Effective Field Theory For Heavy Hadron Molecules (but mostly X(3872))

S.Fleming, M. Kusunoki, T.M., U. van Kolck, PRD 76:034006 (2007)

S.Fleming, T.M., PRD 78:094019 (2008)

H.-W. Hammer, T.M., E. Braaten, PRD 82:034018 (2010)

T.M., R. Springer, PRD 83:094001 (2011)

T.M., J. Powell, arXiv:1109.3479

T.M., S. Fleming, arXiv:1110.0265

Thomas Mehen, Duke U.

INT Seminar, "Frontiers of QCD", 11/17/2011

- Brief Review of XYZ Spectroscopy
- X(3872)

  Case for Molecular State $D^0\bar{D}^{0*} + D^{*0}\bar{D}^0$

  Recent Controversies: Babar measurement of $m_{3\pi}$ and J^{PC} of $X(3872)$

  Production of X(3872) at colliders

- XEFT: Effective theory for X(3872) Production/Decay

  KSW-like theory of DD^* bound states

  Universal Predictions (LO)

  $$X(3872) \to D^0\bar{D}^0\pi^0$$
  $$D^{+0}\bar{D}^{*0} \to X(3872)\pi^+$$
  $$D^{(*)}X(3872) \to D^{(*)}X(3872)$$

  Range, Pion Corrections (NLO)

  $$X(3872) \to D^0\bar{D}^0\pi^0$$

  Factorization Thms. for Decay to $Q\bar{Q}$

  $$X(3872) \to \psi(2S)\gamma$$
  $$\psi(4040) \to X(3872)\gamma$$

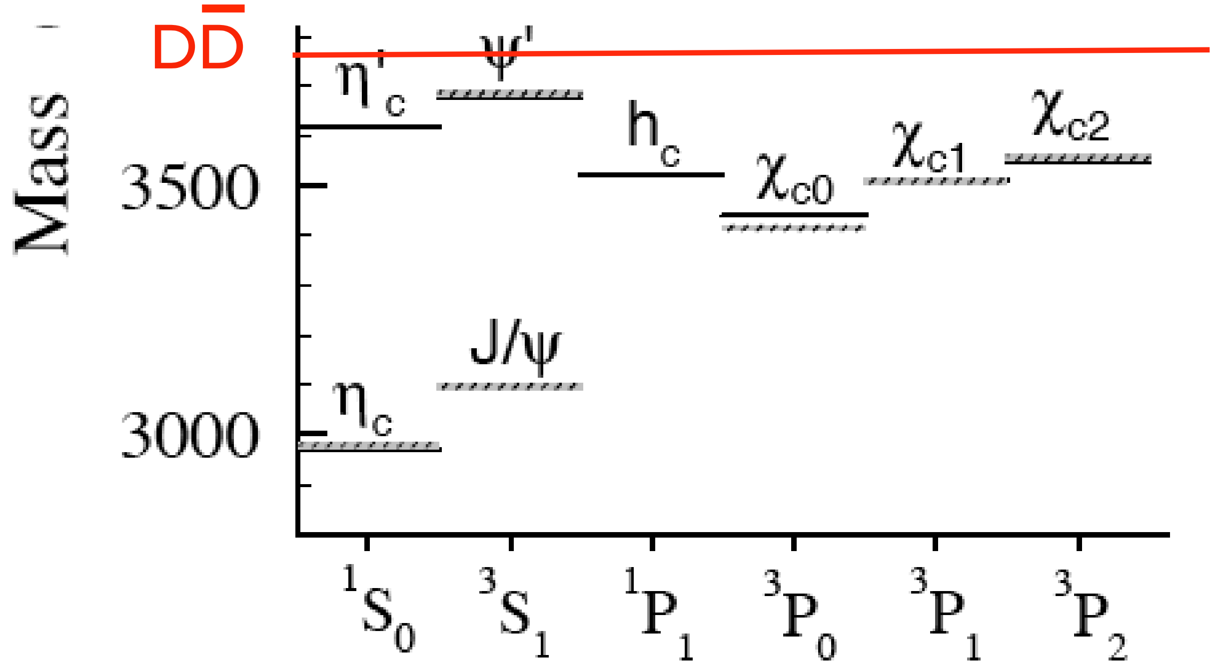
- New Bottomonium Resonances

- Below D$\bar{D}$ threshold:

complete HQS multiplets

$(\eta_c, J/\psi) \quad (\eta_c', \psi')$

(h_c, χ_{cJ})



c$\bar{c}$ meson masses & (most) transitions described by potential model

- Above D$\bar{D}$ threshold:

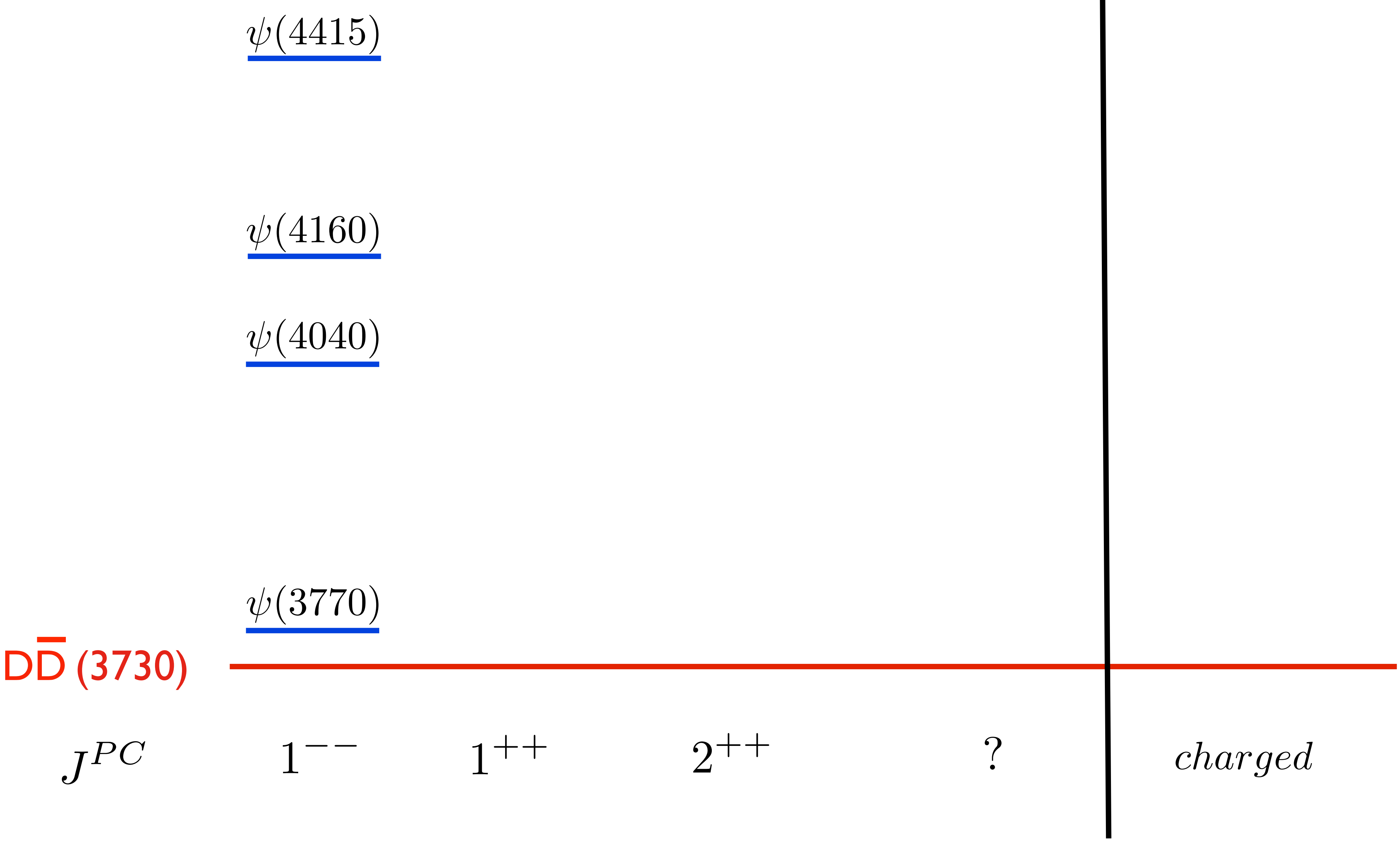
X(3872): bound state of $D^0\bar{D}^{*0} + c.c.$

new 1^{--} states: Y(4008), Y(4260),Y(4360),Y(4660)

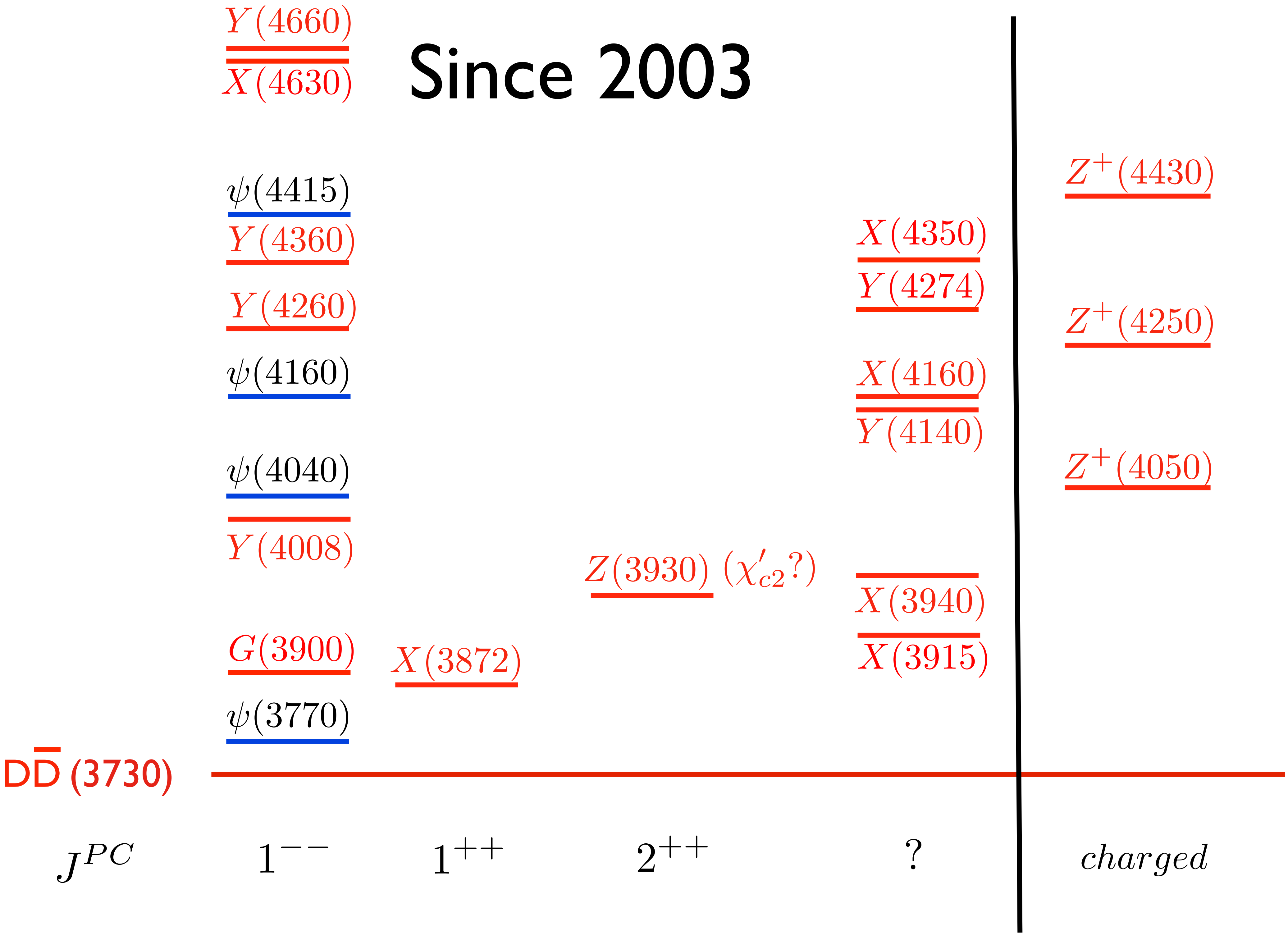
charged states! $Z^+(4430) \to \pi^+\psi' \qquad Z^+(4050), Z^+(4250) \to \pi^+\chi_{c1}$

others whose J^{PC}, nature unclear

Before 2003


$\psi(4415)$
$\psi(4160)$
$\psi(4040)$
$\psi(3770)$
$D\bar{D}$ (3730)
J^{PC}
1^{--}
1^{++}
2^{++}
?
charged

Since 2003

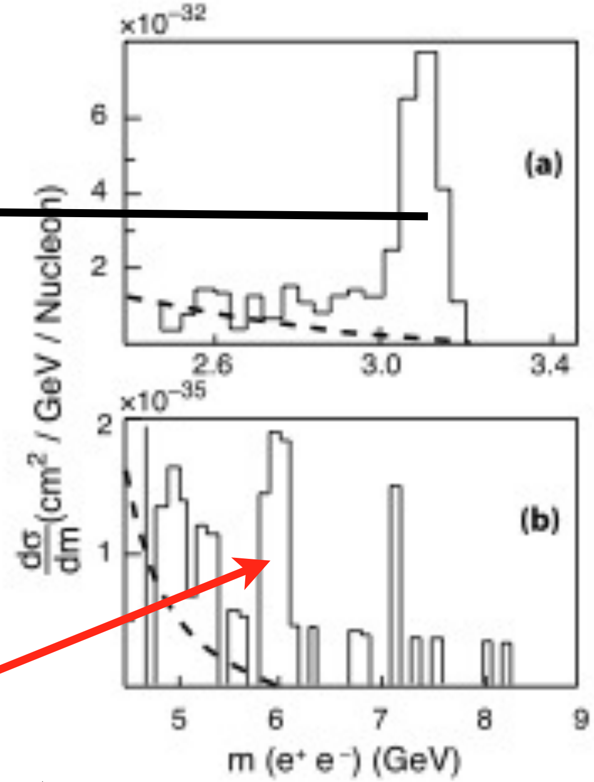


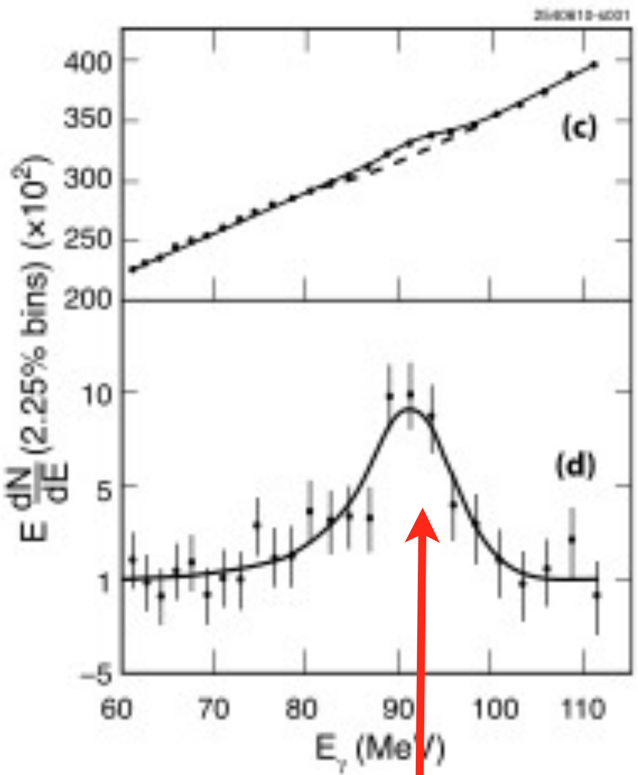
Y(4660)
X(4630)
ψ(4415)
Y(4360)
Y(4260)
ψ(4160)
ψ(4040)
Y(4008)
G(3900)
ψ(3770)
X(3872)
Z(3930) (χ'c2?)
X(4350)
Y(4274)
X(4160)
Y(4140)
X(3940)
X(3915)
Z+(4430)
Z+(4250)
Z+(4050)
DD̄ (3730)
J^PC
1−−
1++
2++
?
charged

