“Improved” holographic vector mesons.

Andreas Karch

(work with Carlos Hoyos and Raul Alvares ; 1108.1191)

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Holography: Top-down.

Holography = Solvable Toy Model

Solvable models of strong coupling dynamics.

- Study Transport, real time
- Study Finite Density
- Explore paradigms "beyond Landau"

(Challenging in real QCD, experimentally relevant)

(this is interesting for a different audience)

Gives us qualitative guidance/intuition.

Not QCD! Expect errors of order 100%

(better than extrapolating perturbation theory to $\alpha_s \sim 1$??)

Holographic Theories:

Examples known:

- in d=1, 2, 3, 4, 5, 6 space-time dimensions
- with our without super-symmetry
- conformal or confining
- with or without chiral symmetry breaking
- with finite temperature and density

Holographic Theories:

Holographic toy models have two key properties:

"Large N": theory is essentially classical

"Large λ": large separation of scales in the spectrum

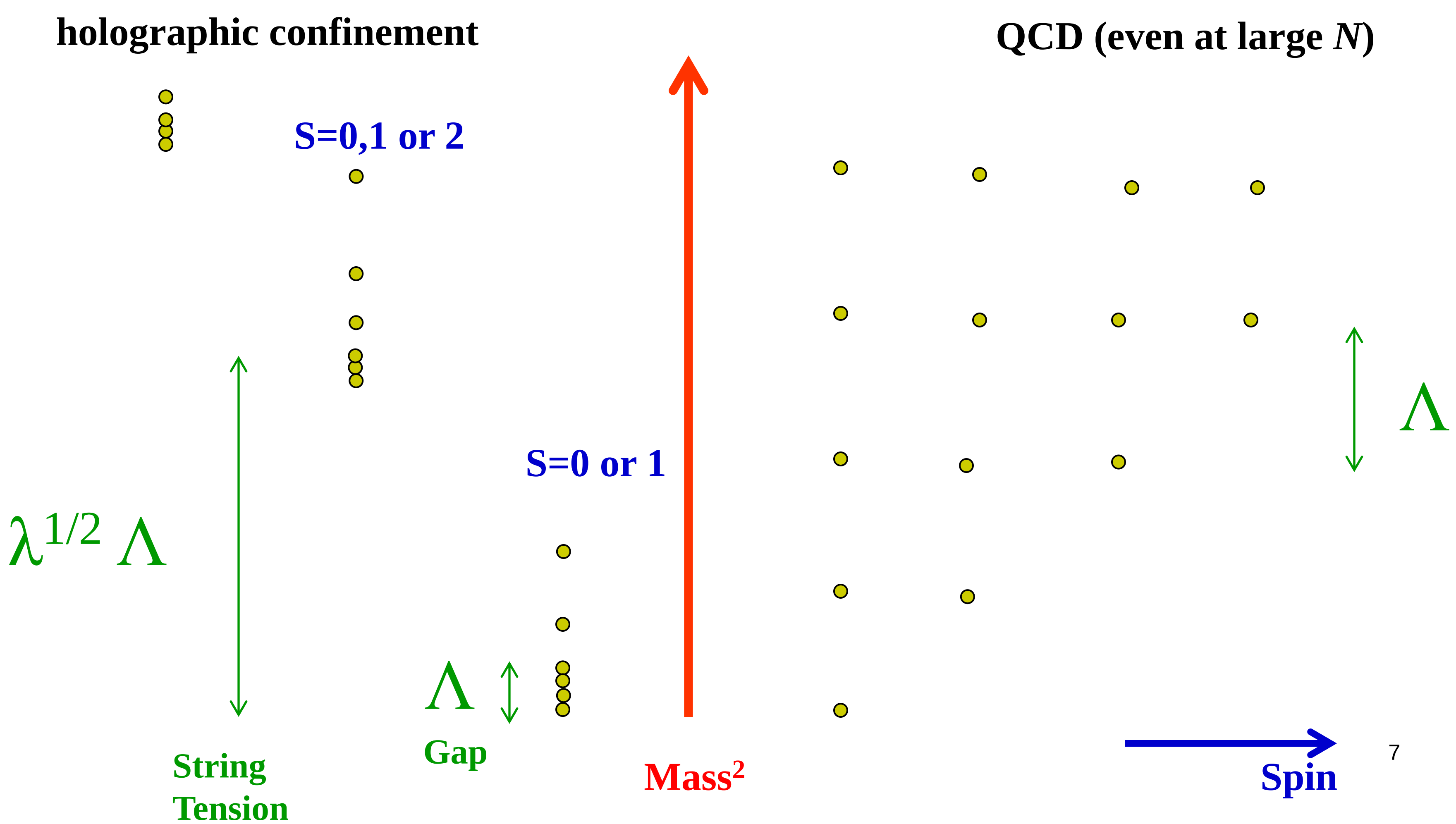
$$m_{\text{spin-2-meson}} \sim \lambda^{1/4}\ m_{\text{spin-1-meson}}$$

