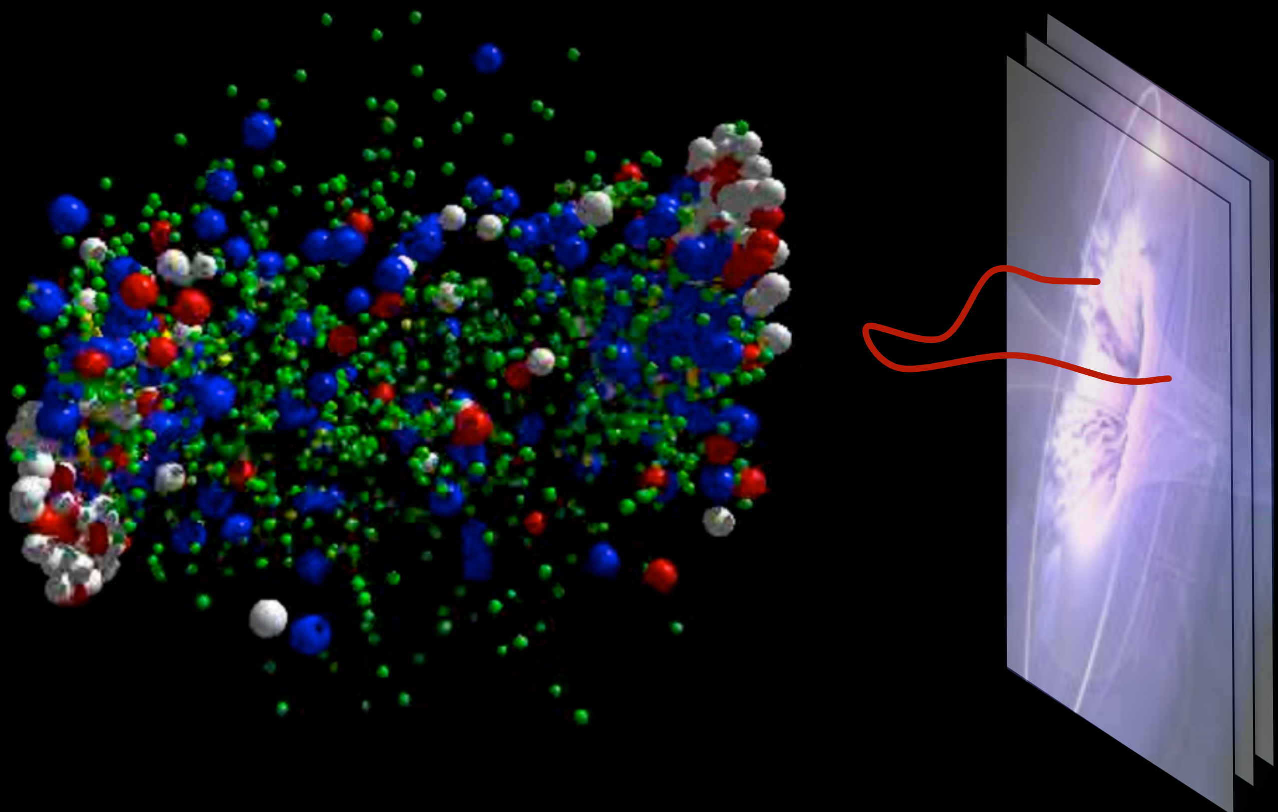
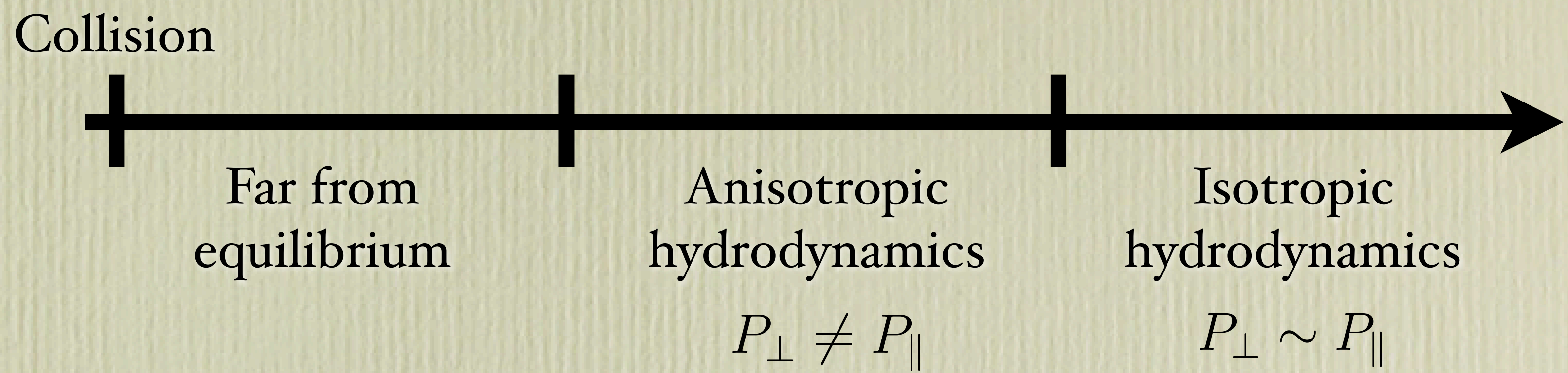
Thermodynamics and Instabilities of a Strongly Coupled Anisotropic Plasma



David Mateos
ICREA & University of Barcelona

Motivation



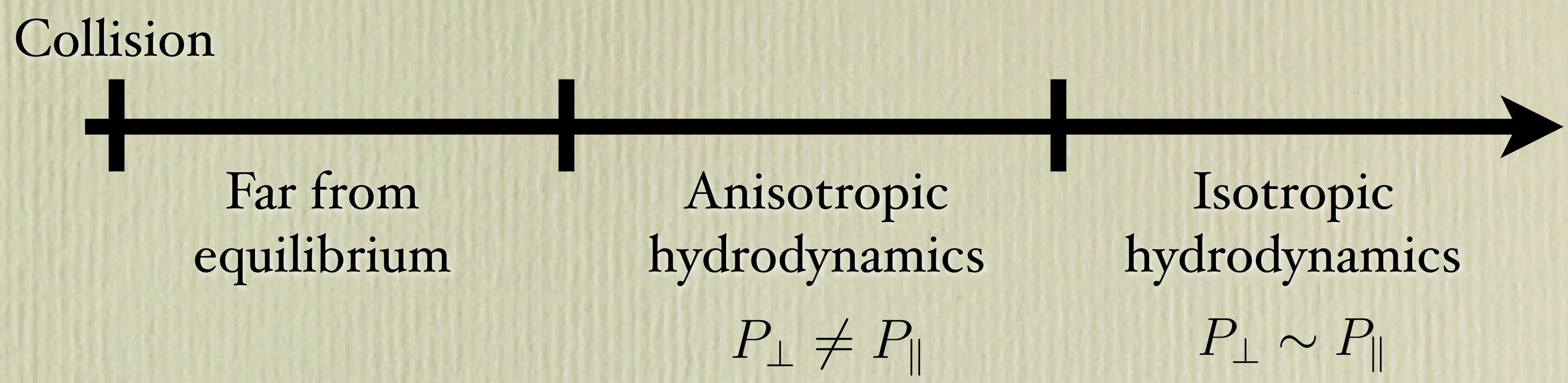
- Anisotropic hydrodynamics.

Florkowski & Ryblewski '11
Martinez & Strickland, ' 11

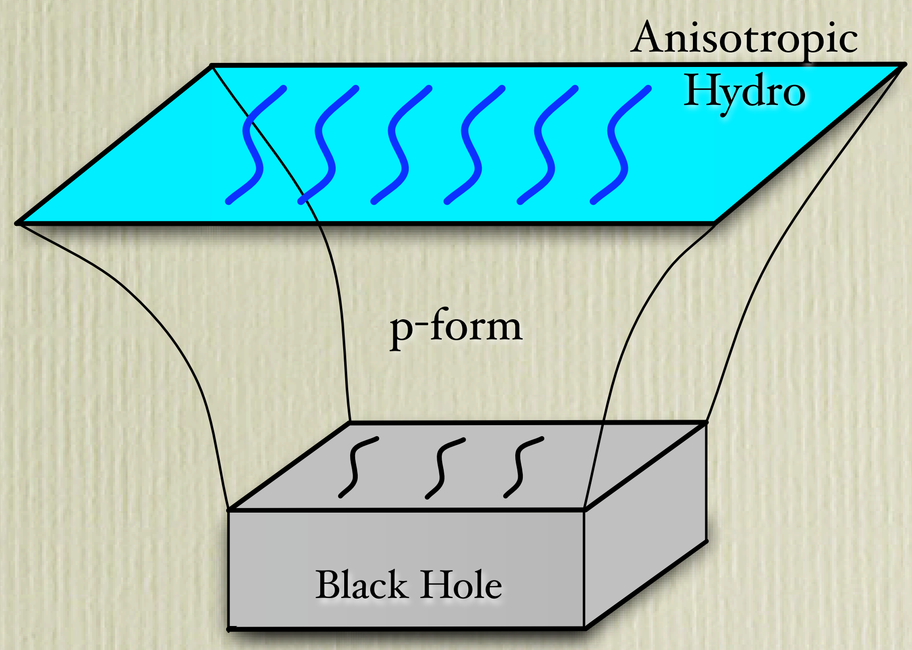
- Weak coupling instabilities.

