External Parameters and Chiral Symmetry Breaking in Holographic Duals of Large N Gauge Theories

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Credits

Primarily based on: in collaboration with M. Sohaib Alam and Vadim Kaplunovsky
to appear

Earlier works done: in collaboration with Tameem Albash, Veselin Filev and Clifford V. Johnson
0709.1547, 0709.1554, 0803.0038

Related works: Karch et al, Erdmenger et al, Evans et al
and many more

Outline

Introduction and Motivation

conventional wisdom

Gauge-gravity duality: specific realizations

Klebanov-Witten background

The dynamics of flavours and chiral symmetry breaking

non-susy D7/anti-D7 branes

External parameters and chiral symmetry breaking: phase structure

Electric and Magnetic field at finite temperature

Conclusions and Outlook

General lessons etc.

Introduction & motivation

We want to learn about strongly coupled systems

e.g., Quark-Gluon Plasma at RHIC, strongly coupled condensed matter systems etc.

Gauge-gravity duality is a remarkable tool

soluble models

String theory provides concrete examples of this duality

specific brane constructions (top down)

Introduction & motivation

We want to learn about strongly coupled systems

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QCD

Gauge-gravity duality is a remarkable tool

soluble models

String theory provides concrete examples of this duality

specific brane constructions (top down)

Conventional wisdom

$3 = \infty$: an useful approximation

The duality works for large N gauge theories (super Yang-Mills)

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An example comparison:

	QCD	SYM	
$T = 0$	N=3, confinement, discrete spectrum, scattering, ...	large N, deconfined, conformal, supersymmetric, ...	
$T = T_c$	strongly coupled plasma of gluons and fundamental matter; deconfined, screening, finite correlation length, ...	strongly coupled plasma of gluons and adjoint + fundamental matter; deconfined, screening, finite correlation length, ...	⇨ Our focus
$T \gg T_c$	becomes weakly coupled	remains strongly coupled	

Elusive QCD features

Confinement

e.g. confining holographic duals: Klebanov-Strassler, global AdS

Confinement/deconfinement transition

e.g. Hawking-Page transition

Chiral symmetry breaking and chiral phase transition

e.g. Sakai-Sugimoto model

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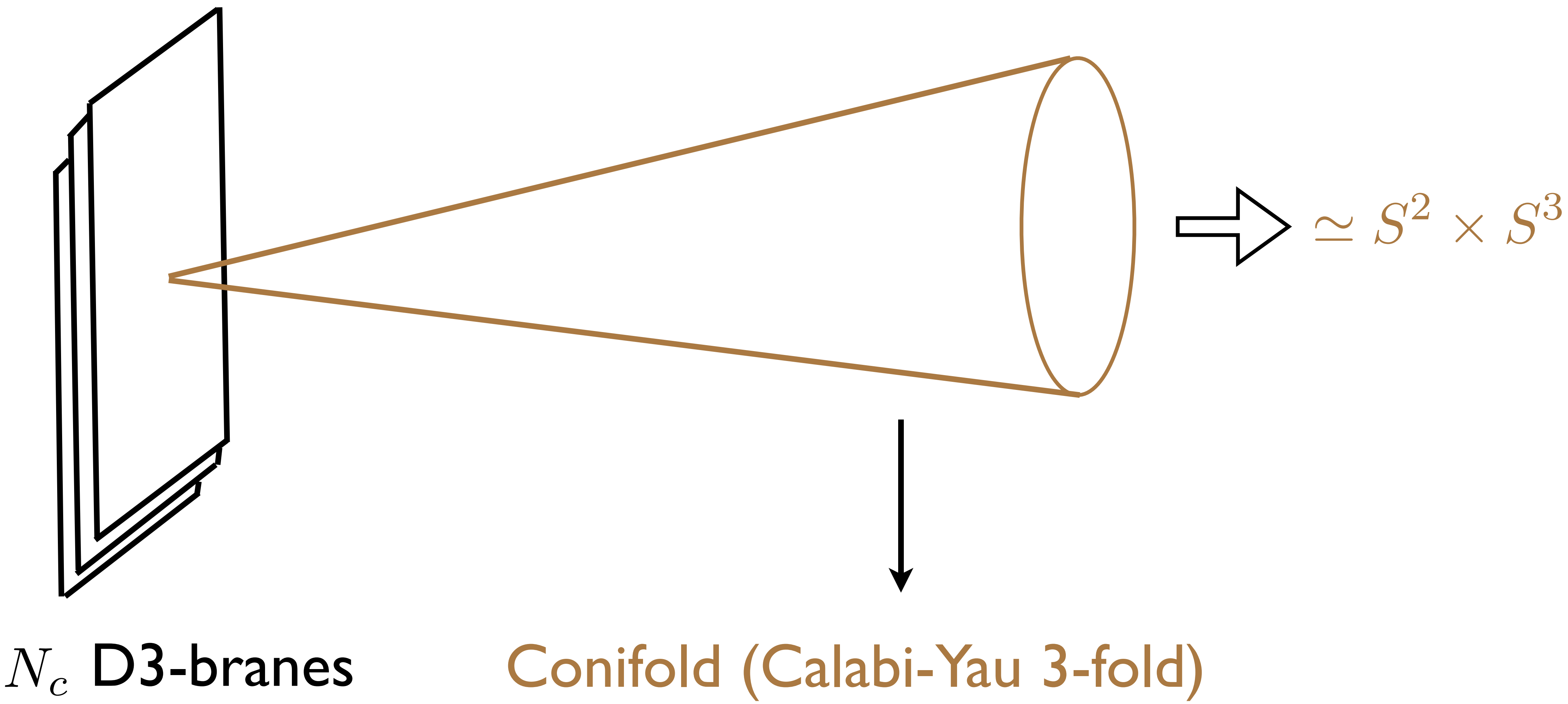
e.g. Sakai-Sugimoto model

Kuperstein-Sonnenschein model

Specific example: the brane picture



Near-horizon geometry is: $AdS_5 \times T^{1,1}$ $(T^{1,1} \simeq S^2 \times S^3)$

Klebanov-Witten background (Romans' solution)

Specific example: the duality

String theory in $AdS_5 \times T^{1,1}$

isometry: $SO(4,2)$

$SU(2) \times SU(2) \times U(1)$

AdS-Schwarzschild geometry

$\mathcal{N} = 1$ quiver gauge theory

superconformal, global R-symmetry

$SU(N_c) \times SU(N_c)$ gauge group

two bi-fundamental chiral superfields

finite temperature
(broken susy)

The gravitational background

The metric: $$ds^2 = -\frac{r^2}{R^2} f(r) dt^2 + \frac{r^2}{R^2} d\vec{x}^2 + \frac{R^2}{r^2}\frac{dr^2}{f(r)} + R^2 ds^2_{T^{1,1}} \ ,$$

$$ds^2_{T^{1,1}} = \frac{1}{3}\left[\frac{1}{4}\left(f_1^2 + f_2^2\right) + \frac{1}{3} f_3^2 + \left(d\theta - \frac{1}{2} f_2\right)^2 + \left(\sin\theta d\phi - \frac{1}{2} f_1\right)^2\right] \ ,$$

$$R^4 = \frac{27}{4}\pi g_s N_c \alpha'^2 = \lambda \alpha'^2 \ . \qquad \{f_i\} \equiv S^3 \ , \quad \{\theta, \phi\} \equiv S^2 \ .$$

α' : string tension

λ : ‘tHooft coupling

g_s : string coupling

$$f(r) = 1 - \left(\frac{r_H}{r}\right)^4 \ , \quad T = \frac{r_H}{\pi R^2} \ .$$

Euclideanize: $t \to i\tau$

Adding flavours

The background is obtained from near-horizon limit of a stack of N_c D-branes

We put N_f flavour-branes in the probe limit, i.e. $N_f \ll N_c$

The classical dynamics is determined by the probe action

$$S = -\mu_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det\left(\mathrm{P}[G+B] + 2\pi\alpha' F\right)} + S_{\mathrm{WZ}}$$

The DBI piece

The Wess-Zumino piece

Here we consider: $p = 7$

Adding flavours

	0	1	2	3	4	5	6	7	8	9
D3	–	–	–	–	•	•	•	•	•	•
D7, $\overline{\text{D7}}$	–	–	–	–	–	–	–	–	•	•