CAVEAT: Bumps can be Fakes

"Heavy quarkonium: progress, puzzles, and opportunities", Brambilla, et. al., arXiv.1010.5827

J/ψ





"Υ" Text

The 6 GeV phenomenon was not confirmed several months later in a dimuon version [3] of the same experiment. The same authors discovered the true $\Upsilon(1S)$ shortly thereafter

"Oops-Leon"

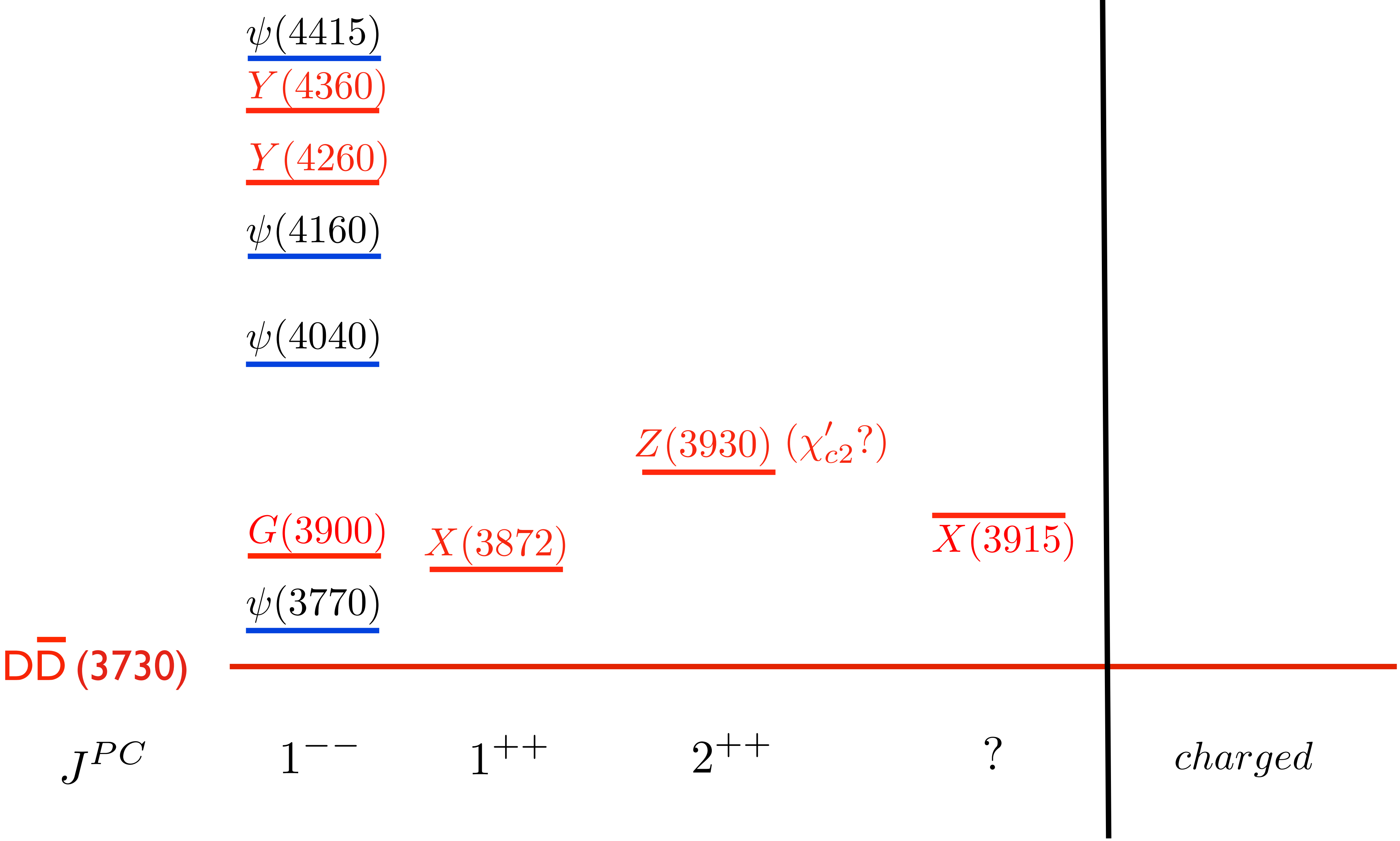
91 MeV peak in $\psi(2S)$ decays

$\eta(2S)$ candidate for 20 years! (refuted)

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.52±0.20	1.3±0.6	$1^{++}/2^{-+}$	$B \to K(\pi^+\pi^- J/\psi)$	Belle [85, 86] (12.8), *BABAR* [87] (8.6)	2003	OK
		(<2.2)		$p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$	CDF [88–90] (np), DØ [91] (5.2)		
				$B \to K(\omega J/\psi)$	Belle [92] (4.3), *BABAR* [93] (4.0)		
				$B \to K(D^{*0}\bar{D}^0)$	Belle [94, 95] (6.4), *BABAR* [96] (4.9)		
				$B \to K(\gamma J/\psi)$	Belle [92] (4.0), *BABAR* [97, 98] (3.6)		
				$B \to K(\gamma\psi(2S))$	*BABAR* [98] (3.5), Belle [99] (0.4)		
$X(3915)$	3915.6 ± 3.1	28±10	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [100] (8.1), *BABAR* [101] (19)	2004	OK
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [102] (7.7)		
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi(D\bar{D}^*)$	Belle [103] (6.0)	2007	NC!
				$e^+e^- \to J/\psi\ (...)$	Belle [54] (5.0)		
$G(3900)$	3943 ± 21	52±11	1^{--}	$e^+e^- \to \gamma(D\bar{D})$	*BABAR* [27] (np), Belle [21] (np)	2007	OK
$Y(4008)$	$4008^{+121}_{-\ 49}$	226±97	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	Belle [104] (7.4)	2007	NC!
$Z_1(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	?	$B \to K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!
$Y(4140)$	4143.4 ± 3.0	$15^{+11}_{-\ 7}$	$?^{?+}$	$B \to K(\phi J/\psi)$	CDF [106, 107] (5.0)	2009	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi(D\bar{D}^*)$	Belle [103] (5.5)	2007	NC!
$Z_2(4250)^+$	$4248^{+185}_{-\ 45}$	$177^{+321}_{-\ 72}$	?	$B \to K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!
$Y(4260)$	4263 ± 5	108±14	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	*BABAR* [108, 109] (8.0)	2005	OK
					CLEO [110] (5.4)		
					Belle [104] (15)		
				$e^+e^- \to (\pi^+\pi^- J/\psi)$	CLEO [111] (11)		
				$e^+e^- \to (\pi^0\pi^0 J/\psi)$	CLEO [111] (5.1)		
$Y(4274)$	$4274.4^{+8.4}_{-6.7}$	32^{+22}_{-15}	$?^{?+}$	$B \to K(\phi J/\psi)$	CDF [107] (3.1)	2010	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0,2^{++}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [112] (3.2)	2009	NC!
$Y(4360)$	4353 ± 11	96±42	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- \psi(2S))$	*BABAR* [113] (np), Belle [114] (8.0)	2007	OK
$Z(4430)^+$	4443^{+24}_{-18}	$107^{+113}_{-\ 71}$	?	$B \to K(\pi^+\psi(2S))$	Belle [115, 116] (6.4)	2007	NC!
$X(4630)$	$4634^{+\ 9}_{-11}$	92^{+41}_{-32}	1^{--}	$e^+e^- \to \gamma(\Lambda_c^+\Lambda_c^-)$	Belle [25] (8.2)	2007	NC!
$Y(4660)$	4664±12	48±15	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- \psi(2S))$	Belle [114] (5.8)	2007	NC!
$Y_b(10888)$	10888.4±3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^- \Upsilon(nS))$	Belle [37, 117] (3.2)	2010	NC!

“Heavy quarkonium: progress, puzzles, and opportunities”, Brambilla, et. al., arXiv.1010.5827

“OK” States



$\psi(4415)$
$Y(4360)$
$Y(4260)$
$\psi(4160)$
$\psi(4040)$
$Z(3930)$ $(\chi'_{c2}?)$
$G(3900)$
$X(3872)$
$X(3915)$
$\psi(3770)$
$D\bar{D}$ (3730)
J^{PC}
1^{--}
1^{++}
2^{++}
?
charged

What Can These Be?

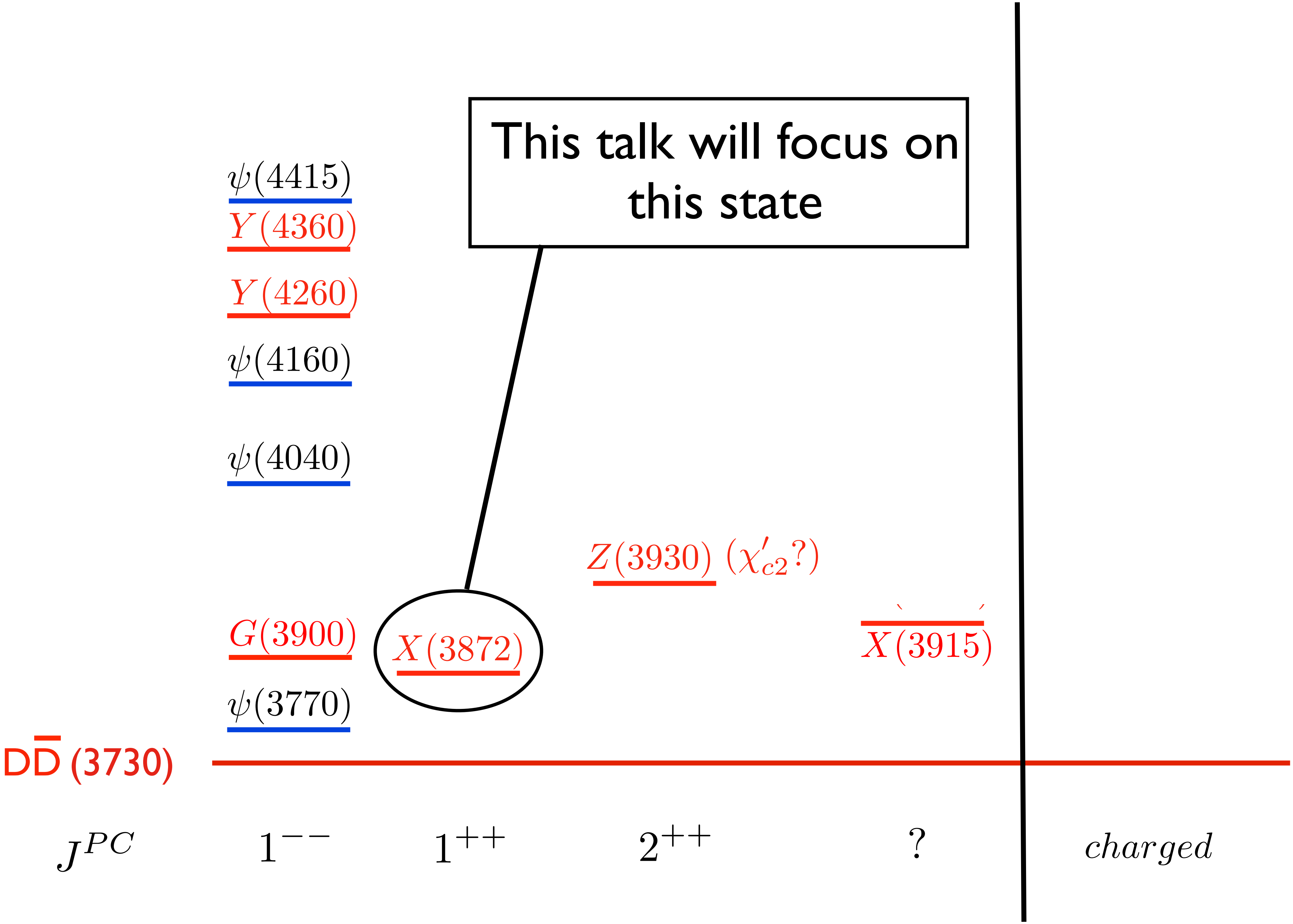
- Charmonium $c\bar{c}$
- Charmonium Hybrids $c\bar{c}g$
- Tetraquarks $c\bar{c}q\bar{q}$

diquarkonium $(cq)_{\bar{3}}(\bar{c}\bar{q})_3$

hadro-charmonium $(c\bar{c})_1(q\bar{q})_1$ (Dubyinskiy-Voloshin)

- Hadronic Molecules - $D^{(*)}\overline{D}^{(*)}$ bound states

baryonium - charm baryon bound states


This talk will focus on this state
ψ(4415)
Y(4360)
Y(4260)
ψ(4160)
ψ(4040)
Z(3930) (χ'c2?)
G(3900)
X(3872)
X(3915)
ψ(3770)
DD̄ (3730)
J^PC
1−−
1++
2++
?
charged

X(3872)

- shallow bound state of $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}$
- Decays: $X(3872) \to J/\psi\pi^+\pi^-$ $\quad X(3872) \to J/\psi\pi^+\pi^-\pi^0$

  $\to D^0\bar{D}^0\pi^0$ $\quad \to J/\psi\gamma$ (C=1)

  $\to D^0\bar{D}^0\gamma$ $\quad \to \psi(2S)\gamma$

- angular distributions in $J/\psi\pi^+\pi^-$ require $J^{PC} = 1^{++}$ or 2^{-+}

  observation of $\psi'\gamma$ and $D^0\bar{D}^0\pi^0$ disfavor $J = 2$

$J^{PC} = 1^{++}$ S-wave coupling to $D\bar{D}^* + \bar{D}D^*$

CAVEAT: Babar measurement of $m_{3\pi}$ $X(3872) \to J/\psi\pi^+\pi^-\pi^0$

prefers $J^{PC} = 2^{-+}$ P. del Amo Sanchez et al. (BABAR) (2010), 1005.5190.

- $$\frac{\mathrm{Br}[X(3872) \to J/\psi\,\pi^+\pi^-\pi^0]}{\mathrm{Br}[X(3872) \to J/\psi\,\pi^+\pi^-]} = 1.0 \pm 0.4 \pm 0.3$$

X(3872) is mixed state w/ I=0 and I=1

- extremely close to threshold:

$$M_X - (M_{D^{*0}} + M_{D^0}) = 0.42 \pm 0.39\,\mathrm{MeV}$$

$m_X = 3871.61 \pm 0.16 \pm 0.19\,\mathrm{MeV}$ CDF, Aaltonen, et. al. 0906.5128 [hep-ex]

$m_{D^0} = 1864.84 \pm 0.18\,\mathrm{MeV}$

$m_{D^{*0}} = 2006.97 \pm 0.19\,\mathrm{MeV}$ PDG

unique among proposed molecules:

$Z^+(4430) : (D_1^0 D^{*+}) \quad E_B = -0.4 \pm 5.4\,\mathrm{MeV}$

$Y(4660) : (\psi' f_0) \quad E_B = 2 \pm 25\,\mathrm{MeV}$

- Universality: $\psi_{DD^*}(r) \propto \frac{e^{-r/a}}{r} \quad a = 10.0^{+\infty}_{-4.2}\,\mathrm{fm} \qquad B.E. = \frac{1}{2\mu_{DD^*} a^2}$

Long distance physics of X(3872) calculable in terms of scattering length, known properties of D mesons - Effective Range Theory (ERT)

(M.B. Voloshin, E. Braaten, et. al.)