QCD: **1275 MeV** **775 MeV**

(note: there are some exotic examples where the same parameter N controls both, classicality and separation of scales in spectrum)

Bottom-up models.

Bottom-up versus top down:



Bottom-up Strategy

- Postulate an effective theory for QCD in terms of a 5d bulk (2-derivative action.)
- Follow **standard holography rules** to fix action and background (comparing to UV free QCD)
- Model is justified by success.
- Systematic expansion relies on 5 >> 3. How good is this approximation?

Bottom-Up Success. (Erlich, Katz, Son, Stephanov)

TABLE II: Results of the model for QCD observables. Model A is a fit of the three model parameters to m_π, f_π and m_ρ (see asterisks). Model B is a fit to all seven observables.

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

Accident?

Bottom-up Motivation

- Even if bottom-up gave only $1/3^2 \sim 10\%$ errors (highly questionable), it would never be competitive with lattice for masses + equilibrium. Why bother?
- Answers are simple and intuitive.
- Can be used to quickly survey large classes of non-QCD theories (e.g. for technicolor or hidden valleys).

Holographic rules:

Holography - Rule 1:

Field theory operator ↔ Bulk field

Implies infinite number of bulk fields, one for each QCD operator:

$$\overline{\psi}\,\psi\,,\ \ \overline{\psi}\,\gamma_\mu\psi\,,\ \ \overline{\psi}\,\gamma_{[\mu}\gamma_{\nu]}\psi\,,\ \ \overline{\psi}\,F_{\mu\nu}\psi\,,\ \ \ldots$$

Holographic Mesons:

(Erlich, Katz, Son, Stephanov; Karch, Katz, Son, Stephanov)

$$\overline{\psi}\,\psi \longrightarrow X$$

$$\overline{\psi}\,\gamma_\mu \psi \longrightarrow A_\mu$$

} **Dimension 3**

$$S = \int d^5x\,\sqrt{g}\;\mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

Drop all fields dual to operators of dimension 4 and higher!
(and hope for the best).

Holographic rules:

Holography - Rule 2:

Correlation functions ↔ Bulk on shell action

Large momentum behavior of bulk propagator (plugged into action) has to reproduce free UV correlators of QCD.

Hard and Soft Wall Models:

(Erlich, Katz, Son, Stephanov; Karch, Katz, Son, Stephanov)

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\Big\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \Big\}$$

↑

fixed by UV behavior of JJ correlator

Guess background geometry

hard wall: simple
soft wall: correct Regge

} **AdS in UV**

Building a better model.

What about:

$$\overline{\psi}\, F_{\mu\nu}\psi \longrightarrow$$

Dimension 5. Can be neglected?
(dimension ~ $\lambda^{1/4}$ in holography)

$$\overline{\psi}\, \gamma_{[\mu}\gamma_{\nu]}\psi \longrightarrow$$

Dimension 3. Massive $B_{\mu\nu}$ should definitely be included.
(dimension ~ $\lambda^{1/4}$ in holography)

Without $B_{\mu\nu}$ operator, we are missing "half" of the vector mesons ($J^{PC} = 1^{+-}$) - does it matter?

Hard- and Softwall predict: no b_1!

