Non-Perturbative Thermal QCD: AdS/QCD and Gluon Condensates

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E. Megías et al. NPB834(2010), 0911.1680(2009), NP P.S.207(2010), PRD83(2011), PLB696(2011), AIP C.P.1343(2011); PRD80(2009),PRD81(2010).

Issues

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1. Motivation
2. Heavy $\bar{q}q$ potential at zero temperature
   - Scale Invariance and Confinement
   - Soft-wall model of AdS/QCD
   - The 5D Einstein-dilaton model
3. Thermodynamics of AdS/QCD
   - Black Holes
   - The 5D Einstein-dilaton model at finite T
   - Thermodynamics
   - Trouble finding the optimal AdS/QCD
4. Dimension Two Gluon Condensate
   - Dimension two Condensates?
   - Power Corrections
   - Duality between perturbative series and power corrections

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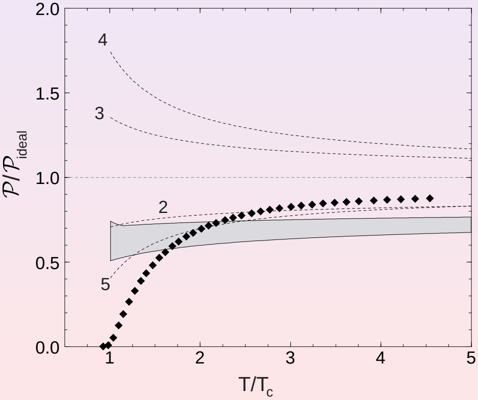
Motivation

Pressure of Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

E. Braaten and A. Nieto (1996), J.O. Andersen et al (1999).



Motivation

Interaction Measure in Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

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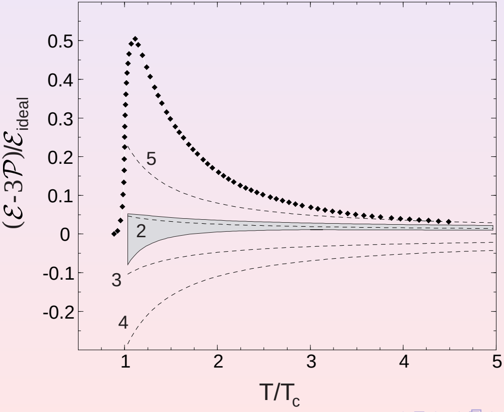
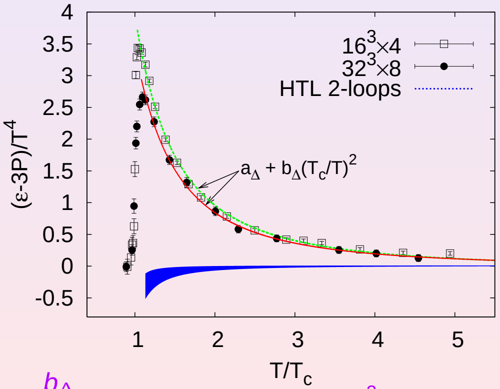


Motivation

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_\Delta + \frac{b_\Delta}{T^2}, \quad b_\Delta = (3.46 \pm 0.13) T_c^2, \qquad 1.13 T_c < T < 4.5 T_c .$$

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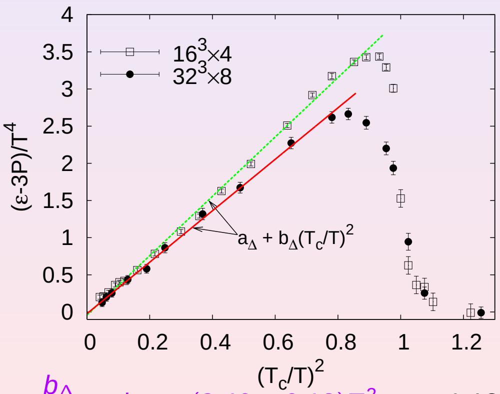
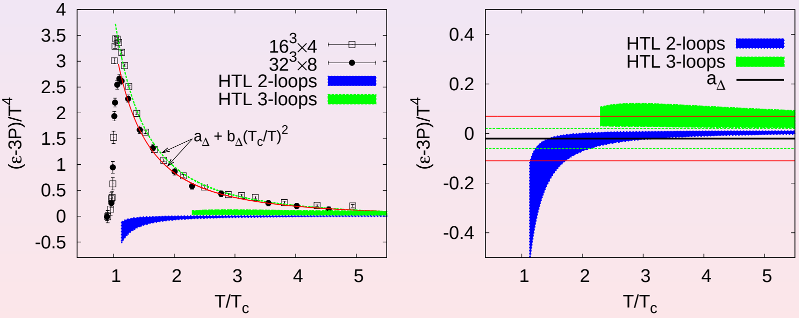


$$\frac{\epsilon - 3P}{T^4} = a_\Delta + \frac{b_\Delta}{T^2}, \quad b_\Delta = (3.46 \pm 0.13) T_c^2, \qquad 1.13 T_c < T < 4.5 T_c .$$

Motivation

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = \underbrace{\Delta_{HTL}(\mu_T)}_{\sim 1/\log T} + \frac{b_\Delta}{T^2}$$



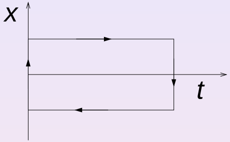
Perturbation Theory and Hard Thermal Loops only yield a_Δ !!.

Issues

Scale Invariance and Confinement

Consider a rectangular Wilson loop:

$$W(\mathcal{C}) = \exp\left(ig\int_{\mathcal{C}} A_\mu dx^\mu\right)$$



It is related to the potential $V_{q\bar{q}}(R)$ acting between charges q and $\bar{q}$:

$$\langle W(\mathcal{C})\rangle \underset{t\to\infty}{\sim} \exp\left(-t\cdot V_{q\bar{q}}(R)\right)$$

Scale transformations: $t \to \lambda t$, $R \to \lambda R$.
The only scale invariant solution is the Coulomb Potential:

$$V_{q\bar{q}} \sim \frac{1}{R}$$

Running coupling and string tension break scale invariance:

$$V_{q\bar{q}}(R) = -\frac{4}{3}\frac{\alpha_s(R)}{R} + \sigma R\,.$$

Issues

Soft-wall model of AdS/QCD

$$ds^2_{QCD} = h(z) \cdot ds^2 = h(z) \underbrace{\frac{\ell^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)}_{AdS_5} .$$

- $h(z) = 1 \Longrightarrow$ Conformal
- $h(z) \neq 1 \Longrightarrow$ Non conformal

Breaking of scaling invariance in QCD is given by the running coupling:

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} = \frac{\beta(\alpha_s)}{4\alpha_s^2}\langle F^2_{\mu\nu}\rangle .$$

where $\beta(\alpha_s) = \mu\frac{d\alpha_s}{d\mu}$ and $\alpha_s(E) \sim 1/\log(E/\Lambda)$.
$\Longrightarrow$Assume an ansatz for conformal symmetry breaking similar to 1-loop running coupling (H.J. Pirner & B. Galow '09):

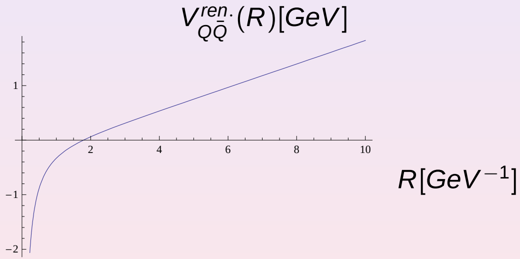
$$h(z) = \frac{\log(\epsilon)}{\log(\epsilon + (\Lambda z)^2)} , \qquad z \sim \frac{1}{E} .$$