- for recent work attempting to extract resonance parameters from line shapes in $X(3872) \to J/\psi\, \pi^+\pi^-$ and $D^0 \bar{D}^0 \pi^0$ see

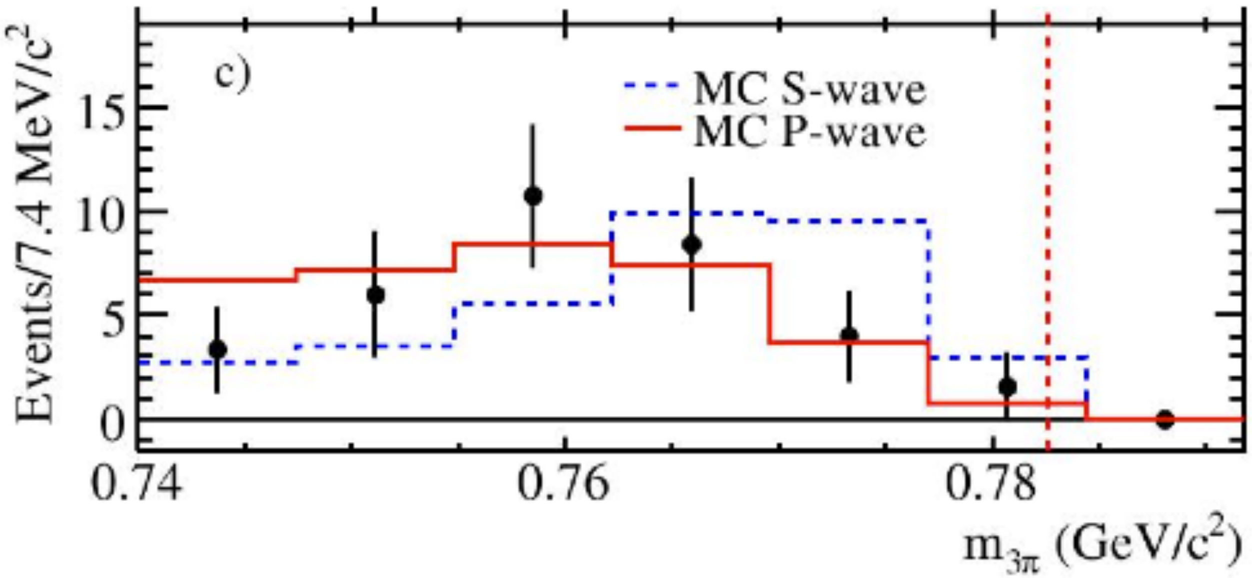
E. Braaten & J. Stapleton, PRD 81:014019 (2010)

C. Hanhart, et. al., PRD 76:034007 (2007)

Y.S. Kalashnikova & A.V. Nefediev, PRD 80:074004 (2009)

Quantum numbers of X(3872)?

- Babar measurement of $m_{3\pi}$ P. del Amo Sanchez et al. (BABAR) (2010), 1005.5190.



- Disaster for molecular interpretation

- also difficult for other scenarios, e.g., conventional quarkonium, tetraquark, etc.

Y.S. Kalashnikova & A.V. Nefediev, arXiv:1008.2895 [hep-ph]
T.J. Burns, et. al., arXiv:1008.0018 [hep-ph]
Y. Jia, et. al., arXiv:1007.4541 [hep-ph]

X(3872) Production in Hadron Colliders

- CDF X(3872) production cross section

$$\sigma_{\text{prompt}}[X(3872)]\ \text{Br}[X \to J/\psi\pi^+\pi^-] \approx 3.1 \pm 0.7\ \text{nb},$$

$$\sigma_{b\text{-decay}}[X(3872)]\ \text{Br}[X \to J/\psi\pi^+\pi^-] \approx 0.59 \pm 0.23\ \text{nb}.$$

$$\sigma_{prompt}[X(3872)] \sim 33 - 72\ \text{nb}$$

- Is this too big for a molecule?

- Bound cross section $\sigma[p\bar{p} \to X(3872)] \leq \sigma[p\bar{p} \to D^0\overline{D}^{*0}(k < k_{max})]$

calculate $\sigma[p\bar{p} \to D\bar{D}^*(k < k_{max})]$ w/ PYTHIA

$\sigma[p\bar{p} \to D\bar{D}^*(k < k_{max})] \propto k_{max}^3$ $\quad k_{max} \sim \gamma \approx 35\ \text{MeV}$

Find $\sigma[p\bar{p} \to X(3872)] \leq 0.1\ \text{nb}\,!$ C. Bignamini, et. al. PRL 103:162001 (2009)

C. Bignamini, et. al. PoS EPS-HEP2009:074 (2009)

- Two objections: E. Braaten & P. Artoisenet, PRD82:014013 (2010)

$i)$ Final State Interactions, Watson-Migdal Theorem

$$\sigma^*(p\bar{p} \to X(3872)) = \sigma(k < \Lambda) \times \frac{6\pi\sqrt{2\mu|\mathcal{E}_0|}}{\Lambda} \quad \propto k_{max} \text{ not } k_{max}^3$$

$ii)$ $k_{max} \sim m_\pi$ (range) not γ

with these corrections estimate CDF X(3872) cross section

$$\sigma[p\bar{p} \to X(3872)] = 1.3 - 23\,\text{nb} \quad (E_X = 0.3\,\text{MeV},\ m_\pi/2 < \Lambda < 2m_\pi)$$

make predictions for cross section for X(3872)
at LHC using NRQCD Factorization Formalism

- Test methods in $\Upsilon \to ggg \to \bar{d}(+d)$

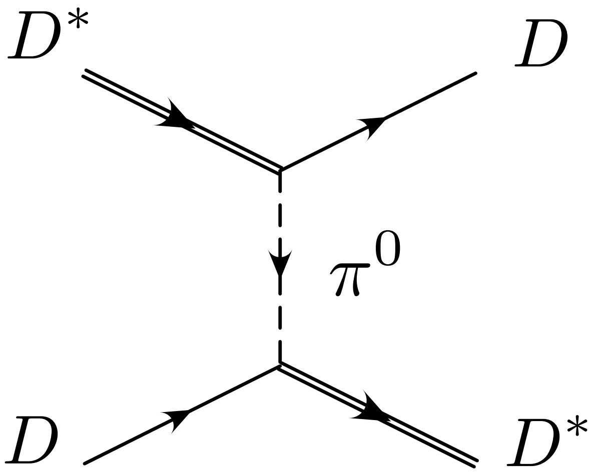
E. Braaten & P. Artoisenet, arXiv:1007.2868

X-EFT and Pions in the X(3872)

Energy Scales from Pion Exchange in the X(3872)

- π^0 exchange

$$\Delta \equiv m_{D^*} - m_D \approx 142\,\mathrm{MeV} \qquad m_{\pi^0} \approx 135\,\mathrm{MeV}$$



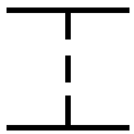
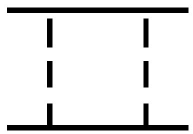
$$\frac{g^2}{2f^2}\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\Delta^2+m_\pi^2} = \frac{g^2}{2f^2}\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\mu^2}$$

- $\mu^2 \equiv \Delta^2 - m_\pi^2 \approx (44\,\mathrm{MeV})^2$ - new long-distance scale
- binding momentum: $\gamma \equiv \sqrt{-2\mu_{DD^*}\mathrm{B.E.}} \leq 34\,\mathrm{MeV}$
- $X(3872) \to D^0 \bar{D}^0 \pi^0$: $\qquad T_\pi \leq 6\,\mathrm{MeV} \quad T_D \leq 3.2\,\mathrm{MeV}$

⟶ Non-relativistic D^0, D^{*0}, π^0

Perturbative Pions and the X(3872)

Nuclear Physics: NN scattering

 $= \frac{g_A^2}{2f^2} A\left(\frac{p}{m_\pi}\right)$,  $= \left(\frac{g_A^2}{2f^2}\right)^2 \frac{M m_\pi}{4\pi} B\left(\frac{p}{m_\pi}\right)$

Expansion parameter: $\frac{g_A^2 M_N m_\pi}{8\pi f^2} \sim \frac{1}{2}$

NLO ~30% accuracy, fails at NNLO

S. Fleming, T.M., I. Stewart, NPA 677, 313 (2000)

X(3872): $g_A = 1.25 \rightarrow g \sim 0.5 - 0.7$ $m_\pi \rightarrow \mu$

$$\frac{g^2 M_D \mu}{8\pi f^2} \sim \frac{1}{20} - \frac{1}{10}$$

XEFT

S.Fleming, M.Kusunoki, T.M., U.van Kolck, PRD76:034006 (2007)

- Non-Relativistic Propagators

D D^* π $\sim \frac{1}{Q^2}$

- Contact interactions, Pion Exchange

$C_0 \sim Q^{-1}$ $C_2\, p^2 \sim Q^0$ $B_1\, \epsilon \cdot p_\pi \sim Q^{-1}$ $\sim Q^0$

- Power Counting

$$p_D \sim p_\pi \sim \mu \sim \gamma \sim Q \qquad \gamma \equiv \sqrt{-2\mu_{DD^*}\text{B.E.}} \leq 34\,\text{MeV}$$

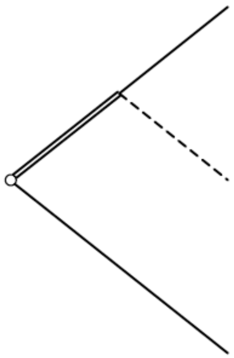
- $m_\pi \approx \Delta_H \approx 140\,\text{MeV}$ are large scales in X-EFT

similar to KSW theory of NN force

D. Kaplan, M. Savage, M. Wise, PLB 424:390 (1998), NPB 534:329 (1998)

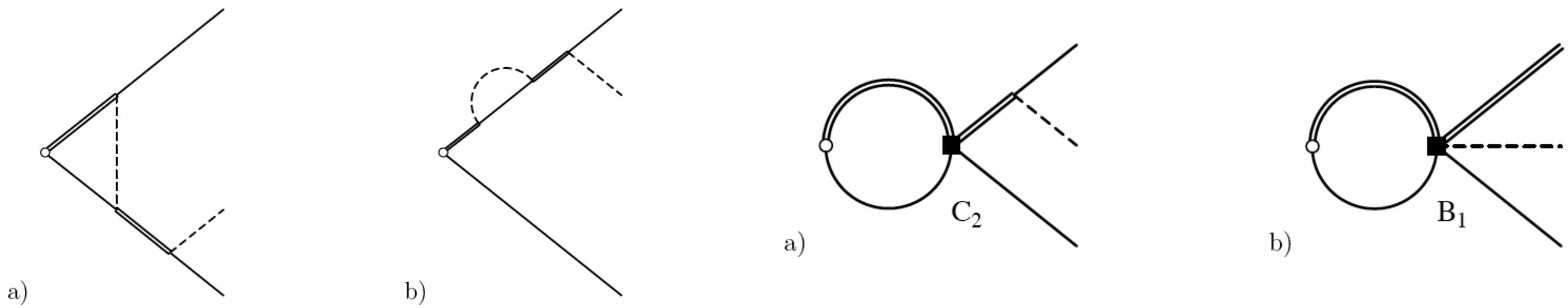
LO - reproduce ERT prediction for $X(3872) \to D^0 \bar{D}^0 \pi^0$

M.B.Voloshin, PLB 579: 316 (2004)

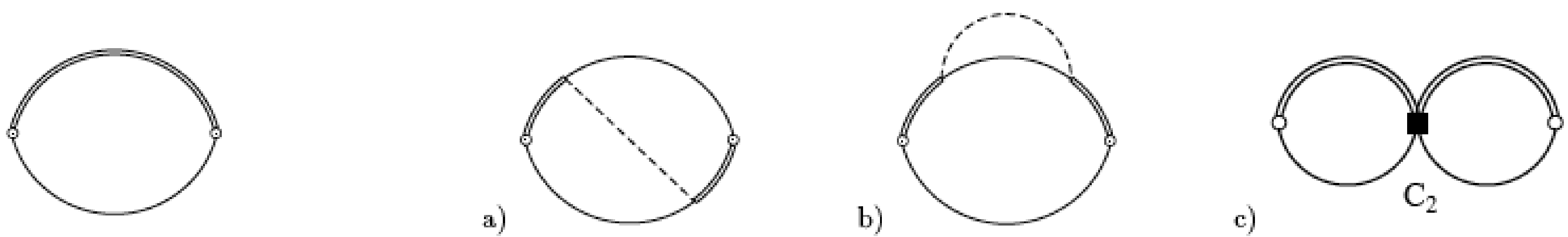


$$\frac{d\Gamma_{\rm LO}}{dp_D^2 dp_{\bar{D}}^2} = \frac{g^2}{32\pi^3 f_\pi^2} 2\pi\gamma (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left[\frac{1}{p_D^2 + \gamma^2} + \frac{1}{p_{\bar{D}}^2 + \gamma^2} \right]^2$$

