Two-hadron correlations at high energy

Jamal Jalilian-Marian

Baruch College, New York NY

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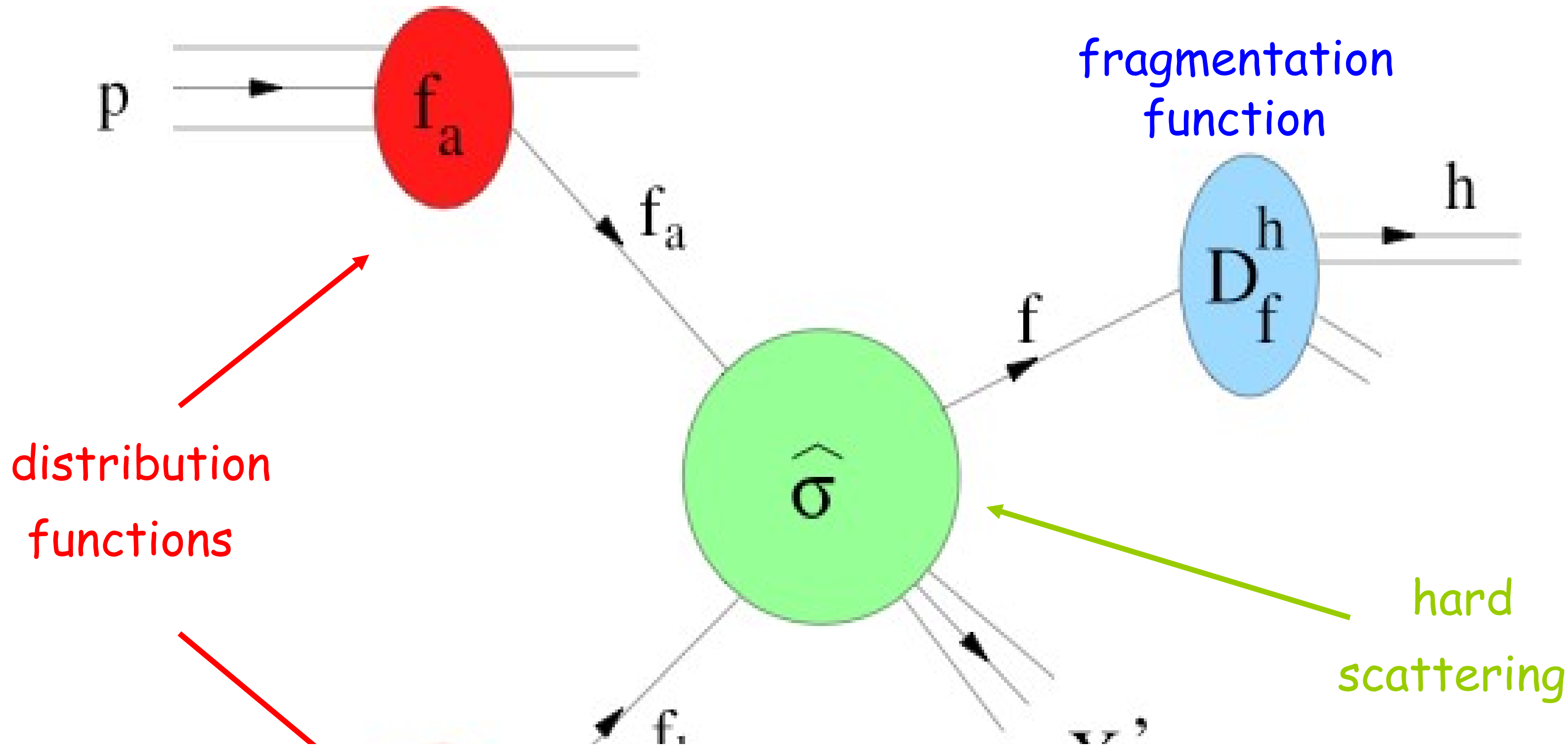
pp collisions at *high* p_t

Collinear factorization: separation of long and short distances

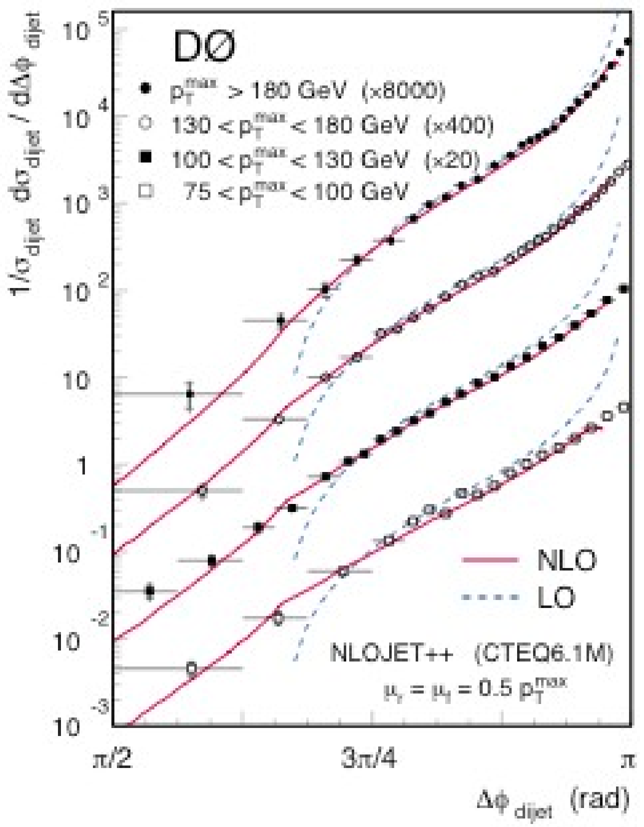


$$d\sigma = \int dx_1\, dx_2\, dz\, f_a^{H1}(x_1, M^2)\, f_b^{H2}(x_2, M^2)\, D_c^h(z, M^2)$$

$$\otimes \quad d\hat{\sigma}_{ab}^c(x_1 P_{H1}, x_2 P_{H2}, P_h/z, M^2)$$

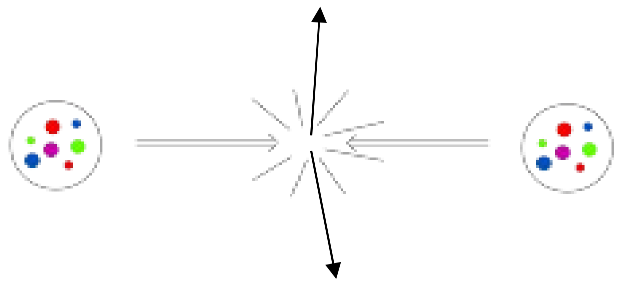
Di-jet correlations in pp: pQCD

in pQCD calculations based on collinear factorization, dijets are back-to-back

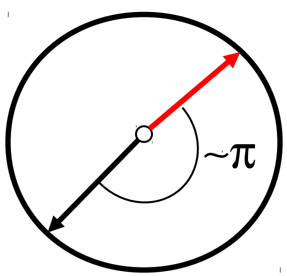


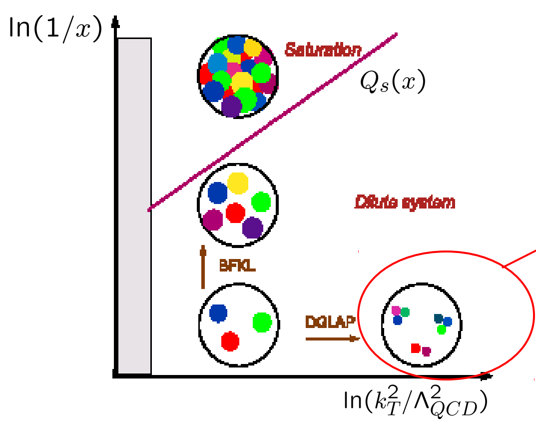
peak narrower with higher p_T

Tevatron ***high p_T*** data



transverse view

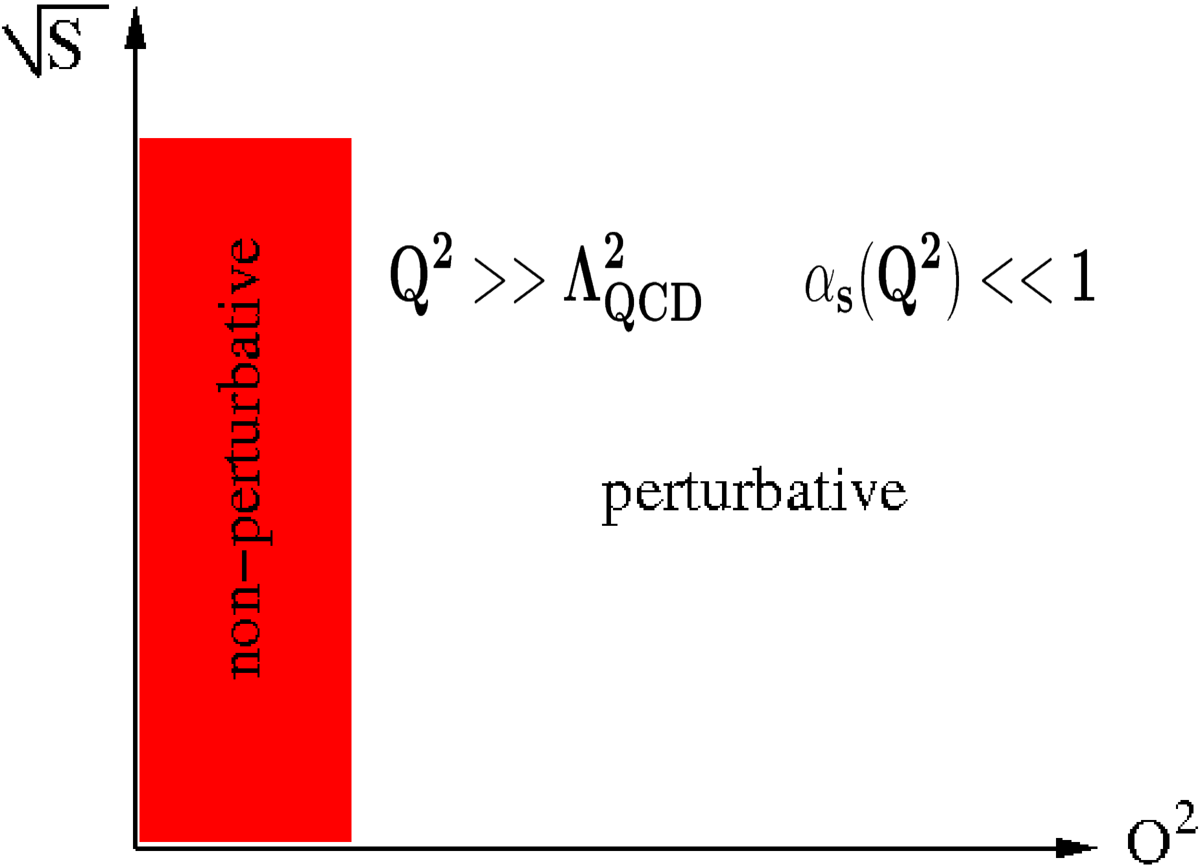




probing Λ_{QCD}/p_T <<1

power corrections are negligible

QCD: the old paradigm



pQCD tools:
twist expansion,
collinear factorization

$\sqrt{S} \to \infty$

$Q^2 \to \infty$

but bulk of QCD phenomena happens at low Q

QCD in the Regge-Gribov limit

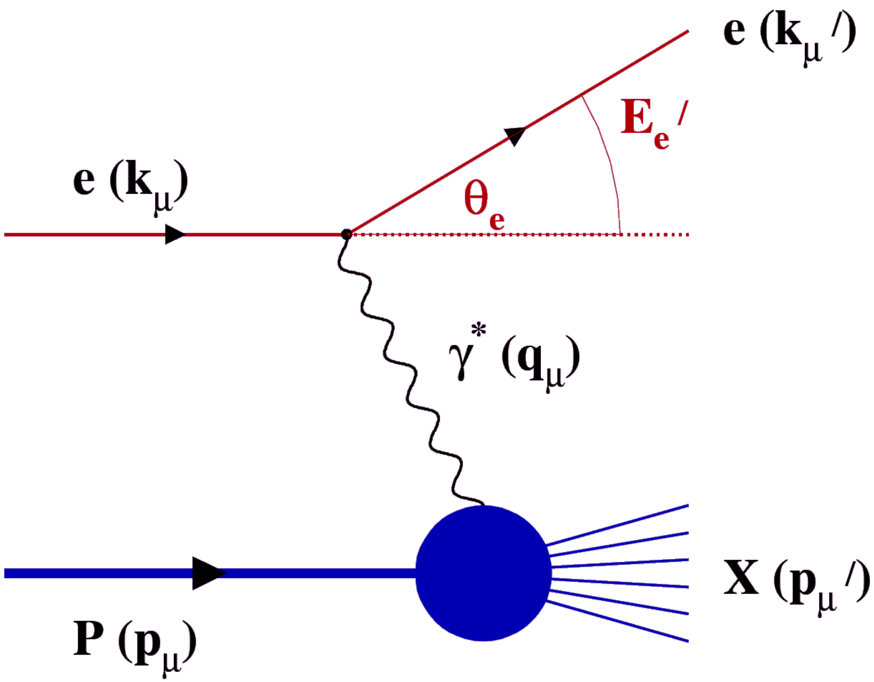
$$\sqrt{S} \to \infty \qquad Q^2 \sim \text{const.} \qquad x \sim \frac{Q^2}{S} \to 0$$

