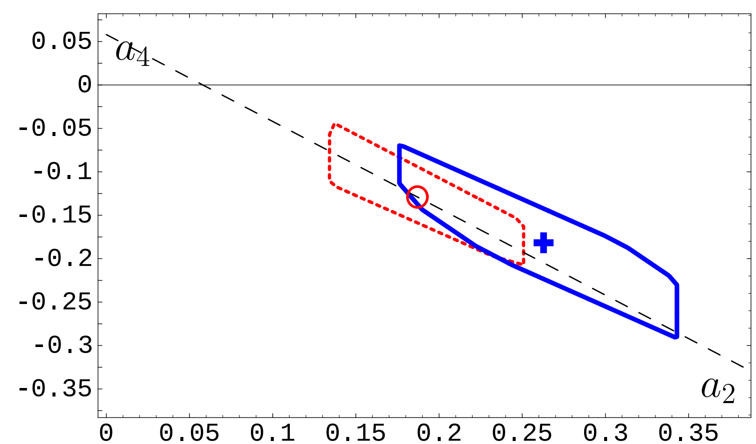
Moments: $\langle \xi^{2N} \rangle \equiv \int_0^1 dx\, \varphi_\pi(x)(2x-1)^{2N}\,, \quad \langle x^{-1} \rangle \equiv \int_0^1 dx\, \varphi_\pi(x) x^{-1}\,.$

SVZ	$\langle \xi^0 \rangle$	LO	local cond.	f_π
CZ	$\langle \xi^{2N} \rangle$, $N = 0, 1$	LO	local cond.	$f_\pi,\, a_2$
BMS	$\langle \xi^{2N} \rangle$, $N = 0, 1, \ldots, 5$	NLO	nonlocal cond.	$f_\pi,\, a_2,\, a_4,\ \langle x^{-1} \rangle$
Here	$[D^{(\nu)} \varphi_\pi](x)$	NLO	nonlocal cond.	$\varphi'_\pi(0)$

Pion DA in a form of Gegenbauer expansion:

$$\varphi_\pi(x;\mu^2) = 6x\bar{x}\left[1 + a_2 C_2^{3/2}(2x-1) + a_4 C_4^{3/2}(2x-1) + \ldots\right]$$

We extract the (a_2, a_4) Gegenbauer coefficients from QCD SRs on the Moments Region for (a_2, a_4) of the pion DA for improved model (solid line) in comparison with minimal result: BMS model ($\circ$) and bunch (dashed line).



QCD SR for $\varphi'_\pi(0)$ in Gaussian model

By differentiating QCD SR for pion DA at $x = 0$. We arrive at SR for $\varphi'_\pi(0)$

$$f_\pi^2\, \varphi'_\pi(0) = \frac{3}{2\pi^2} M^2 \left(1 - e^{-s_0/M^2}\right) - f_{A_1}^2\, \varphi'_{A_1}(0)\, e^{-m_{A_1}^2/M^2} + \frac{144\pi\alpha_S}{81} \langle\bar{q}q\rangle^2 \Phi' \,,$$

where only 4-quark condensate contribution survives.

- Nonperturbative term mainly defined by scalar-quark condensate at large and moderate distances

$$\Phi' = \int_0^\infty d\alpha \frac{f_S(\alpha)}{\alpha^2} = \langle\bar{q}q\rangle^{-1} \int_0^\infty z^2 \langle\bar{q}(0)q(z)\rangle dz^2 \,.$$

- Simplest assumption for scalar condensate model $f_S(\alpha) = \delta(\alpha - \lambda_q^2/2)$ leads to Gaussian behavior $\sim \textbf{exp}\left(\lambda_q^2 x^2/8\right)$ of coordinate dependence and to simple expression for nonperturbative contribution to SR:

$$\Phi' \longrightarrow \Phi'_{\text{Gauss}} = 4/\lambda_q^4 \,.$$

- Then QCD SR result is $\varphi'_\pi(0) = 5.3(5)$, where nonlocality parameter $\lambda_q^2 = 0.4\,\text{GeV}^2$ was used.

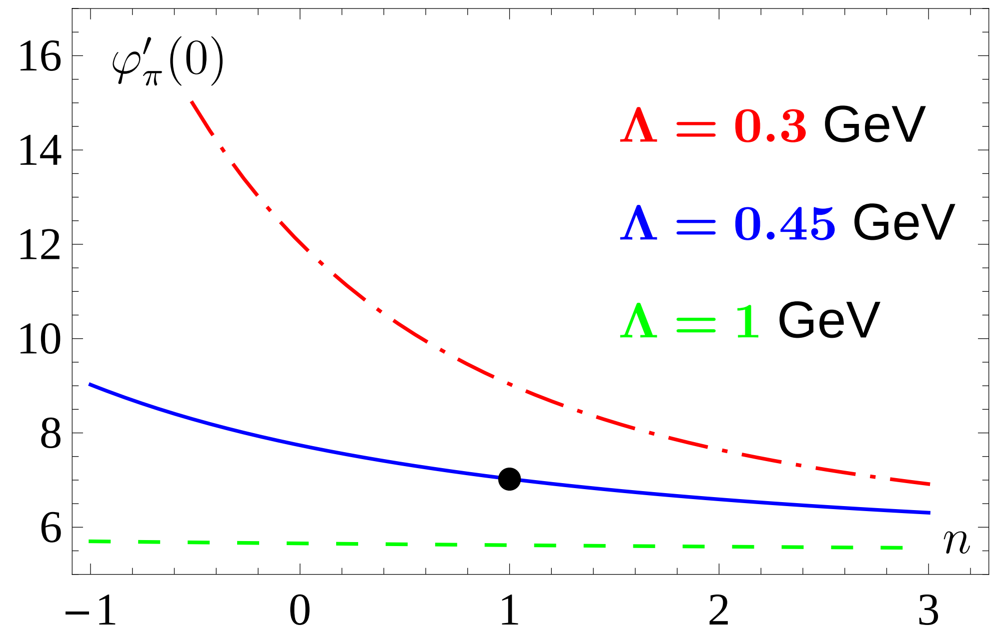
QCD SR for $\varphi'_\pi(0)$ with smooth NLC

- There is an indication from heavy-quark effective theory **[Radyushkin 91]** that in reality quark-virtuality distribution f_S should be parameterized in a different way as to ensure that scalar condensate **decreases exponentially** at large distances. $\langle\bar{q}(0)q(z)\rangle \sim |z|^{-(2n+1)/2} e^{-\Lambda|z|}$.

- This could be realized by model:

$$f_S(\alpha;\Lambda,n,\sigma) \sim \alpha^{n-1} e^{-\Lambda^2/\alpha - \alpha\,\sigma^2}\ .$$

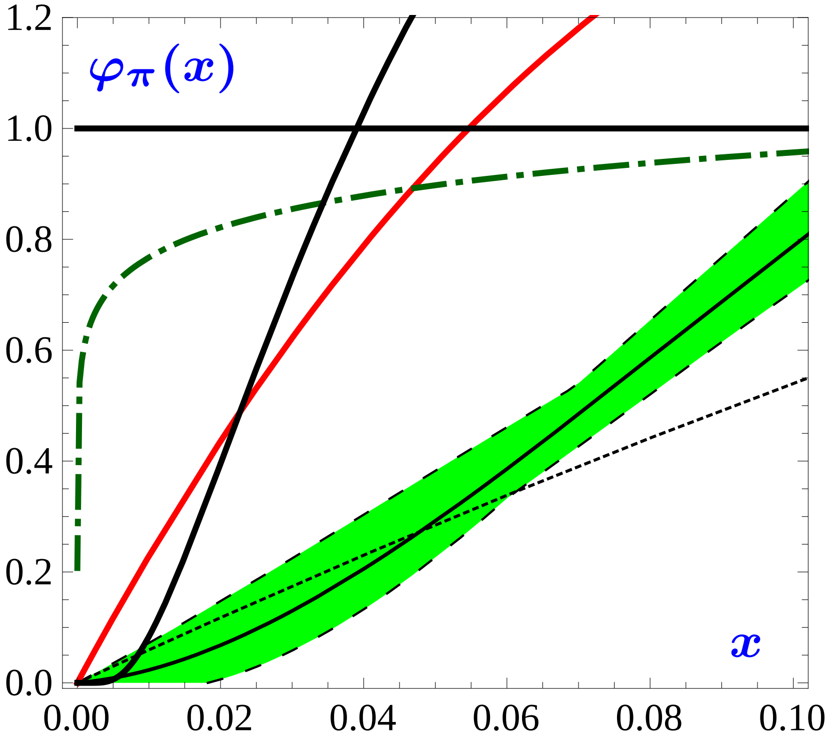
- Analysis of SR for the heavy-light meson, obtained in heavy quark effective theory, leads to values $\Lambda = 0.45$ GeV and $n = 1$. For these parameters we get $\varphi'_\pi(0) = 7.0(7)$ (black point in Fig.).



- Slower decay at large distances, causes an increase of the pion DA slope $\varphi'_\pi(0)$;

Comparison of results with pion DA models

Approach	$[D^{(3)}\varphi_\pi](0.5)$	$\varphi'_\pi(0)$
Integral LO QCD SR	4.7 ± 0.5	5.5 ± 1.5
Differential LO QCD SR, Gaussian decay of NLC	—	5.3 ± 0.5
Differential LO QCD SR, exponential decay of NLC	—	7.0 ± 0.7



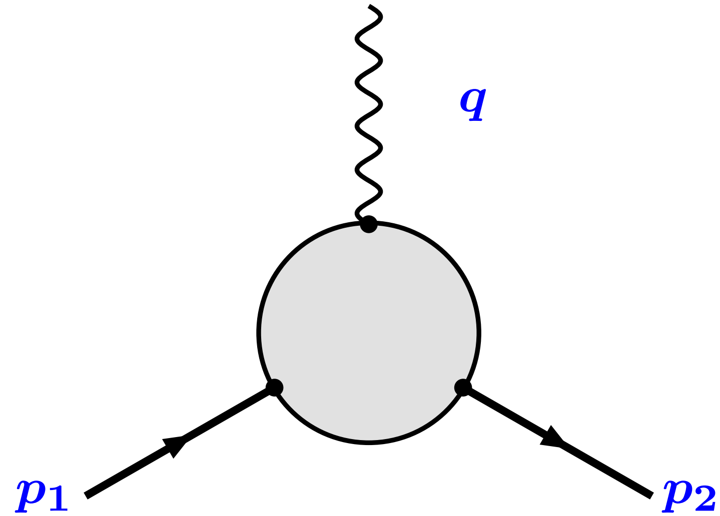
$$[D^{(\nu+2)}\varphi_\pi](x) = \frac{1}{x}\int_0^x dy\, \varphi_\pi(y)\, \frac{(\log x/y)^\nu}{y\Gamma(1+\nu)}$$

Curve	Model	$[D^{(3)}\varphi_\pi](0.5)$	$\varphi'_\pi(0)$
green band with black line	BMS DA	5.7 ± 1.0	1.7 ± 5.3
black dashed	Asy DA	5.25	6
red solid	CZ DA	15.1	26.2
green dash-dotted	$\sim x^{0.1}$	227	$\gg 6$
black solid	[WH10]	14	0

Definition of pion Form Factor

Pion FF F_π is defined by the matrix element

$$\langle \pi^+(p_2)|J_\mu(0)|\pi^+(p_1)\rangle = (p_1 + p_2)_\mu F_\pi(Q^2)\,,$$

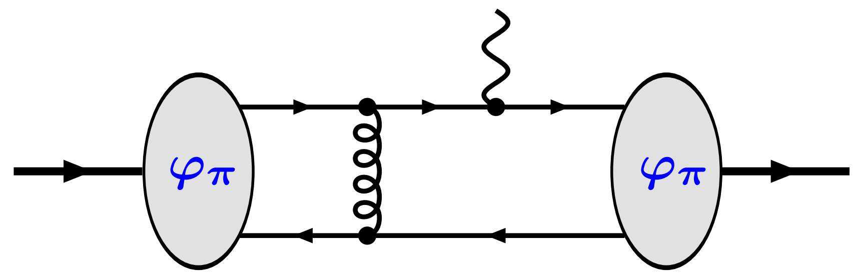


where J_μ is the electromagnetic current, $(p_2 - p_1)^2 = q^2 \equiv -Q^2$ is the photon virtuality, and pion FF is normalized to $F_\pi(0) = 1$.

