Progress on Color-Dual Loop Amplitudes

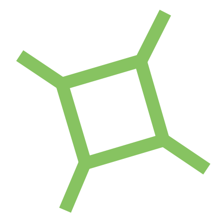
Henrik Johansson

IPhT Saclay

Sept 28, 2011

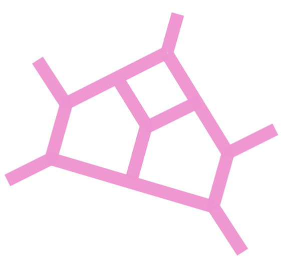
INT: Frontiers in QCD

U. of Washington



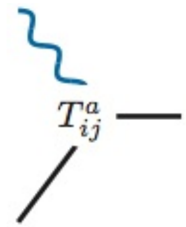
1106.4711 [hep-th], 1107.1935 [hep-th],
(and ongoing work)

Zvi Bern, Camille Boucher-Veronneau,
John Joseph Carrasco,
Lance Dixon, HJ, Radu Roiban

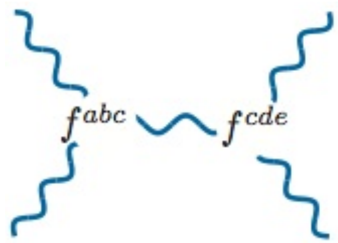


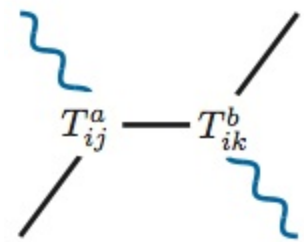
Color diagramatics

Gauge group algebra:

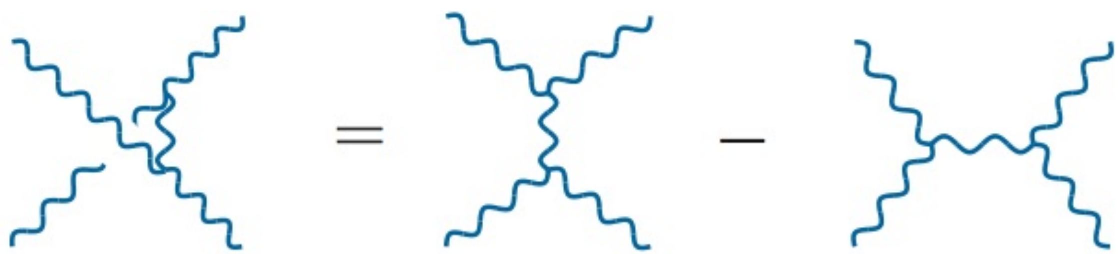


Build color structures





Relations

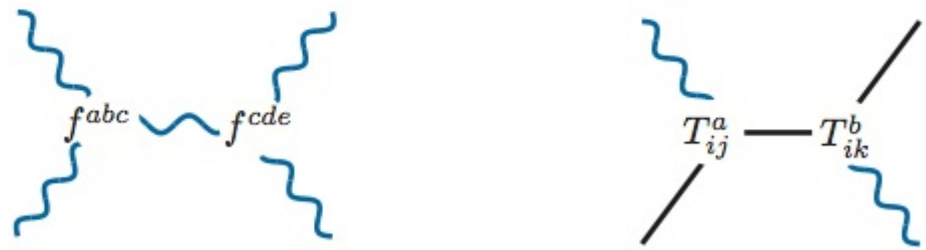


$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$

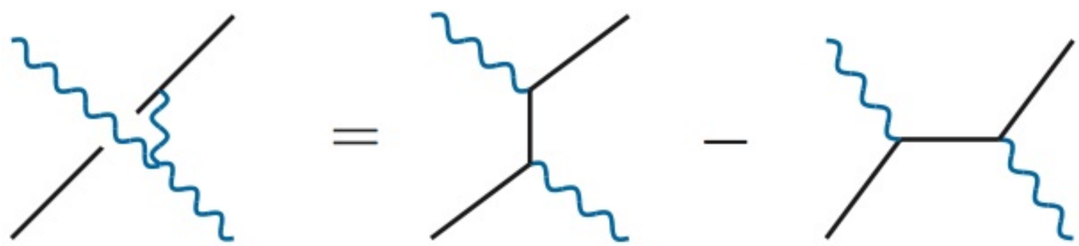
Color diagramatics

Gauge group algebra:

Build color structures

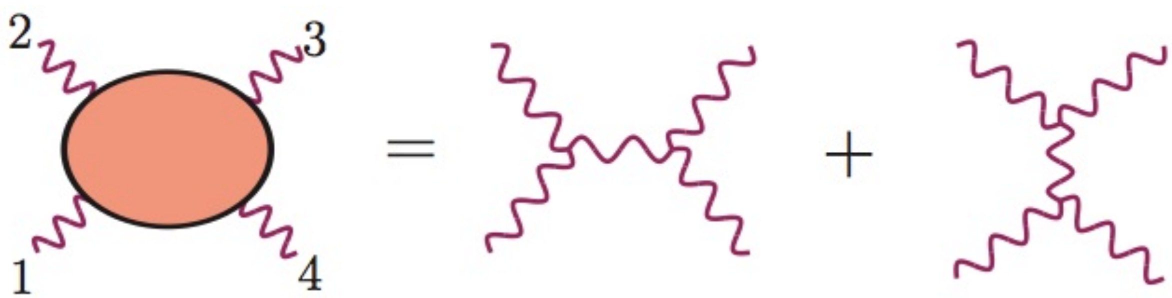


Relations



$$f^{cba}T^c_{ik} = T^b_{ij}T^a_{jk} - T^a_{ij}T^b_{jk}$$

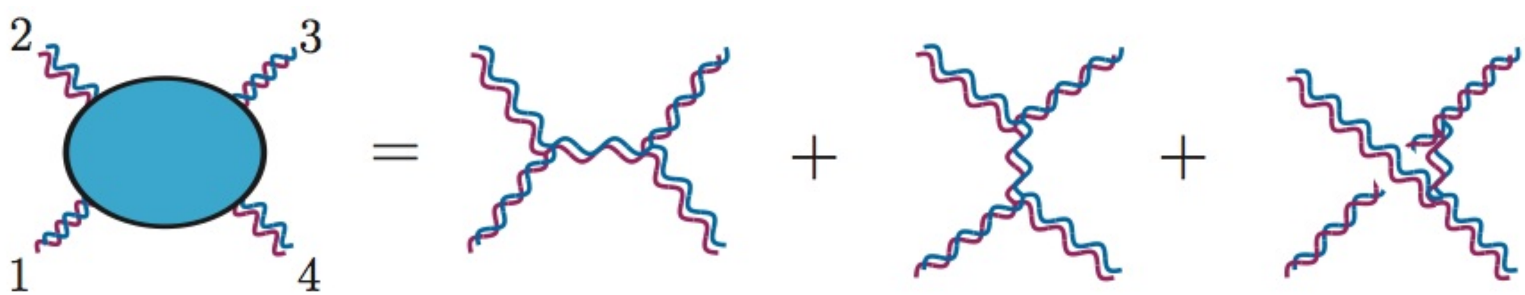
Kinematic diagrams



$$A^{\text{tree}}(1,2,3,4) = \frac{n_s}{s} + \frac{n_t}{t}$$

color-stripped,
color-ordered,
partial ampl.