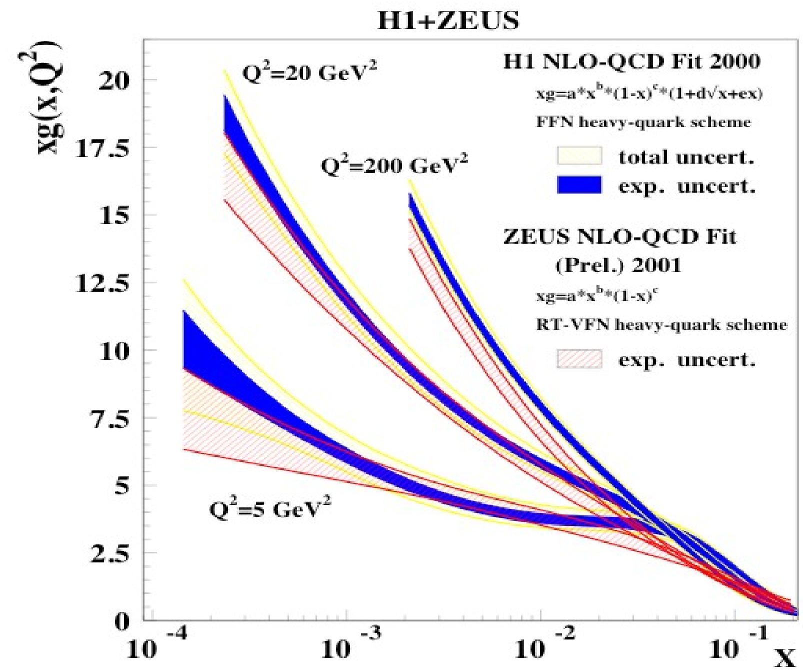
Regge

Gribov

A hadron at small x

DIS: $\mathbf{e\,p \to e\,X}$



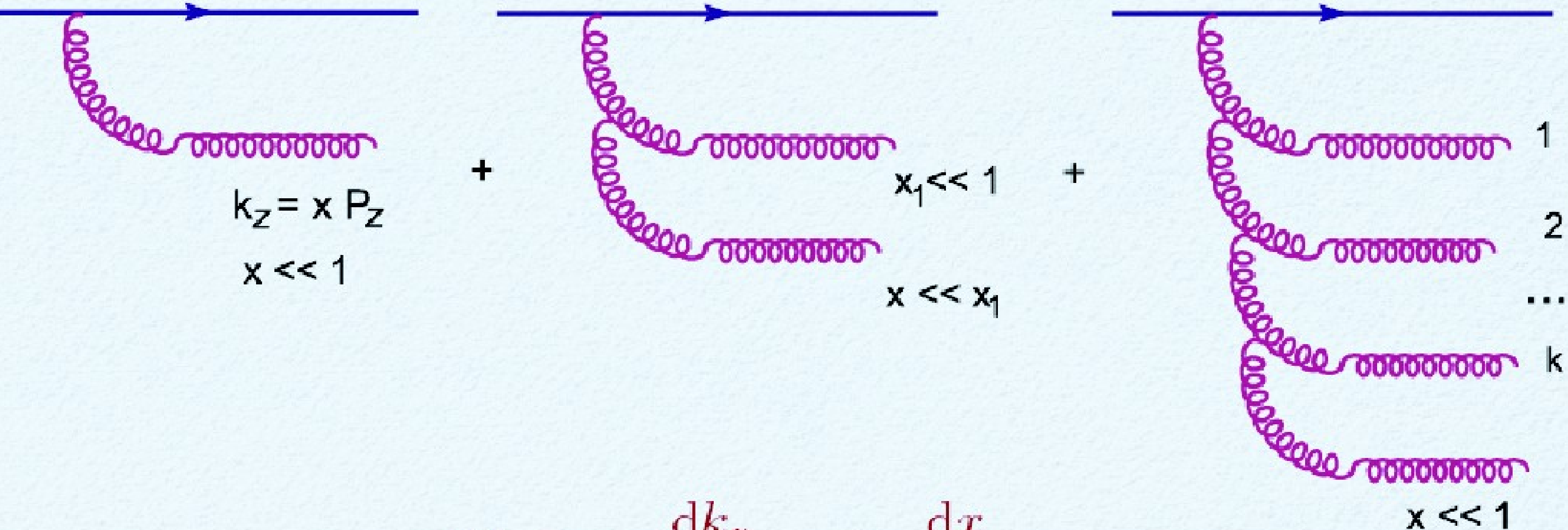


$$\mathbf{x = \frac{p^+}{P^+}}$$ is the fraction of hadron energy carried by a parton

there are a lot of gluons at small x

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons



$$\mathrm{d}\mathcal{P} \;\propto\; \alpha_s \frac{\mathrm{d}k_z}{k_z} \;=\; \alpha_s \frac{\mathrm{d}x}{x}$$

The 'price' of an additional gluon:

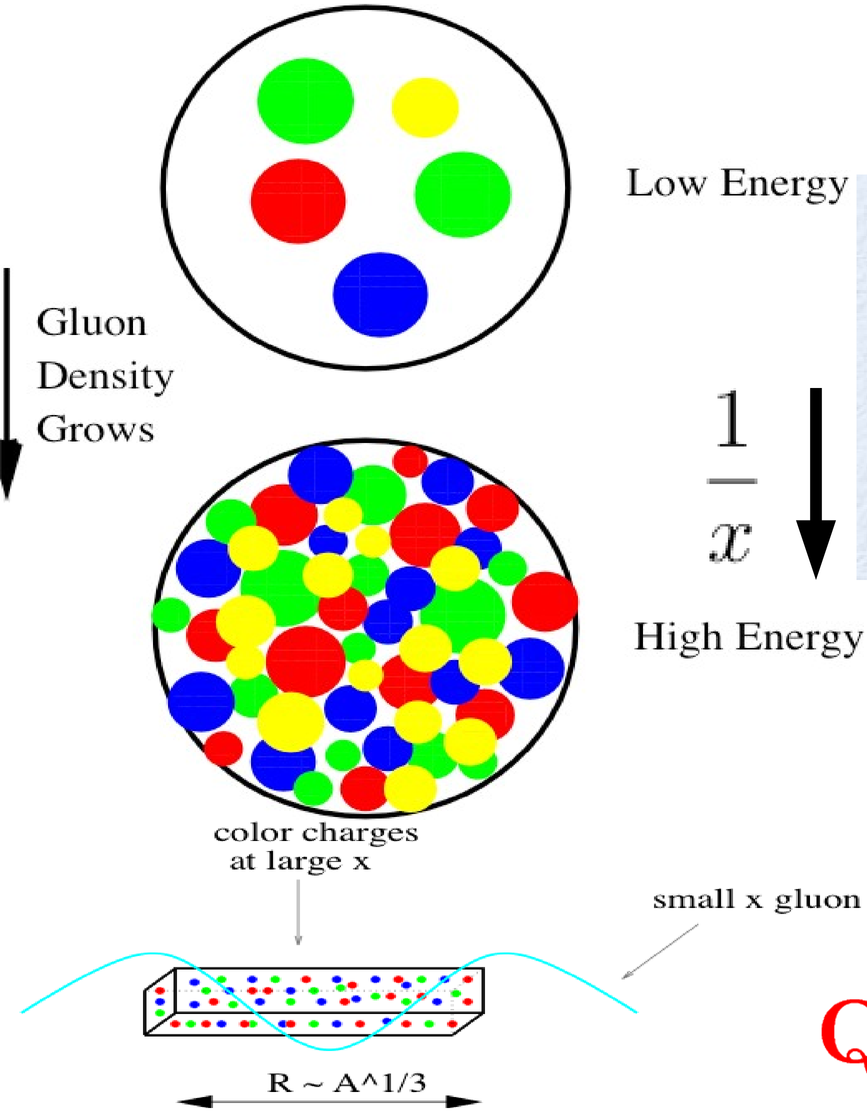
$$\mathcal{P}(1) \;\propto\; \alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} \;=\; \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast

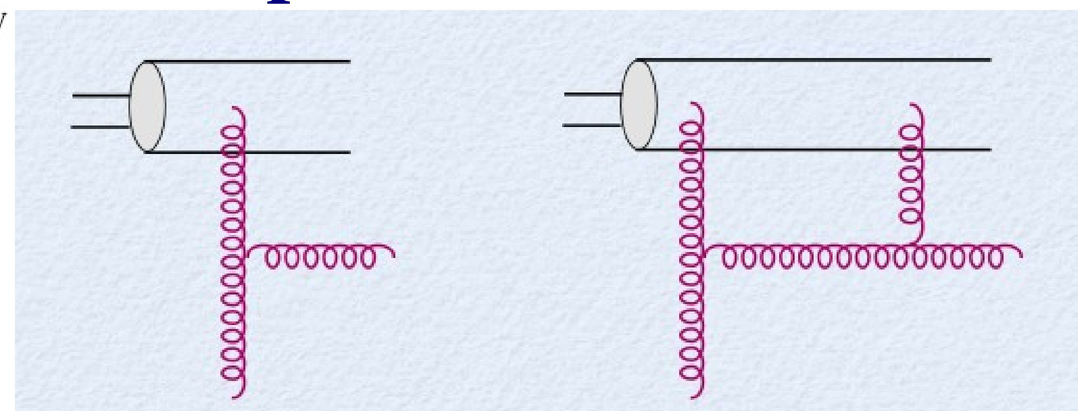
$$n \sim e^{\alpha_s ln 1/x}$$

Gluon saturation

Gribov-Levin-Ryskin



"attractive" bremsstrahlung
vs. "repulsive" recombination



$$\frac{\alpha_s\, xG(x, b_t, Q^2)}{S_\perp\, Q^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

MV effective Action + RGE

$$\mathbf{S}[\mathbf{A},\rho] = -\frac{1}{4}\int \mathbf{d^4x\,F^2_{\mu\nu}} + \frac{\mathbf{i}}{\mathbf{N_c}}\int \mathbf{d^2x_t dx^-}\,\delta(\mathbf{x^-})\mathbf{Tr}[\rho(\mathbf{x_t})\mathbf{U}(\mathbf{A^-})]$$

Large x: color source ρ small x: gluon field $\mathbf{A}^\mu$

$$\mathbf{U}(\mathbf{A^-}) = \hat{\mathbf{P}}\,\mathbf{Exp}\left[\mathbf{ig}\int \mathbf{dx^+}\,\mathbf{A^-_a}\,\mathbf{T_a}\right]$$

$$\mathbf{Z}[\mathbf{j}] = \int [\mathbf{D}\rho]\,\mathbf{W}_{\Lambda^+}[\rho]\left[\frac{\int^{\Lambda^+}[\mathbf{DA}]\delta(\mathbf{A^+})\mathbf{e}^{\mathbf{i}\mathbf{S}[\mathbf{A},\rho]-\int \mathbf{j}\cdot\mathbf{A}}}{\int^{\Lambda^+}[\mathbf{DA}]\delta(\mathbf{A^+})\mathbf{e}^{\mathbf{i}\mathbf{S}[\mathbf{A},\rho]}}\right]$$

$\mathbf{W}_{\Lambda^+}[\rho]$ weight functional: probability distribution of color source ρ at longitudinal scale Λ^+

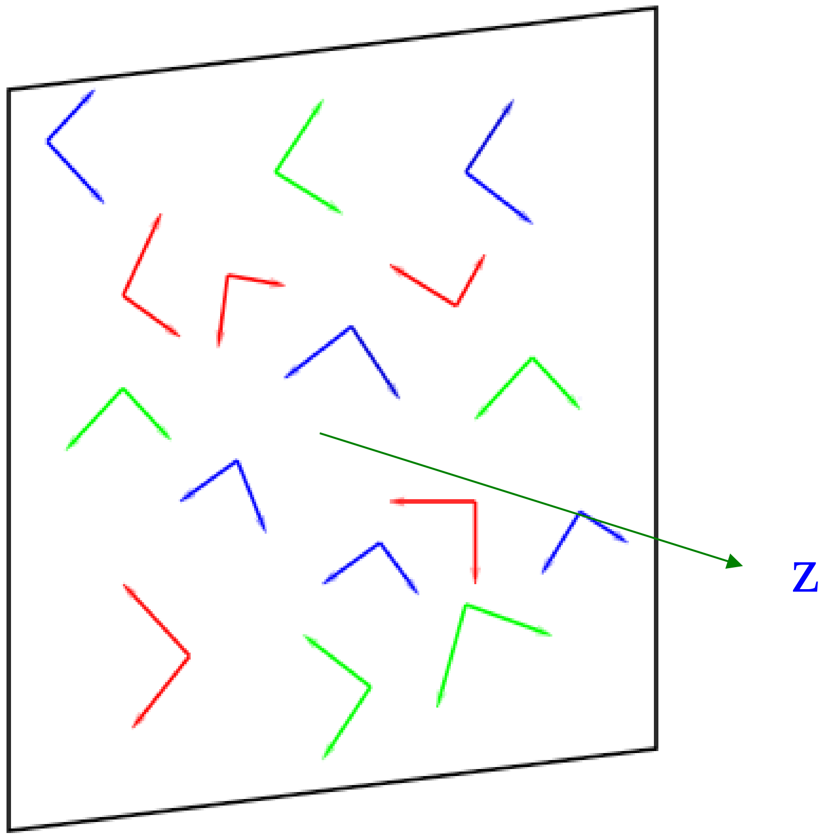
invariance under change of Λ^+ ⟶ RGE for $\mathbf{W}_{\Lambda^+}[\rho]$