$b_1(1235)$ $I^G(J^{PC}) = 1^+(1^{+-})$

Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	dominant		348
[D/S amplitude ratio = 0.277 ± 0.027]			
$\pi^{\pm}\gamma$	$(1.6\pm0.4) \times 10^{-3}$		607
$\eta\rho$	seen		†
$\pi^+\pi^+\pi^-\pi^0$	< 50 %	84%	535
$(K\overline{K})^{\pm}\pi^0$	< 8 %	90%	248
$K^0_S K^0_L \pi^{\pm}$	< 6 %	90%	235
$K^0_S K^0_S \pi^{\pm}$	< 2 %	90%	235
$\phi\pi$	< 1.5 %	84%	147

Is the b_1 parametrically heavy?

LIGHT UNFLAVORED
($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$

Let's look at PDG!

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Isospin singlets mix with glue sector (need extra scalar fields).

Should not expect agreement.

Is the b_1 parametrically heavy?

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Included in Hard/Soft wall model

Is the b_1 parametrically heavy?

	$I^G(J^{PC})$
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Spin 0 or 2 mesons – need extra fields; Do not mix -- not important for understanding vectors + pions.

Is the b_1 parametrically heavy?

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Created by dimension 5 operator:

$$\overline{\psi} F_{\mu\nu} \gamma_5 \psi$$

5 >> 3 implies: 1400 MeV "much" heavier than 1260 MeV

Is the b_1 parametrically heavy?

	$I^G(J^{PC})$
$\pi^\pm$	$1^-(0^-)$
π^0	$1^-(0^{-+})$
$\rho(770)$	$1^+(1^{--})$
$b_1(1235)$	$1^+(1^{+-})$
$a_1(1260)$	$1^-(1^{++})$

But dropping the dimension 3 operator

$$\overline{\psi}\gamma_{[\mu}\gamma_{\nu]}\psi$$

is surely incompatible with data!!!

Building a consistent holographic model.

Building a better model – try I.

$$L = Tr\Big[-\frac{1}{4g_5^2}(F_L^2 + F_R^2) + g_B\Big(\frac{1}{3}|H|^2 - |B|^2\Big) + \lambda(X^\dagger F_L B + B F_R X^\dagger + h.c.)\Big]$$

Building a better model – try I.

$$L = Tr\Big[-\frac{1}{4g_5^2}(F_L^2 + F_R^2) + g_B\big(\frac{1}{3}|H|^2 - |B|^2\big) + \lambda(X^\dagger F_L B + B F_R X^\dagger + h.c.)\Big]$$

The old players. The rho mesons, axial vector mesons and and pions are described by these.

$$X = \frac{1}{2}\left(g_X m\, z + \frac{\langle \overline{\psi}\psi \rangle}{g_X} z^3 \right) \mathbf{1}_{2\times 2}$$

Building a better model – try I.

$$L = Tr\Big[-\frac{1}{4g_5^2}(F_L^2 + F_R^2) + g_B\big(\frac{1}{3}|H|^2 - |B|^2\big) + \lambda(X^\dagger F_L B + B F_R X^\dagger + h.c.)\Big]$$

A new field, massive $B_{\mu\nu}$. Mass fixed by dimension 3. Bifundamental under $SU(2)_L$ x $SU(2)_R$

(Capiello, Cata, Ambrosio)

Building a better model – try I.

$$L = Tr\Big[-\frac{1}{4g_5^2}(F_L^2 + F_R^2) + g_B\Big(\frac{1}{3}|H|^2 - |B|^2\Big) + \lambda(X^\dagger F_L B + B F_R X^\dagger + h.c.)\Big]$$

Chiral symmetry breaking background for X mixes B and F: B describes new 1+- mesons, but also modifies rho spectrum!

$$X = \frac{1}{2}\left(g_X m\, z + \frac{\langle\bar\psi\psi\rangle}{g_X} z^3\right)\mathbf{1}_{2\times 2}$$

Building a better model – try I.

$$L = Tr\Big[-\frac{1}{4g_5^2}(F_L^2 + F_R^2) + g_B\big(\frac{1}{3}|H|^2 - |B|^2\big) + \lambda(X^\dagger F_L B + B F_R X^\dagger + h.c.)\Big]$$

All bulk couplings fixed by matching to UV structure of QCD correlators!

But…..

Bi-fundamental $B_{\mu\nu}$ is a complex field!

What is the role of real and imaginary part?