(absorb contact terms using $1 = s/s$)



$$\mathcal{A}_4^{\text{tree}} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

color dressed

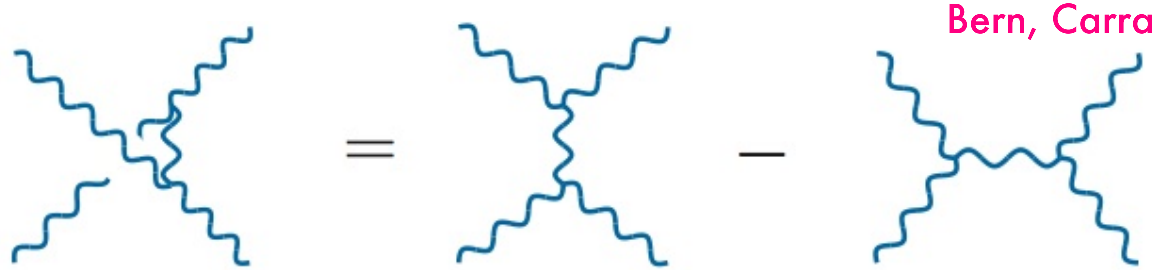
color factors: $c_s = f^{abc} f^{cde}$

kinematic factors: Feynman rules, BCFW etc.

kinematics is dual to color

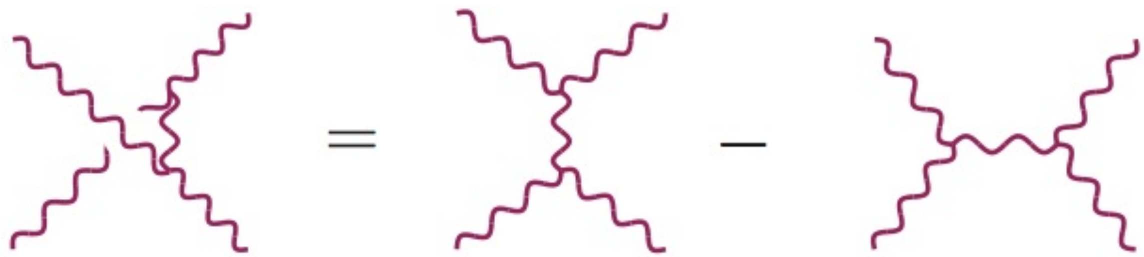
Bern, Carrasco, HJ

color Jacobi



$$c_u = c_t - c_s$$

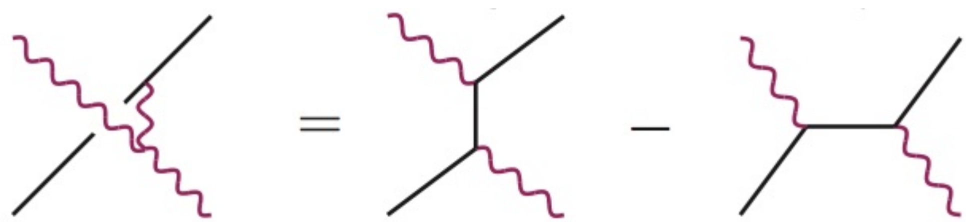
kinematic Jacobi



$$n_u = n_t - n_s$$

can be checked for 4pt on-shell ampl. using Feynman rules Haltzen, Zhu

e.g.



$$\varepsilon_2 \cdot (\overline{u}_1 \not{V} u_3) \cdot \varepsilon_4 = \overline{u}_1 \not{\varepsilon}_4 \not{p}_t \not{\varepsilon}_2 u_3 - \overline{u}_1 \not{\varepsilon}_2 \not{p}_s \not{\varepsilon}_4 u_3$$

Outline

- Simple double-copy structure of gravity
- Duality between color and kinematics
  - Some tree-level warm-up
  - Recent progress on explicit 5pt loop amplitudes with manifest duality
  - UV properties of 5pt $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG
  - One loop and $\mathcal{N}$<8 SG
- Conclusion

Gauge theory structure

Generic D-dimensional Yang-Mills theories have a novel structure

- Use representation of amplitude having only cubic graphs:

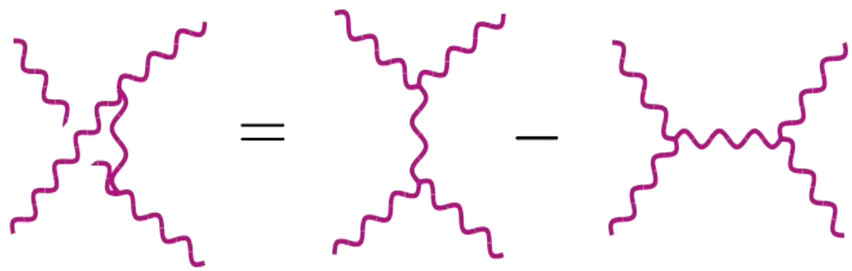
$$\mathcal{A}_m^{(L)} = \sum_{i \in \Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

numerators

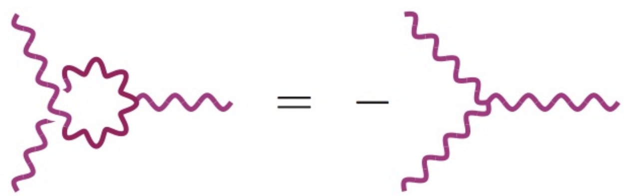
color factors

propagators

Color & kin. numerators satisfy the (Lie) algebra (defining) properties:



Jacobi identity



antisymmetry

Duality: color ↔ kinematics

Bern, Carrasco, HJ

Gravity is a double copy

- Gravity amplitudes are obtained after replacing color by kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

[BCJ]

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

- The two numerators can belong to different theories:

n_i		$\tilde{n}_i$		
($\mathcal{N}$=4)	×	($\mathcal{N}$=4)	→	$\mathcal{N}$=8 sugra
($\mathcal{N}$=4)	×	($\mathcal{N}$=2)	→	$\mathcal{N}$=6 sugra
($\mathcal{N}$=4)	×	($\mathcal{N}$=0)	→	$\mathcal{N}$=4 sugra
($\mathcal{N}$=0)	×	($\mathcal{N}$=0)	→	Einstein gravity + axion+ dillaton

similar to Kawai-Lewellen-Tye but works at loop level

Five-point example

- Decomposing 5pt amplitude in terms of 15 cubic diagrams

$$\mathcal{A}_5^{\text{tree}} = g^3\Big(\frac{n_1c_1}{s_{12}s_{45}} + \frac{n_2c_2}{s_{23}s_{51}} + \frac{n_3c_3}{s_{34}s_{12}} + \frac{n_4c_4}{s_{45}s_{23}} + \frac{n_5c_5}{s_{51}s_{34}} + \frac{n_6c_6}{s_{14}s_{25}} + \frac{n_7c_7}{s_{32}s_{14}} + \frac{n_8c_8}{s_{25}s_{43}} + \frac{n_9c_9}{s_{13}s_{25}} + \frac{n_{10}c_{10}}{s_{42}s_{13}} + \frac{n_{11}c_{11}}{s_{51}s_{42}} + \frac{n_{12}c_{12}}{s_{12}s_{35}} + \frac{n_{13}c_{13}}{s_{35}s_{24}} + \frac{n_{14}c_{14}}{s_{14}s_{35}} + \frac{n_{15}c_{15}}{s_{13}s_{45}}\Big),$$

kinematic numerator

color factor

propagators

$$s_{ij} = (k_i + k_j)^2$$

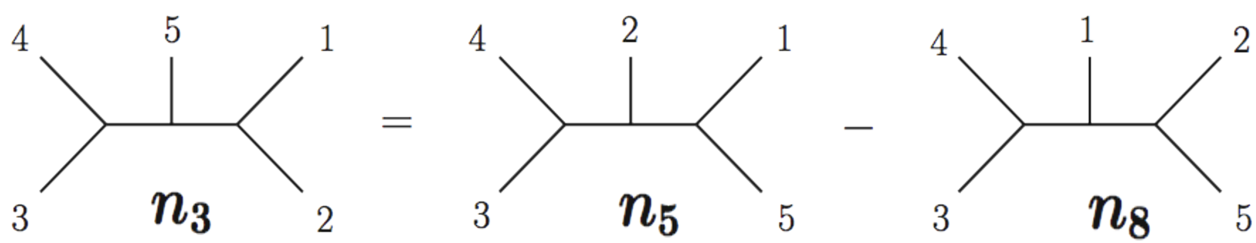
- Equivalent to partial amplitudes

$$A_5^{\text{tree}}(1,2,3,4,5) \equiv \frac{n_1}{s_{12}s_{45}} + \frac{n_2}{s_{23}s_{51}} + \frac{n_3}{s_{34}s_{12}} + \frac{n_4}{s_{45}s_{23}} + \frac{n_5}{s_{51}s_{34}}$$