The Classical Field

saddle point of effective action-> Yang-Mills equations

$$\mathbf{D}_\mu \, \mathbf{F}_{\mathbf{a}}^{\mu\nu} = \delta^{\nu\,+}\delta(\mathbf{x}^-)\,\rho_{\mathbf{a}}(\mathbf{x_t})$$

solutions are non-Abelian Weizsäcker-Williams fields



$$\mathbf{A}^+ = \mathbf{0}$$

$$\mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}_{\mathbf{a}}^{\mathbf{i}} = \theta(\mathbf{x}^-)\,\alpha_{\mathbf{a}}^{\mathbf{i}}(\mathbf{x_t})$$

$$\partial^{\mathbf{i}}\,\alpha_{\mathbf{a}}^{\mathbf{i}} = \mathbf{g}\,\rho_{\mathbf{a}}$$

pure (2d) gauge

color $\mathrm{E}_\perp$, $\mathrm{B}_\perp$ fields

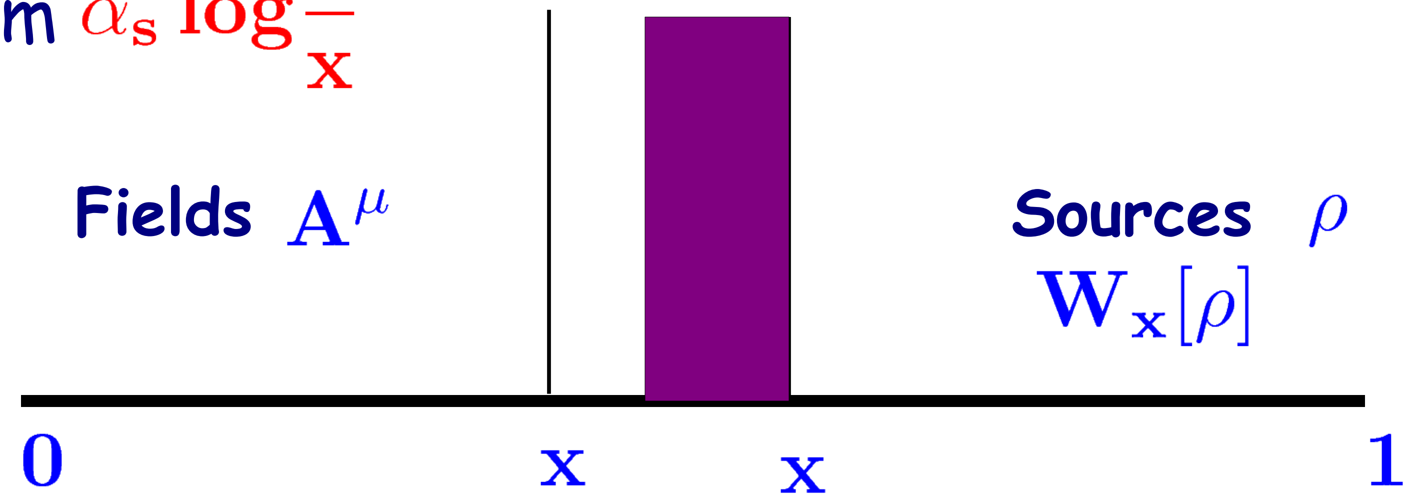
QCD at High Energy: Wilsonian RG

resum $\alpha_s \log \frac{1}{x}$

Fields A^μ

Sources ρ

$W_x[\rho]$



$A^\mu = A^\mu_{class} + \delta A^\mu$

integrate out field fluctuations quadratically

$\rho \rightarrow \rho' = \rho + \delta\rho$

$$\frac{\partial W[\rho]}{\partial \ln 1/x} = \frac{1}{2}\int_{x_t, y_t} \frac{\delta}{\delta \rho^a(x_t)} \chi^{ab}(x_t, y_t)[\rho] \frac{\delta}{\delta \rho^a(y_t)} W[\rho]$$

JIMWLK eq. describes x evolution of observables

CGC:QCD at high gluon density

effective degrees of freedom: Wilson line $V(x_t)$

CGC observables: $< \mathrm{Tr}\, V \cdots\cdots V^\dagger >$ with $\mathbf{V(x_t) = \hat{P} e^{ig \int dx^- A_a^+ t_a}}$

$$\mathbf{A}_{\mathbf{a}}^{\mu}(\mathbf{x_t}, \mathbf{x}^-) \sim \delta^{\mu +}\, \delta(\mathbf{x}^-)\, \alpha_{\mathbf{a}}(\mathbf{x_t}) \qquad \alpha^{\mathbf{a}}(\mathbf{k_t}) = \mathbf{g}\, \rho^{\mathbf{a}}(\mathbf{k_t})/\mathbf{k_t^2}$$

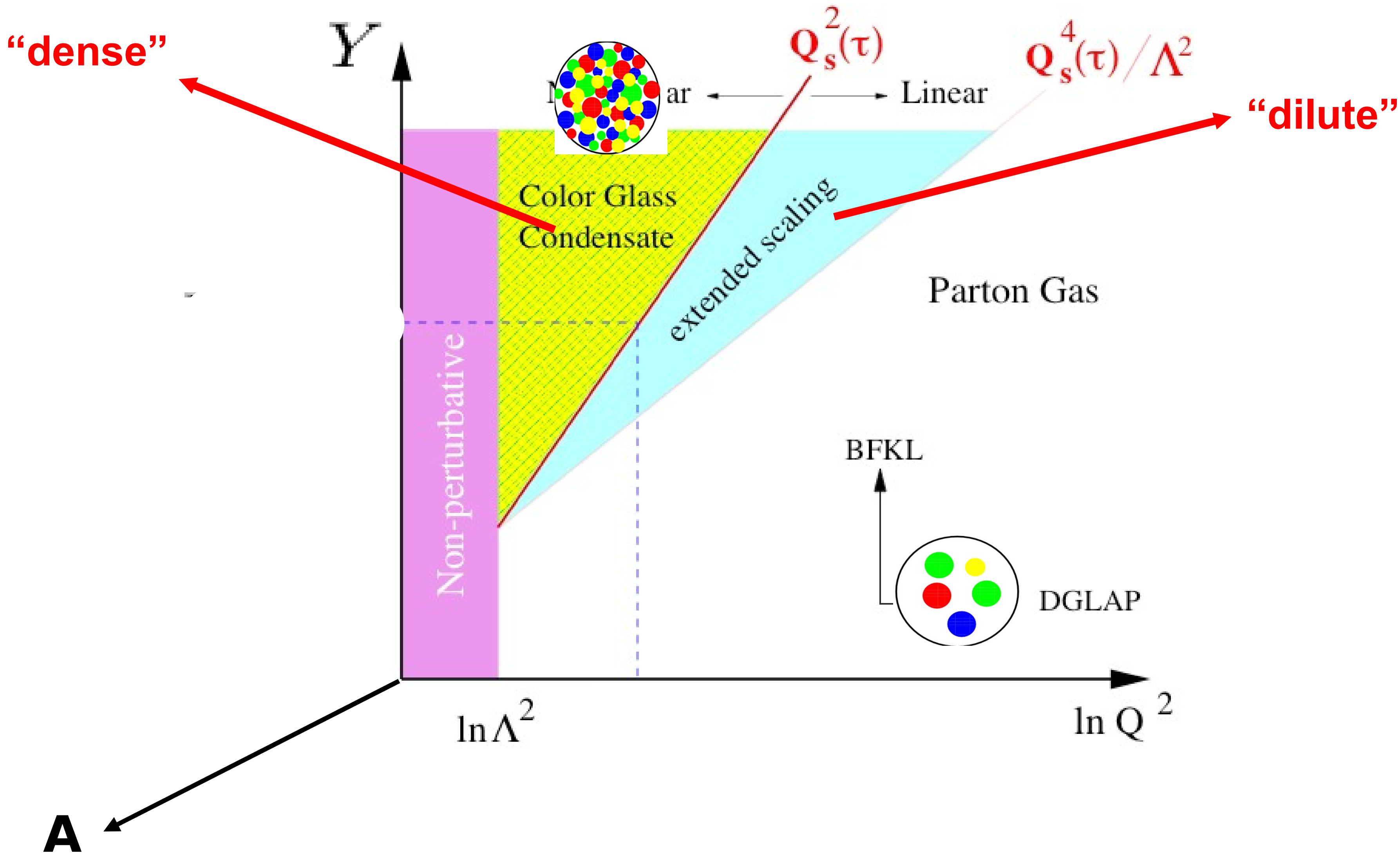
gluon distribution: $\mathbf{xG(x, Q^2)} \sim \int^{\mathbf{Q^2}} \frac{\mathbf{d^2k_t}}{\mathbf{k_t^2}}\, \phi(\mathbf{x}, \mathbf{k_t})$ with $\phi(\mathbf{x}, \mathbf{k_t^2}) \sim < \rho_{\mathbf{a}}^{\star}(\mathbf{k_t})\, \rho_{\mathbf{a}}(\mathbf{k_t}) >$

two main effects:

multiple scatterings

evolution with ln (1/x)

Road Map of QCD Phase Space



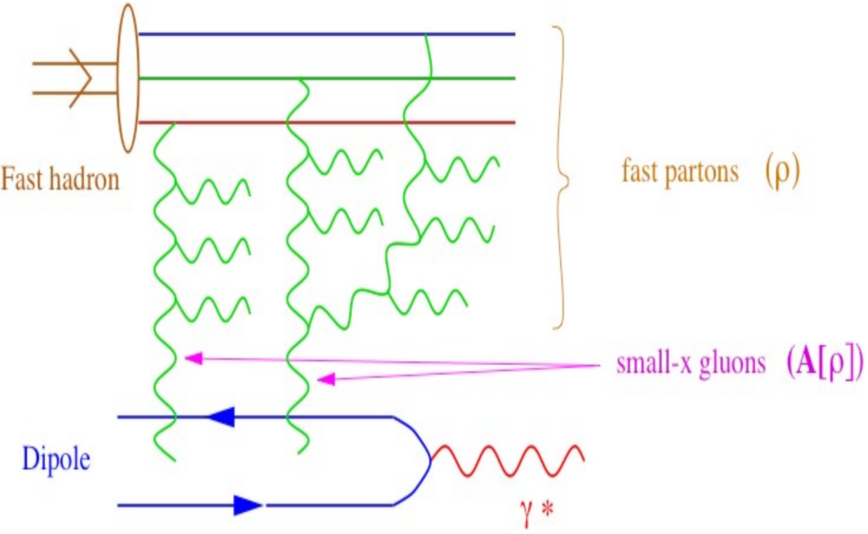
Applications: DIS

high gluon density: "multiple scatterings"

effective degrees of freedom: Wilson line V (x_t)

high energy:evolution of n-point corr. with ln (1/x)

$$\sigma_{\gamma^\star \, p} = \int_0^1 dz \int d^2 r_t d^2 b_t \, |\Psi(z, r_t, Q^2)|^2 \, N_F(x, r_t, b_t)$$



only the 2-pt function contributes

$$N_F \equiv \frac{1}{N_c} < Tr[1 - V^\dagger(x_t) V(y_t)] >$$

where JIMWLK eqs. determine the x dependence of N_F

Applications: single inclusive hadron production in pA

$$\frac{d\sigma^{pA\to hX}}{dY\, d^2P_t\, d^2b} = \frac{1}{(2\pi)^2}\int_{x_F}^{1} dx\, \frac{x}{x_F}$$

$$\Big\{ f_{q/p}(x,Q^2)\;\; N_F[\frac{x}{x_F}P_t, b, y]\;\; D_{h/q}(\frac{x_F}{x}, Q^2) +$$

$$f_{g/p}(x,Q^2)\;\; N_A[\frac{x}{x_F}P_t, b, y]\;\; D_{h/g}(\frac{x_F}{x}, Q^2) \Big\}$$

DHJM, BMTS

2-point function only: same as in DIS and photon,dilepton production in pA (FG and JJM)

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JIMWLK evolution equation

$$\frac{d}{d\,ln 1/x}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\,\eta_{xy}^{bd}\,\frac{\delta}{\delta\alpha_y^d}\,O\right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[1+U_x^\dagger U_y - U_x^\dagger U_z - U_z^\dagger U_y\right]^{bd}$$

Evolution of the 2-point function (dipole)

$$\frac{\mathbf{d}}{\mathbf{dy}} < \mathbf{Tr} \mathbf{V}_{\mathbf{x}}^{\dagger} \mathbf{V}_{\mathbf{y}} > = -\frac{\bar{\alpha}_{\mathbf{s}}}{\mathbf{2}\pi} \int \mathbf{d^2 z} \, \frac{(\mathbf{x} - \mathbf{y})^{\mathbf{2}}}{(\mathbf{x} - \mathbf{z})^{\mathbf{2}} (\mathbf{y} - \mathbf{z})^{\mathbf{2}}} \times \left[< \mathbf{Tr} \mathbf{V}_{\mathbf{x}}^{\dagger} \mathbf{V}_{\mathbf{y}} > - \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr} \mathbf{V}_{\mathbf{x}}^{\dagger} \mathbf{V}_{\mathbf{z}} \mathbf{Tr} \mathbf{V}_{\mathbf{z}}^{\dagger} \mathbf{V}_{\mathbf{y}} > \right]$$

Evolution of 2-point function depends on 4-point function

$$\frac{\mathbf{d}}{\mathbf{dy}} < \mathbf{Tr} \mathbf{V}_{\mathbf{x}}^{\dagger} \mathbf{V}_{\mathbf{z}} \mathbf{Tr} \mathbf{V}_{\mathbf{z}}^{\dagger} \mathbf{V}_{\mathbf{y}} > \sim < \mathbf{V}^4 + \cdots >$$

Infinitely many coupled equations!