NLO - range corrections, non-analytic corr. from π^0 exchange



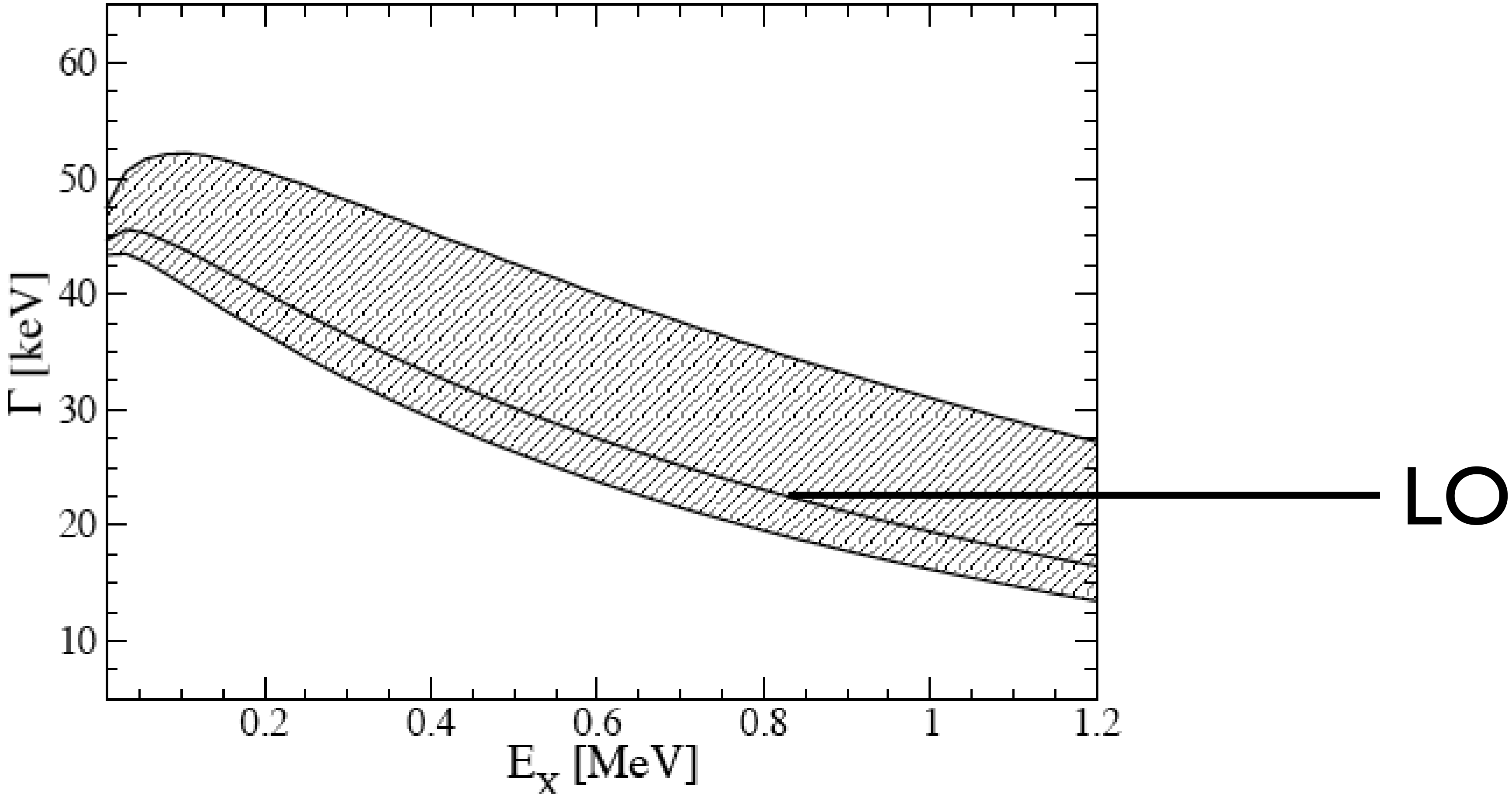
Wavefunction Renormalization



LO

NLO

- $X(3872) \to D^0 \bar{D}^0 \pi^0$ at NNLO



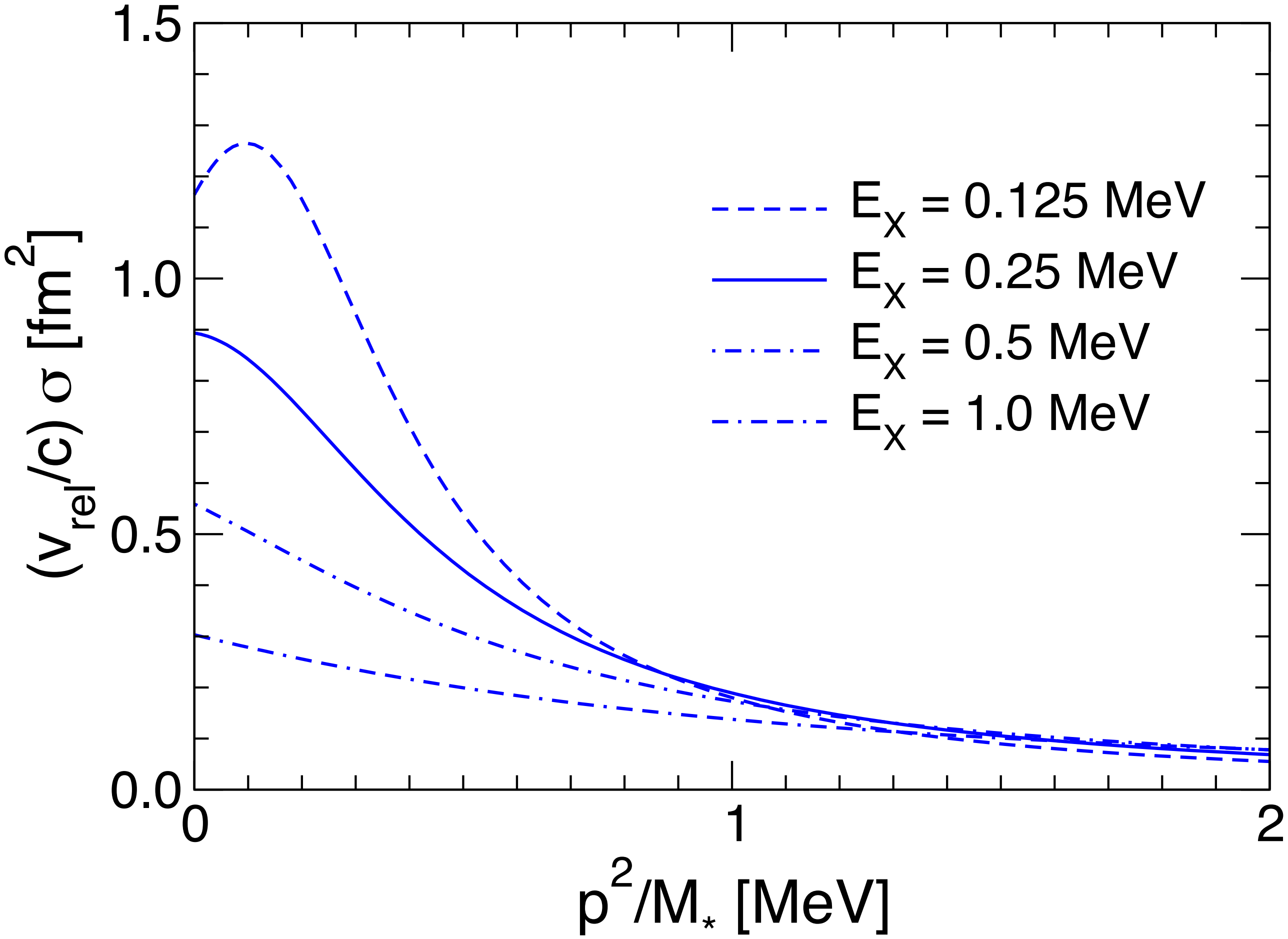
$g = 0.6 \qquad 0 \le r_0 \le (100\,\mathrm{MeV})^{-1} \qquad -1 \le \eta \le 1$

$$\left(\frac{g M_{DD^*}}{f_\pi} C_2(\Lambda_{\mathrm{PDS}}) + B_1(\Lambda_{\mathrm{PDS}})\right)(\Lambda_{\mathrm{PDS}} - \gamma) = \frac{\eta}{(100\,\mathrm{MeV})^3}$$

- Corrections Dominated by Counterterms
- Non-analytic corrections from pion loops are negligible

Other Universal Cross Sections

- **D Meson Coalescence** H.-W. Hammer, T.M., E. Braaten, PRD 82:034018 (2010)



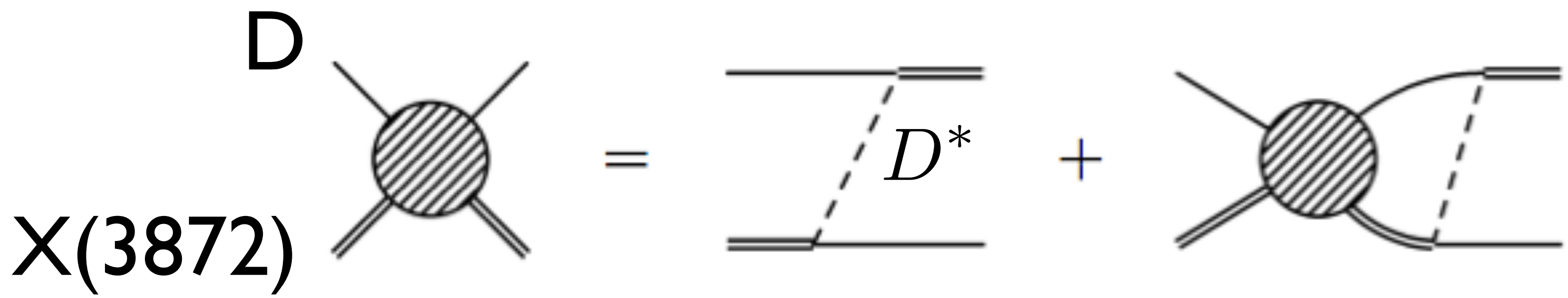
$$D^{+0}\bar{D}^{*0} \rightarrow X(3872)\pi^+$$

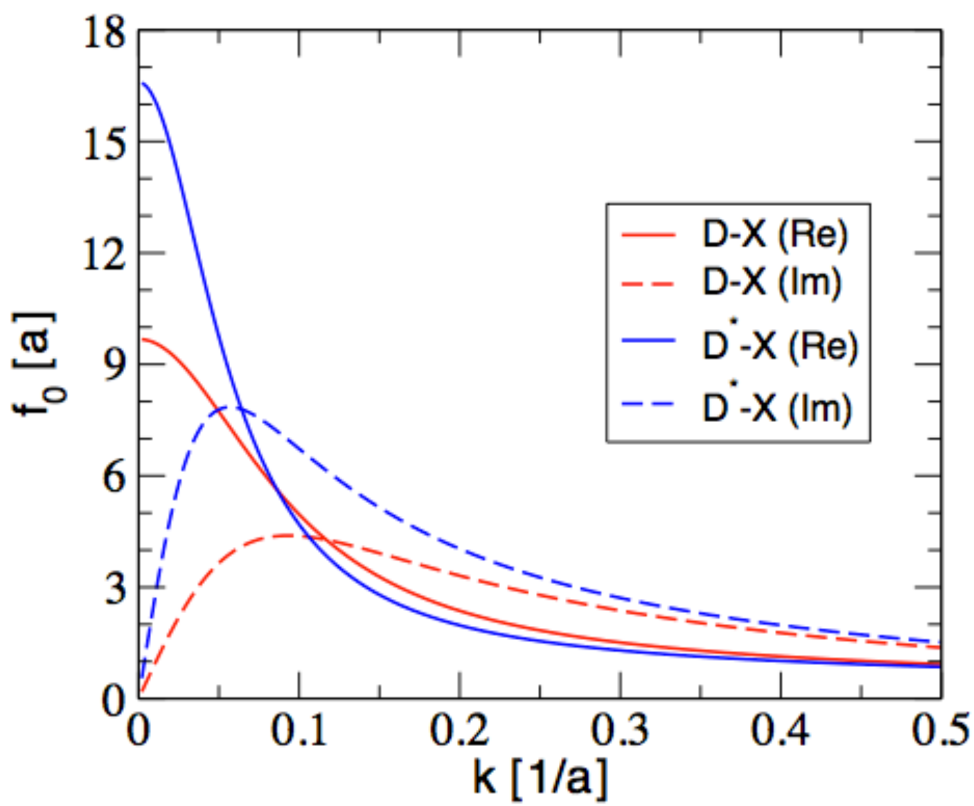
- also $X(3872)\pi \rightarrow D\bar{D}^*, \pi X(3872) \rightarrow \pi X(3872)$
- only input is X(3872) binding energy
- D-X(3872) scattering (three-body calculations)

D. Canham, H.-W. Hammer, R.P. Springer, PRD80:014009 (2009)

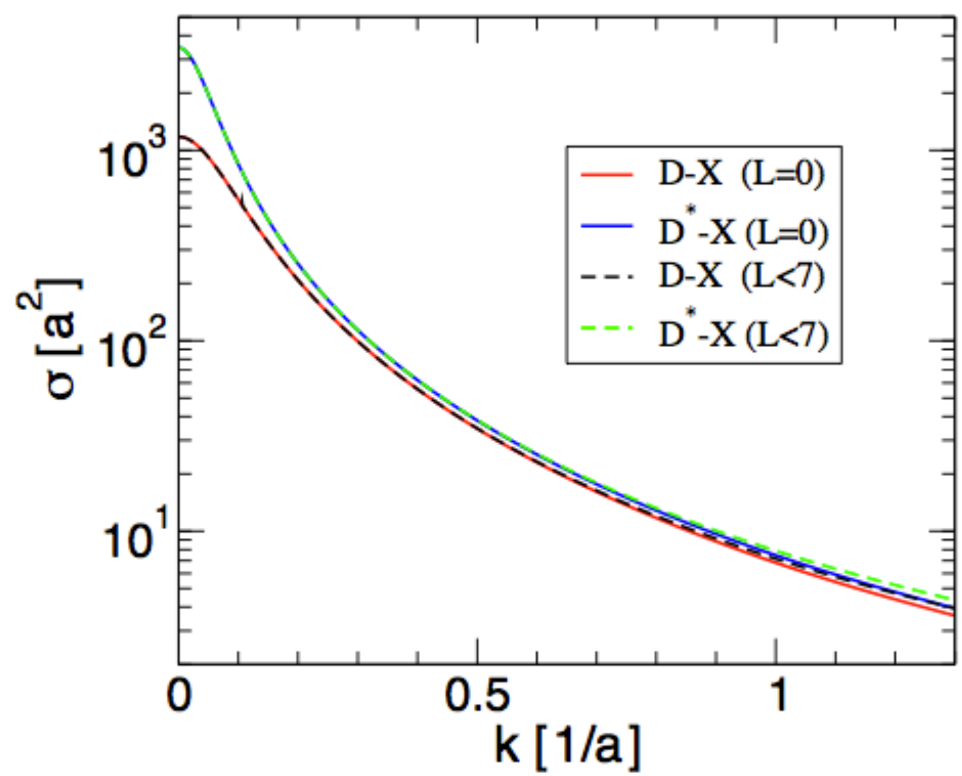
D-X(3872) scattering (three-body calculations)

D. Canham, H.-W. Hammer, R.P. Springer, PRD80:014009 (2009)





scattering length



cross section

X(3872) Decays to Quarkonia

$$X(3872) \to \chi_{cJ}\pi^0, \chi_{cJ}\pi\pi$$

- $X(3872) \to J/\psi + X$ hard to calculate: sensitive to wavefunction at short-distances, hadronic parameters

$$X(3872) \to \chi_{cJ}\pi^0, \chi_{cJ}\pi\pi$$

S. Dubynskiy, M.B. Voloshin, PRD 77:014013 (2008)

- χ_{cJ} - Heavy Quark Symmetry Multiplet: use heavy quark symmetry to predict relative rates, test hypotheses about X(3872)

$$\Gamma_J \equiv \Gamma[X(3872) \to \chi_{cJ}\pi^0]$$

- $2\,^3P_1$: $\Gamma_0 : \Gamma_1 : \Gamma_2 :: 0 : 5p^3_{\pi,1} : 3p^3_{\pi,2}$

$$:: 0.0 : 2.7 : 1.$$

- Molecule: $\Gamma_0 : \Gamma_1 : \Gamma_2 :: 4p^3_{\pi,0} : 3p^3_{\pi,1} : 5p^3_{\pi,2}$

$$:: 2.7 : 0.95 : 1.$$

Calculating $D^0 D^{0*} \to \chi_{cJ}\pi^0$ in $\mathrm{HH}\chi\mathrm{PT}$

- Including χ_{cJ}

$$\chi^i = \sigma^j \chi^{ij} = \sigma^j \left(\chi_2^{ij} + \frac{1}{\sqrt{2}} \epsilon^{ijk} \chi_1^k + \frac{\delta^{ij}}{\sqrt{3}} \chi_0 \right)$$

Casalbuoni, et. al. PLB 309:163, PLB302:95 (1993)

