Momentum broadening in weakly coupled quark-gluon plasma

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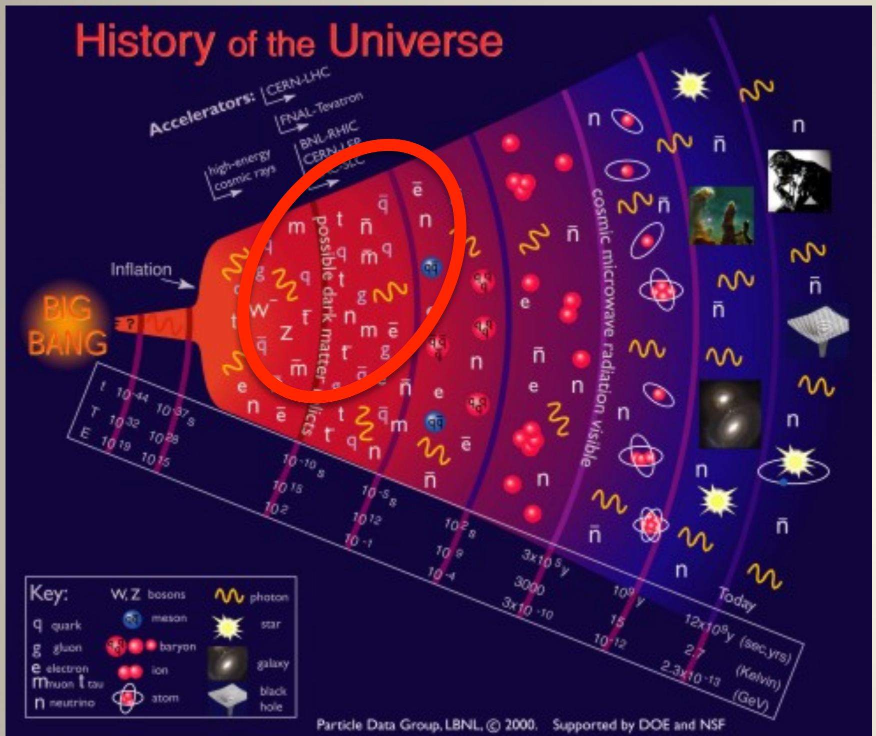
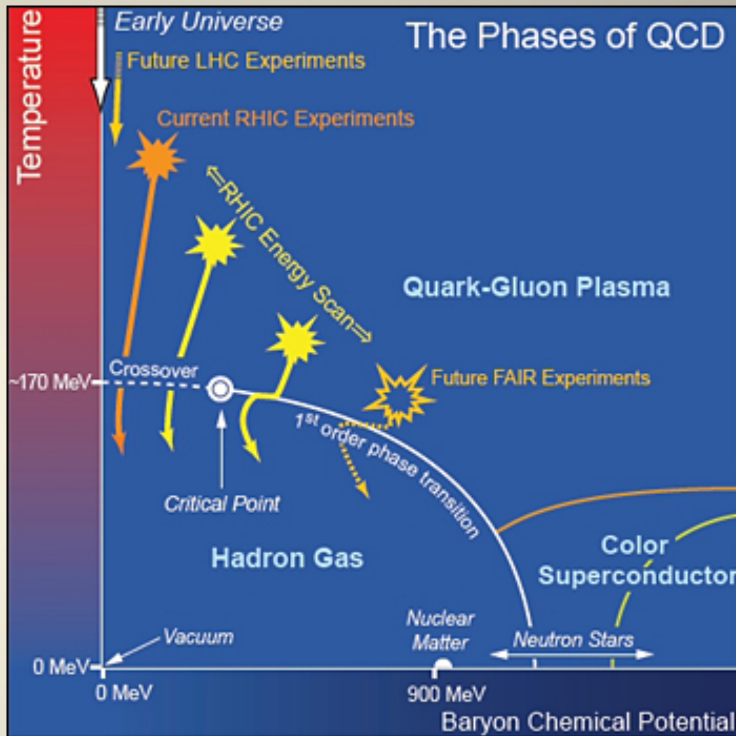
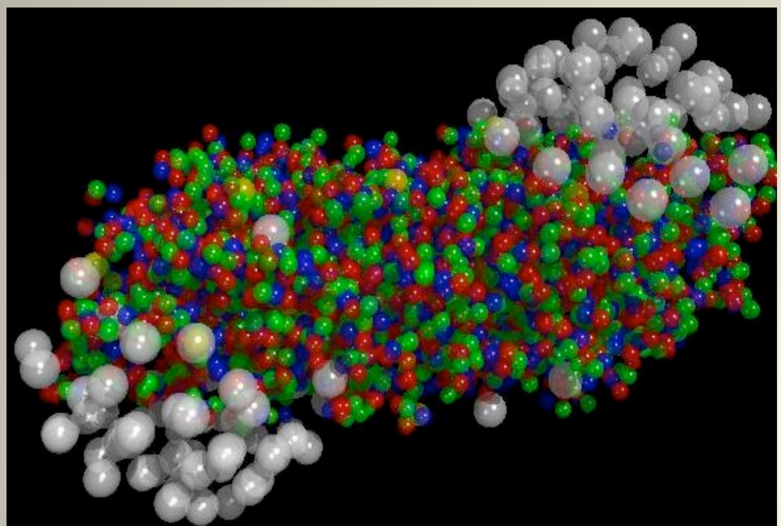
Introduction

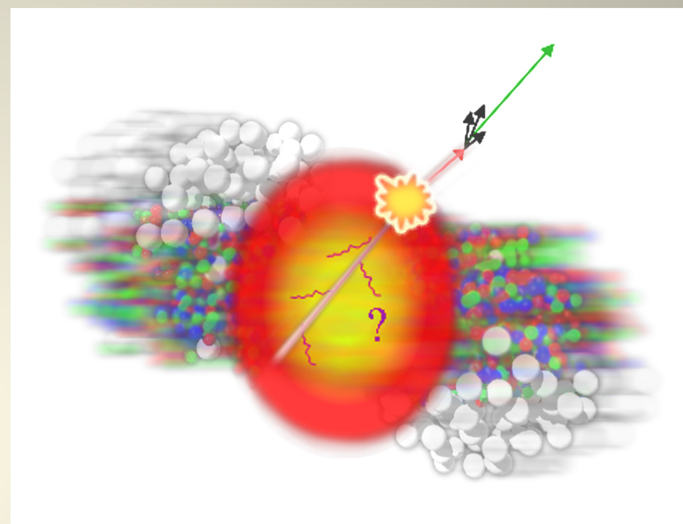




- Heavy Ion Collisions - a way to investigate quark-gluon plasma (QGP)
- Insights into early universe physics
- Understanding the phase diagram of QCD

Introduction

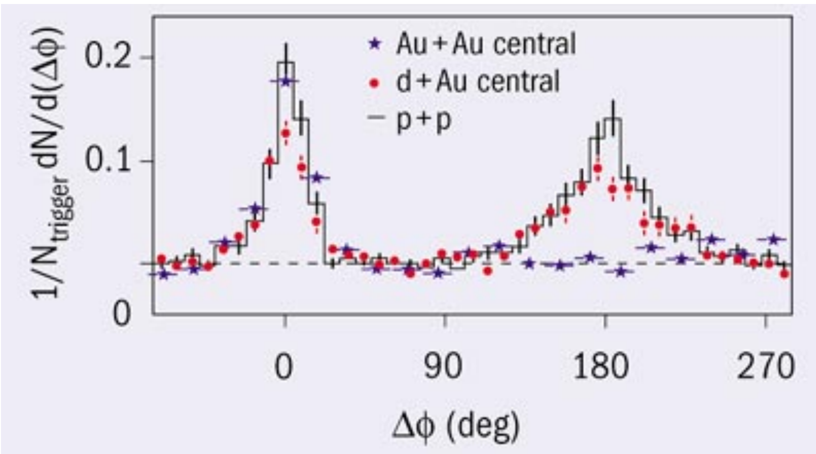




- One of the biggest puzzles in HIC is the energy loss mechanism of a probe quark/gluon that shoots through the medium with high velocity
- One of the quantities to look at – jet quenching parameter
- In some formalisms (*i.e.*, BDMPS) quantifies the energy loss
- Interesting on it’s own, because it can be related to jet broadening