- stands for extended

• stands for point-like

3-3 strings: adjoint sector

3-7 strings: fundamental matter

7-7 strings: global symmetry

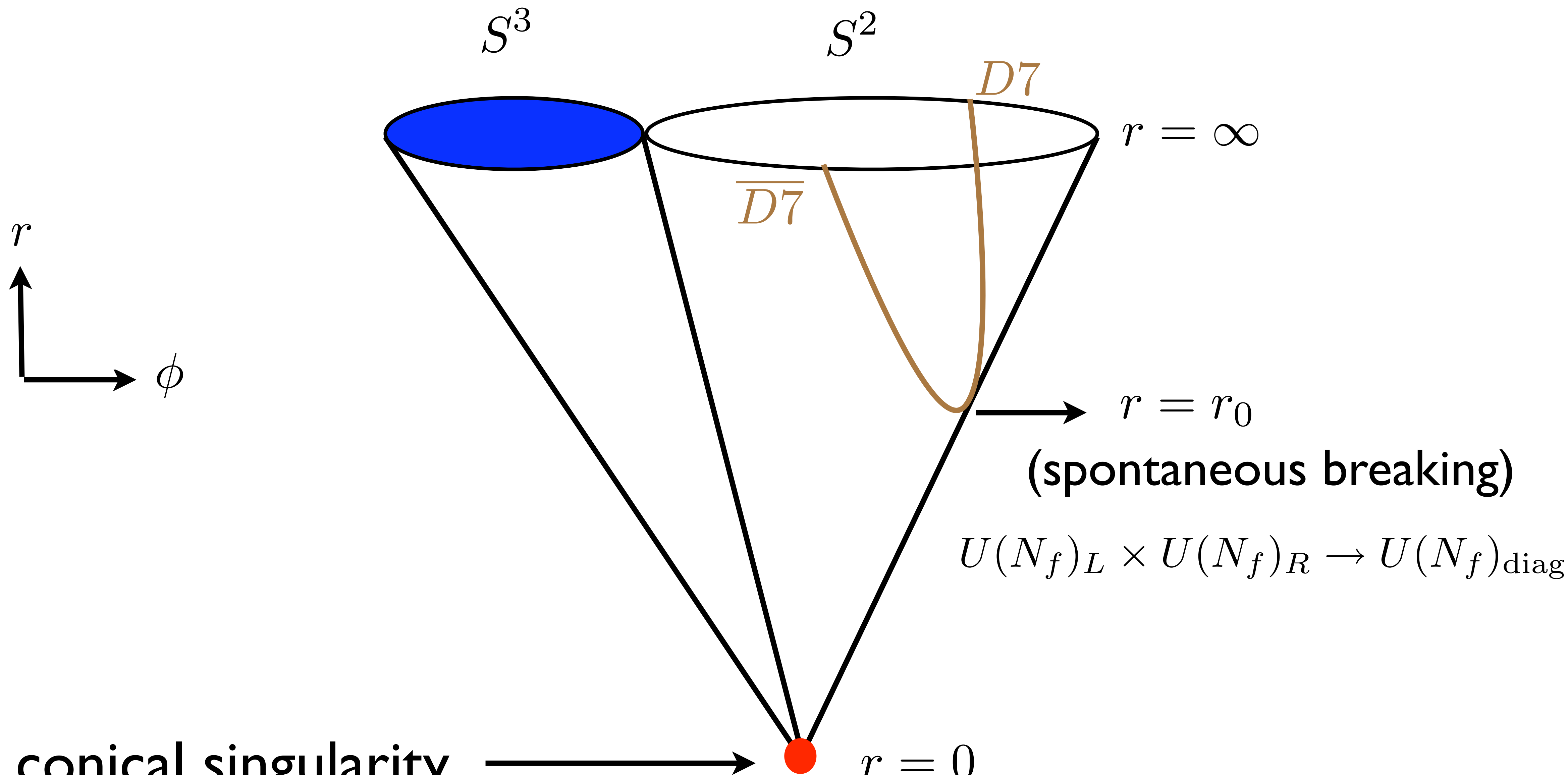
$$ds^2 = \frac{r^2}{R^2} f(r) d\tau^2 + \frac{r^2}{R^2} d\vec{x}^2 + \frac{R^2}{r^2}\frac{dr^2}{f(r)} + R^2 ds^2_{T^{1,1}} \ .$$

2-plane: $\{\theta(r), \phi(r)\}$

Equatorial embedding: $\theta = \frac{\pi}{2} \ , \quad \phi = \phi(r)$

Kuperstein & Sonnenschein '08

T=0 Physics



embedding function: $$\cos\left(\frac{4}{\sqrt{6}}\phi(r)\right) = \left(\frac{r_0}{r}\right)^4 .$$

asymptotic angle separation: $$\Delta\phi_\infty = \frac{\sqrt{6}}{4}\pi .$$

Kuperstein & Sonnenschein '08

Key properties

The probe D7 and anti-D7 each break supersymmetry completely.

non-holomorphic embedding

Conformal theory.

$\Delta\phi_\infty$ *is* r_0 *independent.*

Spontaneous breaking of chiral symmetry.

$U(N_f)_L \times U(N_f)_R \to U(N_f)_{\rm diag}$

Spontaneous breaking of conformal symmetry.

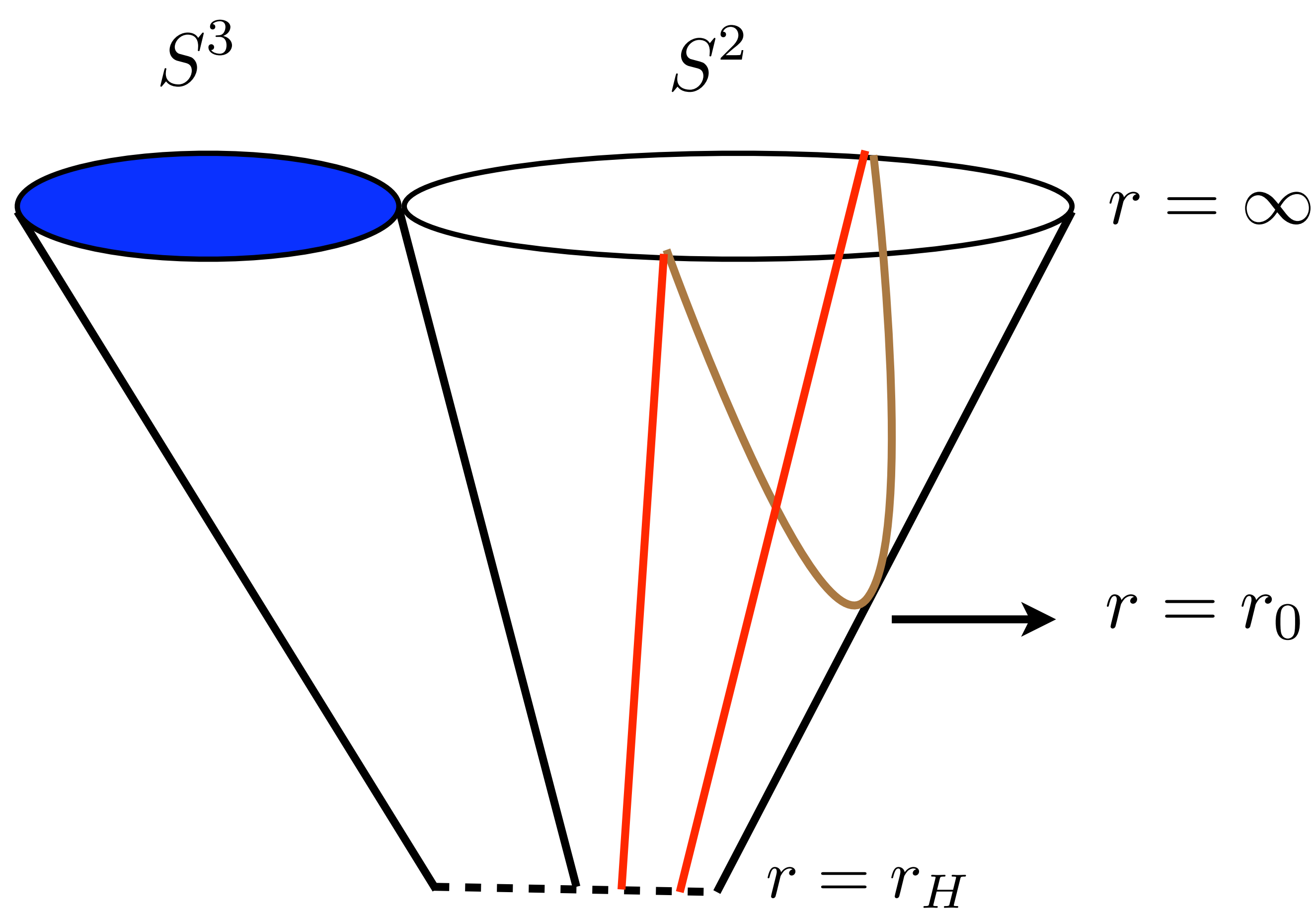
an IR scale (modulus) is generated

Perturbatively stable.

