A massive S-matrix from massless amplitudes

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Princeton University

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based on:

N. Craig, H. Elvang, MK, T. Slatyer 1104.2050

MK 1105.5385

H. Elvang, D.Z. Freedman, MK *to appear*

Motivation

On-shell methods: Successes

Tremendous conceptual progress in

- massless
- planar
- maximally supersymmetric Yang-Mills theory.

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- draw lessons for less "special" theories from massless $\mathcal{N} = 4$ SYM

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Challenges

- draw lessons for less “special” theories from massless $\mathcal{N} = 4$ SYM
- Even better:
  Recycle results in massless $\mathcal{N} = 4$ SYM for other theories?

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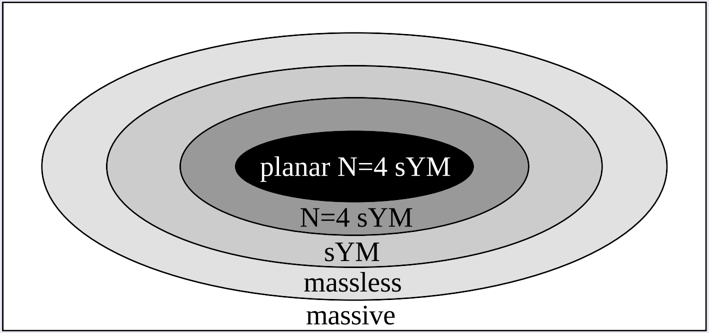
Level of Difficulty [L. Dixon, 1105.0771]



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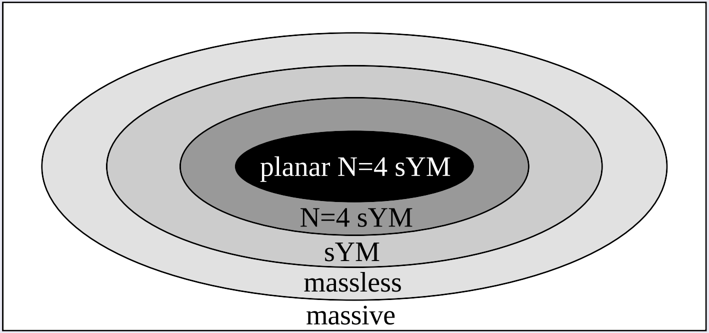

Can we somehow cheat:
Compute massive amplitudes from massless on-shell amplitudes?

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QFT phenomena studied using on-shell methods

- on-shell amplitudes
- light-like Wilson loops
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What about spontaneous symmetry breaking?
Specifically: Can we compute amplitudes in the spontaneously broken theory from on-shell amplitudes in the unbroken theory?

1. Review: The Coulomb-branch of $\mathcal{N} = 4$ SYM

2. Coulomb-branch S-matrix from massless amplitudes

3. Tests of Proposal

4. A CSW-like expansion on the Coulomb branch

Spontaneous symmetry breaking in $\mathcal{N} = 4$ SYM

The origin of moduli space

gauge group $U(M+N)$, R-symmetry group $SU(4)_R$

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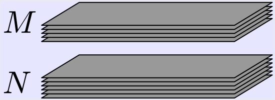
Coulomb branch: the simplest case

scalar vevs: $\langle(\phi_{12})_A{}^B\rangle = \langle(\phi_{34})_A{}^B\rangle = m\,\delta_A{}^B$ for $1 \leq A, B \leq M$.

brane picture:



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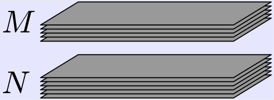

spontaneously breaks gauge and R-symmetry group:

$$U(M+N) \to U(M) \times U(N), \qquad SU(4)_R \to Sp(4) \supset SU(2) \times SU(2).$$

states decompose as: $A_\mu = \begin{pmatrix} (A_\mu)_{N\times N} & (W_\mu)_{N\times M} \\ (\overline{W}_\mu)_{M\times N} & (\tilde{A}_\mu)_{M\times M} \end{pmatrix}$.

Spontaneous symmetry breaking in $\mathcal{N} = 4$ SYM

Recent interest in Coulomb branch amplitudes

- as a regulator of IR divergences in the massless theory
  [Alday, Henn, Naculich, Plefka, Schnitzer, Schuster]
- Coulomb-branch inspired regularization of SUSY Wilson loop?
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Crucial simple properties

- massive and massless phase connected on moduli space of vacua
- masses of any amplitude sum to zero: $\sum_i m_i = 0$.