etc…

- Duality between color and kinematics can be imposed, but not automatic

$$n_3 - n_5 + n_8 = 0 \quad \Leftrightarrow \quad c_3 - c_5 + c_8 = 0$$



$$c_3 \equiv \tilde f^{a_3a_4b}\tilde f^{ba_5c}\tilde f^{ca_1a_2}$$

$$c_5 \equiv \tilde f^{a_3a_4b}\tilde f^{ba_2c}\tilde f^{ca_1a_5}$$

$$c_8 \equiv \tilde f^{a_3a_4b}\tilde f^{ba_1c}\tilde f^{ca_2a_5}$$

checked through 8pts, now all multiplicity solution: Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard Vanhove

Bern, Carrasco, HJ

Applications at Tree-Level

Gauge theory amplitude properties

- Tree level, adjoint representation

$$\mathcal{A}_n^{\rm tree}(1,2,\ldots,n) = g^{n-2} \sum_{\mathcal{P}(2,\ldots,n)} {\rm Tr}[T^{a_1}T^{a_2}\cdots T^{a_n}]\, A_n^{\rm tree}(1,2,\ldots,n)$$

gauge invariant

- Well-known partial amplitude properties

$$A_n^{\rm tree}(1,2,\ldots,n) = A_n^{\rm tree}(2,\ldots,n,1)$$ cyclic symmetry

$$A_n^{\rm tree}(1,2,\ldots,n) = (-1)^n A_n^{\rm tree}(n,\ldots,2,1)$$ reflection symmetry

$(n - 1)!/2$

$$\sum_{\sigma\in{\rm cyclic}} A_n^{\rm tree}(1,\sigma(2,3,\ldots,n)) = 0$$ "photon"-decoupling identity

$$A_n^{\rm tree}(1,\{\alpha\},n,\{\beta\}) = (-1)^{n_\beta} \sum_{\{\sigma\}_i\in {\rm OP}(\{\alpha\},\{\beta^T\})} A_n^{\rm tree}(1,\{\sigma\}_i,n)$$ Kleiss-Kuijf relations

$(n - 2)!$

- *New BCJ relations reduce independent basis to* $(n - 3)!$ Bern, Carrasco, HJ

Duality gives new amplitude relations

In color-ordered tree amplitudes 3 legs can be fixed: $(n\text{-}3)!$ basis

$$A_4^{\rm tree}(1,2,\{4\},3) = \frac{A_4^{\rm tree}(1,2,3,4)s_{14}}{s_{24}}$$

$s_{ij..} = (k_i+k_j+\ldots)^2$

$$A_5^{\rm tree}(1,2,\{4\},3,\{5\}) = \frac{A_5^{\rm tree}(1,2,3,4,5)(s_{14}+s_{45}) + A_5^{\rm tree}(1,2,3,5,4)s_{14}}{s_{24}},$$

$$A_5^{\rm tree}(1,2,\{4,5\},3) = \frac{-A_5^{\rm tree}(1,2,3,4,5)s_{34}s_{15} - A_5^{\rm tree}(1,2,3,5,4)s_{14}(s_{245}+s_{35})}{s_{24}s_{245}}$$

...relations obtained for any multiplicity

These were later found to be equivalent to monodromy relations on the open string worldsheet Bjerrum-Bohr, Damgaard, Vanhove; Stieberger

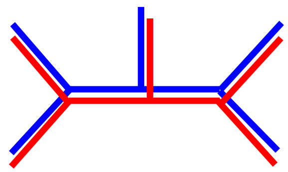
Also field theory proofs through BCFW: Feng, Huang, Jia; Chen, Du, Feng

Critical in the solution of all open string disk amplitudes Mafra, Schlotterer, Stieberger

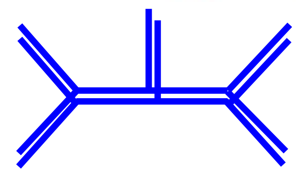
Tree-level gravity checks

- Original conjecture checked through 8 points — Bern, Carrasco, HJ

$$\mathcal{A}_n^{\text{tree}} = \sum_i \frac{n_i c_i}{\prod_\alpha p_\alpha^2} \quad \Leftrightarrow \quad \mathcal{M}_n^{\text{tree}} = \sum_i \frac{n_i \tilde{n}_i}{\prod_\alpha p_\alpha^2}$$



$\Leftrightarrow$



double copy of YM

- All-multiplicity proof assuming gauge theory duality: Bern, Dennen, Huang, Kiermaier

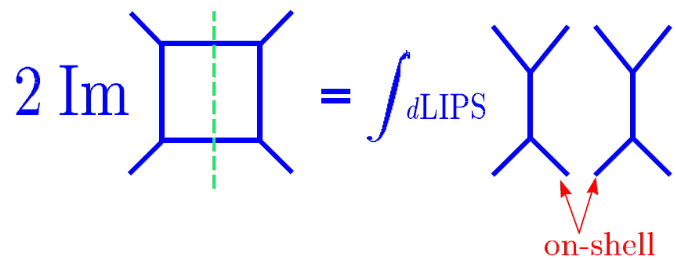
Loop Amplitudes

Unitarity

Optical theorem:

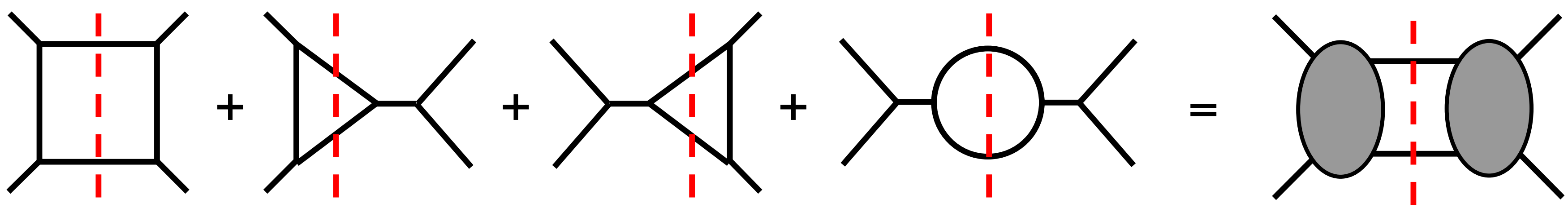
$$1 = S^\dagger S = (1 - iT^\dagger)(1 + iT)$$

$$2\,\mathrm{Im}\,T = T^\dagger T$$



The unitarity method reconstructs the amplitudes avoiding dispersion relations

Bern, Dixon, Dunbar, Kosower (1994)