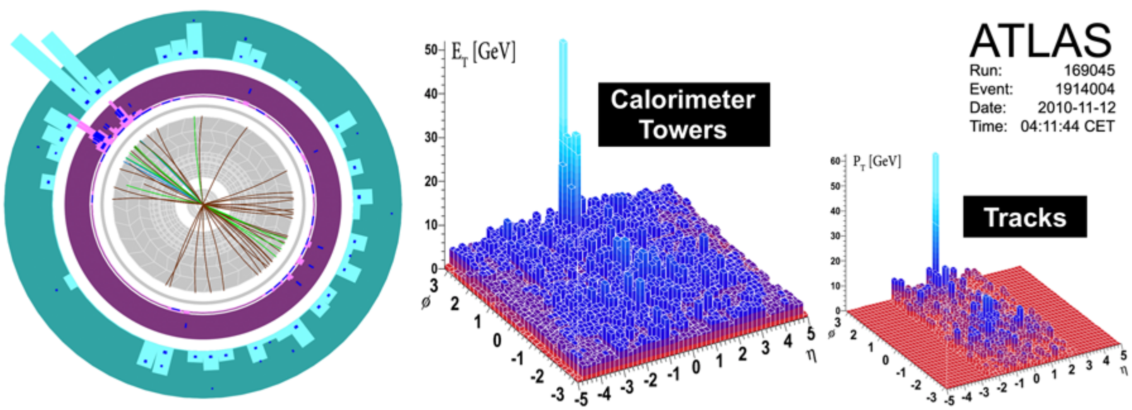
Introduction

STAR, RHIC

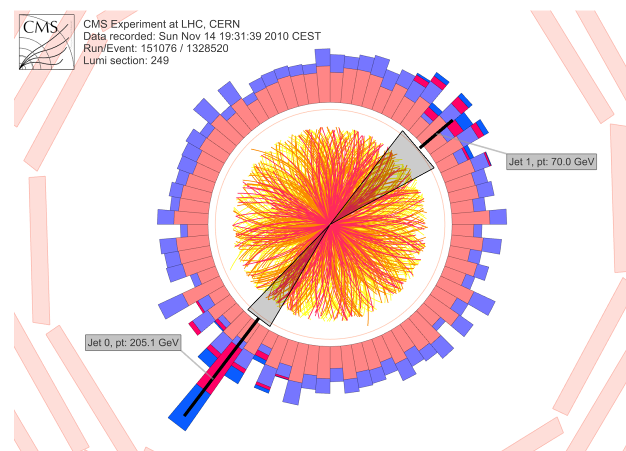


- For di-jet events one of the jets gets suppressed
- Increasing energy of colliding nuclei makes effect more apparent
- RHIC $\sqrt{s} = 200$ GeV
- LHC $\sqrt{s} = 2.76$ TeV

LHC, ATLAS



LHC, CMS



Outline

- Derivation of transverse momentum broadening distribution (jet quenching parameter) in terms of Wilson lines
- Evaluation of the distribution in the weakly coupled equilibrium quark-gluon plasma and comparison to the literature
- Comparison to estimations in strongly coupled SYM theory

Factorization of parton fragmentation function

- Framework: momentum broadening and energy loss occurs at partonic level inside the medium
- Energy loss occurs for partons and not for fragmented hadrons Fragmentation occurs outside of the medium
- For the high energy limit assumption is consistent
- Data suggests (R_{AA}) that energy loss is independent of hardron type

$$R^h_{AA} = \frac{\frac{dN^{AA\to h}_{medium}}{dp_\perp d\eta}}{N^{AB}_{coll}\frac{dN^{pp\to h}_{vacuum}}{dp_\perp d\eta}}$$

$$d\sigma^{AA\to h+\mathrm{rest}}_{\mathrm{(med)}} = \sum_f d\sigma^{AA\to f+X}_{\mathrm{(vac)}} \otimes P_f(\Delta E, L, \hat{q}, ...) \otimes D^{\mathrm{(vac)}}_{f\to h}(z, \mu_F^2)$$

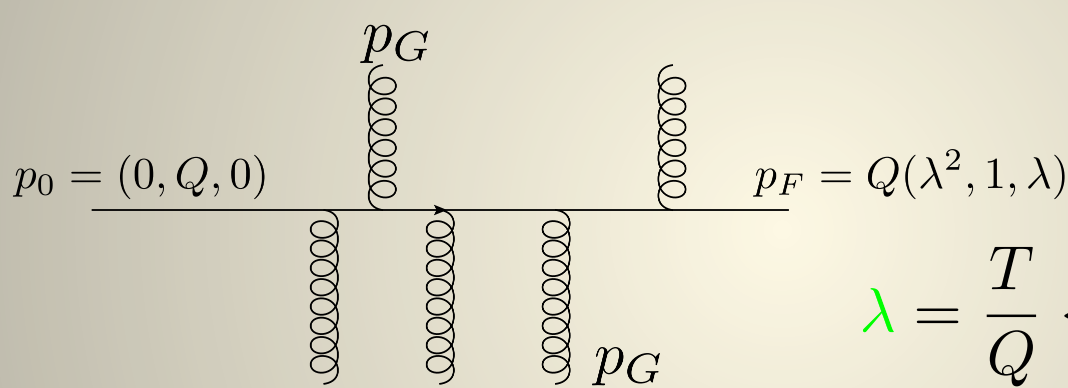
Modified fragmentation function

$$d\sigma^{AA\to f+X}_{\mathrm{(vac)}} = \sum_{ijk} f_{i/A}(x_1, Q^2) \otimes f_{j/A}(x_2, Q^2) \otimes \hat{\sigma}_{ij\to f+k}$$

Hard probe

Energetic parton traveling through the medium experiences:

- Energy loss
- Transverse momentum broadening:



Glauber gluon momentum scaling:

$$p_G = (\lambda^2, \lambda^2, \lambda)$$

$$p^+ \quad p^- \quad p_\perp$$

$$\lambda = \frac{T}{Q} \ll 1$$

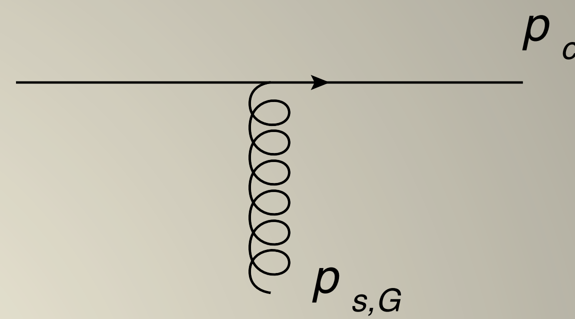
Soft Collinear Effective Theory (SCET) is well suited for problems involving separated scales.
Parton stays on-shell after interaction: Glauber gluons do not induce radiation.
Other applications of SCET for finite T medium:

Idilbi, Majumder (2009); Ovanesyan, Vitev (2011)

Modes of SCET

Off-shell modes with $P^2 \gg Q^2\lambda^2$ are integrated out.

$$\lambda = \frac{T}{Q} \ll 1$$

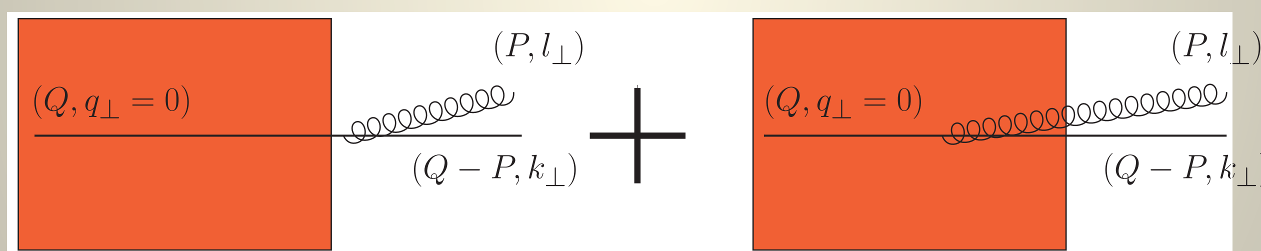


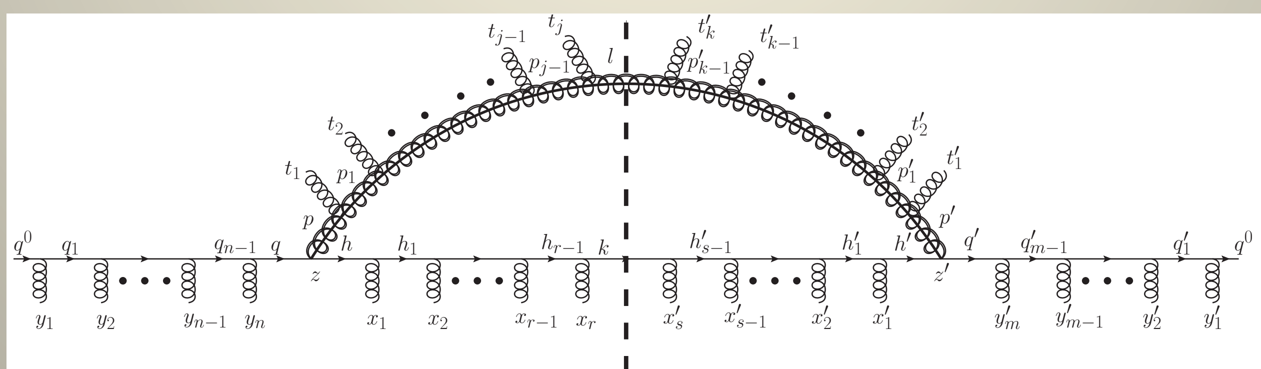
- Collinear modes:
  The mode of energetic parton
  $$p_c = Q(\lambda^2, 1, \lambda)$$
- Soft modes:
  After interaction puts collinear mode off-shell and induces radiation, thus not relevant for momentum broadening
  $$p_s = Q(\lambda, \lambda, \lambda)$$
- Glauber modes:
  Keep collinear mode on-shell, induce momentum broadening only
  $$p_G = Q(\lambda^2, \lambda^2, \lambda)$$

Were shown to be important in specific process Bauer, *et al.* (2010)

Radiation calculation

- Radiation process in the same formalism is attempted to calculate by F. D'Eramo, H. Liu, K. Rajagopal (under progress).
- Check F. D'Eramo talk on 20 September, 2011.
- Interference between "vacuum" and "medium" diagrams.
- Vacuum radiation diagrams (left) evaluated explicitly.





