Towards Precision Jet Mass Calculations

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Frontiers in QCD (INT-11-3)

Oct 5, 2011

References

Resummation of jet mass with a jet veto

arXiv:1102.0561v2

RK, Matthew D. Schwartz, Hau Xing Zhu

The two-loop hemisphere soft function

arXiv:1105.3676

RK, Robert M. Schabinger, Matthew D. Schwartz, Hau Xing Zhu

Outline

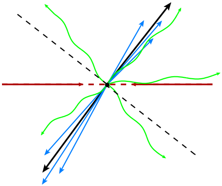
Outline

Introduction

- Very large number of jets at the LHC.
- Jets provide a wealth of information about QCD and exploring new physics.
  - excess in the number of jets could be a sign of new physics
- Substructure may be critical in new physics searches.
  - massive boosted heavy particles can be found in jet
- Jet rate distributions have been calculated to NLO, but little has been said about structure of jets (i.e. m^2, R, angularity, etc.).
- Predictions may be spoiled by large logarithms ($\log^n \frac{m_1}{m_2}, \log R$, etc)
- Effective field theories provide a way to systematically improve calculations.

Factorization (a preview)

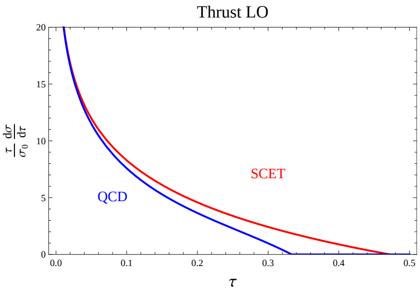
$$\tau = 1 - T \approx \frac{m_L^2 + m_R^2}{Q^2}$$



$$\frac{d\sigma}{dm_L^2 dm_R^2} \sim H(Q,\mu_h) \int dk_L dk_R$$
$$\times\, J(m_L^2 - k_L Q, \mu_j) J(m_R^2 - k_R Q, \mu_j) S(k_L, k_R, \mu_s)$$

(Fleming et al., Schwartz)

- Factorization is achieved using Soft Collinear Effective theory (SCET)
- Use the LO results in SCET to predict the NLO singular piece using renormalization group evolution (RGE).
- Compare α_s^2 results to EVENT2 (Catani and Seymore)



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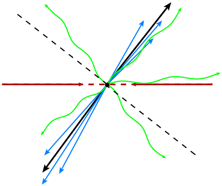

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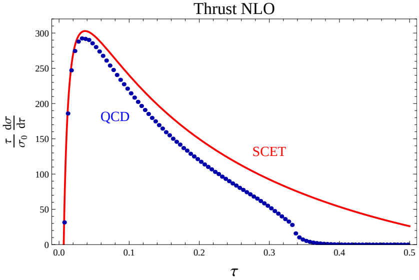

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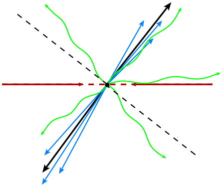

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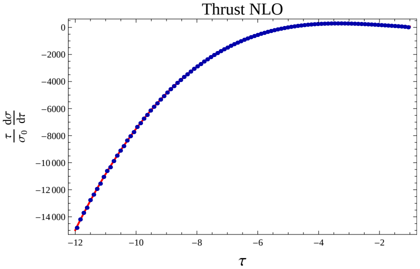


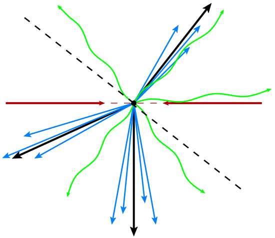
What could go wrong?

- The factorization theorem is valid only when m_L and m_R are small (i.e. the small m^2 region is dominated by IR degrees of freedom)
- SCET does not guarantee $\log m_L^2/m_R^2$ are resummed by RGE (can be calculated by brute force)

- Produce non-global logarithms (Dasgupta and Salem)

$$-C_F C_A \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{16\pi^2}{3} \log^2\left(\frac{m_L^2}{Q^2}\right)$$

- Hard emissions are not included in SCET degrees of freedom (type 1)
- Sharply divided phase space with separated scales $m_L \ll m_R$ (type 2)
- Finite jet size (R), and cutoff scales ($E_{\text{out}} < \omega$) complicate the problem considerably.



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$$\Sigma(\rho_R) = \int_0^\infty dm_1 \int_0^{\rho^R Q^2} dm_2^2 \frac{d^2\sigma}{dm_1^2 dm_2^2}$$

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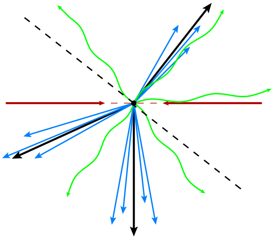

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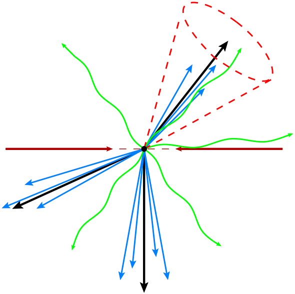
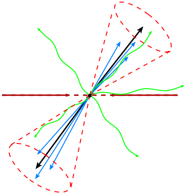


Main Points

- Seek to understand non-global logarithms and how to control them.
- Understand how different jet shapes and jet sizes (R) affect the observables.
- Consider first inclusive and then exclusive observables.
- We perform resummation for a 2-jet observable with jets of size R.

$$\tau_\omega = \frac{m_1^2 + m_2^2}{Q^2}, \qquad E_3 < \omega$$



- Demonstration of factorization of the soft function:

$$S_R(k,\omega,\mu) = S_R^{\text{in}}(k,\mu) S_R^{\text{out}}(\omega,\mu)$$

and discuss limitations.

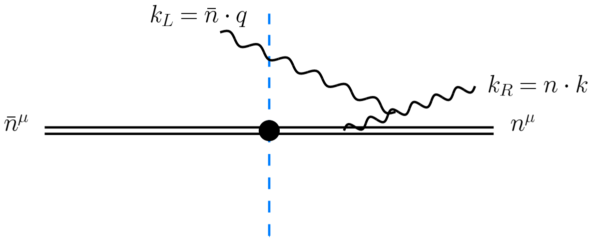
Outline

Hemisphere Jets

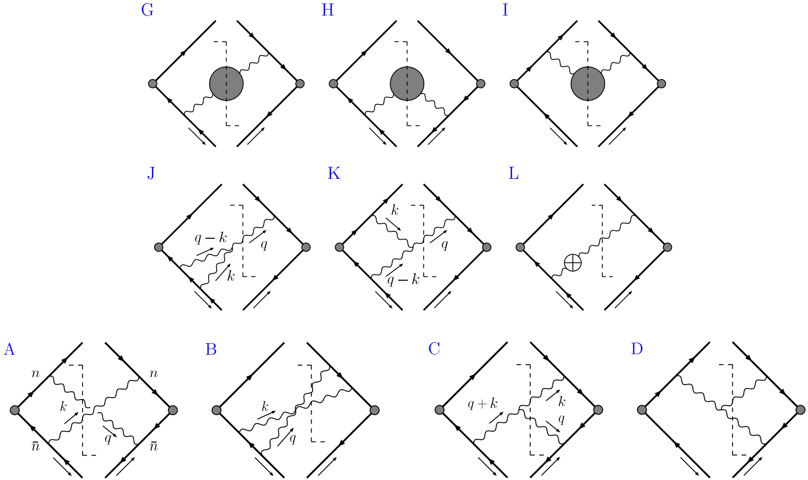
- try calculating:

$$\frac{d\sigma}{dm_L^2 dm_R^2} \sim H(Q,\mu_h) \int dk_L dk_R \times J(m_L^2 - k_L Q, \mu_j) J(m_R^2 - k_R Q, \mu_j) S(k_L, k_R, \mu_s)$$

- RG evolution only resums $\log \frac{m^2}{Q^2}$, but does not say anything about $\log \frac{m_L^2}{m_R^2}$.
- These logs come from k_L/k_R in the soft function.



Calculation of two-loop Soft function



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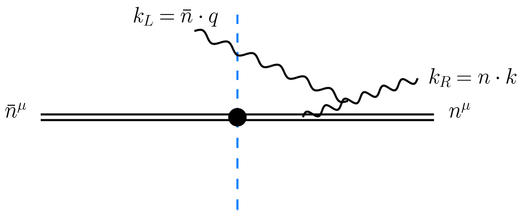

$$S(k_L, k_R) = \delta(k_L)\delta(k_R) + \frac{\alpha_s}{4\pi} S^{(1)}(k_L, k_R, \mu) + \frac{\alpha_s^2}{16\pi^2} S^{(2)}(k_L, k_R, \mu) + \cdots$$

- NLO result

$$S^{(2)} = C_F^2 S_{C_F} + C_F C_A S_{C_A} + C_F n_f T_F S_{n_f}$$

$$S = \frac{\mu^{4\epsilon}}{(k_L k_R)^{1+4\epsilon}} f\left(\frac{k_L}{k_R}, \epsilon\right) + \left(\frac{\mu^{2\epsilon}}{k_R^{1+2\epsilon}} \delta(k_L) + \frac{\mu^{2\epsilon}}{k_L^{1+2\epsilon}} \delta(k_R)\right) g(\epsilon)$$

- There is a different $f(r, \epsilon)$ and $g(\epsilon)$ for each color factor, where $r = k_L/k_R$.
- $f(r, \epsilon)$ was calculated independently by (Hornig et al. 1105.4628)

Cumulative the Soft Function

- Terms of the form $\frac{\mu^{a\epsilon}}{k^{1+a\epsilon}}$, $a = 2, 4$, must be thought of as distributions and integrated.

$$\mathcal{R}(X, Y, \mu) = \int_0^X dk_L \int_0^Y dk_R \; S(k_L, k_R, \mu)$$

- Result is used for integrated heavy jet mass and thrust distributions.
- The singular parts of the thrust and heavy jet mass distributions can be extracted (previously only known numerically)

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) D_\delta^{(\tau)} + \frac{\alpha_s}{4\pi}[D^{(1)}(\tau)]_+ + \left(\frac{\alpha_s}{4\pi}\right)^2 [D^{(2)}(\tau)]_+ + \cdots$$

- Removes a source of theoretical uncertainty in N^3LL result for heavy jet mass, improving fits to α_s.