Other ansatz: $h(z) = e^{\frac{1}{2}cz^2}$ Andreev & Zakharov '07

Soft-wall model of AdS/QCD

H.J.Pirner & B.Galow '09

$$\langle W(\mathcal{C})\rangle \approx \exp\left(-S_{\rm NG}\right) \propto \exp\left(-t \cdot V_{Q\bar{Q}}(R)\right)$$



$$V_{q\bar{q}} \approx -\frac{a}{R} + \sigma R \,, \qquad a = 0.48\,, \quad \sigma = (0.425\,\mathrm{GeV})^2$$

$$\Longrightarrow \quad \epsilon = \Lambda^2 l_s^2 = 0.48\,, \qquad \Lambda = 264\,\mathrm{MeV}\,.$$

Issues

The 5D Einstein-dilaton model

5D Einstein-dilaton model (Gürsoy et al. '08):

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) - \frac{1}{8\pi G_5}\int_{\partial M} d^4x\sqrt{-h}\,K\,.$$

One to one relation between β-function and dilaton potential $V(\phi)$:

$$V(\phi) = -\frac{12}{\ell^2}\left(1 - \left(\frac{\beta(\alpha)}{3\alpha}\right)^2\right)\exp\left[-\frac{8}{9}\int_0^\alpha \frac{\beta(a)}{a^2}da\right]\,, \qquad \alpha = e^\phi\,.$$

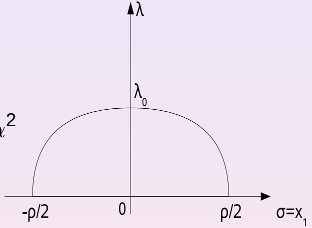
Ansatz (E.Megías et al., NPB834, 2010):

$$\beta(\alpha) = -b_2\alpha + \left[b_2\alpha + \left(\frac{b_2}{\bar\alpha} - \beta_0\right)\alpha^2 + \left(\frac{b_2}{2\bar\alpha^2} - \frac{\beta_0}{\bar\alpha} - \beta_1\right)\alpha^3\right]e^{-\alpha/\bar\alpha}\,.$$

- $\alpha << \bar\alpha \Longrightarrow$ Ultraviolet: $\beta(\alpha) \approx -\beta_0\alpha^2 - \beta_1\alpha^3$
- $\alpha >> \bar\alpha \Longrightarrow$ Infrared: $\beta(\alpha) \approx -b_2\alpha$

The 5D Einstein-dilaton model

$$\langle W(\mathcal{C})\rangle \approx \exp\left(-S_{\rm NG}\right) \propto \exp\left(-t\cdot V_{Q\bar{Q}}(R)\right)$$



$$ds^2 = e^{\frac{4}{3}\phi}e^{2A}\left(dt^2+d\vec{x}^2\right)+e^{\frac{4}{3}\phi}\ell^2 e^{2D}d\alpha^2$$

$$\rho(\alpha_0) = \int_{-\frac{\rho}{2}}^{\frac{\rho}{2}} d\sigma = 2\ell e^{-A_0}\cdot\int_0^{\alpha_0}\frac{e^{D-3\tilde{A}}\cdot\tilde{\alpha}^{-\frac{4}{3}}}{\sqrt{1-\tilde{\alpha}^{-\frac{8}{3}}e^{-4\tilde{A}}}}d\alpha\,,$$

$$\begin{aligned} V^{\rm reg.}_{Q\bar{Q}}(\alpha_0) &= \frac{1}{t}S^{\rm reg.}_{\rm NG} = V - V_s \\ &= \frac{2\ell\alpha_0^{\frac{4}{3}}e^{A_0}}{\pi\bar{l}_s^2}\left[\int_0^{\alpha_0}d\alpha\frac{\tilde{\alpha}^{\frac{4}{3}}e^{D+\tilde{A}}}{\sqrt{1-\tilde{\alpha}^{-\frac{8}{3}}e^{-4\tilde{A}}}} - \int_0^{\infty}d\alpha\,\tilde{\alpha}^{\frac{4}{3}}\cdot e^{D+\tilde{A}}\right]\end{aligned}$$

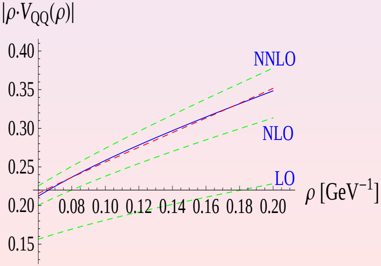
Heavy $\bar{Q}Q$ potential at Short Distances

Analytical result at short distances:

$$V_{Q\bar{Q}}(\rho) = -\frac{2}{\pi}\frac{\ell^2}{\bar{l}_s^2}\frac{\alpha_0^{4/3}(\rho)}{\rho}\Big[\underbrace{0.359}_{\text{LO}} + \underbrace{0.533\beta_0\alpha_0}_{\text{NLO}} + \underbrace{(1.347\beta_0^2 + 0.692\beta_1)\alpha_0^2}_{\text{NNLO}}\Big]$$

where the running coupling is

$$\alpha_0(\rho) = \left[\beta_0 \log\left(\frac{1.32\ell}{\rho}\right) + \frac{\beta_1}{\beta_0}\log\left(\beta_0\log\left(\frac{1.32\ell}{\rho}\right)\right)\right]^{-1} + \mathcal{O}\left(\log^{-3}\right)$$



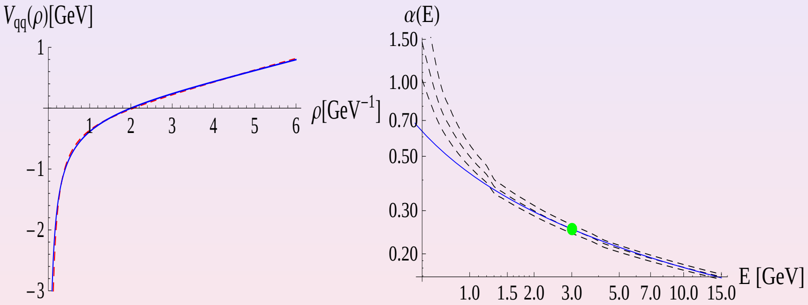
$$V_{Q\bar{Q}}^{\rm PT}(\rho) = -\frac{N_c^2-1}{2N_c}\frac{\alpha_{\rm PT(\rho)}}{\rho}\left(1+\mathcal{O}(\alpha_{\rm PT})\right)$$

Comparison with PT $\Longrightarrow$

$$\frac{\bar{l}_s}{\ell} = 0.33$$

Heavy $\bar{Q}Q$ potential and running coupling

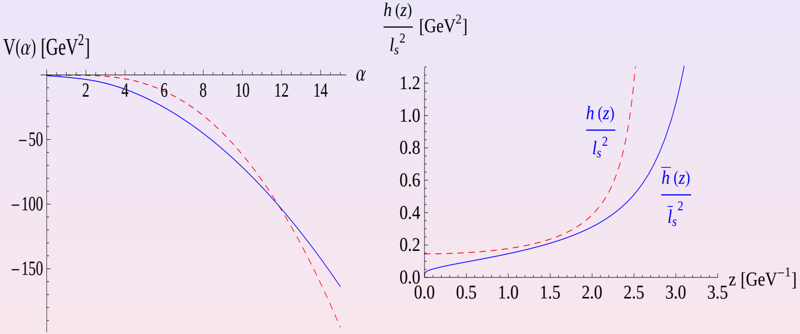
$$\langle W(\mathcal{C})\rangle \approx \exp\left(-S_{\rm NG}\right) \propto \exp\left(-t \cdot V_{Q\bar{Q}}(R)\right)$$