Large N_c :Balitsky-Kovchegov (BK) eq.

$$\frac{d}{dy} < \mathrm{Tr} V_x^\dagger V_y > = -\frac{\bar{\alpha}_s}{2\pi} \int d^2 z \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \times$$

$$\left[< \mathrm{Tr} V_x^\dagger V_y > - \frac{1}{N_c} < \mathrm{Tr} V_x^\dagger V_z > < \mathrm{Tr} V_z^\dagger V_y > \right]$$

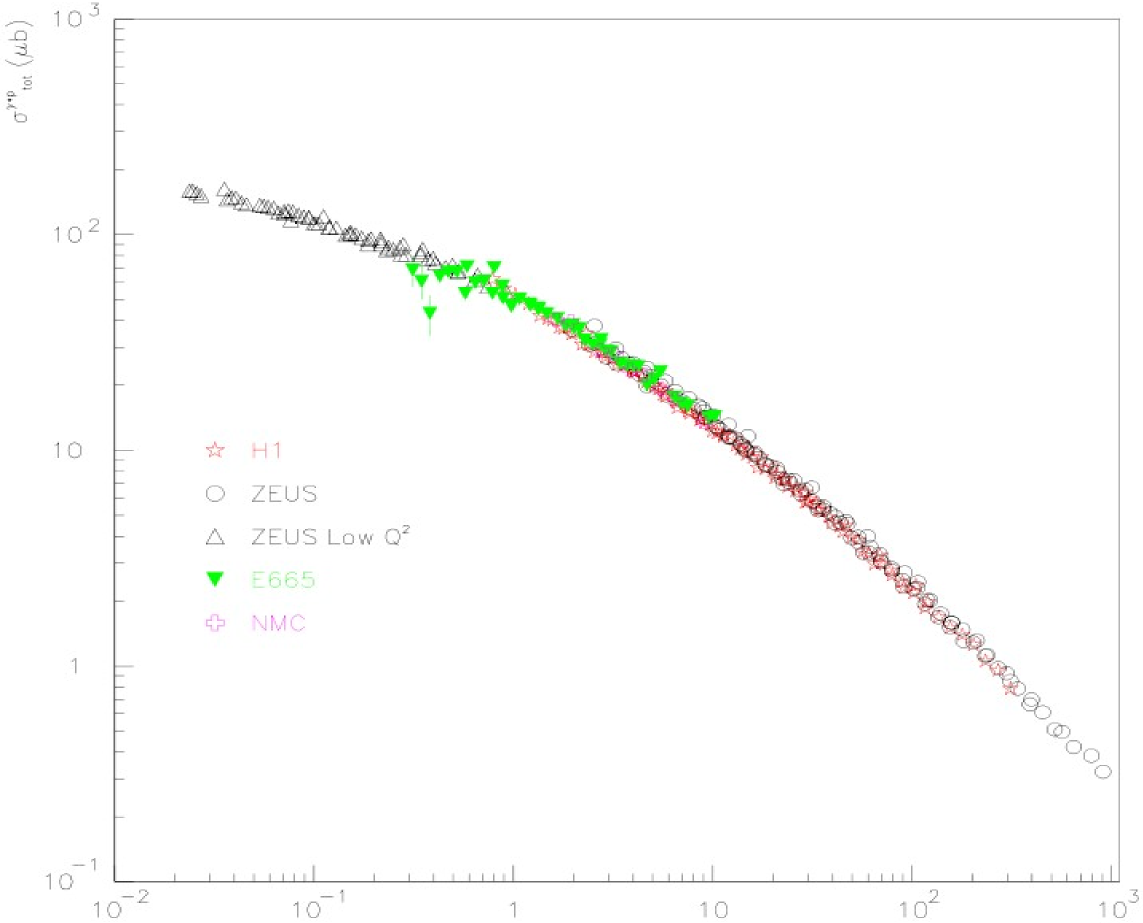
all higher point functions are expressed in terms of the dipole

extended scaling region: $< \mathrm{Tr} V_x^\dagger V_y > \simeq F\left[(x-y) Q_s^2\right]$

IIM, NPA708 (2002) 327

NLO: B-KW-G-BC (2007-2008)

CGC at HERA? Extended scaling



S(G-B)K
PRL86 (2001) 596

Collinear Fact.:

$$\sigma = \sigma(\mathbf{x}, \mathbf{Q^2})$$

CGC:

$$\sigma = \sigma(\mathbf{Q^2}/\mathbf{Q_s^2})$$

$$Q_s << Q << \frac{Q_s^2}{\Lambda}$$

IIM (2002)

$$\mathbf{Q_s^2} = \mathbf{1\,GeV^2}[\mathbf{x_0}/\mathbf{x}]^\lambda$$

$$\mathbf{x_0} = \mathbf{3X10^{-4}}$$

Evolution of the ***dipole***

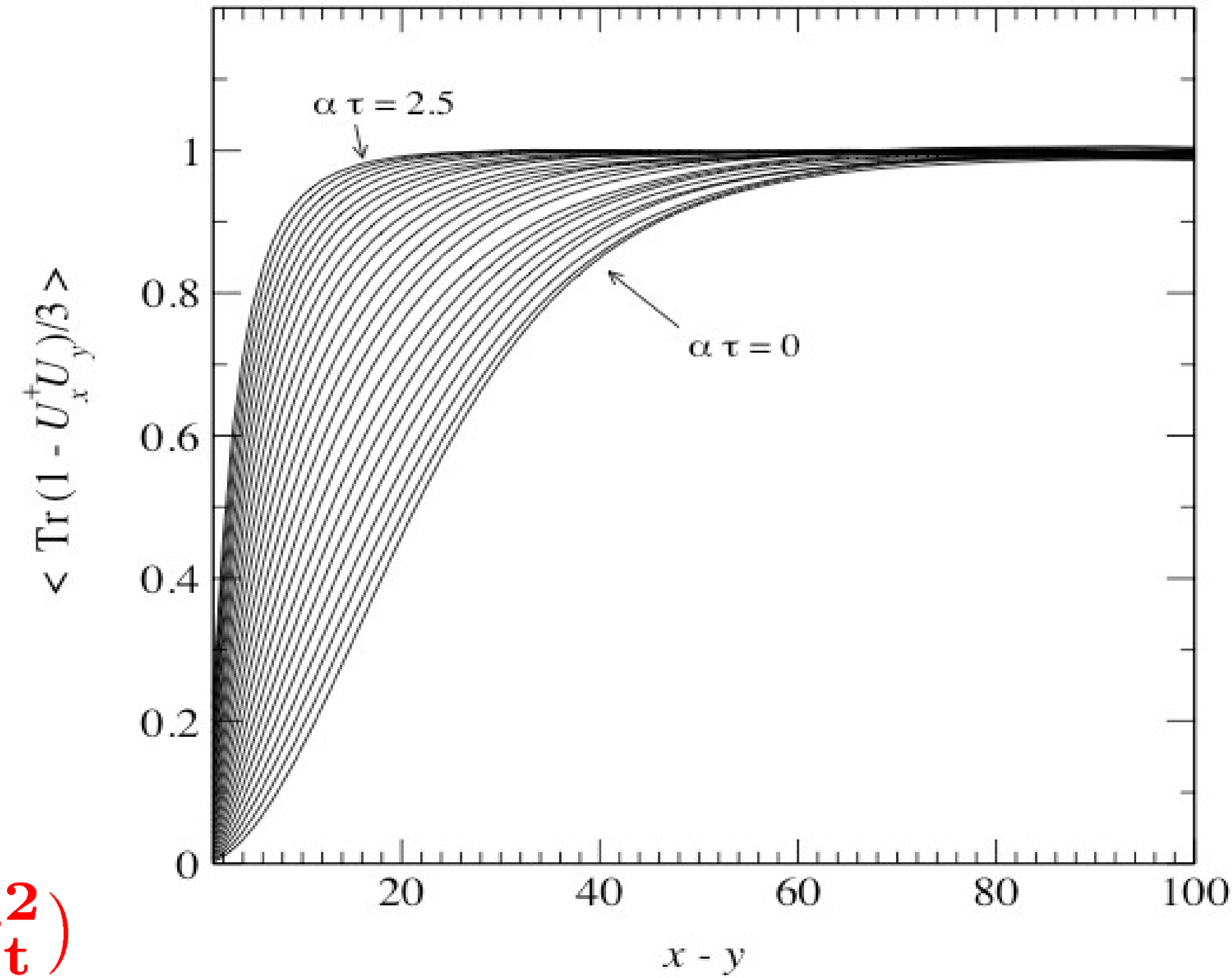
saturation region

dilute region

$$\sim [r_t^2\, Q_s^2]^\gamma$$

pQCD region

$$\sim r_t^2\, xG(x, 1/r_t^2)$$



RW, NPA739 (2004) 183

Two-hadron correlations

away-side correlations in dA: forward rapidity

long-range rapidity correlations: the Ridge

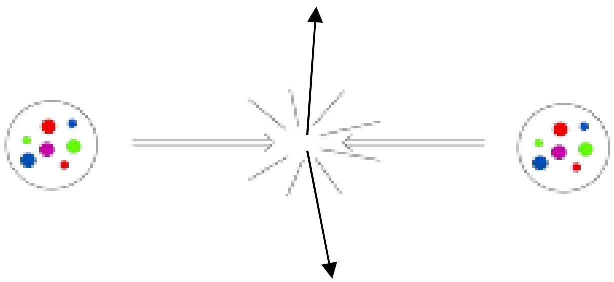
di-jet production in DIS

the role of initial conditions

Di-hadron kinematics in CGC

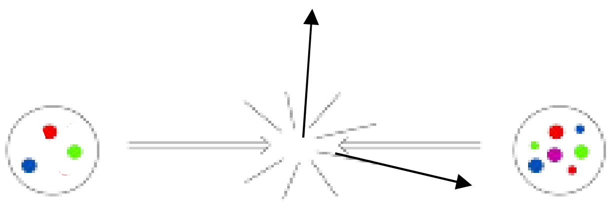
produced partons: $k_1, y_1 \quad k_2, y_2$ $\quad x_p = \frac{k_1\, e^{y_1} + k_2\, e^{y_2}}{\sqrt{s}} \quad x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}}$

scanning the wave-functions $\quad k_1 \sim k_2 \sim k \sim 2\, GeV$



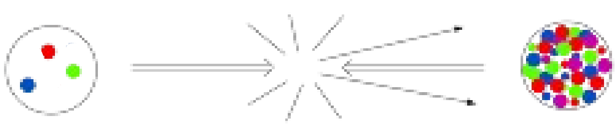
mid-mid correlations
probe moderate x

$$x_p \sim x_A \sim \frac{k}{\sqrt{S}} \sim 10^{-2}$$



mid-forward

$$x_p \sim \frac{k}{\sqrt{S}}\, e^{y_1} \to 1 \quad x_A \sim \frac{k}{\sqrt{S}} \sim 10^{-2}$$

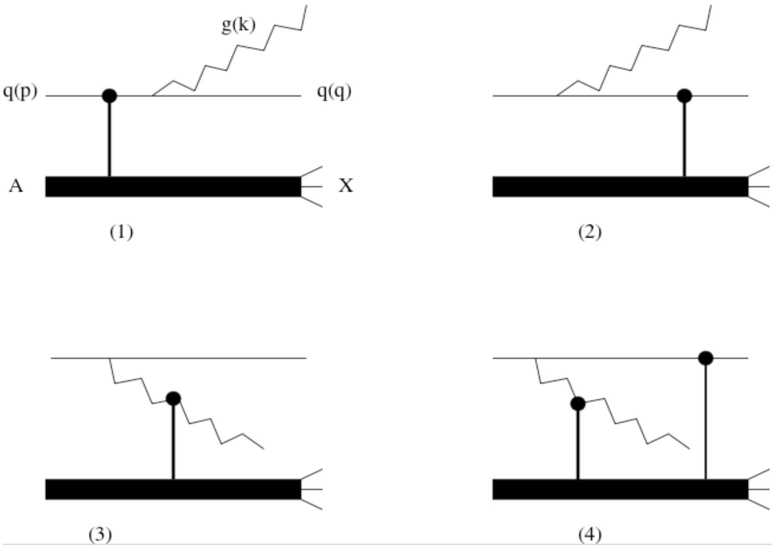


forward-forward

$$x_p \sim \frac{k}{\sqrt{S}}\, e^{y_1} \to 1 \quad x_A \sim \frac{k}{\sqrt{S}}\, e^{-y_2} \sim 10^{-4}$$

Di-jet production: pA

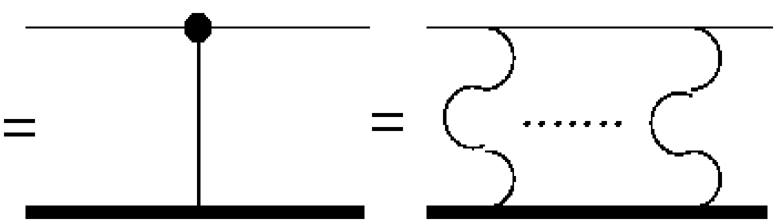
JJM and YK, PRD70 (2004)



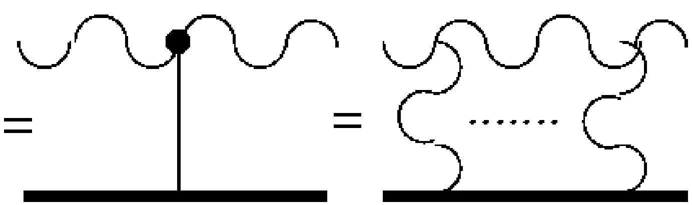
$$\frac{d\sigma^{dA\rightarrow q\,g\,X}}{d\,p_t^2\,dy_1\,d\,q_t^2\,dy_2} \sim \int K \otimes$$

$$[< \mathrm{Tr} V^\dagger\, V > + < \mathrm{Tr} V^\dagger\, V\, V^\dagger\, V >$$