Weibel '59
Mrowczynski, '88
Randrup & Mrowczynski '03
Romatschke & Strickland '03
Arnold, Lenaghan & Moore ' 03
Arnold, Lenaghan, Moore & Yaffe ' 05

Motivation



- Fluid/gravity correspondence with conserved p-form.



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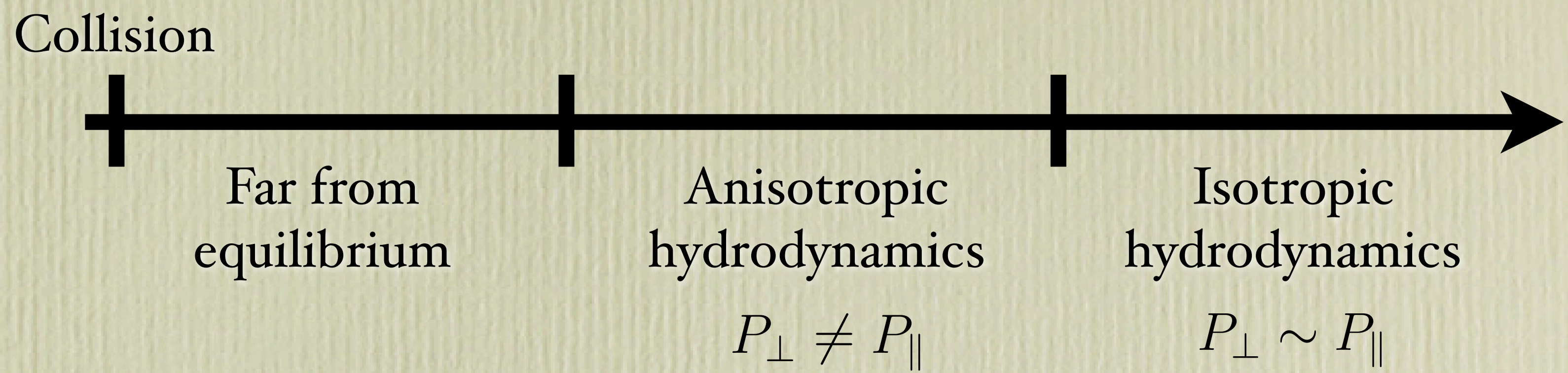
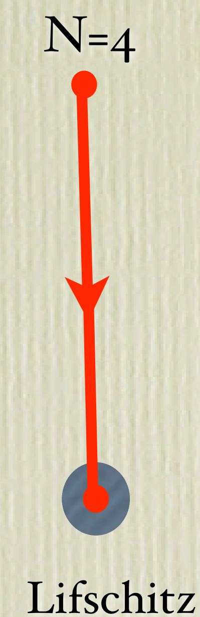


- Fluid/gravity correspondence with conserved p-form.
- Condensed matter applications:
  UV completion of Lifshitz fixed points.

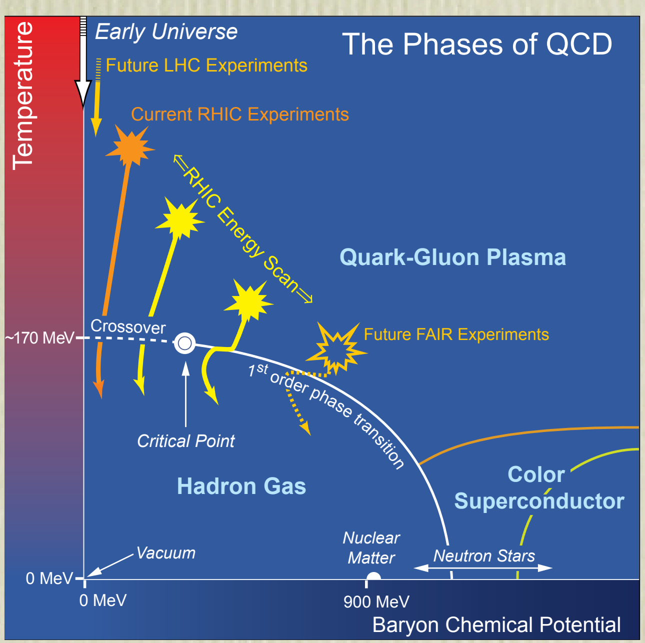


Motivation

Collision

Far from equilibrium | Anisotropic hydrodynamics $P_\perp \neq P_\parallel$ | Isotropic hydrodynamics $P_\perp \sim P_\parallel$

- Fluid/gravity correspondence with conserved p-form.
- Condensed matter applications.
- Connection with QCD at finite density.
- Connection with phenomenon of cavitation.



Solution

- Type IIB supergravity solution with following properties:
  - Is static and anisotropic.
  - Has a smooth horizon.
  - Obeys $AdS_5 \times S_5$ boundary conditions.

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    Finite T

    Calculations unambiguous and well defined.

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  - Is static and anisotropic.

    → Simplicity: Thermodynamics, etc.

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    → Deformation of N=4 SYM
    Solid embedding in string theory

Solution

Inspiration from:
Azeyanagi, Li & Takayanagi '09

- Turn on a position-dependent θ-term:

$$S_{\rm gauge} = S_{\mathcal{N}=4} + \delta S\,, \qquad \delta S = \frac{1}{8\pi^2}\int \theta(x)\,\mathrm{Tr} F\wedge F\,, \qquad \theta(x) = 2\pi n_{\rm D7} z$$

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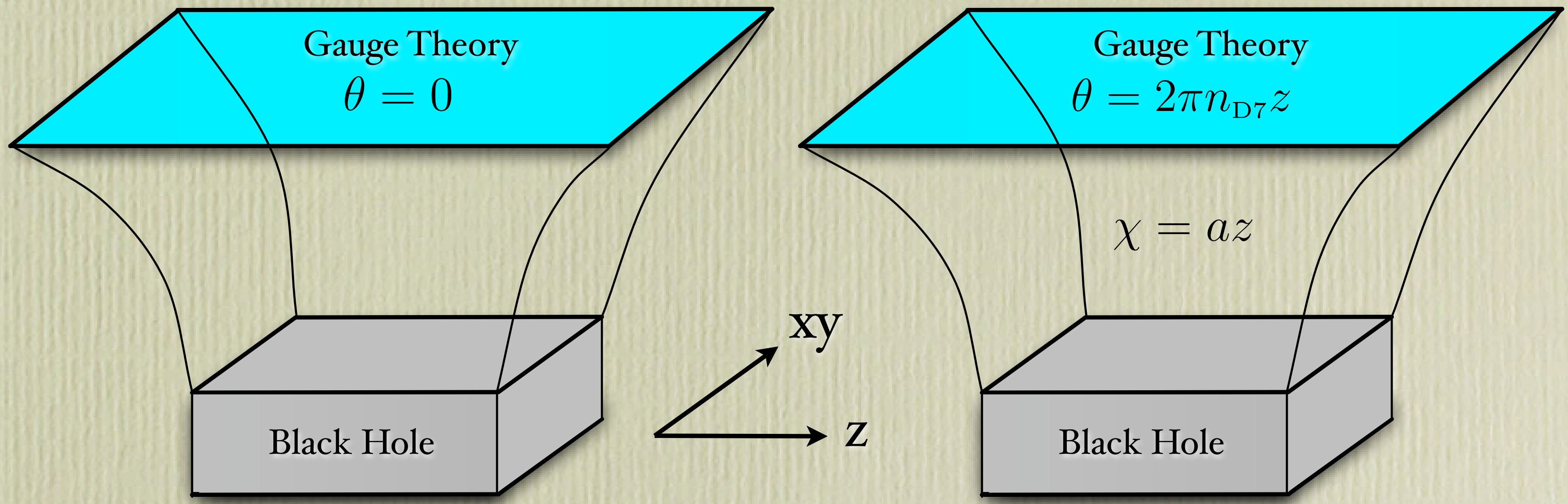
- Translation-invariance is preserved (at least locally):

$$\delta S \propto -\int dz \wedge {\rm Tr}\left(A\wedge F + \frac{2}{3}A^3\right)$$