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$$D_\delta^{(\tau)} = \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ -\frac{3\pi^4}{10} C_F^2 + C_F C_A \left(\frac{638\zeta_3}{9} - \frac{335\pi^2}{54} + \frac{22\pi^4}{45} - \frac{2140}{81}\right) + C_F T_F n_f \left(-\frac{232\zeta_3}{9} + \frac{74\pi^2}{27} + \frac{80\pi^2}{81}\right) \Bigg\}$$

- Removes a source of theoretical uncertainty in N^{3}LL result for heavy jet mass, improving fits to α_s.

Asymptotic behavior and Non-Global Logarithms

- Non-global logs must come from the μ-independent part of the soft function.

$$\mathcal{R}(X,Y,\mu) = \mathcal{R}_\mu\left(\frac{X}{\mu},\frac{Y}{\mu}\right) + \mathcal{R}_f\left(\frac{X}{Y}\right)$$

for $z = \frac{X}{Y} \gg 1$,

$$\mathcal{R}_f^{z\gg 1}(z) = \frac{\pi^4}{2}C_F^2 + \left[\left(\frac{8}{3} - \frac{16\pi^2}{9}\right)|\log z| + -\frac{136}{81} + \frac{154\pi^2}{27} + \frac{184\zeta_3}{9}\right] C_F n_f T_F$$
$$+ \left[-\frac{4}{3}\pi^2 \log^2 z + \left(-8\zeta_3 - \frac{4}{3} + \frac{44\pi^2}{9}\right)|\log z| - \frac{506\zeta_3}{9} + \frac{8\pi^4}{5} - \frac{871\pi^2}{54} - \frac{2032}{81}\right] C_F C_A$$

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for $z = \frac{X}{Y} \sim 1$,

$$\begin{aligned}\mathcal{R}_f^{z\sim 1}(z) = \frac{\pi^4}{2}C_F^2 + \Bigg[\left(-\frac{2}{3} - \frac{4\pi^2}{3} - 4\log^2 2 + \frac{44\log 2}{3}\right)\log^2 z - 32\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{88\zeta_3}{9} \\ -28\zeta_3\log(2) - \frac{2032}{81} - \frac{871\pi^2}{54} + \frac{16\pi^4}{9} - \frac{4\log^4 2}{3} + \frac{4}{3}\pi^2\log^2 z\Bigg] C_F C_A \\ + \left[\left(\frac{4}{3} - \frac{16\log 2}{3}\right)\log^2 z + \frac{154\pi^2}{27} - \frac{136}{81} - \frac{32\zeta_3}{9}\right] C_F n_f T_F + \mathcal{O}(\log^3 z).\end{aligned}$$

- Hoang-Kluth ansatz (0806.3852) only valid at small $\log z$.

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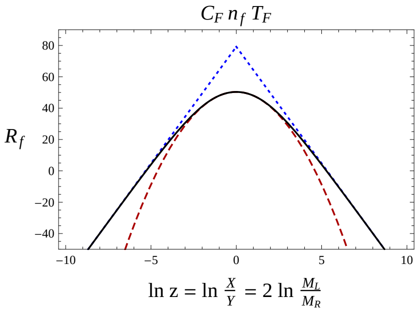
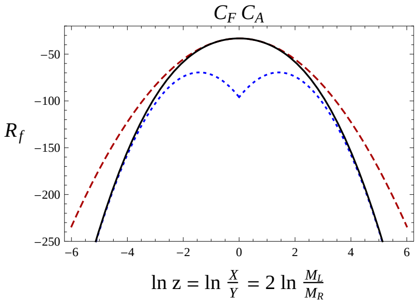
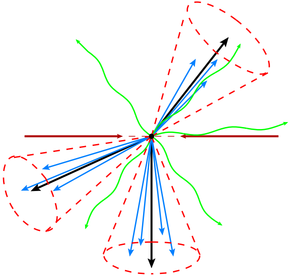




Outline

Jet definition

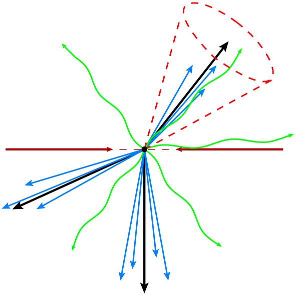


- Cambridge/Aachem algorithm
  - Assign $R_{ij} = \frac{1}{2}(1 - \cos\theta_{ij})$ to each pair of particles
  - For the smallest value of R_{ij}, merge the four vectors of the pair if $R_{ij} < R$.
  - Repeat until there are no pairs with $R_{ij} < R$, then stop.
- Order jets by energy, $E_1 > E_2 > E_3 > \cdots$
- Veto events with $E_3 > \omega$ if interested in dijets.

Inclusive R dependent Jet Shapes

$$\tau_A = \frac{m_{\text{pri}^2} + m^2_{\overline{\text{pri}}}}{Q^2}$$

- $\tau_A \ll 1$ forces dijets
- R-dependent jet shape ($\log R$'s)
- Very sensitive to the choice of the primary jet, sometimes not well defined.
- may be useful in Hadron colliders (dynamical threshold enhancement)



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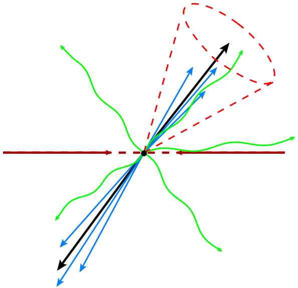
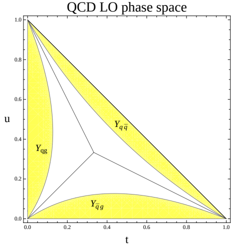


Try τ_{A_1}

$$\tau_{A_1} = \frac{m_1^2 + m_{\bar{1}}^2}{Q^2}$$

- Same as Thrust at $\mathcal{O}(\alpha_s)$
- use $J^{\mathrm{inc}}(p^2)$
- Soft function depends critically on R.



- Soft function is insensitive to jet energy
- If soft gluon is not within R of either jet, which jet is most energetic is ambiguous.

$$\frac{1}{\sigma_0}\frac{d\sigma^{\mathrm{QCD}}}{d\tau_{A_1}} = \delta(\tau_{A_q}) + \frac{\alpha_s}{2\pi}C_F\left[-1+\frac{\pi^2}{3}\right]\delta(\tau_{A_1}) + \frac{\alpha_s}{2\pi}C_F\left[\frac{-4\log\tau_{A_1}-3}{\tau_{A_1}}\right]_+$$

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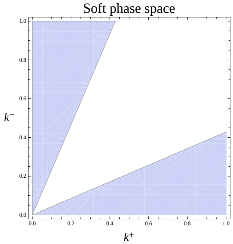

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Try τ_{A_q}

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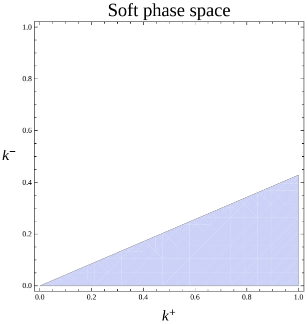
- define quark jet to be "primary"

$$S_R^{\text{in}}(k,\mu) = \delta(k) + \frac{\alpha_s}{2\pi} C_F \left[-\log^2 \frac{R}{1-R} + \frac{\pi^2}{6} \right] \delta(k) + \frac{\alpha_s}{2\pi} C_F \left[\frac{-8\log\frac{k}{\mu} + 4\log\frac{R}{1-R}}{k} \right]_+^{[k,\mu]}$$



- $S_{1-R}^{\text{in}}(k,\mu)$ for other jet

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- at NLO, find negative cross sections since it's not IR safe

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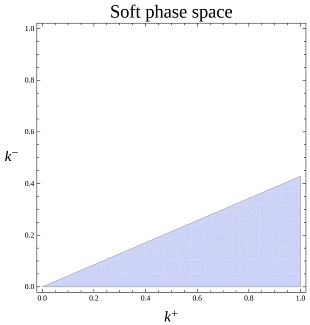

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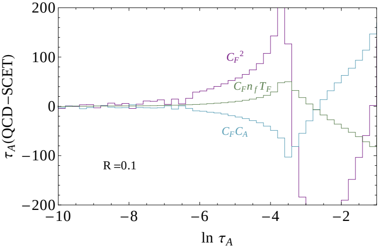
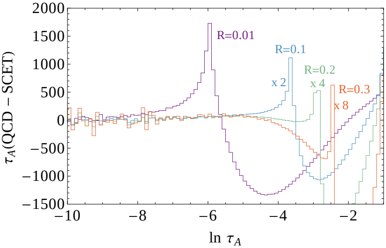
Try an average

$$\frac{d\sigma}{d\tau_{A_1}} + \frac{d\sigma}{d\tau_{A_2}} = \frac{d\sigma}{d\tau_{A_q}} + \frac{d\sigma}{d\tau_{A_{\bar{q}}}}$$

- Agrees with QCD at LO
- at NLO:



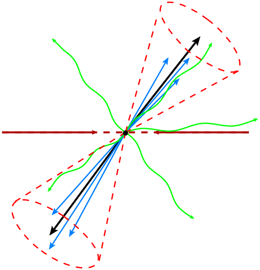


- SCET can resum all large $\log \tau_A$'s, for any R
- For small R, these may not be dominant part. Have not attempted to resum $\log R$'s.