- Symmetries

rotations :	$H_a \to U H_a U^\dagger$	$\bar{H}_a \to U \bar{H}_a U^\dagger$	$\chi^i \to R^{ij} U^\dagger \chi^j U$
heavy quark spin :	$H_a \to S H_a$	$\bar{H}_a \to \bar{H}_a \bar{S}^\dagger$	$\chi^i \to S \chi^i \bar{S}^\dagger$
parity :	$H_a \to -H_a$	$\bar{H}_a \to -\bar{H}_a$	$\chi^i \to \chi^i$
charge conjugation :	$H_a \to \sigma_2 \bar{H}_a^T \sigma_2$	$\bar{H}_a \to \sigma_2 H_a^T \sigma_2$	$\chi^i \to -\sigma_2 (\chi^i)^T \sigma_2 = \chi^i$
$SU_L(3) \times SU_R(3)$:	$H_a \to H_b V_{ba}^\dagger$	$H_a \to V_{ab} \bar{H}_b$	$\chi^i \to \chi^i$

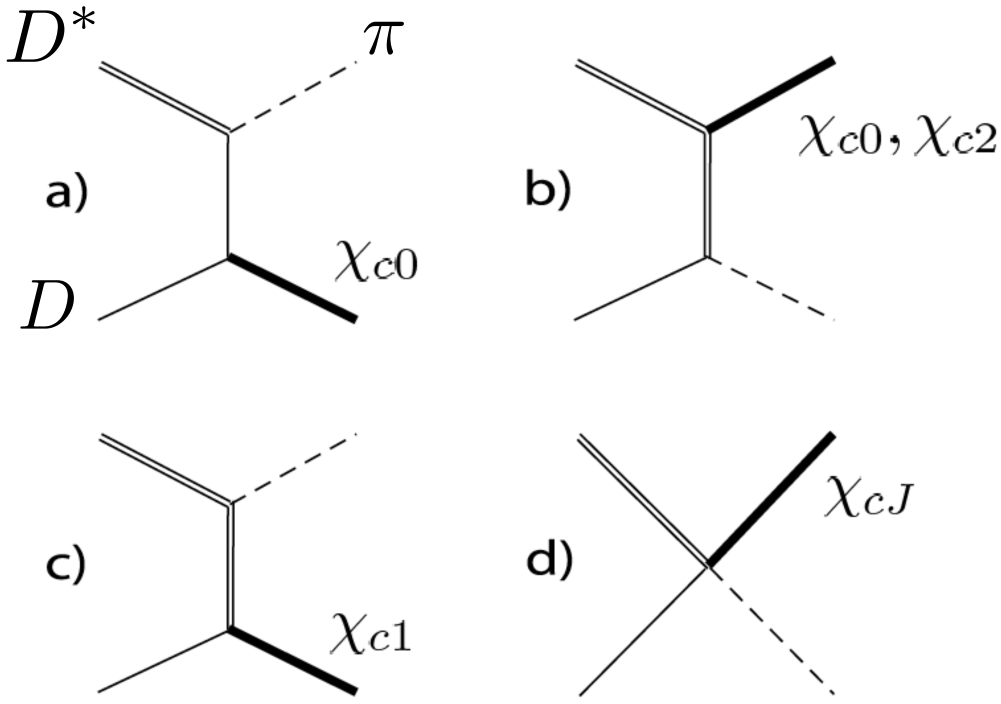
- Lagrangian

$$\mathcal{L}_\chi = i \frac{g_1}{2} \mathrm{Tr}[\chi^{\dagger\, i} H_a \sigma^i \bar{H}_a] + \frac{c_1}{2} \mathrm{Tr}[\chi^{\dagger\, i} H_a \sigma^j \bar{H}_b] \epsilon_{ijk} A_{ab}^k + \mathrm{h.c.}$$

S.Fleming, T.M., PRD 78:094019 (2008)

T.M., S. Fleming, arXiv:1110.0265

- Matching $\mathrm{HH}\chi\mathrm{PT}$ onto X-EFT



LO: a), b), c) $\sim O(Q^0)$

NLO: d) $\sim O(Q)$

$$\mathcal{M}(D^0 D^{*0} \to \chi_{c0}\pi^0) = \frac{\vec{\epsilon}_1^{\,*} \cdot \vec{p}_\pi \times \vec{\epsilon}_D}{\sqrt{2} f_\pi} \left[\frac{\sqrt{2} g g_1}{E_\pi} + \frac{c_1}{\sqrt{2}} \right]$$

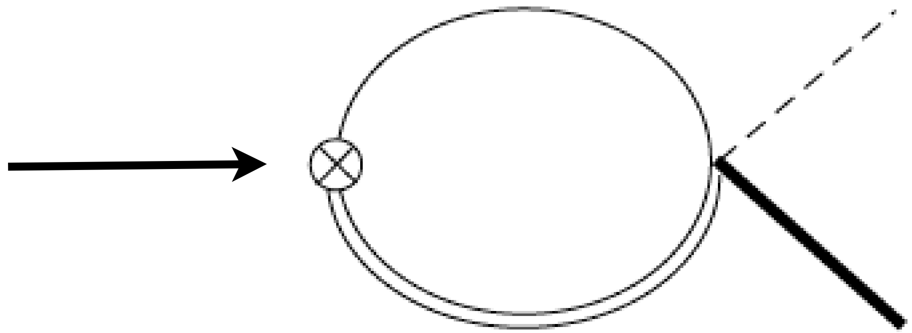
virtual D,D* offshell by $\sim E_\pi$

- Reproduce in X-EFT w/ local operator

$$\mathcal{L} = i \frac{C_{\chi,0}(E_{\pi,0})}{4\sqrt{m_\pi}} (\vec{V}\bar{P} + \vec{\bar{V}} P) \cdot \frac{\vec{\nabla}\pi^0}{f_\pi} \chi_{c0}^\dagger$$

$$C_{\chi,0}(E_{\pi,0}) = \frac{\sqrt{2} g g_1}{E_\pi} + \frac{c_1}{\sqrt{2}}$$

Calculation of $X(3872) \to \chi_{c,J}\pi^0$ in X-EFT



$$\Gamma[X(3872) \to \chi_{c,J}\pi^0] =$$

$$\frac{1}{3}\sum_{\lambda} |\langle 0|\frac{1}{\sqrt{2}}\vec{\epsilon}_{\lambda}\cdot(\vec{V}\bar{P}+\vec{\bar{V}}P)|X,\lambda\rangle|^2 \; \frac{m_{\chi_{cJ}}}{m_X}\frac{p_{\pi,J}^3}{72\pi f_\pi^2}\alpha_J|C_{\chi,J}(E_{\pi,J})|^2$$

long distance matrix element

$$\propto |\mathcal{M}(D^0\bar{D}^{0*} + c.c. \to \chi_{cJ}\pi^0)|^2$$

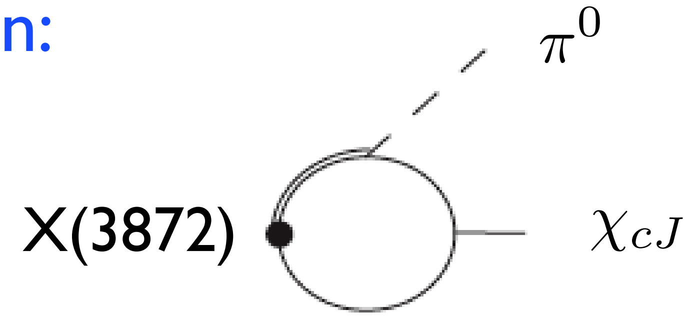
(partially) calculable in $\mathrm{HH}\chi\mathrm{PT}$

- Reproduces X(3872) factorization theorems

E. Braaten, M. Kusunoki, PRD 72:014012 (2005)

E. Braaten, M. Lu, PRD 74:054020 (2006)

- Comparison w/ direct evaluation:



$$\int d^4 l \, \frac{1}{E_X - \Delta + l_0 - \frac{l^2}{2m_{D^*}}} \frac{1}{-l_0 - \frac{l^2}{2m_{D^*}}} \frac{1}{E_X + l_0 - E_\pi - \frac{(l-p_\pi)^2}{2m_D}}$$

$$= \int d^3 l \, \frac{2\mu_{DD^*}}{l^2 + \gamma^2} \frac{1}{E_\pi - \Delta - \frac{l^2}{2m_{D^*}} - \frac{(l-p_\pi)^2}{2m_D}} \approx \frac{1}{E_\pi - \Delta} \int d^3 l \, \frac{2\mu_{DD^*}}{l^2 + \gamma^2}$$

O(Q^2/m_D)

- direct evaluation + multipole expansion is equivalent to matching procedure described above

- drops contributions coming from integrand from

$$l \sim \sqrt{2\mu_{DD^*}(E_\pi - \Delta)} \sim 750 \, \mathrm{MeV}$$

outside range of X-EFT !

- Dubyinskiy-Voloshin

$$\Gamma_0 : \Gamma_1 : \Gamma_2 :: 4p^3_{\pi,0} : 3p^3_{\pi,1} : 5p^3_{\pi,2} :: 2.7 : 0.95 : 1.0$$

- X-EFT

$$\Gamma_0 : \Gamma_1 : \Gamma_2 ::$$

$$4|C_{\chi,0}(E_{\pi,0})|^2 p^3_{\pi,0} : 3|C_{\chi,1}(E_{\pi,1})|^2 p^3_{\pi,1} : 5|C_{\chi,2}(E_{\pi,2})|^2 p^3_{\pi,2}$$

relative rates agree when χ_{cJ} are degenerate (HQS)

- EFT: two distinct processes

a) $D^0 \bar{D}^{0*} \to \chi_{cJ} \pi^0 \propto c_1$ b) $D^0 \bar{D}^{0*} \to D^0 (\bar{D}^0, \bar{D}^{0*})_{\text{virtual}} \pi^0$

$\quad \hookrightarrow \chi_{cJ} \propto g_1$

b) modifies $\propto p^3_{\pi,J}$ expectation for P-wave decay rate

Results

- DV: $\Gamma_0 : \Gamma_1 : \Gamma_2 :: 2.7 : 0.95 : 1.0$
- LO: $(c_1 = 0)$ $\Gamma_0 : \Gamma_1 : \Gamma_2 :: 4.8 : 1.6 : 1.0$
- NLO:

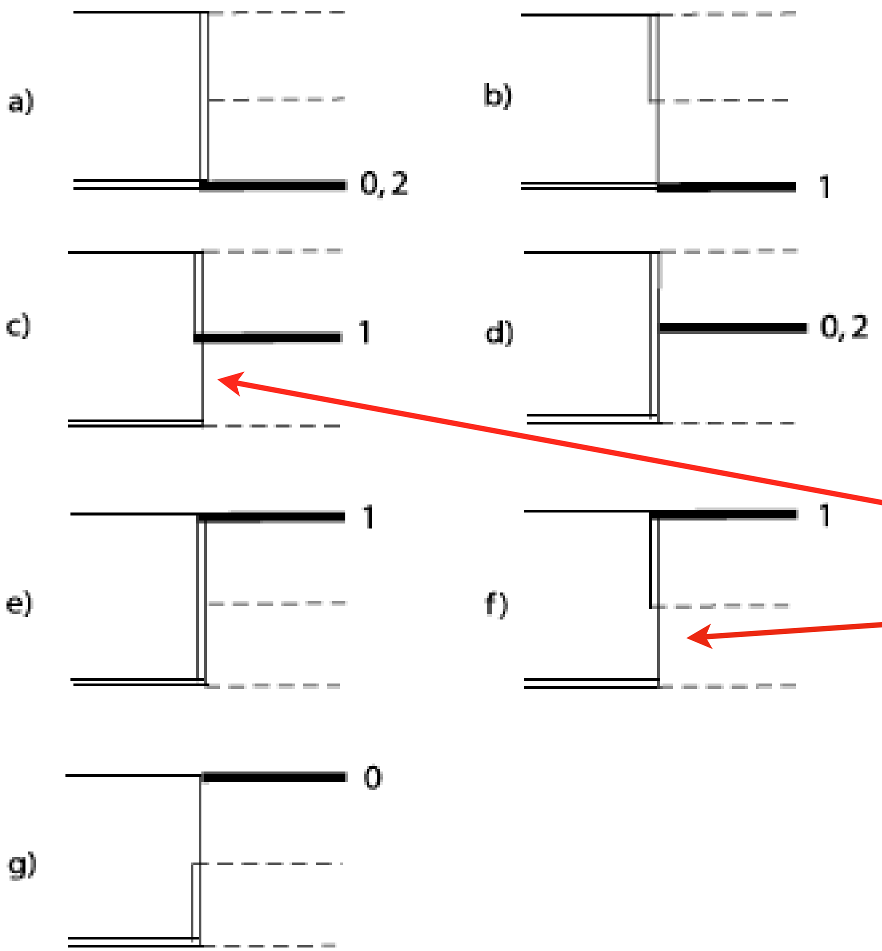
$$\Gamma_0 : \Gamma_1 : \Gamma_2 \;::\; 3.01 : 1.06 : 1 \qquad c_1/g_1 = (100\,\mathrm{MeV})^{-1}$$

$$\Gamma_0 : \Gamma_1 : \Gamma_2 \;::\; 3.49 : 1.20 : 1 \qquad c_1/g_1 = (300\,\mathrm{MeV})^{-1}$$

$$\Gamma_0 : \Gamma_1 : \Gamma_2 \;::\; 3.76 : 1.28 : 1 \qquad c_1/g_1 = (500\,\mathrm{MeV})^{-1}$$

$$\Gamma_0 : \Gamma_1 : \Gamma_2 \;::\; 4.11 : 1.38 : 1 \qquad c_1/g_1 = (1000\,\mathrm{MeV})^{-1}$$

- $X(3872) \to \chi_{cJ}\pi^0\pi^0$ in X-EFT at LO $(c_1 = 0)$



$$\left(\frac{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^0\,\pi^0]}{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^0]}\right)_{\mathrm{LO}} = 9.1\,10^{-6}\,;$$

$$\left(\frac{\mathrm{Br}[X(3872)\to\chi_{c1}\,\pi^0\,\pi^0]}{\mathrm{Br}[X(3872)\to\chi_{c1}\,\pi^0]}\right)_{\mathrm{LO}} \boxed{= 2.8\,10^{-3}\,;}$$

$$\left(\frac{\mathrm{Br}[X(3872)\to\chi_{c2}\,\pi^0\,\pi^0]}{\mathrm{Br}[X(3872)\to\chi_{c2}\,\pi^0]}\right)_{\mathrm{LO}} = 7.9\,10^{-6}$$

virtual D^0 can go on-shell in these graphs! enhanced by

$$\frac{(E_\pi + \Delta_H)^2}{(E_\pi - \Delta_H)^2} \sim 10^{-2}$$

T.M., S. Fleming, arXiv:1110.0265

- Estimates for $X(3872) \to \chi_{cJ}\pi^+\pi^-$

$$\left(\frac{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^+\,\pi^-]}{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^0]}\right)_{\mathrm{LO}} \approx 2\left(\frac{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^0\,\pi^0]}{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^0]}\right)_{\mathrm{LO}} \approx 1.8\,10^{-5} \quad \left(\frac{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^+\,\pi^-]}{\mathrm{Br}[X(3872)\to\chi_{c0}\,\pi^0]}\right)_{\mathrm{LO}} \approx \mathcal{O}(10^{-3})$$

X(3872) Decays to Quarkonia

$$X(3872) \to \psi(2S)\gamma \text{ and } \psi(4040) \to \mathrm{X}(3872)\gamma$$

- Recent measurement of $X(3872) \to \psi(2S)\gamma$

$$\frac{\Gamma[X(3872) \to \psi(2S)\gamma]}{\Gamma[X(3872) \to J/\psi\gamma]} = \begin{cases} 3.5 \pm 1.4 & \text{(BaBar, Phys. Rev. Lett. 102:132001 (2009))} \\ < 2.1 & \text{(Belle, Phys.Rev. Lett.107:091803 (2011))} \end{cases}$$

- Molecular Model Prediction (E. Swanson, Phys. Rept. 429:1243-305 (2006))

$$\frac{\Gamma[X(3872) \to \psi(2S)\gamma]}{\Gamma[X(3872) \to J/\psi\gamma]} = 3.7\,10^{-3}$$

prediction sensitive to short distance structure of X(3872), e.g.,
$|J/\psi\,\omega\rangle, |J/\psi\rho\rangle$ in X(3872) wavefunction of Swanson's model suppresses ratio

- Naive point-like coupling: $g_{(1S,2S)}\, E_\gamma\, \vec{\epsilon}_X \cdot \vec{\epsilon}_\psi \times \vec{\epsilon}_\gamma$

$$\frac{\Gamma[X(3872) \to \psi(2S)\gamma]}{\Gamma[X(3872) \to J/\psi\gamma]} = \frac{g_{2S}^2}{g_{1S}^2}\frac{E_{2S,\gamma}^3}{E_{1S,\gamma}^3} = \frac{g_{2S}^2}{g_{1S}^2} 1.7\,10^{-2}$$

so ratio still somewhat puzzling...