Dual description



$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\rm YM}^2} = \chi + ie^{-\phi}$$

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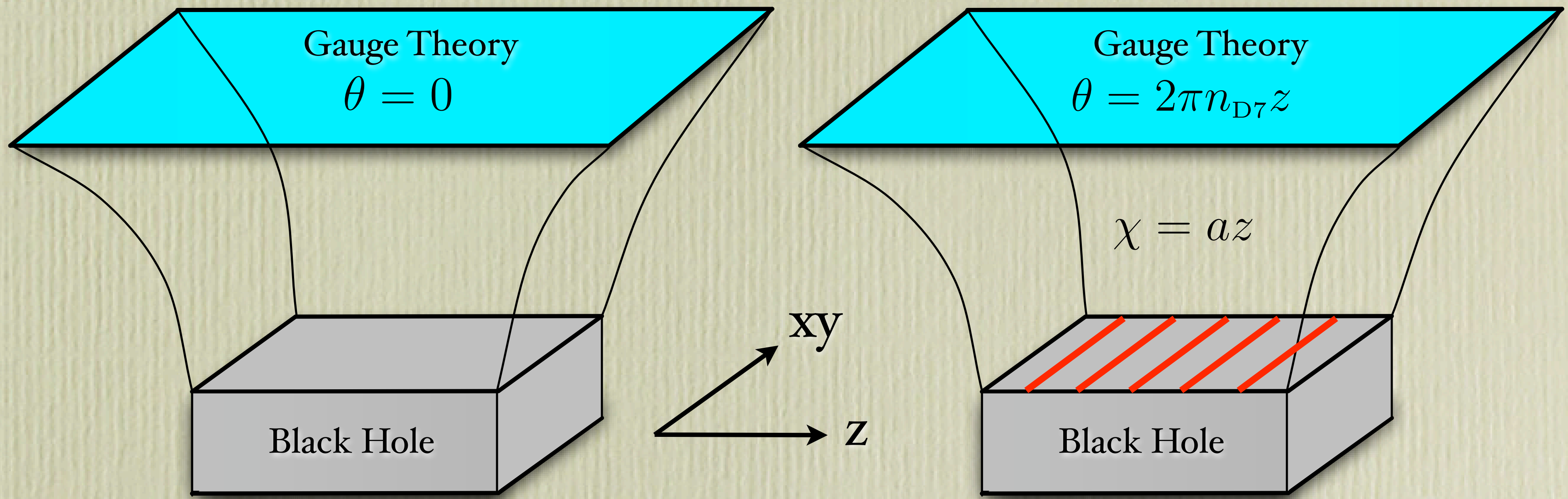

- Axion sourced by D7-brane charge density:

		t	x	y	z	u	S^5
N_{c}	D3	$\times$	$\times$	$\times$	$\times$		
n_{D7}	D7	$\times$	$\times$	$\times$			$\times$

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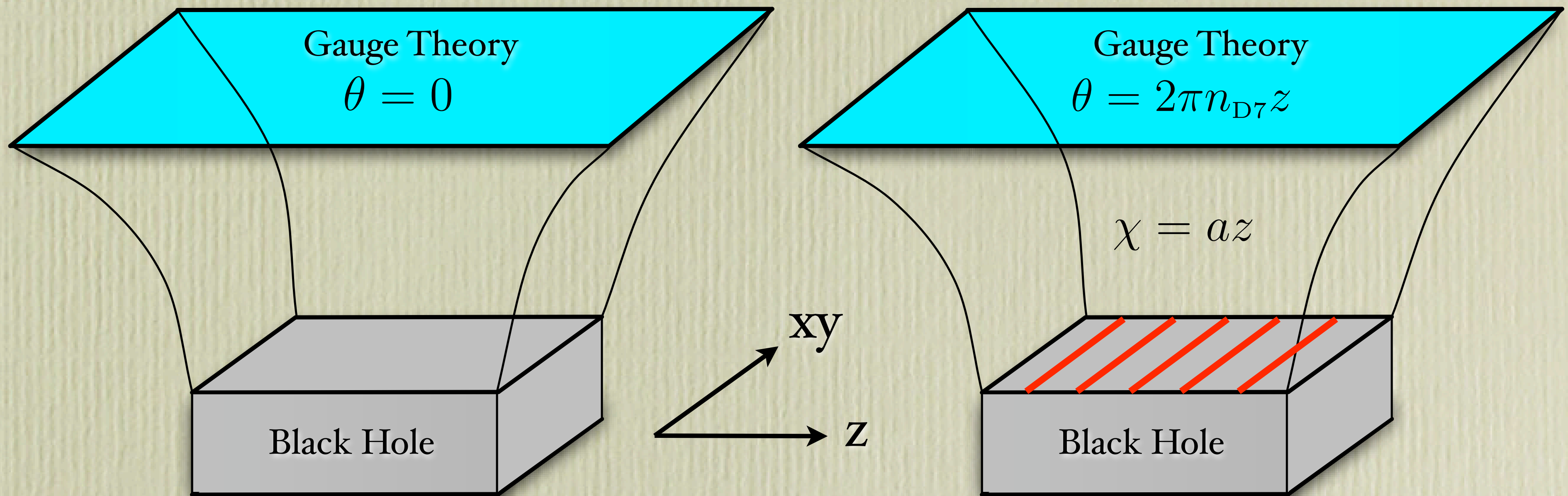

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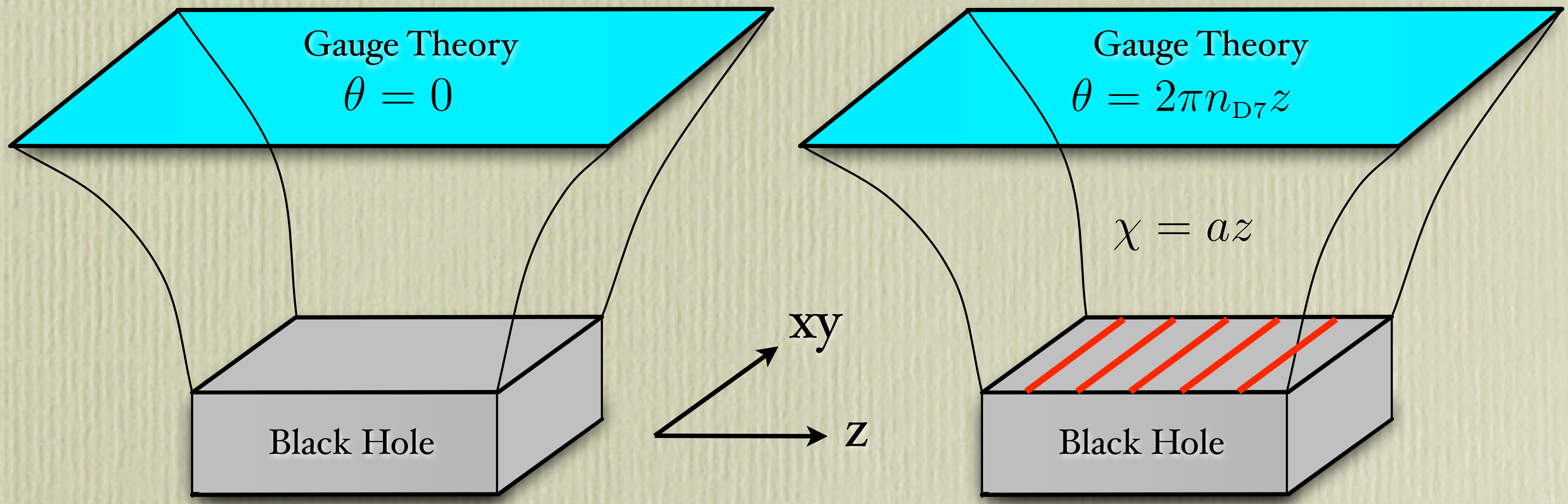
- D7-branes do not reach AdS boundary: No new d.o.f. added.

Dual description



- D7-branes dissolved in geometry (backreaction incorporated).

