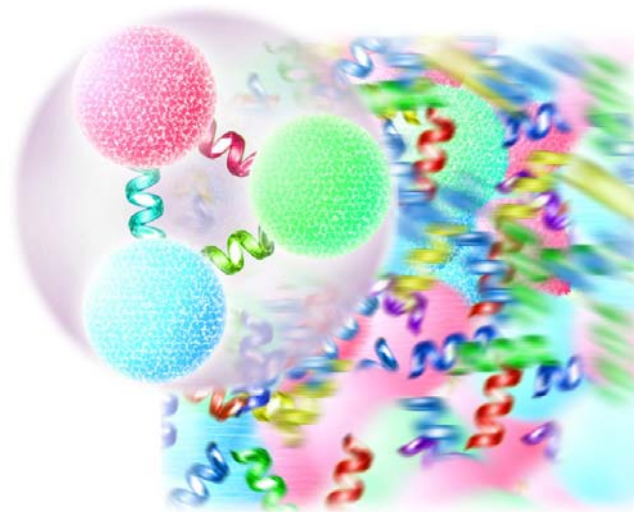


Talk @ INT Program “Frontiers in QCD”

Seattle, NOV.3<sup>rd</sup>, 2011

# Fermion Fluctuations & Correlations in sQGP: Quasi-Particles, or Bound-States, or Holography?



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# OUTLINE

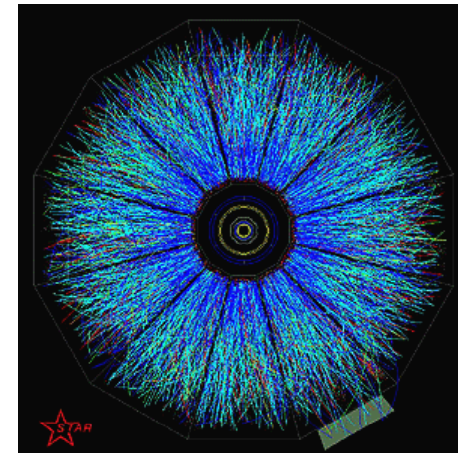
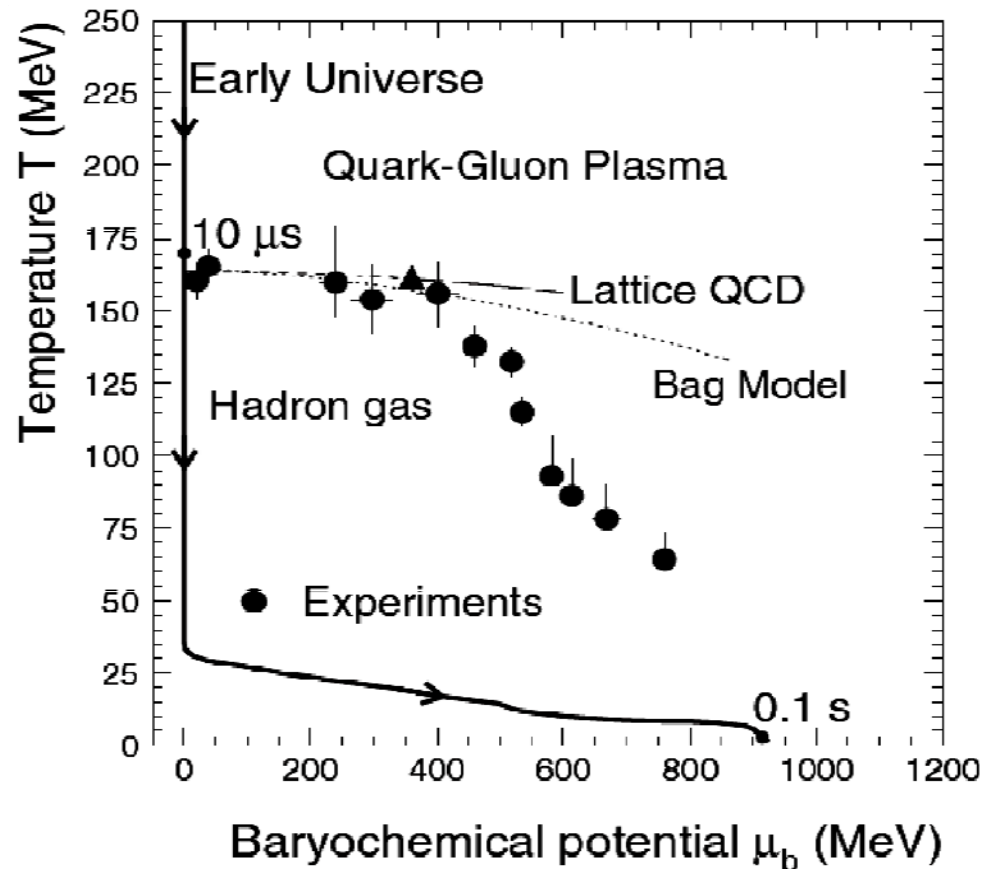
- Fermion fluctuations & correlations: introduction
- Susceptibilities in QCD: quasi-particles? bound states?
- Strongly interacting quarks: results from holography
- Discussions & Summary

References:

*JL & Shuryak, *Phy. Rev. D* 73(2006)014509;*

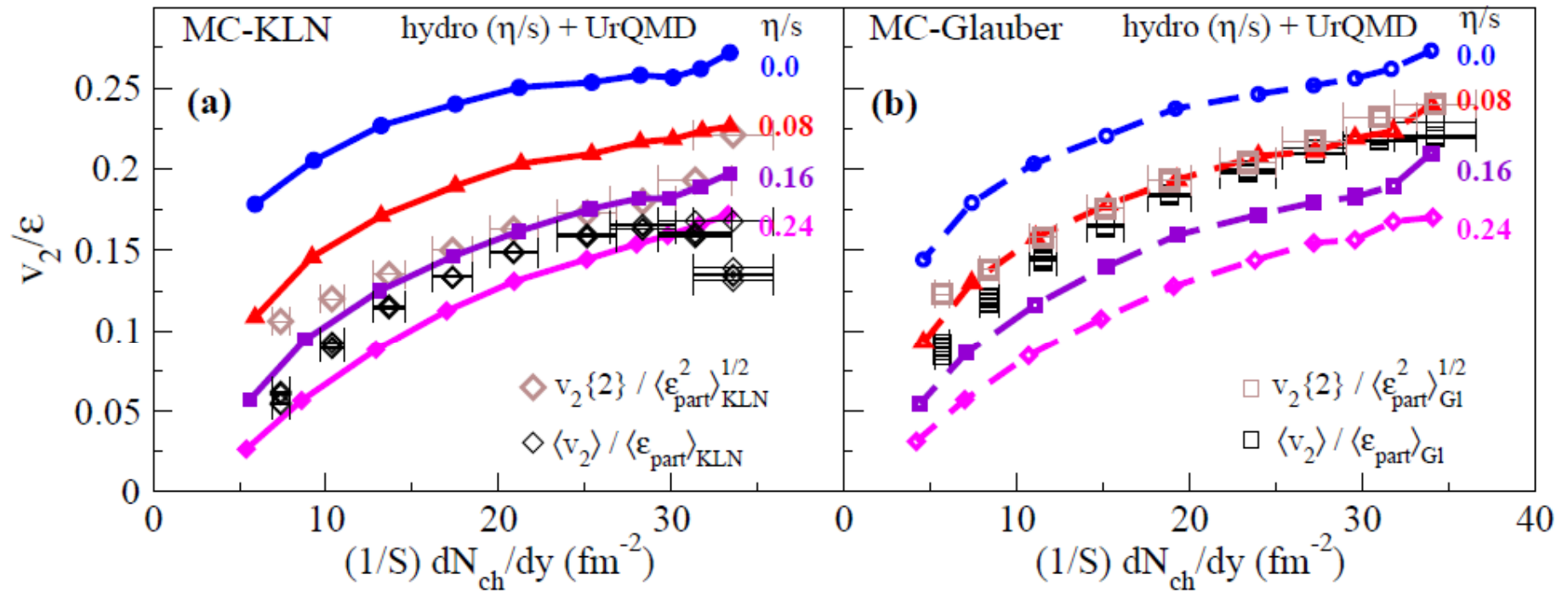
*Kim & JL, *Nucl. Phys. B* 822(2009)201.*

# QUEST FOR THE PHASES OF QCD



Remarkable progresses in the last 10 years:  
RHIC experiments; lattice QCD.  
Now LHC Era