no pathology

Dymarsky, Melnikov & Sonnenschein '10

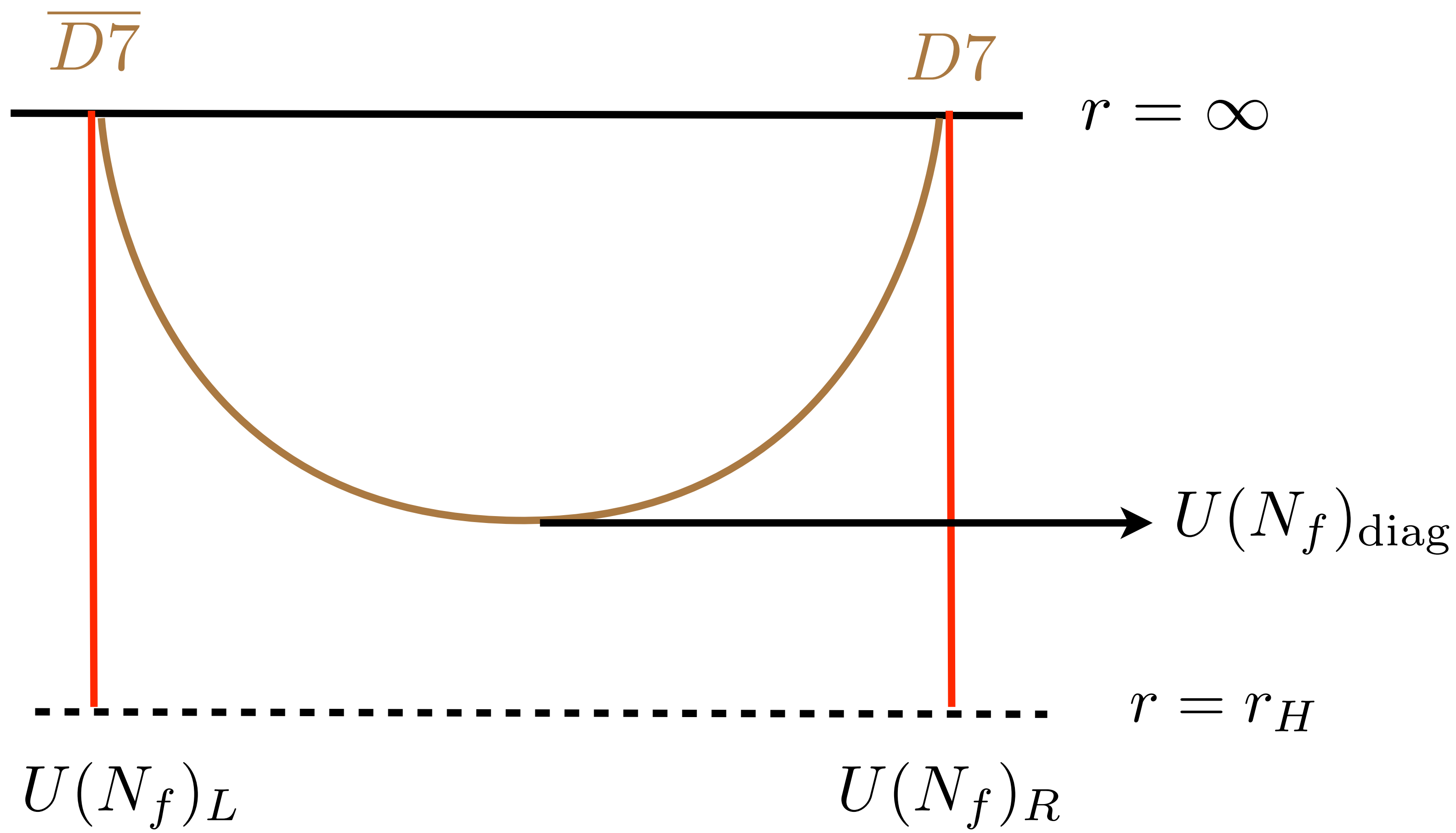
Introducing temperature



Two classes of embeddings: (i) probe branes reach black hole. *parallel-shaped*

(ii) probe branes join above the black hole. *U-shaped*

Introducing temperature



Favoured embedding determined by energetics.

$$S_{\mathrm{probe}}^{\mathrm{on-shell}} \leftrightarrow TF$$

Helmholtz free energy

Parallel shaped are always favoured.

Key features

Conformal symmetry explicitly broken by temperature.

Both chiral symmetry broken and restored phases are available as solutions.

No phase transition: symmetry restoration occurs at any temperature.
Need another scale to have a non-trivial phase structure.

Meson melting transition: quasinormal modes of the bulk fluctuations.

New scale in the game

We can excite gauge fields on the world volume of the probe brane itself

Recall the DBI action contains: $\sqrt{-\det(\mathrm{P}[G+B]+2\pi\alpha' F)}$

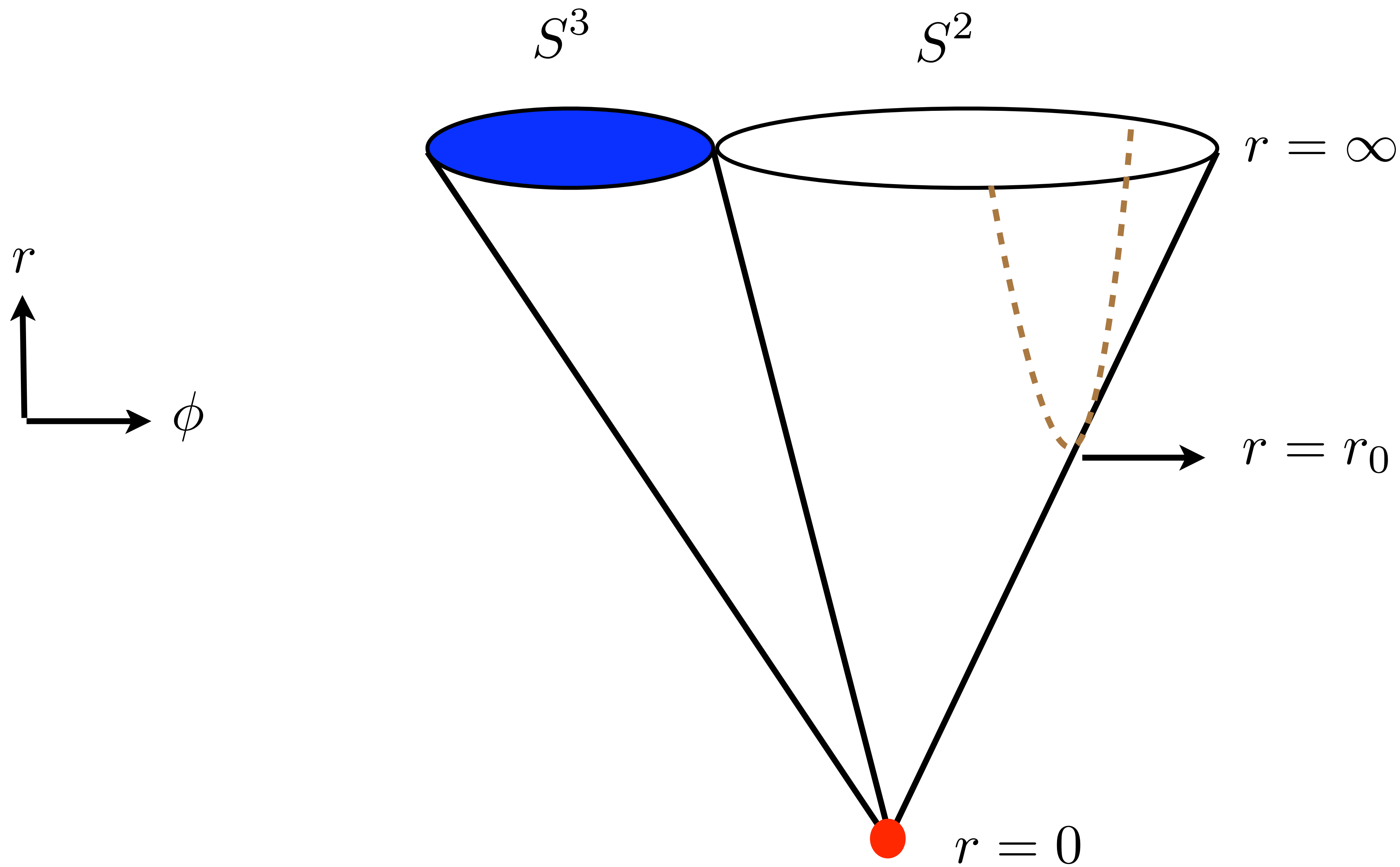
Still in the probe limit, background does not care