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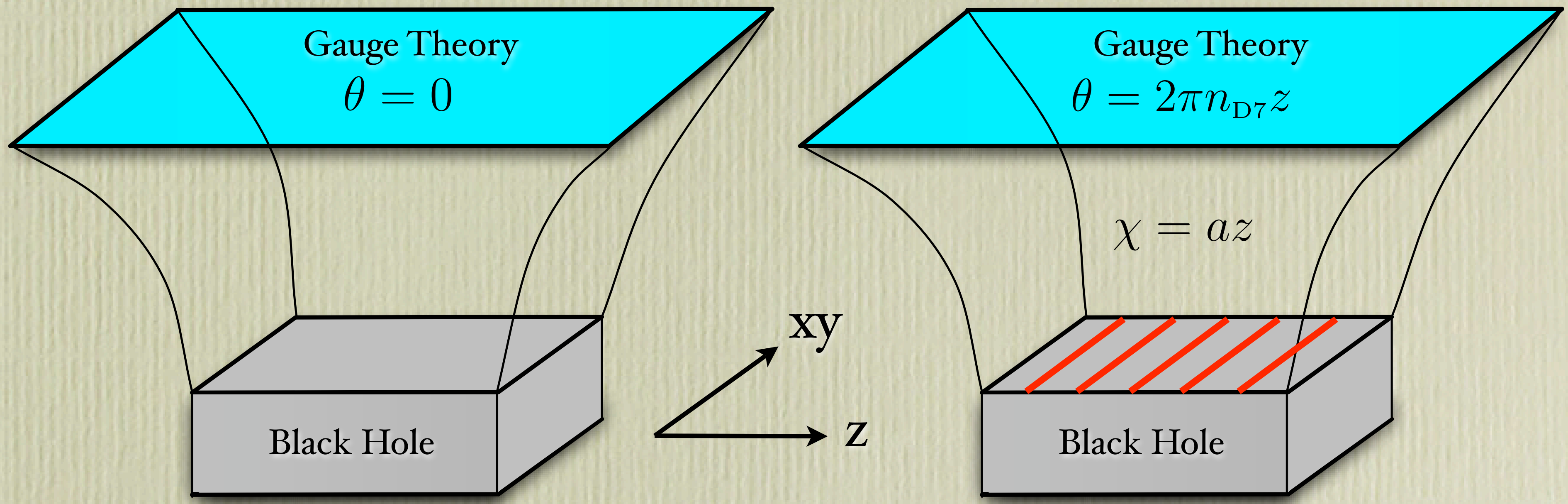
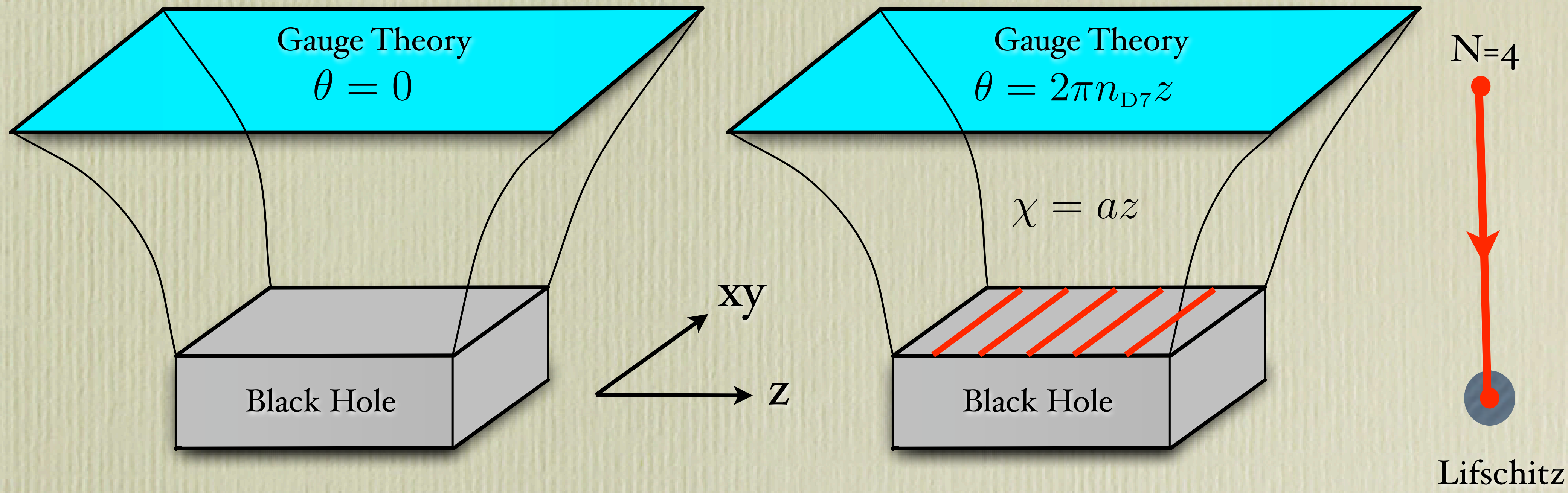


- D7-branes dissolved in geometry (backreaction incorporated).
- Isotropy broken by external source/extended objects.

Dual description



- RG flow between AdS and Lifshitz.

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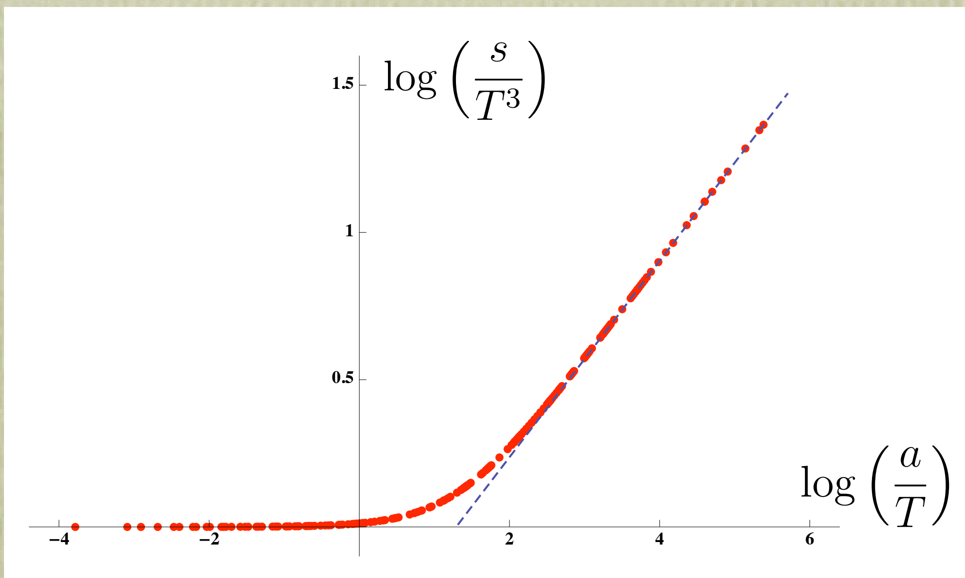

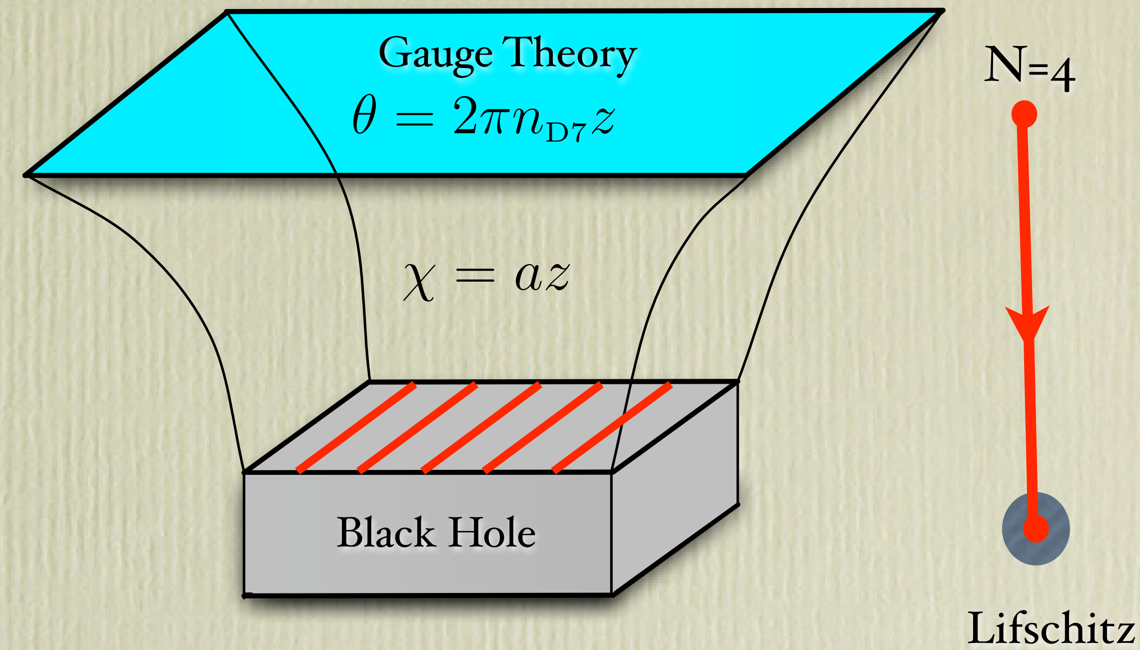

- RG flow between AdS and Lifshitz.