Glauber gluons interacting with collinear quarks

- Separating quark field into big and small components

$\xi(x) = \xi_{\bar{n}}(x) + \xi_n(x)$ $\quad \xi_n(x) = \frac{\not{n}\bar{\not{n}}}{2}\xi(x)$ - small component, integrated out

$\xi_{\bar{n}}(x) = \frac{\bar{\not{n}}\not{n}}{2}\xi(x)$ - collinear component

- Lagrangian for massless quark $\mathcal{L} = \bar{\xi} i \not{D} \xi \qquad D_\mu = \partial_\mu - igA_\mu \qquad A_\mu = A_\mu^a T^a$

from which follows $\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \not{n} (\bar{n}\cdot D)\xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i \not{D}_\perp \frac{1}{2in\cdot D} i \not{D}_\perp \not{n} \xi_{\bar{n}}$

- Decomposing the quark field

$$\xi_{\bar{n}}(x) = e^{-iQx^+} \sum_{q_\perp} e^{iq_\perp \cdot x_\perp} \xi_{\bar{n},q_\perp}(x)$$

Idilbi, Majumder (2009)

- To leading order (λ^4)

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp} \bar{\xi}_{\bar{n},q'_\perp} \left[i\bar{n}\cdot D + \frac{q_\perp^2}{2Q} \right] \not{n} \xi_{\bar{n},q_\perp}$$

$A^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$\xi_{\bar{n}} \sim Q^{\frac{3}{2}} \lambda$

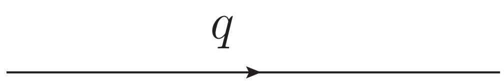
$n \equiv \frac{1}{\sqrt{2}}(1,0,0,1)$

$\bar{n} \equiv \frac{1}{\sqrt{2}}(1,0,0,-1)$

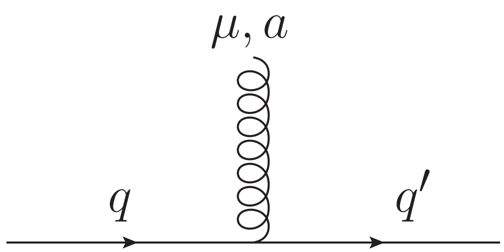
- Feynman rules follow…

Feynman rules involving Glauber gluons

Expanding in powers of λ the following Feynman rules follow

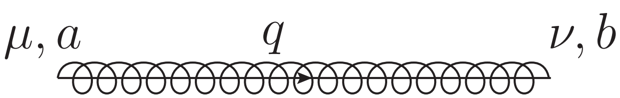


$$= \frac{iQ}{2q^+Q - q_\perp^2 + i\epsilon} \; \not{\bar{n}}$$

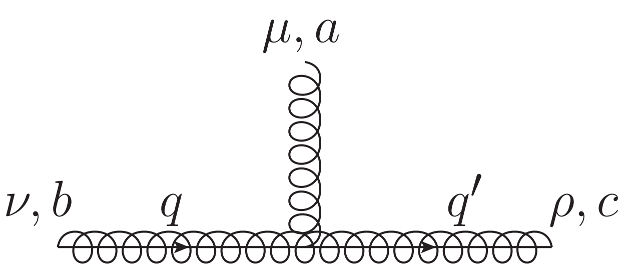


$$= igt^a \, \bar{n}^\mu \; \not{n}$$

In the similar way, Feynman rules involving only Glauber gluons:

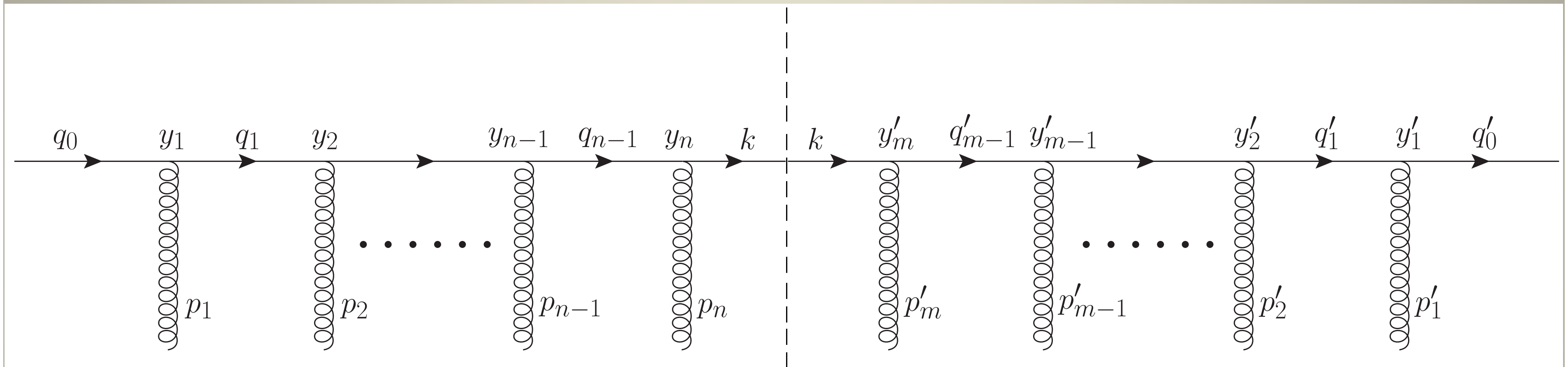


$$= \frac{-i}{2q^+Q - q_\perp^2 + i\epsilon}\left[g_{\mu\nu} - (1-\alpha)\frac{q_\mu q_\nu}{2q^+Q - q_\perp^2}\right]\delta^{ab}$$



$$= -2\,ig\;(t^a_G)_{bc}\,\bar{n}^\mu \times \Big[g^{\nu\rho}Q + n^\nu(q'_\perp - q_\perp)^\rho - n^\rho(q'_\perp - q_\perp)^\nu - \frac{\alpha-1}{2\alpha}\left(n^\rho q^\nu + n^\nu q'^\rho\right)\Big]$$

Summing over all the interactions



- Summing over all possible interactions of propagating quark (gluon) with the medium thermal Glauber gluons.
- Automatically takes care of summing over all possible cuts.
- No radiation processes considered, any self-energy diagrams would induce radiation.

Limits and relating diagrams to distribution

- The limit $Q \to \infty$ or more precisely $Q \gg k_\perp^2 L$
- Glauber gluon fields get summed into Wilson lines
- Unitarity of S matrix $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$ implies $2\mathrm{Im}M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$
- Distribution is related to the matrix elements $P(k_\perp) = L^2 |M_{\beta\alpha}|^2 \quad \beta \neq \alpha$
- Which follows from the identification $\sum_\beta = L^2 \int \frac{d^2 k_\perp}{(2\pi)^2}$
- $P(0)$ is found from the normalization condition.

Transverse momentum broadening

- Momentum broadening of quark (gluon) traveling through medium is calculated using

Casalderrey-Solana and Salgado (2007)
Liang, Wang and Zhou (2008)
D'Eramo, Liu and Rajagopal (2010)

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d\,(\mathcal{R})} \left\langle \mathrm{Tr} \left[\mathrm{W}^\dagger_\mathcal{R}[\mathrm{x}^+ = 0, \mathrm{x}_\perp] \, \mathrm{W}_\mathcal{R}[\mathrm{x}^+ = 0, 0] \right] \right\rangle$$

where $\mathcal{R}$ is the $SU(N)$ representation to which the collinear particle belongs and $d\,(\mathcal{R})$ is the dimension of this representation.

- Normalization condition $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$

- Valid for both weak and strong coupling, general medium
- $\mathcal{N} = 4$ SYM case was considered

Liu, Rajagopal, Wiedemann (2006)

Wilson lines in weakly coupled equilibrium quark-gluon plasma

$$\mathcal{W}_{\mathcal{R}}(x_\perp) = \frac{1}{d(\mathcal{R})}\left\langle \mathrm{Tr}\left[\mathrm{W}^\dagger_{\mathcal{R}}[\mathrm{x}^+ = 0, \mathrm{x}_\perp]\,\mathrm{W}_{\mathcal{R}}[\mathrm{x}^+ = 0, 0]\right]\right\rangle$$

Average is taken over the specific medium, which in our case is weakly coupled equilibrium quark-gluon plasma.