Have to satisfy the equations of motion

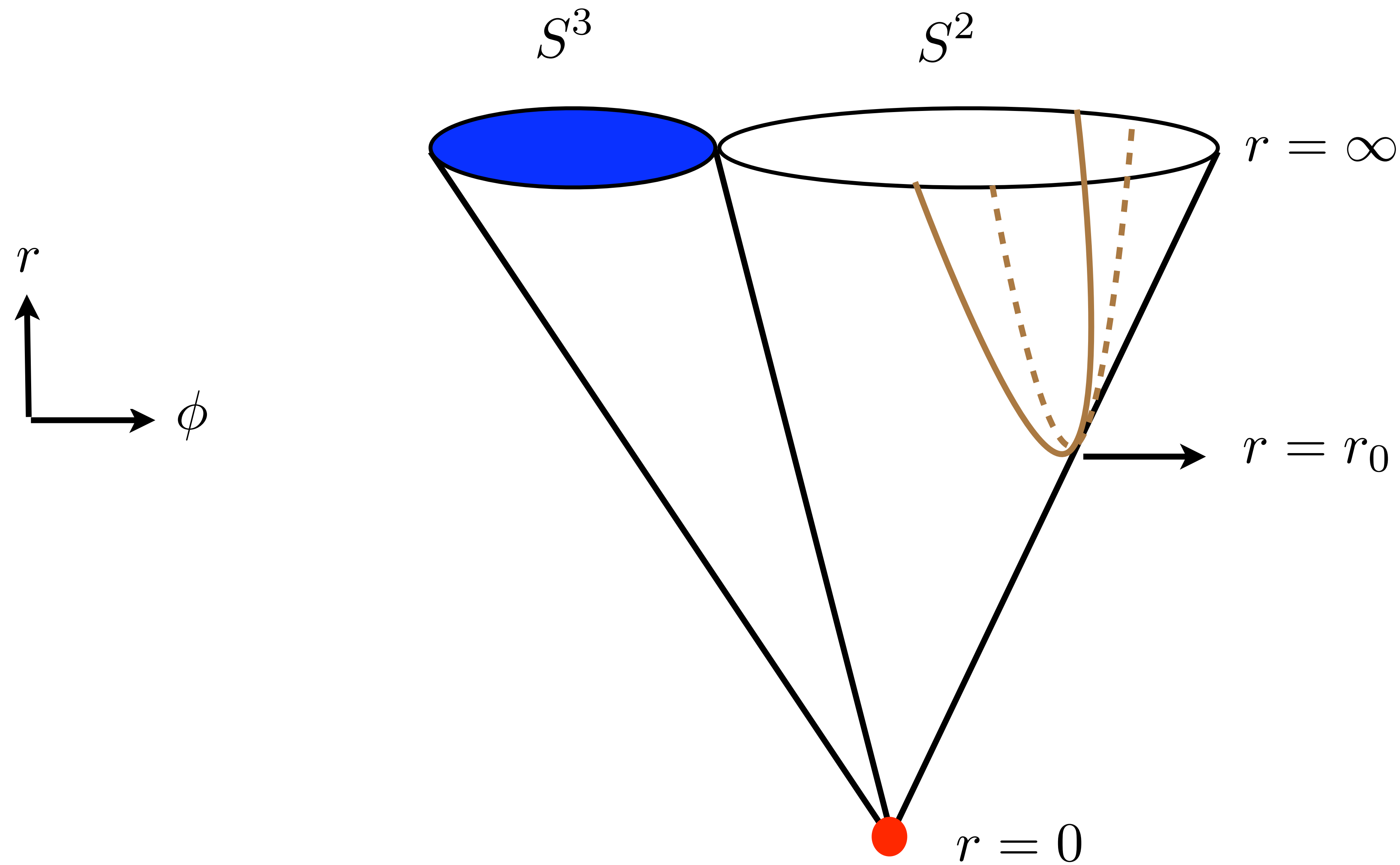
Simplest ansatz: $A_3 = Hx^2$

Introduces a magnetic field to which only the flavours couple: $F_{23} = H$

Physics at T=0



Physics at T=0



The magnetic field increases the (symmetry breaking) coupling:

$$\Delta\phi_\infty(H) > \Delta\phi_\infty(0)$$

Some features

Conformal symmetry explicitly broken by the magnetic field.

Magnetic field enhancing the symmetry breaking mechanism.

magnetic catalysis in chiral symmetry breaking

The coupling is not a constant anymore.

depends on the external parameters introduced.

Two competing scales

Finite temperature alone restores chiral symmetry.

Magnetic field alone enhances chiral symmetry breaking.

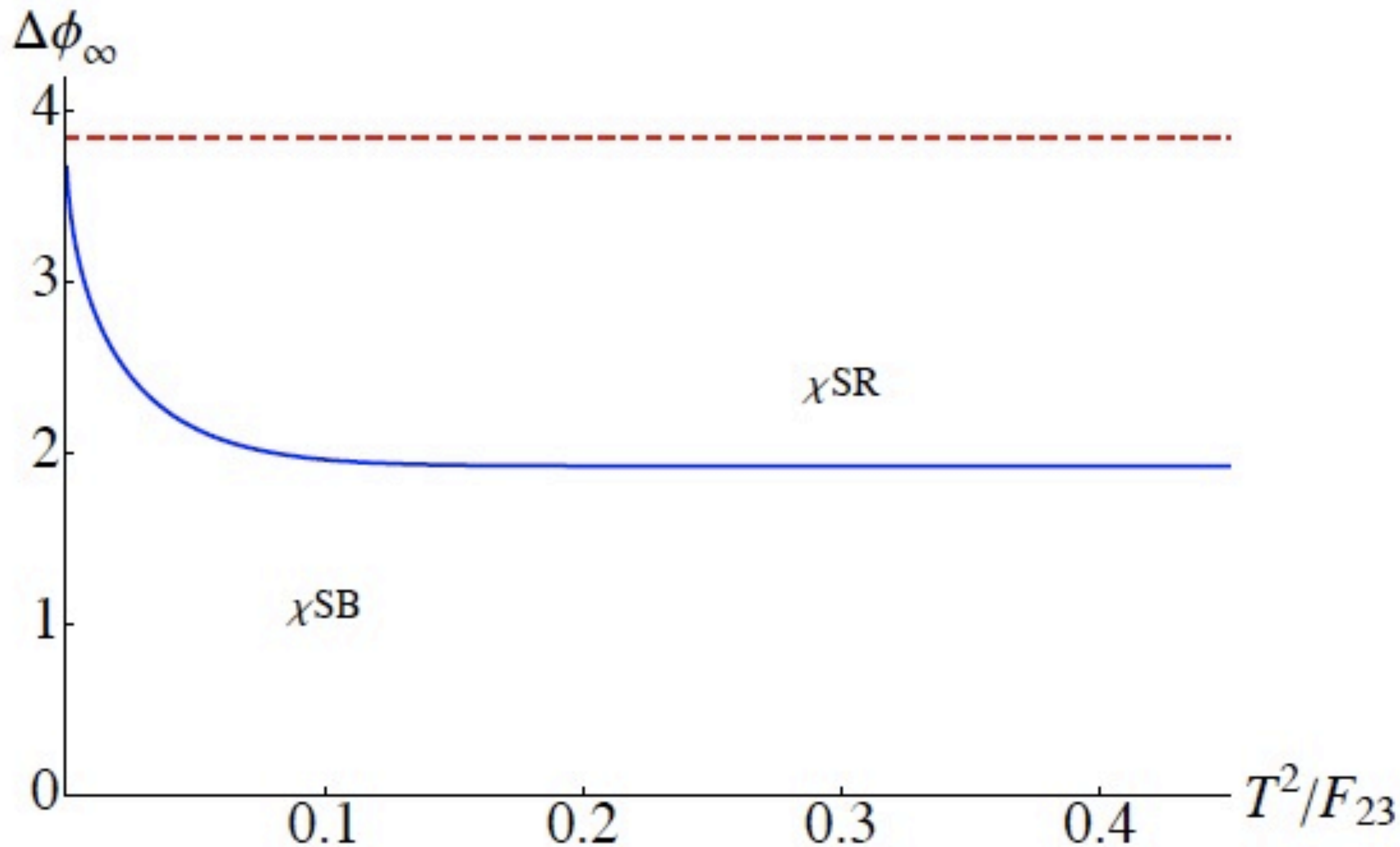
First order phase transition when both are present.

Phase boundary: obtained by looking at the energy difference of the two classes of embeddings.