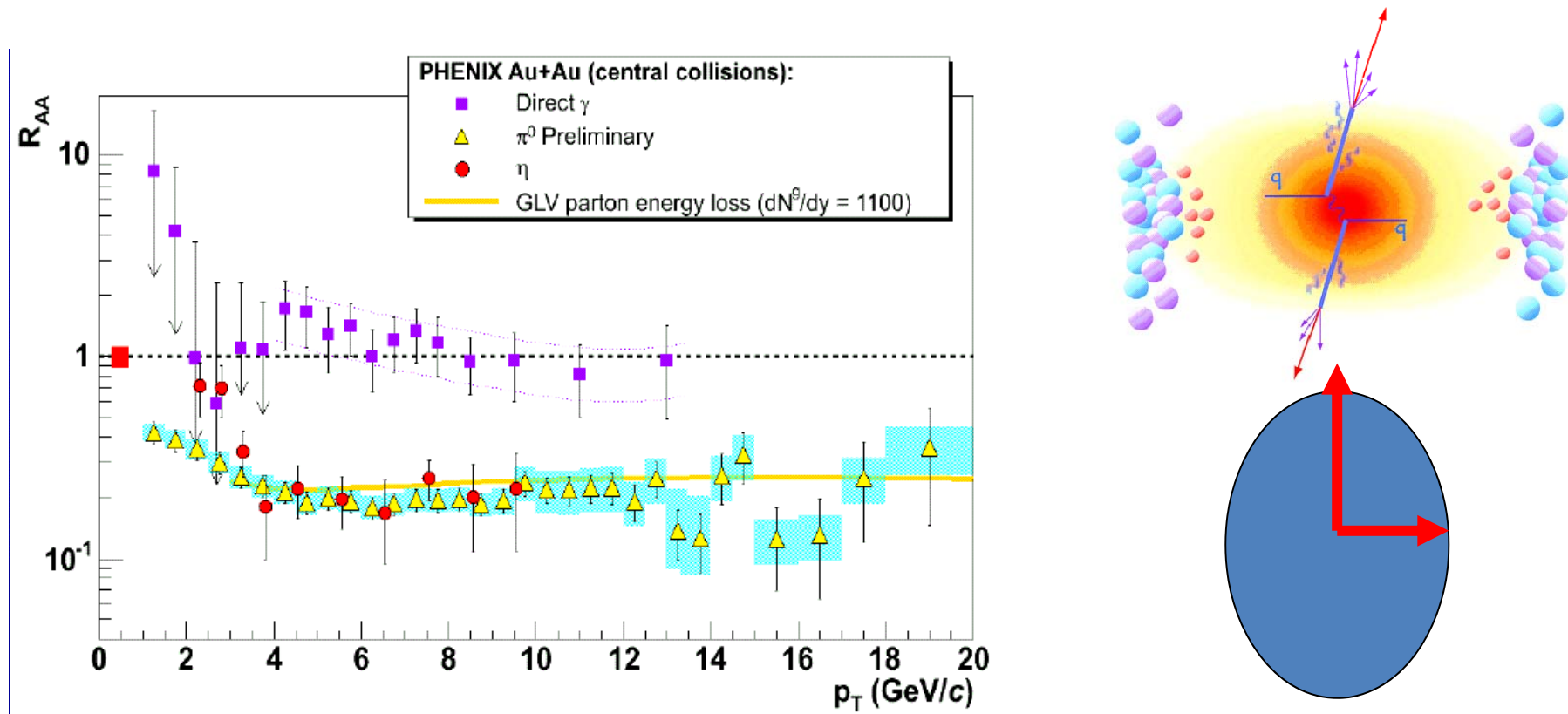
# RHIC: NEARLY PERFECT FLUID



*Song, Bass, Heinz, Hirano, Shen, 2010*

**Created matter's explosion appears nearly ideal  $\rightarrow$  strongly coupled**

# RHIC: STRONG JET QUENCHING



**Jet-medium interaction is very strong  $\rightarrow$  color-opaque matter!**

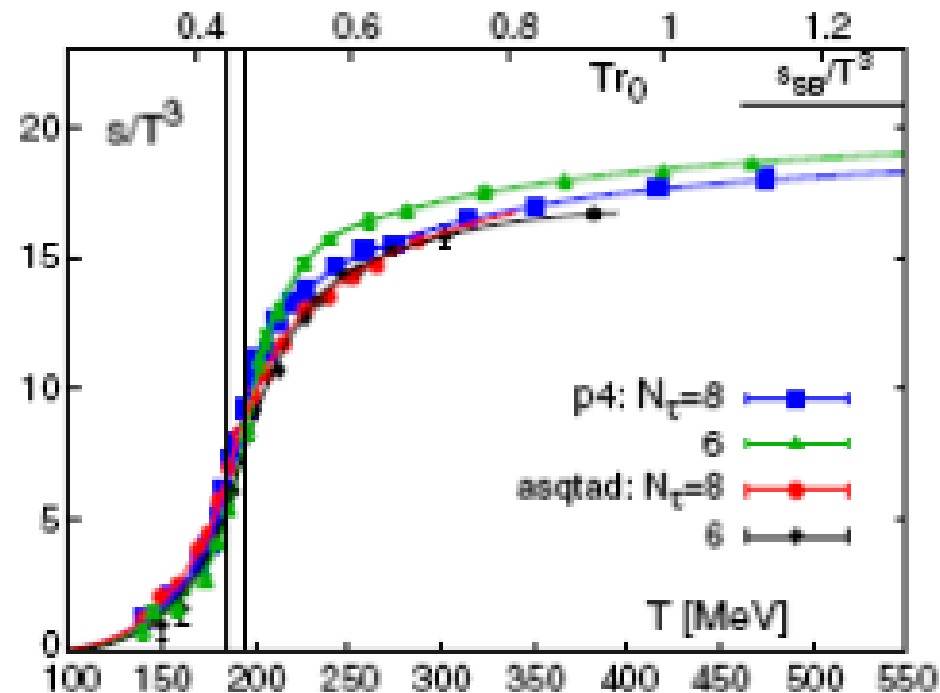
Physics beyond pQCD needed for full account of data

# SQGP: HOW STRONGLY COUPLED?

Coupling scale a la thermodynamics e.g. entropy

Infinity  
( $3/4 * SB$ )

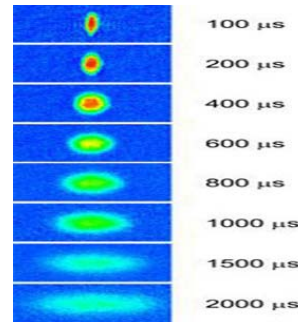
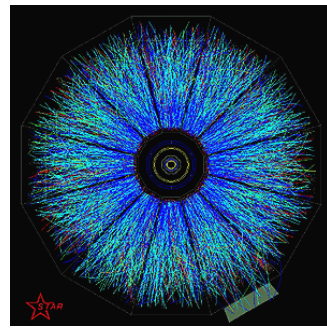
0  
(Stefan-Boltzmann)



# sQGP: HOW STRONGLY COUPLED?

Coupling scale a la transport e.g. shear viscosity

Infinity ————— 0  
 (eta/s=1/4pi) (infinately viscous)  
 (quantum limit?)



$$\frac{\eta}{s} [\text{AdS BH}] \approx \frac{\eta}{s} [\text{sQGP, cold atom}] \ll \frac{\eta}{s} [\text{water}]$$

Caveat: eta/s may NOT be a good fluidity measure everywhere !  
 See: JL & Koch, PRC,10

# FERMIONS IN SQGP

Strongly coupled quark-gluon plasma:

*What would be the pattern for fluctuations & correlations of fermions that carry conserved charges ?*

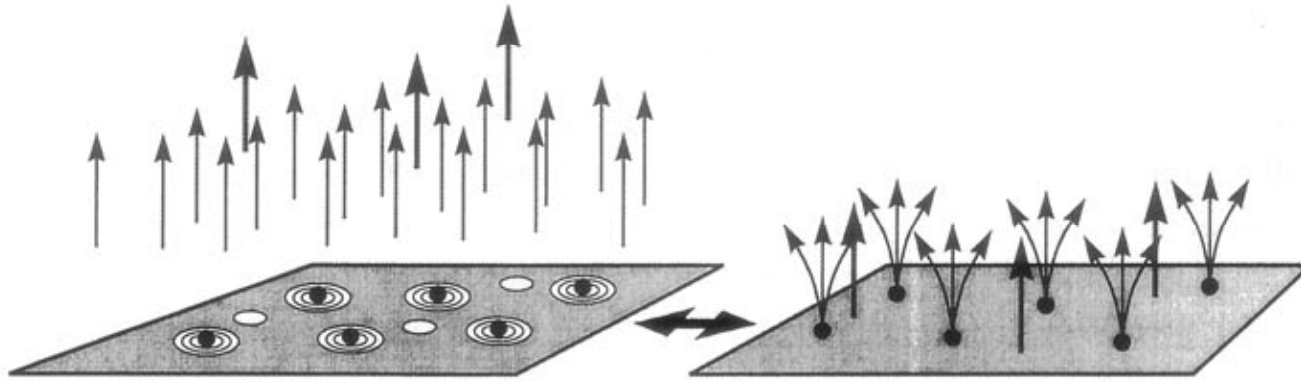
Study of these fluctuations & correlations are important:

--- it may tell **new degrees of freedom**: new unit of conserved charge fluctuations,  
i.e. is the system partonic or hadronic?

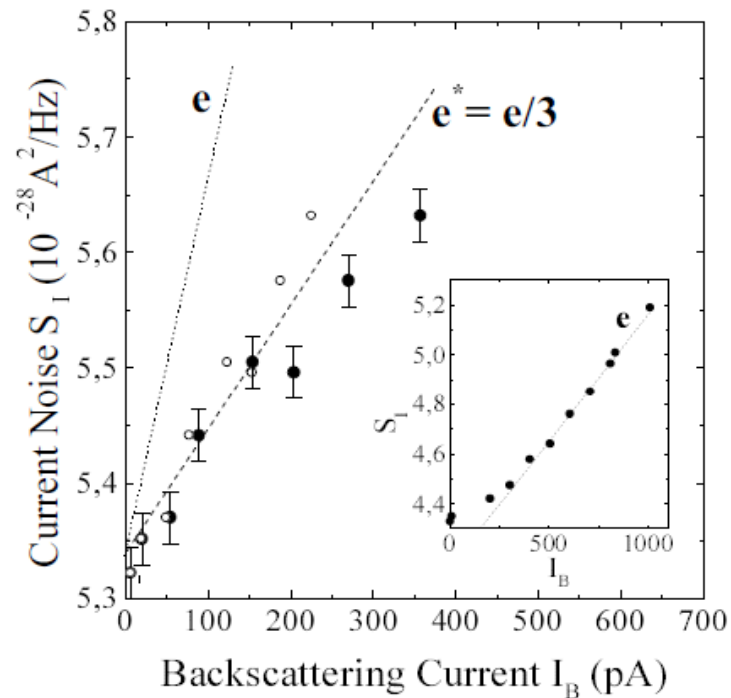
--- it may indicate specific phenomenon like a **critical point**