Wilson lines in weakly coupled equilibrium quark-gluon plasma

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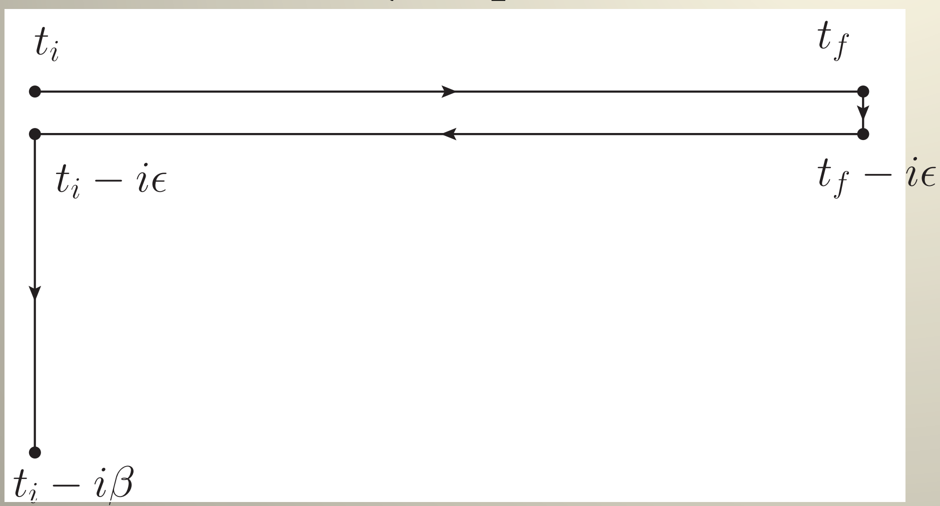
Average is taken over the specific medium, which in our case is weakly coupled equilibrium quark-gluon plasma.

Wilson line takes the form

$$W_{\mathcal{R}}\left[y^+, y_\perp\right] \equiv P\left\{\exp\left[ig\int_0^{L^-} dy^-\, A^+_{\mathcal{R}}(y^+, y^-, y_\perp)\right]\right\}$$

P stands for path ordering



Schwinger-Keldysh countour

Since gluon operators are path ordered, Wilson lines are separated by $i\epsilon$ on the Schwinger-Keldysh countour.

Real time thermal field theory "primer"

- Can be formulated on the Schwinger-Keldysh contour
- Doubling of degrees of freedom
- For scalar (bosonic) theory, for contour separation $\epsilon \to 0$

$$D_{ij}(Q) = \begin{bmatrix} \frac{i}{Q^2+i\epsilon} + n_B(q_0)2\pi\delta(Q^2) & 2\pi\delta(Q^2)(\theta(-q_0) + n_B(q_0)) \\ 2\pi\delta(Q^2)(\theta(q_0) + n_B(q_0)) & \frac{-i}{Q^2-i\epsilon} + n_B(q_0)2\pi\delta(Q^2) \end{bmatrix}$$

- For fermions $n_B(q_0) \to -n_F(q_0)$ and times $\not{Q}$
- For covariant Feynman gauge, times $g_{\mu\nu}\delta^{ab}$
- Vertex functions same as for T = 0, conserving the *i,j* index
- Convenient to switch to Keldysh representation,

$$D_R = D_{11} - D_{12}$$
$$D_A = D_{11} - D_{21}$$
$$D_S = D_{11} + D_{22}$$

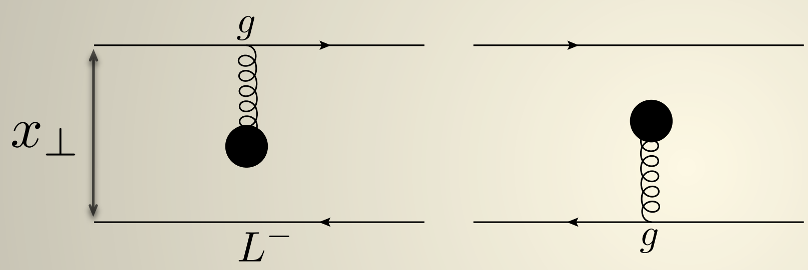
where only three components are independent due to (sum rule)

$$D_{11} + D_{12} + D_{21} + D_{22} = 0$$

Counting powers of g

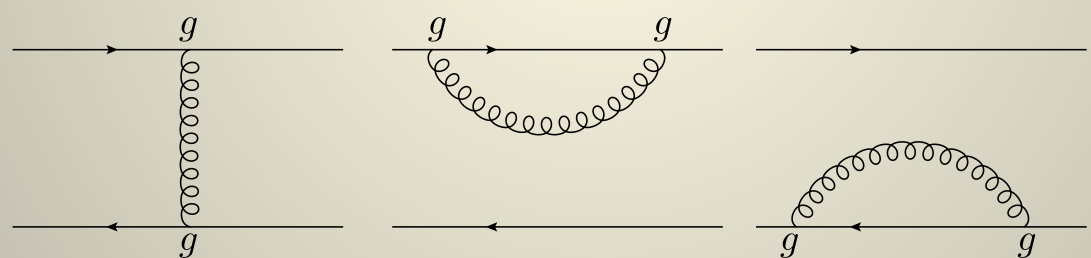
- Let's find the LO contributions by counting powers of explicitly g

g :



Vanish due to $\mathrm{tr}[A] = 0$

g^2 :

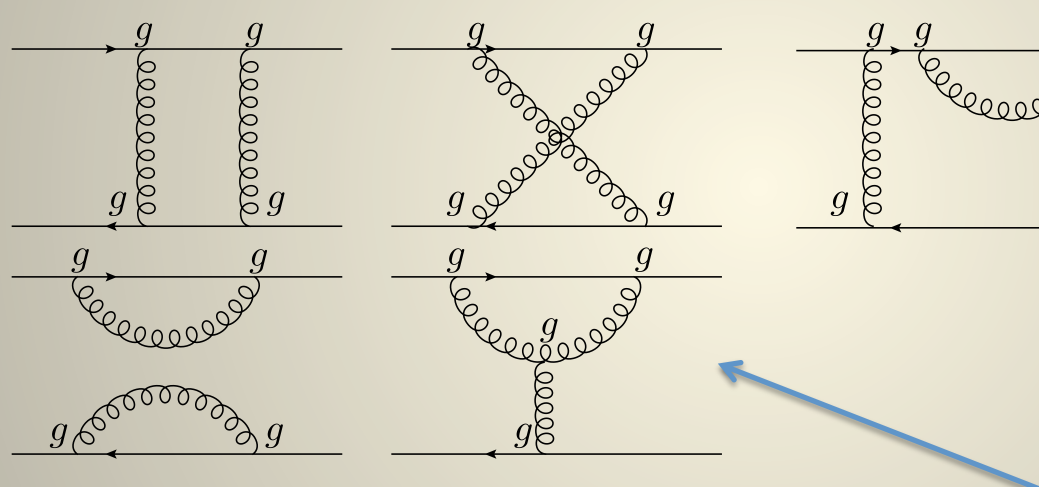


Vanish due to $g_{++} = 0$

g^3 : No diagrams contributing

Counting powers of g

g^4 tree level diagrams:

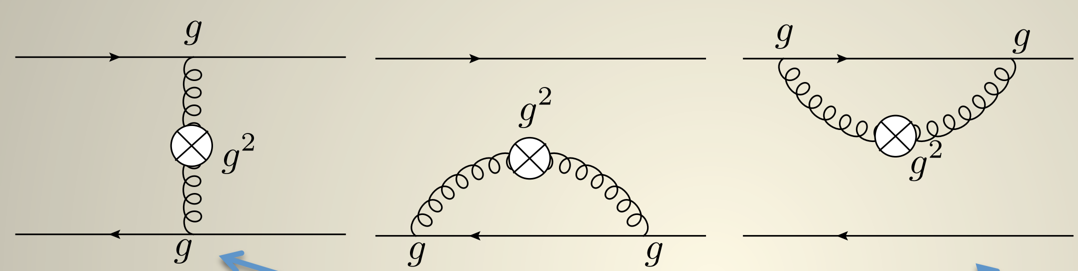


$+$ permutations

Vanish also due to $g_{++} = 0$ and due to $\Gamma^{abc}_{+++} = 0$

Counting powers of g

g^4 diagrams involving an effective propagator:



Non-vanishing!

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) + \delta^2(k_\perp)P_{11}$$

Leading order in the UV is g^4

Probability distribution and "plus" distribution function

- One can express probability distribution

Ligeti, Stewart, Tackman (2008)

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) - \delta^2(k_\perp)\int d^2q_\perp P_>(q_\perp)$$

- $P_>(k_\perp)$ is IR divergent, which is

1. Irrelevant for an evaluation of jet quenching parameter (second moment of distribution)
2. Important if we care about $P(k_\perp)$ itself
3. Solution: use "plus" distribution function to extract delta function part from the second term and show that the divergent part cancels the divergent part of the third term

$$P(k_\perp) = \delta^2(k_\perp)\left((2\pi)^2 - \int_{k_{\perp 0}}^{\infty} dq_\perp 2\pi q_\perp P_>(q_\perp)\right) + [P_>(k_\perp)]_+$$

- Can interpret $[P_>(k_\perp)]_+$ as $P(k_\perp)$, for $k_\perp > T$.