Described by the curve: $\Delta\phi_\infty(T^2/H)$

monotonically decreasing

The phase diagram



Magnetic Catalysis as expected

Introducing an electric field

Excite an appropriate gauge field on the probe: $A_x = -Et + A(r)$ ⁇

Gives a constant electric field: $F_{tx} = -E$

Assume: $A(r) = 0$, $\quad \mathcal{L}_{\rm DBI} \sim \left(1 - \frac{E^2}{r^4 f(r)}\right)^{1/2}$, vanishes at: $r = r_* > r_H$

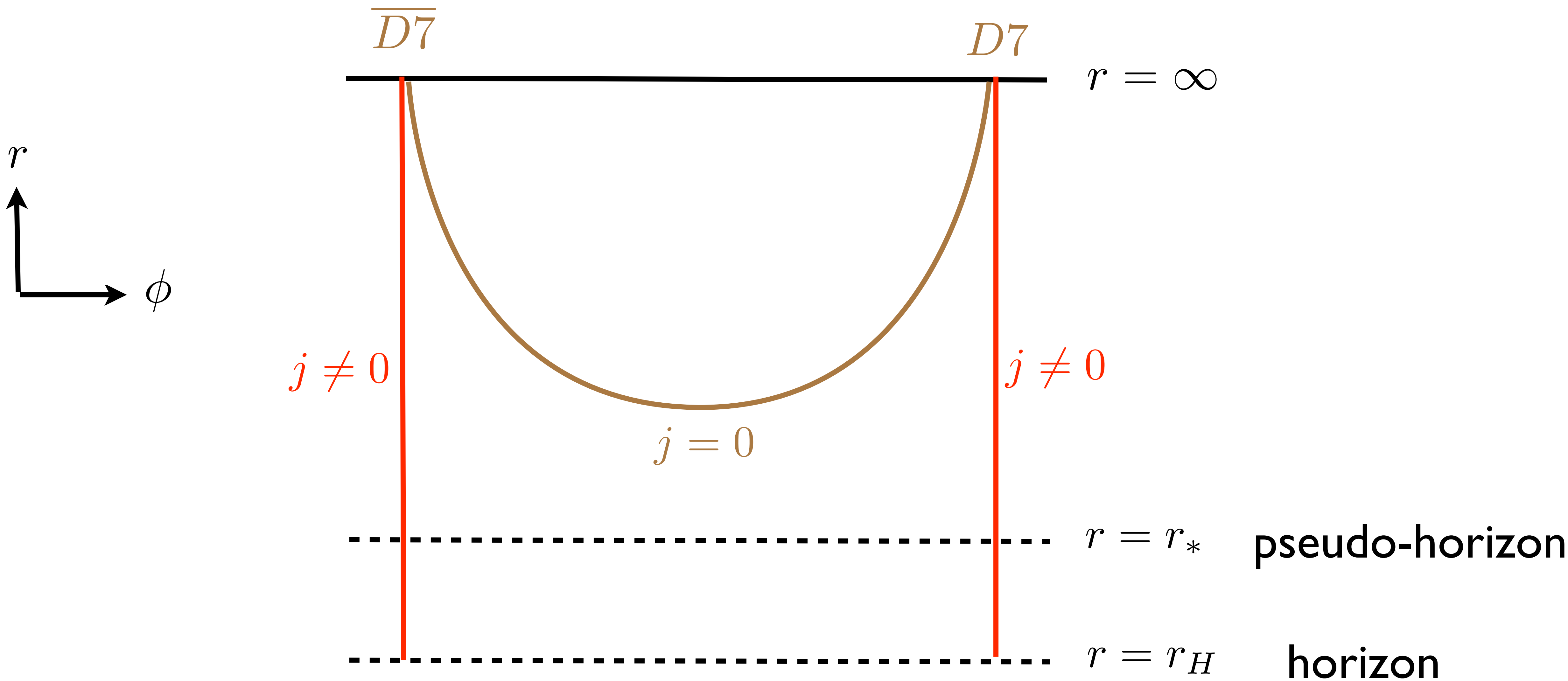
Cannot happen!

If: $A(r) \neq 0$, then E.O.M. $\Longrightarrow \quad \frac{\partial \mathcal{L}}{\partial A'} = j = \text{const}$

AdS/CFT dictionary gives: $j \sim \langle J_x \rangle \equiv$ boundary current

Karch & O'Bannon '07

Introducing an electric field



The condition of parallel branes reach the horizon $\Longrightarrow$ $j(E)$

Ohm's law

Introducing an electric field

Thermodynamic free energy $\longleftrightarrow$ On-shell action

Parallel embeddings are subtle

There is a boundary term: $S_{\mathrm{on-shell}} \sim \int_{r_{\min}}^{\infty} \mathcal{L}(A', r) dr + jA|_{r_{\min}}$.

Usual identification: $r_{\min} = r_H$.

Introducing an electric field

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Usual identification: $r_{\min} = r_H \quad .$

The gauge field blows up at the horizon!

Our proposal: $r_{\min} = r_* = \left(r_H^4 + R^4 E^2\right)^{1/4} \quad .$

The open string metric

Fundamental flavours are ultimately open string degrees of freedom

Open strings in a given gravitational background with an anti-symmetric field "feel" an effective metric

$$\gamma^{ab} = \Big[\underbrace{(G}_{\text{background metric}} + \underbrace{F}_{\text{anti-symmetric 2-form}})_{\rm symm} \Big]^{ab} = \Big[(G+F)^{-1}\, G\, (G-F)^{-1} \Big]^{ab} \ .$$

Seiberg & Witten '99

G : induced metric on the probe

F : gauge field on the probe

γ has a horizon at $r = r_*$

Working with the proposal

Increasing electric field increasingly favours parallel embeddings

Explore the phase diagram with electric and magnetic field
at finite T

Two representative configurations:

$$A_x = -Et + A(r) \ , \quad A_y = Hx \ .$$

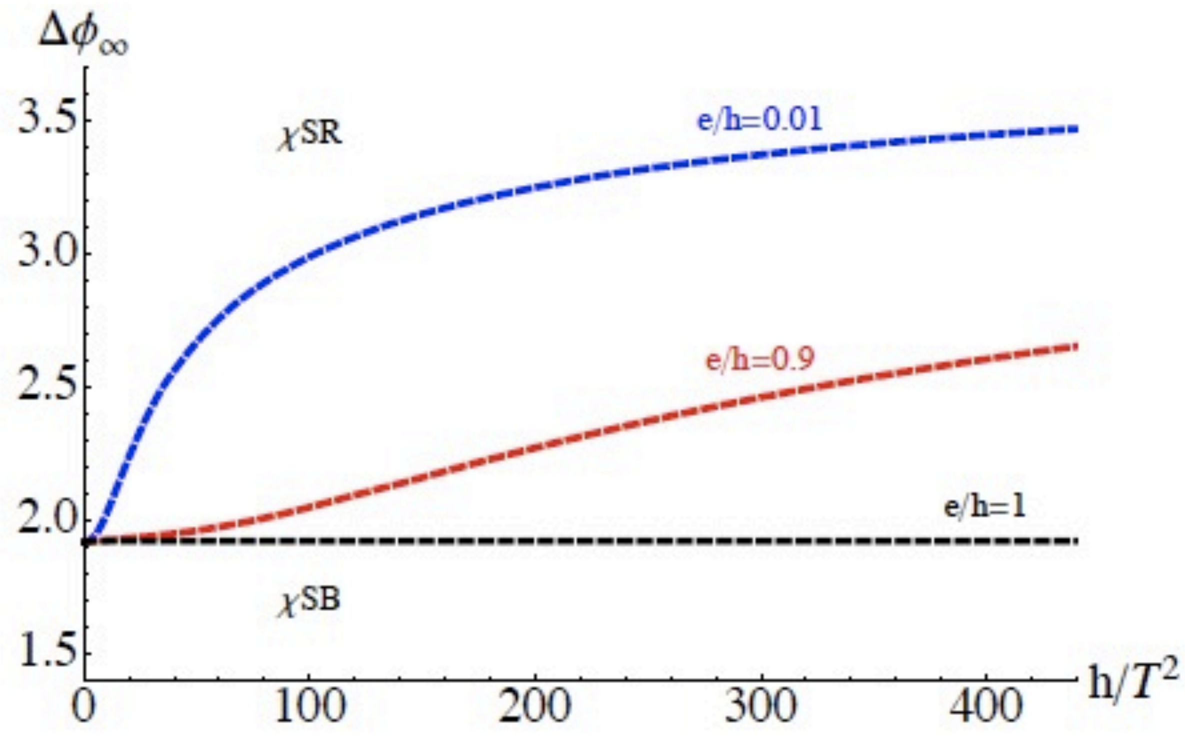
Perpendicular configuration

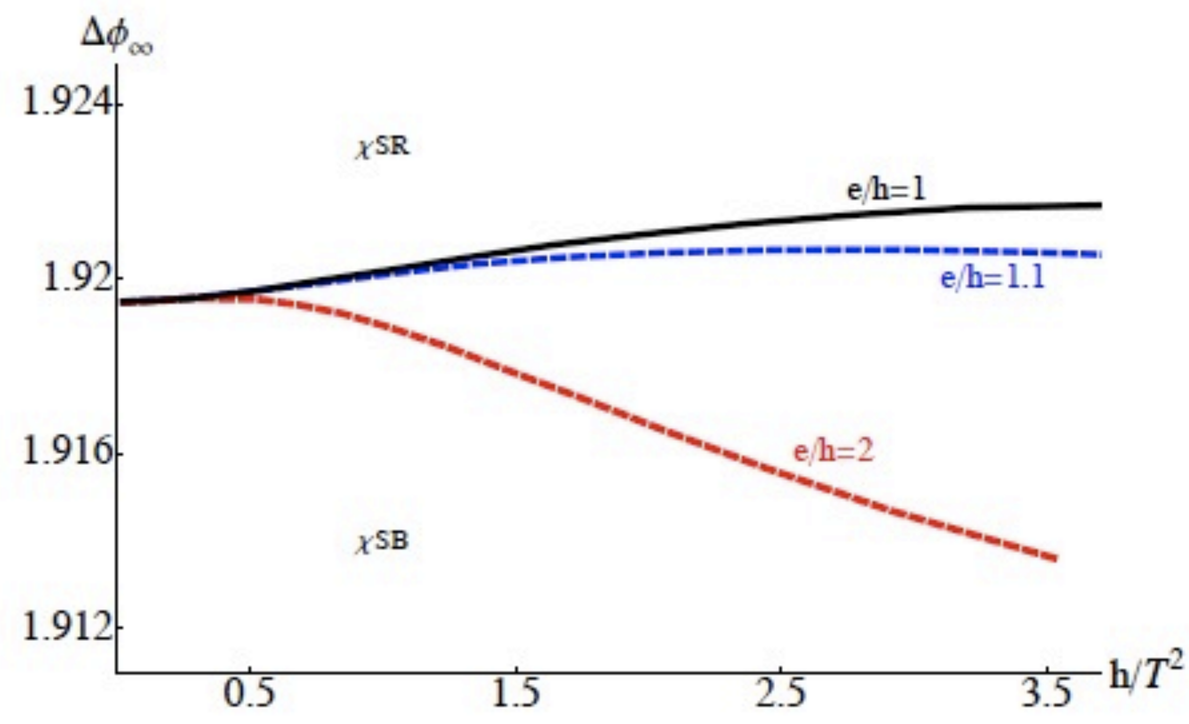
$$A_x = -Et + A(r) \ , \quad A_z = Hy \ .$$

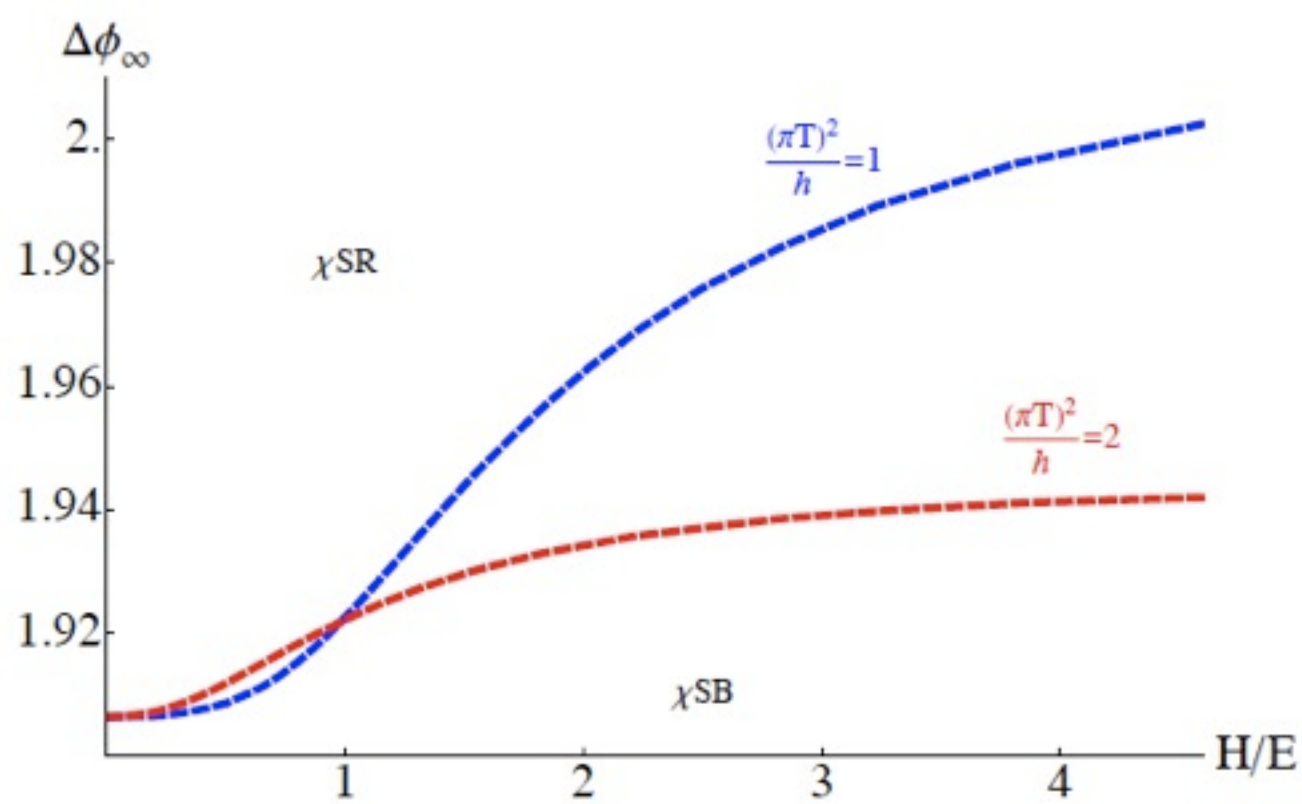
Parallel configuration
(technically more difficult)

r_* depends on both electric and magnetic field

Perpendicular fields







$$e = (2\pi\alpha')\,E$$

$$h = (2\pi\alpha')\,H$$

Parallel fields

The Chern-Simons term contributes $\sim \cos\theta(r)$

Can no longer consider the equatorial embedding

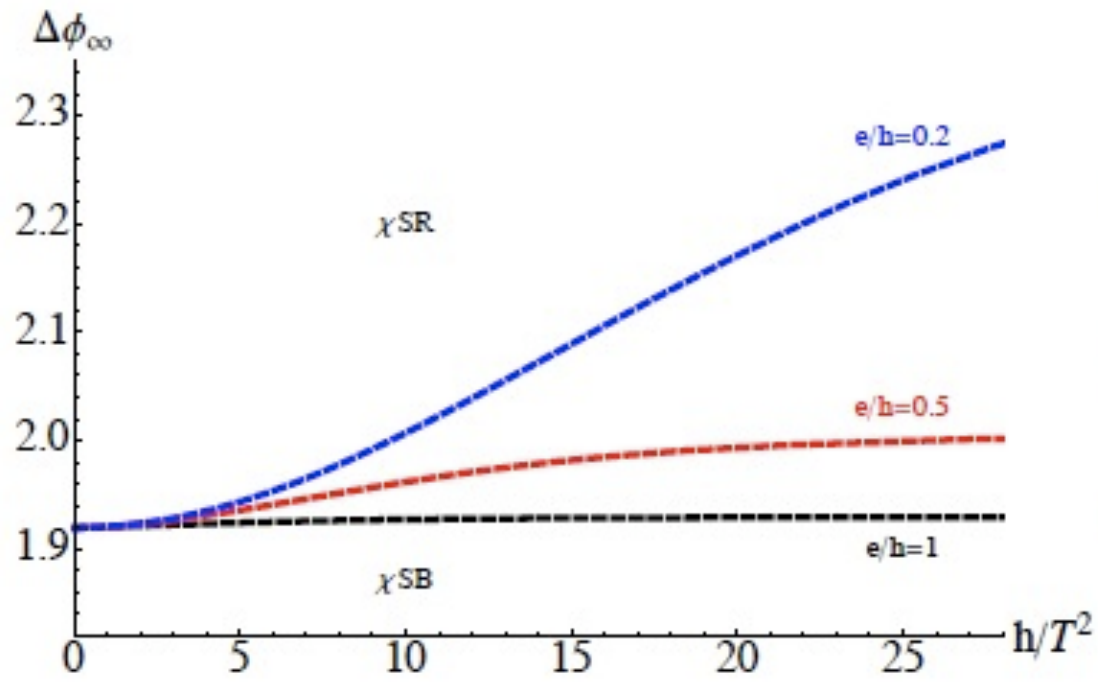
Similar to charged particle in a magnetic monopole potential