- Entropy density interpolates:

$$T \gg a : \qquad s \sim T^3$$

$$T \ll a : \qquad s \sim a^{1/3} T^{8/3}$$

Conformal Anomaly

- Counterterms required for holographic renormalization are:

$$
\begin{aligned}
S_{\mathrm{ct}} &= \text{diff invariant} + \log(\mu r) \int d^4x \sqrt{\gamma}\, \mathcal{A} \\
\mathcal{A} &= \frac{N_{\mathrm{c}}^2 a^4}{48\pi^2}\,, \qquad \mu = \text{arbitrary scale}
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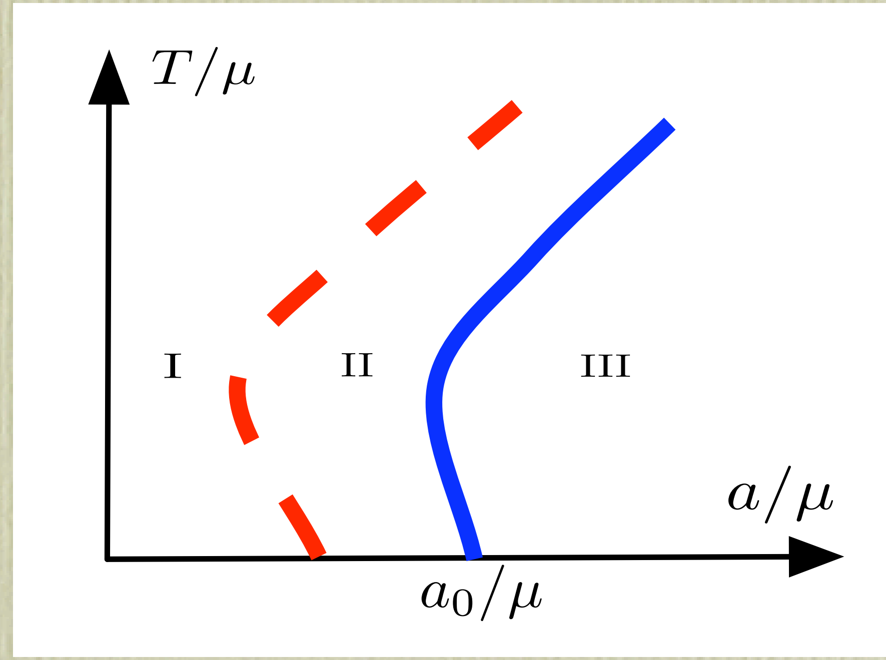
- Calculation of the stress tensor yields:

$$
\langle T_{ij} \rangle = \mathrm{diag}(E, P_x, P_y, P_z)\,, \qquad P_x = P_y \neq P_z
$$

$$
\partial^i \langle T_{ij} \rangle = 0\,, \qquad \langle T^i_i \rangle = \mathcal{A}
$$

Phase diagram



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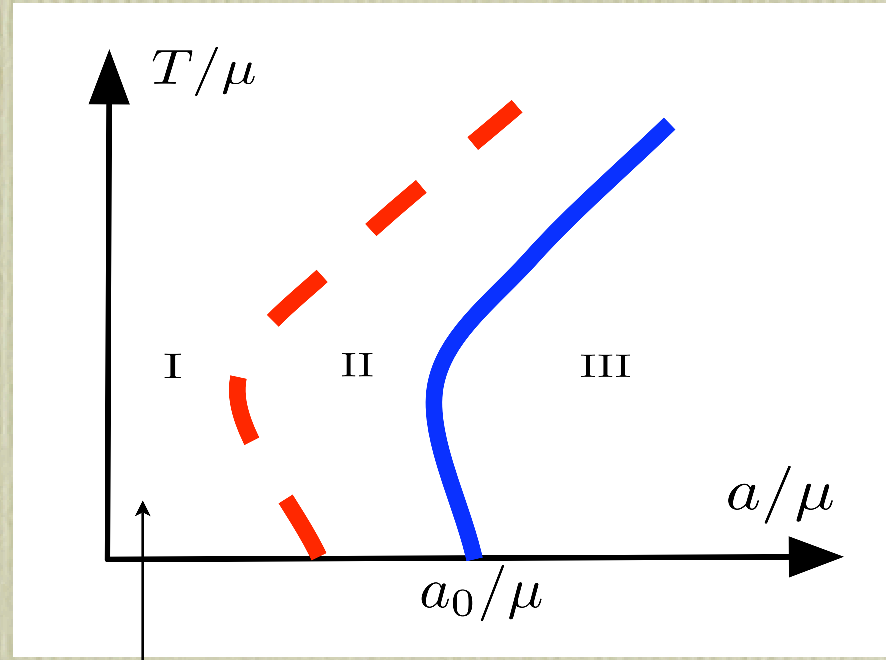
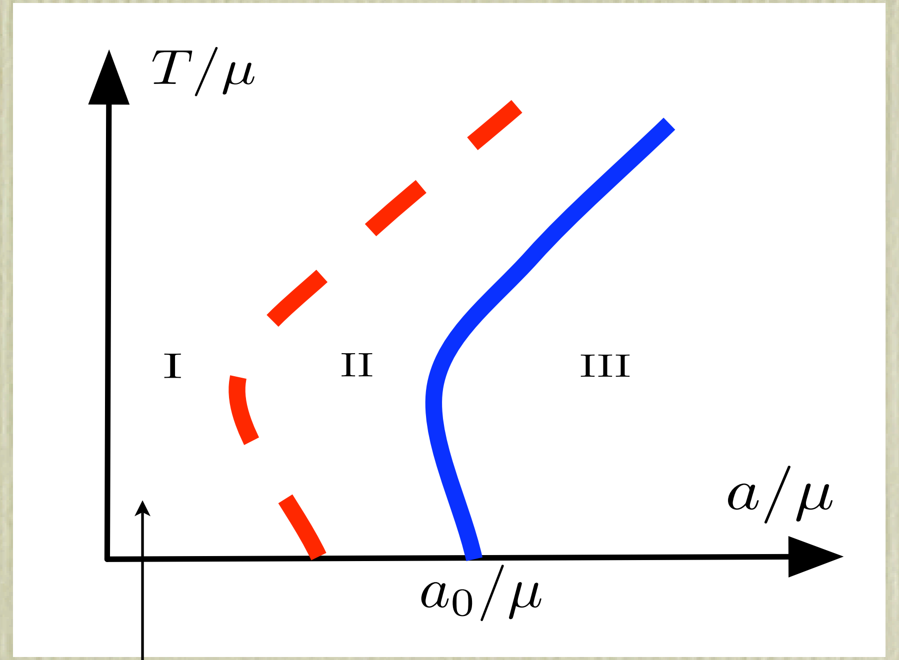


Unstable against infinitesimal charge fluctuations:

$$\left(\frac{\partial^2 F}{\partial a^2}\right)_T < 0$$

Phase diagram



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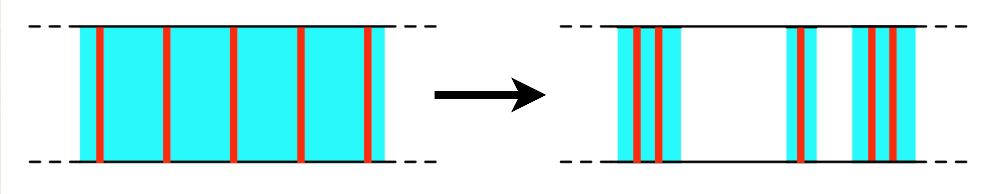
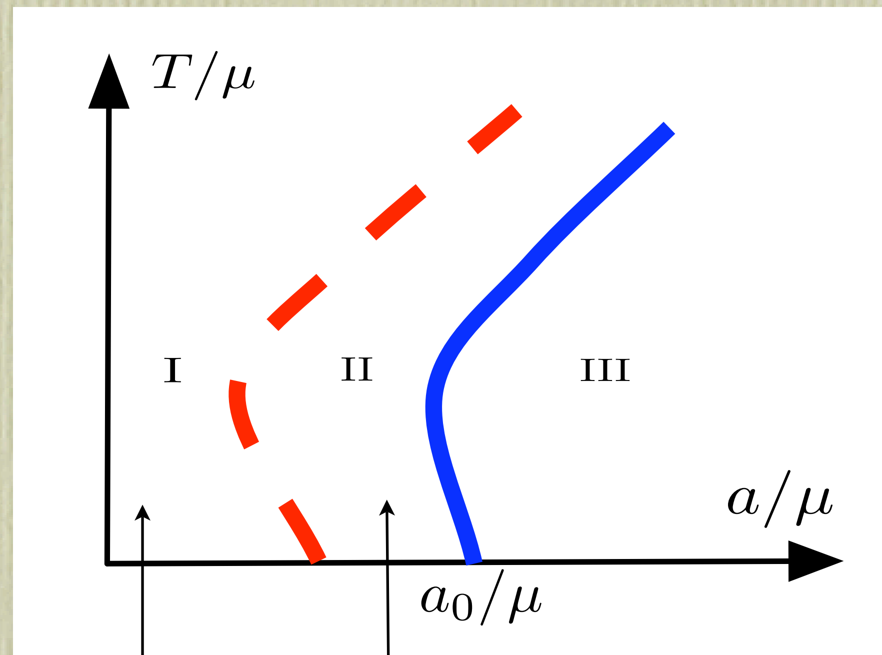