  (obvious from 6d momentum conservation)

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Convenient variables to express Coulomb-branch amplitudes?

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Review: Massless spinor helicity formalism

$$p_i \leftrightarrow |i\rangle[i| \quad \Rightarrow \quad \text{e.g.} \ \left\langle g_1^- g_2^- g_3^+ \cdots g_n^+ \right\rangle = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle} \,.$$

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Reference vector q

- introduced as a technical tool, defines a basis of "helicity amplitudes"
- BUT: has no direct physical significance or interpretation!

Explicit amplitudes

Example: Ultra-helicity violating (UHV) amplitudes

4-point amplitude: $$\left\langle W_1^- \overline{W}_2^+ \, g_3^+ \, g_4^+ \right\rangle = -\frac{m^2 \langle q1^\perp \rangle^2 [34]}{\langle q2^\perp \rangle^2 \langle 34 \rangle (P_{23}^2 + m^2)} .$$

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n-point amplitude: `[see also: Forde, Kosower; Ferrario et al]`

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Compare: Adler zeros, Soft-pion theorems

- Goldstone bosons: soft-limits probe vacuum manifold
- BUT: Goldstone bosons $\Leftrightarrow$ global symmetry $\Leftrightarrow$ all vacua equivalent
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- old theorem with modern applications:
  finiteness of $\mathcal{N} = 8$ supergravity for $L < 7$ loops!
- $\mathcal{N} = 4$ SYM: scalars not GSB
  $\Rightarrow$ vacua not equivalent $\Rightarrow$ non-vanishing soft limits!
  Should teach us about the new vacuum!

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Naive ansatz:

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4. RHS generically ill-defined due to soft divergences!

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decompose massive W momenta as: $p_i = p_i^{\perp} - \frac{m_i^2}{2\, q\cdot p_i} q\,.$

choose $p_i^{\perp}$ as massless gluon momentum on RHS!

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- massive W polarizations $\epsilon^+ = \frac{\sqrt{2}|q\rangle[i^\perp|}{\langle i^\perp q\rangle}\,, \quad \epsilon^- = \frac{\sqrt{2}|i^\perp\rangle[q|}{[i^\perp q]}$

  $\Leftrightarrow$ massless $\pm$-helicity gluons.

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- longitudinal W boson with polarization $\not{\epsilon}^{L} = \frac{1}{m_i}\left(\not{p}_i^{\perp} + \frac{m_i^2}{2q\cdot p_i}\not{q}\right)$
  $\Leftrightarrow$ massless scalar $\frac{1}{\sqrt{2}}(\phi^{12}+\phi^{34})\,.$

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Puzzle 2: How to map W-polarizations to massless states?

- massive W polarizations $\epsilon^+ = \frac{\sqrt{2}|q\rangle[i^\perp|}{\langle i^\perp q\rangle}\,, \quad \epsilon^- = \frac{\sqrt{2}|i^\perp\rangle[q|}{[i^\perp q]}$
  $\Leftrightarrow$ massless $\pm$-helicity gluons.
- longitudinal W boson with polarization $\not{\epsilon}^L = \frac{1}{m_i}\left(\not{p}_i^\perp + \frac{m_i^2}{2q\cdot p_i}\not{q}\right)$
  $\Leftrightarrow$ massless scalar $\frac{1}{\sqrt{2}}(\phi^{12}+\phi^{34})\,.$

LHS now depends on one arbitrary massless reference spinor q $\Leftrightarrow$ RHS?

Massive amplitudes from massless amplitudes

$$\langle\, W_1 \overline{W}_2 \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\text{vev}}_{\varepsilon q_1} \phi^{\text{vev}}_{\varepsilon q_2} \ldots \phi^{\text{vev}}_{\varepsilon q_s}\, g_2 \ldots \rangle \quad .$$

Puzzle 3: How to choose soft-scalar momenta q_i?

Massive amplitudes from massless amplitudes

$$\langle\, W_1 \overline{W}_2 \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\text{vev}}_{\varepsilon q_1} \phi^{\text{vev}}_{\varepsilon q_2} \ldots \phi^{\text{vev}}_{\varepsilon q_s}\, g_2\, \ldots \rangle \quad .$$

Puzzle 3: How to choose soft-scalar momenta q_i?