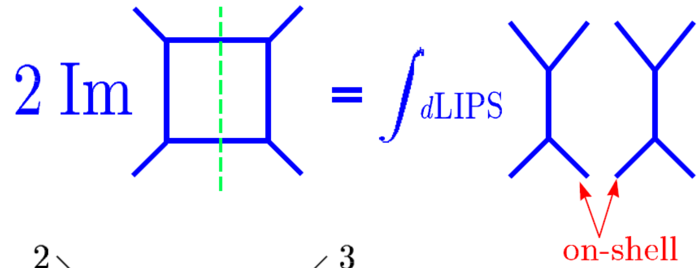
Compute a cut: put loop legs on-shell in amplitude = sew trees amplitudes

checking every cut channel will fix the loop integrals

Unitarity method developments

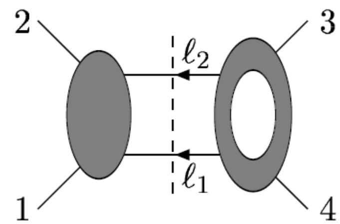
TIME

optical theorem



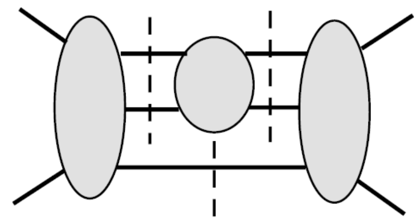
unitarity method

Bern, Dixon, Dunbar and Kosower (1994)



generalized unitarity

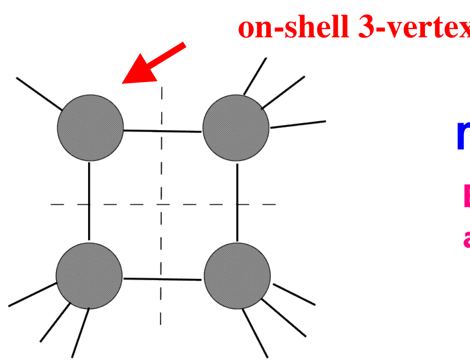
Bern, Dixon and Kosower



quadruple cut
& leading singularity

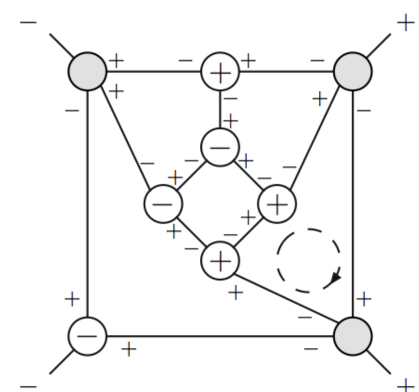
Britto, Cachazo, Feng;
Buchbinder, Cachazo (2004)

Cachazo and Skinner
Cachazo, Spradlin, Volovich
(2008)



maximal cut

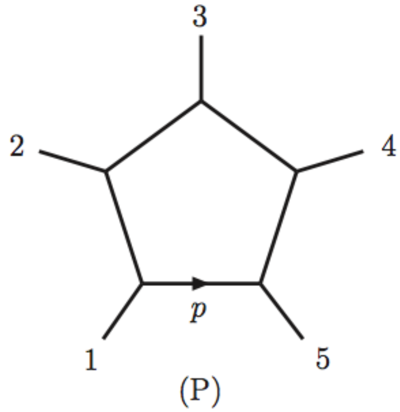
Bern, Carrasco, HJ
and Kosower (2007)

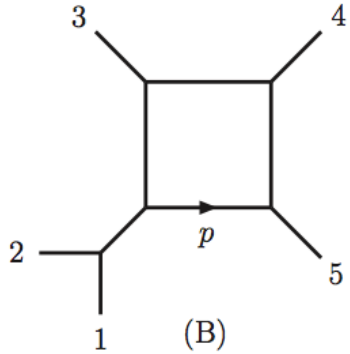


Duality at loop level

1-loop 5-pts $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG

Carrasco, HJ 1106.4711 [hep-th]



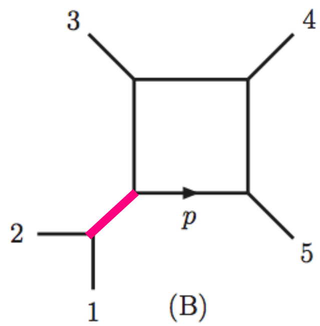


$$\beta_{12345} \equiv N^{(\mathrm{P})} = \delta^{(8)}(Q)\frac{[1\,2]\,[2\,3]\,[3\,4]\,[4\,5]\,[5\,1]}{4i\varepsilon(1,2,3,4)}$$

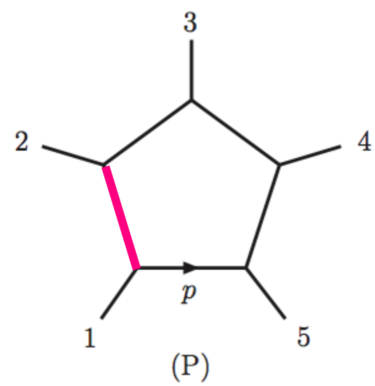
$$\gamma_{12} \equiv N^{(\mathrm{B})} = \delta^{(8)}(Q)\frac{[1\,2]^2\,[3\,4]\,[4\,5]\,[3\,5]}{4i\varepsilon(1,2,3,4)}$$

- The five-point amplitude makes the duality manifest !
- $\mathcal{N}$=8 SG is obtained through the numerator double copy

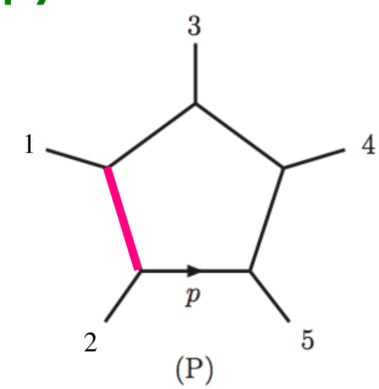
e.g. Jacobi relation:



=



−



$$N^{(\mathrm{B})}(1,2,3,4,p) = N^{(\mathrm{P})}(1,2,3,4,p) - N^{(\mathrm{P})}(2,1,3,4,p)$$

Equivalent amplitudes in:
0803.1988 [hep-th] (Cachazo), hep-ph/9511336 (Bern, Morgan)

All-loop 5pt $\mathcal{N}=4$ ansatz

Extrapolating the one-loop solution we can predict the all-loop structure

All cubic Feynman-like diagrams

$$\mathcal{A}_5^{(L)} = i g^{2L+3} \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{N_i C_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_m}^2}$$

with numerators

$$N_i = \sum_{j,k,n} a_{i;jk;n}\, \gamma_{jk}\, M_n^{(L)}$$

⇒ J.J. Carrasco

constants (the $a_{i;jk;n}$)

non-local state/helicity factors (the γ_{jk})

local momentum invariants, dimension: $2L$-2 (the $M_n^{(L)}$)

gammas and betas interchangeable

$$\gamma_{12} = \beta_{12345} - \beta_{21345}$$

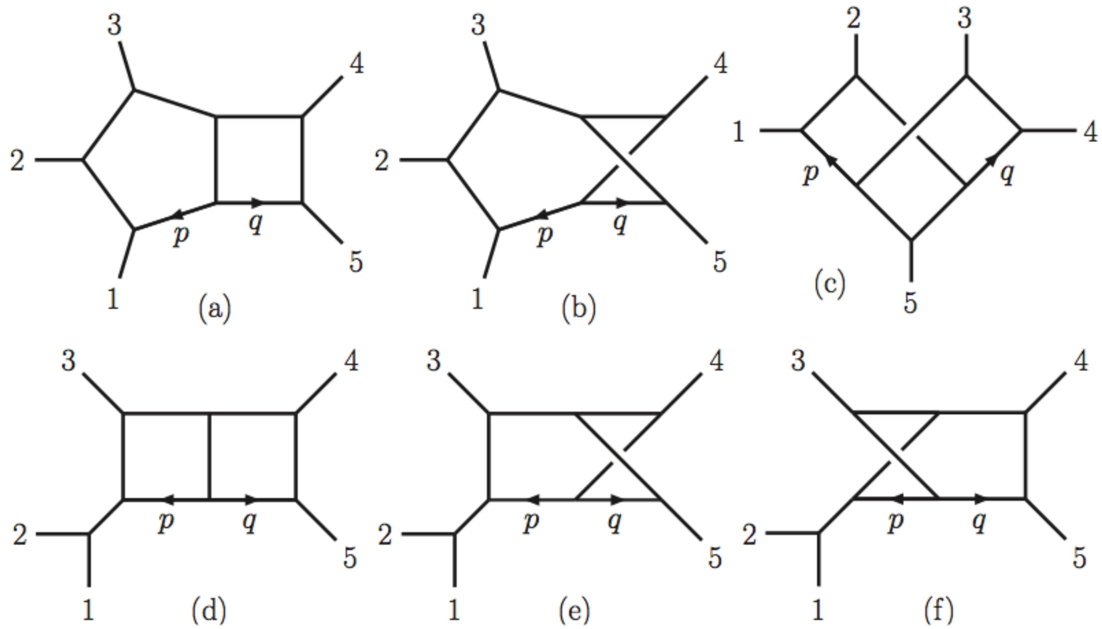
$$\beta_{12345} = \frac{1}{2}(\gamma_{12} + \gamma_{13} + \gamma_{14} + \gamma_{23} + \gamma_{24} + \gamma_{34})$$