We are interested in space-like region $Q^2 > 0$.

At asymptotically large $Q^2 \gtrsim 20$ GeV^2, the pQCD factorization gives the pion FF

$$F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2)f_\pi^2}{9Q^2}\left|\int_0^1 \frac{\varphi_\pi(x,Q^2)}{x}\,dx\right|^2$$

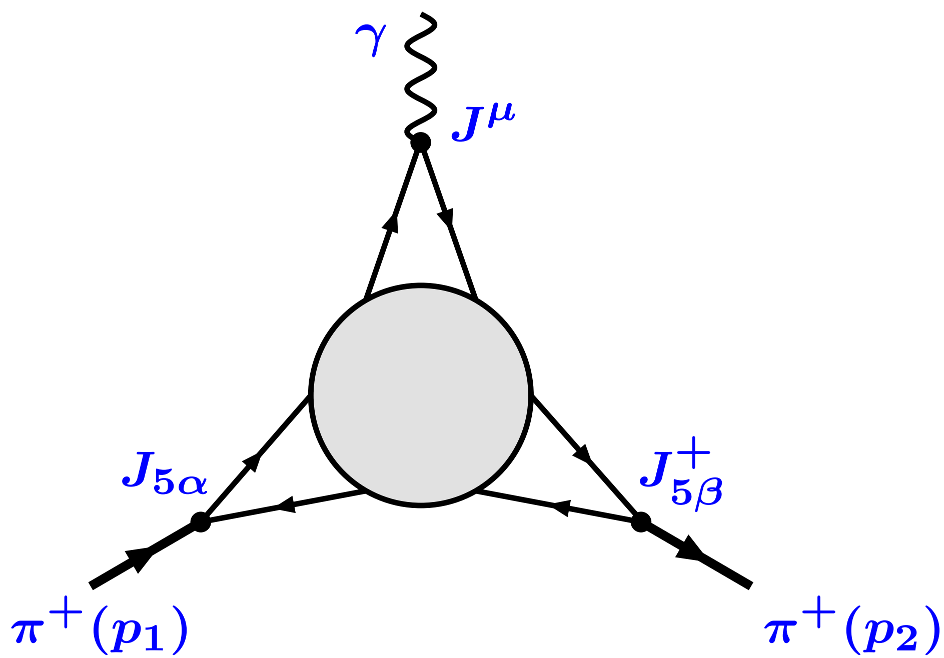


in terms of the pion DA $\varphi_\pi(x, Q^2)$ of the leading twist.

Pion form factor from AAV correlator

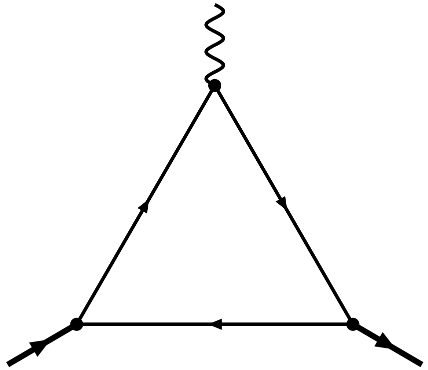
For intermediate momentum transfer 1 GeV$^2 \lesssim Q^2 \lesssim 20$ GeV^2 one cans use QCD SR technique via Axial-Axial-Vector correlator:

$$\int\int d^4x\, d^4y\, e^{i(qx-p_2y)} \langle 0|T\left[J^+_{5\beta}(y) J^\mu(x) J_{5\alpha}(0)\right]|0\rangle$$



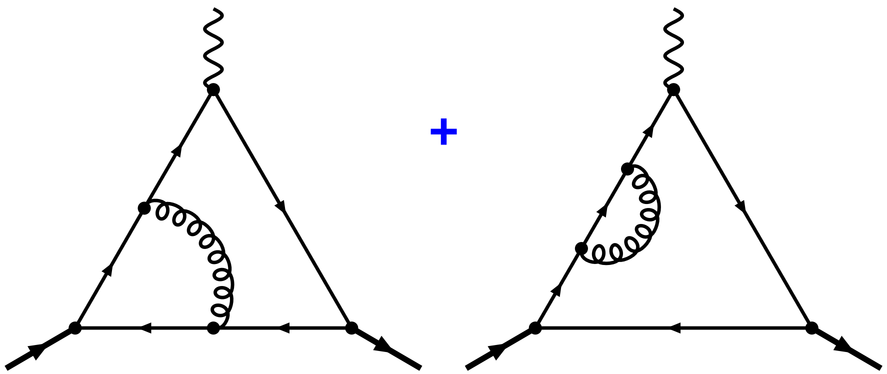
where EM current $J^\mu(x) = e_u\,\overline{u}(x)\gamma^\mu u(x) + e_d\,\overline{d}(x)\gamma^\mu d(x)$ and axial-vector current: $J_{5\alpha}(x) = \overline{d}(x)\gamma_5\gamma_\alpha u(x)$.

Diagramms for AAV-correlator



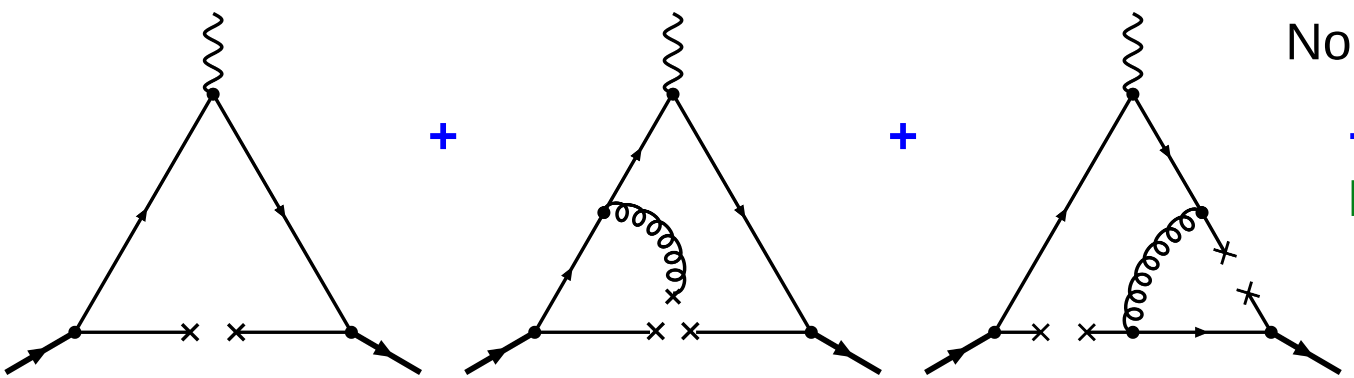
Perturbative LO term

Nesterenko&Radyushkin
$\oplus$**Ioffe&Smilga 1982**



Perturbative NLO terms

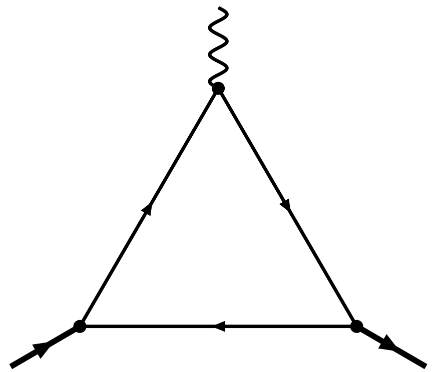
+...
Braguta&Onishchenko 2004



Nonperturbative terms

+...
Nesterenko&Radyushkin
$\oplus$**Ioffe&Smilga 1982**
local condensates

Diagramms for AAV-correlator



Perturbative LO term

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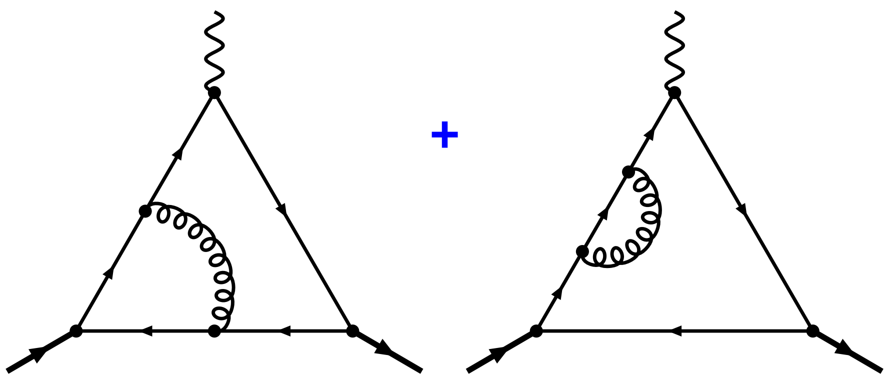
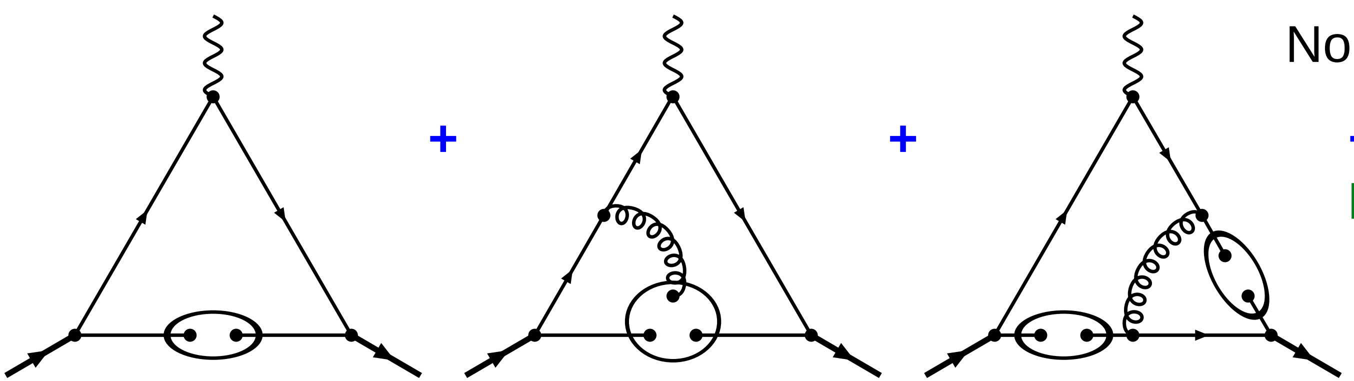


Perturbative NLO terms

+ . . .
Braguta&Onishchenko 2004



Nonperturbative terms

+ . . .
Bakulev&Radyushkin 1991

nonlocal condensates

QCD SR with local condensates

The Borel SR for the pion FF based on three-point AAV correlator:

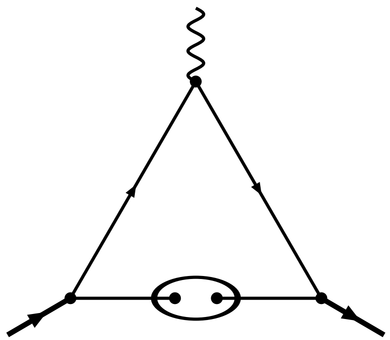
$$f_\pi^2 \, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1 \, ds_2 \, \rho(s_1, s_2, Q^2) \, e^{-(s_1+s_2)/M^2} + \Phi_{\text{nonpert}}(Q^2, M^2) \, .$$

$$\Phi_{\text{nonpert}}(Q^2, M^2) = \frac{\langle \alpha_s GG \rangle}{12 \, \pi \, M^2} + \frac{208 \, \alpha_s \pi \langle \bar{q} q \rangle^2}{81 M^4} \left(1 + \frac{2 \, Q^2}{13 \, M^2} \right) .$$

- Wrong scale behavior of nonperturbative terms at large Q^2.
- SR becomes unstable for $Q^2 > 3$ GeV^2.