Recall: bi-fundamental, complex X dual to

$$\bar{\psi}\psi + i\bar{\psi}\gamma_5\psi$$

So: $B_{\mu\nu}$ dual to:

$$\bar{\psi}\gamma_{[\mu}\gamma_{v]}\psi + i\bar{\psi}\gamma_5\gamma_{[\mu}\gamma_{v]}\psi$$

But…..

$B_{\mu\nu}$ dual to: $\bar{\psi}\gamma_{[\mu}\gamma_{v]}\psi + i\bar{\psi}\gamma_5\gamma_{[\mu}\gamma_{v]}\psi$

$$\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi = \frac{i}{2}\epsilon^{\mu\nu}{}_{\alpha\beta}\bar{\psi}\sigma^{\alpha\beta}\psi.$$

$B_{\mu\upsilon}$ is an imaginary anti-self dual field!

(Domokos, Harvey, Royston)

First order action for self-dual field

(Domokos, Harvey, Royston)

$$S_B = -\int d^5x \left[i(B \wedge dB^\dagger - B^\dagger \wedge dB) + m_B |B|^2 \right]$$

(kinetic terms first order Chern-Simons form)

(m_B=2 for Δ=3)

Equations of motion:

$$\pm\epsilon^{z\alpha\beta\mu\nu} H_{\mp z\alpha\beta} + \frac{m_B}{z} B_\pm^{\mu\nu} = 0,$$

$$\pm\epsilon^{z\alpha\beta\gamma\mu} H_{\mp\alpha\beta\gamma} + \frac{3m_B}{z} B_\pm^{\mu z} = 0$$

constrain source term:

$$B^{(0)}{}_{+\,\mu\nu} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} B^{(0)}{}_{-}^{\alpha\beta} = 0.$$

Building a better model – try 2

$$S_V = \int d^5x\sqrt{-g}\Big[-\frac{1}{4g_5^2}\sum_{i=V,A} F_{i\,MN}F^{i\,MN} + \frac{g_B}{3}\varepsilon^{MNLPQ}\left(B_{-MN}H_{+LPQ} - B_{+MN}H_{-LPQ}\right) -$$

$$- g_B m_B \sum_{\alpha=+,-} B_{\alpha MN}B_{\alpha}^{MN} + \frac{\lambda}{2}v(z)F_{V\,MN}B_{+}^{MN}\Big],$$

$$X = \frac{1}{2}\left(g_X m\, z + \frac{\langle\overline{\psi}\psi\rangle}{g_X}z^3\right)\mathbf{1}_{2\times2} \equiv \frac{g_X}{2}v(z)\mathbf{1}_{2\times2}\,.$$

Building a better model – try 2

$$S_V = \int d^5x\sqrt{-g}\Big[-\frac{1}{4g_5^2}\sum_{i=V,A} F_{i\,MN}F^{i\,MN} + \frac{g_B}{3}\varepsilon^{MNLPQ}(B_{-MN}H_{+LPQ} - B_{+MN}H_{-LPQ}) -$$

$$- g_B m_B \sum_{\alpha=+,-} B_{\alpha MN}B_{\alpha}^{MN} + \frac{\lambda}{2}v(z)F_{V\,MN}B_{+}^{MN}\Big],$$

(m_B=2 for Δ=3)

3 couplings to be fixed!

g_X (= normalization of scalar X) fixed by condensate 2-pt function

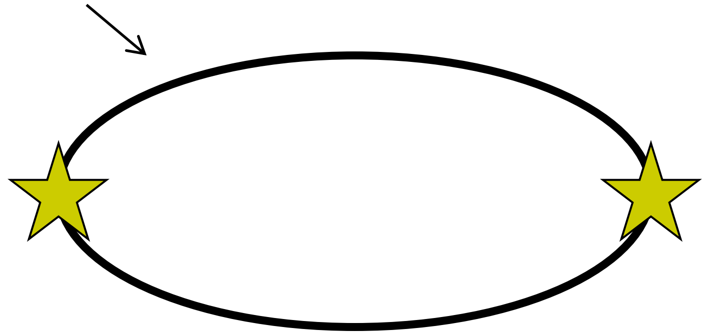
(Cherman, Cohen, Werbos)