$E_{2S,\gamma} = 181\,\mathrm{MeV}, E_{1S,\gamma} = 697\,\mathrm{MeV}$

successfully calculated in mixed charmonium/molecule model

(Y. Dong, et. al., arXiv:0909.0380 [hep-ph])

- $E_{1S,\gamma} = 697\,\mathrm{MeV}$ so $X(3872) \to J/\psi\gamma$ outside range of EFT

- Analysis of $X(3872) \to \psi(2S)\gamma$ T.M., R. Springer, PRD 83:094001 (2011)

$$\mathcal{L} = \frac{e\beta}{2}\mathrm{Tr}[H_1^\dagger H_1\,\vec{\sigma}\cdot\vec{B}\,Q_{11}] + \frac{eQ'}{2m_c}\mathrm{Tr}[H_1^\dagger\,\vec{\sigma}\cdot\vec{B}\,H_1] + h.c.$$
$$+ i\frac{g_2}{2}\mathrm{Tr}[J^\dagger H_1\vec{\sigma}\cdot\overleftrightarrow{\partial}\,\bar{H}_1] + i\frac{ec_1}{2}\mathrm{Tr}[J^\dagger H_1\vec{\sigma}\cdot\vec{E}\bar{H}_1] + h.c.$$

charmonium superfield $J = \eta_c + \vec{\psi}\cdot\vec{\sigma}$

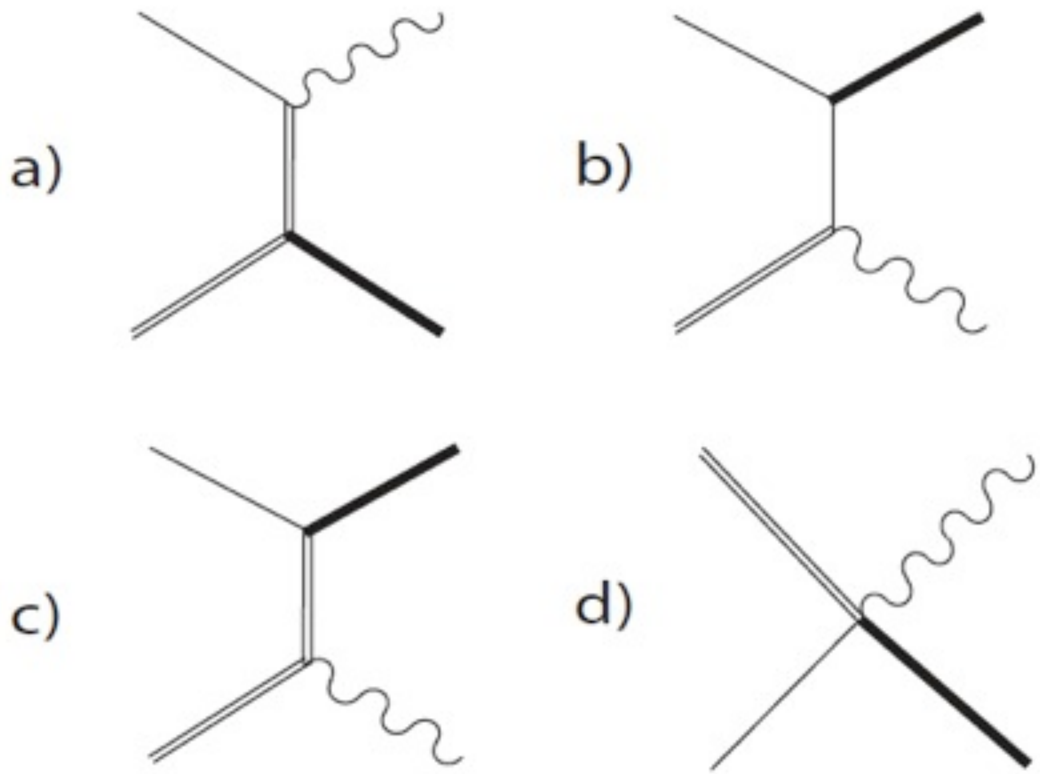
$\beta_+(\beta_-)$ coupling for $D^{*0} \to D^0\gamma\,(D^{*0} \to D^{*0}\gamma)$

$$\beta_\pm = \beta \pm \frac{1}{m_c} \quad r_\beta = \beta_+/\beta_-$$

g_2 P-wave coupling of charmonia to D mesons

c_1 contact interaction coupling charmonia, D mesons, E-field

$D^0 \bar{D}^{*0} + c.c. \to \psi(2S)\gamma$



$$a) = -\frac{g_2\, e\, \beta_+}{3} \frac{1}{E_\gamma + \Delta} (\vec{k} \cdot \vec{\epsilon}^{\,*}_\psi\, \vec{\epsilon}_{D^*} \cdot \vec{k} \times \vec{\epsilon}^{\,*}_\gamma - \vec{k} \cdot \vec{\epsilon}_{D^*}\, \vec{\epsilon}^{\,*}_\psi \cdot \vec{k} \times \vec{\epsilon}^{\,*}_\gamma)$$

$$b) = \frac{g_2\, e\, \beta_+}{3} \frac{1}{\Delta - E_\gamma} \vec{k} \cdot \vec{\epsilon}^{\,*}_\psi\, \vec{\epsilon}_{D^*} \cdot \vec{k} \times \vec{\epsilon}^{\,*}_\gamma$$

$$c) = \frac{g_2\, e\, \beta_-}{3} \frac{1}{E_\gamma} \vec{k} \cdot \vec{\epsilon}_{D^*}\, \vec{\epsilon}^{\,*}_\psi \cdot \vec{k} \times \vec{\epsilon}^{\,*}_\gamma$$

$$d) = -e\, c_1\, E_\gamma\, \vec{\epsilon}_{D^*} \cdot \vec{\epsilon}^{\,*}_\psi \times \vec{\epsilon}^{\,*}_\gamma$$

- all diagrams O(Q) in HHChiPT counting
- contact interaction gives naive coupling, a)-c) give rise to new spin structures
- b) enhanced by $\frac{E_\gamma}{E_\gamma - \Delta} \sim 4.7$ and $\propto \vec{k} \cdot \vec{\epsilon}^{\,*}_\psi$

- Decay Rate

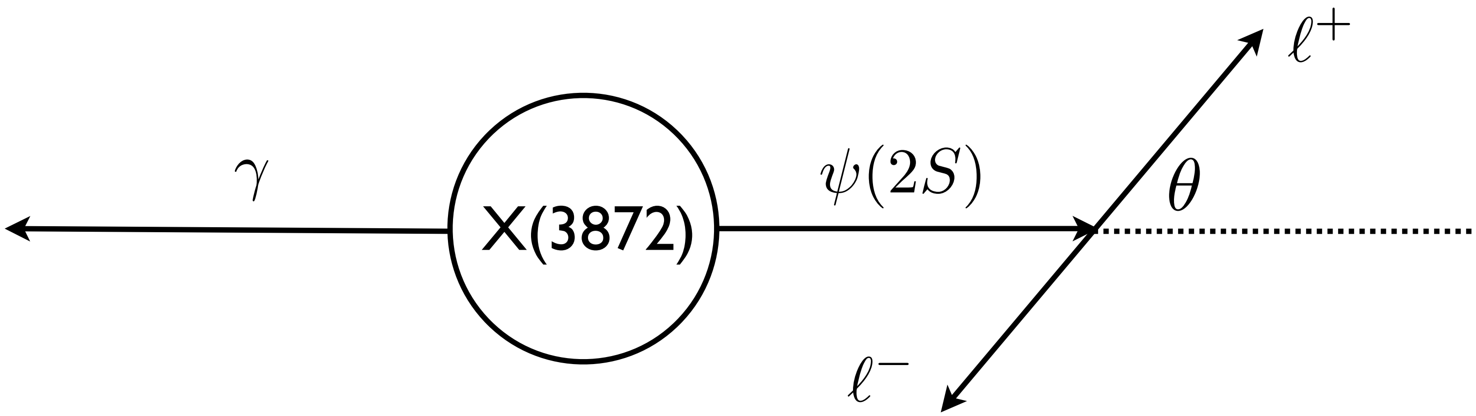
$$\Gamma[X(3872) \to \psi(2S)(\vec{\epsilon}_\psi)\gamma] = \sum_\lambda |\langle 0| \frac{1}{\sqrt{2}} \epsilon^i(\lambda)\,(V^i\,\bar{P} + \bar{V}^i\,P)|X(3872,\lambda)\rangle|^2$$
$$\times \frac{E_\gamma}{36\pi}\frac{m_\psi}{m_X}\left[(A+C)^2 + (B-C)^2\right]$$

$$A = \frac{g_2 e \beta_+}{3}\frac{2E_\gamma^3}{\Delta^2 - E_\gamma^2} \quad B = \frac{g_2 e}{3}\frac{\beta_+ E_\gamma^2 + \beta_- E_\gamma(E_\gamma + \Delta)}{E_\gamma + \Delta} \quad C = -ec_1 E_\gamma$$

- $\Gamma[X \to \psi\gamma]$ no longer $\propto E_\gamma^3$ because of diagrams a)-c)

- Absolute rate unknown

- Polarization $\psi(2S) \to \ell^+\ell^-$ $\quad \frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha\cos^2\theta \quad \alpha = \frac{1-3f_L}{1+f_L}$



contact interaction

i) $g_2\beta \ll c_1$ d) only

$$f_L = \frac{1}{2}, \alpha = -\frac{1}{3}$$

$$\mathcal{M} \propto \vec{\epsilon}_X \cdot \vec{\epsilon}_\psi^{\,*} \times \vec{\epsilon}_\gamma^{\,*}$$

constituent decay

ii) $g_2\beta \gg c_1$ a-c) only b) dominates

$$f_L = \frac{4E_\gamma^4}{4E_\gamma^4 + (2E_\gamma + \Delta)^2(E_\gamma - \Delta)^2} = 0.92$$

$$\alpha = -0.91$$

- Polarization measurement would shed light on relative importance of decay mechanisms

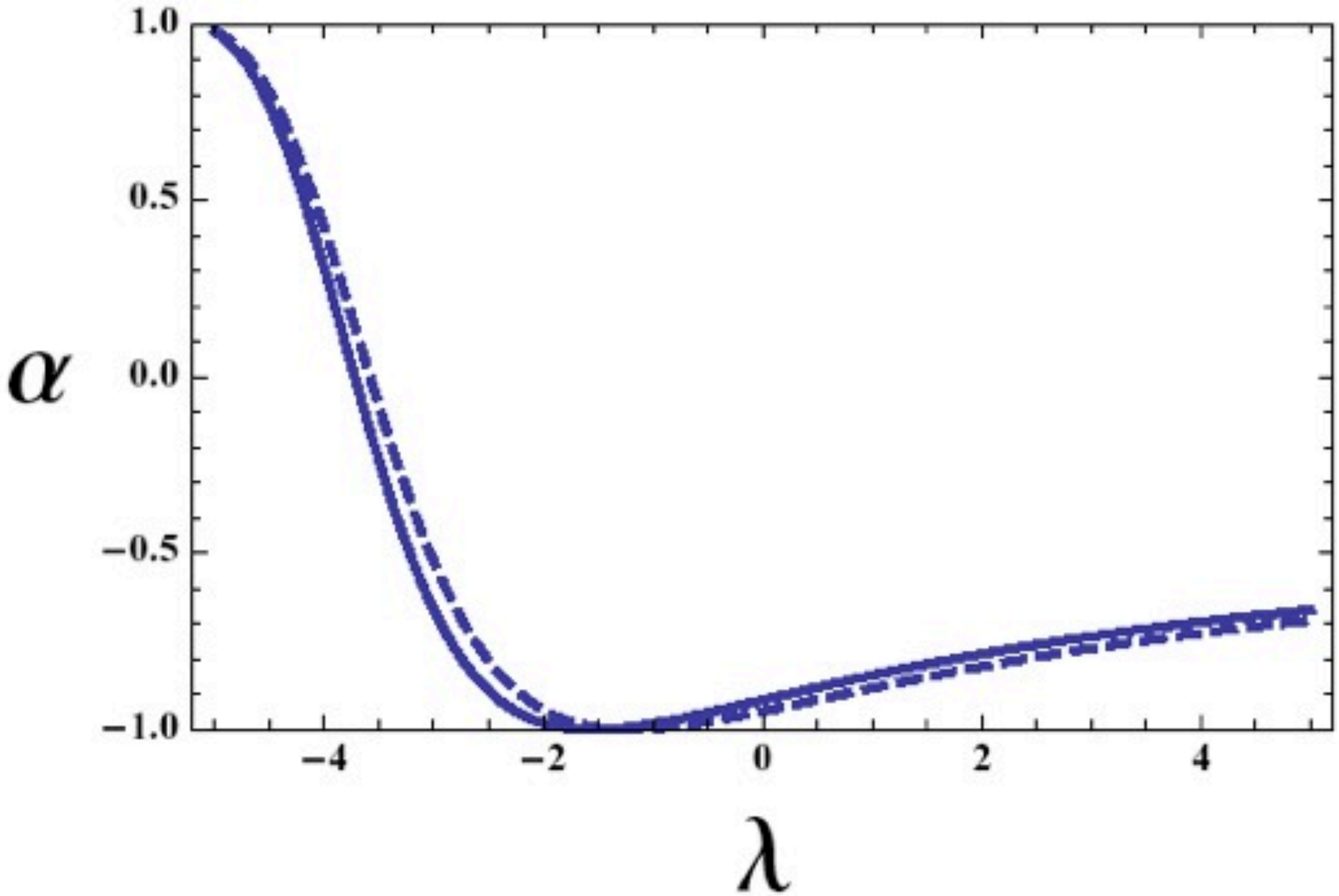
- Polarization as function of $\lambda \equiv \frac{3c_1}{g_2\beta_+} \approx 1.3 \frac{c_1}{\text{GeV}^{-5/2}} \sim O(1)$