Outline

Exclusive Jet Masses

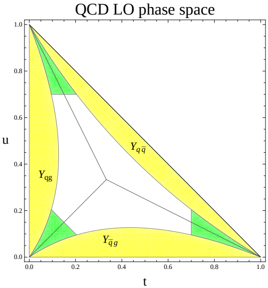
- Veto events with $E_3 > \omega$
- Trivial dependence on m_2 at LO
- Clustered jet always has the most energy
- $f_\omega(R)$ vanishes at $R \to 1/2$ (hemisphere case)



$$\frac{1}{\sigma_0}\left[\frac{d^2\sigma}{dm_1^2 dm_2^2}\right]_{\mathrm{QCD}} = \delta(m_1^2)\delta(m_2^2) + \frac{\alpha}{4\pi}C_F\delta(m_2^2)$$
$$\times\left\{\left(-2+\frac{2\pi^2}{3} - 8\log\frac{R}{1-R}\log\frac{2\omega}{Q} + f_\omega(R)\right)\delta(m_1^2) + \left[\frac{-6+8\log\frac{R}{1-R} - 8\log\frac{m_1^2}{Q^2}}{m_1^2}\right]_* + \cdots\right\}$$

Exclusive Jet Masses

- Veto events with $E_3 > \omega$
- Trivial dependence on m_2 at LO
- Clustered jet always has the most energy
- $f_\omega(R)$ vanishes at $R \to 1/2$ (hemisphere case)



$$\frac{1}{\sigma_0}\left[\frac{d^2\sigma}{dm_1^2 dm_2^2}\right]_{\rm QCD} = \delta(m_1^2)\delta(m_2^2) + \frac{\alpha}{4\pi} C_F \delta(m_2^2)$$

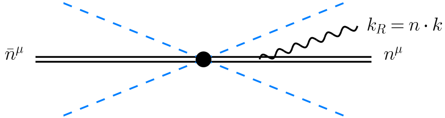
$$\times \left\{ \left(-2 + \frac{2\pi^2}{3} \; -8\log\frac{R}{1-R}\log\frac{2\omega}{Q} + f_\omega(R) \right) \delta(m_1^2) \right.$$

$$\left. + \left[\frac{-6 + 8\log\frac{R}{1-R} - 8\log\frac{m_1^2}{Q^2}}{m_1^2} \right]_* + \cdots \right\}$$

Exclusive Jet Masses

$$\frac{d\sigma}{dm_q^2 dm_{\bar{q}}^2} \sim H(Q,\mu_h) \int dk_q dk_{\bar{q}}$$
$$\times\, J(m_q^2 - k_q Q, \mu_j) J(m_{\bar{q}}^2 - k_{\bar{q}} Q, \mu_j) S_R(k_q, k_{\bar{q}}, \omega, \mu_s)$$

- At order α_s: $S_R(k_L, k_R, \omega, \mu) = S_R^{\text{in}}(k_L,\mu) S_R^{\text{in}}(k_R,\mu) S_R^{\text{out}}(\omega,\mu)$

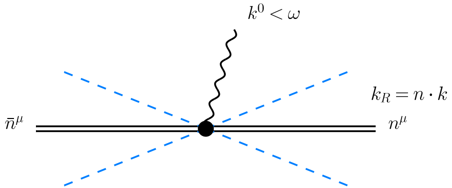


$$S_R^{\text{in}}(k,\mu) = \delta(k) + \frac{\alpha_s}{4\pi} C_F \left(-2\log^2 \frac{R}{1-R} + \frac{\pi^2}{3} \right) \delta(k)$$
$$+ \frac{\alpha_s}{4\pi} C_F \left[\frac{-16 \log \frac{k}{\mu} - 8 \log \frac{R}{1-R}}{k} \right]_*^{[k,\mu]}$$

Exclusive Jet Masses

$$\frac{d\sigma}{dm_q^2 dm_{\bar{q}}^2} \sim H(Q,\mu_h) \int dk_q dk_{\bar{q}}$$
$$\times\, J(m_q^2 - k_q Q, \mu_j) J(m_{\bar{q}}^2 - k_{\bar{q}} Q, \mu_j) S_R(k_q, k_{\bar{q}}, \omega, \mu_s)$$

- At order α_s: $S_R(k_L, k_R, \omega, \mu) = S_R^{\rm in}(k_L,\mu) S_R^{\rm in}(k_R,\mu) S_R^{\rm out}(\omega,\mu)$



$$S_R^{\rm out}(\omega,\mu) = 1 + \frac{\alpha_s}{4\pi} C_F \left[-8 \log \frac{R}{1-R} \log \frac{2\omega}{\mu} + 2 \log^2 \frac{R}{1-R} + f_0(R) \right]_*^{[k,\mu]}$$

QCD vs SCET

$$\frac{1}{\sigma_0}\left[\frac{d^2\sigma}{dm_1^2 dm_2^2}\right]_{\text{QCD}} = \delta(m_1^2)\delta(m_2^2) + \frac{\alpha}{4\pi}C_F\delta(m_2^2)$$

$$\times\left\{\left(-2+\frac{2\pi^2}{3} \quad -8\log\frac{R}{1-R}\log\frac{2\omega}{Q} + f_\omega(R)\right)\delta(m_1^2) + \left[\frac{-6+8\log\frac{R}{1-R} - 8\log\frac{m_1^2}{Q^2}}{m_1^2}\right]_*\right\}$$

Combining the soft function with the hard and inclusive jet functions, we get

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dm_q^2 dm_{\bar q}^2} = \delta(m_q^2)\delta(m_{\bar q}^2) + \frac{\alpha}{4\pi}C_F$$

$$\left\{\left(-2+\frac{2\pi^2}{3} - 8\log\frac{R}{1-R}\log\frac{2\omega}{Q} + f_0(R)\right)\delta(m_q^2)\delta(m_{\bar q}^2)\right.$$

$$\left. + \left[\frac{-6+8\log\frac{R}{1-R} - 8\log\frac{m_q^2}{Q^2}}{2m_q^2}\right]_* \delta(m_{\bar q}^2) + \left[\frac{-6+8\log\frac{R}{1-R} - 8\log\frac{m_{\bar q}^2}{Q^2}}{2m_{\bar q}^2}\right]_* \delta(m_q^2)\right\}$$

QCD vs SCET

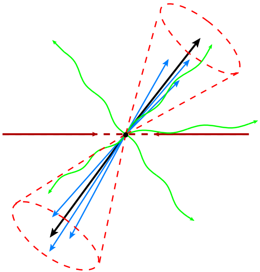
- $\delta(m_q)\delta(m_{\bar{q}})$ matches $\delta(m_q)\delta(m_2)$ with $f_0(R)$ instead of $f_\omega(R)$.
- SCET is symmetric $m_q \leftrightarrow m_{\bar{q}}$, QCD is not
- Mass of the hardest jet is not simply related to any projection of $\frac{d^2\sigma}{dm_q^2 dm_{\bar{q}}^2}$

$$\left[\frac{d\sigma}{dm^2}\right]_{\text{QCD}} = \int_0^{Q^2 R} dm_1^2 \int_0^{Q^2 R} dm_2^2 \, \frac{d^2\sigma}{dm_1^2 dm_2^2} \times \frac{1}{2}\left[\delta(m^2 - m_1^2) + \delta(m^2 - m_2^2)\right]$$

- Will have NGLs of form $\log^n \frac{m_1}{m_2}$

Define τ_ω

- Veto events with $E_3 > \omega$
- Define: $\tau_\omega = \dfrac{m_1^2 + m_2^2}{Q^2}$
- Reproduces QCD as $\tau_\omega \to 0$
- Avoids NGLs of form $\log^n \dfrac{m_1}{m_2}$



$$\frac{1}{\sigma_0}\frac{d\sigma^{\mathrm{QCD}}}{d\tau_\omega} = \delta(\tau_\omega) + \frac{\alpha_s}{2\pi}C_F\left[7 - \frac{5\pi^2}{6} + 4\log\frac{1-R}{R}\log\frac{2\omega}{Q} + f_\omega(R)\right]\delta(\tau_\omega) + \frac{\alpha_s}{2\pi}C_F\left[\frac{-4\log\tau_\omega - 3 - 4\log\frac{1-R}{R}}{\tau_\omega}\right]_+ + \cdots$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\mathrm{SCET}}}{d\tau_\omega} = \delta(\tau_\omega) + \frac{\alpha_s}{2\pi}C_F\left[7 - \frac{5\pi^2}{6} + 4\log\frac{1-R}{R}\log\frac{2\omega}{Q} + f_0(R)\right]\delta(\tau_\omega) + \frac{\alpha_s}{2\pi}C_F\left[\frac{-4\log\tau_\omega - 3 - 4\log\frac{1-R}{R}}{\tau_\omega}\right]_+ + \cdots$$