LHS depends only on one reference null momentum q

Massive amplitudes from massless amplitudes

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LHS depends only on one reference null momentum q

$\Rightarrow$ need to set all $q_i = q$ to match parameters!

Massive amplitudes from massless amplitudes

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New Puzzle: collinear divergences as $q_i \to q$!

Massive amplitudes from massless amplitudes

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Puzzle 3: How to choose soft-scalar momenta q_i?

LHS depends only on one reference null momentum q

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New Puzzle: collinear divergences as $q_i \to q$!

collinear divergences are anti-symmetric in momenta.

Massive amplitudes from massless amplitudes

$$\langle W_1 \overline{W}_2 \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \langle g_1 \, \phi^{\text{vev}}_{\varepsilon q} \phi^{\text{vev}}_{\varepsilon q} \ldots \phi^{\text{vev}}_{\varepsilon q} \, g_2 \ldots \rangle_{sym} \, .$$

Puzzle 3: How to choose soft-scalar momenta q_i?

LHS depends only on one reference null momentum q

$\Rightarrow$ need to set all $q_i = q$ to match parameters!

New Puzzle: collinear divergences as $q_i \to q$!

collinear divergences are anti-symmetric in momenta.

Resolution: Symmetrize in q_i before taking the limit $q_i \to q$.

Makes sense, because vev scalars should not be color-ordered!

Massive amplitudes from massless amplitudes

$$\langle\, W_1 \overline{W}_2 \, \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \langle\, g_1 \, \phi^{\text{vev}}_{\varepsilon q} \phi^{\text{vev}}_{\varepsilon q} \ldots \phi^{\text{vev}}_{\varepsilon q} \, g_2 \, \ldots \rangle_{sym} \,.$$

Puzzle 4: soft-divergences in the limit $\varepsilon \to 0$

Massive amplitudes from massless amplitudes

$$\langle\, W_1 \overline{W}_2 \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\text{vev}}_{\varepsilon q}\, \phi^{\text{vev}}_{\varepsilon q} \,..\, \phi^{\text{vev}}_{\varepsilon q}\, g_2 \,\ldots \rangle_{sym} \,.$$

Puzzle 4: soft-divergences in the limit $\varepsilon \to 0$

Finite soft limits at leading non-vanishing order!

Massive amplitudes from massless amplitudes

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Puzzle 4: soft-divergences in the limit $\varepsilon \to 0$

Finite soft limits at leading non-vanishing order!

$O(m^2)$ term in $\langle W^- \overline{W}^+ \phi^{34} \phi^{34} \rangle$:

$$\langle\, g_1^-\, \phi^{\text{vev}}_{\varepsilon q}\, \phi^{\text{vev}}_{\varepsilon q}\, g_2^+\, \phi_3^{34} \phi_4^{34} \,\rangle_{\text{sym}} \;=\; -\,\frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp] (P_{23}^\perp)^2} + O(\varepsilon^1)\,. \quad \text{(correct!)}\,.$$

Divergent soft limits at subleading orders!

Massive amplitudes from massless amplitudes

$$\left\langle W_1 \overline{W}_2 \ldots \right\rangle \stackrel{?}{=} \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \left\langle g_1\, \phi^{\text{vev}}_{\varepsilon q} \phi^{\text{vev}}_{\varepsilon q} \,..\, \phi^{\text{vev}}_{\varepsilon q}\, g_2 \ldots \right\rangle_{sym} .$$

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Massive amplitudes from massless amplitudes

$$\langle\, W_1 \overline{W}_2 \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\text{vev}}_{\varepsilon q} \phi^{\text{vev}}_{\varepsilon q} \ldots \phi^{\text{vev}}_{\varepsilon q}\, g_2 \ldots \rangle_{sym}\,.$$

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Related problem: $O(m^2)$ violation of momentum conservation on RHS

$$\sum_i p_i^\perp = \sum_i \left(p_i + \frac{m_i^2}{2\, q\cdot p_i} q \right) = \left[\sum_{i=1}^{n} \frac{m_i^2}{2\, q\cdot p_i} \right] q \neq 0\,.$$

Massive amplitudes from massless amplitudes

$$\langle\, W_1 \overline{W}_2 \ldots \rangle \quad \stackrel{?}{=} \quad \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\mathrm{vev}}_{\varepsilon q}\, \phi^{\mathrm{vev}}_{\varepsilon q} \ldots \phi^{\mathrm{vev}}_{\varepsilon q}\, g_2 \, \ldots \rangle_{sym} \,.$$

Special choice of q

We must impose

$$\sum_{i=1}^{n} \frac{m_i^2}{2\, q \cdot p_i} \;=\; 0$$

Massive amplitudes from massless amplitudes

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to

- prevent $1/\varepsilon$ divergence at first subleading order on RHS .
- guarantees momentum conservation on RHS: $\sum_i p_i^{\perp} = 0$

Massive amplitudes from massless amplitudes

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only constrains choice of helicity basis!

any Coulomb-branch amplitude expressible in any q-helicity basis!