$g_2 \approx 0.81\,\text{GeV}^{-3/2}$ from $\psi' \to J/\psi\,\pi^0\,(\eta)$

(Guo, et. al. arXiv: 0907.0521[hep-ph])

$\beta = (356\,\text{MeV})^{-1}$ from $D^* \to D\gamma$

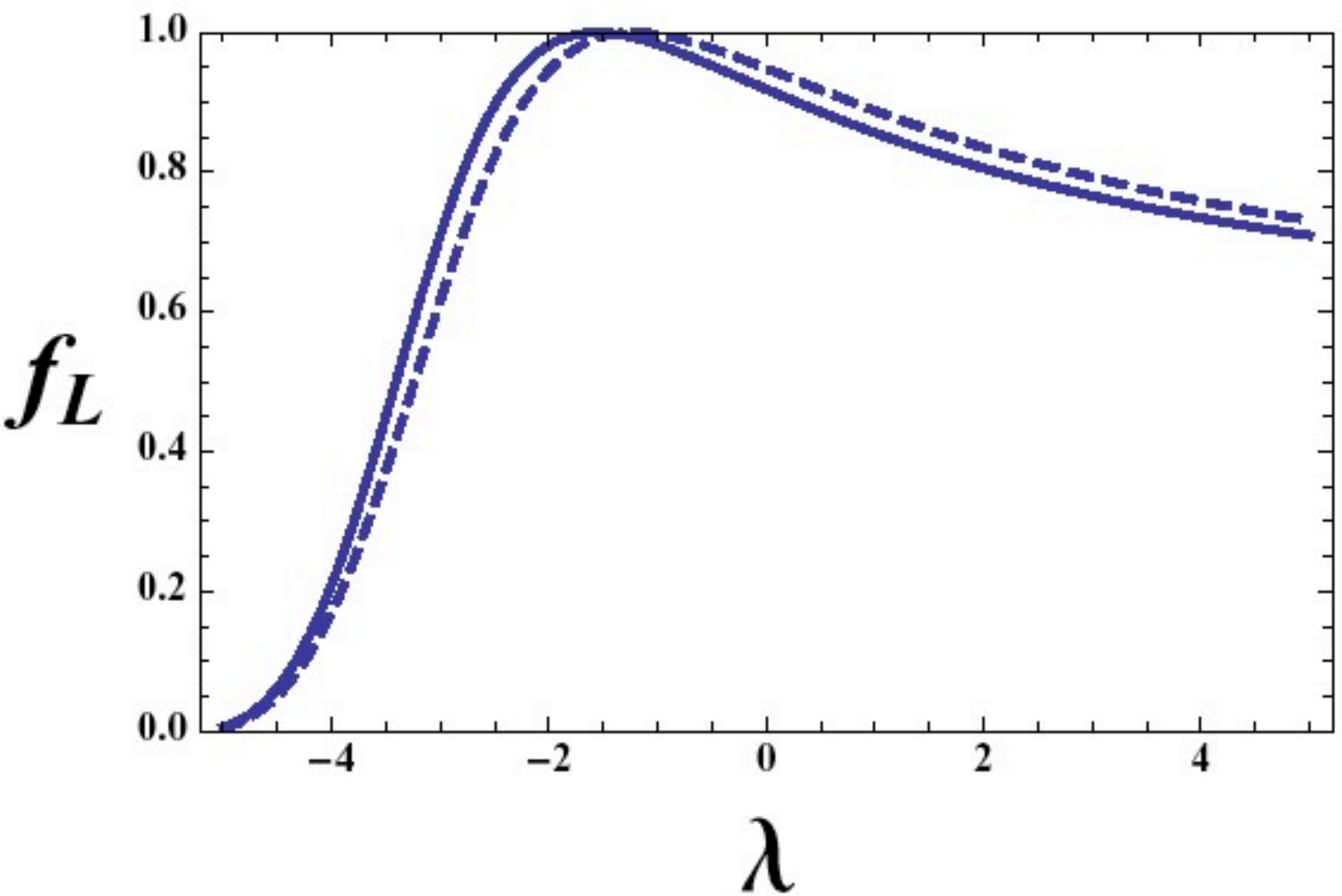
(Hu & T.M., et. al. PRD73:054003 (2006))



- Longitudinal Polarization $(\alpha < -0.5)$ for $-3.5 \leq \lambda \leq 5$

(solid line - $r_\beta = 1.0$, dotted line - $r_\beta = 0.66$, includes Λ/m_c corrections)

- Longitudinal Polarization vs. λ



Expectations for $J^{PC} = 2^{-+}$ assignment

- EFT Lagrangian $\mathcal{L} = g' \, \mathrm{Tr}[X^{ij} J^\dagger \sigma^i] B^j$

$$\mathcal{M}[X(3872) \to \psi(2S)(\vec{\epsilon}_\psi)\gamma] \propto \vec{\epsilon}_\psi^{\,*i} (\vec{k} \times \vec{\epsilon}_\gamma^{\,*})^j h^{ij}$$

Predict: $f_L = 0.3 \;\; \alpha = 0.08$

- Model as $\eta_c(^1D_2)$ (including M1 + higher multipoles)

(Y. Jia, W.L. Sang, and J. Xu, arXiv:1007.4541 [hep-ph])

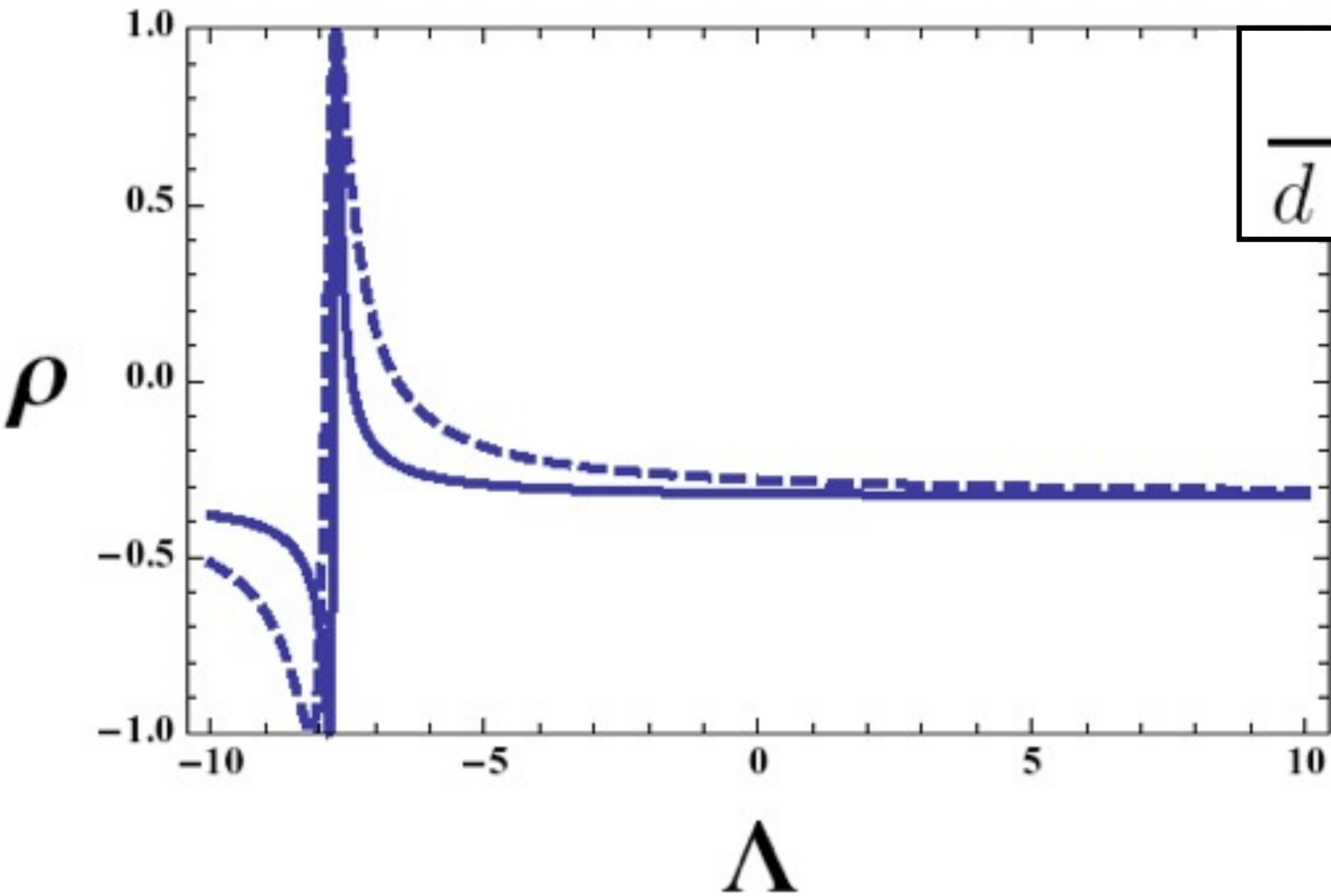
$$f_L = 0.11 - 0.28 \;\; \alpha = 0.13 - 0.6$$

- $J^{PC} = 2^{-+}$ predicts negligible or slightly transverse polarization

- $e^+e^- \to \psi(4040) \to X(3872)\gamma$ (BES?)

$\psi(4040)$ produced with polarization transverse to beam axis (LO)

same (crossed) graphs as $X(3872) \to \psi(2S)\gamma$



$$\frac{d\sigma}{d\cos\theta} \propto 1 + \rho\cos^2\theta$$

θ - angle from beam axis

$$\Lambda \equiv \frac{3\tilde{c}_1}{\tilde{g}_2\beta_+}$$

- $J^{PC} = 2^{-+}$ predicts $\rho = 0.08$

molecule predicts $\rho \approx -1/3$ for most of parameter space

Summary

- plethora of XYZ states above the open charm threshold

X(3872) : weakly bound DD* molecule

- XEFT: low energy effective theory for nonrelativistic $D^0 \bar{D}^{*0} \pi^0$

can be used to systematically study X(3872) decays

- Matching $\mathrm{HH}\chi\mathrm{PT}$ amplitudes for $D^0 \bar{D}^{*0} \to \mathrm{Quarkonia} + X$

reproduces fact. theorems for $X(3872) \to \mathrm{Quarkonia} + X$

- Heavy quark symmetry predicts relative rates for

$X(3872) \to \chi_{cJ}\pi^0, \chi_{cJ}\pi\pi$ test molecular hypothesis

- XEFT: $\Gamma_0 : \Gamma_1 : \Gamma_2 :: 4.8 - 3.0 : 1.6 - 0.95 : 1.0$

$0 < c_1/g_1 < (100\,\mathrm{MeV})^{-1}$

- $X(3872) \to \psi(2S)\gamma$ and $\psi(4040) \to X(3872)\gamma$

$$\frac{\Gamma[X(3872) \to \psi(2S)\gamma]}{\Gamma[X(3872) \to J/\psi\gamma]}$$ hard to calculate

If constituent decays dominate $X(3872) \to \psi(2S)\gamma$

expect longitudinally polarized $\psi(2S)$

Predictions for angular distributions in $\psi(4040) \to X(3872)\gamma$

Polarization and angular distribution measurements can shed light on relative importance of decay mechanisms, as well as differentiate J^{PC} assignments for X(3872)

- Calculations of universal cross sections/decays

$$X(3872) \to D^0 \bar{D}^0 \pi^0 \qquad D^{+0} \bar{D}^{*0} \to X(3872)\pi^+$$

$$D^{(*)}X(3872) \to D^{(*)}X(3872)$$

- Future Work: incorporating $D^+ D^{*-}$ threshold

  more universal predictions, e.g. $\pi X(3872) \to \pi X(3872)$

  higher order corrections

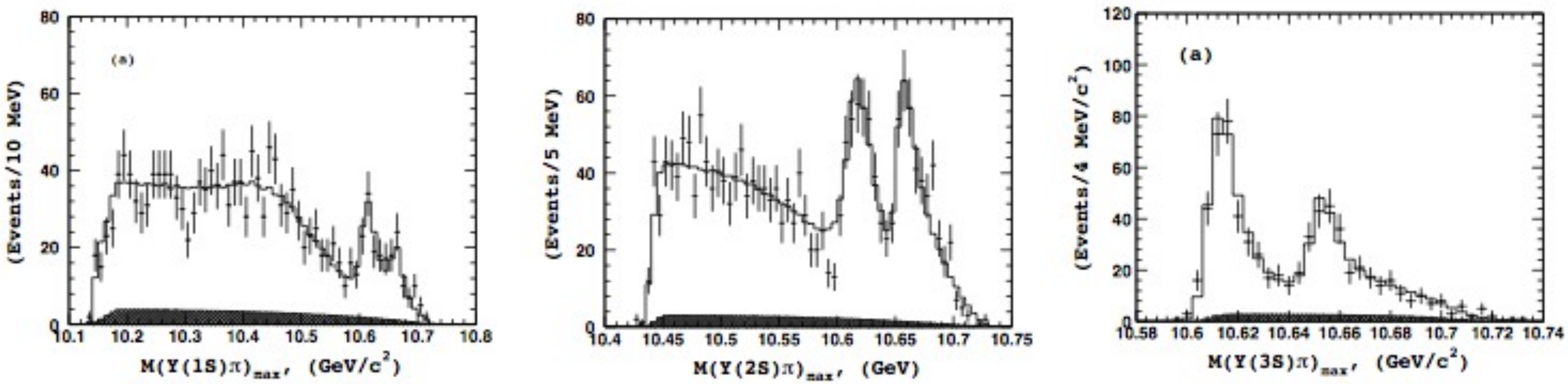
New Bottomonium Resonances

Belle, arXiv:1110.3934

- Z(10160) and Z(10650): resonant structures in

$$\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^- \;\; (n = 1, 2, \text{ or } 3)$$

$$\Upsilon(5S) \to h_b(mP)\pi^+\pi^- \;\; (m = 1 \text{ or } 2)$$



$$\Upsilon(5S) \to Z_b\pi \to \Upsilon(nS)\pi^+\pi^-$$

- likely quantum numbers: $I^G(J^P) = 1^+(1^+)$
- $B\bar{B}^*$ threshold: 10558 MeV $B^*\bar{B}^*$ threshold: 10604 MeV
- large widths ~ 15 MeV (unlike X(3872))

Heavy Quark Spin Symmetry Predictions

M.B Voloshin, PRD 84: 031502 (2011)   A.E. Bondar, et.al., PRD 84: 054010 (2011)

Hamiltonian

$$H_s = \mu\,(\vec{s}_b \cdot \vec{s}_{\bar{q}}) + \mu\,(\vec{s}_{\bar{b}} \cdot \vec{s}_q) = \frac{\mu}{2}\,(\vec{S}_H \cdot \vec{S}_{SLB}) - \frac{\mu}{2}\,(\vec{\Delta}_H \cdot \vec{\Delta}_{SLB})\ ,$$

Quark Model Wavefunctions $S_{Q\bar{Q}} \otimes S_{q\bar{q}}$

$$W_2: \quad 1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=2}$$

$$W_1: \quad 1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=1}$$

$$W'_{b0}: \quad \frac{\sqrt{3}}{2}\,0_{Q\bar{Q}} \otimes 0_{q\bar{q}} + \frac{1}{2}\,1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=0}$$

$$W_0: \quad \frac{\sqrt{3}}{2}\,1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=0} - \frac{1}{2}\,0_{Q\bar{Q}} \otimes 0_{q\bar{q}}$$