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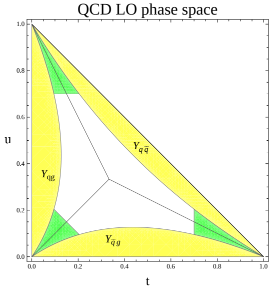


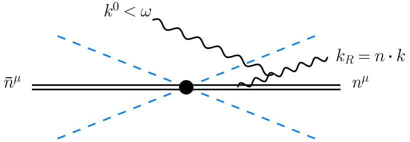
$$\frac{1}{\sigma_0}\frac{d\sigma^{\rm QCD}}{d\tau_\omega} = \delta(\tau_\omega) + \frac{\alpha_s}{2\pi}C_F\left[7 - \frac{5\pi^2}{6} + 4\log\frac{1-R}{R}\log\frac{2\omega}{Q} + f_\omega(R)\right]\delta(\tau_\omega)$$
$$+ \frac{\alpha_s}{2\pi}C_F\left[\frac{-4\log\tau_\omega - 3 - 4\log\frac{1-R}{R}}{\tau_\omega}\right]_+ + \cdots$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\rm SCET}}{d\tau_\omega} = \delta(\tau_\omega) + \frac{\alpha_s}{2\pi}C_F\left[7 - \frac{5\pi^2}{6} + 4\log\frac{1-R}{R}\log\frac{2\omega}{Q} + f_0(R)\right]\delta(\tau_\omega)$$
$$+ \frac{\alpha_s}{2\pi}C_F\left[\frac{-4\log\tau_\omega - 3 - 4\log\frac{1-R}{R}}{\tau_\omega}\right]_+ + \cdots$$

Outline

1. Introduction
2. 2-loop Hemisphere Soft function
3. Inclusive R dependent Jet Shapes
4. Exclusive Jet Masses
5. Factorization of the Soft Function

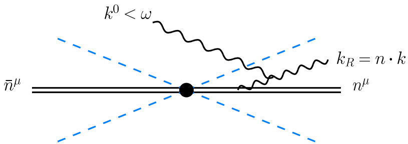
Factorization of the Soft Function



$$S_R(k, \omega, \mu) = S_R^{\text{in}}(k, \mu) S_R^{\text{out}}(\omega, \mu) S_R^F\left(\frac{\omega}{k}\right)$$

- All NGL's are in $S_R^F\left(\frac{\omega}{k}\right)$
- When can we neglect ω/k dependence?

Factorization of the Soft Function



- For small R, we can show

$$S_R(k, \omega, \mu) = S_R^{\text{in}}(k, \mu) S_R^{\text{out}}(\omega, \mu)$$

  for $\omega/Q \lesssim k/Q \ll R \ll 1$

- Later, we discuss $\log \omega/k$ terms which violate this factorization.
- SCET requires ω, k to be small, but they can be far apart
- For small R, k is in the cone and has collinear scaling.

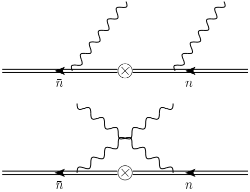
$$k^+ < \frac{R}{1-R} k^- \qquad (k^+, k^-, k_\perp) \sim \frac{k}{R}(R, 1, \sqrt{R})$$

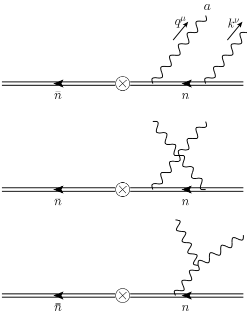
- q is outside of either cone with $E_q < \omega$.

$$(q^+, q^-, q_\perp) \sim (\omega, \omega, \omega)$$

Factorization of the Soft Function

diagrammatic proof





For small R, and $\omega \lesssim k_R$

$$\sum \mathcal{M}_i = \left(g^2 \frac{n^\mu n^\nu}{k^- q^-} T^b T^a - g^2 \frac{\bar{n}^\mu n^\nu}{k^- q^+} T^b T^a \right) \varepsilon^a_\mu(q) \varepsilon^b_\nu(k)$$

Equivalent to the the following refactorization

$$Y^\dagger_{\bar{n}} Y_n \to (Y^{sc}_{\bar{n}})^\dagger (Y^{us}_{\bar{n}})^\dagger (Y^{us}_n)(Y^{sc}_n).$$

Similar to the factorization using SCET$_+$ in Bauer et al. 1106.6047

Soft Anomalous dimension

$$\Gamma_s = \frac{\alpha_s}{\pi} C_F \Gamma_{\text{cusp}} \log \frac{k_L k_R}{\mu^2} + \gamma_S^{\text{out}} + \gamma_S^{\text{in}}$$

- Extract γ_S from the α_s calculation

$$\gamma_S^{\text{out}} = -\frac{\alpha_s}{4\pi} C_F \Gamma_{\text{cusp}} \log \frac{R}{1-R}$$

$$\gamma_S^{\text{in}} = \gamma_S^{\text{hemi}} + \frac{\alpha_s}{4\pi} C_F \Gamma_{\text{cusp}} \log \frac{R}{1-R}$$

- RG invariance requires the R dependence to cancel in the sum to all orders Ellis et al. 0912.062, JHEP 1011,101 (2010)
- Holds at two loops, suspect it holds at all orders.
- Refactorization gives predictive power through separating scales
- As $R \to \frac{1}{2}$, $\gamma_S^{\text{in}} \to \gamma_S^{\text{hemi}}$ and $\gamma_S^{\text{out}} \to 0$.
- At order α_s^2, this form contributes terms to the expression

$$\Gamma_1 \log \frac{R}{1-R} \log \tau_\omega$$

Soft Anomalous dimension

$$\Gamma_s = \frac{\alpha_s}{\pi} C_F \Gamma_{\text{cusp}} \log \frac{k_L k_R}{\mu^2} + \gamma_S^{\text{out}} + \gamma_S^{\text{in}}$$

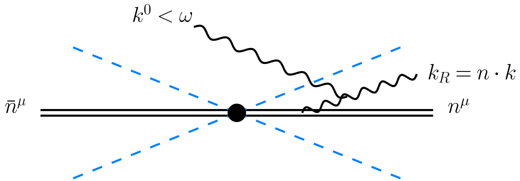
- Extract γ_S from the α_s calculation

$$\gamma_S^{\text{out}} = -\frac{\alpha_s}{4\pi} C_F \Gamma_{\text{cusp}} \log \frac{R}{1-R} - \gamma_R(R)$$

$$\gamma_S^{\text{in}} = \gamma_S^{\text{hemi}} + \frac{\alpha_s}{4\pi} C_F \Gamma_{\text{cusp}} \log \frac{R}{1-R} + \gamma_R(R)$$

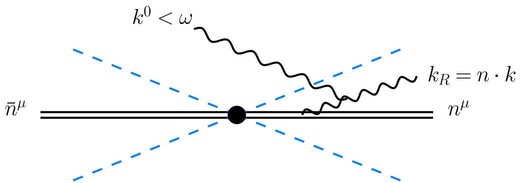
- $\gamma_R(R)$ should approach a constant in small R
- The structure of $\gamma_R(R)$ is not known beyond 1 loop

Predictions from Refactorization



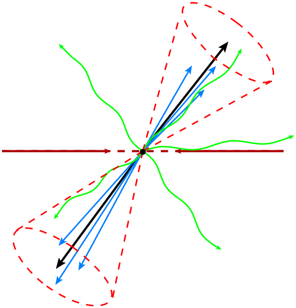
- Now consider $\omega \not\approx \tau_\omega Q$
- SCET agrees with QCD up to powers in ω/Q and τ_ω (brute force if necessary)
- Neglecting powers of $\omega/\tau_\omega Q$ is consistent with numerics.
- Could be important $\log \frac{\omega}{\tau_\omega Q}$ terms

Predictions from Refactorization



- When R is not small, "in" jet radiation is not small and there is no obvious factorization.
- $R \to \frac{1}{2}$ (hemisphere case), the ω dependence vanishes
- Factorization captures the $\log R \log \frac{\tau_\omega Q}{2\omega}$, but not the terms constant in R wrong.
- The factorization holds at small and large R and is a good approximation for moderate R.
- Much of the R dependence of full QCD is captured by the small R limit.