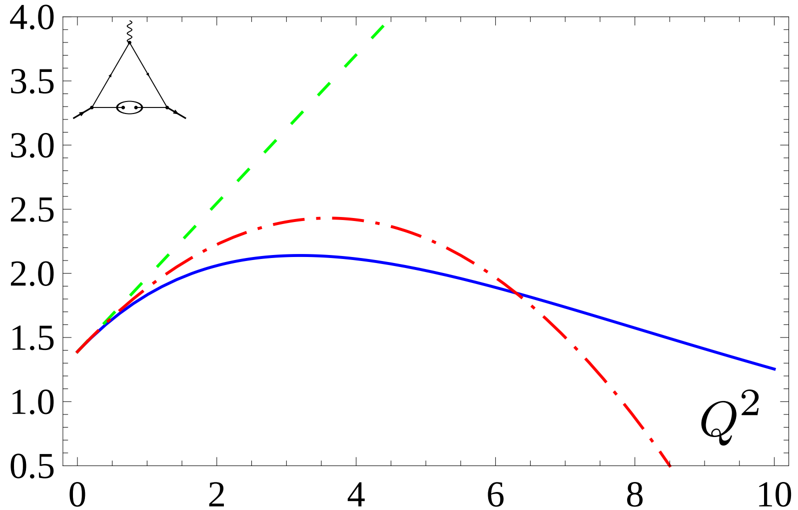
QCD SR with nonlocal condensates

The difference between local ($\lambda_q^2 \to 0$) and nonlocal case could be shown on an example of the vector quark condensate contribution to three-point AAV correlator:



$$= \frac{16\,\alpha_s \pi \langle \bar{q}q \rangle^2}{81 M^4} \left(2 + \frac{Q^2}{M^2 - \lambda_q^2}\right) \exp\left[\frac{-Q^2\,\lambda_q^2}{2\,M^2\,(M^2 - \lambda_q^2)}\right]$$

$$\sim \frac{1}{M^4}\left(2 + \frac{Q^2}{M^2}\right) - \frac{\lambda_q^2}{2}\frac{Q^4}{M^{10}} + \ldots \quad \text{for} \quad \lambda_q^2 \to 0\,.$$



- The taking into account the nonlocality λ_q^2 expands the admissible region in QCD SR up to $Q^2 \sim 7$.
- For momentum $Q^2 < 7$ the results is weekly depending on modeling.

QCD SR with nonlocal condensates

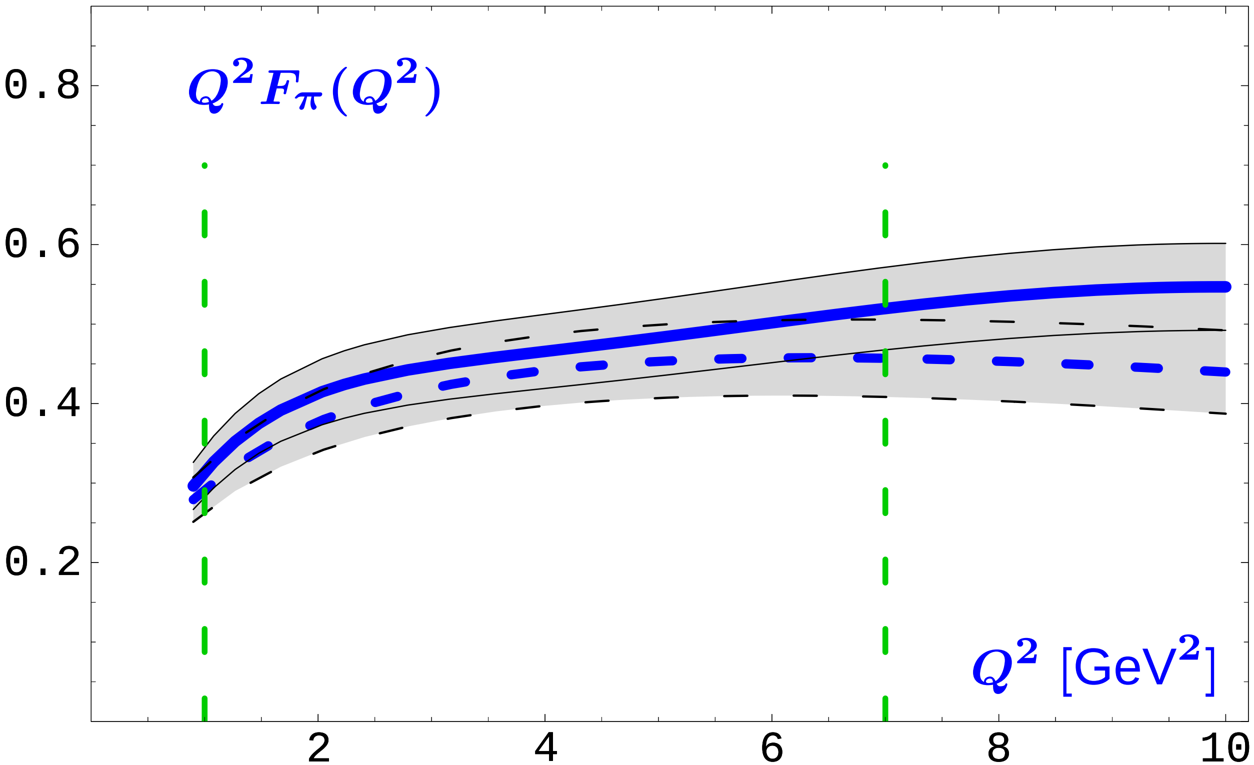
The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 \, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1 \, ds_2 \, \rho(s_1, s_2, Q^2) \, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2) \, .$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
Standard SR	LO	local	$c_1 + Q^2/M^2$ where $c_i \neq f\left(Q^2\right)$
SR with NLC	LO	local $+$ nonlocal	$\left(c_2 + \frac{Q^2}{M^2}\right)\left(e^{-c_3 Q^2 \lambda_q^2/M^4} + c_4\right)$
LD SR ($M^2 \to \infty$)	NLO	NO	0
Here	NLO	nonlocal	$\left(c_5 + Q^2/M^2\right) \, e^{-c_6 Q^2 \lambda_q^2/M^4}$

- Using nonlocal condensates improves Q^2 behavior of OPE and as a result widens region of applicability up to $Q^2 \simeq 10$ **GeV^2**.
- We use model-independent expression for Φ_{OPE}-term obtained by **Bakulev&Radyushkin**, but significantly different model of condensate's nonlocality.

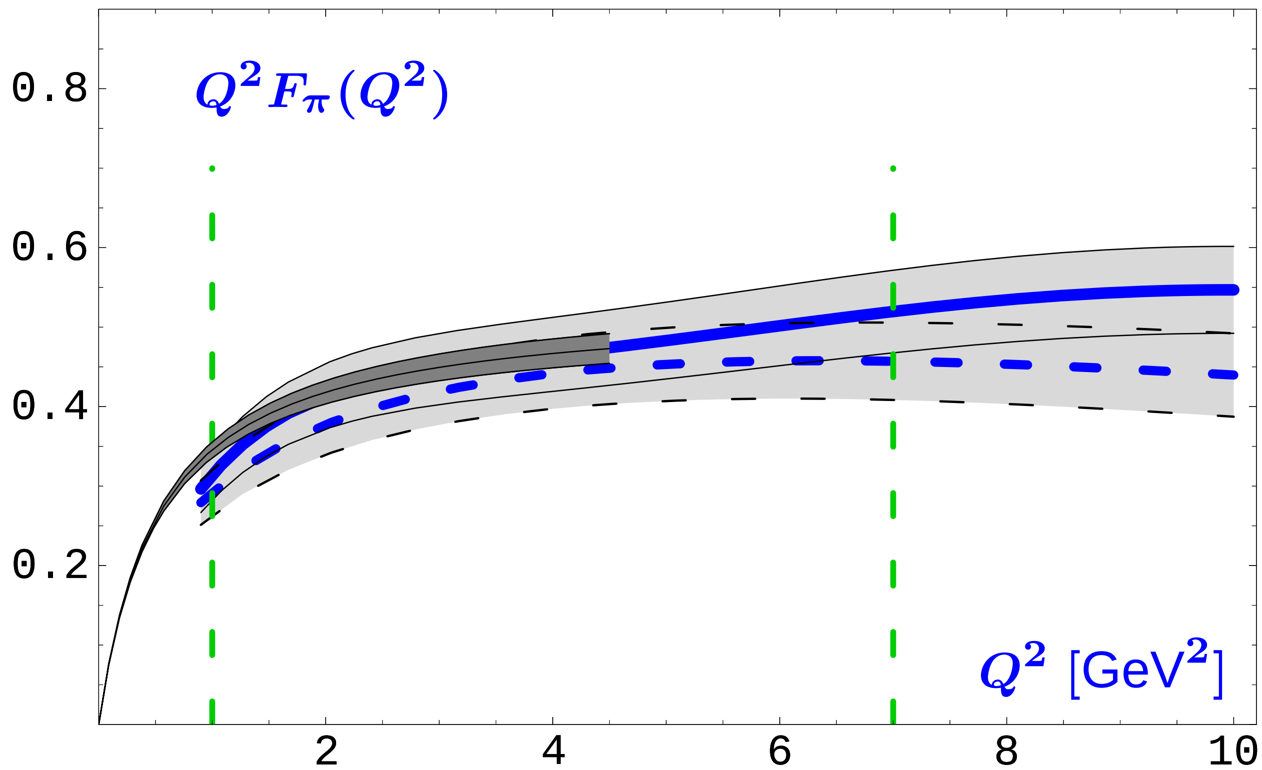
NLC QCD SR Result



curve	approach
	Minimal
	Improved
	Lattice **[Brommel08]**
	[Cornell 78]
	[JLab 08]
	QCD SR **[NR,IS 82]**
	LD SR **[BLM07]**
	AdS/QCD **[GR08]**
	AdS/QCD **[BT07]**

- Pion FF in QCD SR with nonlocal condensates in minimal and improved models in comparison with lattice simulations, experimental results and other approaches.
- We wait for the data up to 6 GeV^2 from JLab 12 GeV^2 Upgrade!