# IN SEARCH OF FRACTIONAL CHARGE



**Fractional Quantum Hall State: quasi-particle with charge (1/3) of electron's**



**New state of matter**

→→

**New degrees of freedom  
& conserved charge carrier**

→→

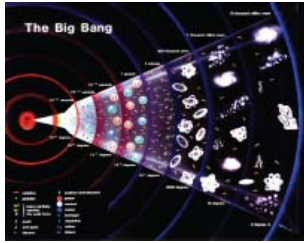
**New basic UNIT for transmission  
of conserved charge**

→→

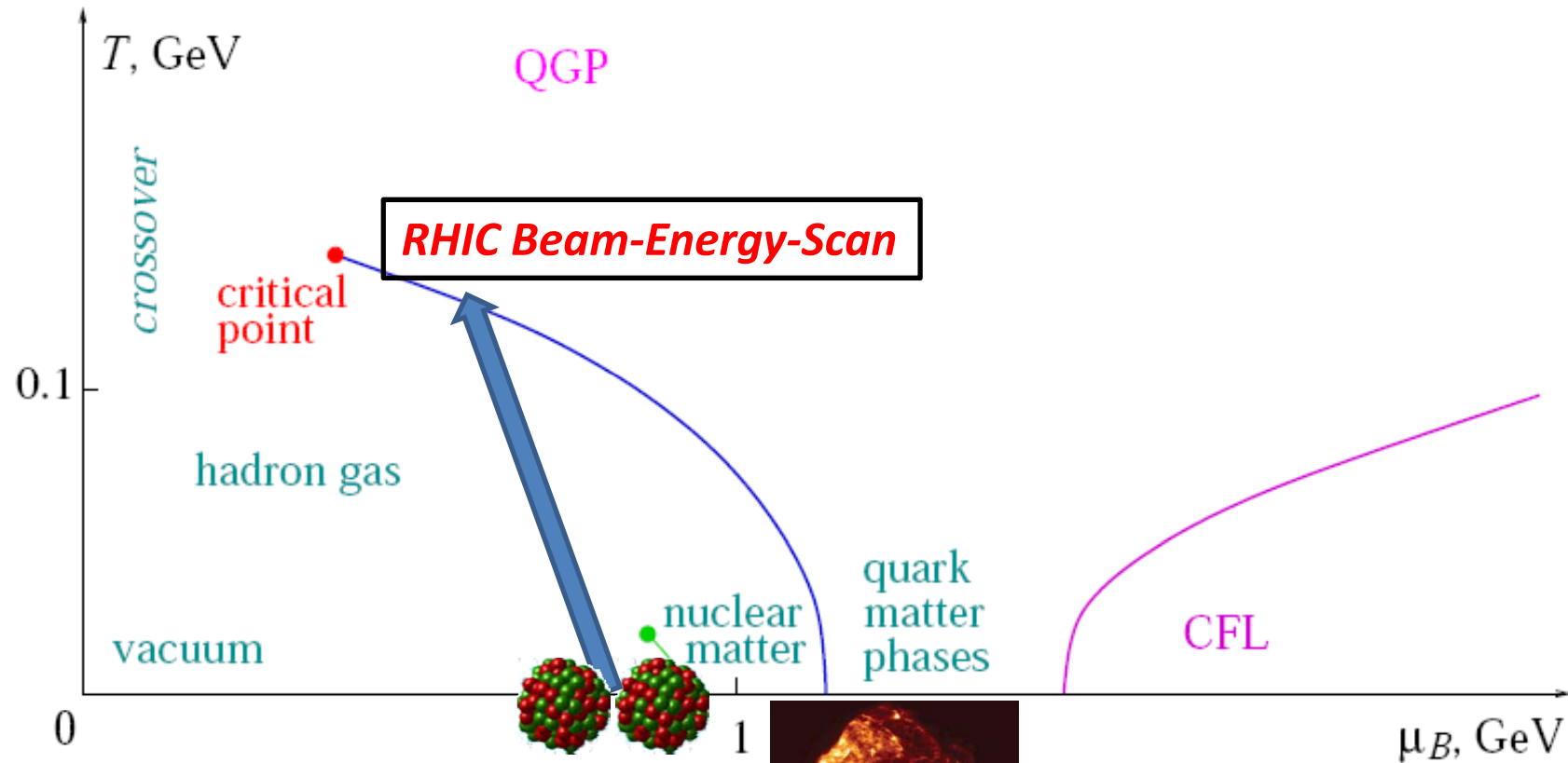
**New behavior of fluctuation & correlation**

**Is the baryon number to be transmitted  
in its broken fraction, 1/3, in QGP?**

# PHASES OF QCD



Looking for distinctive patterns of fluctuations & correlations:  
Can we spot the critical fluctuations?

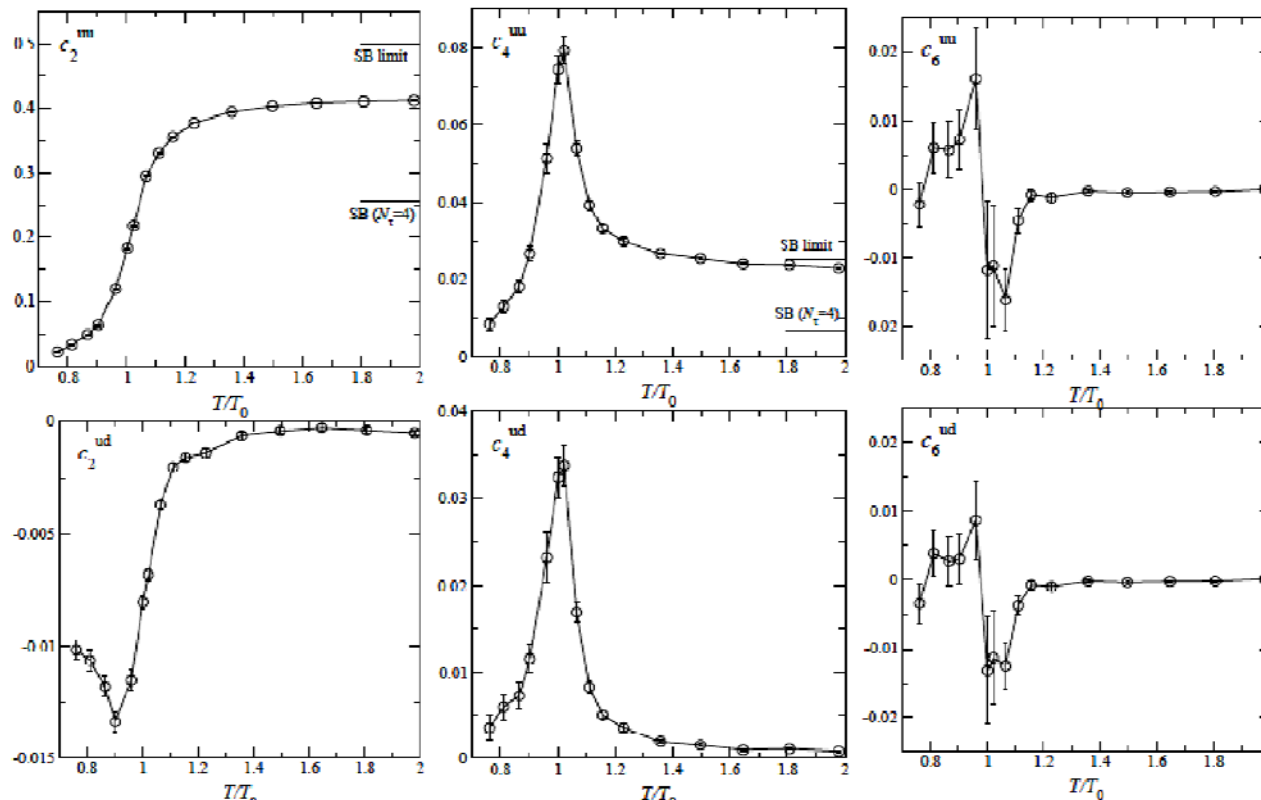


*from Stephanov, arXiv:0701002*

# SUSCEPTIBILITIES

$$\chi^{(n_i, n_j, n_k)} \equiv \frac{1}{VT} \frac{\partial^{n_i}}{\partial(\mu_i/T)^{n_i}} \frac{\partial^{n_j}}{\partial(\mu_j/T)^{n_j}} \frac{\partial^{n_k}}{\partial(\mu_k/T)^{n_k}} \log Z.$$

Can be defined for various conserved charges & their mixture (non-diagonal).