Thrust-like jets

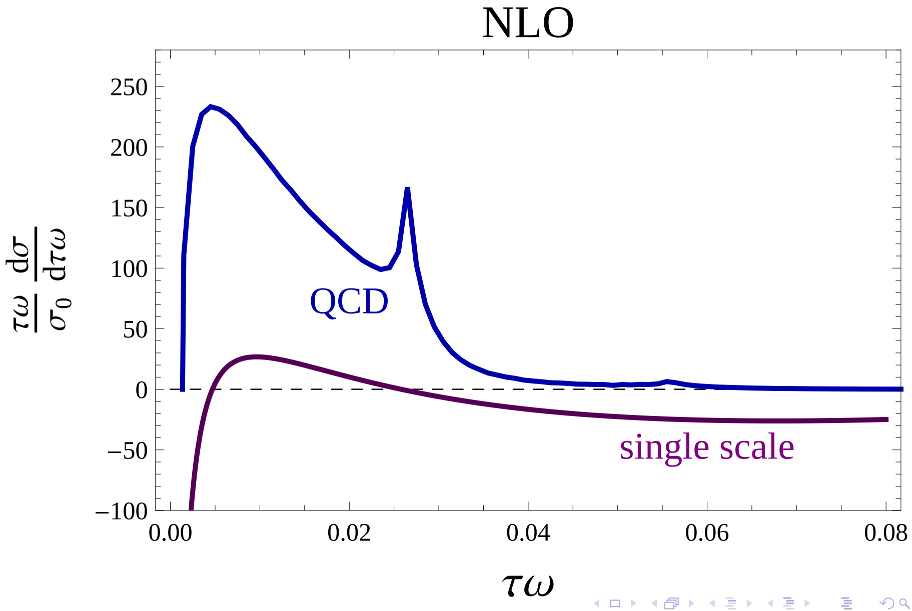


- Find thrust axis
- Cluster particles within R of thrust axis
- Same as Cambridge/Aachem at α_s, similar at α_s^2
- NGL's structure is different than CA (Hornig et al. 1110.0004)

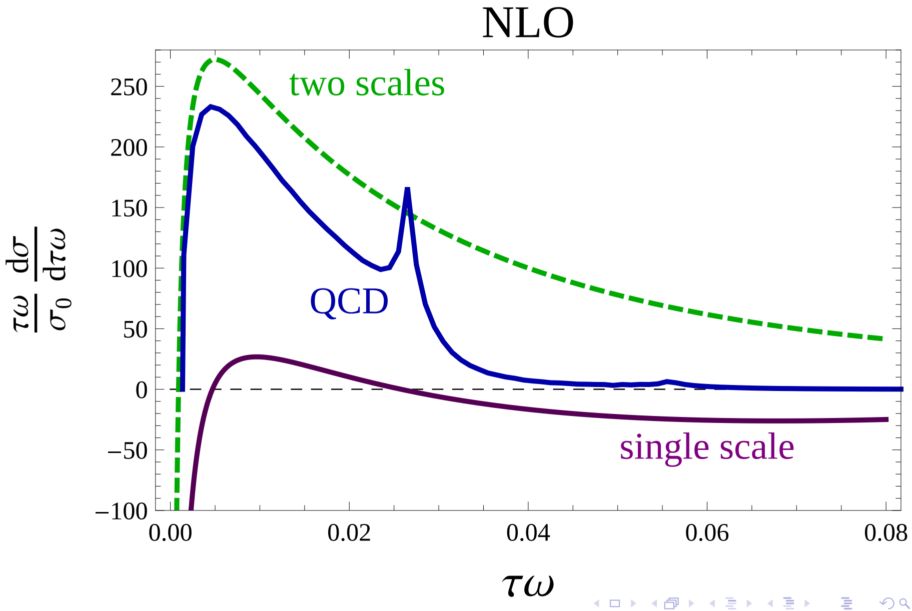
Numerical Check of Ansatz

- The α_s^2 predictions from SCET where compared to EVENT2 (Catani and Seymore)
- Checked both Cambridge/Aachem jets and Thrust-like jets
- We expect SCET to agree with EVENT2 up to powers in τ_ω and ω/Q.
- Highly non-trivial check of the factorization theorem
- Holds independently various color factors C_F^2, $C_A C_F$ and $C_F n_f T_F$.
- Checked for a large range of R values

Comparison with EVENT2



Comparison with EVENT2



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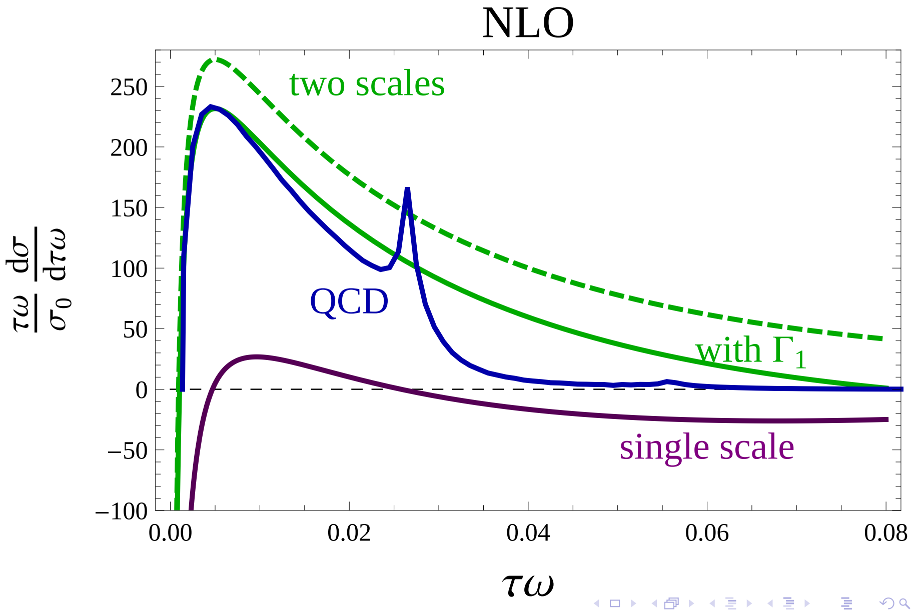

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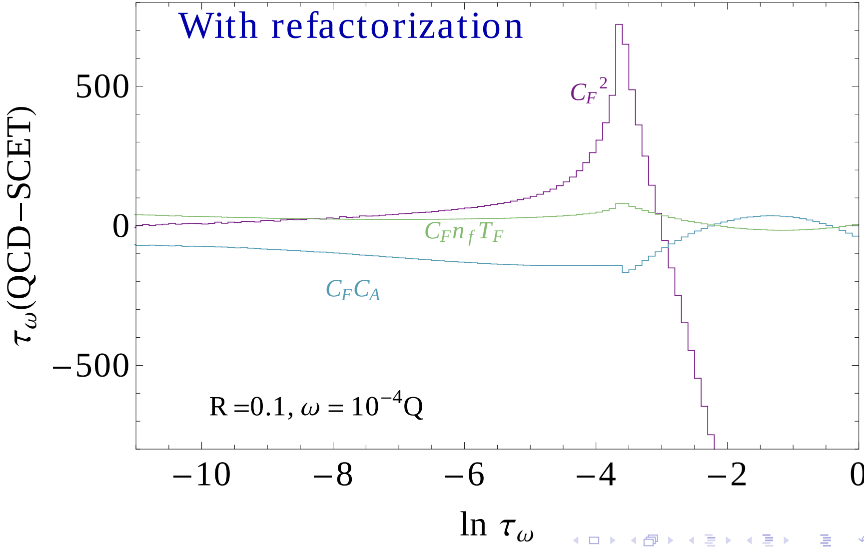

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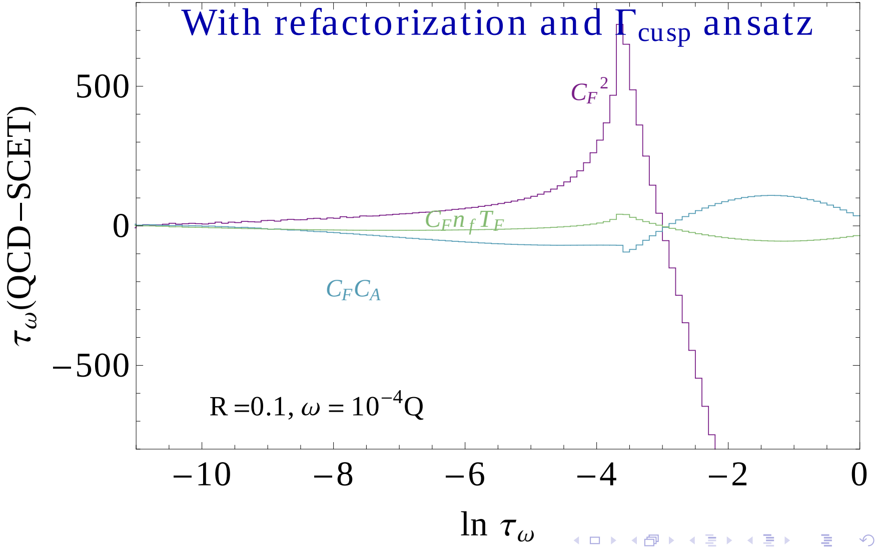
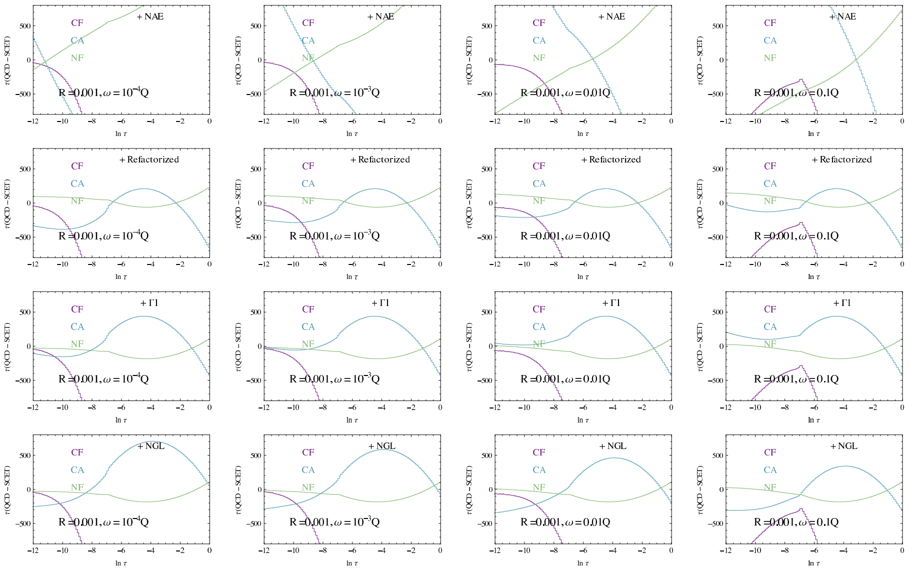