Homogeneous Mixed

Phase diagram



Metastable: Unstable against finite fluctuations: $P_z < P_{\rm iso}$

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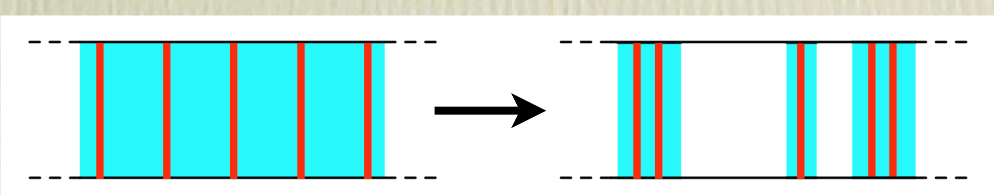

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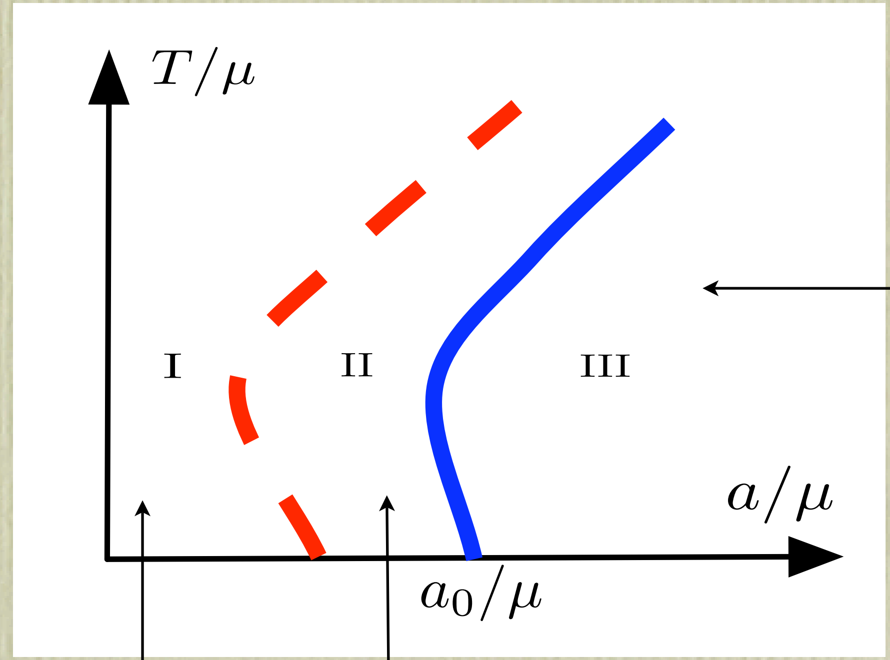

Stable

Metastable: Unstable against finite fluctuations: $P_z < P_{\rm iso}$

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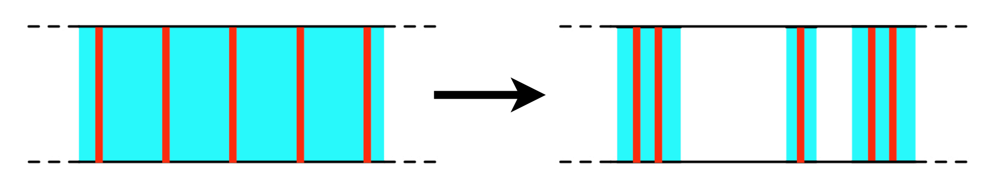
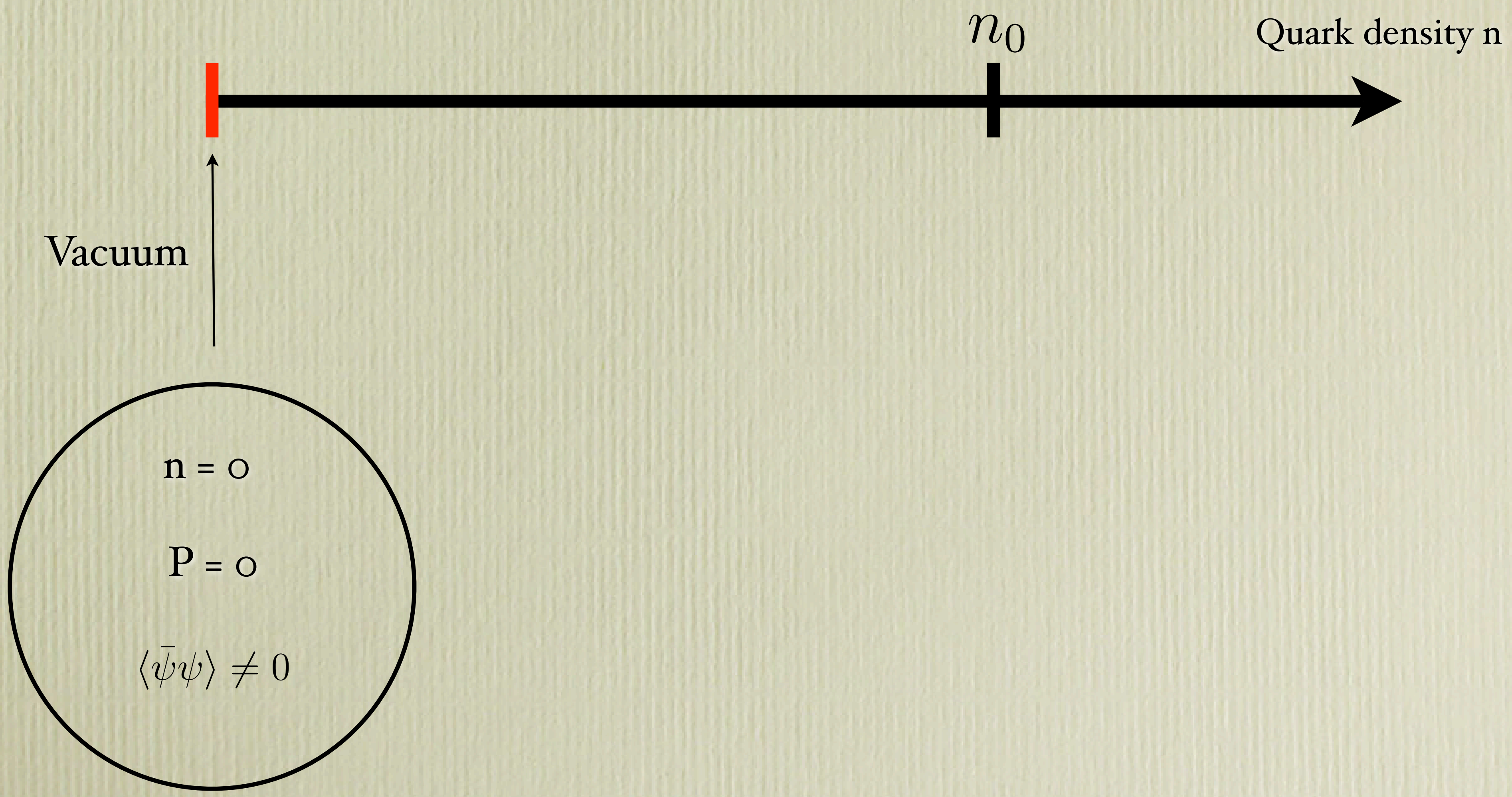


Homogeneous

Mixed

Discussion

- Similarity with QCD at finite density and low T:



