Thermodynamics and Instabilities of a Strongly Coupled Anisotropic Plasma

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Collision

Far from equilibrium

Anisotropic hydrodynamics $P_{\perp} \neq P_{\parallel}$

Isotropic hydrodynamics $P_{\perp} \sim P_{\parallel}$

● Anisotropic hydrodynamics.

Martinez & Strickland, ' 11 ! 70⁰ (3) Florkowski & Ryblewski '11

• Weak coupling instabilities.

Mrowczynski, '88 *Mondatorine et strictature 55*
Arnold, Lenaghan & Moore ' 03 *A*hor d Mrowczynski, '88
Randrup & Mrowczynski '03 Arnold, Lenaghan & Moore ' 03
Arnold, Lenaghan, Moore & Yaffe ' 05 Weibel '59 Romatschke & Strickland '03

Collision Anisotropic hydrodynamics Isotropic hydrodynamics Far from equilibrium $P_{\perp} \sim P_{\parallel}$ $P_{\perp} \neq P_{\parallel}$

16 (10 **Expondence with conserved p-form.** *s* ldence wi th co ∼ ^λ (2) • Fluid/gravity correspondence with conserved p-form.

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- *E*(*x*⊥) *E*(*x*⊥) *cos* de la contra • Condensed matter applications: UV completion of Lifshitz fixed points. $\begin{array}{|c|c|c|c|c|}\hline \textbf{1} & \textbf{2} & \textbf{3} & \textbf{3$

 $N=4$

 $\frac{N}{2}$

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a phase transition: a physical change

- *E*(*x*⊥) *E*(*x*⊥) *cos* extent ter applications n explored by n • Condensed matter applications.
	- Connection with QCD at finite density. h ()(|) at suddenly changes from liquid to vapor
	- curve in its phase diagram, a graph of temper at the versus pressure in the curve of the curve of the curve of the curve of the curve traces the boundary between solid and liquid. And depending on the substance, s_{on} phenomenon of cavitation. • Connection with

- Type IIB supergravity solution with following properties:
	- Is static and anisotropic.

- Has a smooth horizon.

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Deformation of N=4 SYM Solid embedding in string theory

Inspiration from: Azeyanagi, Li & Takayanagi '09 ≈ 2 − 8 GeV/fm (15)

• Turn on a position-dependent θ -term: δ*S* = −*a* θ -term:

$$
S_{\text{gauge}} = S_{\mathcal{N}=4} + \delta S \,, \qquad \delta S = \frac{1}{8\pi^2} \int \theta(x) \operatorname{Tr} F \wedge F \,, \qquad \theta(x) = 2\pi n_{\text{D7}} z
$$

NY 1873

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 $\frac{488}{48}$ *s*

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$$

• Translation-invariance is preserved (at least locally):

$$
\delta S \propto -\int dz \wedge \text{Tr}\left(A \wedge F + \frac{2}{3}A^3\right)
$$

Dual description **but the contract of the CP** description parts your

S^{*n*} B₇⁻brane charge dens $Ity:$ of the D3- and D7-branes that give rise to our solution can be summarized in a standard • Axion sourced by D7-brane charge density:

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D7-branes do not reach AdS boundary: No new d.o.f. added. just like the *^N*^c D3-branes that give rise to *AdS*5×*S*5. We also emphasize that, unlike the case

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- 1 Isotropy broken by external source/extended objects.

• RG flow between AdS and Lifshitz.

Dual description *f*100 ⁴^π *.* (13) *n* 1 *i* *e*− 5</sub> *i e e e e e e e e e* The isotropic black D3-brane solution is a solution of the equations above with *a* = 0 and ^φ = 0 *, ^B* ⁼ *^H* = 1 *, ^F* = 1 [−] *^u*⁴ *, u*^H ⁼ ¹ ^π*^T .* (15)

^c *T*³ *.* (16)

*^s*⁰ ⁼ ^π²

between the two limiting behaviours.

The dashed blue line is a straight line with slope 1*/*3.

*N*²

 \overline{C} $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{2}$ • RG flow between Ads and Lifshitz. 3 Stress tensor ● RG flow between AdS and Lifshitz.

geometry in the UV and an Lifshitz-like geometry in the IR [34]. The radial position at which

 Γ at α procedure can be regularized by a procedure calcular called holographic renormalization. • Entropy density interpolates: *T* ≫ *a* : *s* ∼ *T*³ ⁴*^G* (9) ■ *Entropy der* cos ^θ*^c* ⁼ *^v*lim

T ≪ *a* : $s \sim a^{1/3} T^{8/3}$ ¹⁶π*^G* (10) (11) $T \ll a$: $\epsilon \sim a^{1/3}T^8$

Conformal Anomaly

• Counterterms required for holographic renormalization are:

$$
S_{\rm ct} = \text{diff invariant} + \log(\mu r) \int d^4x \sqrt{\gamma} \mathcal{A}
$$

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- Implies physics depends on *two* ratios a/μ and T/μ . ϵ Implies physics depends on *tave rating a (u and* $\frac{1}{2}$ implies physics depends on *two* ratios ω/μ and
- *The serves censor J* refers. *S*ct = diff invariant + log(*µr*) • Calculation of the stress tensor yields: anomaly, which when evaluated on our solution takes the **• Calculation of the stres**

T $\{T_x = P_y \neq P_z\}$ P*" P" P" P"* (6) P" (6) P $\partial^i \langle T_{ij} \rangle = 0 \, , \qquad \langle T^i_i \rangle =$ *a*₂, ⁴⁸π² *, µ* = arbitrary scale (3) *S*ct = diff invariant + log(*µr*) *N*² *a*4 ⁴⁸π² *, µ* = arbitrary scale (3) F the stress tensor is seen to be stress tensor is seen to be stress tensor is seen to be seen to be seen to be F $\langle I_{ij} \rangle = \text{diag}(E, \Gamma_x, \Gamma_y, \Gamma_z),$ $\langle T^i_i \rangle = \mathcal{A}$ $\langle T_{ij} \rangle = \text{diag}(E, P_x, P_y, P_z), \qquad P_x = P_y \neq P_z$

Unstable against infinitesimal charge fluctuations:

$$
\left(\frac{\partial^2 F}{\partial a^2}\right)_T<0
$$

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and no separation between hard and soft degrees of freedom (see e.g. [8] and references

• Superficial similarity with weak coupling instabilities:

Tendency of similarly oriented currents tend to cluster together. Figure 11: In weakly coupled plasmas parallel currents tend to cluster together.

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. Obvious differences: In weakly coupled plasmas anisotropy is dynamical. a homogeneous, static, anisotropic and unstable background (for certain values of (*a, T*)). allisotropy is dynamical. and that of weakly coupled plasmas. In the description of the latter one often considers

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- Conjectured to take place in QGP due to bulk viscosity. \mathcal{L} ^{\mathcal{L}}
- $\frac{1}{2}$ $\frac{1}{2}$ • Again obvious differences: In flowing liquid $P < P_{\text{vapor}}$ arises from dynamical contributions.