V(x) =



U(x) =



$$U^{ab}(x_t)\, t^b = V^\dagger(x_t)\, t^a\, V(x_t)$$

AK and ML, JHEP (2006), FGV, NPA (2006), CM, NPA (2007)
KT, NPA (2010), DMXY, PRD (2011), SXY (2011)

di-jet production in pA

$O_2(r,\bar{r}) \equiv Tr V_r\, V_{\bar{r}}^\dagger$ **dipole** → **F2 in DIS, single hadron in pA**

$$O_4(r,\bar{r}:s) \equiv Tr V_r^\dagger\, t^a\, V_{\bar{r}}\, t^b\, [U_s]^{ab} = \frac{1}{2}\left[Tr V_r^\dagger\, V_s\;\; Tr V_{\bar{r}}\, V_s^\dagger - \frac{1}{N_c} Tr V_r^\dagger\, V_{\bar{r}}\right]$$

$$O_6(r,\bar{r}:s,\bar{s}) \equiv Tr V_r\, V_{\bar{r}}^\dagger\, t^a\, t^b\, [U_s\, U_{\bar{s}}^\dagger]^{ba} = \frac{1}{2}\left[Tr V_r\, V_{\bar{r}}^\dagger\, V_{\bar{s}}\, V_s^\dagger\, Tr V_s\, V_{\bar{s}}^\dagger - \frac{1}{N_c} Tr V_r\, V_{\bar{r}}^\dagger\right]$$

quadrupole

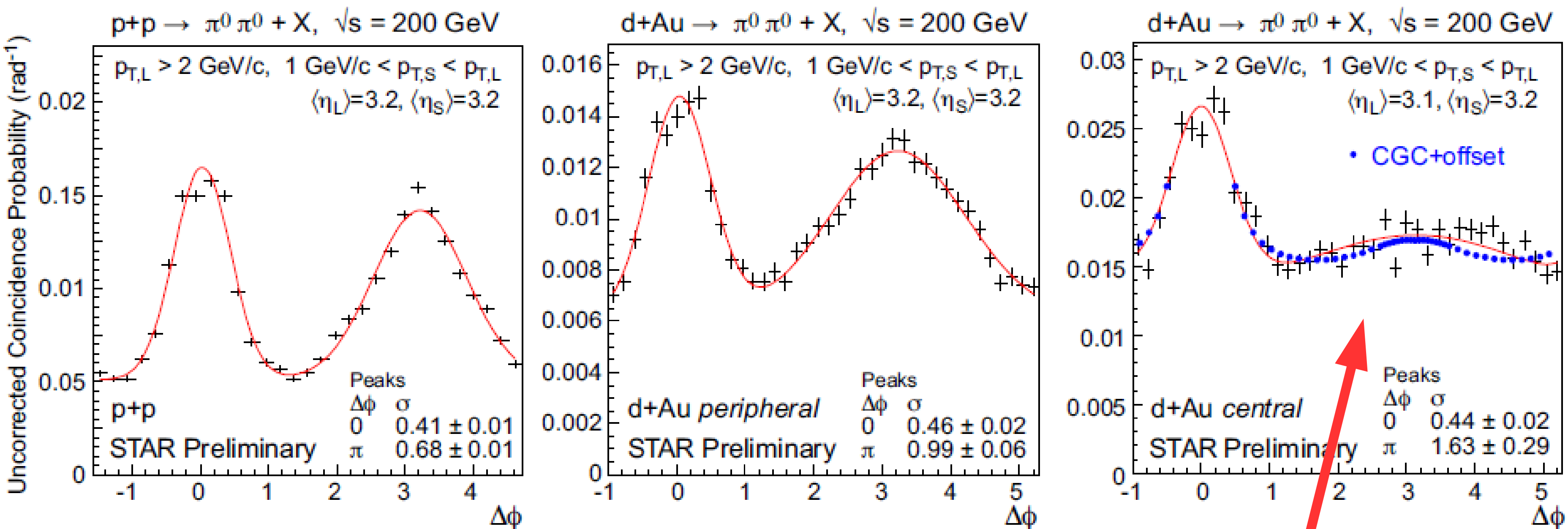
Dipole + large N_c approximation?

$$\langle O_4(r,\bar{r}:s)\rangle \simeq \langle O_2(r-s)\rangle\, \langle O_2(s-\bar{r})\rangle$$

$$\langle O_6(r,\bar{r}:s,\bar{s})\rangle \simeq \langle O_2(r-s)\rangle\, \langle O_2(\bar{r}-\bar{s})\rangle\, \langle O_2(s-\bar{s})\rangle + \langle O_2(r-\bar{r})\rangle\, \langle O_2(\bar{s}-s)\rangle\, \langle O_2(s-\bar{s})\rangle$$

disappearance of back to back jets

Recent STAR measurement (arXiv:1008.3989v1):



CGC fit from
Albacete + Marquet, PRL (2010)
using running coupling BK solution,
Also by Tuchin, NPA846 (2010)

multiple scatterings
de-correlate the hadrons

JIMWLK: Beyond dipole + large Nc

Recall evolution of O2 is sensitive to O4 only

$$\frac{d}{dy}\langle O_4(r,\bar{r}:s)\rangle = -\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z\,\Big\langle 2\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right]O_4(r,\bar{r}:s)$$

$$-\frac{1}{N_c}\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}\,TrV_r^\dagger\,V_z\;\;TrV_s^\dagger\,V_{\bar{r}}\;\;TrV_z^\dagger\,V_s\right.$$

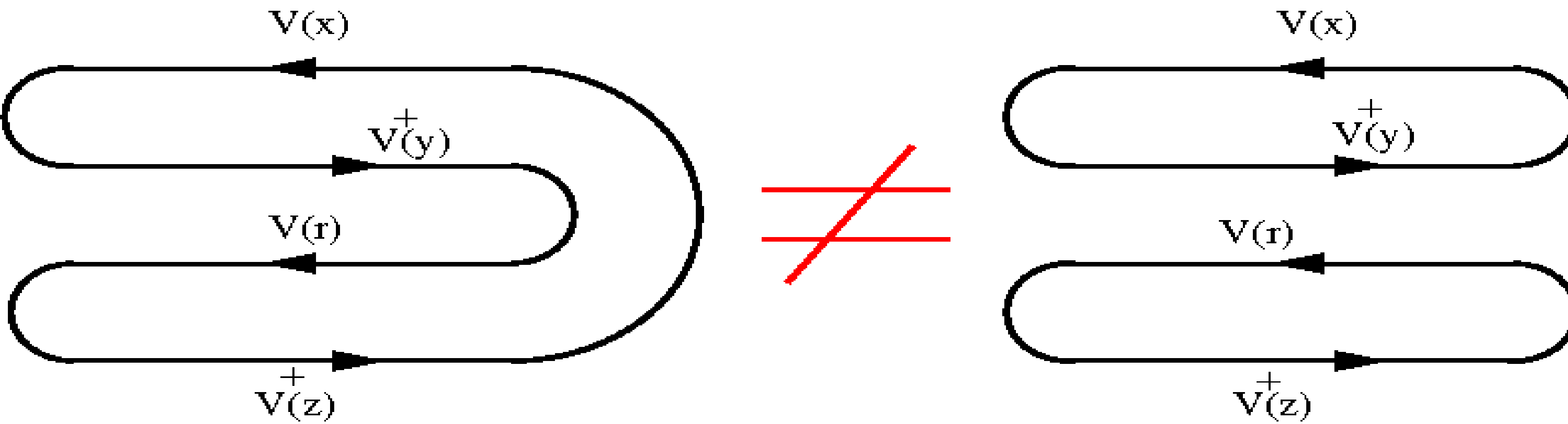
$$\left.+\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}TrV_r^\dagger\,V_s\;\;TrV_z^\dagger\,V_{\bar{r}}\;\;Tr\,V_s^\dagger\,V_z\right]\Big\rangle+\cdots\cdots$$

$$\mathbf{\frac{d}{dy}S_4(r,\bar{r}:s)\simeq\frac{d}{dy}[S_2(s-\bar{r})\,S_2(r-s)]+O(\frac{1}{N_c^2})}$$

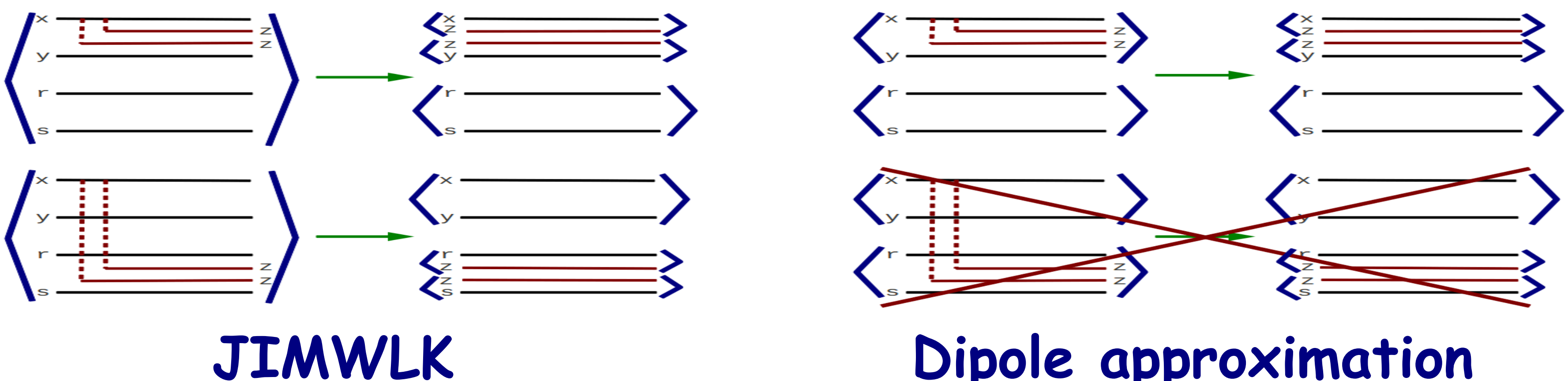
$$with\quad S_4\equiv\frac{1}{C_A\,C_F}\,\langle O_4\rangle\quad and\quad S_2\equiv\frac{1}{C_A}\,\langle O_2\rangle$$

DIS structure functions, single inclusive production in pA probe dipoles

Quadrupole vs. dipoles



and they evolve differently even at large N_c



jjm-yk,PRD70 (2004) 114017, ad-jjm,PRD82 (2010) 074023

Evolution of quadrupole from JIMWLK

$$\frac{d}{dy}\left\langle Q(r,\bar{r},\bar{s},s)\right\rangle$$

$$= \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z\Bigg\{\Bigg\langle\left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right]Q(z,\bar{r},\bar{s},s)\,S(r,z)$$

$$+ \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right]Q(r,z,\bar{s},s)\,S(z,\bar{r})$$

$$+ \left[\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}-\frac{(\bar{r}-s)^2}{(s-z)^2(\bar{r}-z)^2}\right]Q(r,\bar{r},z,s)\,S(\bar{s},z)$$

$$+ \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right]Q(r,\bar{r},\bar{s},z)\,S(z,s)$$

$$- \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}\right]Q(r,\bar{r},\bar{s},s)$$

$$- \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right]S(r,s)\,S(\bar{r},\bar{s})$$

$$- \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right]S(r,\bar{r})\,S(\bar{s},s)\Bigg\rangle\Bigg\}$$

$$\frac{d}{d\,y}Q = \int P_1\,[Q\,S] - P_2\,[Q] + P_3[S\,S]$$

with

$$P_1 - P_2 + P_3 = 0$$

very recent attempts to solve this analytically: EI-DT

Evolution of quadrupole in the linear region

expand all Wilson lines in gA and ignore non-linear terms

O $(gA)^2$ → BFKL (evolution of a 2-reggeized gluon state)

O $(gA)^4$ → BJKP (evolution of a 4-reggeized gluon state)

JIMWLK evolution of n-Wilson lines *may* contain the BJKP hierarchy as its linear limit (in progress)

Dijet production poses new challenges to CGC

but

every challenge can become an opportunity

What is the energy dependence of quadrupoles ?

How large are the N_c suppressed terms ?

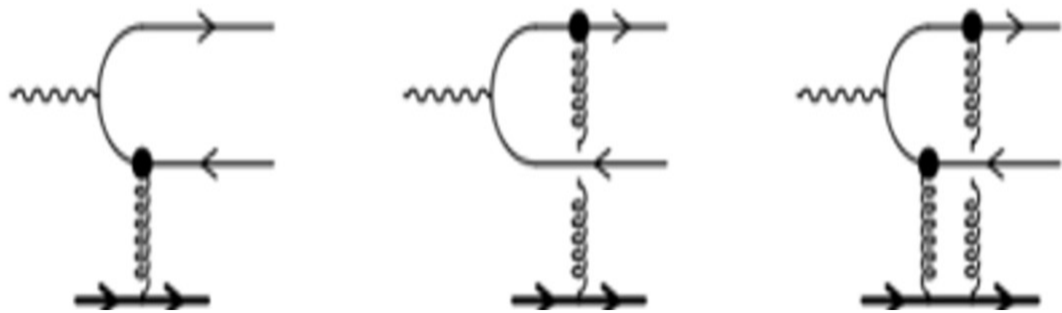
Evolution of higher point functions depends on lower point functions!