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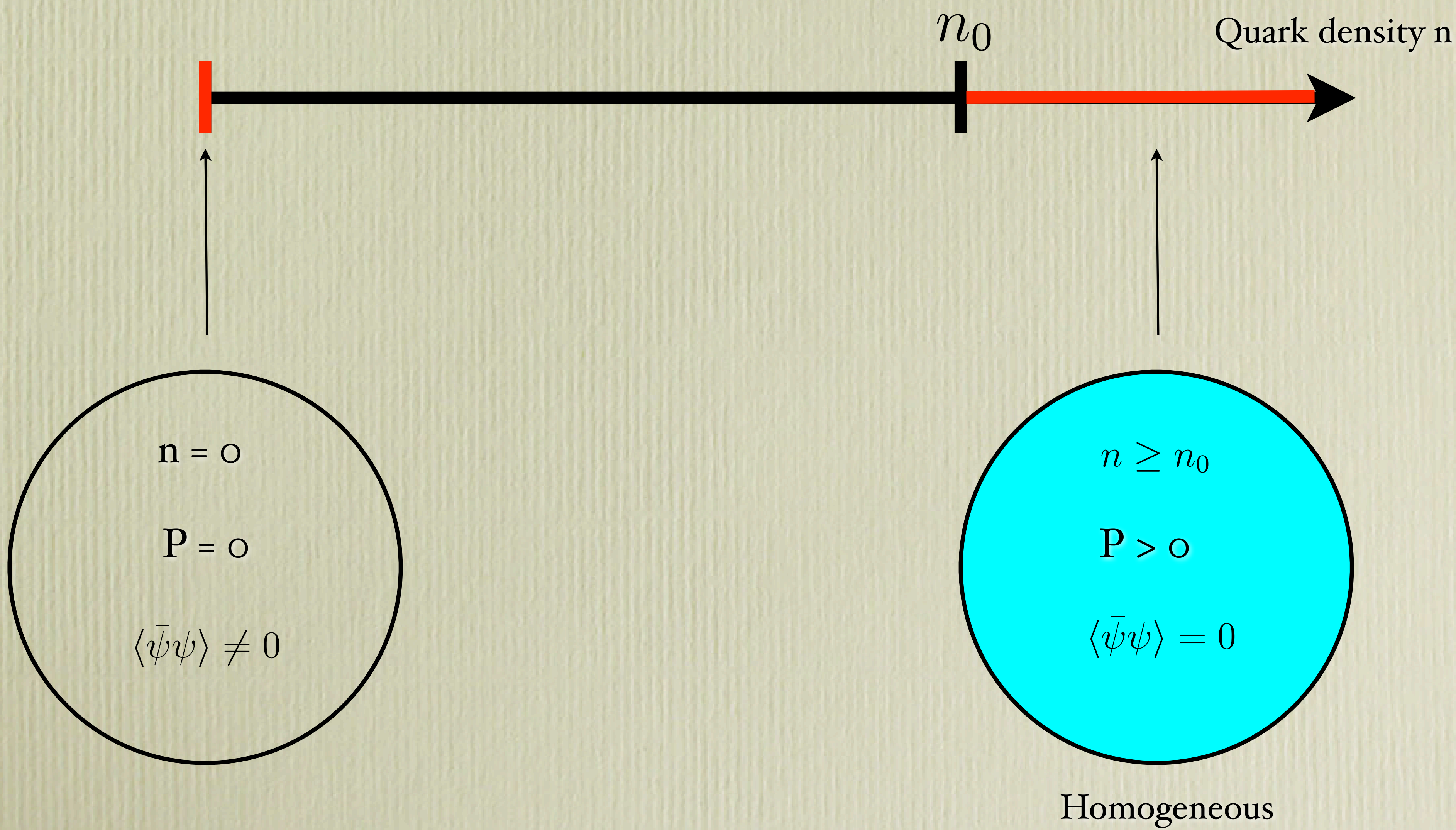

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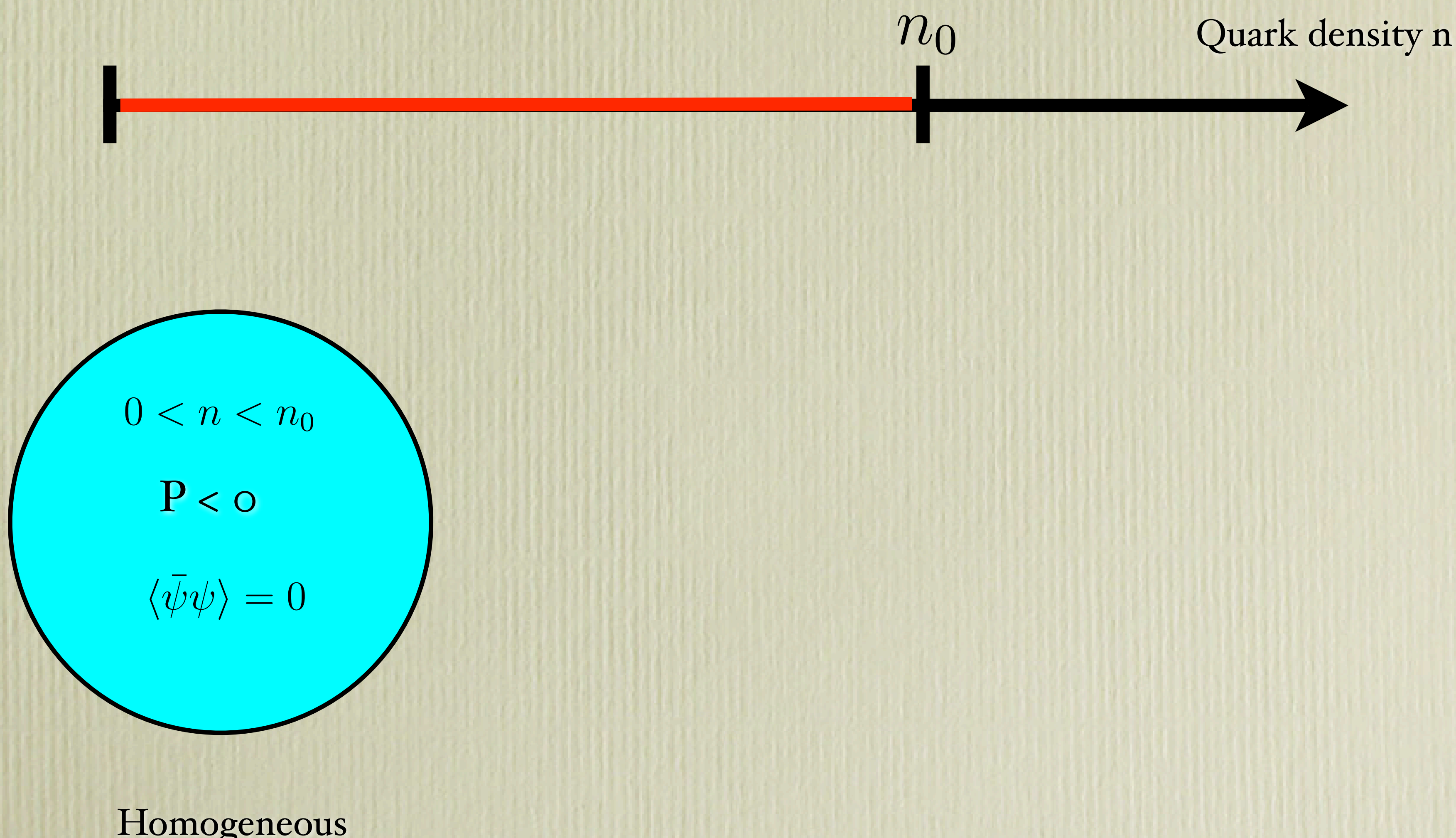