- $\sigma = (0.425\,\mathrm{GeV})^2 \Longrightarrow \frac{b_2}{\bar{\alpha}} = 3.51\mathrm{GeV}\cdot \bar{l}_s$
- Fit of running coupling $\Longrightarrow \frac{b_2}{\bar{\alpha}} = 5.09$

$$\bar{l}_s = 1.45\,\mathrm{GeV}^{-1}$$

Dilaton potential and warp factor

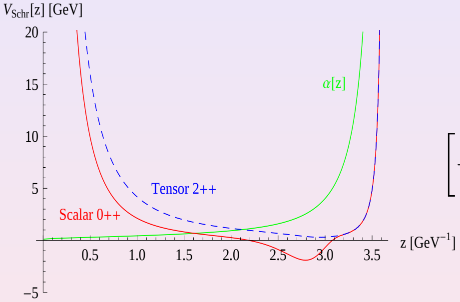


$$V(\alpha) \sim -(b_2^2 - 9)\alpha^{\frac{8}{9}b_2}\,, \quad \alpha \to \infty;$$

$$\alpha(z) \sim \frac{1}{(z_{\rm IR} - z)^{\frac{b_2}{\frac{4}{9}b_2^2 - 1}}}\,, \quad z \to z_{\rm IR}$$

Confinement and 'good' IR singularity

Effective Schrödinger potentials for glueballs 0^{++} and 2^{++}:



$$\left[-\frac{\partial^2}{\partial z^2} + V_{\text{Schr.}}(z)\right]\psi_n(z) = m_n^2\psi_n(z)$$

- Confining theory $\Longrightarrow 1.5 < b_2$
- IR singularity repulsive to physical modes $\Longrightarrow b_2 < 2.37$

Best choice of parameters:

$$b_2 = 2.3\,, \quad \bar{\alpha} = 0.45\,, \quad \ell = 4.39\,\text{GeV}^{-1}\,, \quad \bar{l}_s = 1.45\,\text{GeV}^{-1}\,.$$

Issues

Schwarzschild black hole

General Relativity with no source $\Longrightarrow$ Einstein-Hilbert action

$$S_{EH} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g}\, R \,, \qquad R = g^{\mu\nu} R_{\mu\nu}$$

Classical solution $\frac{\delta}{\delta g_{\mu\nu}} \Longrightarrow$ Einstein equations

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \underset{\text{spherical}}{\Longrightarrow} R_{\mu\nu} = 0$$

Schwarzschild solution in spherical coordinates (1915):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega_2^2 \,, \qquad f(r) = 1 - \frac{r_h}{r}$$

r_h is the horizon. Not physical singularity: $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 12 \frac{r_h^2}{r^6}$.
Large distance limit: $g_{tt}(r) \underset{r\to\infty}{\sim} -(1 + 2V_{Newton}(r)) \Longrightarrow r_h = 2G_4 M$.

Black hole thermodynamics

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right), \qquad \beta = \frac{1}{T}$$

Periodicity in euclidean time ($\tau = it$): $\Phi(\tau+\beta) = \Phi(\tau)$

*Regularity: Expansion around the horizon $f(r_h) = 0$; $r = r_h(1+\rho^2)$:

$$ds^2_{\rm BH} = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_2^2 \underset{\rho\to 0}{\sim} 4r_h^2\left(d\rho^2 + \rho^2 \underbrace{\left(\frac{d\tau}{2r_h}\right)^2}_{d\theta^2} + \frac{1}{4}d\Omega_2^2\right)$$

$\Longrightarrow$ Periodicity: $\frac{\tau}{2r_h} \to \frac{\tau}{2r_h} + 2\pi \Longrightarrow \tau \to \tau + 4\pi r_h =: \tau + \beta$

$$T = \frac{1}{8\pi M G_4}$$

*Thermodynamical interpretation of black holes:

$$dM = TdS \Longrightarrow S = \int \frac{dM}{T} = 4\pi G_4 M^2$$

$$\mathcal{A} = 4\pi r_h^2 = 16\pi(G_4 M)^2 \Longrightarrow S_{\text{Black Hole}}(T) = \frac{\mathcal{A}(r_{\text{horizon}})}{4G_D} \quad \text{Bek-Hawking}$$

Issues

The 5D Einstein-dilaton model at finite temperature

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) - \frac{1}{8\pi G_5}\int_{\partial M} d^4x\sqrt{-h}\,K$$

Finite temperature solutions (E. Kiritsis et al. JHEP (2009) 033):

- Thermal gas solution (confined phase):

$$ds^2_{\rm th} = b_0^2(z)(-dt^2 + d\vec{x}^2 + dz^2)\,, \qquad t \sim t + i\beta$$

- Black hole solution (deconfined phase):

$$ds^2_{\rm BH} = b^2(z)\left[-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right]$$

In the UV ($z \simeq 0$): flat metric $b(z) \simeq \ell/z$ and $f(0) = 1$.
There exists an horizon $f(z_h) = 0$.
Regularity at the horizon $\Longrightarrow T = \frac{|\dot{f}(z_h)|}{4\pi}$.

The 5D Einstein-dilaton model

Einstein equations $\frac{\delta}{\delta g_{\mu\nu}}$:

$$\underbrace{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)}_{E_{\mu\nu}} - \underbrace{\left(\frac{4}{3}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\left(\frac{4}{3}(\partial\phi)^2 + V(\phi)\right)\right)}_{T_{\mu\nu}} = 0$$

$$(a) \qquad \frac{\ddot{f}}{\dot{f}} + 3\frac{\dot{b}}{b} = 0\,, \Longrightarrow f(z) = 1 - \frac{\int_0^z \frac{du}{b(u)^3}}{\int_0^{z_h} \frac{du}{b(u)^3}}$$

$$(b) \qquad 6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\dot{\phi}^2\,,$$

$$(c) \qquad 6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f} V(\phi)$$

Conformal solution:

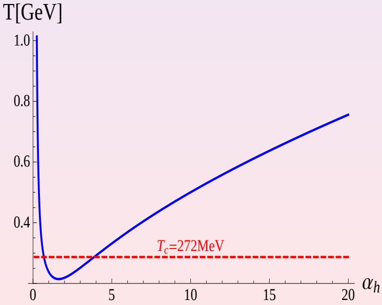
$$V(\phi) = -\frac{12}{\ell^2}\,, \quad \dot{\phi} = 0 \Longrightarrow b(z) = \frac{\ell}{z}\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^4\,, \quad T = \frac{1}{\pi z_h}$$

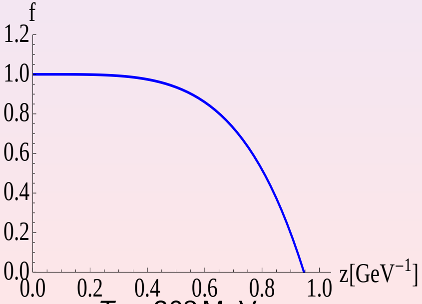
The 5D Einstein-dilaton model

Input:

$$\beta(\alpha) = -b_2\alpha + \left[b_2\alpha + \left(\frac{b_2}{\bar{\alpha}} - \beta_0\right)\alpha^2 + \left(\frac{b_2}{2\bar{\alpha}^2} - \frac{\beta_0}{\bar{\alpha}} - \beta_1\right)\alpha^3\right] e^{-\alpha/\bar{\alpha}}$$

$$\Longrightarrow V(\alpha) = -\frac{12}{\ell^2}\left(1 - \left(\frac{\beta(\alpha)}{3\alpha}\right)^2\right) \exp\left[-\frac{8}{9}\int_0^\alpha \frac{\beta(a)}{a^2} da\right] \quad , \text{ E.Megías NPB'10.}$$