and satisfies relations

$$\sum_{i=1}^{5} \gamma_{ij} = 0 \qquad \gamma_{ij} = -\gamma_{ji}$$

only 6 independent gammas!

2-loop 5-pts $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG



Carrasco, HJ
1106.4711 [hep-th]

The 2-loop 5-point amplitude with duality exposed

$\mathcal{I}^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a),(b)	$\frac{1}{4}\Big(\gamma_{12}(2s_{45}-s_{12}+\tau_{2p}-\tau_{1p})+\gamma_{23}(s_{45}+2s_{12}-\tau_{2p}+\tau_{3p})$ $+2\gamma_{45}(\tau_{5p}-\tau_{4p})+\gamma_{13}(s_{12}+s_{45}-\tau_{1p}+\tau_{3p})\Big)$
(c)	$\frac{1}{4}\Big(\gamma_{15}(\tau_{5p}-\tau_{1p})+\gamma_{25}(s_{12}-\tau_{2p}+\tau_{5p})+\gamma_{12}(s_{34}+\tau_{2p}-\tau_{1p}+2s_{15}+2\tau_{1q}-2\tau_{2q})$ $+\gamma_{45}(\tau_{4q}-\tau_{5q})-\gamma_{35}(s_{34}-\tau_{3q}+\tau_{5q})+\gamma_{34}(s_{12}+\tau_{3q}-\tau_{4q}+2s_{45}+2\tau_{4p}-2\tau_{3p})\Big)$
(d)-(f)	$\gamma_{12}s_{45}-\frac{1}{4}\Big(2\gamma_{12}+\gamma_{13}-\gamma_{23}\Big)s_{12}$

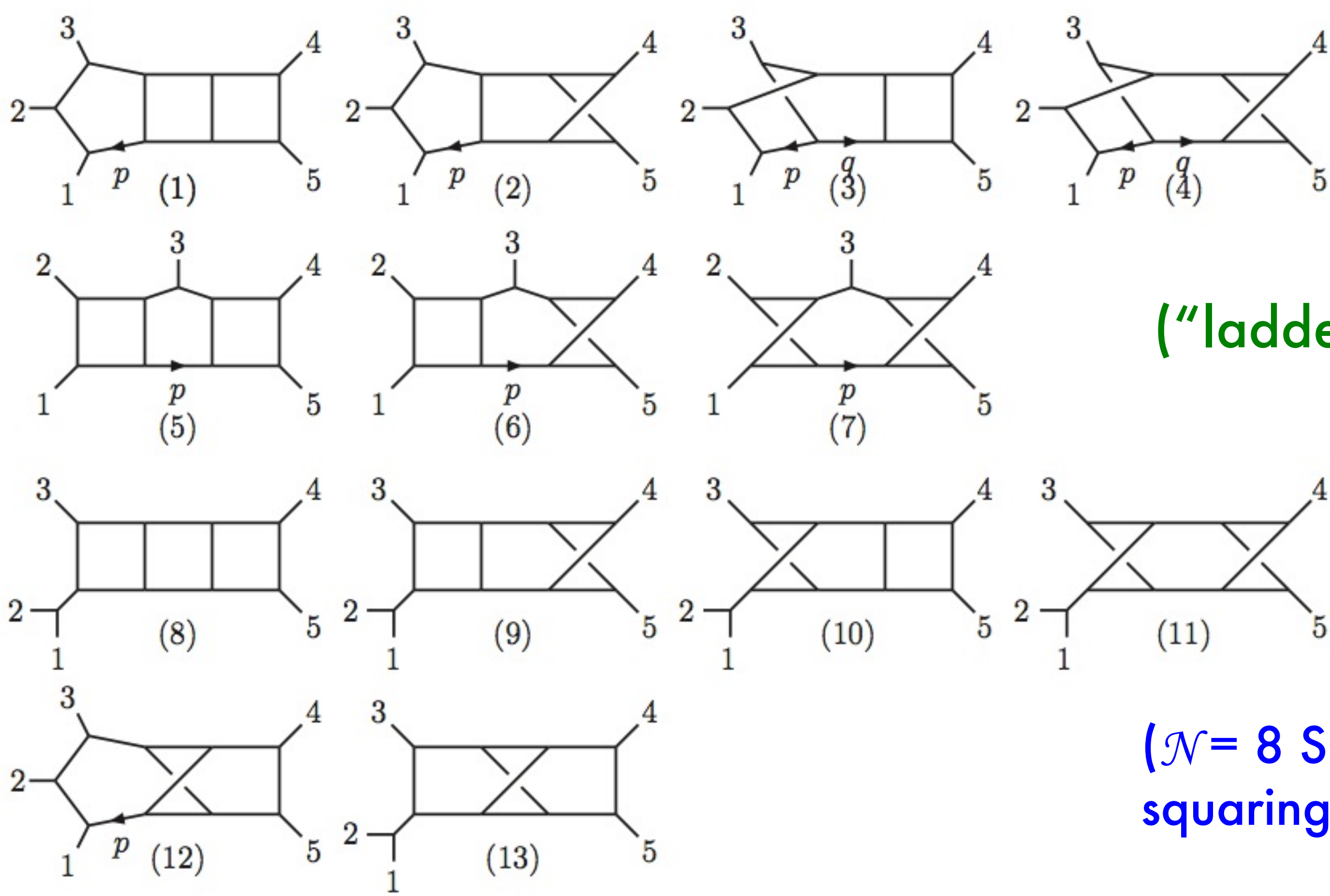
$\mathcal{N}$= 8 SG obtained from numerator double copies

$$\tau_{ip}=2k_i\cdot p$$

3-loop 5-point SYM and $\mathcal{N}$=8 SG

Again the color-dressed D-dimensional amplitude admits a representation with manifest duality

Carrasco, HJ
(to be published)



("ladder-like" diagrams)