Susceptibilities:  
Taylor coefficients  
 for expanding the  
 pressure in terms of  
 chemical potentials

See reviews in e.g. V. Koch, [arXiv:0701002](https://arxiv.org/abs/0701002)

# SUSCEPTIBILITIES CONNECTED WITH DATA

$$\chi_q^{(1)} = \frac{1}{VT^3} \langle \delta N_q \rangle$$

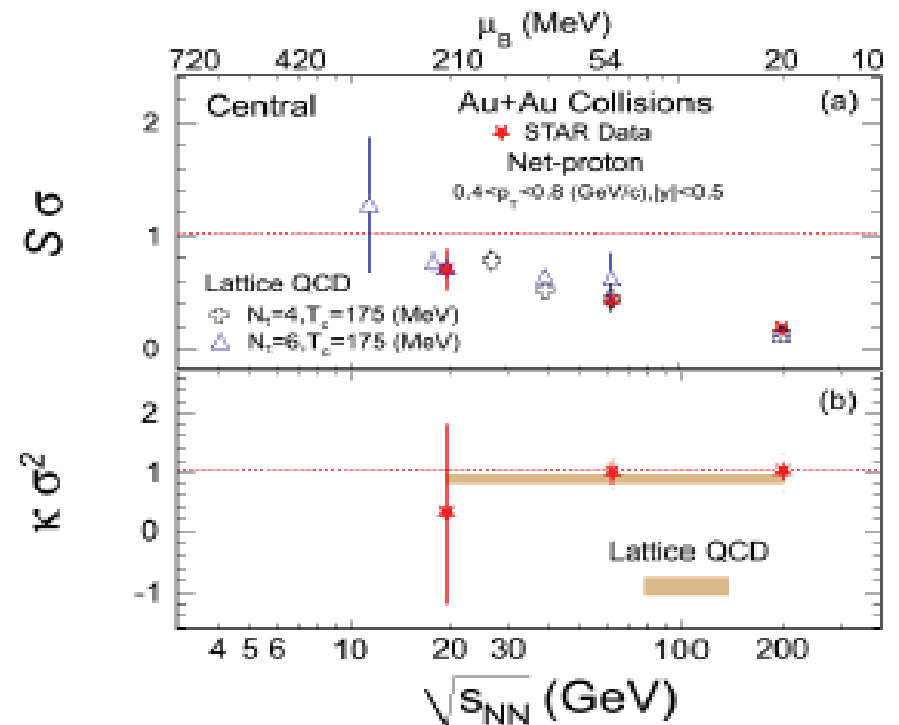
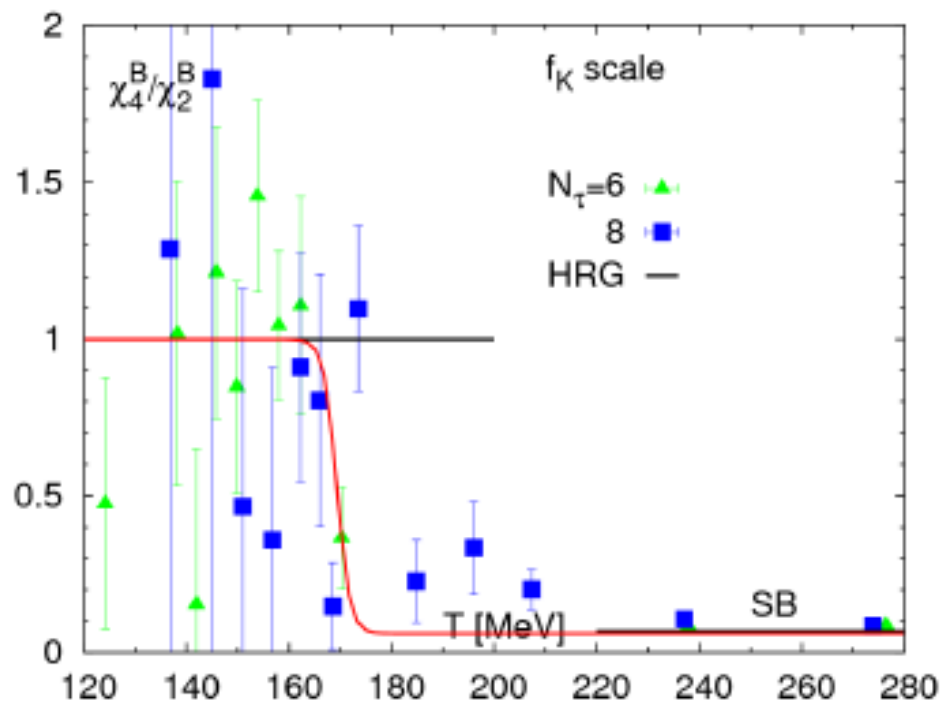
$$\chi_q^{(2)} = \frac{1}{VT^3} \langle (\delta N_q)^2 \rangle$$

$$\chi_q^{(3)} = \frac{1}{VT^3} \langle (\delta N_q)^3 \rangle$$

$$\chi_q^{(4)} = \frac{1}{VT^3} \left( \langle (\delta N_q)^4 \rangle - 3 \langle (\delta N_q)^2 \rangle^2 \right)$$

$$\frac{T^2 \chi_q^{(4)}}{\chi_q^{(2)}} = \kappa \sigma^2$$

$$\frac{T \chi_q^{(3)}}{\chi_q^{(2)}} = S \sigma$$



# SUSCEPTIBILITIES: BENCHMARK I

$$P(T, \mu) = T^4 \sum_{n=0}^{\infty} \frac{d_n(T)}{n!} \left( \frac{\mu}{T} \right)^n$$

Consider a free gas of heavy particles (non-relativistic, N.R.), with baryon number  $B$ , and mass  $M \gg T$ :

$$d_n^{\text{free}}|_{\text{NR}} = N_i \left( \frac{M}{2\pi T} \right)^{\frac{3}{2}} e^{-\frac{M}{T}} \times 2B^n \equiv \mathcal{F} \left[ \frac{M}{T} \right] B^n.$$

- Same  $T$ -dependence for all orders
- $d_n$  is positive, proportional to  $B^n$
- at any order the ratio  $d_{(n+2)}/d_n = B^2$

# SUSCEPTIBILITIES: BENCHMARK II

$$P(T, \mu) = T^4 \sum_{n=0}^{\infty} \frac{d_n(T)}{n!} \left( \frac{\mu}{T} \right)^n$$

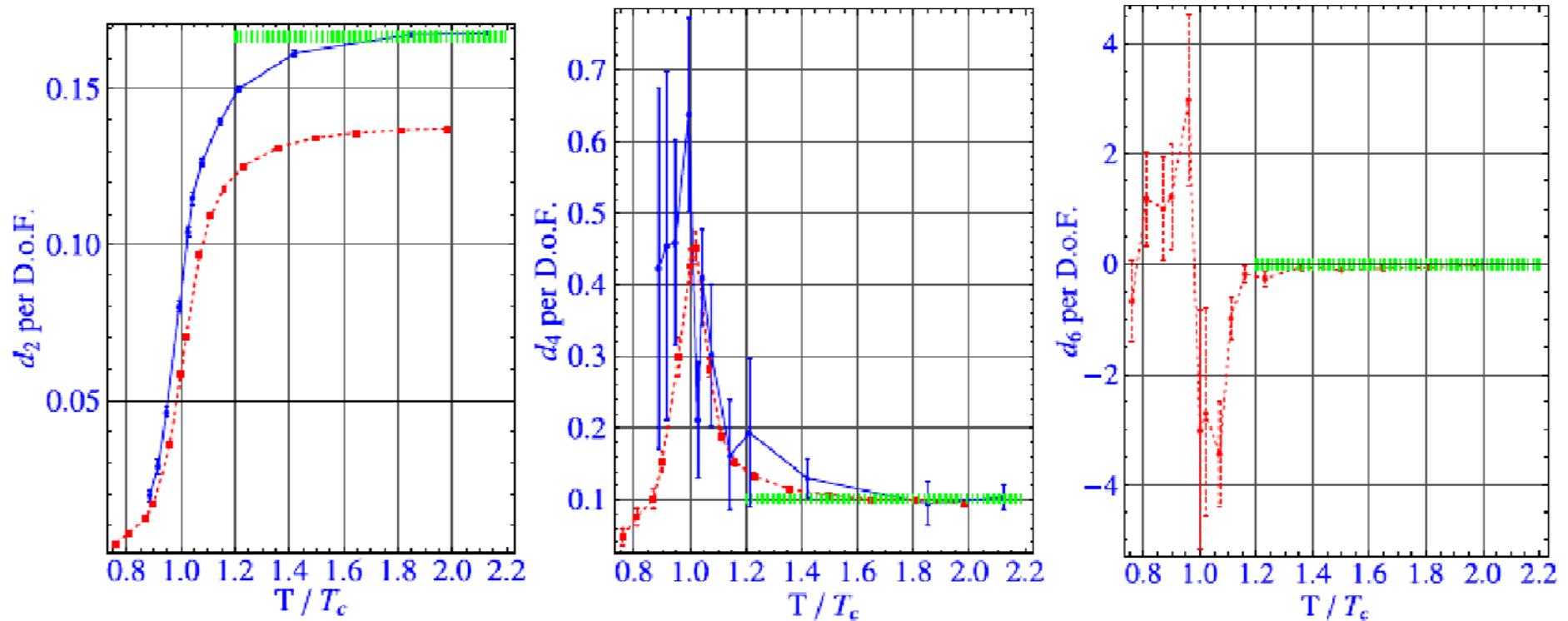
Consider a free gas of massless particles (Stefan-Boltzmann, S.B.), with baryon number  $B$ , and mass  $M=0$ :

$$d_2^{\text{free}}|_{\text{UR}} = N_i \frac{B^2}{6}, \quad d_4^{\text{free}}|_{\text{UR}} = N_i \frac{B^4}{\pi^2}, \quad d_{n>4}^{\text{free}}|_{\text{UR}} = 0.$$

- No T-dependence, up to  $n=4$
- $d_n$  is positive, proportional to  $B^n$
- $d_4/d_2 \sim B^2$

# SUSCEPTIBILITIES FROM LATTICE QCD

LQCD data: Bielefeld-BNL 2005 2-flavor with heavy pion; 2009 “almost physical” pion mass