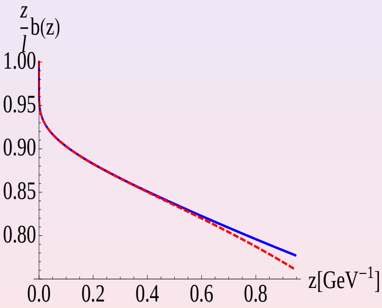


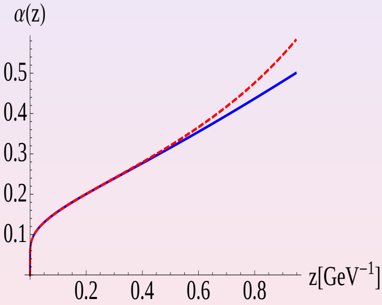
$T = 368\,\text{MeV}$

The 5D Einstein-dilaton model

Input:

$$\beta(\alpha) = -b_2\alpha + \left[b_2\alpha + \left(\frac{b_2}{\bar{\alpha}} - \beta_0\right)\alpha^2 + \left(\frac{\beta_2}{2\bar{\alpha}^2} - \frac{\beta_0}{\bar{\alpha}} - \beta_1\right)\alpha^3\right] e^{-\alpha/\bar{\alpha}}$$





$T = 368\text{MeV}$

Issues

Thermodynamics

Postulate: Entropy of gauge theories is equal to the Bekenstein-Hawking entropy of their string duals.

$$S(T) = \frac{\mathcal{A}(z_h)}{4G_5} = \frac{V_3 b^3(z_h)}{4G_5}, \qquad z_h \simeq \frac{1}{\pi T}$$

High temperature limit: $s(T) \underset{T\to\infty}{\sim} \frac{\pi^3\ell^3}{4G_5} T^3 = \frac{32}{45}\pi^2 T^3 =: s_{\text{ideal}}(T)$

One can compute all the thermodynamic quantities:

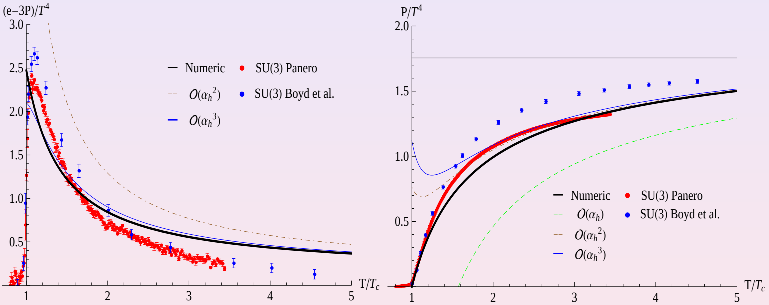
$$s(T) = \frac{d}{dT} p(T), \qquad \Delta(T) \equiv \frac{\epsilon - 3p}{T^4} = \frac{s}{T^3} - \frac{4p}{T^4}.$$

In the free energy $\Longrightarrow$ contributions from big and small black holes:

$$p(T) = p(T_0) + \int_{T_0}^{T} d\tilde{T} s(\tilde{T}) = \int_{+\infty}^{\alpha_h} d\tilde{\alpha}_h \left(\frac{dT}{d\tilde{\alpha}_h}\right) s(\tilde{\alpha}_h).$$

Thermodynamics

Input $\beta(\alpha)$.



$$N_c = 3\,, \qquad N_f = 0\,, \qquad b_2 = 2.3\,, \qquad \bar{\alpha} = 0.45$$

$$\frac{p(T)}{T^4} = \frac{\pi^3 \ell^3}{16 G_5}\left(1 - \frac{4}{3}\beta_0 \alpha_h + \frac{2}{9}(4\beta_0^2 - 3\beta_1)\alpha_h^2 + \dots\right)\,, \quad \alpha_h \simeq \frac{1}{\beta_0 \log(\pi T/\Lambda)}$$

Thermodynamics

Free Energy from: Bekenstein-Hawking entropy $\underset{\text{Classically}}{\Longleftrightarrow}$ Free Energy from: Gibbons-Hawking action

$$S = \frac{\mathcal{A}}{4G_5} \qquad \beta\mathcal{F} = \mathcal{S}_{\text{reg}} := \mathcal{S}_{BH} - \mathcal{S}_{th} \quad \Longrightarrow$$

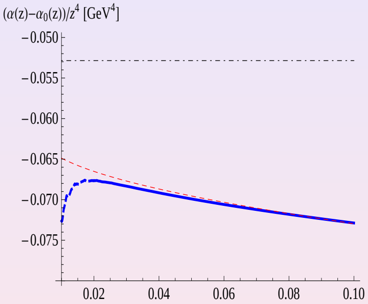
$$\Longrightarrow \mathcal{F} = \frac{V_3}{16\pi G_5}\left(15G - \frac{C_f}{4}\right)$$

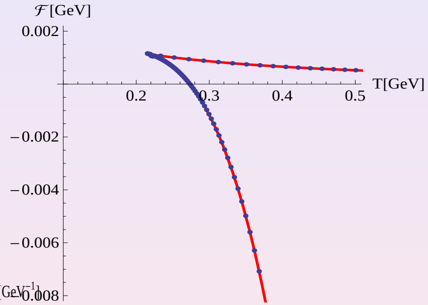
E. Kiritsis et al. JHEP (2009) 033.

$$G = \frac{\pi G_5}{15}\frac{\beta(\alpha)}{\alpha^2}\langle \text{Tr}F_{\mu\nu}^2\rangle\,, \qquad C_f = 4\pi T b^3(z_h) \sim T\cdot s$$

$$b_T(z) = b_0(z)\left[1 + \frac{G}{\ell^3}z^4\left(1 + \frac{19}{12}\beta_0\alpha_0(z) + c_2^b\alpha_0^2(z) + \cdots\right) + \cdots\right]\,,\ z\to 0$$

Thermodynamics

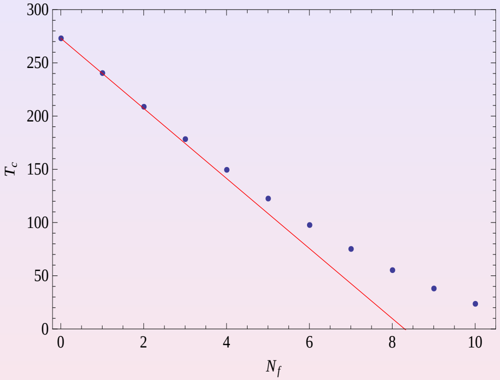




$$\frac{\alpha_T(z)-\alpha_0(z)}{z^4} = -\frac{45}{8}\frac{G}{\ell^3\beta_0}\left(1+\left(\frac{11}{6}\beta_0-\frac{\beta_1}{\beta_0}\right)\alpha_0(z)+c_2^{\alpha}\alpha_0^2(z)+\cdots\right)$$

$\Longrightarrow$ Corrections in α_0 are very important for agreement.