Comparison with EVENT2: Thrust-Axis clustering



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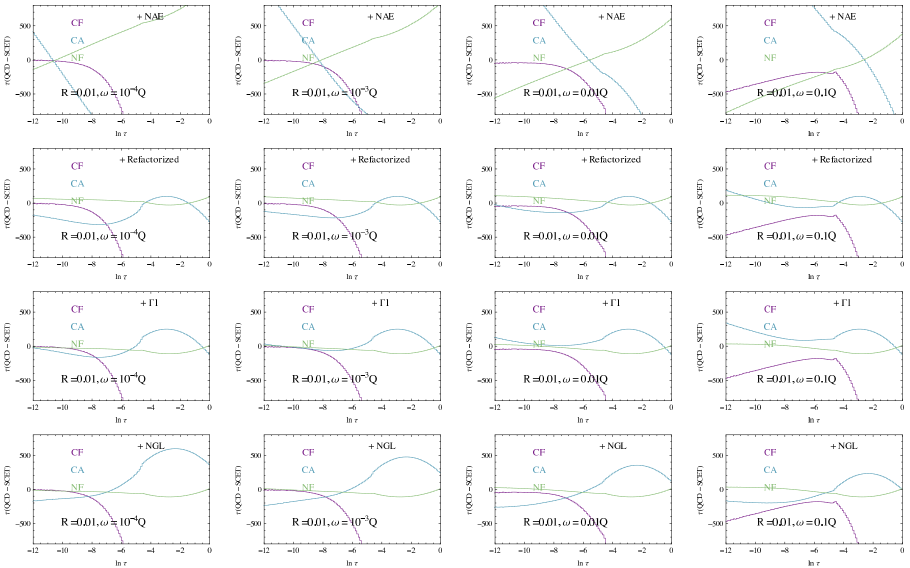

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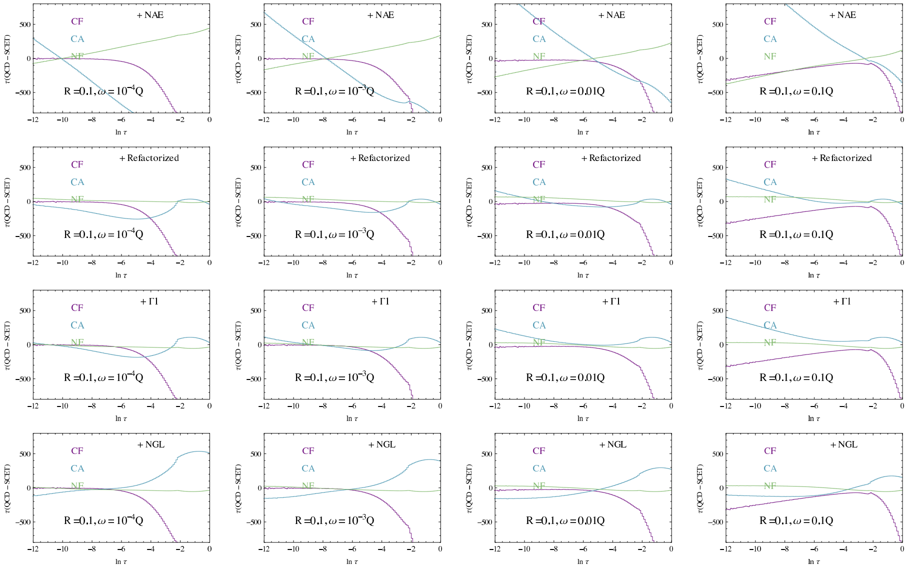

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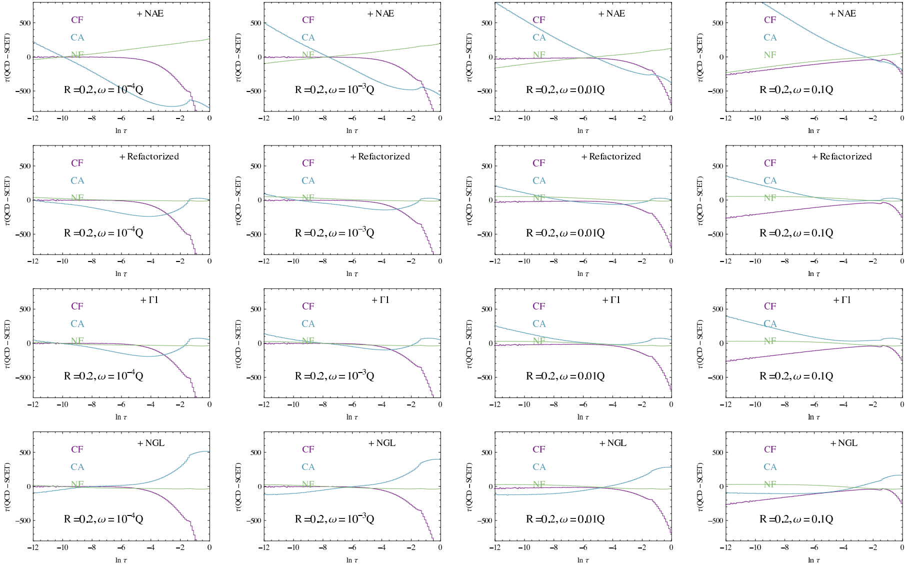

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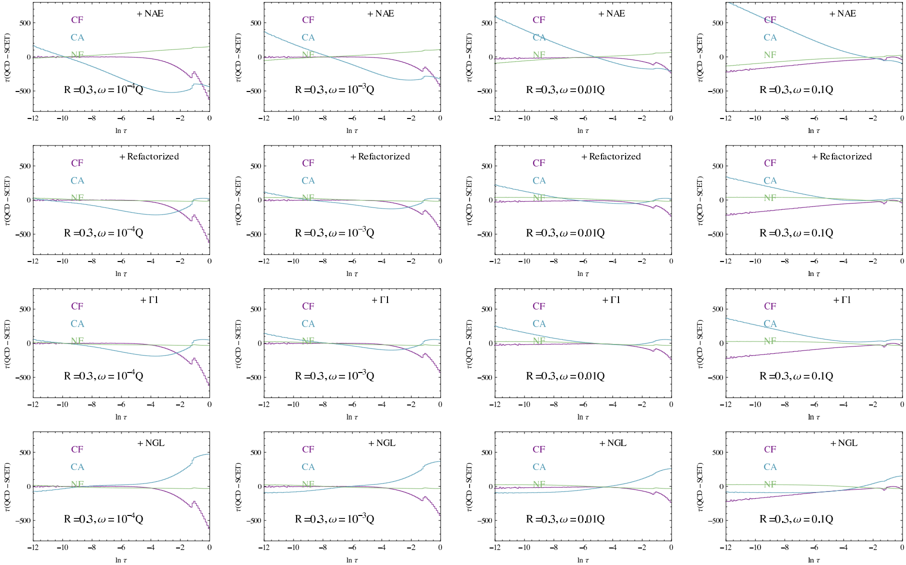

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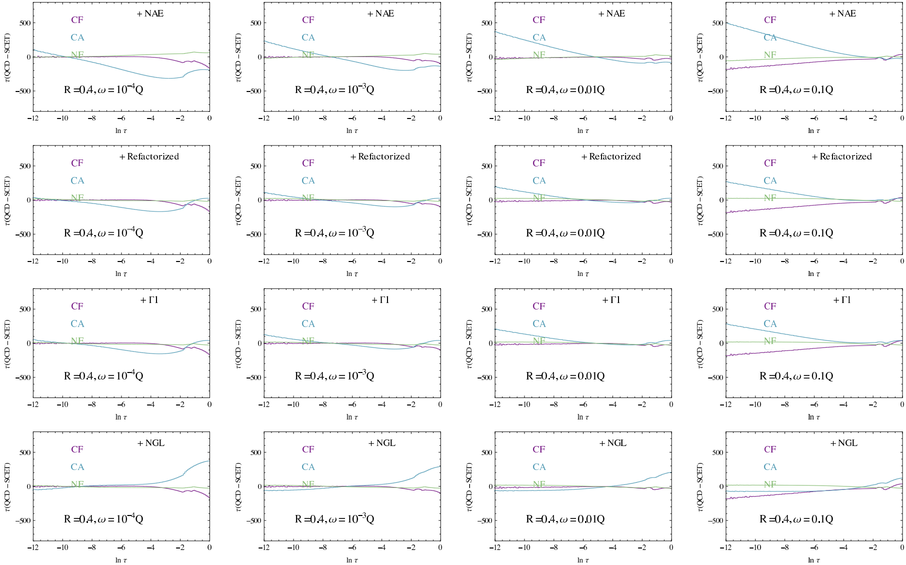