**Below  $T_c$ : behavior close to the N.R.-benchmark , ratios  $\sim B=1$**

**i.e. hadronic(baryonic) resonance gas (verified by many)**

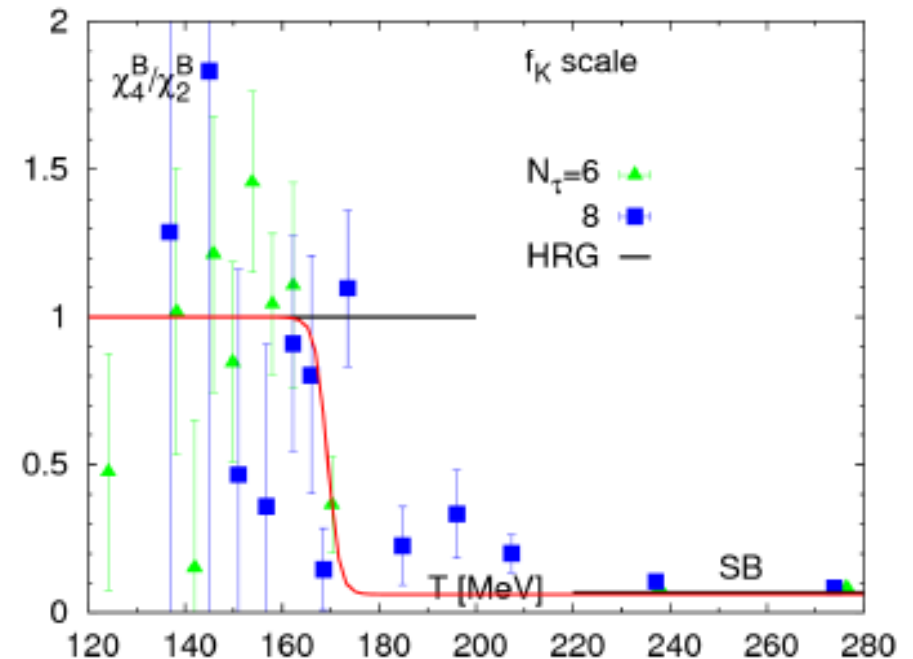
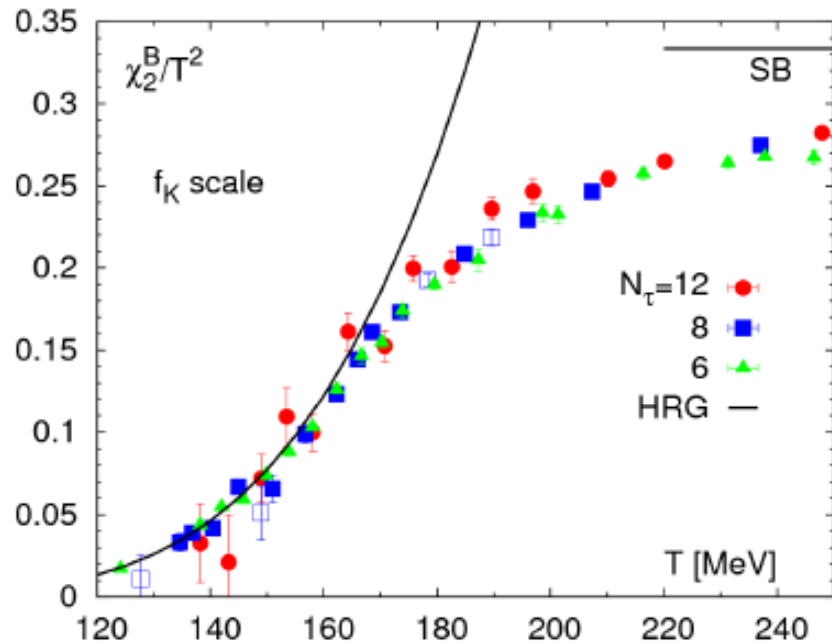
**Well above  $T_c$  (at least  $1.5T_c$  up): behavior close to S.B., ratios  $\sim B=1/3$**

**i.e. seemingly (quasi-)quarks that are VERY light ! ?**

**[caution: stay tuned till conclusive and agreed data from varied groups]**

# SUSCEPTIBILITIES FROM LATTICE QCD

Newest HotQCD data



*Yet, a few nontrivial questions to be answered:*

- **what about the near- $T_c$  plasma? The behavior there resembles neither!**
- **what are those nontrivial structures --- peaks and wiggles ---- due to?**
- **is the nearly S.B. behavior at higher  $T$  in accord with our integrated picture of QGP in this  $T$ -region?**

*Different models have varied answers --- let's examine them.*



# QUASI-PARTICLE MODELS

*Free gas of quark quasi-particles with medium generated mass:  $M(T, \mu)$*

$$d_2 = \left. \frac{\partial(n_B/T^3)}{\partial \tilde{\mu}} \right|_{\mu=0} = -\frac{2g}{2\pi^2} \int dx x^2 n^2 F^{(1)}(\epsilon_0),$$

$$d_4 = \left. \frac{\partial^3(n_B/T^3)}{\partial \tilde{\mu}^3} \right|_{\mu=0} \\ = -\frac{2g}{2\pi^2} \int dx x^2 \left[ n^4 F^{(3)}(\epsilon_0) + 3n^2 F^{(2)}(\epsilon_0) \frac{\tilde{m}_0}{\epsilon_0} \right. \\ \left. \times \left( \left. \frac{\partial^2 \tilde{m}}{\partial \tilde{\mu}^2} \right|_{\mu=0} \right) \right],$$

$$d_6 = \left. \frac{\partial^5(n_B/T^3)}{\partial \tilde{\mu}^5} \right|_{\mu=0} \\ = -\frac{2g}{2\pi^2} \int dx x^2 \left[ n^6 F^{(5)}(\epsilon_0) + 10n^4 F^{(4)}(\epsilon_0) \frac{\tilde{m}_0}{\epsilon_0} \right. \\ \left. \times \left( \left. \frac{\partial^2 \tilde{m}}{\partial \tilde{\mu}^2} \right|_{\mu=0} \right) + 15n^2 F^{(3)}(\epsilon_0) \frac{\tilde{m}_0^2}{\epsilon_0^2} \left( \left. \frac{\partial^2 \tilde{m}}{\partial \tilde{\mu}^2} \right|_{\mu=0} \right)^2 \right. \\ \left. + 5n^2 F^{(2)}(\epsilon_0) \left( \frac{\tilde{m}_0}{\epsilon_0} \left( \left. \frac{\partial^4 \tilde{m}}{\partial \tilde{\mu}^4} \right|_{\mu=0} \right) \right. \right. \\ \left. \left. + \frac{3x^2}{x^2 + \tilde{m}_0^2} \left( \left. \frac{\partial^2 \tilde{m}}{\partial \tilde{\mu}^2} \right|_{\mu=0} \right)^2 \right) \right].$$

**The main message:**

$$d_2 (T) \rightarrow \{M\}_{T, \mu=0}$$

$$d_4 (T) \rightarrow$$

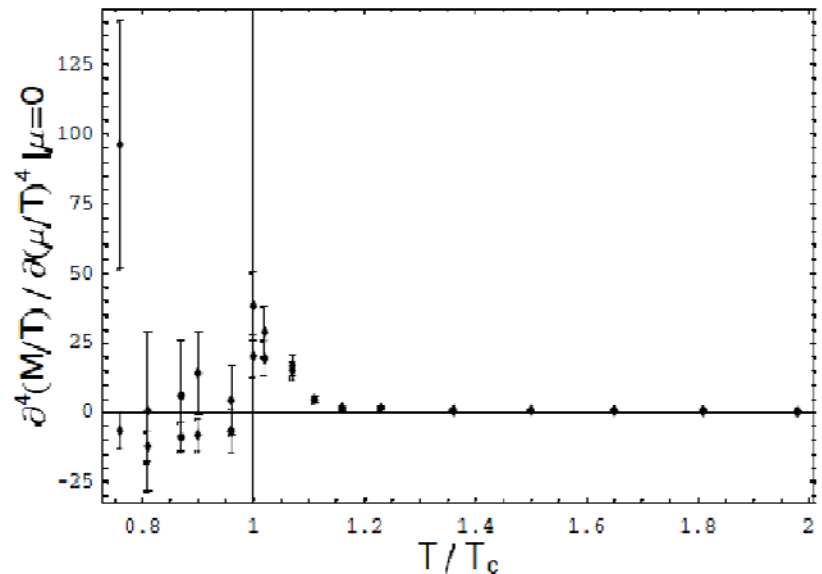
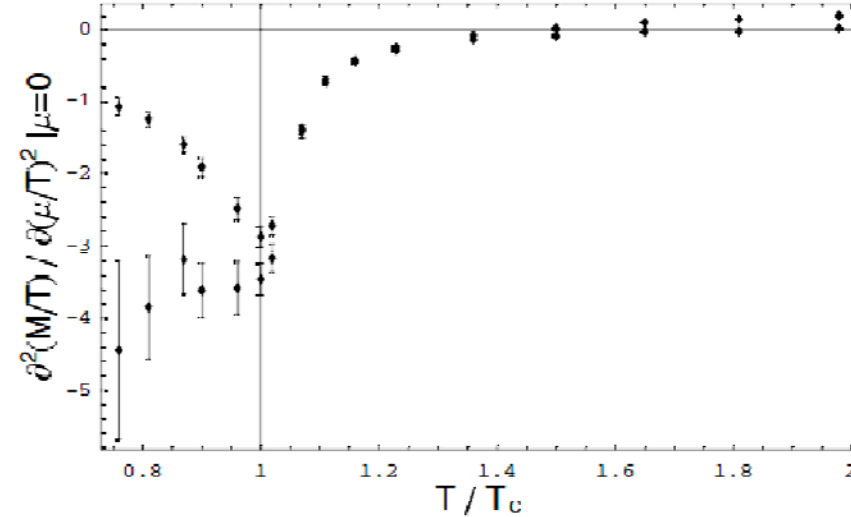
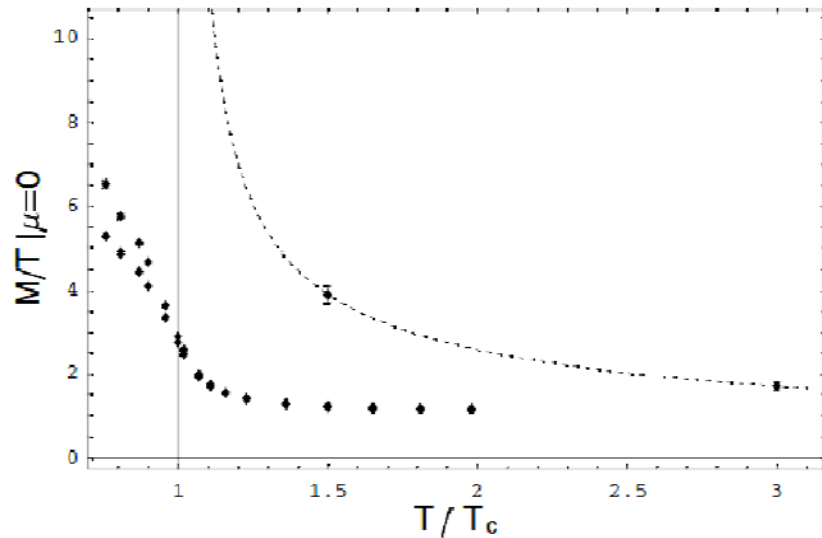
$$\left\{ M, \frac{d^2 M}{d\mu^2} \right\}_{T, \mu=0}$$

$$d_6 (T)$$

$$\rightarrow \left\{ M, \frac{d^2 M}{d\mu^2}, \frac{d^4 M}{d\mu^4} \right\}_{T, \mu=0}$$