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- guarantees momentum conservation on RHS: $\sum_i p_i^{\perp} = 0$

only constrains choice of helicity basis!

any Coulomb-branch amplitude expressible in any q-helicity basis!

Special choice of q for two massive lines

Simple orthogonality relation: $q\cdot(p_1 + p_2) \;=\; 0\,.$

Massive amplitudes from massless amplitudes

Summary: Refined proposal

$$\left\langle\, W_1 \overline{W}_2 \ldots \right\rangle \quad = \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \left\langle\, g_1\, \phi_{\varepsilon q}^{\mathrm{vev}}\, \phi_{\varepsilon q}^{\mathrm{vev}} \,..\, \phi_{\varepsilon q}^{\mathrm{vev}}\, g_2 \,\ldots \right\rangle_{sym} .$$

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W-boson, pol $\epsilon_q^{\pm}$		$\pm$-helicity gluon
W-boson, pol ϵ_q^{L}	$\Longleftrightarrow$	scalar $\frac{1}{\sqrt{2}}(\phi^{12}+\phi^{34})$

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Open questions:

- show that proposal is free of soft divergences to all orders
- verify proposal for explicit Coulomb-branch amplitudes

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Open questions:

- show that proposal is free of soft divergences to all orders
- verify proposal for explicit Coulomb-branch amplitudes

BUT: Infinite sum, vev-scalar symmetrization... daunting task in practice?

Convenient representation for massless amplitudes

$$\left\langle\, W_1 \overline{W}_2 \,\ldots \right\rangle \quad = \quad \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \left\langle\, g_1\, \phi_{\varepsilon q}^{\mathrm{vev}} \phi_{\varepsilon q}^{\mathrm{vev}} \,..\, \phi_{\varepsilon q}^{\mathrm{vev}}\, g_2 \,\ldots \right\rangle_{sym}.$$

- need a convenient representation for massless amplitudes

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- proposal introduces reference vector $q \Rightarrow$ CSW expansion natural!

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The CSW expansion for massless amplitudes

MHV vertices, connected by scalar propagators:

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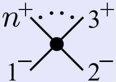

$$\frac{\langle 1^\perp 2^\perp \rangle^4}{\langle 1^\perp 2^\perp \rangle \cdots \langle n^\perp 1^\perp \rangle}$$

Convenient representation for massless amplitudes

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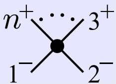

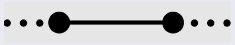

$$\frac{\langle 1^\perp 2^\perp \rangle^4}{\langle 1^\perp 2^\perp \rangle \cdots \langle n^\perp 1^\perp \rangle} \qquad\qquad \frac{1}{(P_I^\perp)^2}\,, \qquad P_I^\perp \equiv \sum_{i\in I} p_i^\perp$$

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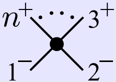
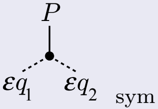


$$\frac{\langle 1^\perp 2^\perp\rangle^4}{\langle 1^\perp 2^\perp\rangle \cdots \langle n^\perp 1^\perp\rangle} \qquad\qquad \frac{1}{(P_I^\perp)^2}\,, \qquad P_I^\perp \equiv \sum_{i\in I} p_i^\perp$$

holomorphic in $|i^\perp\rangle$, CSW prescription $|P_I^\perp\rangle \equiv P_I^\perp|q]$ for internal $P_I^\perp$

Simplification through vev-scalar symmetrization

Vanishing vertices



$$\propto \quad \frac{1}{\langle P q_1 \rangle \langle q_1 q_2 \rangle \langle q_2 P \rangle} + \frac{1}{\langle P q_2 \rangle \langle q_2 q_1 \rangle \langle q_1 P \rangle} \; = \; 0$$