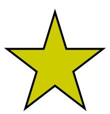
$$X = \frac{1}{2}\left(g_X m\, z + \frac{\langle\bar{\psi}\psi\rangle}{g_X}z^3\right)1_{2\times 2} \equiv \frac{g_X}{2}v(z)1_{2\times 2}\,.$$

QCD Correlation Functions

in the UV: 1-loop!

quark propagator



 =

$$\left\{ \begin{array}{l} V^{\mu\, a}(x) \;=\; \bar{\psi}(x)\gamma^{\mu}\tau^{a}\,\psi(x) \\ \\ T^{\mu\nu\, a}(x) \;=\; \bar{\psi}(x)\sigma^{\mu\nu}\tau^{a}\,\psi(x) \end{array} \right.$$

QCD Correlation Functions

$$\Pi^{\mu\nu,\,ab}_{VV}(q^2) = i \int d^4x\, e^{iqx} \langle \Omega | T\{ V^{\mu\, a}(x) V^{\nu\, b\,\dagger}(0) \} | \Omega \rangle ,$$

$$\Pi^{\mu;\nu\rho,\,ab}_{VT}(q^2) = i \int d^4x\, e^{iqx} \langle \Omega | T\{ T^{\nu\rho\, a}(x) V^{\mu\dagger\, b}(0) \} | \Omega \rangle ,$$

$$\Pi^{\mu\nu;\alpha\beta,\,ab}_{TT}(q^2) = i \int d^4x\, e^{iqx} \langle \Omega | T\{ T^{\mu\nu\, a}(x) T^{\alpha\beta\dagger\, b}(0) \} | \Omega \rangle$$

3 types of correlators: VV, TV, TT

(in addition: axial axial; does not mix with T)

QCD Correlation Functions

$$\Pi_{VV}^{\mu\nu,\,ab}(q^2) = \delta^{ab}(q^\mu q^\nu - q^2\eta^{\mu\nu})\Pi_{VV}(q^2),$$

$$\Pi_{TT}^{\mu\nu;\alpha\beta,\,ab}(q^2) = \delta^{ab}\Pi_{TT}^{+}(q^2)F_{+}^{\mu\nu;\alpha\beta} + \delta^{ab}\Pi_{TT}^{-}(q^2)F_{-}^{\mu\nu;\alpha\beta},$$

$$\Pi_{VT}^{\mu;\nu\rho,\,ab}(q^2) = i\delta^{ab}(\eta^{\mu\nu}q^\rho - \eta^{\mu\rho}q^\nu)\Pi_{VT}(q^2),$$

$q \cdot \Pi = 0$ (conserved current)

→ unique tensor structure in TV and VV

QCD Correlation Functions

$$\Pi_{VV}^{\mu\nu,\,ab}(q^2) = \delta^{ab}(q^\mu q^\nu - q^2\eta^{\mu\nu})\Pi_{VV}(q^2),$$

$$\Pi_{TT}^{\mu\nu;\alpha\beta,\,ab}(q^2) = \delta^{ab}\Pi_{TT}^{+}(q^2)F_{+}^{\mu\nu;\alpha\beta} + \delta^{ab}\Pi_{TT}^{-}(q^2)F_{-}^{\mu\nu;\alpha\beta},$$

$$\Pi_{VT}^{\mu;\nu\rho,\,ab}(q^2) = i\delta^{ab}(\eta^{\mu\nu}q^\rho - \eta^{\mu\rho}q^\nu)\Pi_{VT}(q^2),$$

2 tensor structures in TT!

$$q^2 P_{\mu\nu} = q^2\eta_{\mu\nu} - q_\mu q_\nu$$

$$P^{\alpha}_{[\mu}P^{\beta}_{\nu]} = \frac{1}{q^2}F^{\alpha\beta}_{+\mu\nu}$$ **positive parity projector**

$$F_{-}^{\mu\nu;\alpha\beta} = F_{+}^{\mu\nu;\alpha\beta} - q^2(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha})$$ **negative parity projector**