NLC QCD SR Result



curve	approach
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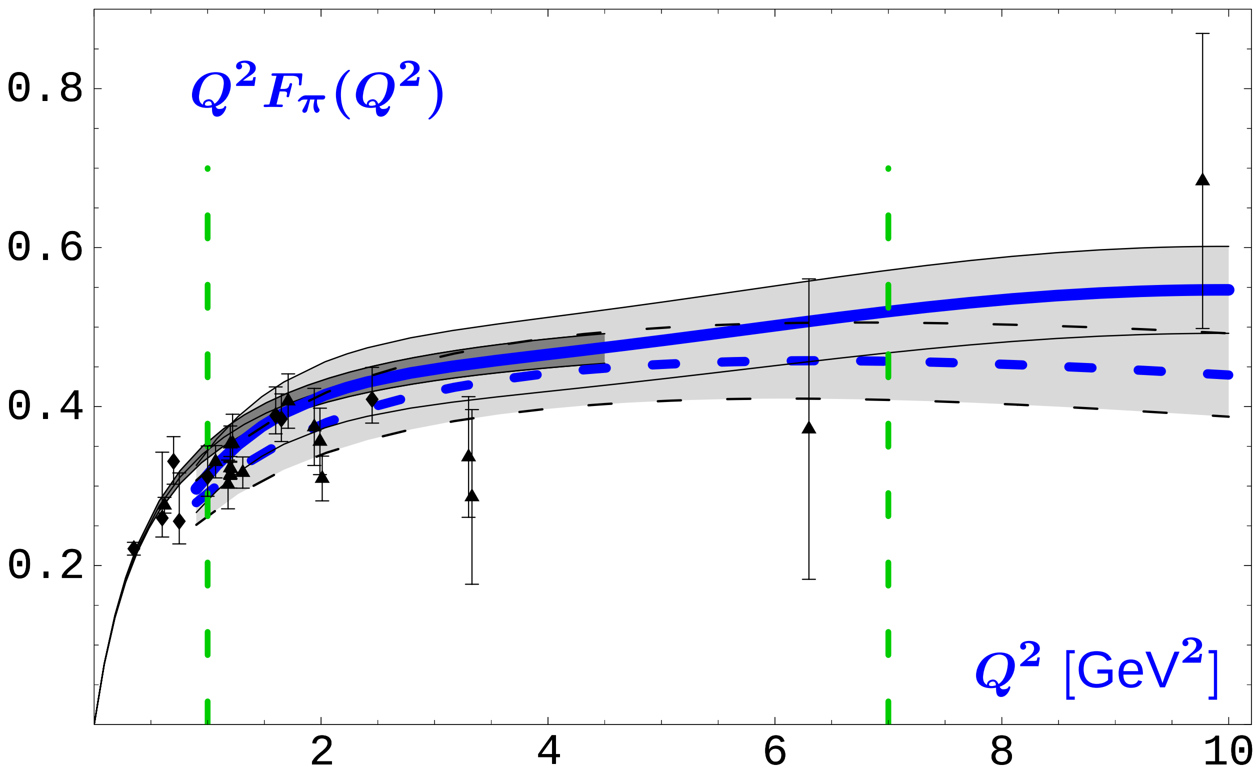

curve	approach
	Minimal
	Improved
	Lattice **[Brommel08]**
▲	**[Cornell 78]**
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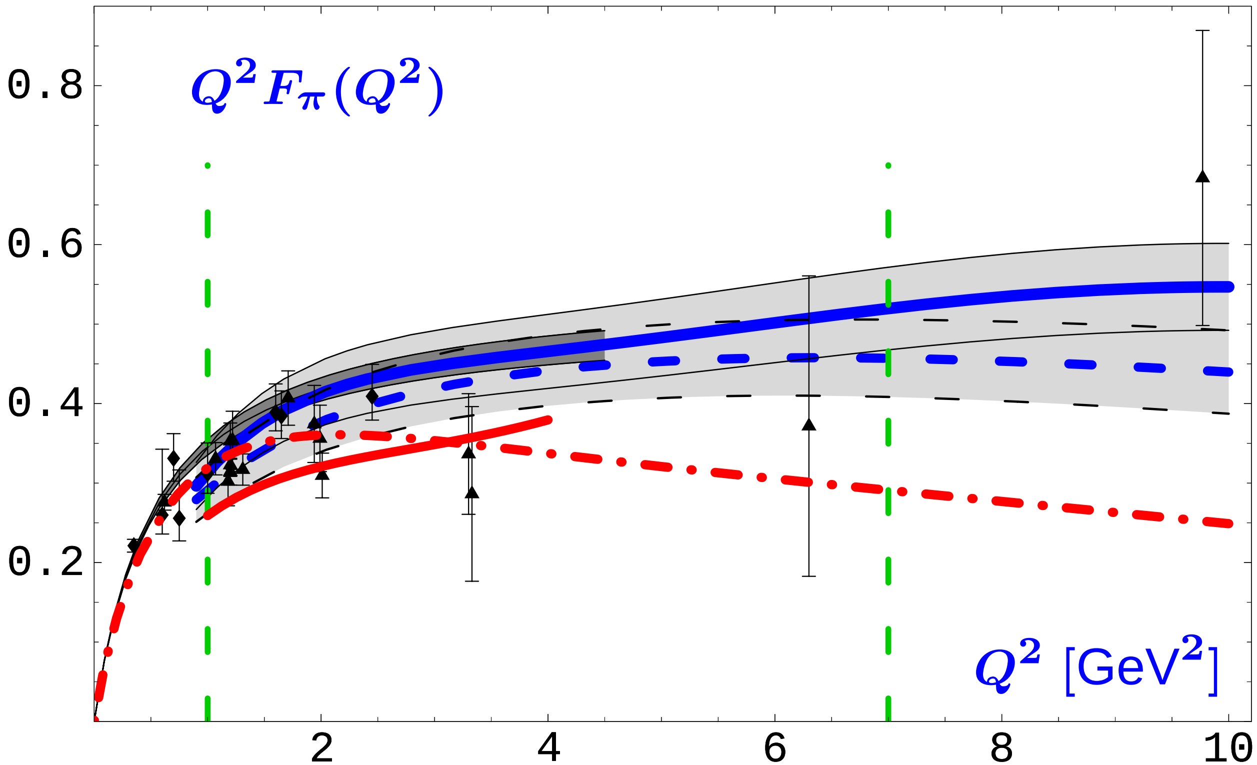

curve	approach
	Minimal
	Improved
	Lattice **[Brommel08]**
	[Cornell 78]
	[JLab 08]
	QCD SR **[NR,IS 82]**
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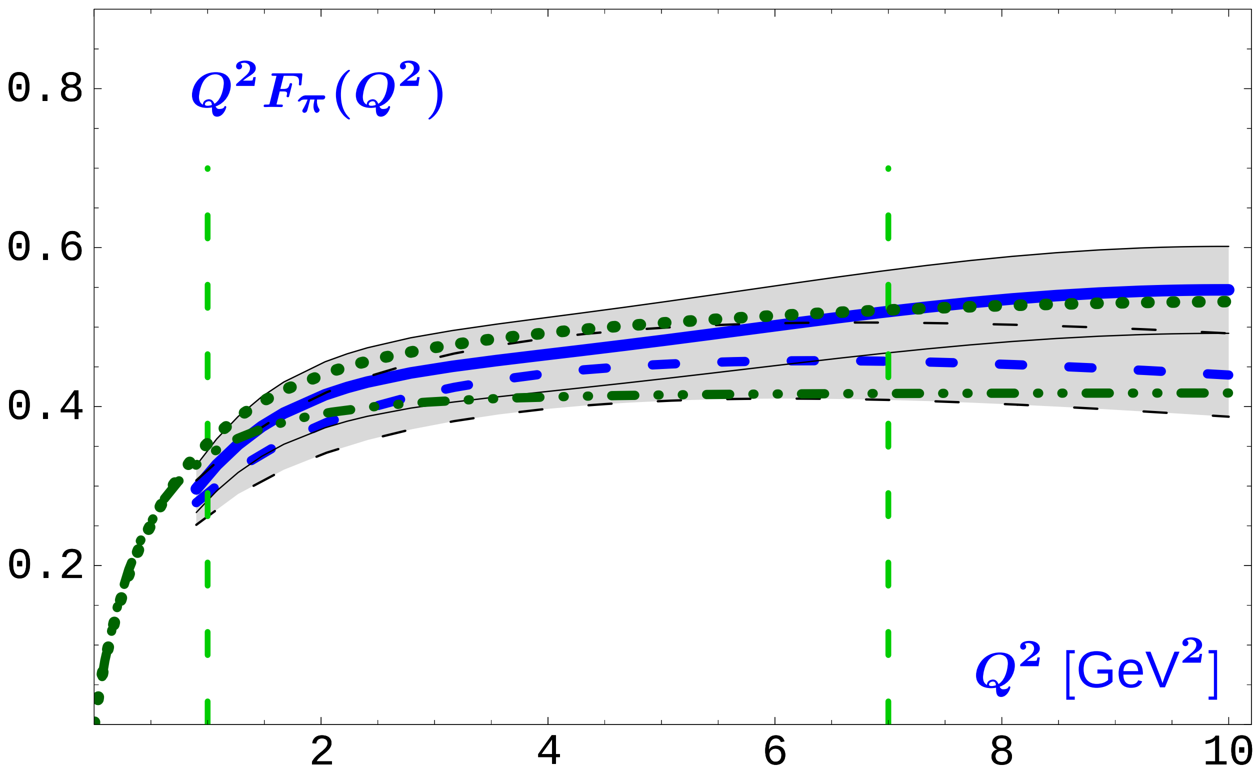
NLC QCD SR Result



curve	approach
blue dashed	Minimal
blue solid	Improved
	Lattice **[Brommel08]**
	[Cornell 78]
	[JLab 08]
	QCD SR **[NR,IS 82]**
	LD SR **[BLM07]**
green dash-dot	AdS/QCD **[GR08]**
green dotted	AdS/QCD **[BT07]**

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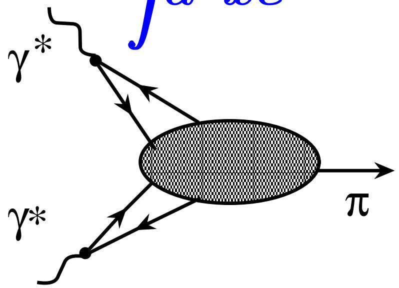
“Factorization” $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(P)$ in pQCD

$$\int d^4x e^{-iq_1\cdot z}\langle \pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot F^{\gamma^*\gamma^*\pi}(Q^2,q^2)\,,$$

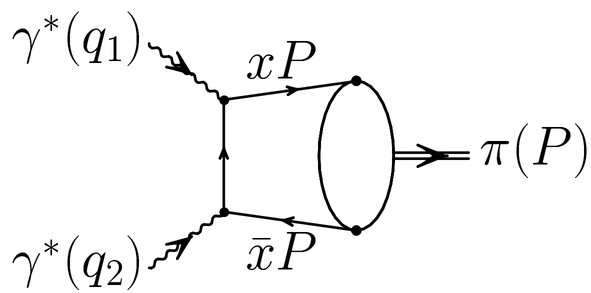


where $-q_1^2 = Q^2 > 0,\ \ -q_2^2 = q^2 \geq 0$

Collinear factorization at $Q^2, q^2 \gg$ (hadron scale $\sim m_\rho^2$)

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = T(Q^2,q^2,\mu_F^2;x)\otimes\varphi_\pi(x;\mu_F^2) + O(\frac{1}{Q^4})\,,$$

where μ_F^2 – boundary between large scale and hadronic one.



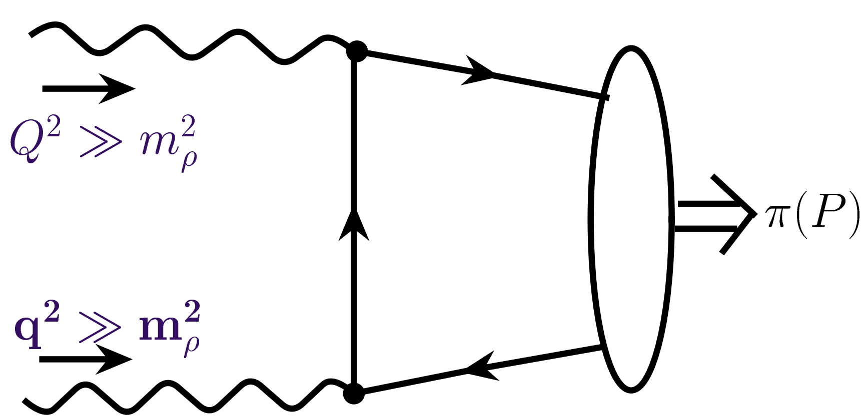
$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}}{3}f_\pi\int_0^1 dx\frac{1}{Q^2x+q^2\bar{x}}\ \varphi_\pi(x)$$

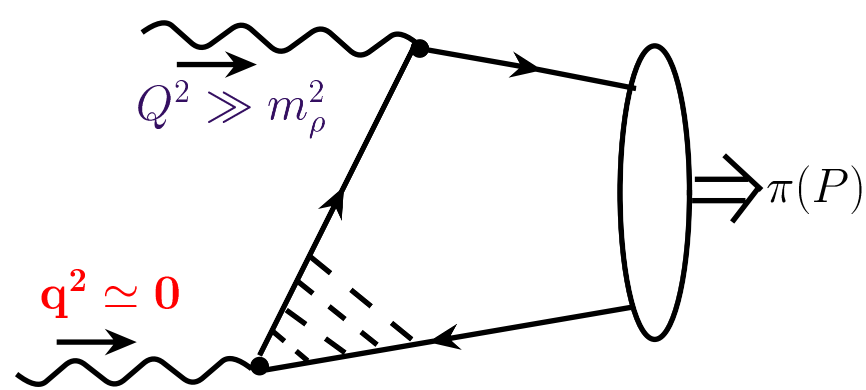
$$Q^2F^{\gamma^*\gamma\pi}(Q^2,q^2\to 0) = \frac{\sqrt{2}}{3}f_\pi\int_0^1\frac{dx}{x}\varphi_\pi(x) \equiv \frac{\sqrt{2}}{3}f_\pi\langle x^{-1}\rangle_\pi$$

$\gamma^*\gamma \to \pi$: Why Light-Cone Sum Rules?