(obvious from antisymmetry of 3-point ampliutde)

Simplification through vev-scalar symmetrization

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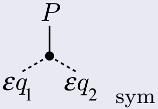

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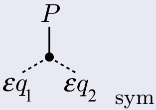
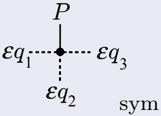


$= 0\,,$



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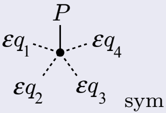
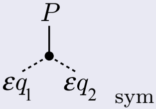


$= 0\,.$

(obvious from $U(1)$-decoupling identity)

Vanishing vertices



$$\propto \frac{1}{\langle P q_1\rangle\langle q_1 q_2\rangle\langle q_2 P\rangle} + \frac{1}{\langle P q_2\rangle\langle q_2 q_1\rangle\langle q_1 P\rangle} = 0$$

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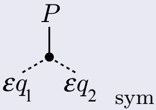
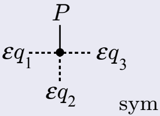
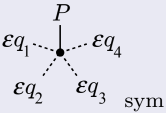


$= 0\,,$

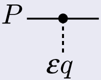


$= 0\,,$

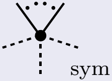


$= 0\,.$

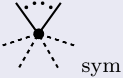
(obvious from $U(1)$-decoupling identity)
vanishing vertices with more non-vev lines:



$= 0\,,$



$= 0\,,$



$= 0\,.$

A four-point tree-level example

The 4-point amplitude $\langle W^- \overline{W}^+ \phi^{34} \phi^{34} \rangle$

$$\langle W^- \overline{W}^+ \phi^{34} \phi^{34} \rangle = -\frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp] (P_{23}^2 + m^2)} = -\frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp] \left[(P_{23}^\perp)^2 - m^2 \frac{q \cdot P_{23}}{q \cdot p_2} + m^2 \right]}$$

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leading order

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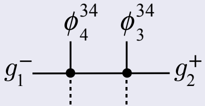
leading order

Only one diagram contributes to the massless NMHV amplitude:

$$\lim_{\varepsilon \to 0} \left\langle g_1^- \, \phi_{\varepsilon q}^{\text{vev}} \phi_{\varepsilon q}^{\text{vev}} \, g_2^+ \, \phi_3^{34} \, \phi_4^{34} \right\rangle_{\text{sym}} =$$



$$= -\frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp] (P_{23}^\perp)^2} \, .$$

A four-point tree-level example

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Subleading order: $\left\langle\, g_1^- \, \phi_{\varepsilon q}^{\text{vev}} \phi_{\varepsilon q}^{\text{vev}} \phi_{\varepsilon q}^{\text{vev}} \phi_{\varepsilon q}^{\text{vev}} \, g_2^+ \, \phi_3^{34} \, \phi_4^{34} \,\right\rangle_{\text{sym}}$

A four-point tree-level example

The 4-point amplitude $\langle W^- \overline{W}^+ \phi^{34}\phi^{34}\rangle$

$$\langle W^- \overline{W}^+ \phi^{34}\phi^{34}\rangle = -\frac{m^2\langle 1^\perp|q|2^\perp]}{\langle 2^\perp|q|1^\perp](P_{23}^2+m^2)} = -\frac{m^2\langle 1^\perp|q|2^\perp]}{\langle 2^\perp|q|1^\perp]\left[(P_{23}^\perp)^2 - m^2\frac{q\cdot P_{23}}{q\cdot p_2} + m^2\right]}$$

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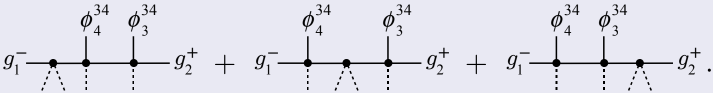

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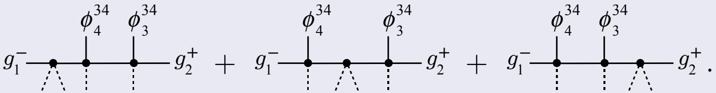
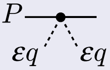


crucial vertex:



$$\Bigg|_{\mathrm{sym}} = -\frac{1}{2} \langle \phi_{ab} \rangle \langle \phi^{ab} \rangle = -m^2 .$$

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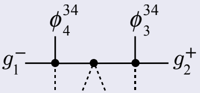
$\lim_{\varepsilon \to 0}$



$$= \frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp] (P_{23}^\perp)^2} \times \frac{m^2}{(P_{23}^\perp)^2} \, .$$

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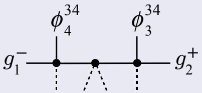
$\lim_{\varepsilon \to 0}$ 

$= \frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp] (P_{23}^\perp)^2} \times \frac{m^2}{(P_{23}^\perp)^2} .$

Finite contribution, builds up '$+m^2$' in propagator!