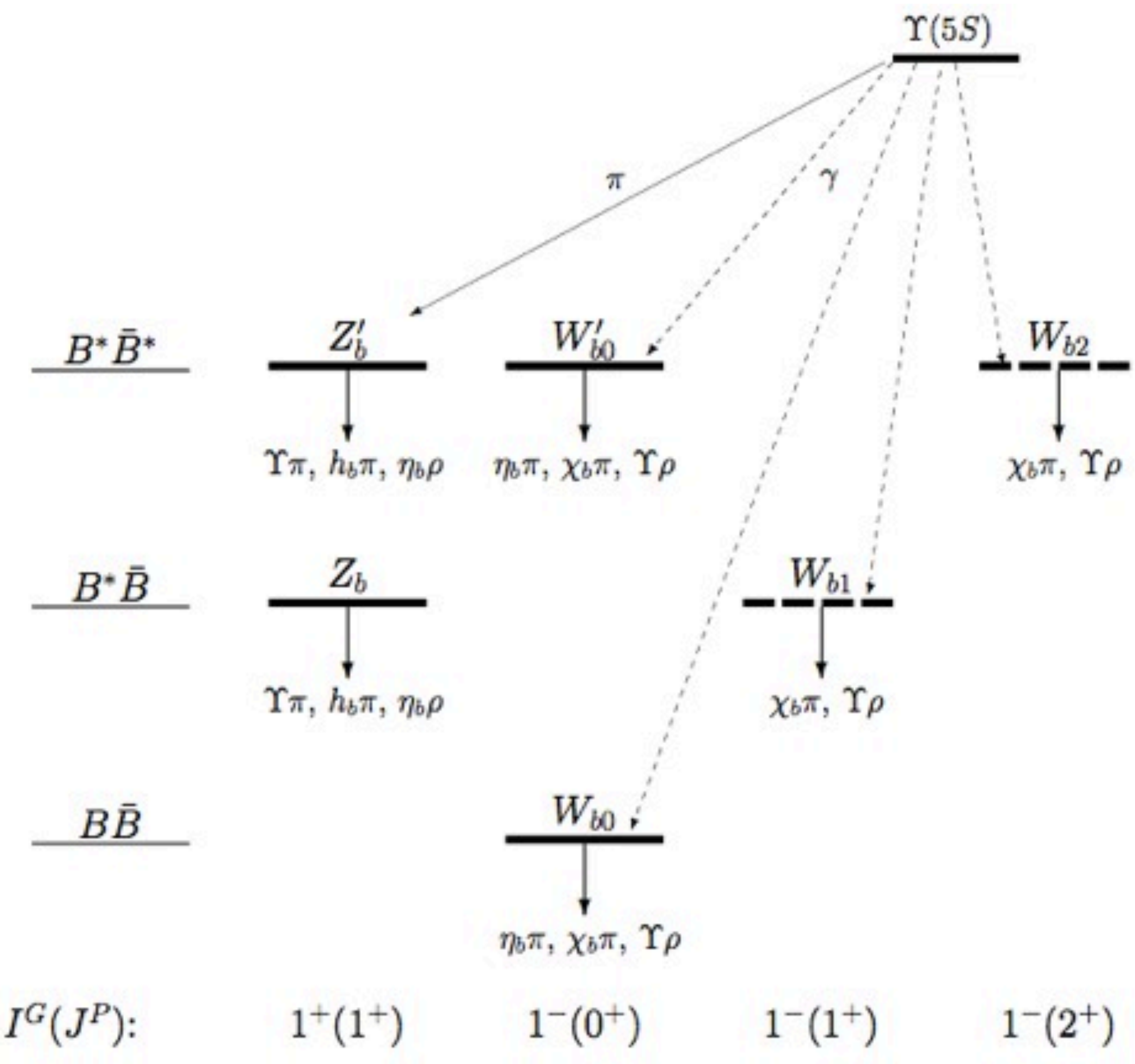
$$Z': \quad \frac{1}{\sqrt{2}}\,0_{Q\bar{Q}} \otimes 1_{q\bar{q}} - \frac{1}{\sqrt{2}}\,1_{Q\bar{Q}} \otimes 0_{q\bar{q}}$$

$$Z: \quad \frac{1}{\sqrt{2}}\,0_{Q\bar{Q}} \otimes 1_{q\bar{q}} + \frac{1}{\sqrt{2}}\,1_{Q\bar{Q}} \otimes 0_{q\bar{q}}\,.$$

binding should only depend on $S_{q\bar{q}}$

expect similar states in other channels

Spectrum and Transitions



Strong Decay Widths

$$\Gamma(W_{b2}) = \Gamma(W_{b1}) =$$
$$= \frac{3}{2}\Gamma(W_{b0}) - \frac{1}{2}\Gamma(W'_{b0})$$

Radiative Decays

$$f(W_{b0}\gamma) : f(W'_{b0}\gamma) : f(W_{b1}\gamma) : f(W_{b2}\gamma) = \frac{3}{4}\omega_0^3 : \frac{1}{4}\omega_2^3 : 3\,\omega_1^3 : 5\omega_2^3$$

Effective Field Theory

T.M., J. Powell, arXiv:1109.3479, accepted in PRD

$$
\begin{aligned}
\mathcal{L} =\; & \mathrm{Tr}[H_a^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M} \right)_{ba} H_b] + \frac{\Delta}{4}\mathrm{Tr}[H_a^\dagger \sigma^i H_a \sigma^i] \\
& + \mathrm{Tr}[\bar{H}_a^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M} \right)_{ab} \bar{H}_b] + \frac{\Delta}{4}\mathrm{Tr}[\bar{H}_a^\dagger \sigma^i \bar{H}_a \sigma^i] \\
& - \frac{C_{00}}{4}\mathrm{Tr}[\bar{H}_a^\dagger H_a^\dagger H_b \bar{H}_b] - \frac{C_{01}}{4}\mathrm{Tr}[\bar{H}_a^\dagger \sigma^i H_a^\dagger H_b \sigma^i \bar{H}_b] \\
& - \frac{C_{10}}{4}\mathrm{Tr}[\bar{H}_a^\dagger \tau^A_{aa'} H_{a'}^\dagger H_b \tau^A_{bb'} \bar{H}_{b'}] - \frac{C_{11}}{4}\mathrm{Tr}[\bar{H}_a^\dagger \tau^A_{aa'} \sigma^i H_{a'}^\dagger H_b \tau^A_{bb'} \sigma^i \bar{H}_{b'}]\,. \\
=\; & -2\,C_{11}\left(W_{0+}^{A\dagger} W_{0+}^A + Z_+^{Ai\dagger} Z_+^{Ai} + W_1^{Ai\dagger} W_1^{Ai} + \sum_\lambda W_{2\lambda}^{A\dagger} W_{2\lambda}^A \right) \\
& -2\,C_{10}\left(W_{0-}^{A\dagger} W_{0-}^A + Z_-^{Ai\dagger} Z_-^{Ai} \right),
\end{aligned}
$$

interpolating fields

$$
W_{0+}^A = \tfrac{1}{2} W_0'^{\,A} + \tfrac{\sqrt{3}}{2} W_0^A \qquad W_{0-}^A = \tfrac{\sqrt{3}}{2} W_0'^{\,A} - \tfrac{1}{2} W_0^A
$$

$$
Z^{Ai} = \frac{1}{\sqrt{2}}(V_a^i \tau^A_{ab} \bar{P}_b - P_a \tau^A_{ab} \bar{V}_b^i) \quad W_0^A = P_a \tau^A_{ab} \bar{P}_b \quad W_1^{Ai} = \frac{1}{\sqrt{2}}(V_a^i \tau^A_{ab} \bar{P}_b + P_a \tau^A_{ab} \bar{V}_b^i)
$$

$$
Z'^{\,Ai} = \frac{i}{\sqrt{2}}\,\epsilon^{ijk} V_a^j \tau^A_{ab} \bar{V}_b^k \quad W_0'^{\,A} = \frac{1}{\sqrt{3}} V_a^i \tau^A_{ab} \bar{V}_b^i \quad W_2^{A\lambda} = \epsilon^\lambda_{ij} V_a^i \tau^A_{ab} \bar{V}_b^j\,,
$$

Solve coupled channel problem in EFT

$$T_{Z'Z'} = \frac{4\pi}{M}\frac{-\gamma_+ + \sqrt{M(\Delta - E) - i\epsilon}}{(\gamma_+ - \sqrt{M(\Delta - E) - i\epsilon})(\gamma_+ - \sqrt{M(2\Delta - E) - i\epsilon}) - \gamma_-^2}$$

$$T_{Z'Z} = T_{ZZ'} = \frac{4\pi}{M}\frac{\gamma_-}{(\gamma_+ - \sqrt{M(\Delta - E) - i\epsilon})(\gamma_+ - \sqrt{M(2\Delta - E) - i\epsilon}) - \gamma_-^2}$$

$$T_{ZZ} = \frac{4\pi}{M}\frac{-\gamma_+ + \sqrt{M(2\Delta - E) - i\epsilon}}{(\gamma_+ - \sqrt{M(\Delta - E) - i\epsilon})(\gamma_+ - \sqrt{M(2\Delta - E) - i\epsilon}) - \gamma_-^2},$$

HQSS predictions

Binding momenta

$$\gamma_Z = \gamma_{Z'} = \gamma_+ \qquad B.E. = -\gamma^2/M$$

$$\gamma_{W_1} = \gamma_{W_2} = \gamma_{11}$$

$$\gamma_{W_0} = \frac{\gamma_{10} + 3\gamma_{11}}{4} = \frac{\gamma_Z + \gamma_{W_1}}{2}$$

$$\gamma_{W_0'} = \frac{3\gamma_{10} + \gamma_{11}}{4} = \frac{3\gamma_Z - \gamma_{W_1}}{2}$$

Decay Rates

$$\Gamma[W_1] = \Gamma[W_2] = \frac{3}{2}\Gamma[W_0] - \frac{1}{2}\Gamma[W_0']$$

$$\Gamma[Z] = \Gamma[Z'] = \frac{1}{2}(\Gamma[W_0] + \Gamma[W_0'])$$

(new)

Explicit calculations of 2-body Decays

$$\begin{aligned}\mathcal{L}_{\mathrm{HH}\chi\mathrm{PT}} &= g\,\mathrm{Tr}[\bar{H}_a^\dagger\,\sigma^i\,\bar{H}_b]A^i_{ab} - g\,\mathrm{Tr}[H_a^\dagger H_b\,\sigma^i]A^i_{ba} \\ &+ \frac{1}{2}g_{\pi\Upsilon,n}\,\mathrm{tr}[\Upsilon_n^\dagger H_a\bar{H}_b]A^0_{ab} + \frac{1}{2}g_{\Upsilon,n}\,\mathrm{tr}[\Upsilon_n^\dagger H_a\sigma^j i\overleftrightarrow{\partial}_j\bar{H}_a] + \mathrm{h.c.} \\ &+ \frac{1}{2}g_{\pi\chi,n}\,\mathrm{tr}[\chi_{n,i}^\dagger H_a\sigma^j\bar{H}_b]\epsilon_{ijk}A^k_{ab} + \frac{i}{2}\,g_{\chi,n}\,\mathrm{tr}[\chi_{n,i}^\dagger H_a\sigma^i\bar{H}_a] + \mathrm{h.c.}\end{aligned}$$

$$\begin{aligned}\Gamma[W_0\to\pi\eta_b] &= \frac{m_\eta k_\pi E_\pi^2}{8\pi m_{W_0}f_\pi^2}\left[g_{\pi\Upsilon} - 2gg_\Upsilon\frac{k_\pi^2}{E_\pi(E_\pi+\Delta)}\right]^2\times\mathcal{O}_1 \qquad (36)\\ \Gamma[W_0'\to\pi\eta_b] &= \frac{3m_\eta k_\pi E_\pi^2}{8\pi m_{W_0'}f_\pi^2}\left[g_{\pi\Upsilon} - 2gg_\Upsilon\frac{k_\pi^2}{E_\pi^2}\left(1+\frac{1}{3}\frac{\Delta}{E_\pi-\Delta}\right)\right]^2\times\mathcal{O}_2 \\ \Gamma[Z\to\pi\Upsilon] &= \frac{m_\Upsilon k_\pi E_\pi^2}{4\pi m_Z f_\pi^2}\left[\left[g_{\pi\Upsilon} - 2gg_\Upsilon\frac{k_\pi^2}{E_\pi^2}\left(1-\frac{\Delta}{3}\frac{E_\pi-2\Delta}{E_\pi^2-\Delta^2}\right)\right]^2 + \frac{2}{9}\left[gg_\Upsilon\frac{k_\pi^2}{E_\pi^2}\frac{\Delta}{E_\pi-\Delta}\right]^2\right]\times\mathcal{O}_3 \\ \Gamma[Z'\to\pi\Upsilon] &= \frac{m_\Upsilon k_\pi E_\pi^2}{4\pi m_{Z'}f_\pi^2}\left[\left[g_{\pi\Upsilon} - 2gg_\Upsilon\frac{k_\pi^2}{E_\pi^2}\left(1+\frac{1}{3}\frac{\Delta}{E_\pi-\Delta}\right)\right]^2 + \frac{2}{9}\left[gg_\Upsilon\frac{k_\pi^2}{E_\pi^2}\frac{\Delta}{E_\pi-\Delta}\right]^2\right]\times\mathcal{O}_4\,.\end{aligned}$$

corrections to HQSS from phase space, kinematics

Predictions

$$\Gamma[W_0 \to \pi\eta_b(3S)] \;:\; \Gamma[W_0' \to \pi\eta_b(3S)] : \Gamma[Z \to \pi\Upsilon(3S)] : \Gamma[Z' \to \pi\Upsilon(3S)]$$
$$= 0.26 : 2.0 : 0.62 : 1 \qquad (\lambda_\Upsilon = 0)\,,$$

$$\Gamma[W_0 \to \pi\eta_b(3S)] \;:\; \Gamma[W_0' \to \pi\eta_b(3S)] : \Gamma[Z \to \pi\Upsilon(3S)] : \Gamma[Z' \to \pi\Upsilon(3S)]$$
$$= 0.12 : 2.1 : 0.41 : 1 \qquad (|\lambda_\Upsilon| = \infty)\,.$$

$$\Gamma[W_0 \to \pi\chi_{b1}(2P)] \;:\; \Gamma[W_0' \to \pi\chi_{b1}(2P)] : \Gamma[Z \to \pi h_b(2P)] : \Gamma[Z' \to \pi h_b(2P)]$$
$$= 0.72 : 0.57 : 0.66 : 1 \qquad (g_{\pi\chi}/g_\chi = 0\,\mathrm{GeV}^{-1})\,,$$

$$\Gamma[W_1 \to \pi\chi_{bJ}(2P)] \;:\; \Gamma[W_2 \to \pi\chi_{bJ}(2P)] : \frac{3}{2}\Gamma[W_0 \to \pi\chi_{b1}(2P)] - \frac{1}{2}\Gamma[W_0' \to \pi\chi_{b1}(2P)]$$
$$= 0.81 : 1 : 0.43 \qquad (g_{\pi\chi}/g_\chi = 0\,\mathrm{GeV}^{-1})\,. \qquad (42)$$