Solve JIMWLK numerically

Di-jet correlations: DIS

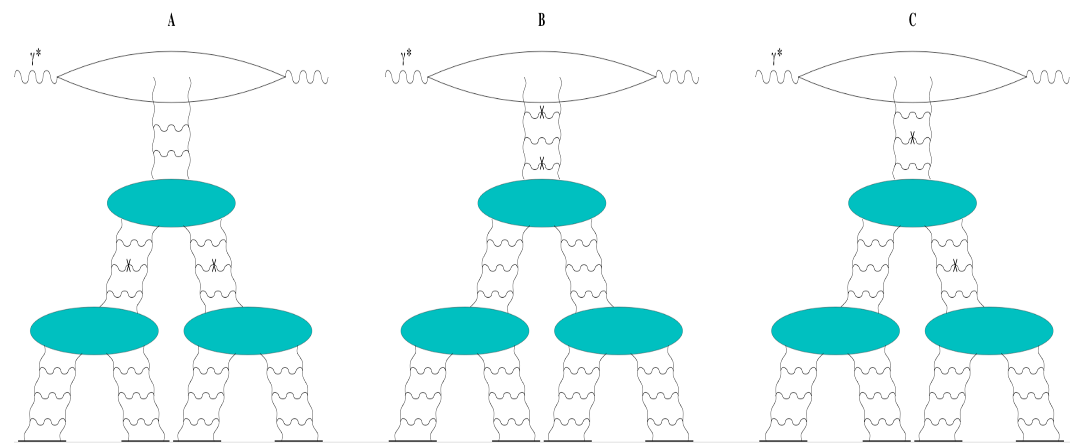
$$\gamma^\star \, \mathbf{p}(\mathbf{A}) \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{X}$$

FG & JJM, PRD67 (2003)



$$\gamma^\star \, \mathbf{p}(\mathbf{A}) \rightarrow \mathbf{g}\,\mathbf{g}\,\mathbf{X}$$

JJM & YK, PRD70 (2004)
AK & ML, JHEP (2006)

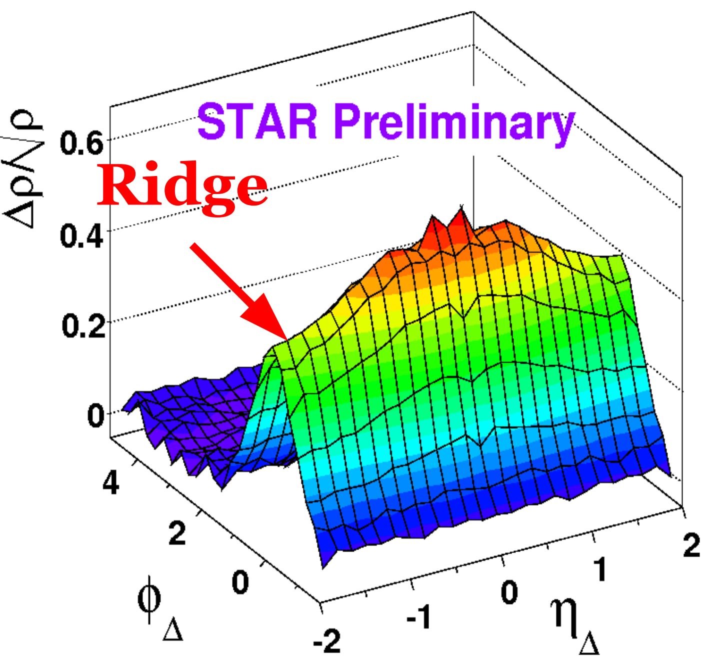


di-jet production in pA and DIS probes quadrupoles

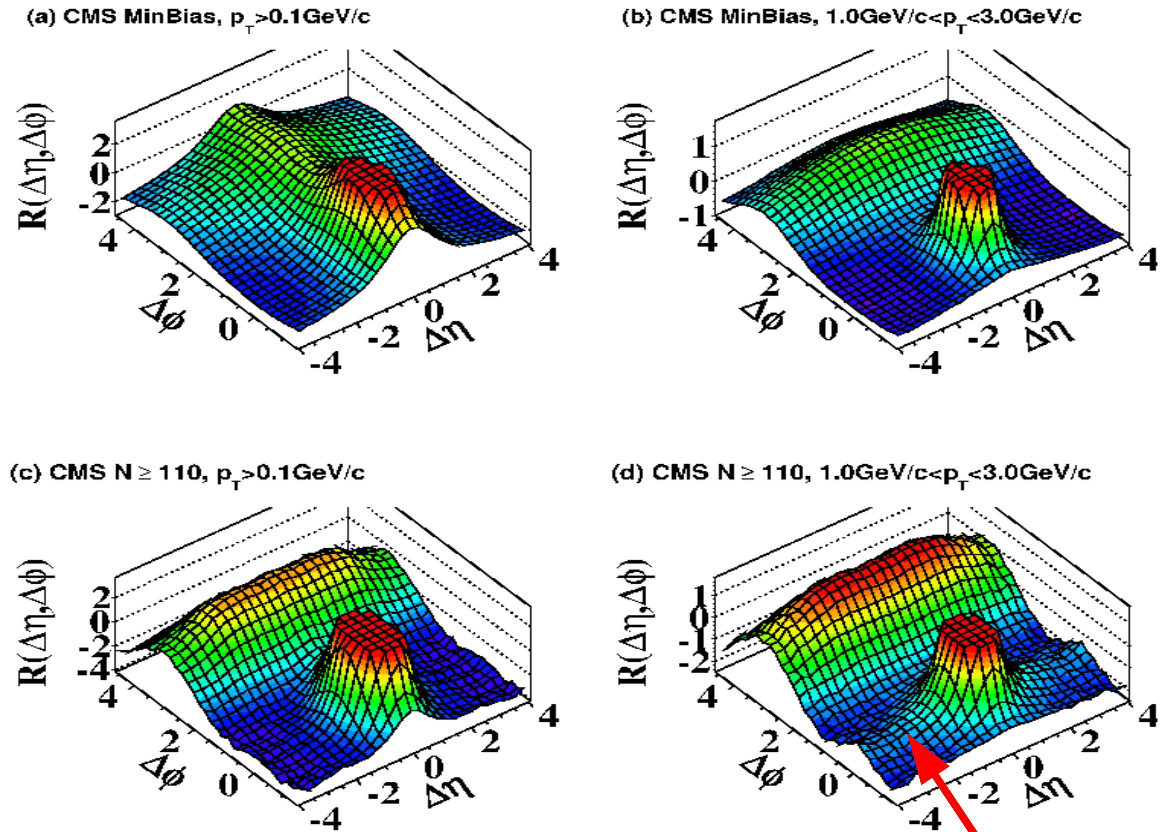
The Ridge

near-side long-range rapidity correlations

The Ridge



AA at RHIC



PP at LHC

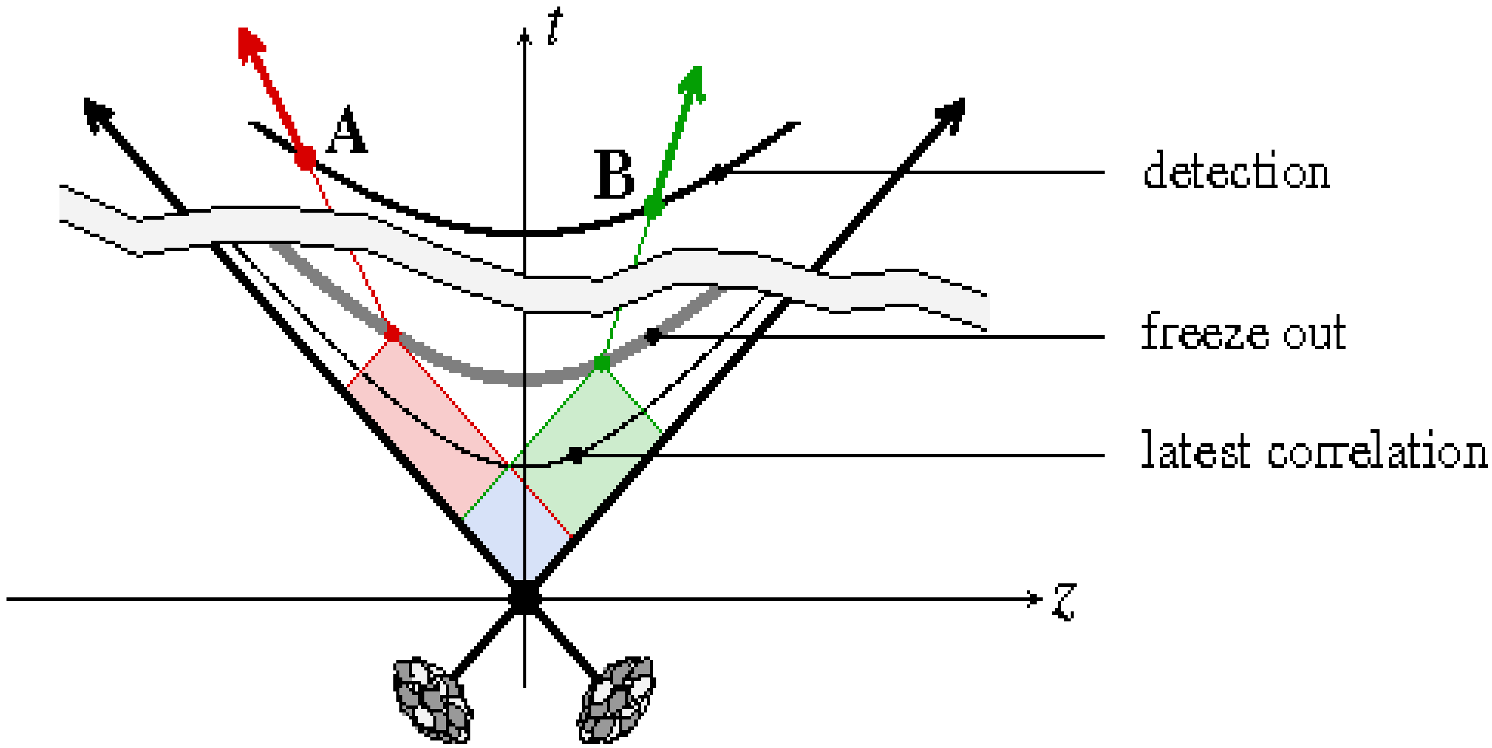
Ridge

long-range rapidity ***correlations***

multi-gluon correlations in the nucleus:

need to go beyond single parton distributions

The Ridge



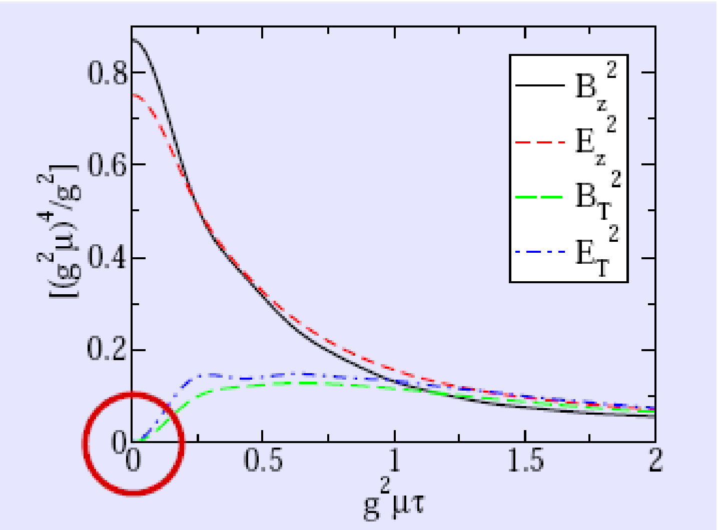
$$\tau \leq \tau_{\mathrm{fo}}\, e^{-\frac{1}{2}|y_A - y_B|}$$

DGMV: NPA810 (2008) 91

late time interactions can not affect long-range rapidity correlations

GLASMA:

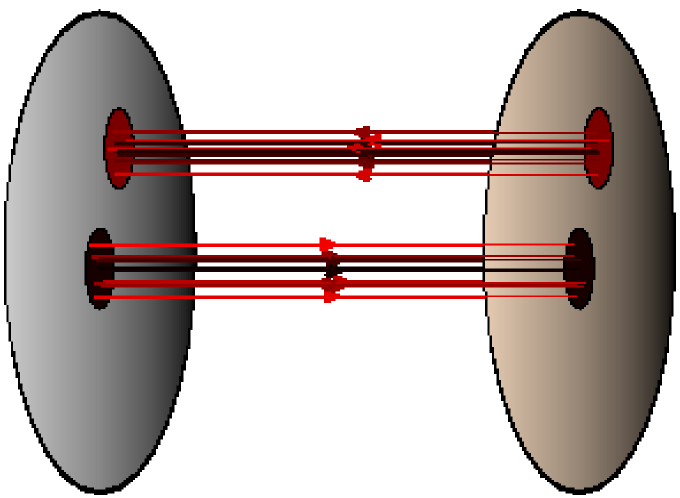
gluon fields produced in collision of two sheets of color glass