A four-point tree-level example

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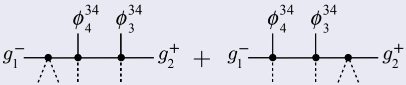

$$= \frac{m^2 \langle 1^\perp | q | 2^\perp]}{\langle 2^\perp | q | 1^\perp](P_{23}^\perp)^2} \times \left[\frac{1}{\varepsilon} \left(\frac{m^2}{4\, q \cdot p_1} + \frac{m^2}{4\, q \cdot p_2} \right) - \frac{m^2 \, q \cdot P_{23}}{(P_{23}^\perp)^2 \, q \cdot p_2} + O(\varepsilon) \right].$$

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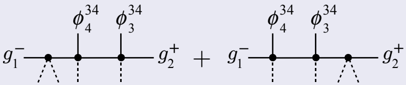

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Finite sum of divergent diagrams, builds up $p_2^\perp \to p_2$ in propagator!

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All orders in m

- Internal 4-point vertices  build up '$+m^2$' in propagator
- External 4-point vertices  build up '$p_2^\perp \to p_2$' in propagator

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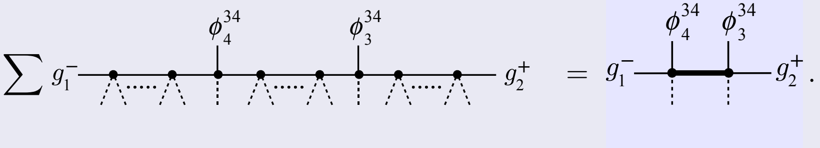

Checks and Generalizations of the proposal

$$\langle\, W_1 \overline{W}_2 \,\ldots \rangle \quad = \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\text{vev}}_{\varepsilon q}\, \phi^{\text{vev}}_{\varepsilon q} \,..\, \phi^{\text{vev}}_{\varepsilon q}\, g_2 \,\ldots \rangle_{sym} \,.$$

Non-trivial Checks

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Non-trivial Checks

- An all-n amplitude, to all orders in m:

$$\langle W_1^- \overline{W}_2^+ \phi_3^{34} \ldots \phi_n^{34} \rangle \; =$$

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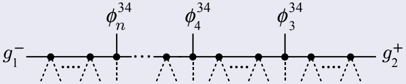

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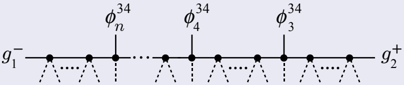
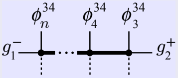


$$=$$



$$= \frac{-m^{n-2}\langle 1^\perp|q|2^\perp]}{\langle 2^\perp|q|1^\perp](P_{23}^2+m^2)\cdots(P_{23\ldots n\text{-}1}^2+m^2)} \,.$$

Checks and Generalizations of the proposal

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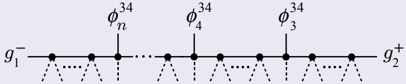

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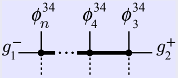

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- Proof of finite soft limit, to all orders in m, for any amplitude!

Checks and Generalizations of the proposal

Generalizations of proposal

- natural proposal for CB amplitudes with arbitrary masses.

  breaking $U(N) \to \prod_k U(M_k) \;\Rightarrow\; \langle\phi\rangle \sim v_k \;\Rightarrow\; m_X = v_{k_1} - v_{k_2}$

  with particle X in bifundamental of $U(M_{k_1}) \times U(M_{k_2})$.