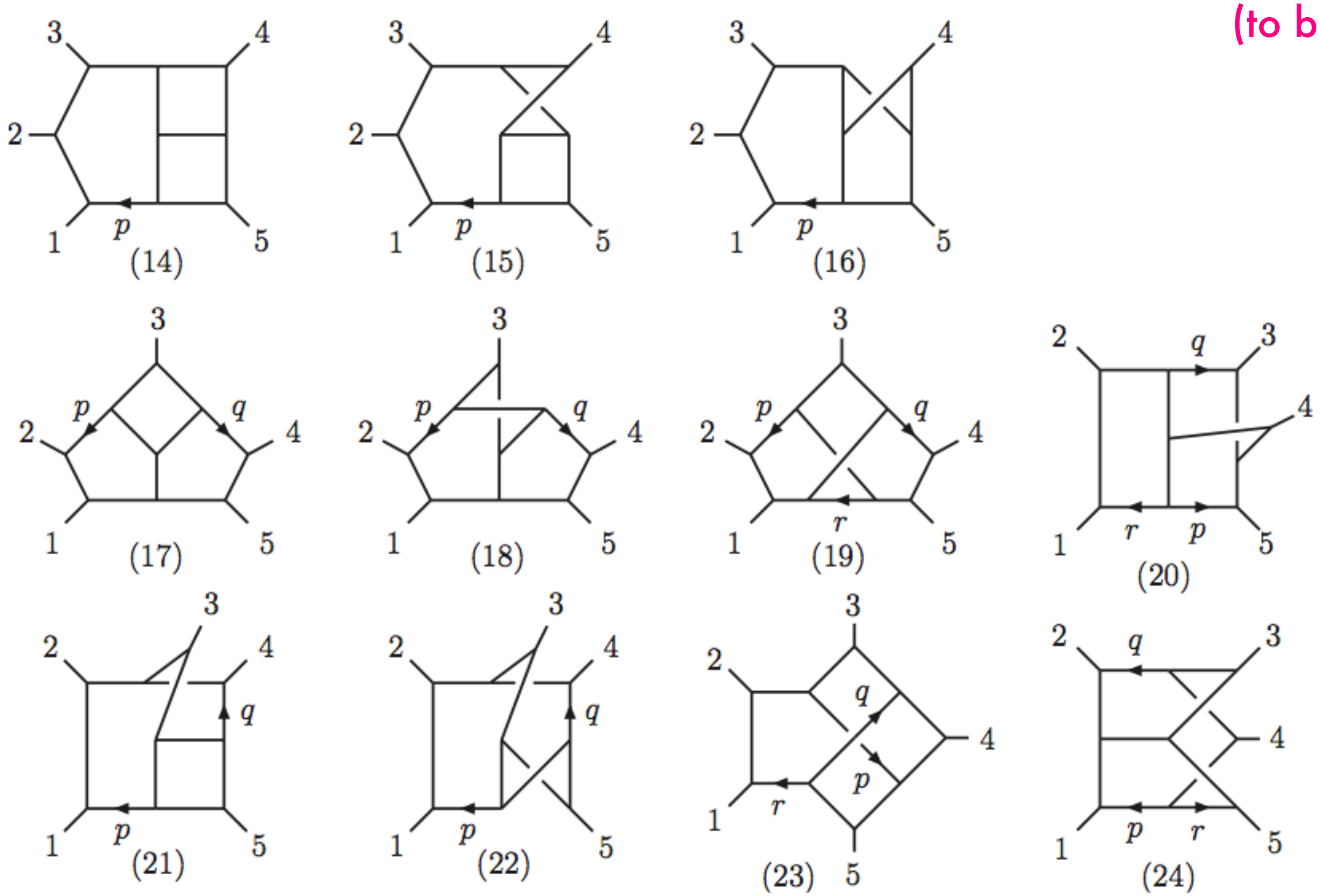
($\mathcal{N}$= 8 SG obtained from squaring the numerators)

$$N_8 = \gamma_{12} s_{45}^2 - \frac{1}{12} s_{12} \Big(\gamma_{13}(2s_{13} + 12 s_{23} - s_{12}) - \gamma_{23}(2 s_{23} + 12 s_{13} - s_{12}) - \gamma_{12}(7 s_{12} - 11 s_{45}) \Big)$$

3-loop 5-point SYM and $\mathcal{N}$=8 SG

some "Mercedes-like" diagrams…

Carrasco, HJ
(to be published)



$$N_{14} = \frac{1}{2}\gamma_{45}(\tau_{1p}^2 + \tau_{2p}^2 + \tau_{3p}^2 + \tau_{4p}^2 + \tau_{5p}^2) + \text{ subleading in } p$$

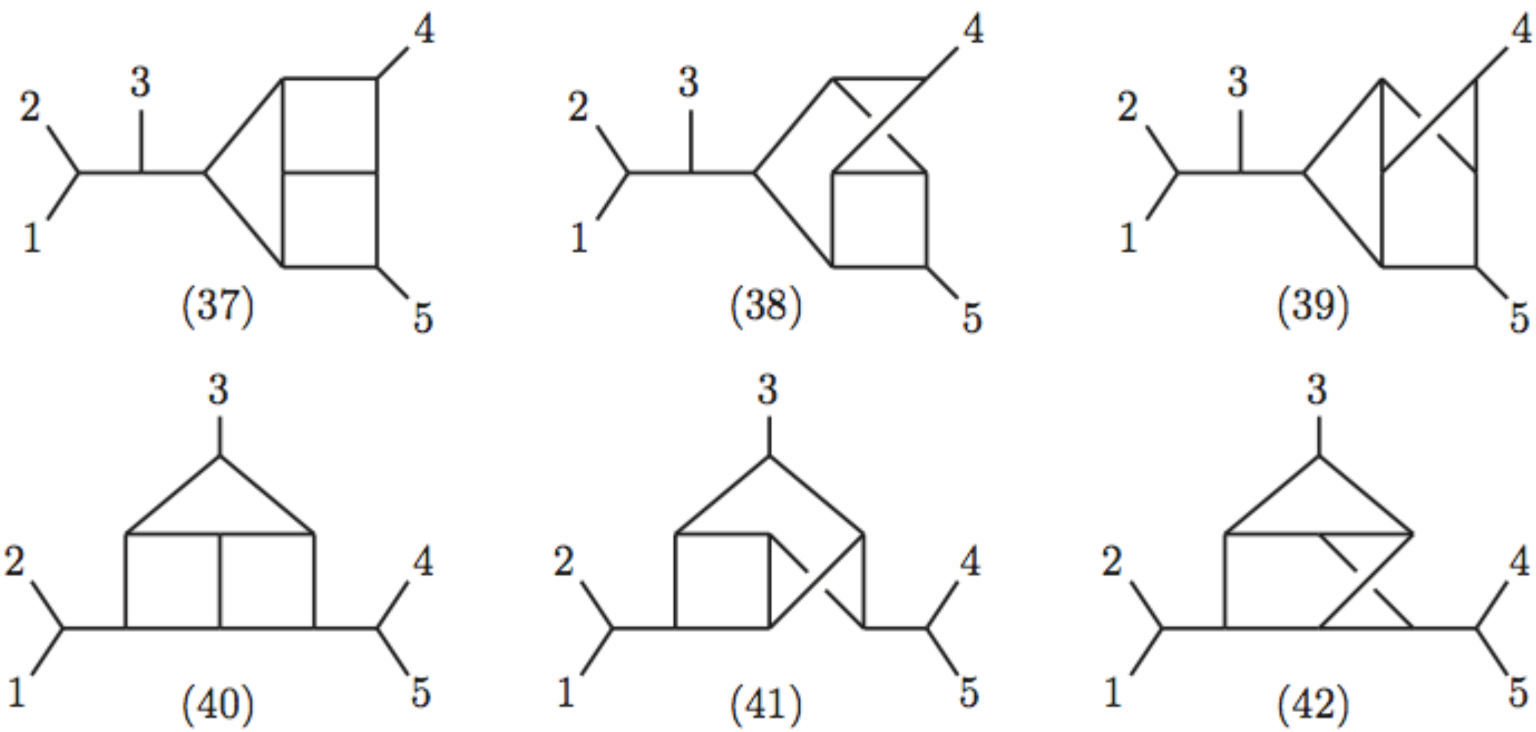
$$\tau_{ip}^2 = 2k_i \cdot p$$

3-loop 5-point SYM and $\mathcal{N}$=8 SG

...in total 42 diagrams.