# QUASI-PARTICLE MODELS

*Free gas of quark quasi-particles with medium generated mass:  $M(T, \mu)$*

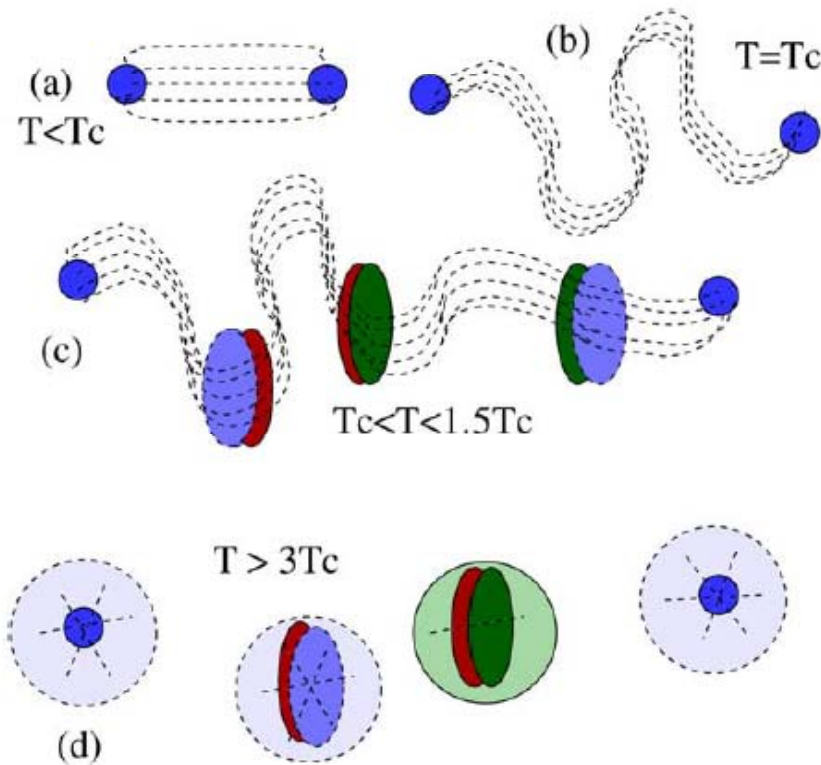


**There are problems with quasi-particle models:**

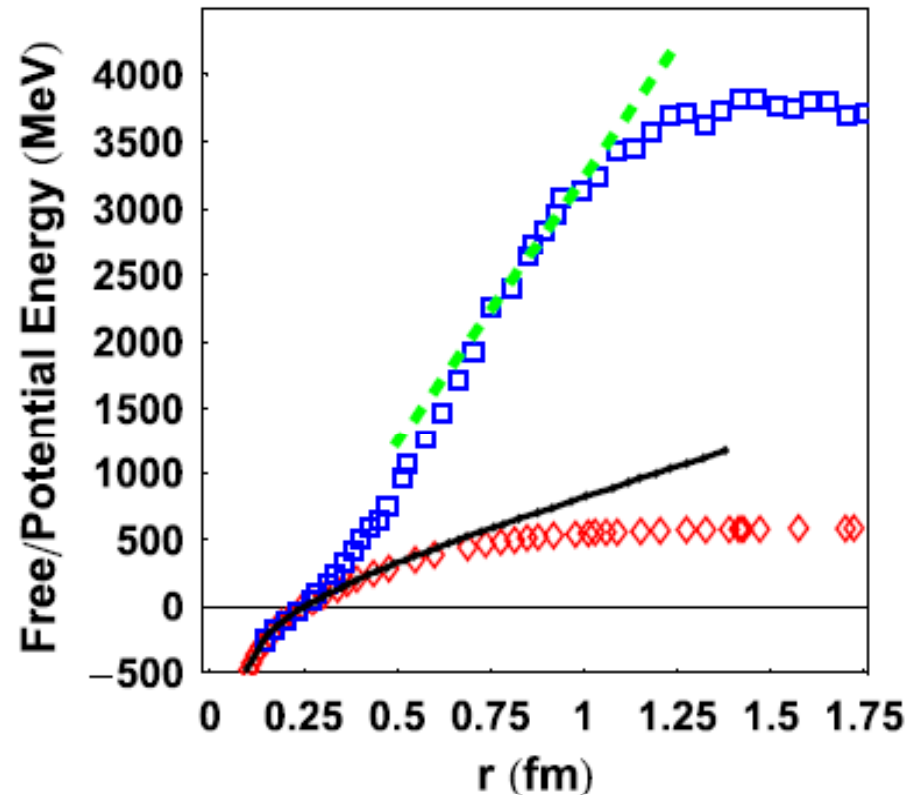
- dynamical explanation of such dependence?  
e.g. HTL not generating these...
- masses compared with direct lattice results?  
e.g.  $M_g/q \sim 1-3 T$  (Petreczky...; Karsch...)
- Polyakov loop suppression ?!
- what about the “perfect fluid” and quenching?  
→ susceptibilities of quarks at strong coupling?

# BACK TO BARYONS: DEAD OR ALIVE IN SQGP?

- RHIC phenomenology → collective flow, jet quenching,... → strongly coupled!
- Lattice QCD → strong screening kicks in late; Polyakov line restores late;  
VERY strong potential between color charges!



[JL&Shuryak,](#)  
[Nucl. Phys. A 775\(2006\)224.](#)



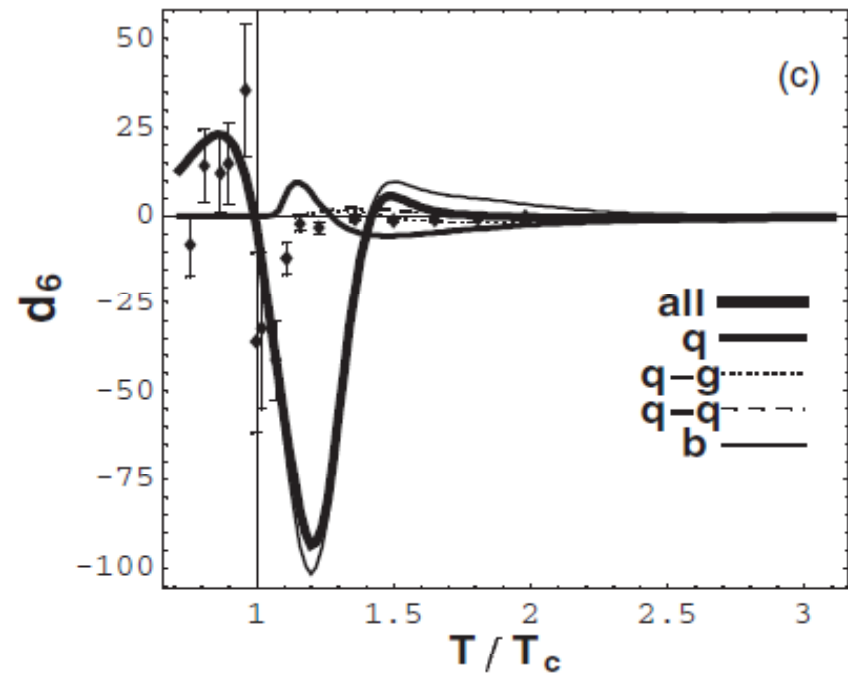
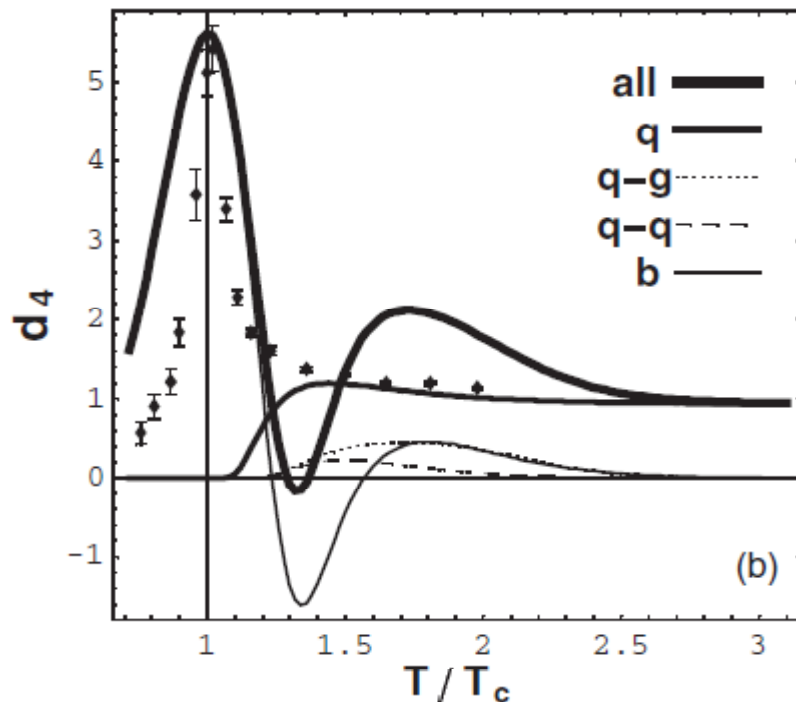
[JL&Shuryak,](#)  
[Phy. Rev. D 82\(2010\)094007.](#)