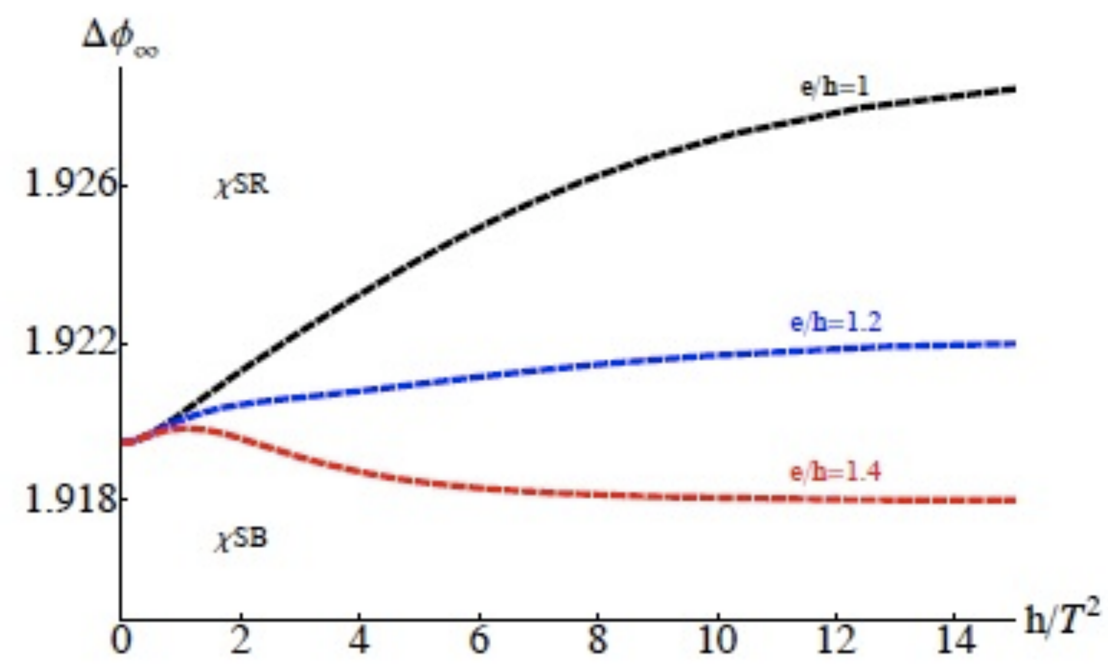
there is a conserved angular momentum

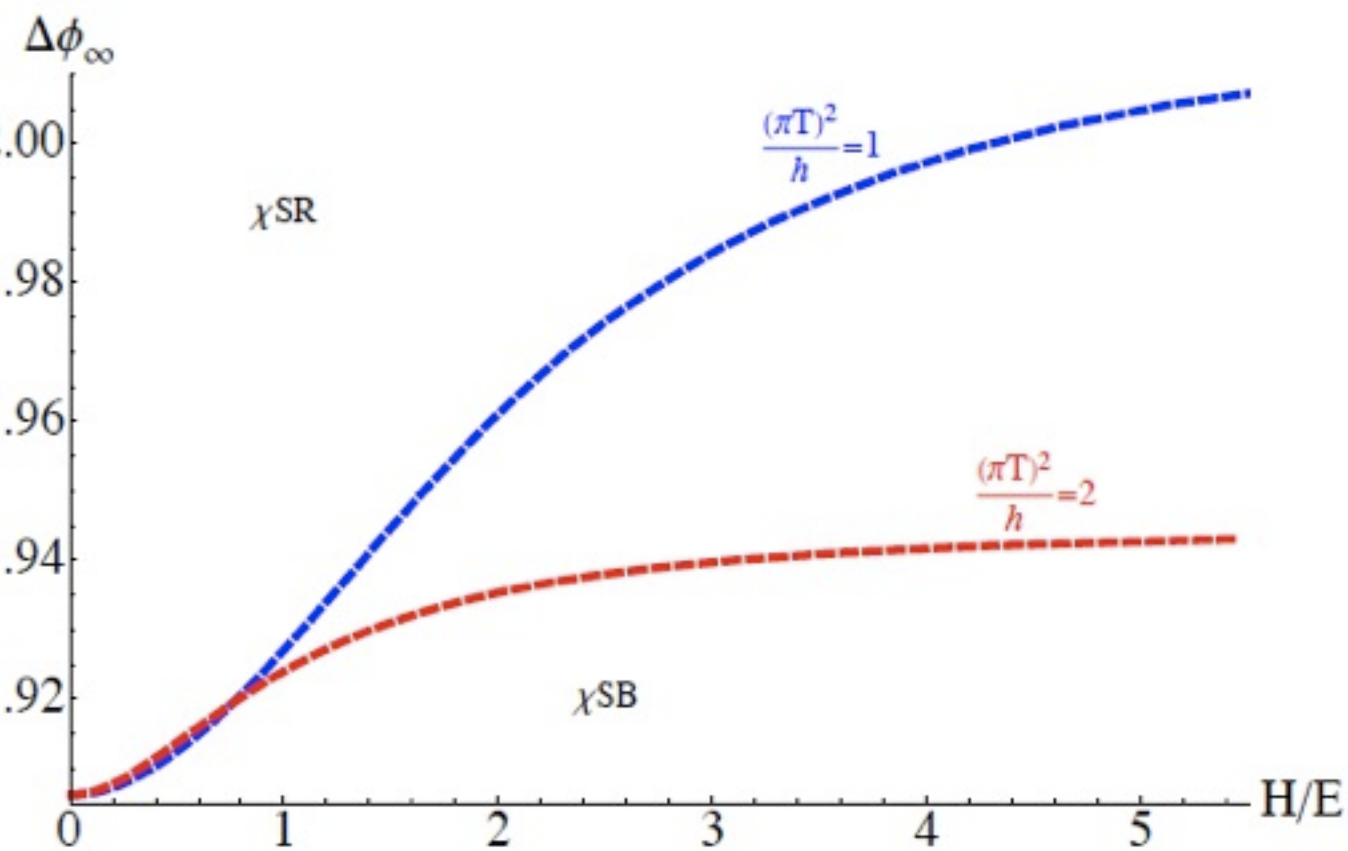
$$\tan\theta_c = -\frac{\mathcal{A}(r)\sqrt{\mathcal{B}(r)}}{k\alpha} \ ,$$

$$k = \frac{2}{3}eh \ , \quad \alpha = \frac{\sqrt{3}}{R} \ .$$

Parallel fields







A brief summary

Ubiquitous effect of magnetic catalysis in chiral symmetry breaking persists at strong coupling

for field theory studies, see e. g. Miransky et al.

Electric field restores the symmetry, drives a flavour current

non-linear conductivity. Karch & O'Bannon

Emergence of pseudo-horizon: a natural way to define thermodynamics in a steady-state system

an intriguing feature

Other holographic models

D7-probes in $AdS_5 \times S^5$ $\longleftrightarrow$ $\mathcal{N}=4$ SYM $+$ $\mathcal{N}=2$ hypers

transverse 2-plane $\longleftrightarrow$ $SO(2) \simeq U(1)$

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T=0 physics is supersymmetric, no chiral symmetry breaking

Babbington et al, Mateos et al, Albash et al, Karch et al ... long list

T=0, non-zero magnetic field induces a chiral condensate

Filev et al '07

non-trivial phase diagram when both T and H are present

Albash et al, Erdmenger et al '07

Other holographic models

N_c D4-branes wrapped on a circle

add D8 and anti-D8 (probe) branes

$\Longleftrightarrow$

(4+1)-dim Yang-Mills with flavours

$U(N_f)_L \times U(N_f)_R$

(Sakai-Sugimoto model)

spontaneous breaking of chiral symmetry

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spontaneous breaking of chiral symmetry