Flavor dependence of the phase transition



Agrees quite well with J. Braun & H. Gies JHEP 1005 (2010),

$$\kappa = 0.107$$

$$T_c = T_{N_f=0}\left(1 - \kappa N_f + \mathcal{O}(N_f^2)\right)$$

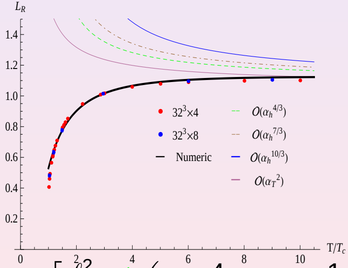
$$\kappa = 0.1205\,, \qquad T_{N_f=0} = 273.0\,\mathrm{MeV}\,, \qquad T_{N_f=4} = 149.5\,\mathrm{MeV}\,,$$

$$T_{N_f=0}^{\mathrm{Lattice}} = 270 \pm 2\,\mathrm{MeV}\,, \qquad T_{N_f=4}^{\mathrm{Lattice}} = 140 - 170\,\mathrm{MeV}\,.$$

Polyakov Loop

$$L(T) := \langle P \rangle = \int DX \, e^{-S_w} \underset{\text{semiclassically}}{\Longrightarrow} \langle P \rangle = \sum_i w_i e^{-S_i} \simeq e^{-S_0}$$

$$S_0^{\text{reg}} = \frac{1}{2\pi l_s^2 T} \left[\int_0^{z_h} dz \, \alpha^{4/3}(z) b^2(z) - \int_0^{z_c} dz \, \alpha_0^{4/3}(z) b_0^2(z) \right]$$



Lattice Gluodynamics SU(3): S.Gupta et al., PRD77(2008).

$$l_s = 2.36 \, \text{GeV}^{-1} \,, \quad z_c = 0.43 \, \text{GeV}^{-1}$$

$$L(T) = \exp\left[\frac{\ell^2}{2 l_s^2} \alpha_h^{\frac{4}{3}} \left(1 + \frac{4}{9} \beta_0 \alpha_h + \frac{1}{81} \left(161 \beta_0^2 + 72 \beta_1 \right) \alpha_h^2 + \mathcal{O}(\alpha_h^3) \right) \right] ,$$

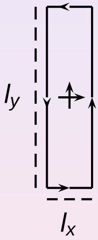
In contradiction with PT: $L_{\text{PT}}(T) = \exp\left(\frac{4}{3} \sqrt{\pi} \alpha^{\frac{3}{2}} + \mathcal{O}(\alpha^2) \right)$.

Spatial Wilson Loops

Rectangular Wilson loop in (x,y) plane:

$$\langle W(\mathcal{C})\rangle = \left\langle \exp\left(ig \int_{\mathcal{C}} A_\mu dx^\mu \right) \right\rangle \overset{l_y\to\infty}{\simeq} e^{-l_y \cdot V(d)} ,$$

$$V(d) \overset{d\to\infty}{\simeq} \sigma_s \cdot d$$

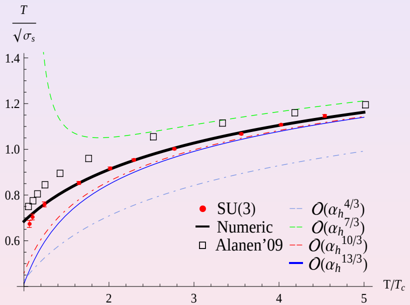


$$S_{\rm NG} = \frac{\ell^2}{2\pi l_s^2} l_y \int_{-l_x/2}^{l_x/2} dx \frac{h(z)}{z^2} \sqrt{1 + \frac{(\partial_x z)^2}{f(z)}} , \quad X(x,y) = (x,y,0,0,z(x))$$

$$\sigma_s(T) = \frac{1}{2\pi l_s^2} \alpha_h^{4/3} b_h^2$$

Spatial Wilson Loops

Spatial string tension: Lattice data: G. Boyd et al. NPB469 (1996)



$$l_s = 1.94\,\mathrm{GeV}^{-1}\,,$$
$$\chi^2/\mathrm{d.o.f.} = 0.87\,.$$

$$\sigma_s(T) = \frac{1}{2\pi l_s^2}\alpha_h^{\frac{4}{3}} b^2(z_h) = \frac{\ell^2}{2l_s^2}\pi T^2 \alpha_h^{\frac{4}{3}}\left[1 - \frac{8}{9}\beta_0\alpha_h + \frac{2}{81}(25\beta_0^2 - 2\beta_1)\alpha_h^2 + \mathcal{O}(\alpha_h^3)\right]$$

This is in contradiction with pQCD: $\sigma_{\mathrm{pQCD}} \sim T^2\alpha_s^2(T)$.
See also Kajantie et al. PRD80 (2009).

Issues

Pressure and Running Coupling

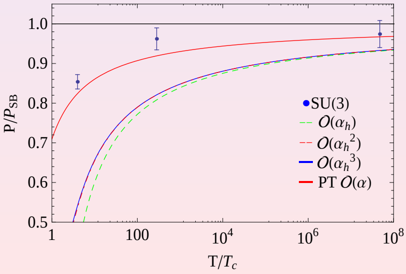
- Pressure from AdS/QCD (UV expansion):

$$P_{AdS/QCD} = P_{SB} \cdot (1 - 2.33\alpha_h + 1.86\alpha_h^2 + \mathcal{O}(\alpha_h^3))$$

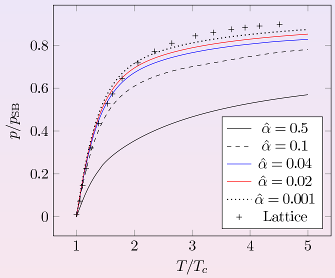
- Pressure from perturbation theory:

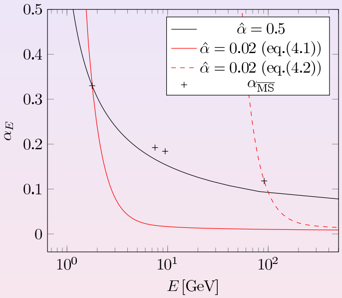
$$P_{pQCD} = P_{SB} \cdot (1 - 1.19\alpha_T + 5.4\alpha_T^{3/2} + \mathcal{O}(\alpha_T^2))$$

A comparison at high T demands $\alpha_{AdS/QCD} \simeq \alpha_{pQCD}/2$.



Pressure and Running Coupling





Simple model: $$\beta(\alpha) = \begin{cases} \beta_{\text{pert}}(\alpha) - \hat{\beta}_2 \alpha^4 & \text{if } \alpha \leq \hat{\alpha} \\ \beta_{\text{pert}}(\hat{\alpha}) - \hat{\beta}_2 \hat{\alpha}^4 - 3(\alpha - \hat{\alpha}) & \text{if } \alpha > \hat{\alpha} \end{cases}$$

Debye Screening

$$S_{AdS/QCD} = -\frac{1}{4G_5^2}\int dz \int d^4x \sqrt{-g}(F^V_{MN}F^{VMN} + F^A_{MN}F^{AMN})$$

Debye mass: $m_D^2 = g^2\Pi_{00}(\omega = 0, \vec{k}^2 = -m_D^2)$
Screening mass in the absence of an external field (Gorsky et al. '10):

$$S_{AdS/QCD} = \frac{V_{4D}}{2G_5^2}\int dz\, b(z)(\partial_z V_0(z))^2$$

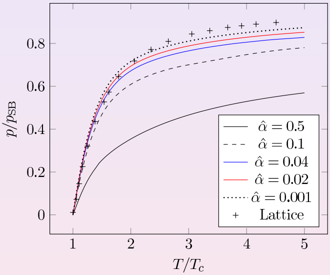
e.o.m.: $\partial_z(b(z)\partial_z V_0(z)) = 0\,, \qquad V_0(z_h) = 0 \ \ \& \ \ V_0(z=0) = \mu.$

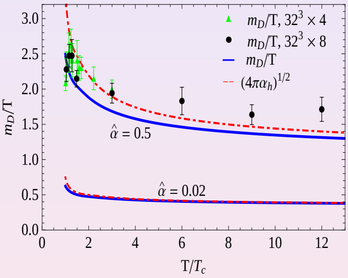
$$V_0(z) = \mu \cdot \left(1 - C_h \int_0^z du\, b^{-1}(u)\right)$$

$$\Longrightarrow \Pi_{00} = \frac{1}{V_{4D}}\frac{\partial^2 S}{\partial \mu^2} \Longrightarrow m_D^2 = \frac{4\pi}{G_5^2}\alpha_h \cdot \left[\int_0^{z_h} du\, b^{-1}(u)\right]^{-1} \underset{T\to\infty}{\simeq} g_h^2 T^2$$

Debye Screening

Debye Mass (E. Megías et al. PLB696(2011))





$$\frac{m_D}{T} \underset{T\to\infty}{\simeq} (4\pi\alpha_h)^{1/2}\left(1 - \frac{2}{9}\beta_0\alpha_h + \mathcal{O}(\alpha_h^2)\right)$$

- $\hat{\alpha} = 0.02 \Longrightarrow$ “Good” P and “bad” m_D.
- $\hat{\alpha} = 0.5 \Longrightarrow$ “Bad” P and “good” m_D.