- For $Q^2 \gg m_\rho^2, \;\; q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance**[Radyushkin–Ruskov, NPB (1996)]**.

- Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances $\sim O(1/\sqrt{q^2})$





pQCD is OK **LCSRs should be applied**

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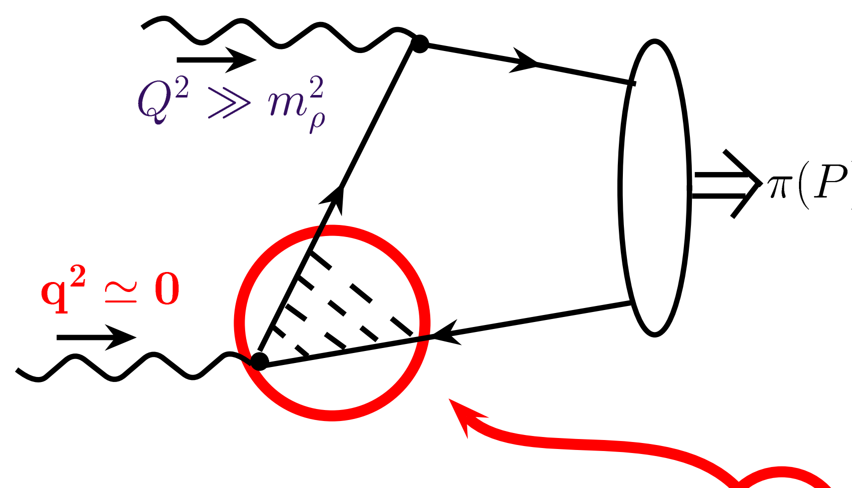


- To account for long-distance effects in pQCD one needs for light-cone DA of real photon

$\gamma^*\gamma \to \pi$: Light-Cone Sum Rules!

Khodjamirian **[EJPC (1999)]**: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) \;=\; \int_0^{s_0} \frac{\rho^{\text{PT}}(Q^2,s)}{m_\rho^2+q^2} e^{(m_\rho^2-s)/M^2} ds + \int_{s_0}^{\infty} \frac{\rho^{\text{PT}}(Q^2,s)}{s+q^2} ds\,,$$

where $s_0 \simeq 1.5$ GeV^2 – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).
Real-photon limit $q^2 \to 0$ can be easily done.

Spectral density is defined by Im-part of FF for two virtual photons:

$$\rho^{\text{PT}}(Q^2,s) = \text{Im}F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2,-s-\imath\varepsilon) = \text{Tw-2} + \text{Tw-4} + \text{Tw-6} + \ldots\,,$$

where twists contributions given in a form of convolution with pion DA:

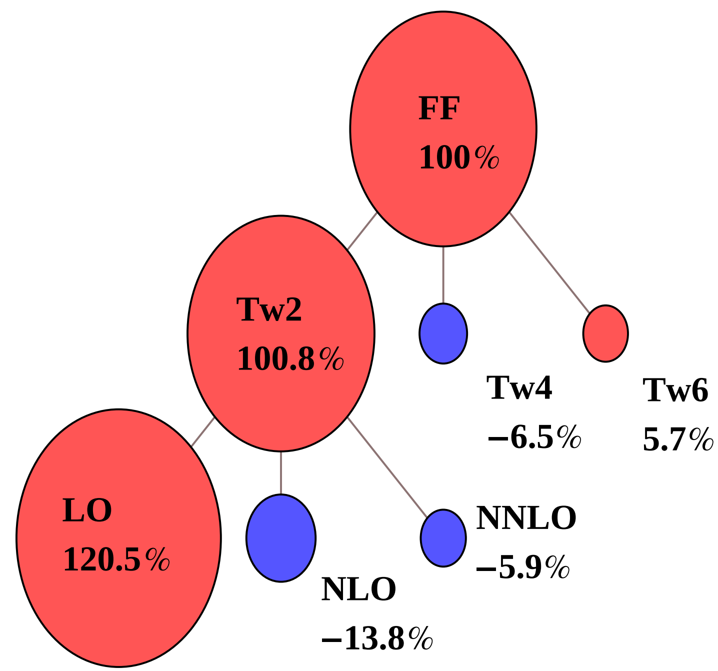
$$\text{Tw-2} \sim (T_{\text{LO}} + T_{\text{NLO}} + T_{\text{NNLO}_{\beta_0}} + \ldots) \otimes \varphi_\pi^{\text{Tw2}}(x)\,.$$

Main Ingredients of Spectral Density

- **LO** Spectral Density, **Tw-4** term — **Khodjamirian[EJPC (1999)]**
- **NLO** Spectral Density — in **[Mikhailov&Stefanis(2009)]**
- **NNLO$_{\beta_0}$** Spectral Density — in **[M&S(2009)]**
- **Tw-6** contribution — in **[Agaev et.al.–PRD83(2011)0540020]**

Terms of Pion-Photon FF at $Q^2 = 8\,\text{GeV}^2$

- Result is dominated by Hard Part of Twist-2 LO contribution.
- Twist-6 contribution is taken into account together with **NNLO$_{\beta_0}$** one — they has close absolute values and opposite signs.



Blue - negative terms

Red - positive terms

Parameters of LC SR

From PDG:

- $\alpha_s(m_Z^2)$
- Masses m_ρ, m_ω
- Decay Widths Γ_ρ, Γ_ω

From QCD SR:

- Borel parameter M^2_{LCSR}
- Vector Chan. Threshold s_0
- Twist-4 $\delta^2 \pm 20\%$
- Twist-6 ($\alpha_S \langle \bar{q} q \rangle$)

Light-Cone Sum Rules:

$\text{FF} = (\text{LO} + \text{NLO}) \otimes (\pi\text{-DA}) + \text{Tw-4} + (\text{NNLO}_{\beta_0} + \text{Tw-6})$

π-DA model

Data on FF

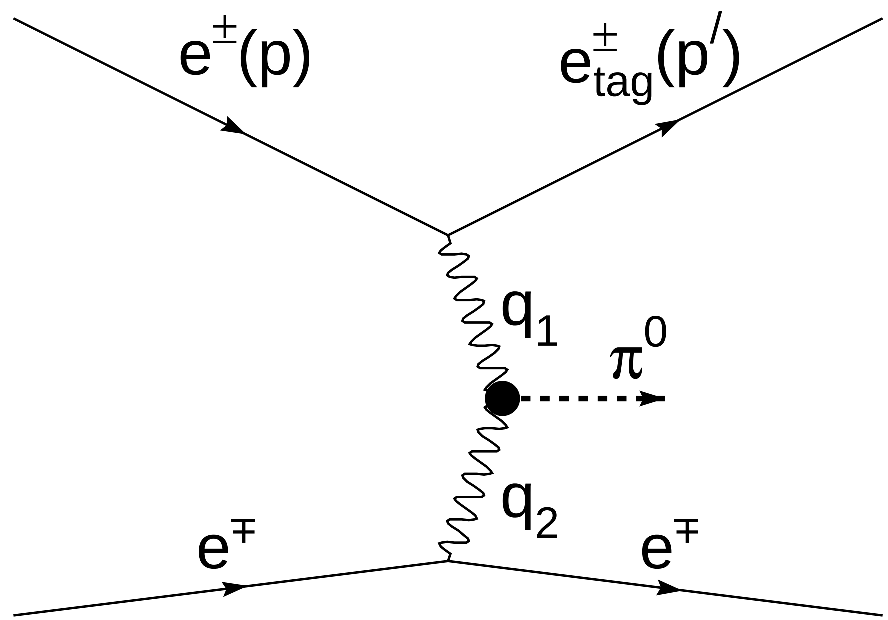
FF Prediction

Fitting π-DA (a_n)

Feynman diagram for $e^+e^- \to e^+e^-\pi^0$

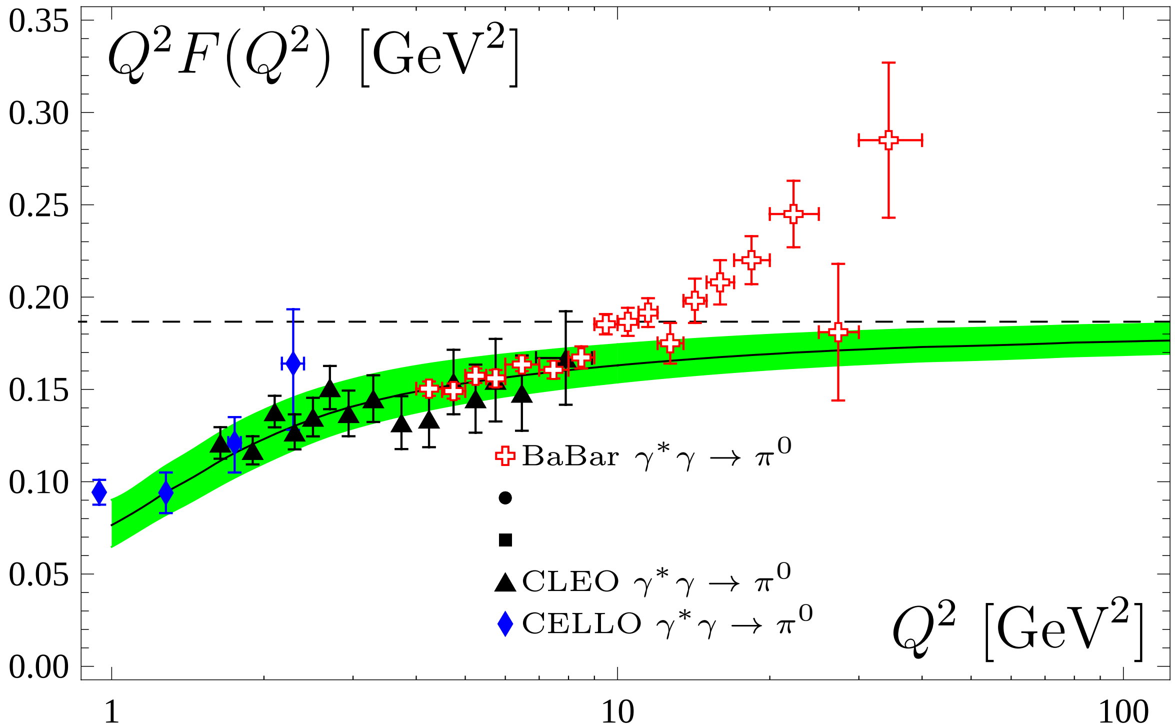
One of the most accurate data on exclusive reactions is data on transition FF $F^{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ provided by series of experiments $e^+e^- \to e^+e^-\pi^0$ with $q_2^2 \approx 0$.

CELLO (1991) $0.7 - 2.2$ GeV^2 ,
CLEO (1998) $1.6 - 8.0$ GeV^2 ,
BaBar (2009) $4 - 40$ GeV^2 .



Pion-gamma FF *vs* Experimental Data

Comparison with all data: **CELLO**, CLEO and **BaBar**



curve	DA
– – – – –	Asymp.QCD
(green band)	**BMS bunch**
	GR-**PRD77-115024**

- **BMS bunch** describes very good all data for $Q^2 \leq 9$ **GeV^2**.