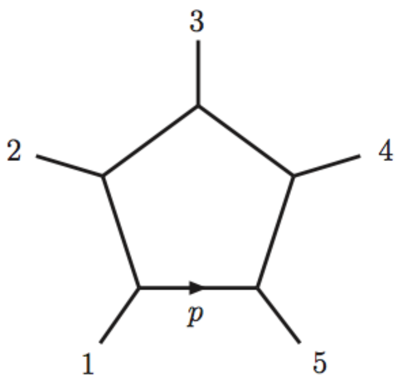
Carrasco, HJ
(to be published)

Conveniently the UV divergent diagrams (in D=6) are very simple:

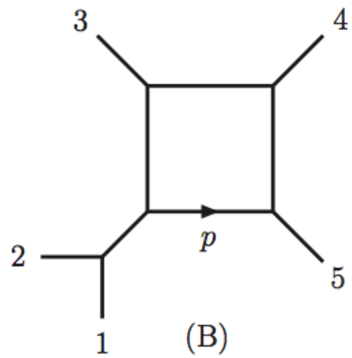


(for SG the UV div. comes from the other diagrams as well)

1-loop 5-pts UV divergences



(P)



(B)

Carrasco, HJ 1106.4711 [hep-th]

$$\beta_{12345} \equiv N^{(\mathrm{P})} = \delta^{(8)}(Q)\frac{[1\,2]\,[2\,3]\,[3\,4]\,[4\,5]\,[5\,1]}{4\,\varepsilon(1,2,3,4)}$$

$$\gamma_{12} \equiv N^{(\mathrm{B})} = \delta^{(8)}(Q)\frac{[1\,2]^2\,[3\,4]\,[4\,5]\,[3\,5]}{4\,\varepsilon(1,2,3,4)}$$

SYM UV div in *D*=8:

$\Rightarrow$ R. Roiban

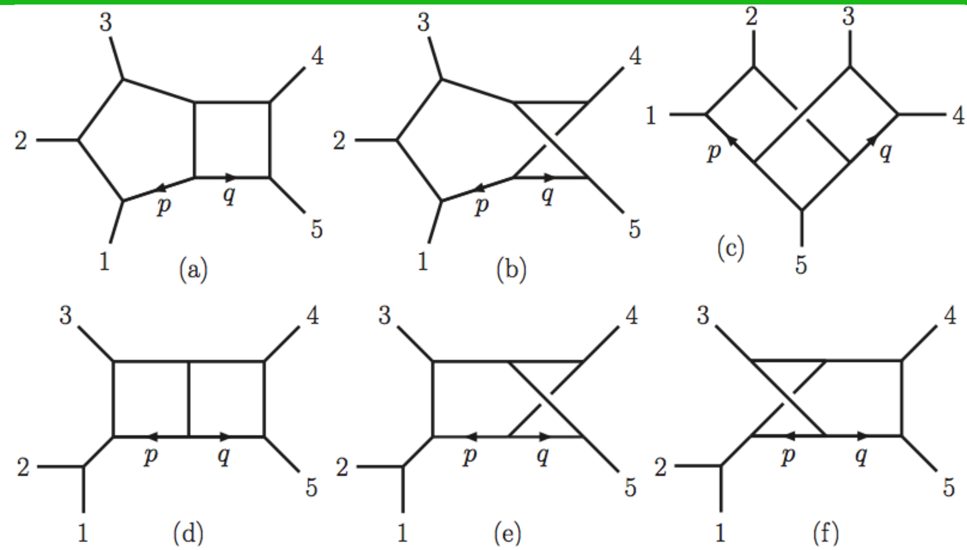
$$\mathcal{A}_5^{(1)}\Big|_{\mathrm{UV}} = -g^5\frac{1}{6(4\pi)^4\epsilon}\Big[N_c\,\mathrm{Tr}_{12345}\Big(\frac{\gamma_{12}}{s_{12}}+\frac{\gamma_{23}}{s_{23}}+\frac{\gamma_{[34]}}{s_{34}}+\frac{\gamma_{45}}{s_{45}}+\frac{\gamma_{51}}{s_{15}}\Big) + 6\,\mathrm{Tr}_{123}\,\mathrm{Tr}_{45}\Big(\frac{\gamma_{12}}{s_{12}}+\frac{\gamma_{23}}{s_{23}}+\frac{\gamma_{31}}{s_{13}}\Big) + \text{ perms}\Big]$$

SG UV div in *D*=8:

$$\mathcal{M}_5^{(1)}\Big|_{\mathrm{UV}} = -i\left(\frac{\kappa}{2}\right)^5\frac{1}{6(4\pi)^4\epsilon}\Big[\frac{\gamma_{12}^2}{s_{12}}+\frac{\gamma_{13}^2}{s_{13}}+\frac{\gamma_{14}^2}{s_{14}}+\frac{\gamma_{15}^2}{s_{15}}+\frac{\gamma_{23}^2}{s_{23}}+\frac{\gamma_{24}^2}{s_{24}}+\frac{\gamma_{25}^2}{s_{25}}+\frac{\gamma_{34}^2}{s_{34}}+\frac{\gamma_{35}^2}{s_{35}}+\frac{\gamma_{45}^2}{s_{45}}\Big]$$

agree with UV calc. at 4pt Green, Schwarz, Brink

2-loop 5-pts UV divergences



Carrasco, HJ 1106.4711 [hep-th]

agree with UV calc. at 4pt

Marcus, Sagnotti; Bern, Dixon, Dunbar, Perelstein, Rozowsky

SYM UV div in *D*=7:

$$\mathcal{A}_5^{(2)}\Big|_{\rm UV} = -g^7\Big[(N_c^2 V^{(\rm P)} + 12(V^{(\rm P)}+V^{(\rm NP)}))\,{\rm Tr}_{12345}\Big(5\beta_{12345} + \frac{\gamma_{12}}{s_{12}}(s_{35}-2s_{12}) + \frac{\gamma_{23}}{s_{23}}(s_{14}-2s_{23}) + \frac{\gamma_{34}}{s_{34}}(s_{25}-2s_{34}) + \frac{\gamma_{45}}{s_{45}}(s_{13}-2s_{45}) + \frac{\gamma_{51}}{s_{15}}(s_{24}-2s_{15})\Big) - 12N_c(V^{(\rm P)}+V^{(\rm NP)})\,{\rm Tr}_{123}\,{\rm Tr}_{45}\,s_{45}\Big(\frac{\gamma_{12}}{s_{12}}+\frac{\gamma_{23}}{s_{23}}+\frac{\gamma_{31}}{s_{13}}\Big) + {\rm perms}\Big]$$

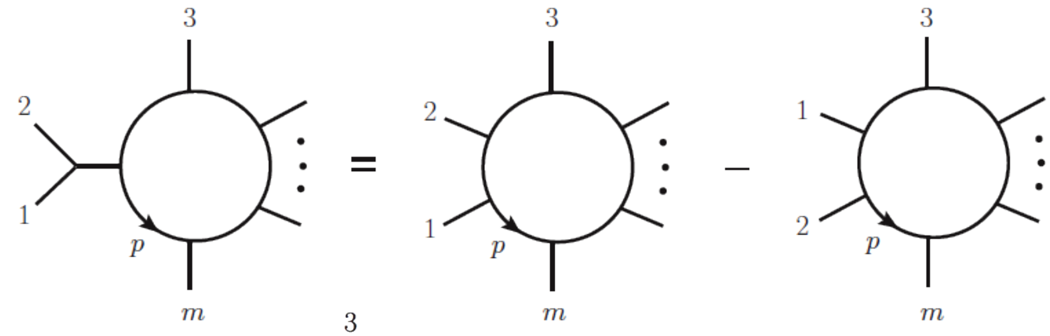
SG UV div in *D*=7:

$$\mathcal{M}_5^{(2)}\Big|_{\rm UV} = i\left(\frac{\kappa}{2}\right)^7 \frac{1}{6}(V^{(\rm P)}+V^{(\rm NP)})\sum_{S_5}\frac{\gamma_{12}^2}{s_{12}}(s_{34}^2+s_{35}^2+s_{45}^2-3s_{12}^2)$$