# SUSCEPTIBILITIES: BARYONS V.S. QUARKS

Thermal factor :  $e^{-M_q/T}$  v.s.  $e^{-M_B/T}$

Charge factor :  $\left(B_q = \frac{1}{3}\right)^n$  v.s.  $\left(B_B = \frac{1}{3}\right)^n$

**Baryons, or baryonic correlations, can contribute much more prominently in higher order susceptibilities, particularly the 4-th and 6-th orders!**

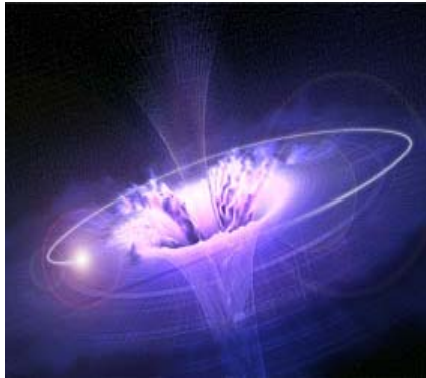


**Baryons dominate the near  $T_c$  peaks and wiggles in the 4-th and 6-th order.**

[JL & Shuryak, \*Phys. Rev. D\* 73\(2006\)014509.](https://arxiv.org/abs/2506.19222)

# SUSCEPTIBILITIES FROM HOLOGRAPHY

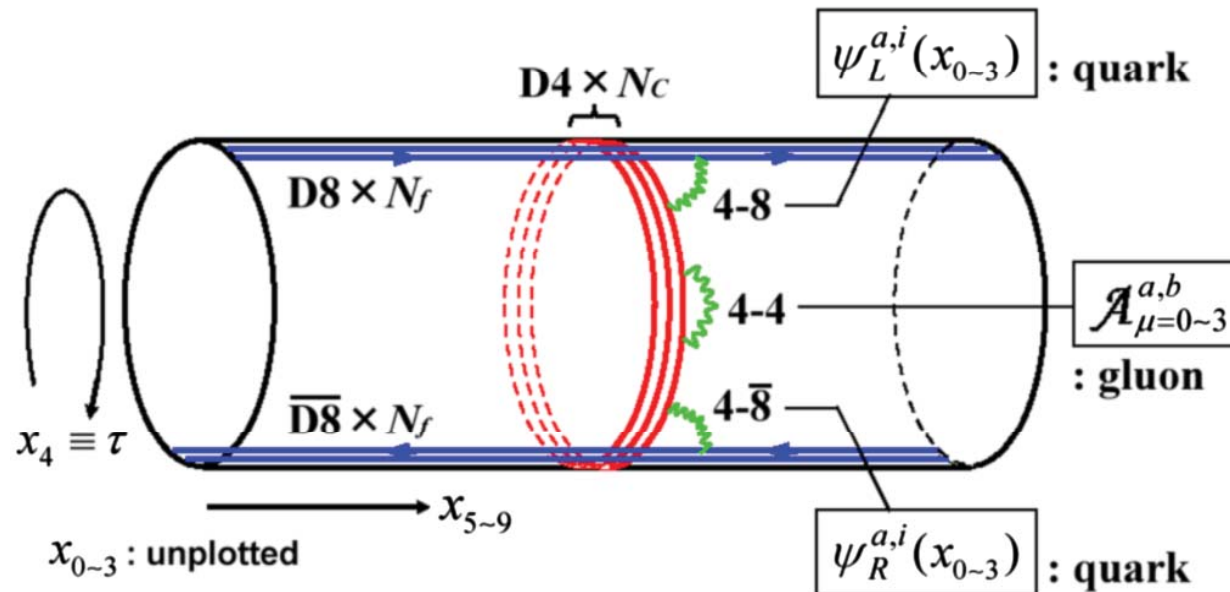
RHIC produces the sQGP: we need **benchmarks of susceptibilities at strong coupling!**  
 May holography provide a useful benchmark as in the case of e.g. shear viscosity?



$$s(\lambda \rightarrow \infty) / s(\lambda = 0) = 3 / 4$$

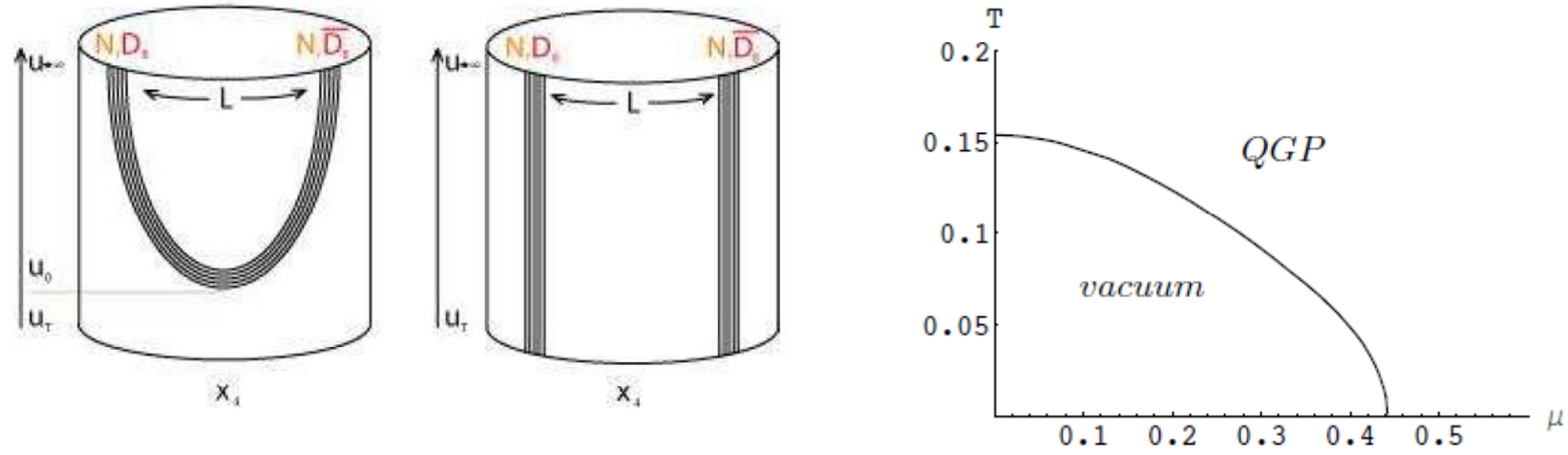
$$\eta / s \geq 1 / (4 \pi)$$

**For baryonic susceptibilities: we Sakai-Sugimoto model (D4/D8 branes)**



# THERMODYNAMICS IN SAKAI-SUGIMOTO

There are different phases ---- we focus on the **high-T and low-density phase**, QGP phase



**Thermodynamics of QGP phase in Sakai-Sugimoto Model with nonzero baryonic density:**

$$P_{\text{QGP}}[T, d(T, \mu)] = \left[ \frac{2}{7} \Gamma_A d^{\frac{7}{5}} + \frac{2}{7} u_T (d^2 + u_T^5)^{\frac{1}{2}} - \frac{2}{7} u_T d {}_2F_1\left(\frac{1}{5}, \frac{1}{2}; \frac{6}{5}; -\frac{u_T^5}{d^2}\right) \right],$$

$$\Gamma_A d^{\frac{2}{5}} - u_T {}_2F_1\left(\frac{1}{5}, \frac{1}{2}; \frac{6}{5}; -\frac{u_T^5}{d^2}\right) - \mu = 0, \quad \Gamma_A = \frac{\Gamma(\frac{3}{10})\Gamma(\frac{6}{5})}{\sqrt{\pi}}.$$

# SUSCEPTIBILITIES FROM HOLOGRAPHY

*Results for susceptibilities in QGP phase from such a holographic model of QCD:*