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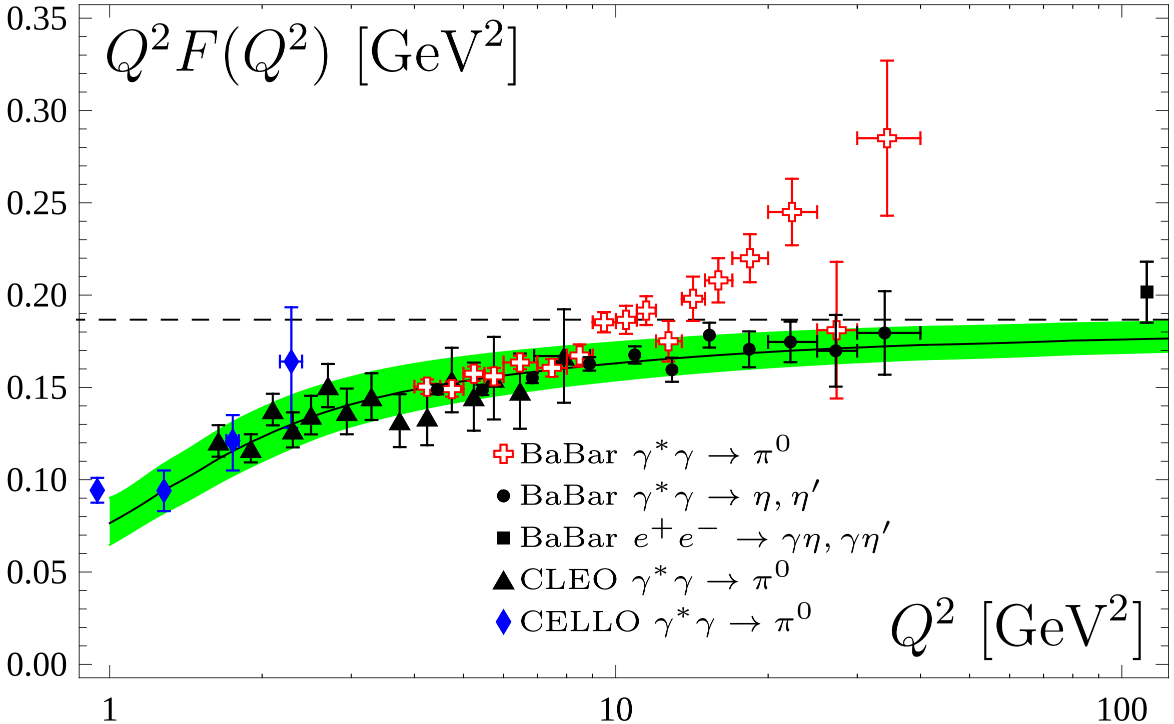

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- Note presented by BaBar rotation of $\gamma^*\gamma \to \eta, \eta'$ and $e^+e^- \to \gamma\eta, \gamma\eta'$ data (**1101.1142[hep-ex]**) to pion FF using $\eta - \eta'$ mixing scheme agrees with **BMS strip**!

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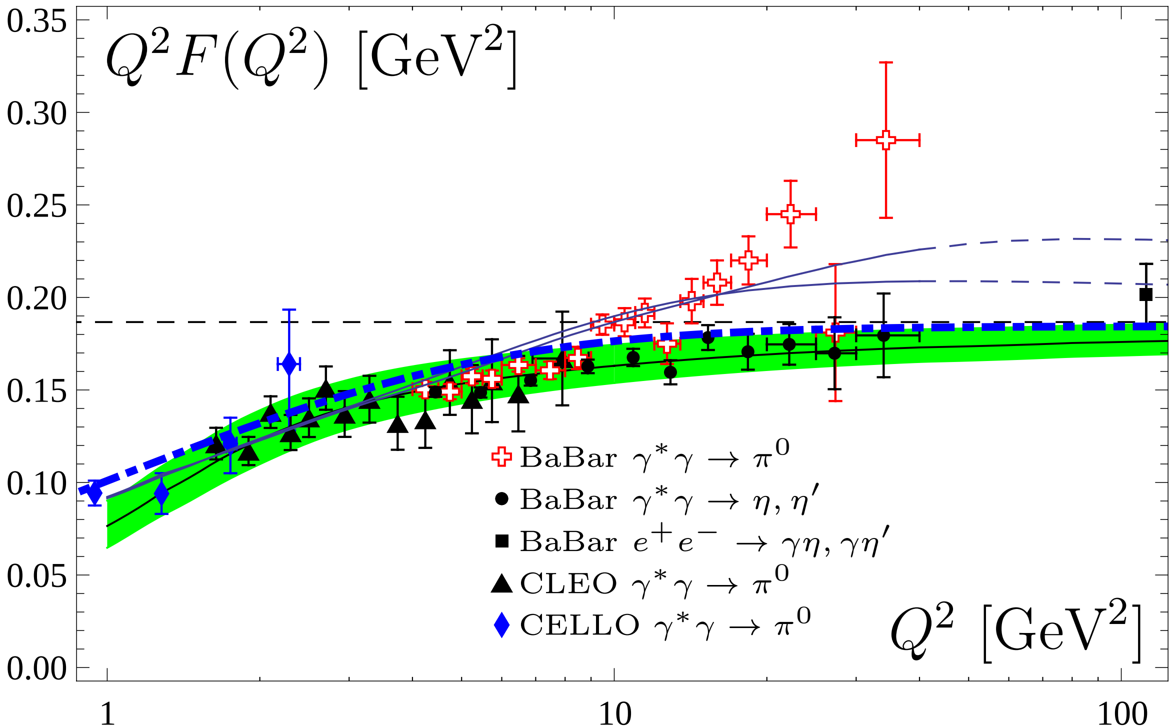

curve	DA
– – – – –	Asymp.QCD
(green band)	**BMS bunch**
———	**ABOP-1,3**
Agaev et al	**PRD83-054020**
– · – · –	GR-**PRD77-115024**

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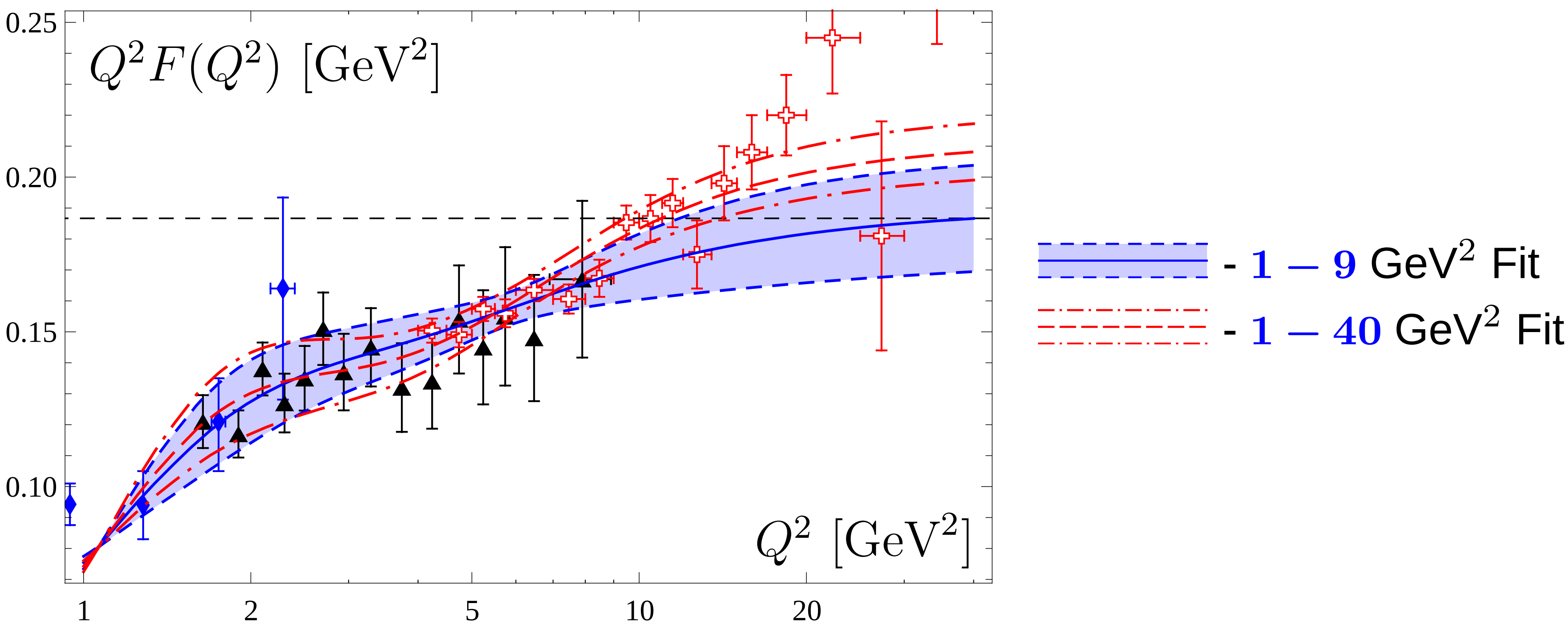
- Note presented by BaBar rotation of $\gamma^*\gamma \to \eta, \eta'$ and $e^+e^- \to \gamma\eta, \gamma\eta'$ data (**1101.1142[hep-ex]**) to pion FF using $\eta - \eta'$ mixing scheme agrees with **BMS strip**!
- ABOP models are in between two sets of BaBar data.

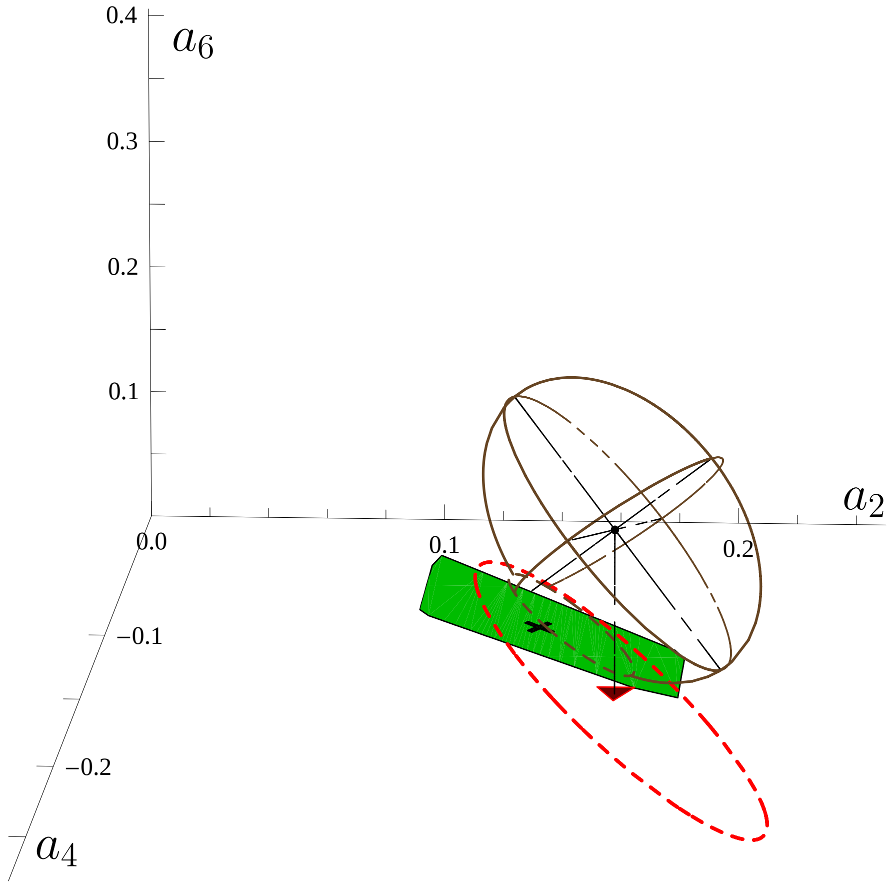
Fitting pion DA under LCSR

- We fitted experim. data on $\pi\gamma$ TFF by varying Gegenbauer coefficients of Pion DA.
- Two sets of experim. data ($1-9$ GeV^2 & $1-40$ GeV^2) were analyzed to show the influence of **BaBar** Data on Pion DA.
- To have an agreement with all data at the level $\chi^2_{\text{ndf}} \approx 1$ we need to take at least 3 terms of pion DA Gegenbauer expansion with corresponding coefficients a_2, a_4, a_6.