DIMENSION TWO GLUON CONDENSATE

In collaboration with: E. Ruiz Arriola and L.L. Salcedo (U. Granada, Spain).

Issues

Dimension 2 condensates?

Standard wisdom: There are no local gauge invariant operators of dimension 2.
But, are there operators of dimension 2 relevant for phenomenology?
There exists a nonlocal gauge invariant condensate

$$\langle A^2_{\rm min}\rangle = \frac{1}{VT}\min_g \int d^4x \langle \left(gA_\mu g^\dagger + g\partial_\mu g^\dagger\right)^2\rangle,$$

which reduces to the $\langle A^2_\mu\rangle$ condensate in the Landau gauge.
Example: Can f_π^2 be written as the expectation value of a gauge invariant local operator?

Dimension 2 condensates?

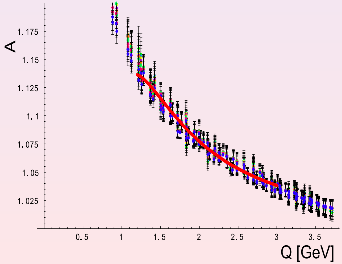
- In perturbation theory one must fix the gauge.
- There is no complete gauge fixing. Thus, there is always a dimension 2 local operator which is invariant under the residual gauge invariance.
- Dimension-2 objects have been seen in fixed gauge lattices: (Skullerad, Rodriguez-Quintero, Oliviera 2006).

Quark propagator on the Lattice in Landau Gauge

P. Bowman, W. Broniowski and E. Ruiz Arriola, PRD70:097505 (2004).

$$S^{-1}(p) = \not{p}A(p) - B(p)$$

$$A(Q) = 1 + \frac{\pi\alpha_s(\mu^2)\langle A^2\rangle_\mu}{N_c Q^2} - \frac{\pi\alpha_s(\mu^2)\langle G^2\rangle_\mu}{3N_c Q^4} + \cdots ,$$



Issues

Dimension Two Gluon Condensate and Polyakov loop

$$D^{ab}_{\mu\nu}(p^2) = \frac{\delta^{ab}\delta_{\mu\nu}}{k^2} \to \frac{\delta^{ab}\delta_{\mu\nu}}{k^2}\left(1 + \frac{m_G^2}{k^2}\right)$$ Narison & Zakharov '99

At finite temperature: $D_{00}(\vec{k}) = \frac{1}{\vec{k}^2+m_D^2}\left(1+\frac{m_G^2}{\vec{k}^2+m_D^2}\right)$ Megías '06

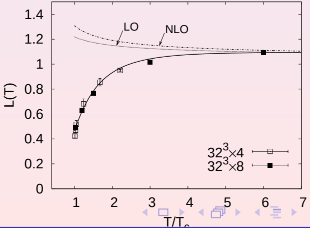
$$\langle A_{0,a}^2\rangle = (N_c^2-1)T\int\frac{d^3k}{(2\pi)^3}D_{00}(\vec{k}) = \underbrace{-\frac{2}{\pi}m_D T}_{\sim T^2} + \underbrace{\frac{m_G^2 T}{\pi m_D}}_{\sim T^0} \equiv \langle A_{0,a}^2\rangle^{\mathrm{P}} + \langle A_{0,a}^2\rangle^{\mathrm{NP}}$$

Polyakov loop:

$$L = \left\langle \frac{1}{N_c}\mathrm{Tr}_c e^{igA_{0,a}/T}\right\rangle \longrightarrow$$

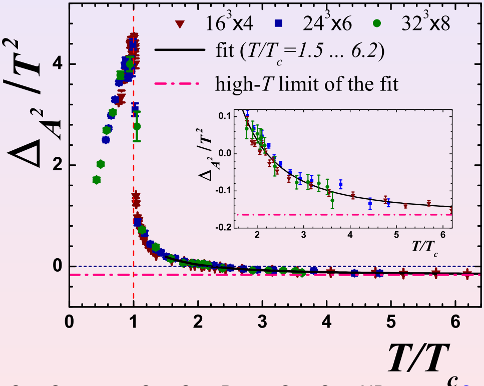
$$\longrightarrow e^{-g^2\langle A_{0,a}^2\rangle/(4N_cT^2)} + \mathcal{O}(g^6)$$

$$g^2\langle A_{0,a}^2\rangle^{NP} = (0.84(6)\ \mathrm{GeV})^2$$



Gluon Asymmetry: Lattice data

Gluon asymmetry $N_c = 2, N_f = 0$
Lattice data (M. Chernodub, E.M. Ilgenfritz, PRD78 (2008))



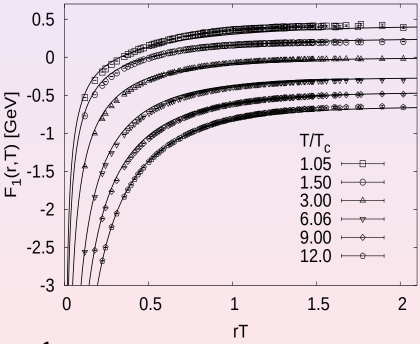
$$\Delta_{A^2} = g^2\langle A_{0,a}^2\rangle - \frac{1}{3}\sum_i g^2\langle A_{i,a}^2\rangle$$

$$\Delta_{A^2}/T^2 \sim 1/T^2$$

$$g^2\langle A_{0,a}^2\rangle = \underbrace{g^2\langle A_{0,a}^2\rangle^P}_{\sim T^2} + \underbrace{g^2\langle A_{0,a}^2\rangle^{NP}}_{\sim \text{cte}} \Longrightarrow g^2\langle A_{0,a}^2\rangle/T^2 = c_P + c_{NP}/T^2 \,.$$

Heavy $\bar{q}q$ Free Energy

Singlet Free Energy $N_c = 3, N_f = 0$
Lattice data (O. Kaczmarek PRD70 (2004))



$$e^{-F_{q\bar{q}}(\vec{x},T)/T + c(T)} = \frac{1}{N_c^2}\langle \mathrm{tr}_c \Omega(\vec{x})\, \mathrm{tr}_c \Omega^\dagger(\vec{0})\rangle\,, \quad g^2\langle A_{0,a}^2\rangle^{NP} = (0.90(5)\ \mathrm{GeV})^2$$

Thermodynamics

From the gluon propagator, one gets (Megías '09):

$$\langle (F^a_{\mu\nu})^2 \rangle = 2\left\langle (B^a_i)^2 - (E^a_i)^2 \right\rangle \simeq 2\langle \partial_i A_{0,a} \partial_i A_{0,a} \rangle = -\frac{6}{\pi} m_G^2 m_D T \sim T^2 \,.$$