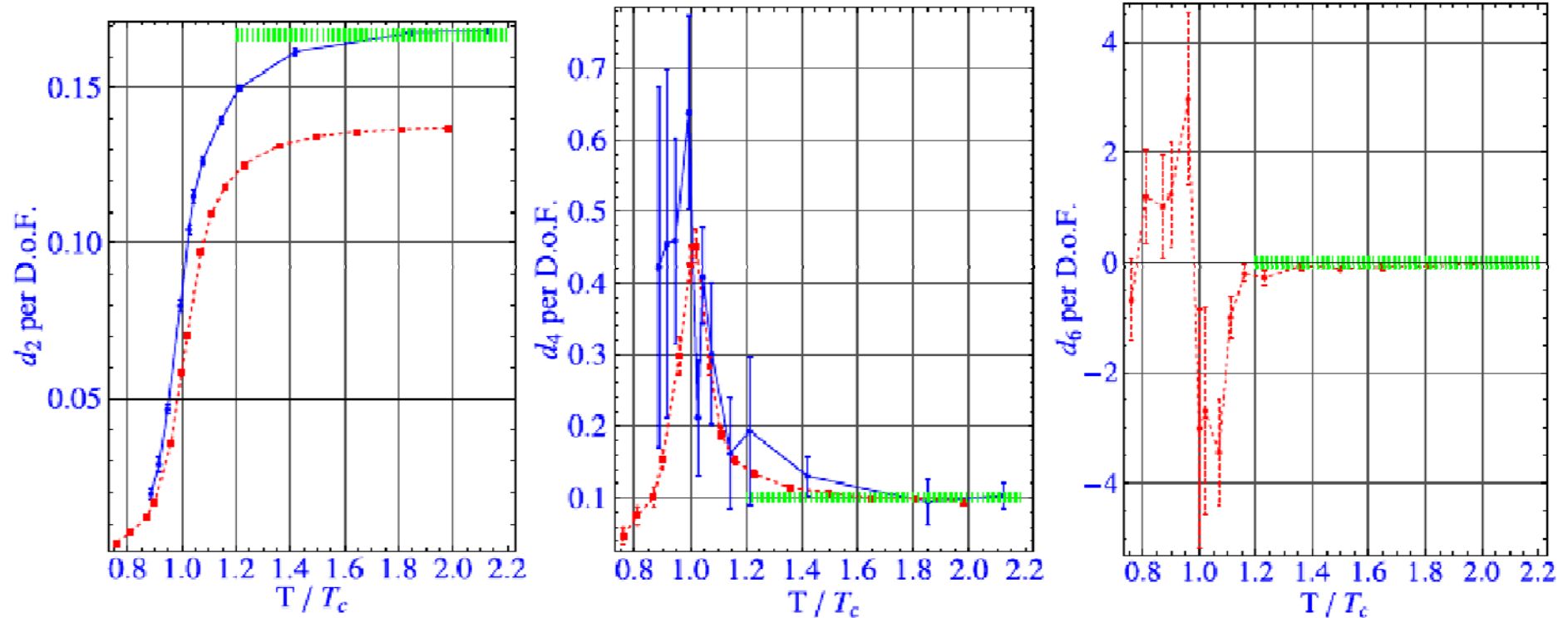
$$d_n = \xi_n N_s N_c N_f \left( \frac{1}{\lambda \tilde{T}} \right)^{n-3} \quad \begin{aligned} \lambda &= g^2 N_c \\ \tilde{T} &= T / T_c \end{aligned}$$

$$\frac{d_2}{N_s N_f N_c} \approx 0.012 \cdot \lambda \tilde{T}, \quad \frac{d_4}{N_s N_f N_c} \approx \frac{0.37}{\lambda \tilde{T}}, \quad \frac{d_6}{N_s N_f N_c} \approx -\frac{26}{\lambda^3 \tilde{T}^3}.$$

Very interesting dynamical feature:

- alternating signs (beyond 6-th order as we checked)
- strong coupling suppressing higher fluctuations while enhancing leading order
- temperature dependence is unusual as well

# A QUICK COMPARISON



$$\frac{d_2}{N_s N_f N_c} \approx 0.012 \cdot \lambda \tilde{T}, \quad \frac{d_4}{N_s N_f N_c} \approx \frac{0.37}{\lambda \tilde{T}}, \quad \frac{d_6}{N_s N_f N_c} \approx -\frac{26}{\lambda^3 \tilde{T}^3}.$$

- qualitatively similar near  $T_c$ , but not quantitatively, and need further understanding... (as in all holographic calculations)
- a useful (but only a) benchmark for how quarks might contribute to susceptibilities at strong coupling

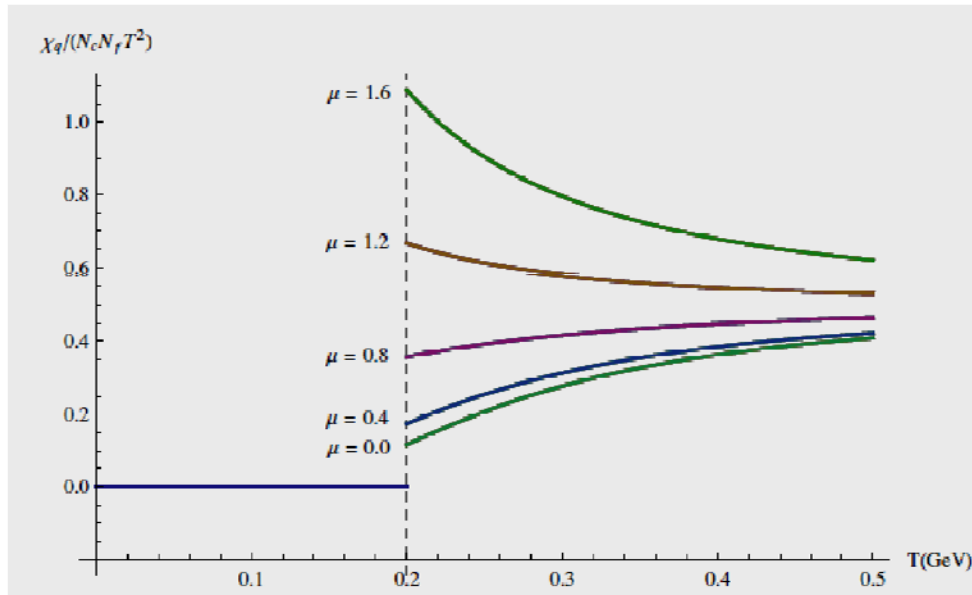


# DIFFERENT HOLOGRAPHIC RESULTS

Our D4-D8 setup results:  $d_n = \xi_n N_s N_c N_f \left( \frac{1}{\lambda \tilde{T}} \right)^{n-3}$

$$\frac{d_2}{N_s N_f N_c} \approx 0.012 \cdot \lambda \tilde{T}, \quad \frac{d_4}{N_s N_f N_c} \approx \frac{0.37}{\lambda \tilde{T}}, \quad \frac{d_6}{N_s N_f N_c} \approx -\frac{26}{\lambda^3 \tilde{T}^3}.$$

A D3-D7 setup result (for the second susceptibility only) :  $\chi_q(T) = \frac{2\pi^2 T^2}{g_5^2} \left( \frac{c}{e^c - 1} \right)$   
[Kim-Matsuo-Sim-Takeuchi-Tsukioka, JHEP05\(2010\)038.](#)



$$\chi_q \sim T^2, \quad \text{D3/D7,}$$

$$\chi_q \sim T^3, \quad \text{D4/D8.}$$

# CROSS-FLAVOR SUSCEPTIBILITIES

*There are more conserved charges (within QCD): Baryon , Isospin , Strangeness  
 → Provides additional probes of charge carriers*

**N** :  $I = 1 / 2$  ,  $B = 1$

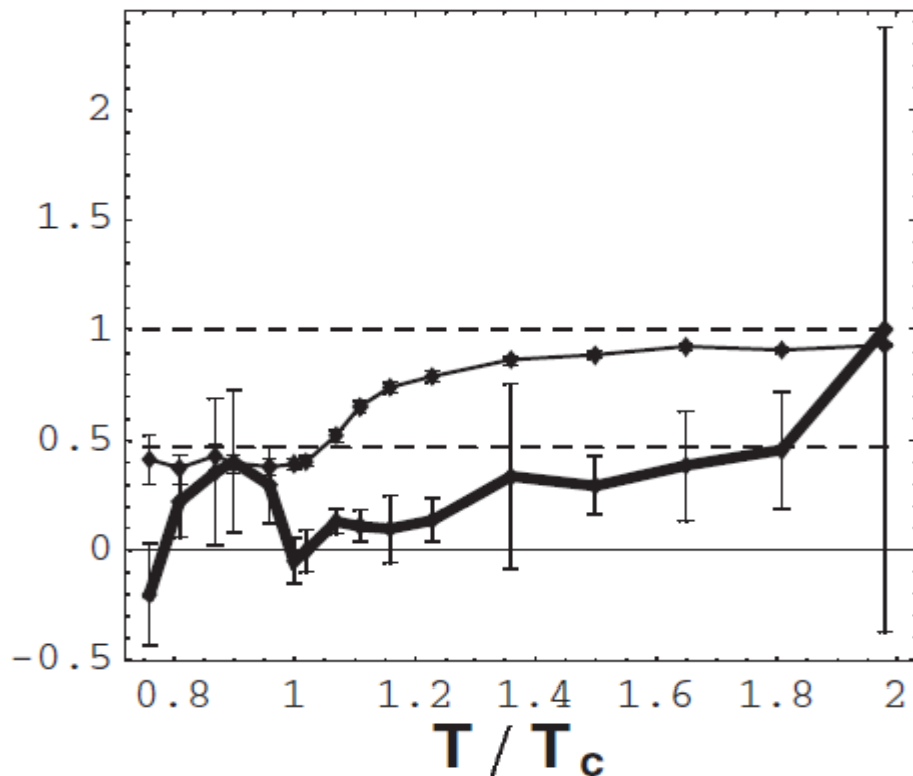
**Δ** :  $I = 3 / 2$  ,  $B = 1$

**Q (u, d)** :

$I = 1 / 2$  ,  $B = 1 / 3$

*An example of mixed-susceptibilities*

$$\frac{d_4^I}{d_4} \sim \frac{I^2}{B^2} \qquad \frac{d_6^I}{d_6} \sim \frac{I^2}{B^2}$$



← QUARKs

← BARYONs

*What could be said about these non-diagonal susceptibilities from holography?*

Need to turn on new external probe flux ...

*Other example: B-S correlations by Koch, et al.*

# SUMMARY

- Susceptibilities of conserved charges provide sensitive probe to the *new D.o.F in new phases of matter*, as well as *possible critical point*.
- Three models are examined: *quasi-particles, bound states, & holography*.
- The *near-Tc structure is intriguing* --- need more accurate lattice data as well as theoretical calculations (e.g. holography)  
in *higher order susceptibilities* and in *cross-flavor non-diagonal susceptibilities*;
- Holographic models to calculate critical fluctuations and correlations that might be related to the measurements from RHIC Low Energy Scan??

*Thank you!*