chiral symmetry restoration transition at finite temperature

Aharony et al, Parnachev et al '06

once again, magnetic catalysis in chiral symmetry breaking

Bergman et al, Johnson et al '08

not an honest (3+1)-dim gauge theory

running dilaton, lacks UV completion

Conclusions and Outlook

Interesting phenomenological consequences of a magnetic field

universal magnetic catalysis at strong coupling + more

Our proposal of thermodynamic free energy in an electric field

intriguing possibility for non-equilibrium systems

More relevant parameters to be explored

chemical potential

Going beyond the probe limit

work in progress

More general lessons of strong coupling physics

perhaps applicable elsewhere

Pictures speak a thousand dualities

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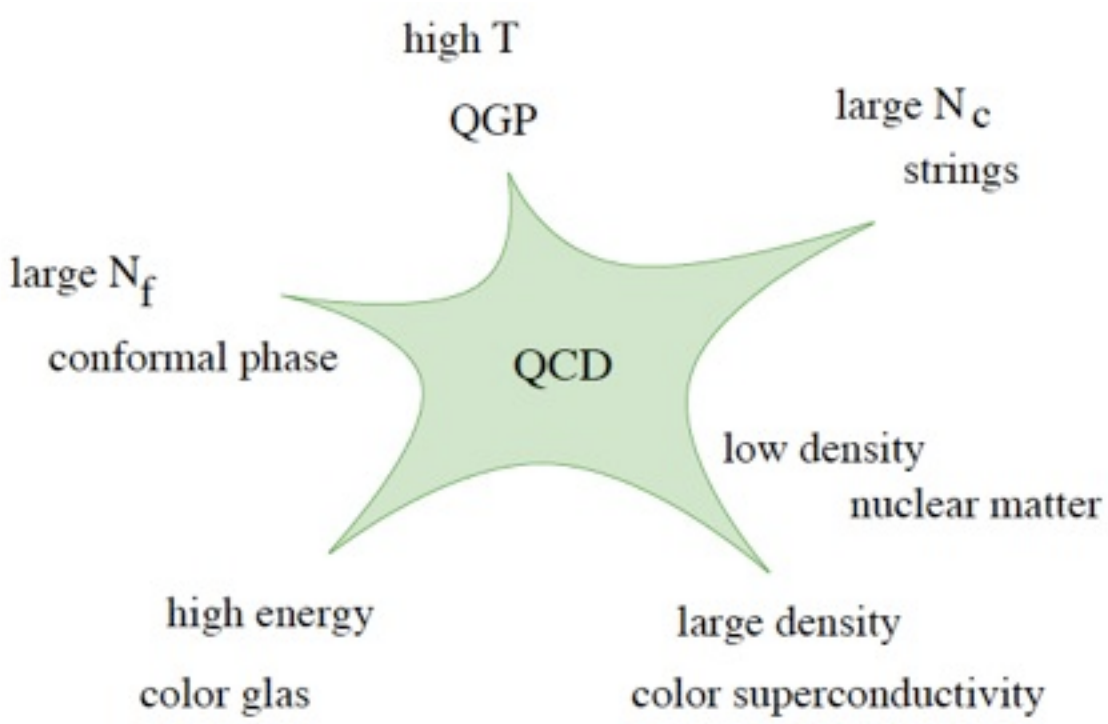
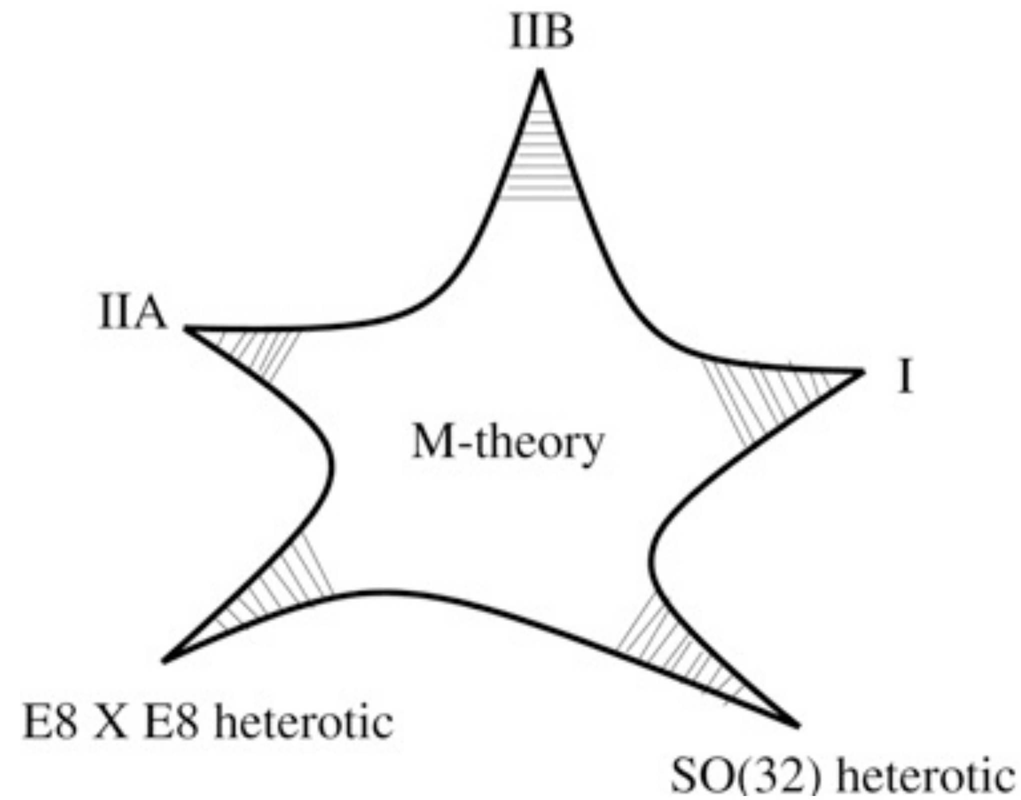
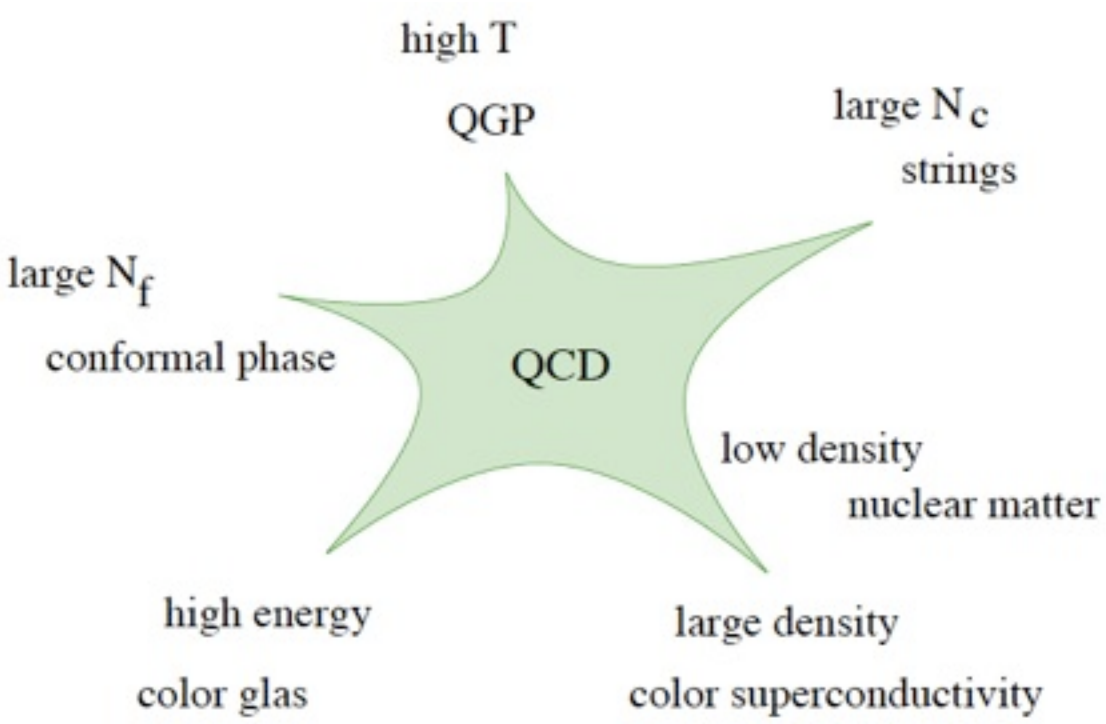


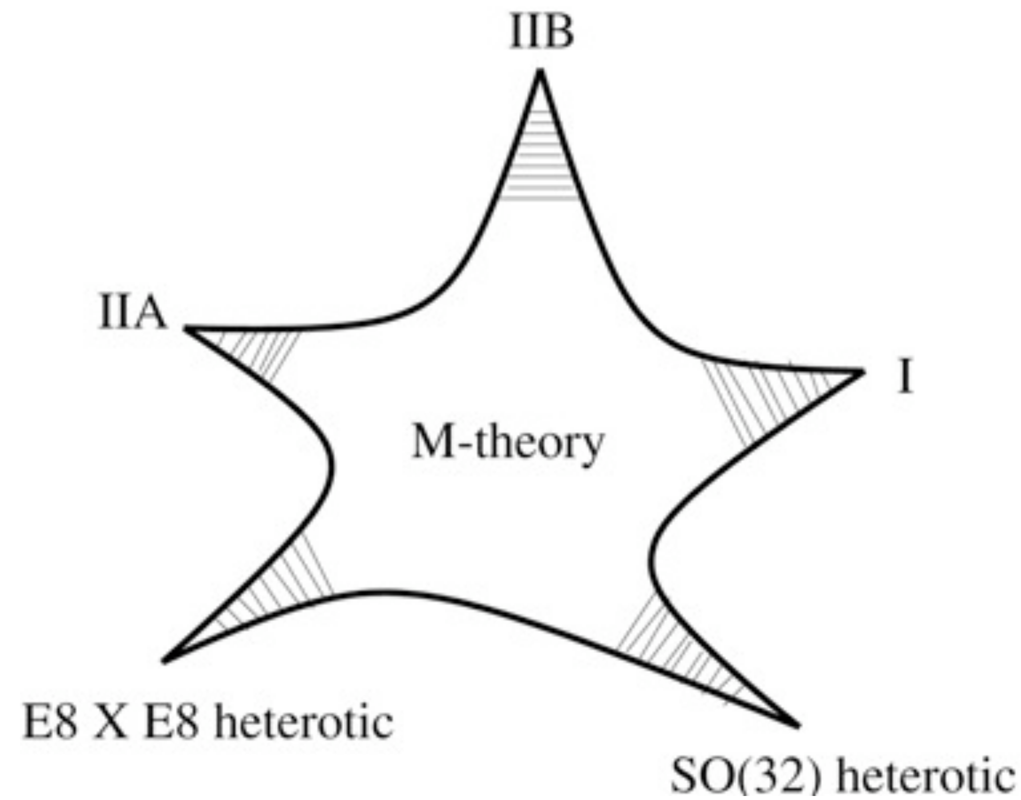


Picture courtesy: Phases of QCD by T. Schafer and web

Don't they look alike?

Pictures speak a thousand dualities





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Thank You