HTL approximation and effective theory

- For soft external momentum, need to use resummed effective theory – Hard Thermal Loops.
- Soft momentum: $q_0, |\vec{q}| \approx gT$
- Hard momentum: $q_0 \approx T$ or $|\vec{q}| \approx T$

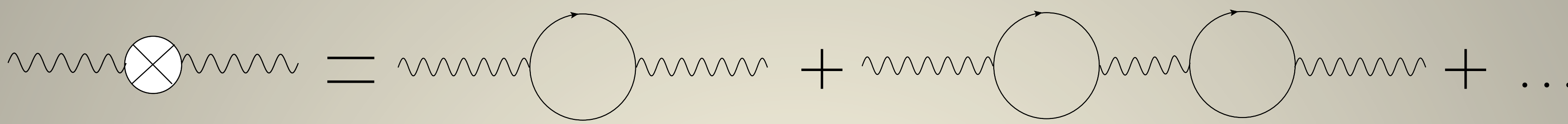
Braaten and Pisarski (1990)
Frenkel and Taylor (1990)
Le Bellac (1996) for a pedagogical review

- Loop corrections are of order $(gT)^2/q^2$. For soft external momentum: corrections comparable to tree level propagator.
- In such case hard loop momentum gives the main contribution, self-energies simplify.
- Satisfies Ward identities.
- Valid only for soft external momentum!
- Need to use HTL resummed propagator and vertices to have valid perturbative expansion in powers of g in IR limit.
- Longitudinal and transverse parts of self-energies in HTL are given by (for $Q^2 < 0$)

$$F_{HTL}(Q) = \frac{m_D^2 Q^2}{q_\perp^2}\left(1 - \frac{q_0}{2q}\log\frac{|q_0+q|}{|q_0-q|} - i\pi\frac{q_0}{2q}\right), \quad G_{HTL}(Q) = \frac{m_D^2 - F_{HTL}}{2}$$

Resummation

- Resummation for QED and analogous for QCD:



- In covariant Feynman gauge self-energies satisfy transversality condition $Q_\mu \Pi_R^{\mu\nu} = 0$ thus $i\Pi_R^{\mu\nu} = F P_L^{\mu\nu} + G P_T^{\mu\nu}$
- Enough to calculate two components of self-energies, which is not necessarily the case for the general gauge
- F and G are longitudinal and transverse self energies (gauge independent) which in static limit correspond to electric and magnetic masses (?)
- Retarded propagator in covariant Feynman gauge at finite temperature:

$$(-i) D^R_{\mu\nu}(Q) = \frac{P^L_{\mu\nu}}{Q^2 - F} + \frac{P^T_{\mu\nu}}{Q^2 - G} - \frac{Q_\mu Q_\nu}{Q^4}$$

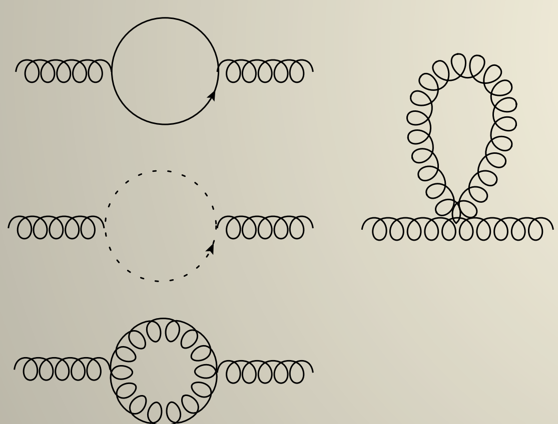
$$F = \frac{Q^2}{q^2}\Pi_R^L \quad G = \Pi_R^T$$

$$P^L_{\mu\nu} + P^T_{\mu\nu} = -g_{\mu\nu} + \frac{Q_\mu Q_\nu}{Q^2}$$

IR and UV limits

IR limit

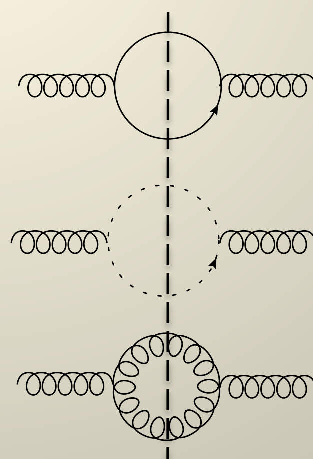
- Resummed propagator. Due to HTL approximation, real and imaginary parts of F_{HTL} and G_{HTL} are known analytically.



$$D_> = -\frac{1}{1-e^{-\beta k_0}} 2\mathrm{Re} D_R$$

UV limit

- In UV limit it is enough to calculate non-amputated propagator, resummation is not necessary, propagator is proportional to imaginary part of self energies.

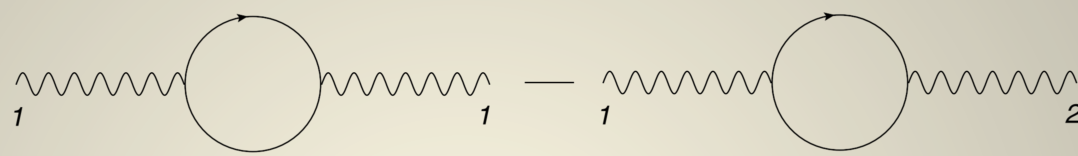


$$D_> = -\frac{1}{k^2 k_\perp^2}\frac{1}{1-e^{-\beta k_0}}\left(\mathrm{Im}\Pi_R^T + \frac{k_\perp^2}{k^2}\mathrm{Im}\Pi_R^L\right)$$

Transverse momentum broadening

$$P_>(k_\perp) = g^2 C_R L^- \sqrt{2} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} D_>^{++}(k^+ = 0, k_0, k_\perp)$$

$\Pi_R =$



- We calculated full self-energies in covariant Feynman gauge

  ImΠ_R – analytically

  ReΠ_R – numerically
- In the regime of soft momentum ($k_0, k_\perp \sim gT$) reproduce HTL result as expected, but Π_R is valid for all momentum space and not restricted to soft momentum region.
- In the UV limit $P_>(k_\perp) \propto 1/k_\perp^4$

  which must be the case according to general arguments.

Full form of self-energies for covariant Feynman gauge

$$\mathrm{Re}(\Pi_R^L)^{T\neq 0} = -\frac{g^2T^2}{6}\left(\frac{N_f}{2}+C_A\right) + \left(\frac{g^2}{8\pi^2 p}\int_0^\infty dk \log\frac{|p_\perp^2+2k(p_0-p)|}{|p_\perp^2+2k(p_0+p)|}\right.$$
$$\left.\left[N_F n_F(k)(4k^2-4kp_0-p_\perp^2) - C_A n_B(k)(2p_\perp^2-4k^2+4p_0k+p_0^2)\right] + (p_0\to -p_0)\right)$$

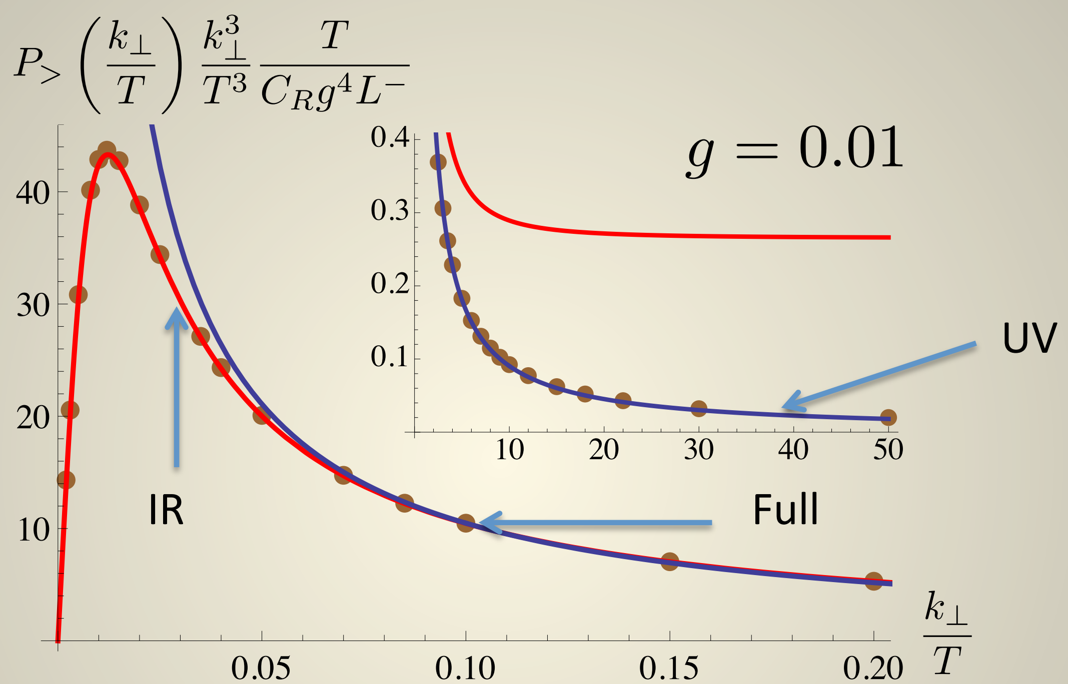
$$\mathrm{Im}(\Pi_R^L)^{T\neq 0} = \frac{N_f}{2}\frac{g^2T^2}{\pi}\left[\mathrm{Li}_2\left(-e^{\frac{-p+p_0}{2T}}\right) - \mathrm{Li}_2\left(-e^{-\frac{p+p_0}{2T}}\right) + \frac{2T}{p}\mathrm{Li}_3\left(-e^{\frac{-p+p_0}{2T}}\right) - \frac{2T}{p}\mathrm{Li}_3\left(-e^{-\frac{p+p_0}{2T}}\right)\right]$$
$$-\frac{1}{24p\pi}C_A g^2\left[5p_0^3+8p_0\pi^2+6p_0p_\perp^2+3p^2\left(\log\left[1-e^{\frac{p-p_0}{2T}}\right]-\log\left[1-e^{\frac{p+p_0}{2T}}\right]\right)\right.$$
$$\left.-12p\left(\mathrm{Li}_2\left(e^{\frac{p-p_0}{2T}}\right)-\mathrm{Li}_2\left(e^{\frac{p+p_0}{2T}}\right)\right)+24\mathrm{Li}_3\left(e^{\frac{p-p_0}{2T}}\right)-24\mathrm{Li}_3\left(e^{\frac{p+p_0}{2T}}\right)\right]$$