It reproduces the thermal behaviour of the **trace anomaly**:

$$T^4 \Delta \equiv \epsilon - 3P = \frac{\beta(g)}{2g} \langle (F^a_{\mu\nu})^2 \rangle = \underbrace{(\mathrm{Pert.})}_{\sim T^4} - \underbrace{\frac{3}{\pi}\frac{\beta(g)}{g} m_G^2 m_D T}_{\sim T^2} \,.$$

It is straightforward to compute all the thermodynamic quantities:

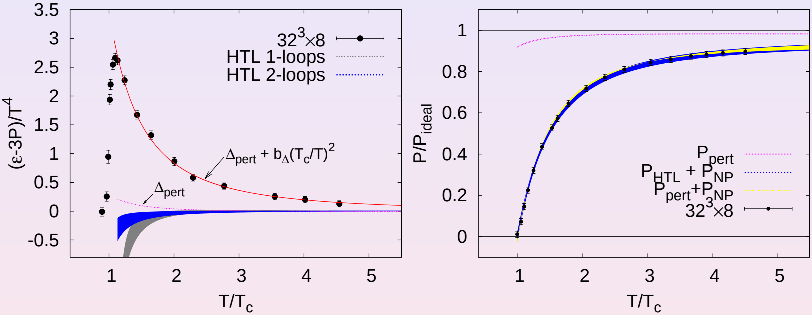
$$\textbf{Pressure}: \quad \frac{P(T)}{T^4} = \int^T \frac{dT'}{T'} \Delta(T') = \underbrace{(\mathrm{Pert.})}_{\sim T^0} + \underbrace{\frac{3}{2\pi}\beta(g)\frac{m_G^2}{T^2}}_{\sim 1/T^2}$$

$$\textbf{Entropy}: \quad \frac{s}{T^3} = \Delta + \frac{4P}{T^4} = \underbrace{(\mathrm{Pert.})}_{\sim T^0} + \underbrace{\frac{3}{\pi}\frac{\beta(g)}{g}\frac{m_G^2 m_D}{T^3}}_{\sim 1/T^2} \,.$$

See also O.Andreev's talk.

Thermodynamics

Lattice data: G. Boyd et al. NPB469 (1996).



Values of the dimension two gluon condensate from a fit of:

Observable	$g^2\langle A^2_{0,a}\rangle_{NP}$
Polyakov loop	$(3.22 \pm 0.07T_c)^2$
Heavy $\bar{q}q$ free energy	$(3.33 \pm 0.19T_c)^2$
Trace Anomaly	$(2.86 \pm 0.24T_c)^2$

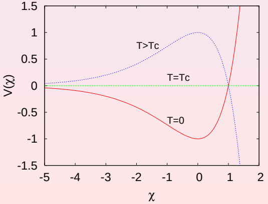
Power Corrections from ET of scale invariance

- Scale invariance $x_\mu \to \lambda x_\mu$ is broken by quantum effects. Effective theory of scale invariance D.Kharzeev '08.

$$\mathcal{L} = \frac{|\epsilon_v|}{m^2}\frac{1}{2}e^{\chi/2}(\partial_\mu \chi)^2 + \underbrace{\left(|\epsilon_v| + \frac{c}{4}(F^a_{\mu\nu})^2\right) e^{\chi}(1-\chi)}_{-V(\chi)} - \frac{1}{4}(F^a_{\mu\nu})^2$$

Properties:

- $\chi = 1$: No coupling between dilatons and gluons $\Longrightarrow$ dynamics of color fields is pert. It happens at some scale $M_0 \sim 5 - 6T_c$.
- No color fields $\Longrightarrow$ minimum of dilaton pot. at $V(\chi = 0) = -|\epsilon_v|$.



$$|\epsilon_v| + \frac{c}{4}(F^a_{\mu\nu})^2 \begin{cases} > 0 & T < T_c \\ = 0 & T = T_c \\ < 0 & T > T_c \end{cases}$$

Power Corrections from ET of scale invariance

D. Kharzeev, E. Levin & E. Megías '09

- Classical equation for A_μ gives:

$$E_i^a = -\partial_i A_{0,a} = \frac{E_0}{F(\chi)} e^{-m_D r}\hat{n} + \frac{gC_a}{F(\chi)4\pi r^2}(1 + m_D r)e^{-m_D r}\hat{r}$$

- Unlike in pQCD $(F_{\mu\nu}^a)^2|_{\rm min} = 0$, the minimun of the effective theory corresponds to a finite chromoelectric field E_0.

$$(F_{\mu\nu}^a)^2|_{\rm min} = 2(\vec{B}^{a2} - \vec{E}^{a2}) \overset{B\simeq 0}{\simeq} -2E_0^2 = -4|\epsilon_v|/c$$

In momentum space: $A_{0,a}(\vec{k}) = -\frac{g}{F(\chi)(\vec{k}^2+m_D^2)} - \frac{8\pi E_0}{F(\chi)(\vec{k}^2+m_D^2)^2}$,

so the gluon propagator is

$$D_{00}(\vec{k}) = \frac{1}{\vec{k}^2 + m_D^2} + \frac{m_G^2}{(\vec{k}^2 + m_D^2)^2}, \qquad m_G^2 = \frac{8\pi E_0}{g}.$$

Issues

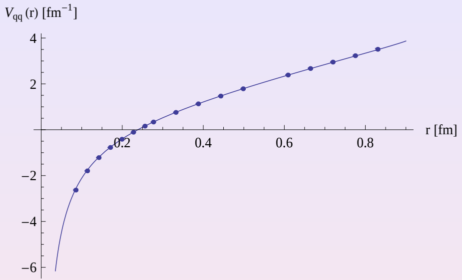
Duality between perturbative series and power corrections

Perturbative-Non perturbative duality (Narison & Zakharov, PLB679 (2009)): Large order perturbative series are dual to non-perturbative contributions.

$$V_{q\bar{q}}(r) \approx -\frac{4}{3}\frac{\alpha_s}{r} + \sigma \cdot r$$

$$V_{q\bar{q}}(r) \approx \frac{1}{r}\sum_{n=1}^{N} a_n \alpha_s^n(r) + \sigma_N \cdot r$$

Both formulas fit lattice data very well.
$\Longrightarrow$ Perturbative resummation could be written in terms of condensates, starting from dimension two.



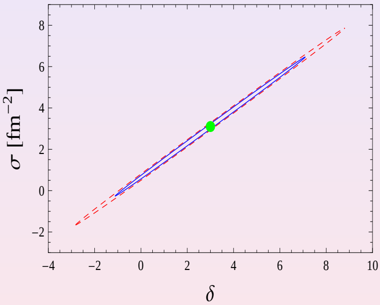
$$N_c = 3\,,\; N_f = 0 : \text{Tree Level} \quad \alpha_s = 0.19\,, \quad \sigma = 4.99\,\text{fm}^{-2}\;(\text{dim-2})\,.$$

(E.Megías, E.Ruiz Arriola & L.L.Salcedo, PRD81(2010)):
In a fit including Pert. and Non Pert. contributions, one should see:

- Smaller contributions from condensates at increasing perturbative orders. $\sigma_N \to 0$ (duality).
- Rather strong statistical correlation between $\Lambda_{\rm QCD}$ and condensates ($\Lambda_{\rm QCD}$ is the only scale in QCD). $r(\sigma_N, \Lambda_{\rm QCD}) \sim \pm 1$.