$\mathcal{N} < 8$ supergravity at one loop

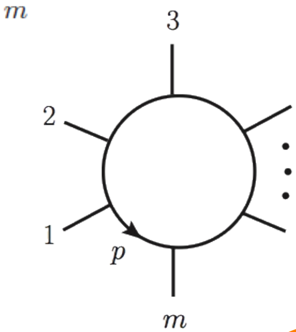
Bern, Boucher-Veronneau, HJ

Eliminate 1PR diagrams:



only ring diagrams remain

color basis of Del Duca, Dixon and Maltoni



$\mathrm{Tr}_A[f^{a_1} f^{a_2} f^{a_3} \cdots f^{a_m}]$

Yang-Mills: $$\mathcal{A}_m^{\text{1-loop}} = g^m \sum_{S_m/(Z_m \times Z_2)} \int \frac{d^D p}{(2\pi)^D} c_{123\ldots m} \underbrace{\mathscr{A}_m(1,2,\ldots,m;p)}_{\text{"integrand"}}$$

need one copy of duality satisfying numerators:

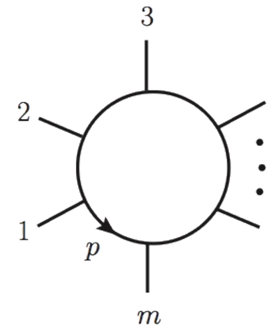
Gravity: $$\mathcal{M}_m^{\text{1-loop}} = \left(\frac{\kappa}{2}\right)^m \sum_{S_m/(Z_m \times Z_2)} \int \frac{d^D p}{(2\pi)^D} \boxed{\tilde{n}_{123\ldots m}(p)} \boxed{\mathscr{A}_m(1,2,\ldots,m;p)}$$

$(\mathcal{N}=4) \times (\mathcal{N}=0,1,2,4)$

Relations between integrated ampls.

$$\mathcal{M}_m^{\text{1-loop}} = \left(\frac{\kappa}{2}\right)^m \sum_{S_m/(Z_m \times Z_2)} \int \frac{d^D p}{(2\pi)^D} \tilde{n}_{123\ldots m}(p) \mathscr{A}_m(1,2,\ldots,m;p)$$

($\mathcal{N}$=4 SYM) × ($\mathcal{N}$= p SYM)



Assume $\tilde{n}_{123\ldots m}(p) = \tilde{n}_{123\ldots m}$ (true for m = 4,5)

Relations between integrated amplitudes:

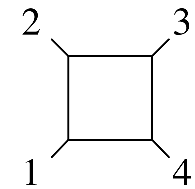
$$\mathcal{M}_m^{\text{1-loop}} = \left(\frac{\kappa}{2}\right)^m \sum_{S_m/(Z_m \times Z_2)} \tilde{n}_{123\ldots m} A^{\text{1-loop}}_{\mathcal{N}\ \text{susy}}(1,2,\ldots,m)$$

$\mathcal{N}$=4 SYM numerators
4pt: Green, Schwarz, Brink
5pt: Carrasco, HJ

$\mathcal{N}$=0,1,2 amplitudes
4pt: Bern and Morgan
5pt in hep-ph/9302280
(Bern, Dixon, Kosower)

⇒ It works! reproduce $\mathcal{N}$=4,5,6 supergravity ampl. Dunbar, Ettle, Perkins, Dunbar and Norridge

Four-point check details



Green, Schwarz, Brink: $n_{1234} = n_{1243} = n_{1423} = istA^{\text{tree}}(1,2,3,4)$

$$\mathcal{M}^{\text{1-loop}}_{\mathcal{N}+4\text{ susy}}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^4 istA^{\text{tree}}(1,2,3,4)\Big(A^{\text{1-loop}}_{\mathcal{N}\text{ susy}}(1,2,3,4) + A^{\text{1-loop}}_{\mathcal{N}\text{ susy}}(1,2,4,3) + A^{\text{1-loop}}_{\mathcal{N}\text{ susy}}(1,4,2,3)\Big)\,,$$

Bern, Morgan in D dim.

Can work with the simpler matter multiplet contributions, we get

$$\mathcal{M}^{\text{1-loop}}_{\mathcal{N}=6,\text{mat.}}(1^-,2^-,3^+,4^+) = -\frac{ic_\Gamma}{2}\left(\frac{\kappa}{2}\right)^4 \frac{\langle 1\,2\rangle^4\,[3\,4]^4}{s^2}\left[\ln^2\left(\frac{-t}{-u}\right) + \pi^2\right] + \mathcal{O}(\epsilon)$$

$$\mathcal{M}^{\text{1-loop}}_{\mathcal{N}=4,\text{mat.}}(1^-,2^-,3^+,4^+) = \frac{1}{2}\left(\frac{\kappa}{2}\right)^4 \frac{\langle 1\,2\rangle^2\,[3\,4]^2}{[1\,2]^2\,\langle 3\,4\rangle^2}\Big[ic_\Gamma s^2 + s(u-t)\big(I_2(t) - I_2(u)\big) - 2I_4^{D=6-2\epsilon}(t,u)stu\Big] + \mathcal{O}(\epsilon)\,,$$

From this one gets any $\mathcal{N} \geq 4$ supergravity theory ampl.

Agrees with Dunbar and Norridge

Summary

- Kinematic numerators of ampl. diagrams behave as if they were on equal footing with color factors.
- Making the duality manifest: Gravity becomes double copy of Yang-Mills theory, order by order in the S-matrix.
- Nontrivial 5-point multi-loop evidence supports the duality. For more evidence: see talk by J.J. Carrasco
- Duality should be a key tool for nonplanar gauge theory and gravity calculations.
  - Examples: UV divergence calculation at 5pts, 1 and 2-loop
  - for $\mathcal{N}$=8 supergravity finiteness issue: see talk by R. Roiban
  - Calc. of $\mathcal{N}$<8 supergravity amplitudes at one loop.
- Outlook:

  More one-loop calculations needed. Evidence & Applications

  What is the kinematic Lie algebra ?

Extra slides

Lagrangian and Lie Algebra

- First attempt at Lagrangian with manifest duality

1004.0693 [hep-th]
Bern, Dennen, Huang, Kiermaier

YM Lagrangian receives corrections at 5 points and higher

$$\mathcal{L}_{YM} = \mathcal{L} + \mathcal{L}'_5 + \mathcal{L}'_6 + \ldots$$

corrections proportional to the Jacobi identity (thus equal to zero)

$$\mathcal{L}'_5 \sim \mathrm{Tr}\,[A^\nu, A^\rho]\frac{1}{\Box}\big([[\partial_\mu A_\nu, A_\rho], A^\mu] + [[A_\rho, A^\mu], \partial_\mu A_\nu] + [[A^\mu, \partial_\mu A_\nu], A_\rho]\big)$$

Introduction of auxiliary "dynamical" fields gives local cubic Lagrangian

$$\mathcal{L}_{YM} = \tfrac{1}{2}A^{a\mu}\Box A^a_\mu - B^{a\mu\nu\rho}\Box B^a_{\mu\nu\rho} - g f^{abc}\underbrace{(\partial_\mu A^a_\nu + \partial^\rho B^a_{\rho\mu\nu})A^{b\mu}A^{c\nu} + \ldots}_{\text{kinematical structure constants}}$$

- Monteiro and O'Connell (1105.2565 [hep-th]) identifies a Lie algebra in the self-dual YM sector $\Rightarrow$ kin. structure constants for MHV tree amplitudes.