Early on glasma fields (E and B) are longitudinal

Lappi+McLerran. NPA772 (2006) 200

Classical solutions are boost invariant



Transverse size of these flux tubes is $\sim \frac{1}{Q_s}$

can be solved numerically

Two-gluon correlation: dilute region

DGMV, NPA810 (2008) 91

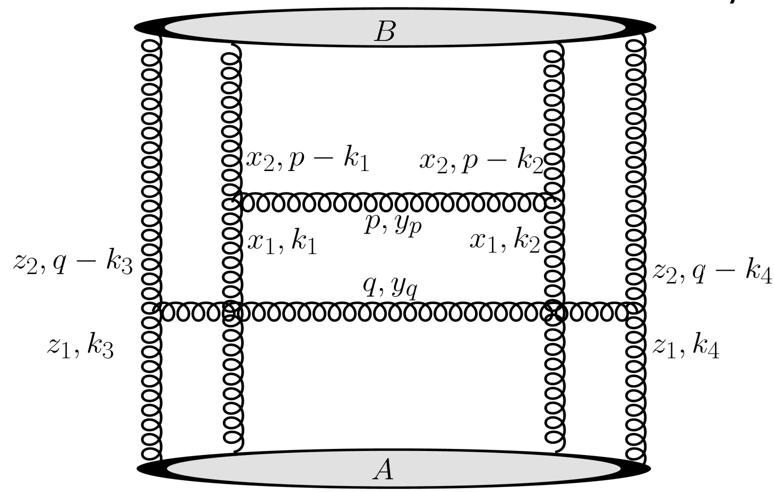
$$C(p_\perp, q_\perp) = \frac{g^{12}}{64(2\pi)^6} \left(f_{abc} f_{a'\bar{b}\bar{c}} f_{a\hat{b}\hat{c}} f_{a'\tilde{b}\tilde{c}}\right) \int \prod_{i=1}^{4} \frac{d^2 k_{i\perp}}{(2\pi)^2 k_{i\perp}^2}$$

$$\times \frac{L_\mu(p_\perp, k_{1\perp}) L^\mu(p_\perp, k_{2\perp})}{(p_\perp - k_{1\perp})^2 (p_\perp - k_{2\perp})^2} \frac{L_\nu(q_\perp, k_{3\perp}) L^\nu(q_\perp, k_{4\perp})}{(q_\perp - k_{3\perp})^2 (q_\perp - k_{4\perp})^2}$$

$$\times \left\langle {\rho^*}_1^{\hat{b}}(k_{2\perp}) {\rho^*}_1^{\tilde{b}}(k_{4\perp}) {\rho_1}^b(k_{1\perp}) {\rho_1}^{\bar{b}}(k_{3\perp}) \right\rangle$$

$$\times \left\langle {\rho^*}_2^{\hat{c}}(p_\perp - k_{2\perp}) {\rho^*}_2^{\tilde{c}}(q_\perp - k_{4\perp}) {\rho_2}^c(p_\perp - k_{1\perp}) {\rho_2}^{\bar{c}}(q_\perp - k_{3\perp}) \right\rangle$$

Gaussian averaging



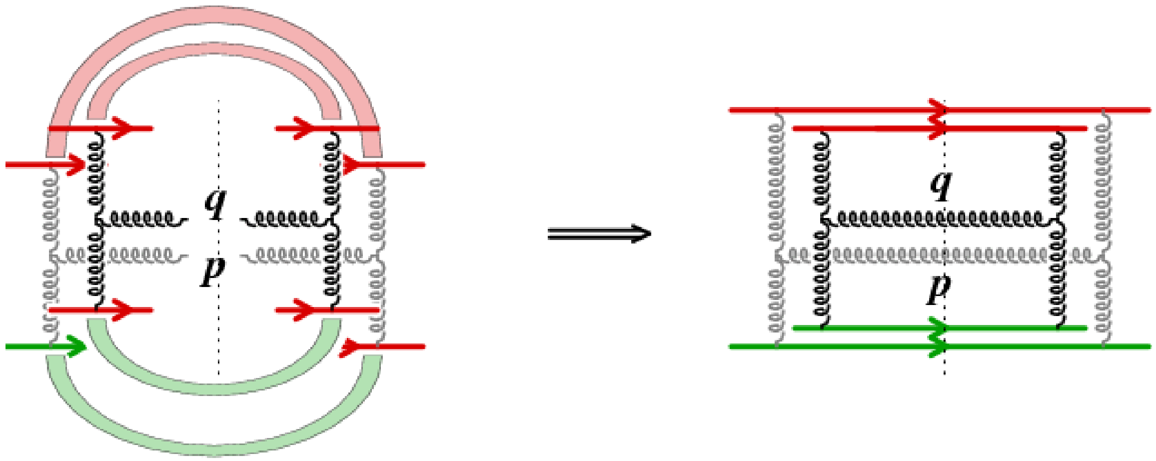
assume *Gaussian factorization*

$$< \rho \cdots\cdots \rho > \sim < \rho^2 > \cdots < \rho^2 >$$

un-integrated gluon distribution

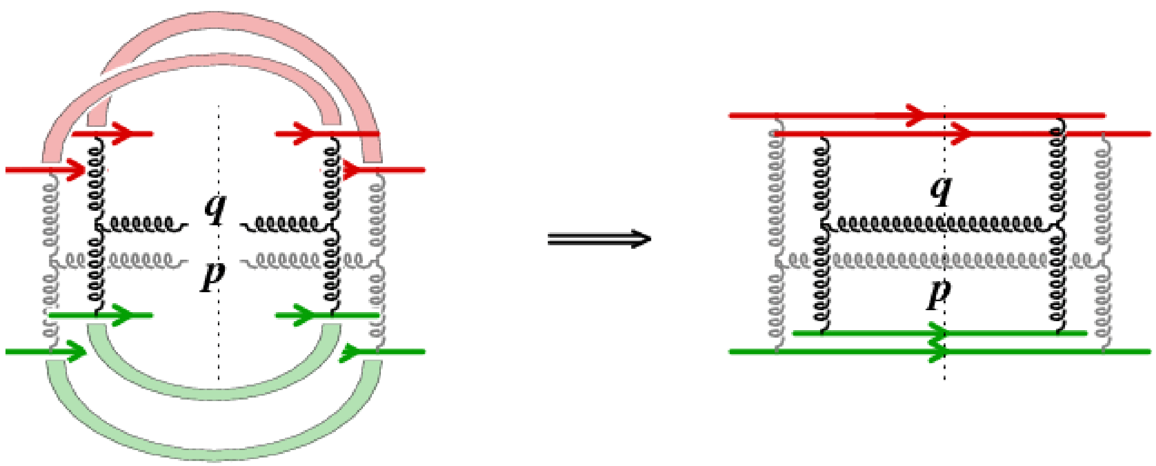
$$\phi(\mathbf{x}, \mathbf{k_t^2}) \sim < \rho^2 >$$

Independent production of two gluons (*subtracted*):



$\sim \mathbf{N_c^2 (N_c^2 - 1)^2}$

Correlated two-gluon production:

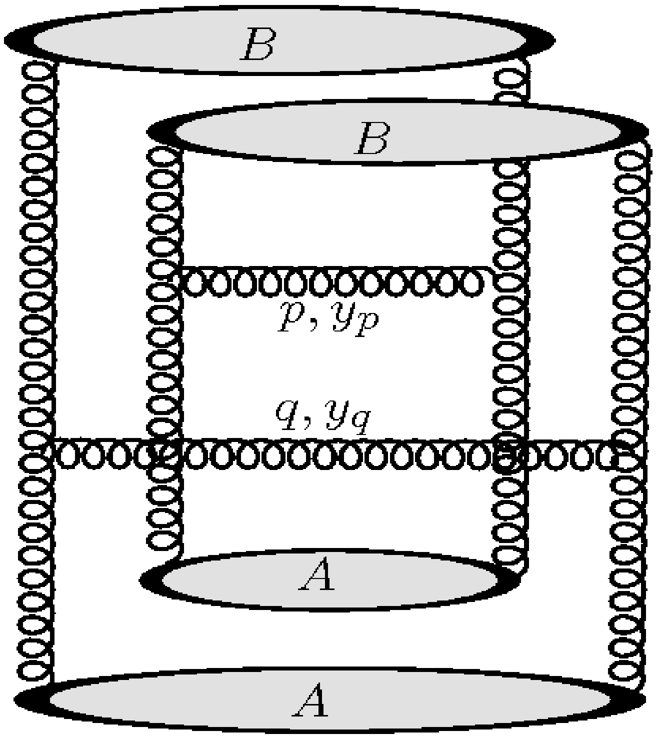


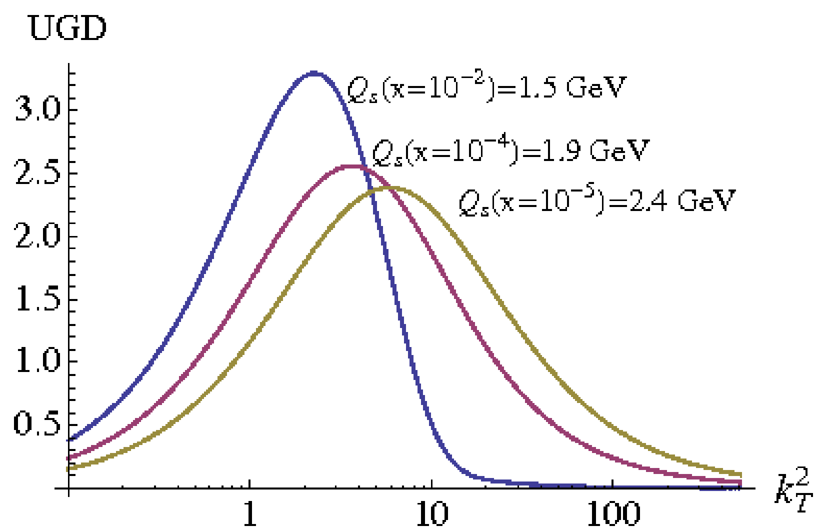
$\sim \mathbf{N_c^2 (N_c^2 - 1)}$

Correlated production is suppressed by N_c^2

Two-gluon production in AA/pp

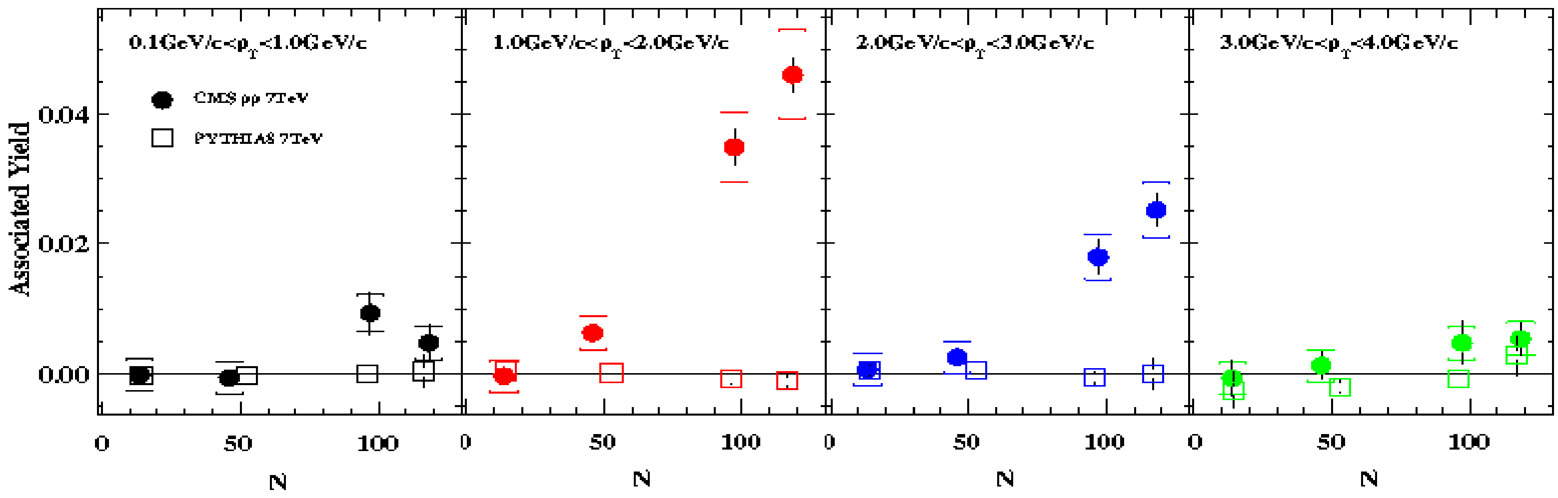
$$\frac{\mathrm{d}N_2}{\mathrm{d}^2\mathbf{p}_\perp \mathrm{d}y_p \mathrm{d}^2\mathbf{q}_\perp \mathrm{d}y_q} = \frac{\alpha_s^2}{16\pi^{10}} \frac{N_c^2 S_\perp}{(N_c^2-1)^3\, \mathbf{p}_\perp^2 \mathbf{q}_\perp^2}$$
$$\times \int \mathrm{d}^2\mathbf{k}_\perp \Big\{ \Phi_A^2(y_p,\mathbf{k}_\perp)\Phi_B(y_p,\mathbf{p}_\perp-\mathbf{k}_\perp)$$
$$\times\left[\Phi_B(y_q,\mathbf{q}_\perp+\mathbf{k}_\perp)+\Phi_B(y_q,\mathbf{q}_\perp-\mathbf{k}_\perp)\right]$$
$$+\Phi_B^2(y_q,\mathbf{k}_\perp)\Phi_A(y_p,\mathbf{p}_\perp-\mathbf{k}_\perp)$$
$$\times\left[\Phi_A(y_q,\mathbf{q}_\perp+\mathbf{k}_\perp)+\Phi_A(y_q,\mathbf{q}_\perp-\mathbf{k}_\perp)\right]\Big\}$$