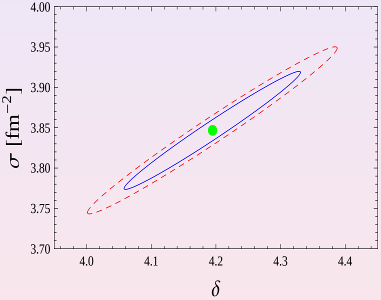
Correlations: Perturbative - Non Perturbative

Fit using PT at N^{3}LL, N. Brambilla et al. PRD80 (2009).



$0.085\text{fm} < r < 0.17\text{fm}$

$r(\delta,\sigma) = 0.9986$



$0.085\text{fm} < r < 0.83\text{fm}$

$r(\delta,\sigma) = 0.984$

$$\alpha_s(\mu): \quad \delta = \log\left(\mu/\Lambda_{QCD}\right)$$

Correlations: Perturbative - Non Perturbative

Order	δ	$\sigma[\mathrm{fm}^{-2}]$	$r(\sigma,\delta)$	χ^2/dof
Tree Level	–	4.99(11)	0.991	0.98
1-loop	3.66(7)	4.25(6)	0.974	0.20
2-loop	4.54(10)	4.12(6)	0.978	0.79
N^3LO	4.31(13)	4.07(6)	0.980	0.93
N^3LL	4.19(14)	3.85(7)	0.984	0.76

- σ_N decreases with N.
- Very high correlation $r(\sigma,\Lambda_{\mathrm{QCD}}) \sim 1$.

More examples of non-perturbative corrections:

- Polyakov loop (Megías et al. JHEP 0601 (2006)):

$$L(T) = L_{\rm PT}(T) \exp\left(-\frac{\mathcal{C}_2}{4N_c T^2}\right) .$$

- Heavy $q\bar{q}$ free energy (Megías et al. PRD75 (2007)):

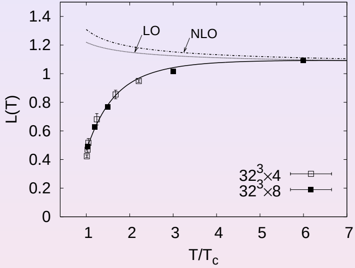
$$F_{q\bar{q}}(r,T) = \underbrace{-\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r} - \frac{2}{3}\alpha_s m_D + \dots}_{\rm PT} \underbrace{-\frac{\sqrt{\pi\alpha_s}}{6m_D}\mathcal{C}_2\left(1-2e^{-m_D r}\right)}_{\rm NP} .$$

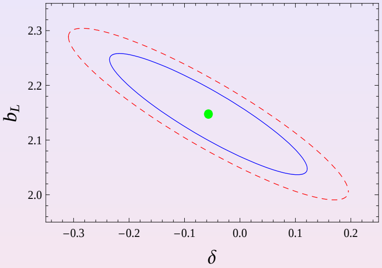
- Trace anomaly (Megías et al. PRD80 (2009)):

$$\frac{\epsilon - 3p}{T^4} = \underbrace{\frac{11}{3}\alpha_s^2 + \mathcal{O}(\alpha_s^{5/2})}_{\rm PT} + \underbrace{\frac{33}{4\pi}\alpha_s\frac{\mathcal{C}_2}{T^2}}_{\rm NP} .$$

Dimension two gluon condensate: $\mathcal{C}_2 \equiv g^2\langle A_{0,a}^2\rangle$.

Polyakov loop





$$\text{PT}: \quad \delta = \log(\mu/2\pi T)$$

$$\text{NP}: \quad b_L = \frac{\mathcal{C}_2}{6T_c^2}$$

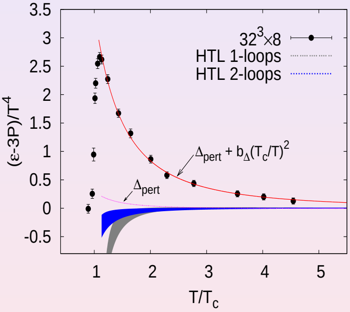
$$1.03 < T/T_c < 6$$

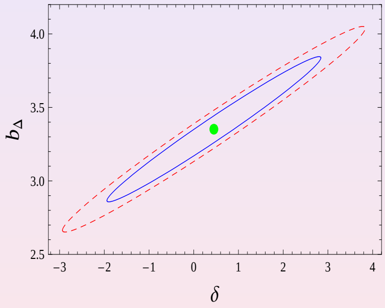
$$r(\delta, b_L) = -0.828$$

Order	$L_P(6T_c)$	δ	b_L	$r(b_L,\delta)$	χ^2/dof
$L_P = \text{const}$	1.121(8)	–	1.72(5)	−0.472	0.45
$\mathcal{O}(\alpha^{3/2})$	1.125(11)	−0.72(20)	2.23(16)	−0.957	1.22
$\mathcal{O}(\alpha^2)$	1.123(9)	−0.06(18)	2.15(11)	−0.901	1.44

Trace Anomaly

$$\frac{(\epsilon - 3P)}{T^4} = \frac{(\epsilon - 3P)_{\rm pert}}{T^4} + b_\Delta \left(\frac{T_c}{T}\right)^2$$





$$\text{PT}: \quad \delta = \log(\mu/2\pi T)$$

$$\text{NP}: \quad b_\Delta = -\frac{3\beta}{g}\frac{\mathcal{C}_2}{T_c^2}$$

$$1.13 < T/T_c < 4.54$$

$$r(\delta, b_\Delta) = 0.983$$

Trace Anomaly

Fit using HTLpt at 1-loop Andersen'00, 2-loops Andersen'02, 3-loops Andersen'09

Order	$\Delta_{\rm HTL}/T^4\vert_{T=4.5T_c}$	δ	b_Δ	$r(b_\Delta,\delta)$
1-loop	$-0.03^{+0.03}_{-0.04}$	-0.03 ± 0.69	3.69(40)	0.975
2-loops	$-0.004^{+0.05}_{-0.004}$	-0.42 ± 0.48	3.57(28)	0.949
3-loops	0.08(5)	2.3 ± 9.7	2.3 ± 1.5	0.977

Order	δ	b_Δ	c_Δ	$r(b_\Delta,\delta)$	$r(b_\Delta,c_\Delta)$
1-loop	0.7 ± 1.8	4.3 ± 1.0	-0.6 ± 1.1	0.932	-0.976
2-loops	$-0.47(80)$	3.58(40)	-0.1 ± 1.0	0.411	-0.145
3-loops	2.3 ± 9.3	2.37 ± 1.6	1.3 ± 1.3	0.964	-0.898

$$\frac{(\epsilon-3P)}{T^4}=\frac{(\epsilon-3P)_{\rm pert}}{T^4}+b_\Delta\left(\frac{T_c}{T}\right)^2+c_\Delta\left(\frac{T_c}{T}\right)^4 .$$

Conclusions:

- AdS/QCD serves as a powerful tool to study the non-perturbative (NP) regime of QCD at zero and finite temperature.
- Model of conformal symmetry breaking in 5D based on dilatons.
- Numerical results of e.o.s. of QCD from Bekenstein-Hawking entropy formula and Gibbons-Hawking action agree.
- Good and unified description for:
  - Equation of State (trace anomaly, pressure, entropy, ...)
  - Flavor dependence for T_c.
  - Polyakov loop and Spatial Wilson loops.
- Dilemma:

  Equation of State $\overset{?}{\Longleftrightarrow}$ Running coupling & Debye screening.
- The NP behaviour of QCD near and above T_c is characterized by power corrections in T. These power corrections are signals of the contribution of a dimension two gluon condensate.
- Study of correlations gives us a tool to understand the duality between perturbative series and power corrections. Lattice data of observables at $T = 0$ and $T \neq 0$ seem to confirm this duality.