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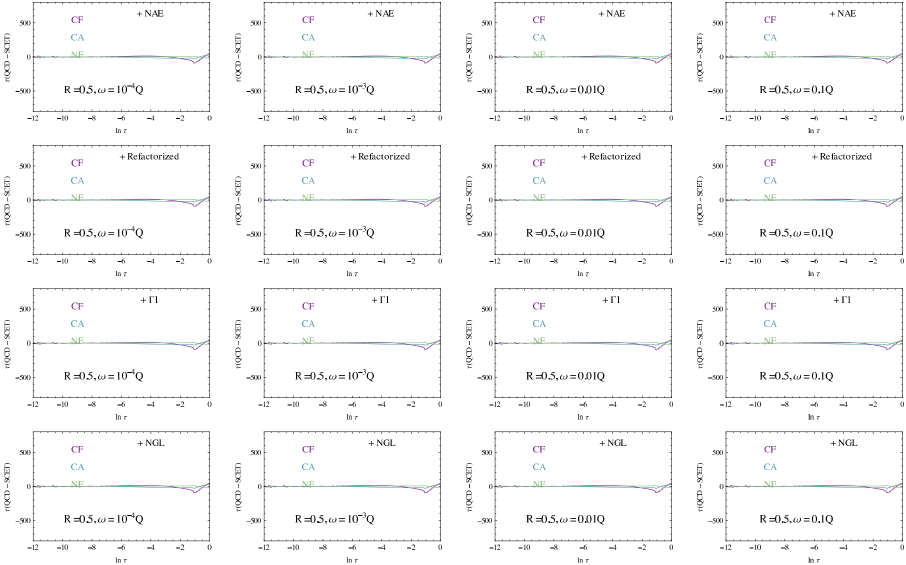
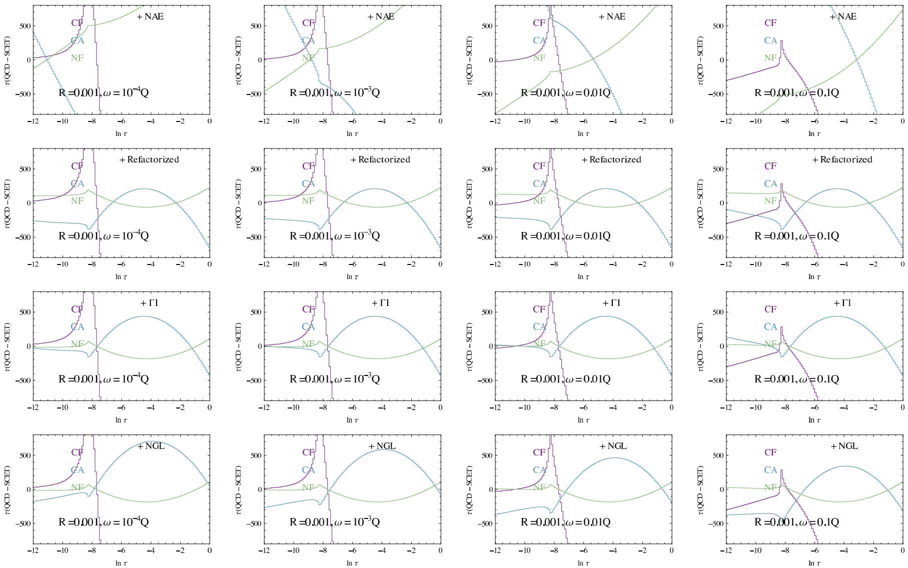


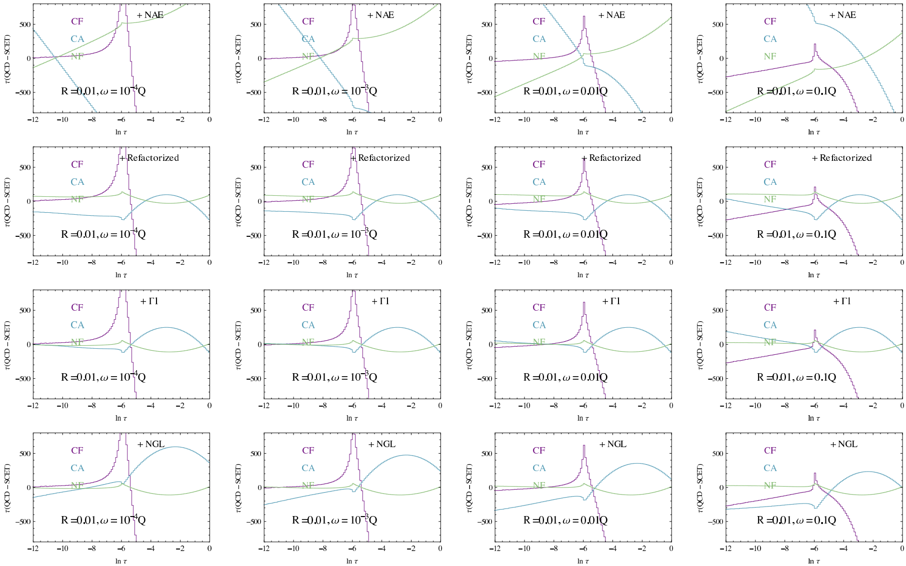
Comparison with EVENT2: Thrust-Axis clustering

too far!

Comparison with EVENT2 Cambridge/Aachem clustering



Comparison with EVENT2 Cambridge/Aachem clustering



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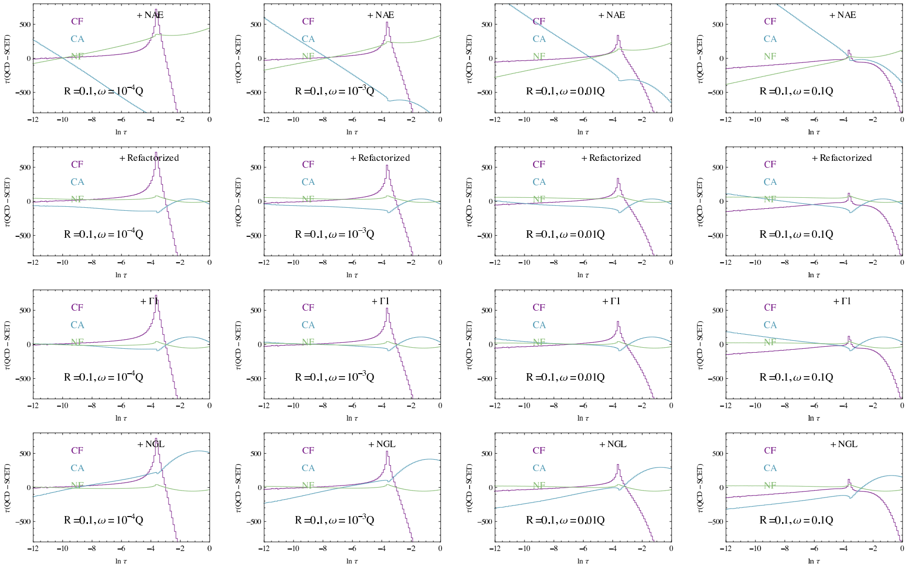

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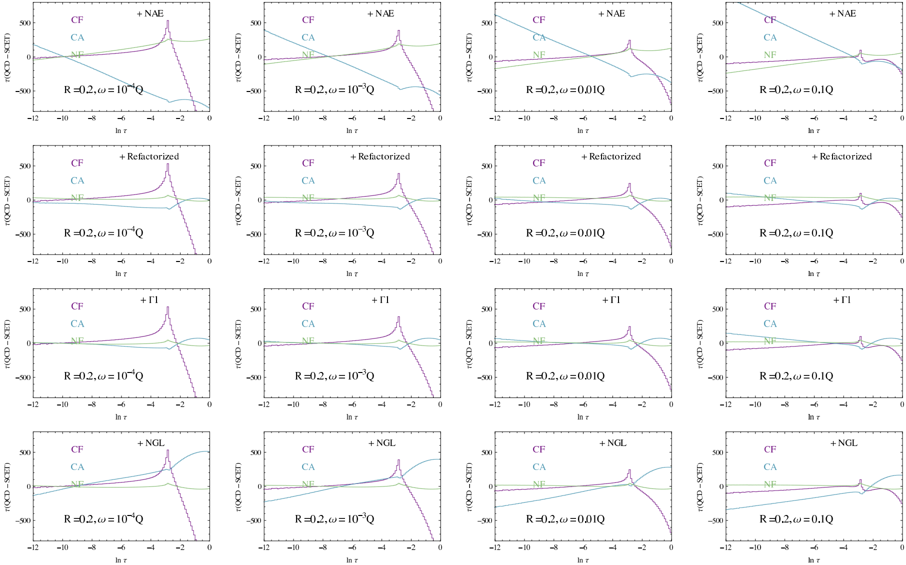