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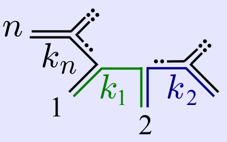
  Double-line notation useful:



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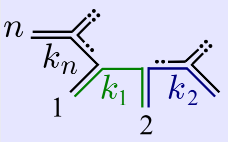

$$\big\langle X_1 X_2 \,\ldots\, X_n \big\rangle = \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \sum_{s_1+\ldots+s_n=s} \Big\langle Y_1 \underbrace{\phi^{\text{vev}}_{\varepsilon q} \ldots \phi^{\text{vev}}_{\varepsilon q}}_{s_1 \text{ times}} Y_2 \underbrace{\phi^{\text{vev}}_{\varepsilon q} \ldots \phi^{\text{vev}}_{\varepsilon q}}_{s_2 \text{ times}} Y_3 \ldots Y_n \underbrace{\phi^{\text{vev}}_{\varepsilon q} \ldots \phi^{\text{vev}}_{\varepsilon q}}_{s_n \text{ times}} \Big\rangle_{\text{sym}} .$$

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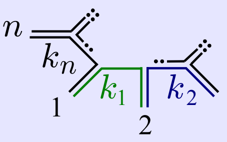


$$\langle X_1 X_2 \ldots X_n \rangle$$

$$= \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \sum_{s_1+\ldots+s_n=s} \langle\, Y_1 \underbrace{\phi_{\varepsilon q}^{\mathrm{vev}} \ldots \phi_{\varepsilon q}^{\mathrm{vev}}}_{s_1 \text{ times}} Y_2 \underbrace{\phi_{\varepsilon q}^{\mathrm{vev}} \ldots \phi_{\varepsilon q}^{\mathrm{vev}}}_{s_2 \text{ times}} Y_3 \ldots Y_n \underbrace{\phi_{\varepsilon q}^{\mathrm{vev}} \ldots \phi_{\varepsilon q}^{\mathrm{vev}}}_{s_n \text{ times}} \,\rangle_{\mathrm{sym}} \,.$$

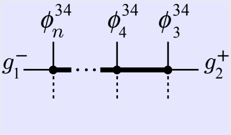
- similar proposal for loop integrand (SUSY important!)

Convenient representation for massless amplitudes

$$\langle\, W_1 \overline{W}_2 \, \ldots \rangle \quad = \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \langle\, g_1 \, \phi^{\mathrm{vev}}_{\varepsilon q} \phi^{\mathrm{vev}}_{\varepsilon q} \, .. \, \phi^{\mathrm{vev}}_{\varepsilon q} \, g_2 \, \ldots \rangle_{sym} \, .$$

- infinite sum over massless amplitudes, complicated symmetrization
- in simple examples:
  infinite sum $\Rightarrow$ single diagram with massive propagators:

$$\langle W_1^- \overline{W}_2^+ \phi_3^{34} \ldots \phi_n^{34} \rangle \; = $$



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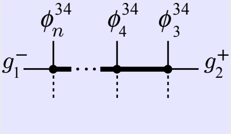

- is this always possible? what are the massive Feynman rules?

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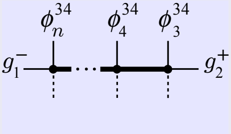
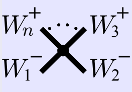


- is this always possible? what are the massive Feynman rules?
  $\Rightarrow$ on-shell derivation of Feynman rules in the broken phase?

Convenient representation for massless amplitudes

$$\langle\, W_1 \overline{W}_2 \,\ldots \rangle \quad = \quad \lim_{\varepsilon \to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\mathrm{vev}}_{\varepsilon q}\, \phi^{\mathrm{vev}}_{\varepsilon q} \,..\, \phi^{\mathrm{vev}}_{\varepsilon q}\, g_2 \,\ldots \rangle_{sym} .$$

Resummation ⇒ Massive CSW-like rules:

- massive scalar propagators:  $= \frac{1}{P_I^2 + m_I^2}$, $\quad m_I = \sum_{i \in I} m_i$.
- MHV vertex: 

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Convenient representation for massless amplitudes

$$\langle\, W_1 \overline{W}_2 \,\ldots \rangle \quad = \quad \lim_{\varepsilon\to 0} \sum_{s=0}^{\infty} \langle\, g_1\, \phi^{\text{vev}}_{\varepsilon q}\, \phi^{\text{vev}}_{\varepsilon q} \,..\, \phi^{\text{vev}}_{\varepsilon q}\, g_2 \,\ldots \rangle_{sym} \,.$$

Resummation $\Rightarrow$ Massive CSW-like rules:

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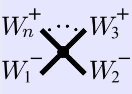
- MHV vertex: 

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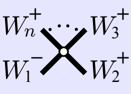
- UHV vertex: 