NLC SR Results vs 3D Constraints

BMPS [arXiv:1105.2753 [hep-ph]]: 3D 1σ-error ellipsoid at $\mu_{\text{SY}} = 2.4$ GeV scale without theoretical $\Delta\delta^2_{\text{tw4}}$ uncertainties.



$1-9$ GeV^2 Data		
red line	$\Leftrightarrow$	**2D projection of 1σ-error ellipsoid**
▼	$\Leftrightarrow$	$\chi^2_{\text{ndf}} \approx 0.4$
×	$\Leftrightarrow$	**BMS model with** $\chi^2_{\text{ndf}} \approx 0.5$

Best-fit $= (0.17, -0.14, 0.12 \pm 0.14)$
BMS $= (0.14, -0.09)$

Good agreement with all data at $Q^2 \leq 9$ **GeV^2**

At 68.3% CL we have good intersection 2D∩3D∩4D$\neq \oslash$

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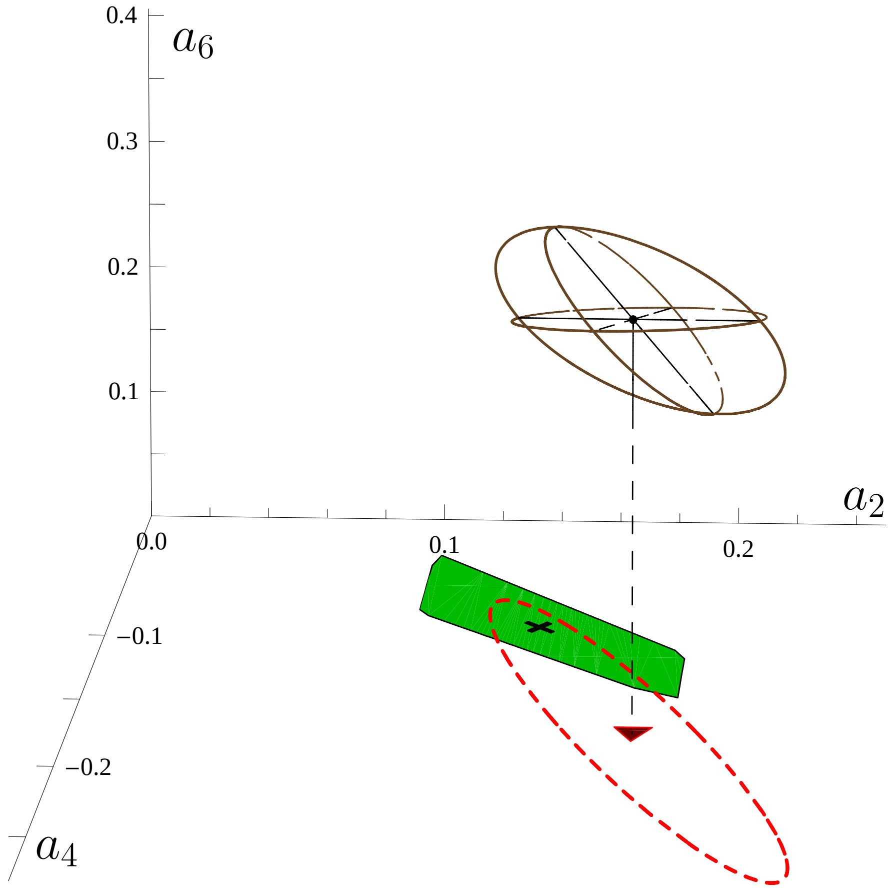


$\mathbf{1-40}$ GeV^2 Data	
— ⇔	**2D projection of 1σ-error ellipsoid**
▼ ⇔	$\chi^2_{\text{ndf}} \approx 1.0$
× ⇔	**BMS model with** $\chi^2_{\text{ndf}} \approx 3.1$

Best-fit $= (\mathbf{0.18, -0.17, 0.31 \pm 0.1})$

BMS $= (\mathbf{0.14, -0.09})$

Good agreement with all data at $Q^2 \leq 9$ **GeV**^{2}

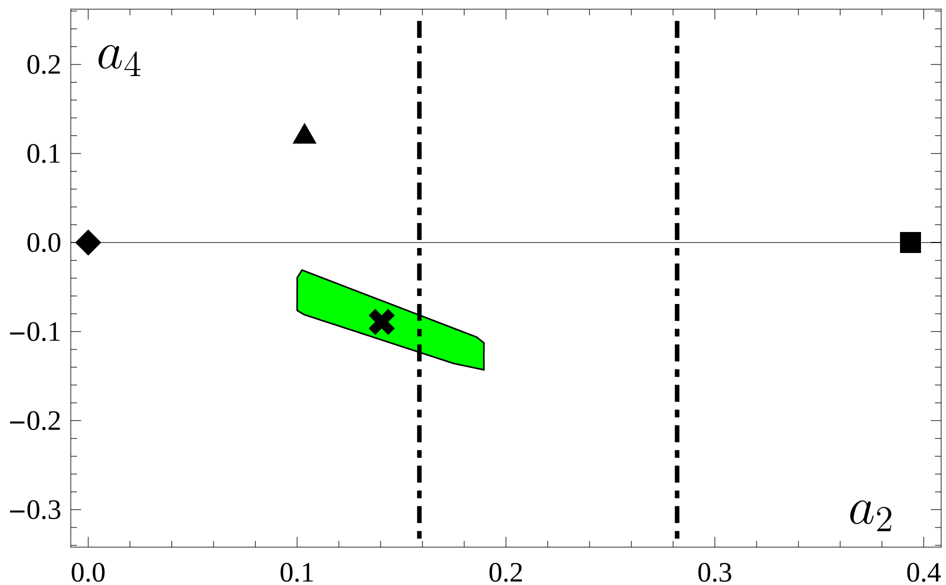
At $\mathbf{68.3\%}$ CL we have good intersection 2D∩3D∩4D$\neq \oslash$

NLC SR Results vs 2D Constraints

NLC-bunch and lattice prediction at $\mu_{\text{SY}} = 2.4$ GeV scale
with $\Delta\delta^2_{\text{tw4}}$ error

DAs: ◆ ⇔ Asymp., ▲ ⇔ ABOP-3, ✖ ⇔ **BMS**, ■ ⇔ CZ

Lattice'10 estimate of a_2 are shown by vertical lines.



$1-9$ GeV^2 Data

BMS bunch agrees well with the lattice data

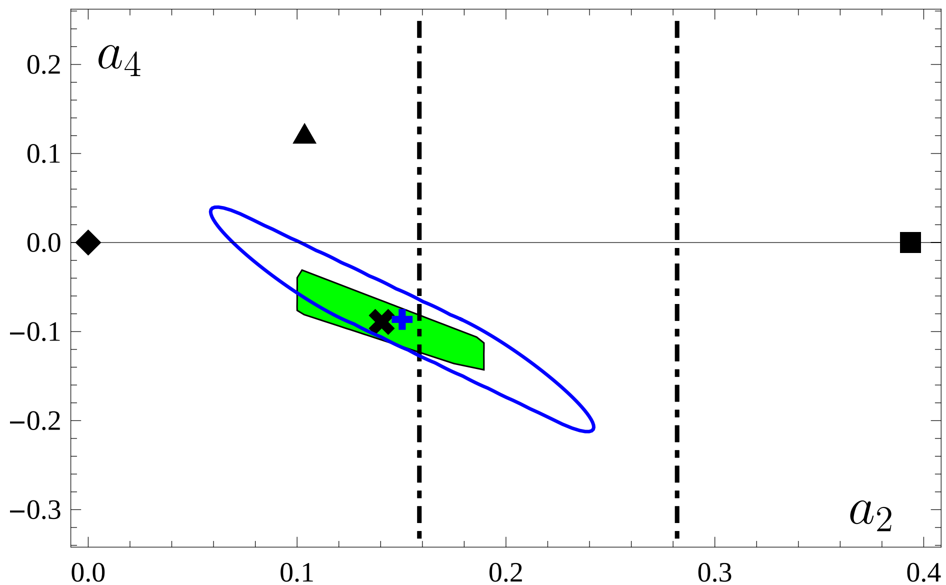
NLC SR Results vs 2D Constraints

2D-Analysis of the data at $\mu_{\text{SY}} = 2.4$ GeV scale

with $\Delta\delta^2_{\text{tw4}}$ error

DAs: ◆ ⇔ Asymp., ▲ ⇔ ABOP-3, ✖ ⇔ **BMS,** ■ ⇔ CZ

Lattice'10 estimate of a_2 are shown by vertical lines.



$1 - 9$ GeV^2 Data

⇔**2D 1σ-error ellipse**

BMS bunch agrees well with the lattice data

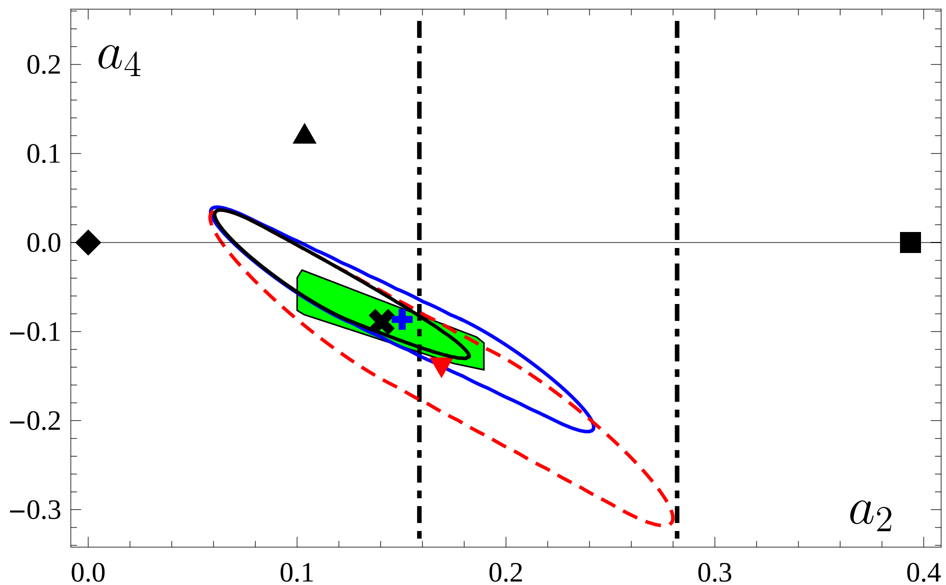
BMS bunch has better agreement with data up 9 **GeV^2** than with CLEO data only.

NLC SR Results vs 2D Constraints

2D cut of 3D ellipsoid of the data analysis at $\mu_{\text{SY}} = 2.4$ GeV scale with $\Delta\delta^2_{\text{tw4}}$ error

DAs: ◆ ⇔ Asymp., ▲ ⇔ ABOP-3, ✖ ⇔ **BMS,** ■ ⇔ CZ

Lattice'10 estimate of a_2 are shown by vertical lines.