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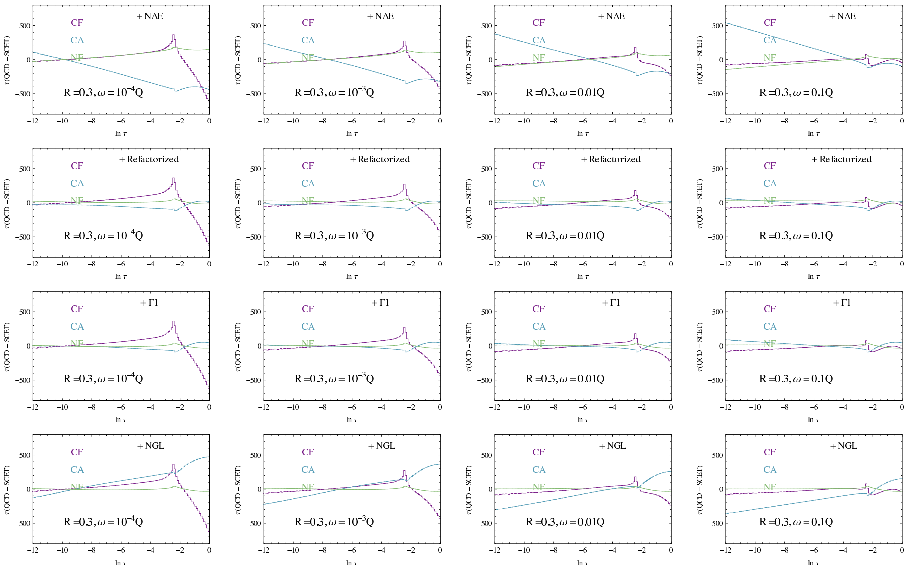

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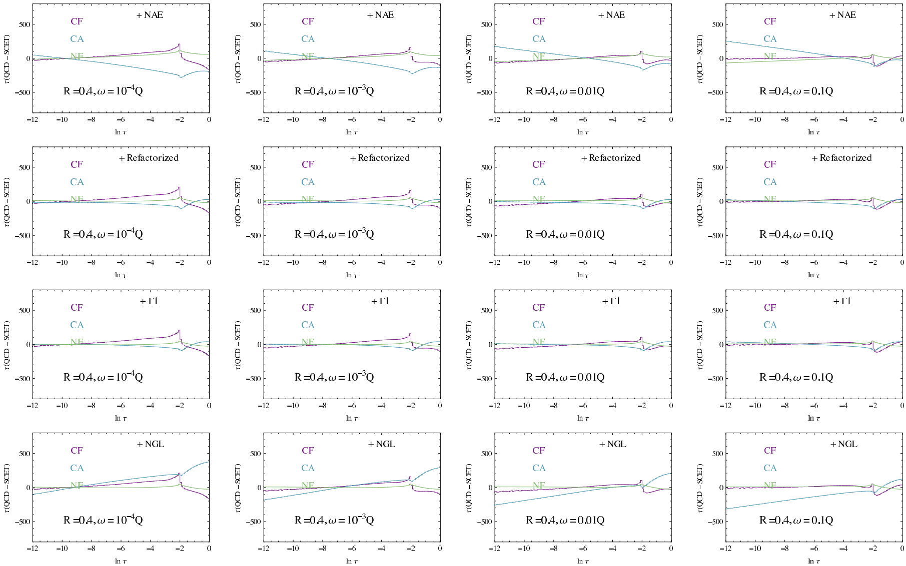

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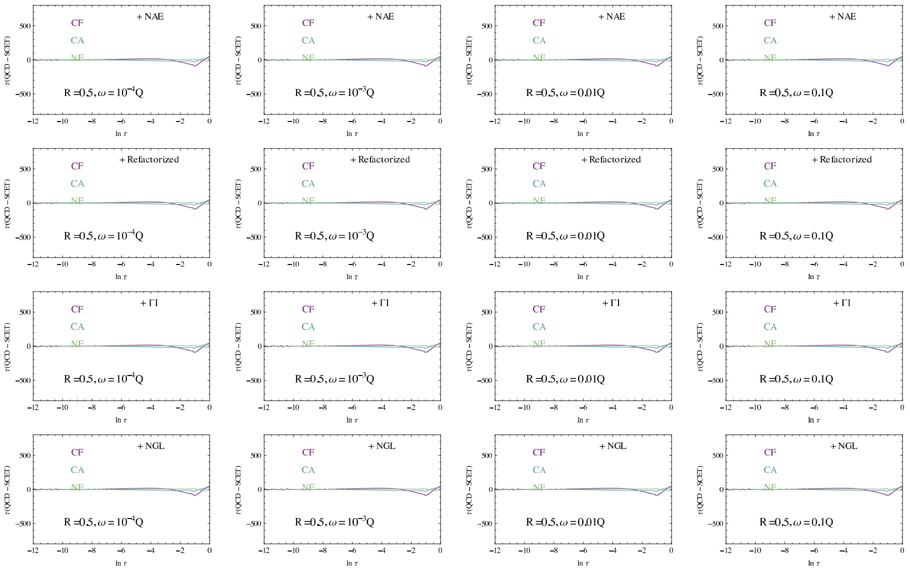


Two loop τ_ω Soft Function preliminary

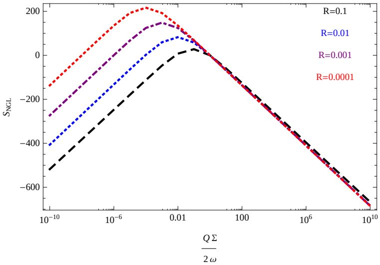
- General form of two loop soft function

$$S_R(k,\omega,\mu) = S_R^{\text{in}}\left(\frac{k}{\mu}\right) S_R^{\text{out}}\left(\frac{\omega}{\mu}\right) S_R^F\left(\frac{k}{\omega}\right)$$

- $S_R^F = S_R^{NGL} + \text{finite}$
- consider cumulative distribution instead ($\tau_\omega \to \sigma$)

- $C_F n_f T_F$ channel
- $R \to 0$ then $\frac{\Sigma Q}{2\omega} \to 0$ (recall $\Sigma < R$)

$$\left(-\frac{32\pi^2}{9} + \frac{16}{3}\right) \log \frac{\Sigma Q}{2\omega}$$



- NGL in $R \to 0$ is twice hemisphere case

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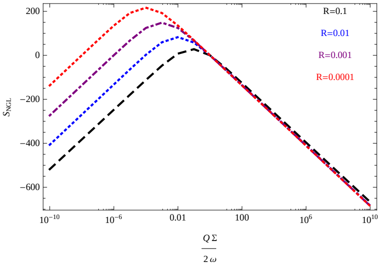
- $S_R^F = S_R^{NGL} + \text{finite}$
- consider cumulative distribution instead ($\tau_\omega \to \sigma$)

- $C_F n_f T_F$ channel
- $\dfrac{\Sigma Q}{2\omega} \to 0$ then $R \to 0$

$$\left(-\frac{32\pi^2}{9} + \frac{16}{3}\right) \log \frac{\Sigma Q}{2\omega R^2}$$



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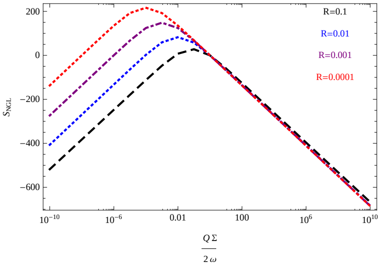
- $S_R^F = S_R^{NGL} + \text{finite}$
- consider cumulative distribution instead ($\tau_\omega \to \sigma$)

- $C_F n_f T_F$ channel
- $\frac{\Sigma Q}{2\omega} \to \infty$, no R dependence.

$$-\left(-\frac{32\pi^2}{9} + \frac{16}{3}\right)\log\frac{\Sigma Q}{2\omega}$$

- NGL in $R \to 0$ is twice hemisphere case



Two loop τ_ω Soft Function preliminary

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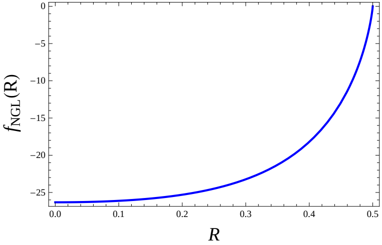

$$\left[-\frac{8\pi^2}{3} + 16\text{Li}_2\left(\frac{R^2}{(1-R)^2}\right)\right]\log^2\frac{\Sigma Q}{2\omega}$$

$$+\left(-16\zeta_3 - \frac{8}{3} + \frac{88\pi^2}{9} + \cdots\right)\log\frac{\Sigma Q}{2\omega}$$

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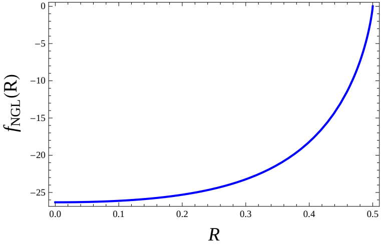

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- Possible $\log R$ dependence in leading NGL missed.

Conclusions

- Inclusive observables (e.g. τ_A) seemed amenable to resummation
- Soft function factorization held in limit $\omega/Q \lesssim \tau_\omega \ll R \ll 1$ but was not a bad approximation elsewhere.
- Non-global structures are present, but numerically small for a large choice of parameters
- The results extrapolated away from $R \to 0$ limit provides good agreement with QCD.
- Calculation of τ_ω soft function almost finished.