$= K^2 \frac{\langle q^\perp 1^\perp\rangle^4}{\langle 1^\perp 2^\perp\rangle \cdots \langle n^\perp 1^\perp\rangle}, \quad K = \sum_i \frac{m_i \langle 1^\perp i^\perp\rangle}{\langle 1^\perp q\rangle \langle i^\perp q\rangle}\,.$

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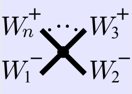
- MHV vertex: 

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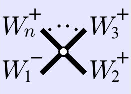
- UHV vertex: 

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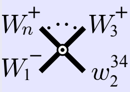
- UHV×MHV vertex: 

$= K \frac{\langle q^\perp 1^\perp \rangle^2 \langle 1^\perp 2^\perp \rangle^2}{\langle 1^\perp 2^\perp \rangle \cdots \langle n^\perp 1^\perp \rangle} \, .$

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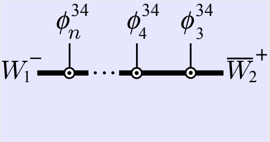

$$= \frac{-m^{n-2}\langle 1^\perp|q|2^\perp]}{\langle 2^\perp|q|1^\perp](P_{23}^2+m^2)\cdots(P_{23\ldots n\text{-}1}^2+m^2)}\,.$$

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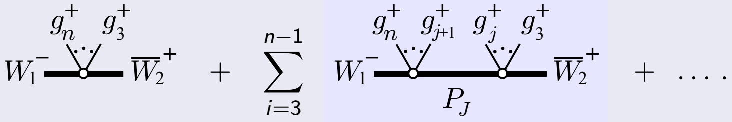

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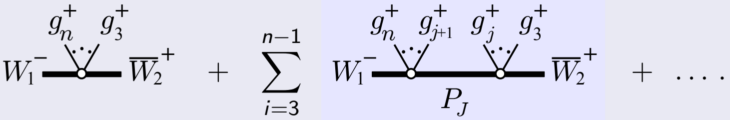

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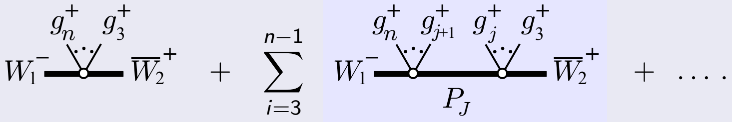

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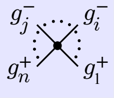
- should be compared to BCFW form of same amplitude:

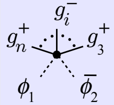
$$-\frac{m^2 \langle q\, 1^\perp \rangle^2}{\langle q\, 2^\perp \rangle^2 \, \langle 34 \rangle \langle 45 \rangle \cdots \langle n-1, n \rangle (P_{n1}^2 + m^2)} \times [3| \prod_{j=3}^{n-2} \left[1 + \frac{P_J |j+1\rangle [j+1|}{P_J^2 + m^2} \right] |n] .$$

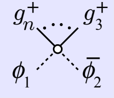
- similar complexity, but no recursion to solve!

Application: Integrands of rational terms in QCD

Simple truncation: Massive scalar coupled to massless gluons [Boels]







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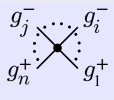

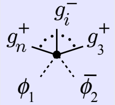

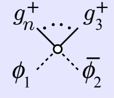

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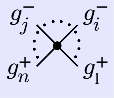

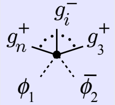

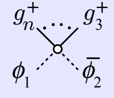

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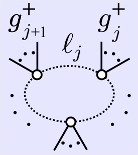
$$I_{\mathrm{CSW}}^{++\cdots+} = \sum$$  .

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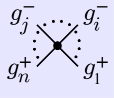

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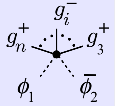

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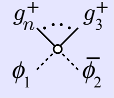

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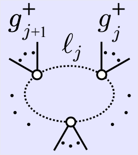

$$=\;\frac{\mathrm{Tr}'\prod_{j=1}^{n}\left[1-\frac{\mu^2|\ell_j\rangle\langle j,j{+}1\rangle\langle\ell_j|}{(\ell_j^2+\mu^2)\langle\ell_j,j\rangle\langle j{+}1,\ell_j\rangle}\right]}{\langle 12\rangle\cdots\langle n1\rangle}\,.$$

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