$1-9$ GeV^2 Data

- ⇔ **2D 1σ-error ellipse**
- ⇔ **2D-Proj. 3D-ellipsoid**
- ⇔ $a_6 = 0$ cut of 3D-ellipsoid

BMS bunch agrees well with the lattice data

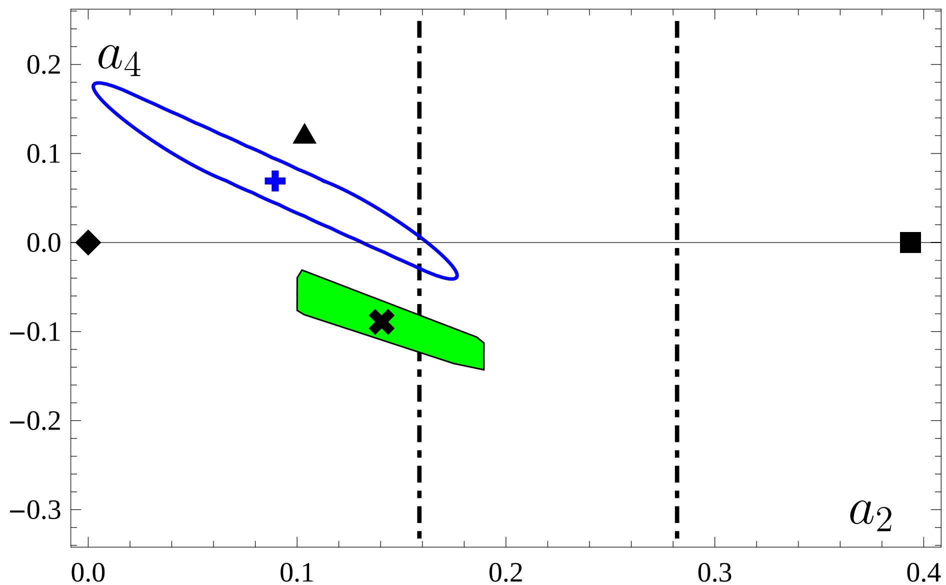
BMS bunch has better agreement with data up 9 **GeV^2** than with CLEO data only.

NLC SR Results vs 2D Constraints

BMPS [arXiv:1105.2753 [hep-ph]]: 2D $\mathbf{1\sigma}$-error ellipses at $\mu_{\text{SY}} = \mathbf{2.4}$ GeV scale with $\Delta\delta^2_{\text{tw4}}$ error

DAs: ◆ ⇔ Asymp., ▲ ⇔ ABOP-3, ✖ ⇔ **BMS**, ■ ⇔ CZ

Lattice'10 estimate of a_2 are shown by vertical lines.



$\mathbf{1 - 40}$ GeV^2 Data

⇔ **2D $\mathbf{1\sigma}$-error ellipse**

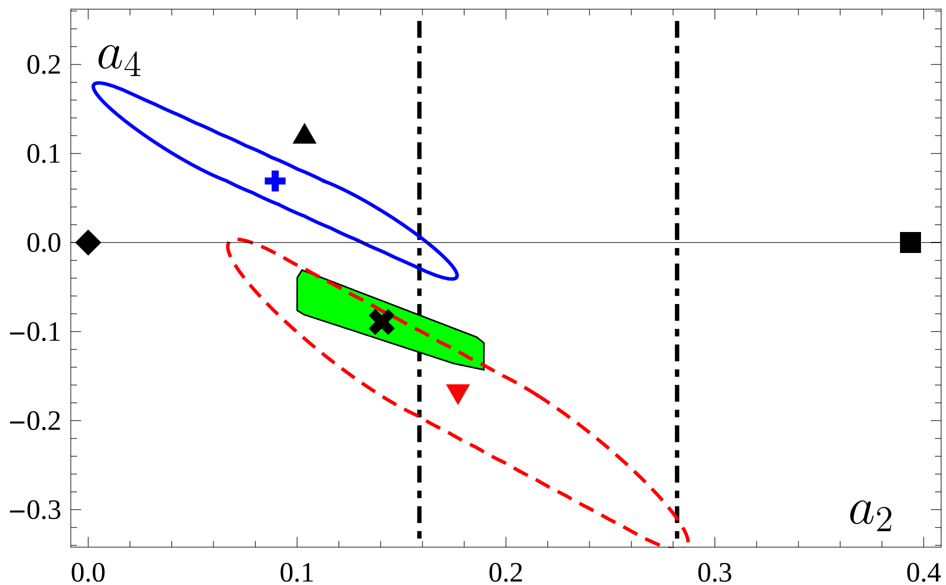
- **Bad agreement** with **2D $\mathbf{1\sigma}$-error ellipse**
- **no cross-section** with $a_6 = 0$ plane.

NLC SR Results vs 2D Constraints

BMPS [arXiv:1105.2753 [hep-ph]]: 2D 1σ-error ellipses at $\mu_{\text{SY}} = 2.4$ GeV scale with $\Delta\delta^2_{\text{tw4}}$ error

DAs: ◆ ⇔ Asymp., ▲ ⇔ ABOP-3, ✖ ⇔ **BMS,** ■ ⇔ CZ

Lattice'10 estimate of a_2 are shown by vertical lines.



$1 - 40$ GeV^2 Data

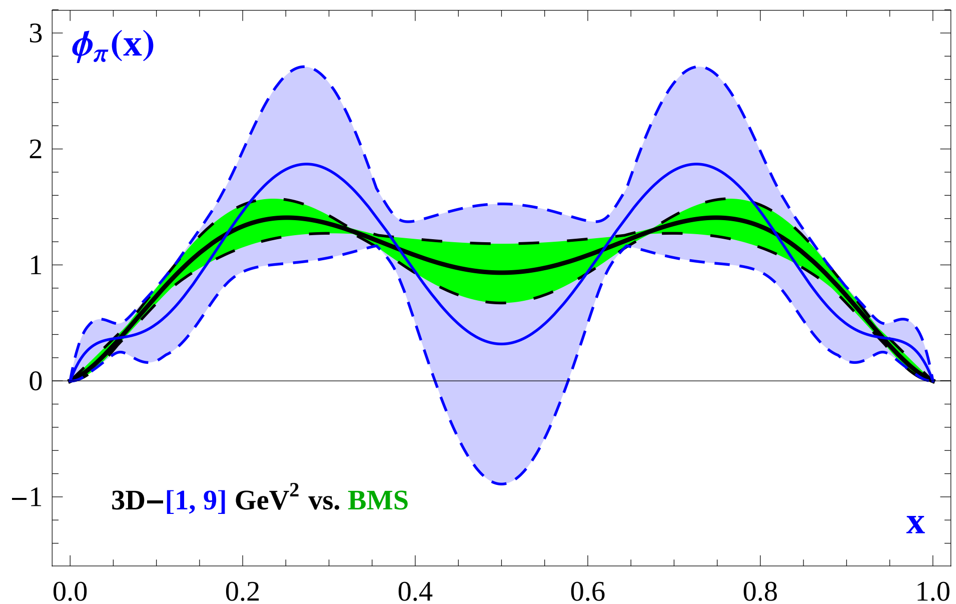
—— ⇔ **2D 1σ-error ellipse**

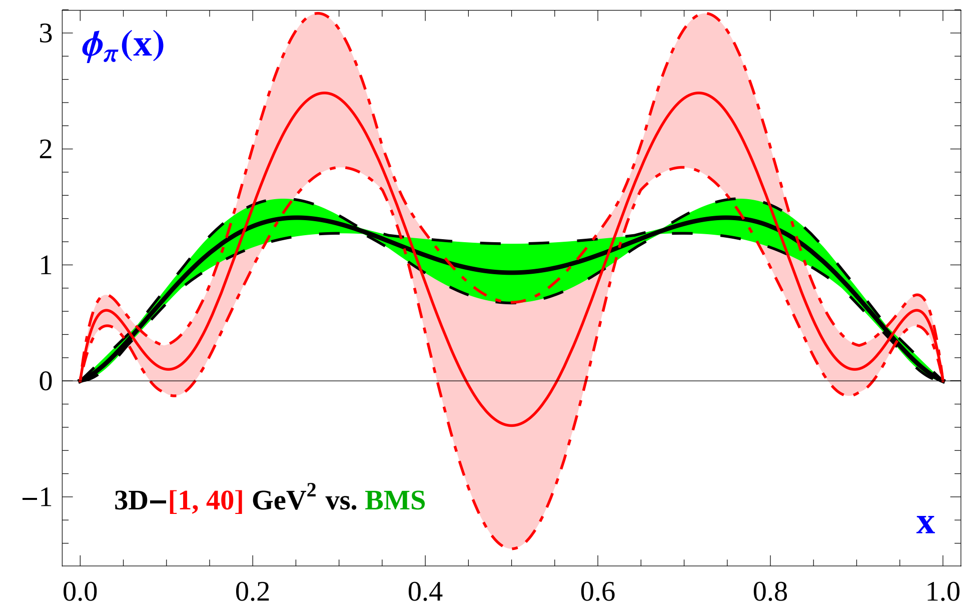
- - - ⇔ **2D-Proj. 3D-ellipsoid**

- **Bad agreement** with **2D 1σ-error ellipse**
- **no cross-section** with $a_6 = 0$ plane.

Data fit of pion DA vs QCD SR

→ BMS, → $\mathbf{1-9}$ GeV^2, → $\mathbf{1-40}$ GeV^2 at $\mu_{SY} = \mathbf{2.4}$ GeV scale





- BMS bunch agrees well with $\mathbf{1-9}$ GeV^2
- New BaBar data does not agree with BMS bunch based on NLC QCD SR.
- Both data sets does not match each other only at the end point region.
- $\mathbf{1-9}$ GeV^2 based **DA** and $\mathbf{1-40}$ GeV^2 based **DA** separated near origin.
- High BaBar data demands the end-point enhanced behavior from pion DA.

Comparison of fit with pion DA models

Model/Fit	Values of a_n	χ^2_{ndf} ($1-9$ GeV^2)	χ^2_{ndf} ($1-40$ GeV^2)
a_2, a_4, a_6 fit	$(0.18, -0.17, 0.31)$	0.4	1.0
NLC QCD SR, **BMS**	$(0.141, -0.089)$	0.5	3.1
Agaev et al	$(0.084, 0.137, 0.088)$	≥ 2.8	≥ 2.4
Modif. fact. fit, Kroll	$(0.21, 0.009)$	3.8	4.4
AdS/QCD, Brodsky et al	$0.15, 0.06, 0.03$	2.3	2.8
CZ	(0.394)	32.3	25.5
Asympt.	$(0, 0)$	4.7	7.9

All a_n values given at $\mu_{\text{SY}} = 2.4$ GeV scale.

- BMS DA gives best description in LC SR of FF for momentum up to $9\ GeV^2$.
- Result of all data fit in LC SR is far from all considered model of pion DA.

$[1-9]$ vs $[1-40]$ GeV^2 data analyses

momentum regions	$[1-9]$ GeV^2	$[1-40]$ GeV^2
BMS bunch	**Agreement**	**No!**
number of harmonics n	$2, 3$	$3, 4$
best χ^2_{ndf}	$0.53,\ \ 0.44$	$1.0,\ \ 0.77$
Slope $\varphi'_\pi(0)$	20.2 ± 20.9	48.5 ± 11.8
Slope $D^{(3)}\varphi_\pi(0.5)$	8.3 ± 3.2	12.7 ± 1.6

NLC Model	gaussian	exponential
$\varphi'_\pi(0)$	5.3 ± 0.5	7.0 ± 0.7
$D^{(3)}\varphi_\pi(0.5)$	4.7 ± 0.5	

Conclusions

- Slope of pion DA at the origin is limited by "speed" of quark condensate decay at large distances. Slower decay at large distances, causes an increase of the pion DA slope $\varphi'_\pi(0)$.
- LO QCD sum rules with natural choices of NLC lead to behavior at the origin **close to asymptotic DA** and contradicting **flat-type pion DAs**.
- Taking into account nonlocality of condensates enlarge the region of applicability of SR towards momenta as high as 10 GeV^2. Result on EM pion FF is in a good agreement with existing experimental data between $1-10$ GeV^2.
- The result from CELLO, CLEO, and BaBar data up to 9 **GeV^2** is in good agreement with previous CLEO based fit and prefers a end-point suppressed pion DA, like **BMS bunch**;
- Beyond 9 **GeV^2**, the best fit to all data on $F_{\gamma^*\gamma\to\pi}(Q^2)$ including higher BaBar points requires a sizeable coefficient a_6, while the a_2 and a_4 remain the same. All data fit prefers a end-point enhanced pion DA.