QCD Correlation Functions

$$\Pi^{\mu\nu,\,ab}_{VV}(q^2) \;=\; \delta^{ab}(q^\mu q^\nu - q^2\eta^{\mu\nu})\Pi_{VV}(q^2),$$
$$\Pi^{\mu\nu;\alpha\beta,\,ab}_{TT}(q^2) \;=\; \delta^{ab}\Pi^{+}_{TT}(q^2)F^{\mu\nu;\alpha\beta}_{+} + \delta^{ab}\Pi^{-}_{TT}(q^2)F^{\mu\nu;\alpha\beta}_{-},$$
$$\Pi^{\mu;\nu\rho,\,ab}_{VT}(q^2) \;=\; i\delta^{ab}(\eta^{\mu\nu}q^\rho - \eta^{\mu\rho}q^\nu)\Pi_{VT}(q^2),$$

2 tensor structures in TT!

$$q^2 P_{\mu\nu} = q^2\eta_{\mu\nu} - q_\mu q_\nu$$

$$P^{\alpha}_{[\mu}P^{\beta}_{\nu]} = \frac{1}{q^2}F^{\alpha\beta}_{+\mu\nu}$$ **positive parity projector**

$$F^{\mu\nu;\alpha\beta}_{-} = F^{\mu\nu;\alpha\beta}_{+} - q^2(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha})$$ **negative parity projector**

QCD Correlation Functions:

$$\Pi_{VV}(q^2) = \sum_n \frac{f_{\rho,n}^2}{M_{\rho,n}^2 - q^2}; \quad \Pi_{TT}^-(q^2) = \sum_n \frac{(f_{\rho,n}^T)^2}{M_{\rho,n}^2 - q^2}$$

$$\Pi_{TT}^+(q^2) = \sum_n \frac{f_{b,n}^2}{M_{b,n}^2 - q^2}; \quad \Pi_{VT}(q^2) = \sum_n \frac{f_{\rho,n} f_{\rho,n}^T}{M_{\rho,n}^2 - q^2}$$

4 independent functions. Large q: match to QCD poles: meson spectrum (vector and bs).

QCD Correlators – large q:

(Reinders, Rubinstein, Yazaki; Craigie, Stern, Cata, Mateu)

$$\lim_{Q^2\to\infty} \Pi_{VV}(Q^2) = -\frac{N_c}{24\pi^2}\log\frac{Q^2}{\mu^2} + \mathcal{O}\left(\frac{\alpha_s}{Q^4}\right),$$

$$\lim_{Q^2\to\infty} \Pi^{\pm}_{TT}(Q^2) = -\frac{N_c}{48\pi^2}\log\frac{Q^2}{\mu^2} \mp \frac{N_c}{8\pi^2}\frac{m^2}{Q^2}\log\frac{Q^2}{\mu^2} + \mathcal{O}\left(\frac{\alpha_s}{Q^4}\right)$$

$$\lim_{Q^2\to\infty} \Pi_{VT}(Q^2) = \frac{N_c}{16\pi^2} m \log\frac{Q^2}{\mu^2} - \frac{\langle\bar{\psi}\psi\rangle}{Q^2} + \mathcal{O}\left(\frac{\alpha_s}{Q^4}\right),$$

QCD Correlators – large q:

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Order m^2 term should come from (bulk) dimension 6 terms involving X^2. Not included in simple model.

QCD Correlators – large q:

$$\lim_{Q^2\to\infty} \Pi_{VV}(Q^2) = -\frac{N_c}{24\pi^2}\log\frac{Q^2}{\mu^2} + \mathcal{O}\left(\frac{\alpha_s}{Q^4}\right),$$

$$\lim_{Q^2\to\infty} \Pi^{\pm}_{TT}(Q^2) = -\frac{N_c}{48\pi^2}\log\frac{Q^2}{\mu^2} \mp \frac{N_c}{8\pi^2}\frac{m^2}{Q^2}\log\frac{Q^2}{\mu^2} + \mathcal{O}\left(\frac{\alpha_s}{Q^4}\right)$$

$$\lim_{Q^2\to\infty} \Pi_{VT}(Q^2) = \frac{N_c}{16\pi^2} m \log\frac{Q^2}{\mu^2} - \frac{\langle\psi\psi\rangle}{Q^2} + \mathcal{O}\left(\frac{\alpha_s}{Q^4}\right),$$

4 numerical coefficients that can be matched

Holographic matching.