$$\mathrm{Re}(\Pi_R^T)^{T\neq 0} = \ldots$$

$$\mathrm{Im}(\Pi_R^T)^{T\neq 0} = \ldots$$

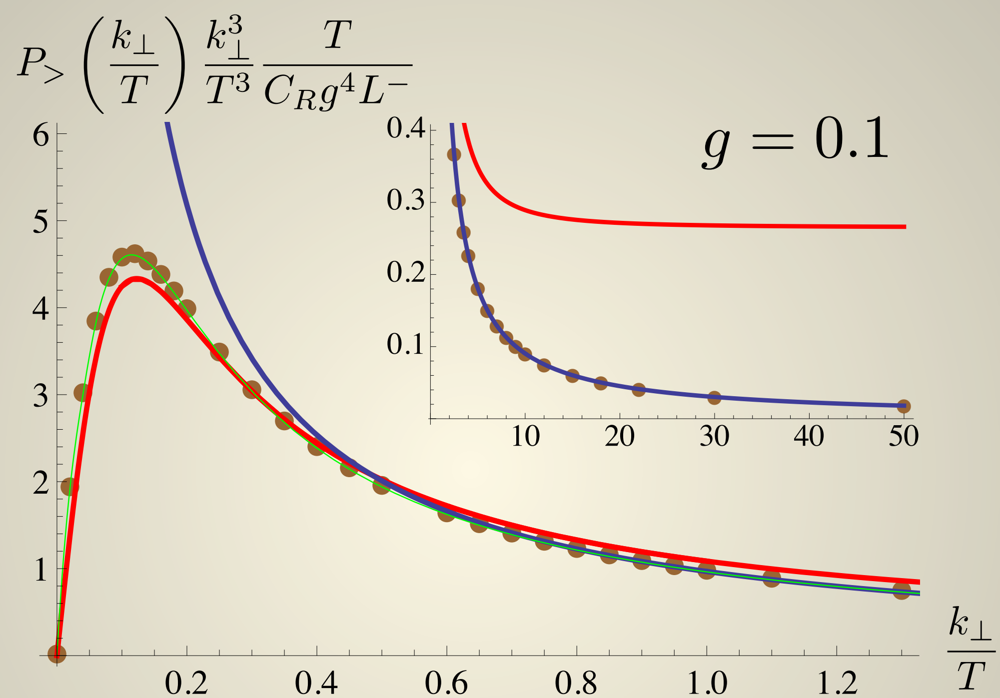
- Analogous expressions for the transverse part
- For light-cone case $p^+ = 0, P^2 < 0$
- Easily generalizable for any P^2
- HTL self-energies obtainable for $p_0, p_\perp \ll T$

Transverse momentum broadening, QCD



- Full expression is obtained with no approximations on Π_R
- UV and IR limits smoothly overlap
- Can integrate over to obtain jet quenching parameter

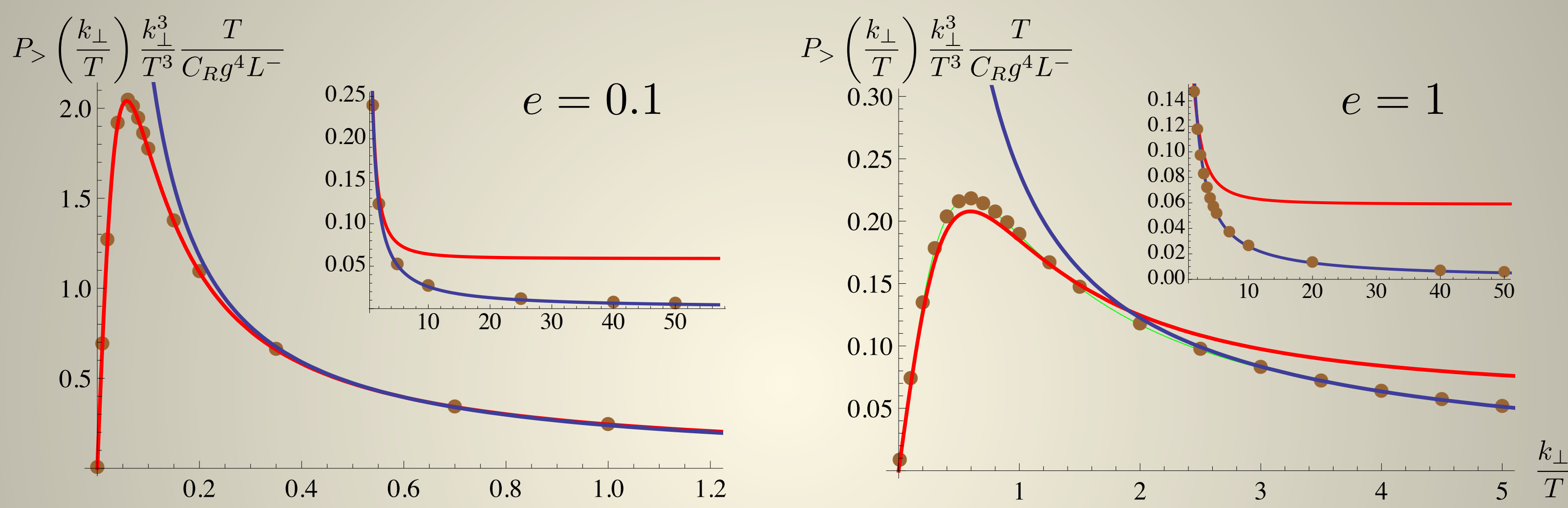
Transverse momentum broadening, QCD



$$P_>(k_\perp) = g^2 C_R L^- \sqrt{2}\int_{-\infty}^{\infty}\frac{dk_0}{2\pi}D_>^{++}(k^+=0,k_0,k_\perp)$$

- For bigger values of g, for the IR region, HTL self-energies are not sufficient.
- Corrections originate from the high k_0 region, where HTL approximated self-energies are not valid.

Transverse momentum broadening, QED



- Can consider QED plasma as a specific case of QCD plasma
- Gluon ➔ Photon. Only contribution to self-energies is from from the fermionic loop, just $g \to \sqrt{2}e$
- For QED plasma, HTL approximation works for higher values of coupling constant.

Short history of transverse momentum broadening

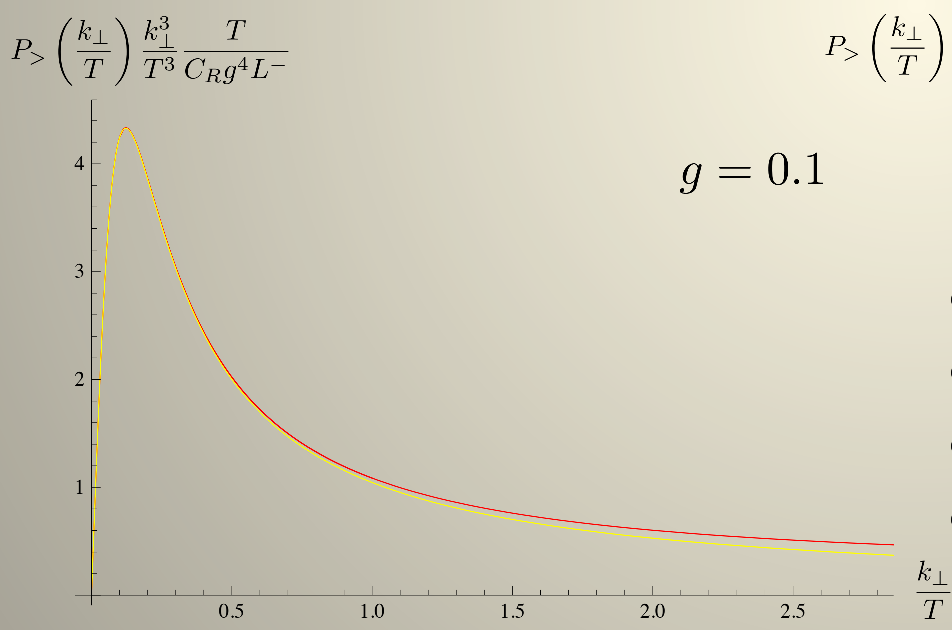
- Different notations, the quantity is the same as $P_>(k_\perp)$
- AMY calculated in the IR limit, used HTL approximated self-energies and "Soft approximation" $$n_B(k_0) = \frac{1}{e^{\beta k_0} - 1} \to \frac{1}{\beta k_0}$$ Arnold, *et al.* (2002)
- AGZ applied Sum Rules for AMY rate to get simple analytical expression in the IR Aurenche, *et al.* (2002)
- Caron-Huot used "Electric QCD" to calculate within HTL and soft approximation regimes to reproduce AGZ result and extended calculation to higher order in g Caron-Huot (2010)
- Arnold+Dogan used 2-body scattering process for thermal medium to calculate momentum broadening distribution in the UV Arnold, Dogan (2008)
- Vitev+Ovanesyan used SCET upon treating the medium in an opacity expansion involving only one or two gluon insertions from the medium obtaining Gaussian distribution in the IR Ovanesyan, Vitev (2011)