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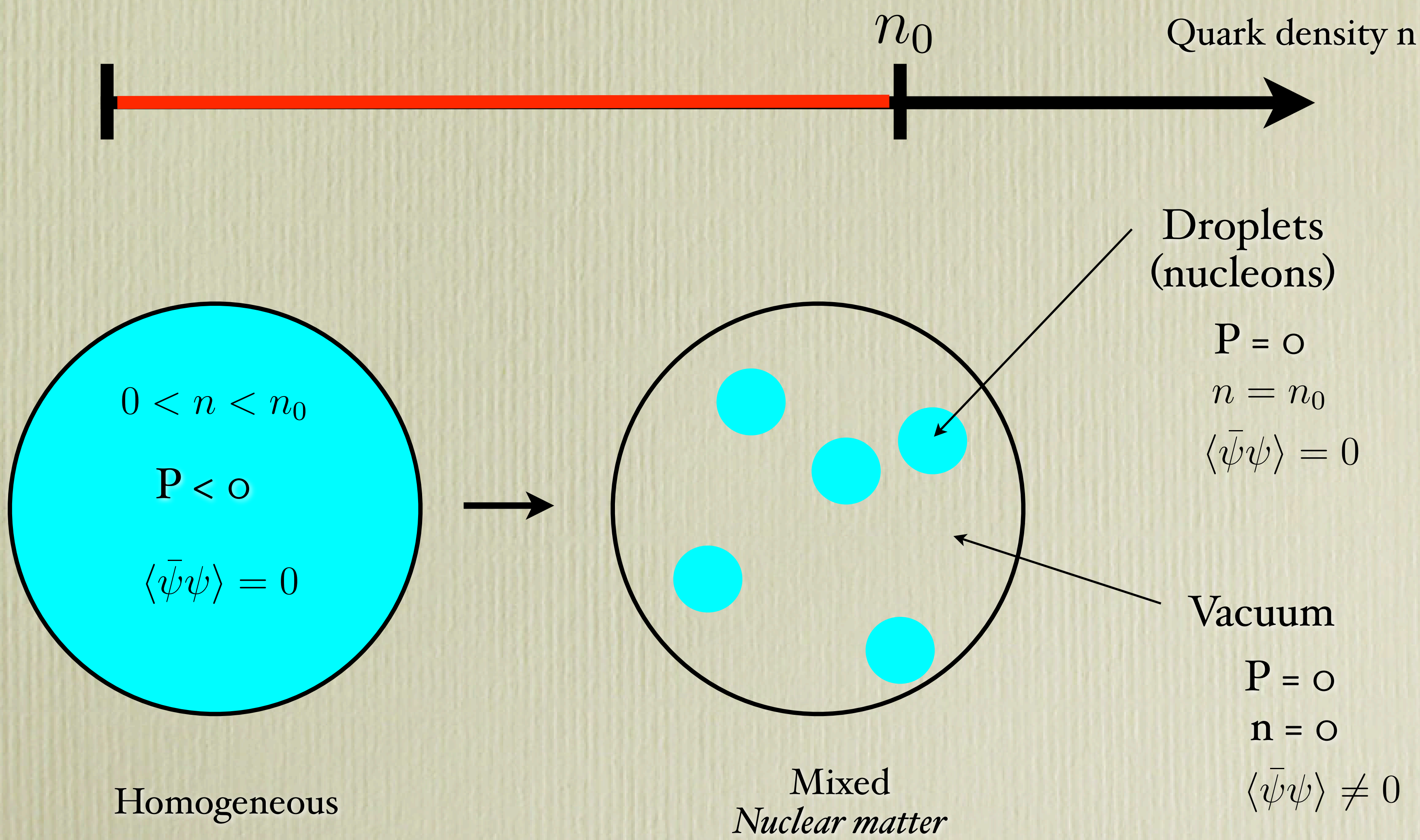
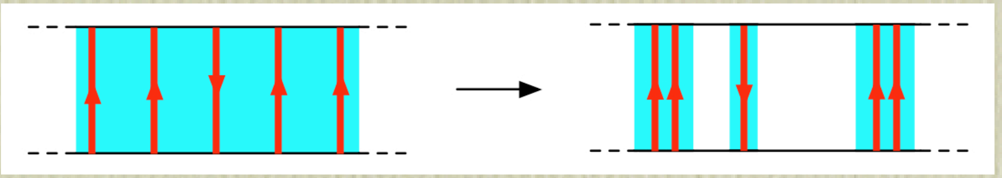


Discussion

- Superficial similarity with weak coupling instabilities:



Tendency of similarly oriented currents tend to cluster together.

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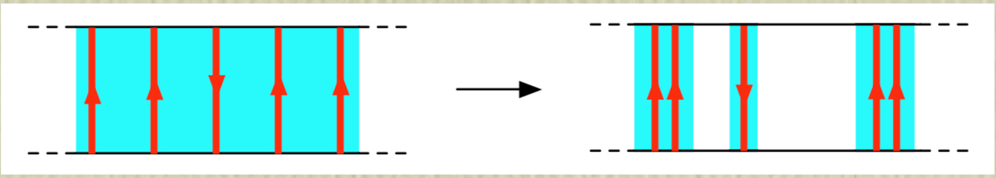
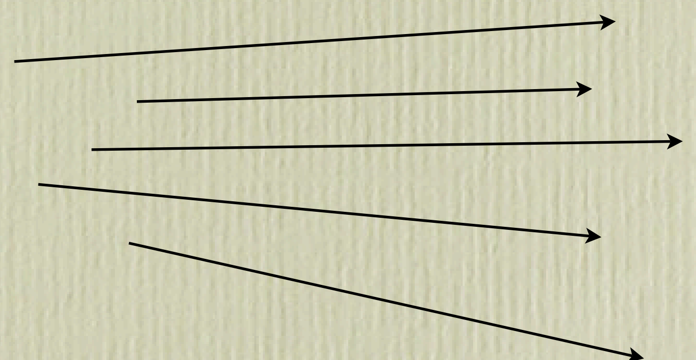


Tendency of similarly oriented currents tend to cluster together.

- Obvious differences: In weakly coupled plasmas anisotropy is dynamical.

Discussion

- Similarity with cavitation in flowing liquids:



Viscosity contributions
can make $P < P_{\mathrm{vapor}}$

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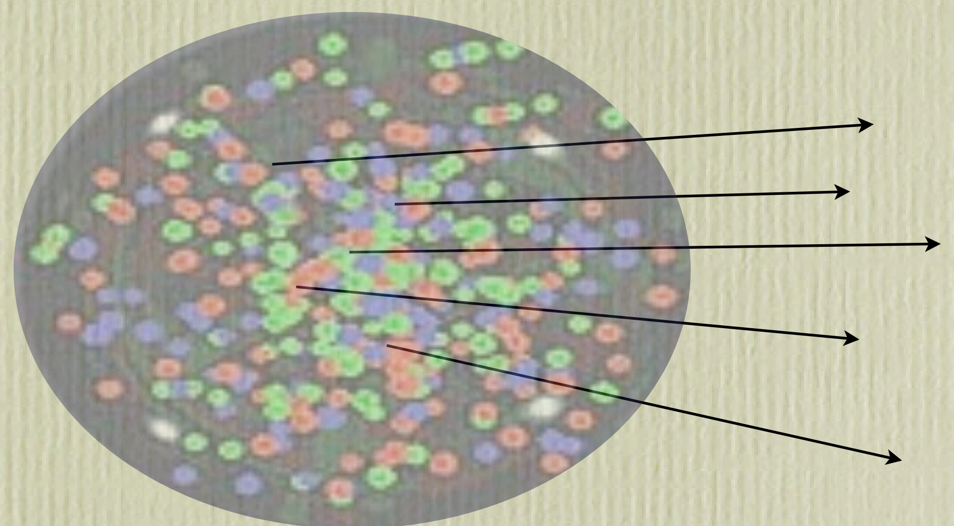

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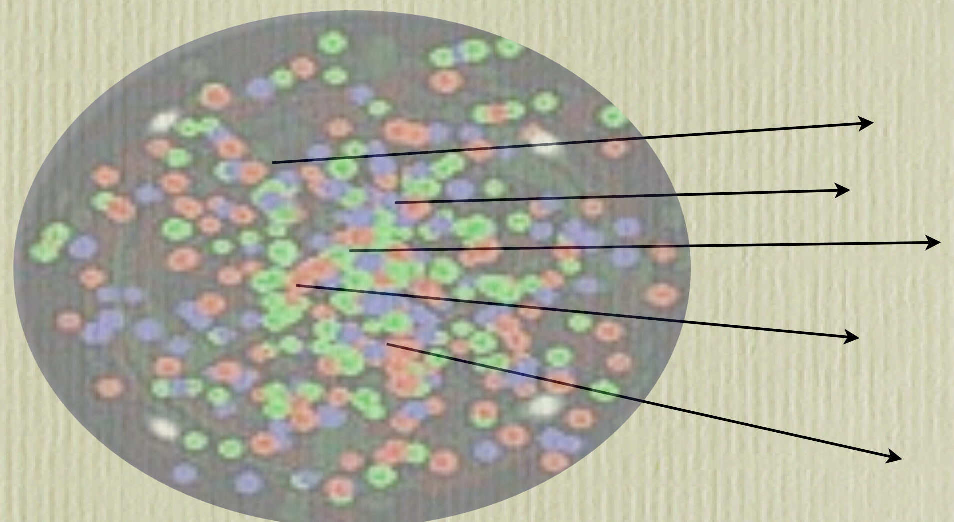

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- Conjectured to take place in QGP due to bulk viscosity.

- Again obvious differences: In flowing liquid $P < P_{\text{vapor}}$ arises from dynamical contributions.