4 coefficients in large q correlators

3 coupling constants in bulk action.

$$S_V = \int d^5x\sqrt{-g}\Big[-\frac{1}{4g_5^2}\sum_{i=V,A} F_{i\,MN}F^{i\,MN} + \frac{g_B}{3}\varepsilon^{MNLPQ}\left(B_{-MN}H_{+LPQ} - B_{+MN}H_{-LPQ}\right) -$$
$$- g_B m_B \sum_{\alpha=+,-} B_{\alpha MN}B_{\alpha}^{MN} + \frac{\lambda}{2}v(z)F_{V\,MN}B_{+}^{MN}\Big],$$

All couplings fixed + 1 consistency check.

Holographic Matching

$$\frac{1}{g_5^2} = \frac{N_c}{12\pi^2}, \quad g_B = \frac{N_c}{16\pi^2}, \quad \lambda = -\frac{3N_c}{4\pi^2}.$$

(these 3 couplings reproduce the 4 pieces in the correlation function!)

A “better” model.

No new free input parameter.

Lots of new predictions possible
(masses and couplings of 1^{+-} mesons).

But will also shift the masses of the standard
vector mesons.

Question: was original success an accident?

Caveat:

We are still guessing the geometry!

Our results refer to hard wall model only.

Results:

Meson spectrum doesn't work!

Main problem: typically b_1 wants to be lighter than ρ.

If we squeeze parameters we can force:

$$m_\rho \simeq 753.95 \text{ MeV}, \; m_{a_1} \simeq 1238.24 \text{ MeV and } m_{b_1} \simeq 1237.87 \text{ MeV}.$$

But: $f_\pi \simeq 4.07 \text{ MeV}.$

Source of mismatch:

Extra singularity in vector eom's from coupling to X:

$$\partial_z B_+^{\nu z} - \frac{f(z)}{g(z)} B_+^{\nu z} - i q_\mu B_+^{\mu\nu} = \frac{\lambda}{8 g_B} \frac{v'(z)}{g(z)} \partial_z V^\nu$$

$$\partial_z^2 V^\nu - \frac{f(z)}{g(z)} \partial_z V^\nu + q^2 V^\nu = \frac{\lambda g_5^2 v'(z)}{g(z)} B_+^{\nu z}$$

$$\partial_z B_-^{\nu z} - \frac{1}{z} B_-^{\nu z} - i q_\mu B_-^{\mu\nu} = 0$$

$$g(z) = 1 - \chi\, v(z)^2$$

$$\chi = \lambda^2 g_5^2 / (8 g_B).$$

For choice of m and condensate that gives physical f_π:

effectively creates second wall outside the hard wall!

ρ's live in a "smaller box" than the b's.

Origin of Mismatch

Direct consequence of using the UV form of the X profile

$$X = \frac{1}{2}\left(g_X m\, z + \frac{\langle\overline{\psi}\psi\rangle}{g_X} z^3\right)\mathbf{1}_{2\times 2}$$

for all values of z.

Can one find a better IR wall (soft wall?) that gives good spectrum?

Summary

Summary

- Bottom-up QCD models most likely not a systematic approximation to anything. Higher order derivative corrections at best suppressed by n/n+1 for some integer n.
- Bottom-up QCD models not including all dimension 3 operators surely not a consistent approximation, as clear from PDG.
- Hard wall model including all dimension 3 operators fails miserably at meson spectrum.

3 Options:

1) We made a mistake.

But: 2 out of 3 collaborators did the calculations independently (3rd one is going around giving talks). Also recall that our results passed one non-trivial internal consistency check!

3 Options:

2) We have a good Lagrangian, but a bad wall.

Someone should do the soft wall! Or find a better wall.

3 Options:

3)

Holography = Solvable Toy Model

Solvable models of strong coupling dynamics.

- Study Transport, real time (Challenging in real QCD, experimentally relevant)
- Study Finite Density
- Explore paradigms "beyond Landau"

(this is interesting for a different audience)

Gives us qualitative guidance/intuition.

Not QCD! Expect errors of order 100%

(better than extrapolating perturbation theory to $\alpha_s \sim 1$??)