Comparison to literature

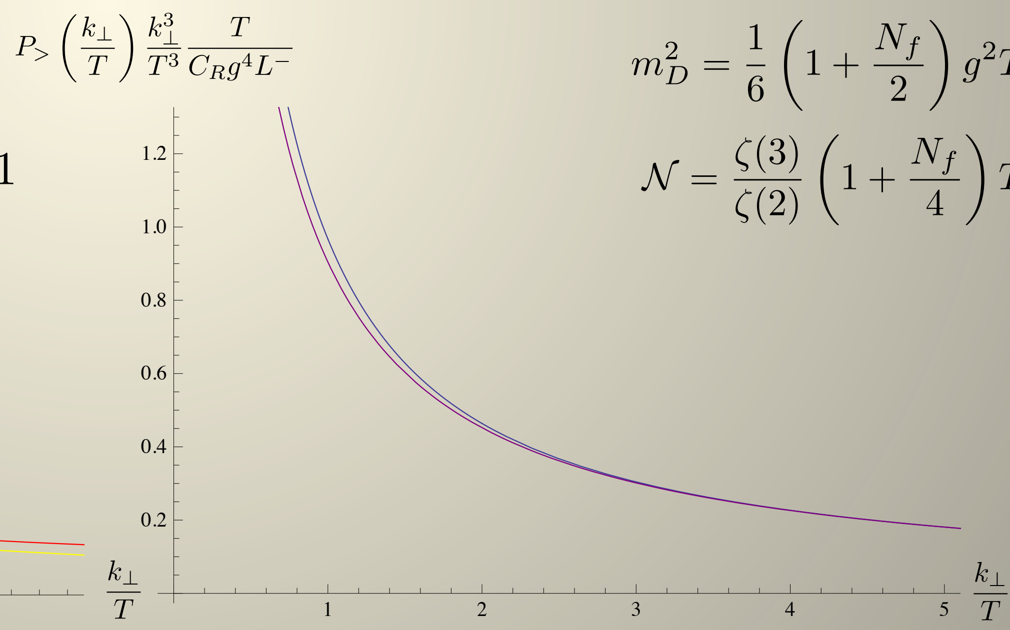
- In the UV limit we reproduce Arnold+Dogan.
- In the IR limit, agree with literature for HTL self-energies and thus estimate corrections when using full self-energies.

$$P_{IR}(k_\perp) = C_R \frac{g^2 T m_D^2}{k_\perp^2 (k_\perp^2 + m_D^2)}, \quad k_\perp \ll T$$ Aurenche, *et al.* (2002), Caron-Huot (2010)

$$P_{UV}(k_\perp) = C_R \frac{g^4 \mathcal{N}}{k_\perp^4}, \quad k_\perp \gg T$$ Arnold, Dogan (2008)



$g = 0.1$

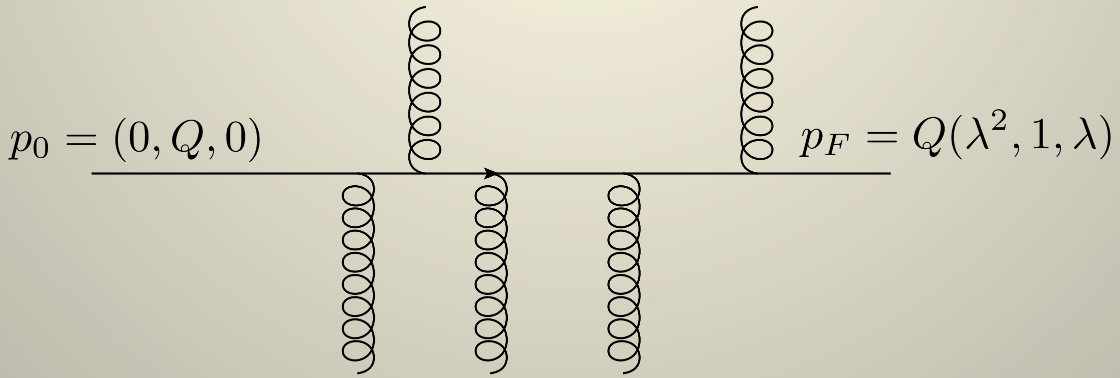


$$m_D^2 = \frac{1}{6}\left(1 + \frac{N_f}{2}\right) g^2 T^2$$

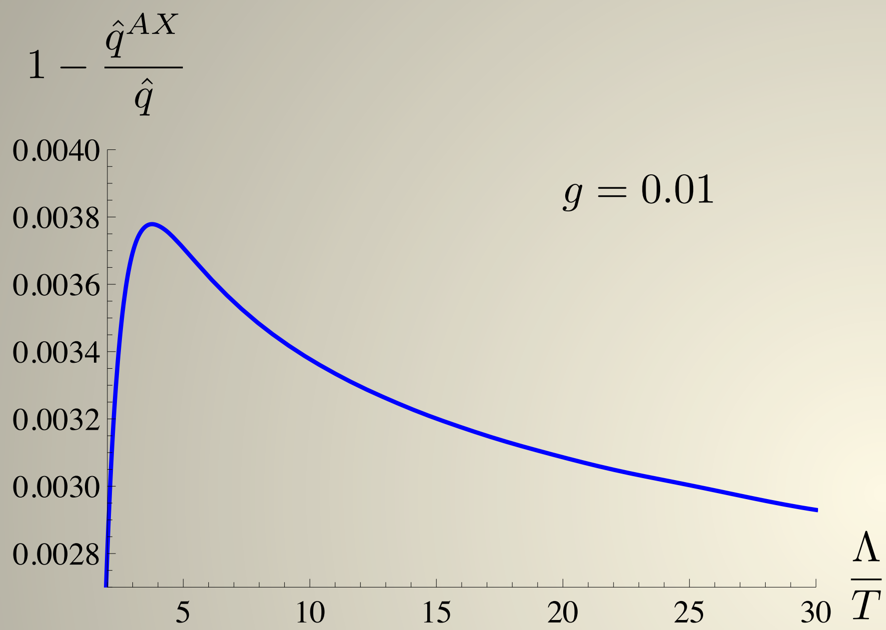
$$\mathcal{N} = \frac{\zeta(3)}{\zeta(2)}\left(1 + \frac{N_f}{4}\right) T^3$$

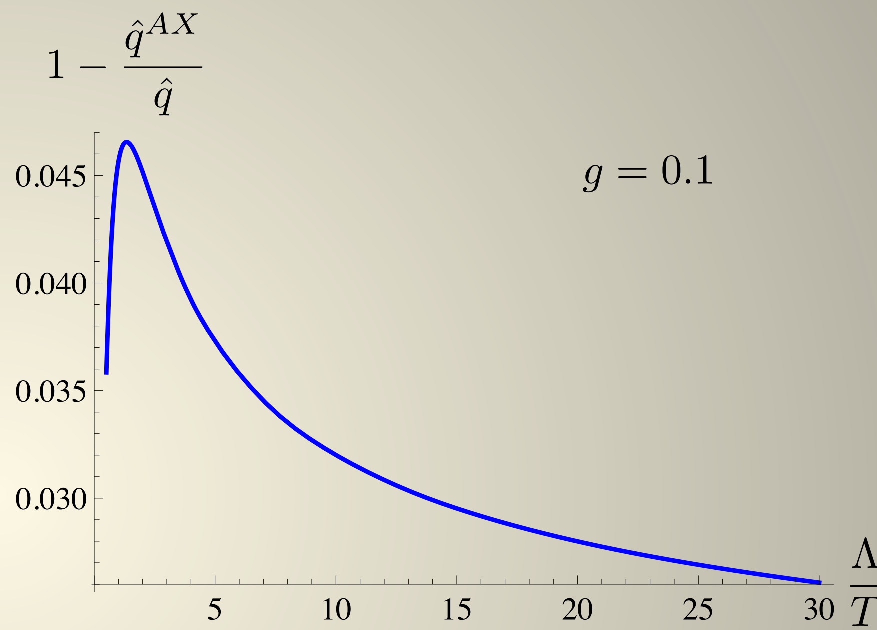
Jet quenching parameter

- Defined by $\hat{q} = \frac{1}{L}\int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$
- Estimates transverse momentum picked-up by particle per distance traveled.
- Second moment of probability distribution $P(k_\perp)$ (other moments might be of interest too).
- "Clean" field theoretical definition, experimentally definition is more elaborate.



Jet quenching parameter





$\hat{q}^{AX}$ – Arnold, Xiao (2008)

$\hat{q}$ – Second moment of $P_>(k_\perp)$

- In the UV limit logarithmically divergent, integrate up to scale Λ.
- Agrees well with our result, corrections originate from the IR region.
- First moment of distribution, which is finite in the UV, might be of interest.

AdS/CFT estimation

- If we turn to the estimation of transverse momentum broadening distribution in the regime of strong coupling, the only tool to use is AdS/CFT correspondence.
- For the adjoint representation

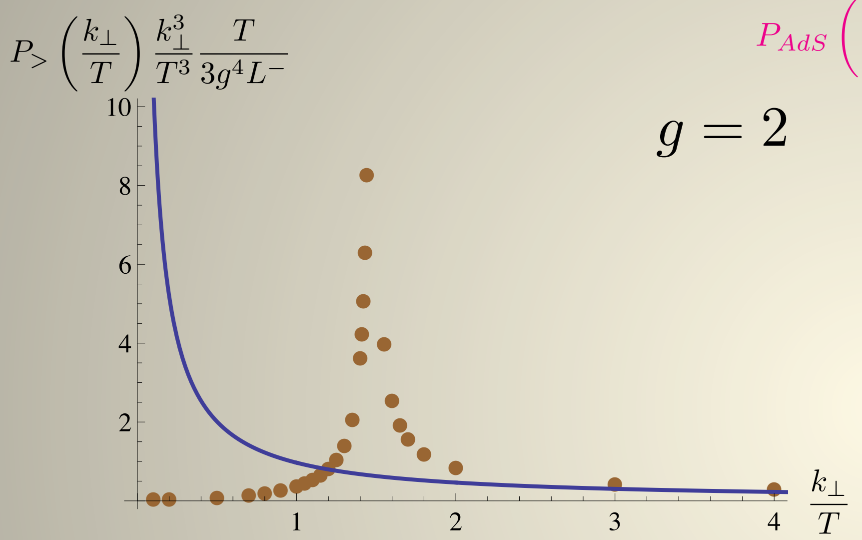
D'Eramo, Liu and Rajagopal (2010)

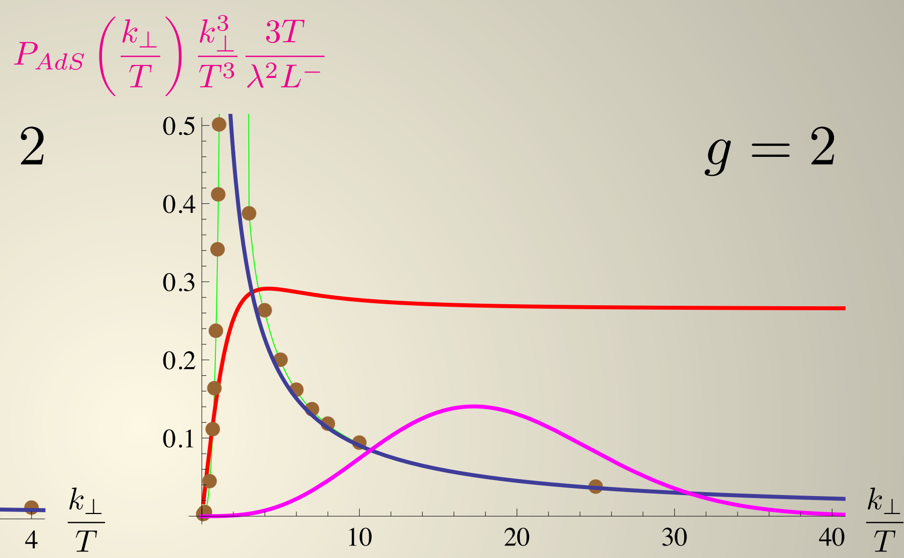
$$P_{AdS}(k_\perp) = \frac{4\sqrt{2}a}{\pi\sqrt{\lambda}T^3L^-}\exp\left[-\frac{\sqrt{2}ak_\perp^2}{\pi^2\sqrt{\lambda}T^3L^-}\right]$$

$a \approx 1.311$
$\lambda = g^2 N_c$

- Distribution is Gaussian and therefore probability to pick up high transverse momentum is small.
- This is very different from the case of weak coupling which behaves as $1/k_\perp^4$ in the UV limit.
- Thus probability to pick up $k_\perp \gg T$ is much bigger for weak coupling estimations as opposed to strong coupling and this might be ascribed to the presence of the quasiparticles in the weak coupling case.

Comparison to AdS/CFT for $g = 2$ ($\alpha \sim 0.3$), L = 5 fm, T = 300 MeV





$$\frac{\int_{k_{\perp min}}^{\infty} dk_\perp k_\perp P_>(k_\perp)}{\int_{k_{\perp min}}^{\infty} dk_\perp k_\perp P_{AdS}(k_\perp)} = \begin{cases} 8.6945, & \text{for } k_{\perp min} = 7.5 \text{ GeV} \\ 56.4725, & \text{for } k_{\perp min} = 10 \text{ GeV} \\ 2.72977 * 10^8, & \text{for } k_{\perp min} = 20 \text{ GeV} \end{cases}$$

- Illustrates at the qualitative level that in the weak coupling case it is much more likely to receive a high transverse momentum kick.

Brown dots – full calculation (green line – interpolation)
Blue line – Expression in the UV limit
Red line – with HTL approximated self-energies
Magenta line – AdS/CFT implied distribution

Summing up

- Wilson line expression for momentum broadening obtained using SCET is taken one step further by evaluating leading order contribution in weakly coupled equilibrium plasma.
- Full field theoretical calculation of transverse momentum broadening beyond HTL and "soft" approximations.
- $P_>(k_\perp)$ valid for all transverse momentum region.
- We reproduce $P_>(k_\perp)$ in the UV limit obtained previously and estimate corrections in the IR limit for previous results in the literature.
- Much more likely to pick up the kick of high transverse momentum than in strong coupling AdS/CFT estimations.

Thank you for the attention!

Back-up slides

Explicit expressions

- Probability distribution function

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) - (2\pi)^2\delta^2(k_\perp)2g^2C_R\int_0^{L^-} \mathrm{d}y_1^- \int_0^{y_1^-} \mathrm{d}y_2^- \mathrm{Re}\left[D_{11}^{++}(0, y_1^- - y_2^-, 0)_{g^2}\right] +$$

$$g^2C_R\int_0^\infty \mathrm{d}^2x_\perp \int_0^{L^-} \mathrm{d}y_1^- \int_0^{L^-} \mathrm{d}y_2^- e^{-ik_\perp\cdot x_\perp} D_>^{++}(0, y_1^- - y_2^-, x_\perp)_{g^2}$$

$$\equiv (2\pi)^2\delta^2(k_\perp) + (2\pi)^2 P_{11}(k_\perp) + P_>(k_\perp)$$

- Taking the limit $L^- \to \infty$

$$P(k_\perp) \equiv (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) + (2\pi)^2 P_{11}(k_\perp)$$

$$= (2\pi)^2\delta^2(k_\perp) + g^2C_R L^- \int \frac{dq_-}{2\pi} D_>^{++}(0, q_-, k_\perp)_{g^2} -$$

$$\delta^2(k_\perp) g^2 C_R L^- \int \frac{dq_- d^2q_\perp}{2\pi} \mathrm{Re} D_{11}^{++}(0, q_-, q_\perp)_{g^2}$$

Hard Thermal Loops

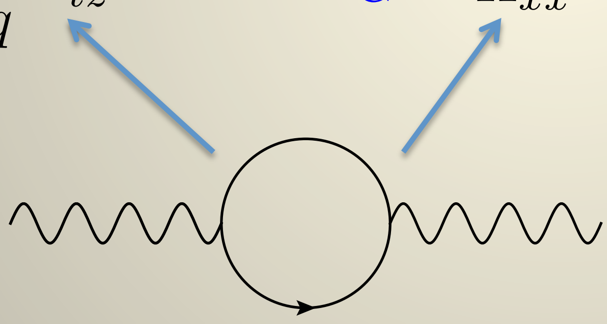
- Retarded propagator for photon

$$D^R_{\mu\nu}(q_\mu) = \frac{i}{Q^2 - G} P^T_{\mu\nu} + \frac{i}{Q^2 - F} P^L_{\mu\nu} - i\zeta \frac{q_\mu q_\nu}{Q^2}$$

ζ - gauge fixing parameter

- HTL "self energies"

$$F = \frac{iQ^2}{q_0 q} \Pi_{tz} \qquad G = \Pi_{xx} = m^2 - \frac{F}{2} \qquad q \equiv |\vec{q}|$$



- It turns out that for gluon field $D_R^{\mu\nu} \to \delta^{ab} D_R^{\mu\nu}$ with thermal mass

$$m^2 \to m^2_{QCD} = \frac{3}{4}(gT)^2$$

Probability distribution and "plus" distribution function

- The Plus Distribution Function for some function $g(x)$ is defined by

$$[\theta(x)g(x)]_+ = \lim_{\beta \to 0} \frac{d}{dx}[\theta(x-\beta)G(x)] \quad \text{with} \quad G(x) = \int_{x_0}^{x} dx' g(x')$$

Ligeti, Stewart, Tackman (2008)
Stewart, Tackman, Waalewijn (2010)

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) - \delta^2(k_\perp)\int d^2q_\perp P_>(q_\perp)$$

- Extract $\delta^2(k_\perp)$ contribution from the second term to see cancelation.
- Introducing the scale $k_{\perp 0}$, we see the finite IR behavior:

$$P(k_\perp) = \delta^2(k_\perp)\left((2\pi)^2 - \int_{k_{\perp 0}}^{\infty} dq_\perp 2\pi q_\perp P_>(q_\perp)\right) + [P_>(k_\perp)]_+$$

- Can interpret $[P_>(k_\perp)]_+$ as $P(k_\perp)$, for $k_\perp > T$.

Satisfy

$$\int_0^{k_{\perp 0}} dk_\perp 2\pi k_\perp [P_>(k_\perp)]_+ = 0$$