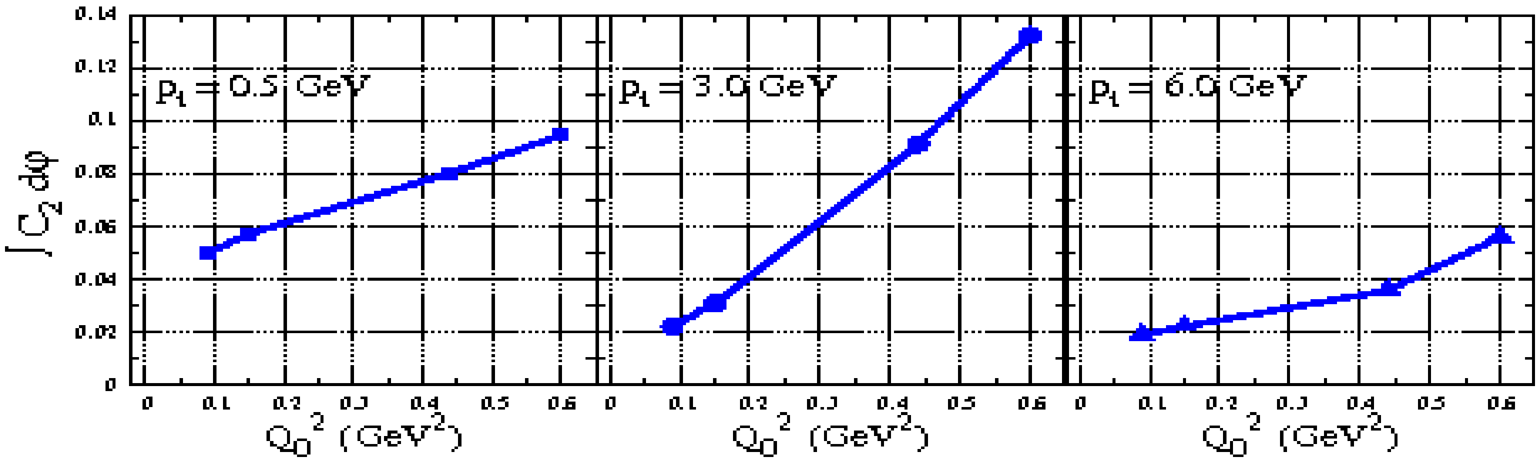




solutions of rcBK
angular collimation

The CMS ridge at LHC





Dumitru et al.,PLB697 (2011) 21

Evolution of gluon 4-pt function

$$\frac{d}{dY}\langle\alpha^a_r\alpha^b_{\bar r}\alpha^c_s\alpha^d_{\bar s}\rangle = \frac{g^2 N_c}{(2\pi)^3}\int d^2z$$

$$\left\langle \frac{\alpha^a_z\alpha^b_{\bar r}\alpha^c_s\alpha^d_{\bar s}}{(r-z)^2} + \frac{\alpha^a_r\alpha^b_z\alpha^c_s\alpha^d_{\bar s}}{(\bar r-z)^2} + \frac{\alpha^a_r\alpha^b_{\bar r}\alpha^c_z\alpha^d_{\bar s}}{(s-z)^2} + \frac{\alpha^a_r\alpha^b_{\bar r}\alpha^c_s\alpha^d_z}{(\bar s-z)^2} - 4\frac{\alpha^a_r\alpha^b_{\bar r}\alpha^c_s\alpha^d_{\bar s}}{z^2}\right\rangle$$

$$+\frac{g^2}{\pi}\int\frac{d^2z}{(2\pi)^2}$$

$$\Bigg\langle f^{e\kappa a}f^{f\kappa b}\frac{(r-z)\cdot(\bar r-z)}{(r-z)^2(\bar r-z)^2}\left[\alpha^e_r\alpha^f_{\bar r} - \alpha^e_r\alpha^f_z - \alpha^e_z\alpha^f_{\bar r} + \alpha^e_z\alpha^f_z\right]\alpha^c_s\alpha^d_{\bar s}$$

$$+f^{e\kappa a}f^{f\kappa c}\frac{(r-z)\cdot(s-z)}{(r-z)^2(s-z)^2}\left[\alpha^e_r\alpha^f_s - \alpha^e_r\alpha^f_z - \alpha^e_z\alpha^f_s + \alpha^e_z\alpha^f_z\right]\alpha^b_{\bar r}\alpha^d_{\bar s}$$

$$+f^{e\kappa a}f^{f\kappa d}\frac{(r-z)\cdot(\bar s-z)}{(r-z)^2(\bar s-z)^2}\left[\alpha^e_r\alpha^f_{\bar s} - \alpha^e_r\alpha^f_z - \alpha^e_z\alpha^f_{\bar s} + \alpha^e_z\alpha^f_z\right]\alpha^b_{\bar r}\alpha^c_s$$

$$+f^{e\kappa b}f^{f\kappa c}\frac{(\bar r-z)\cdot(s-z)}{(\bar r-z)^2(s-z)^2}\left[\alpha^e_{\bar r}\alpha^f_s - \alpha^e_{\bar r}\alpha^f_z - \alpha^e_z\alpha^f_s + \alpha^e_z\alpha^f_z\right]\alpha^a_r\alpha^d_{\bar s}$$

$$+f^{e\kappa b}f^{f\kappa d}\frac{(\bar r-z)\cdot(\bar s-z)}{(\bar r-z)^2(\bar s-z)^2}\left[\alpha^e_{\bar r}\alpha^f_{\bar s} - \alpha^e_{\bar r}\alpha^f_z - \alpha^e_z\alpha^f_{\bar s} + \alpha^e_z\alpha^f_z\right]\alpha^a_r\alpha^c_s$$

$$+f^{e\kappa c}f^{f\kappa d}\frac{(s-z)\cdot(\bar s-z)}{(s-z)^2(\bar s-z)^2}\left[\alpha^e_s\alpha^f_{\bar s} - \alpha^e_s\alpha^f_z - \alpha^e_z\alpha^f_{\bar s} + \alpha^e_z\alpha^f_z\right]\alpha^a_r\alpha^b_{\bar r}\Bigg\rangle$$

Gaussian factorization breaks down!

AD-JJM, PRD81:094015 (2010)

The role of initial conditions

McLerran-Venugopalan (93)

$$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2 A}}{\mathbf{S}_\perp}$$

$$\mathbf{S}(\mathbf{y_t}, \mathbf{z_t}) \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\, \mathbf{V}^\dagger_{\mathbf{y}}\, \mathbf{V_z} > \sim \mathbf{e}^{-\#\,(\mathbf{y_t}-\mathbf{z_t})^{\mathbf{2}}\, \mathbf{Q_s^2}}$$

how about higher order terms in ρ ?

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \left[\frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}} - \frac{\mathbf{d^{abc}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})}{\kappa_{\mathbf{3}}} + \frac{\mathbf{F^{abcd}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})\rho^{\mathbf{d}}(\mathbf{x_t})}{\kappa_{\mathbf{4}}} \right]}$$

these higher order terms may make the single inclusive spectra steeper and give leading N_c correlations (ridge)

AD-JJM-EP, PRD84 (2011)

Two-hadron correlations

qualitative **agreement with CGC expectations/predictions**

A *quantitative* description of two-hadron correlation requires going beyond dipole approximation

Photon-Hadron correlations:dA

another process to test CGC formalism

less inclusive than single inclusive particle production

one less hadron fragmentation function

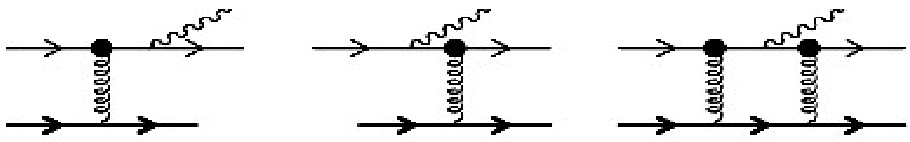
theoretically cleaner: 2-point function only

lower rates compared to two hadron production

photons are hard to measure

will help distinguish between different approaches

$$\mathbf{q(p)\,T \rightarrow q(q)\,\gamma(k)\,X}$$



$$\frac{d\sigma^{d\,A\rightarrow h\,\gamma\,X}}{d^2b_t\,dq_t^2\,dk_t^2\,dy_\gamma\,dy_h\,d\theta} = a\int_{z_{min}}^{1}\frac{dz}{z^5}\,f_{q/d}(x_p, Q^2)$$

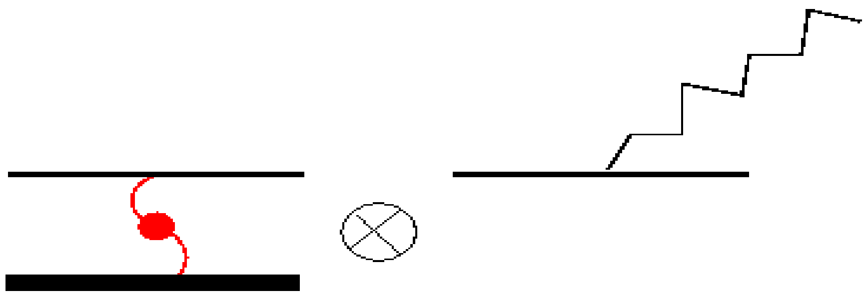
$$D_{h/q}(z, Q^2)[z^2 + (\frac{q^-}{q^- + zk^-})^2]\,\frac{(\tilde{q}_t + z\tilde{k}_t)^2}{(k^-\tilde{q}_t - q^-\tilde{k}_t)^2}\,N_F(|\tilde{q}_t/z + \tilde{k}_t|)$$

FG-JJM, PRD66 (2002) 014021
JJM, EPJC61 (2009) 789

Kopeliovich et al.,
Rezaeian 2010

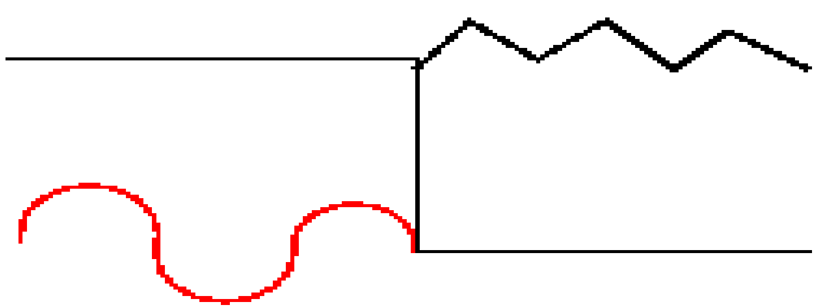
pQCD limit

near side: collinear divergence $\theta \rightarrow 0$



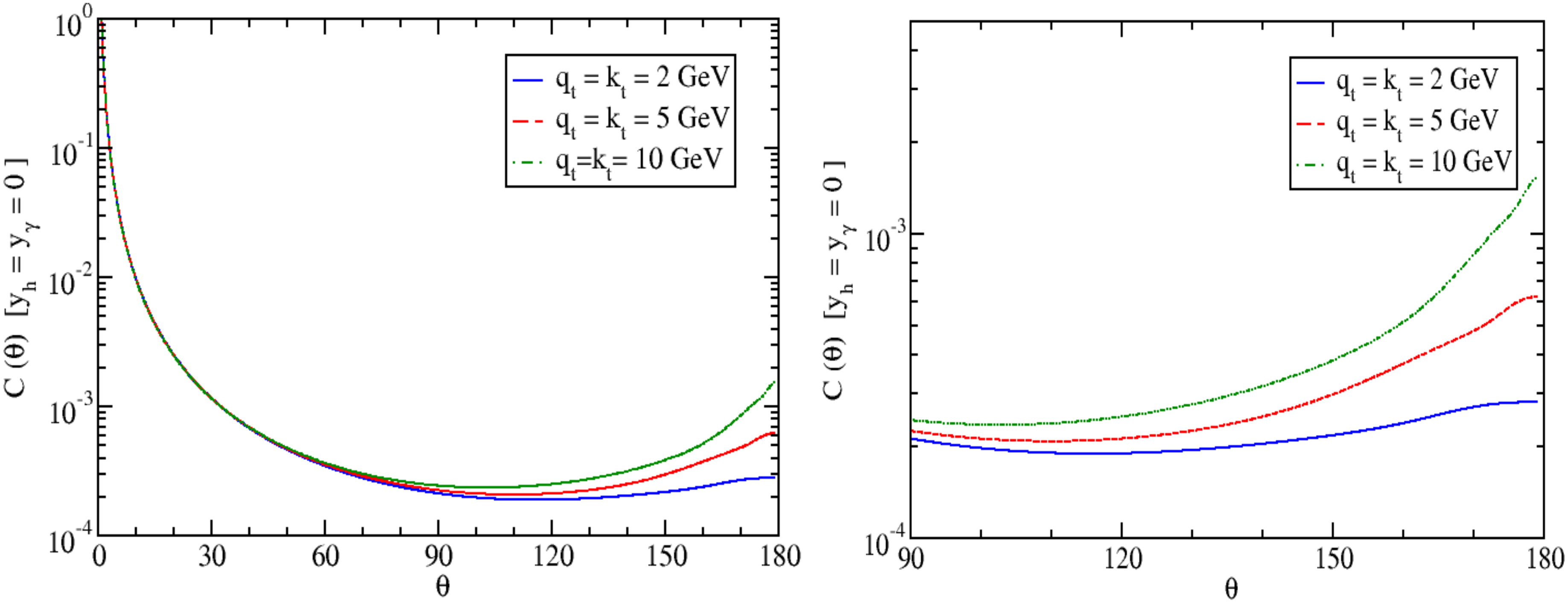
$$N_F \otimes D_{\gamma/q}$$

away side: $\theta \rightarrow \pi$



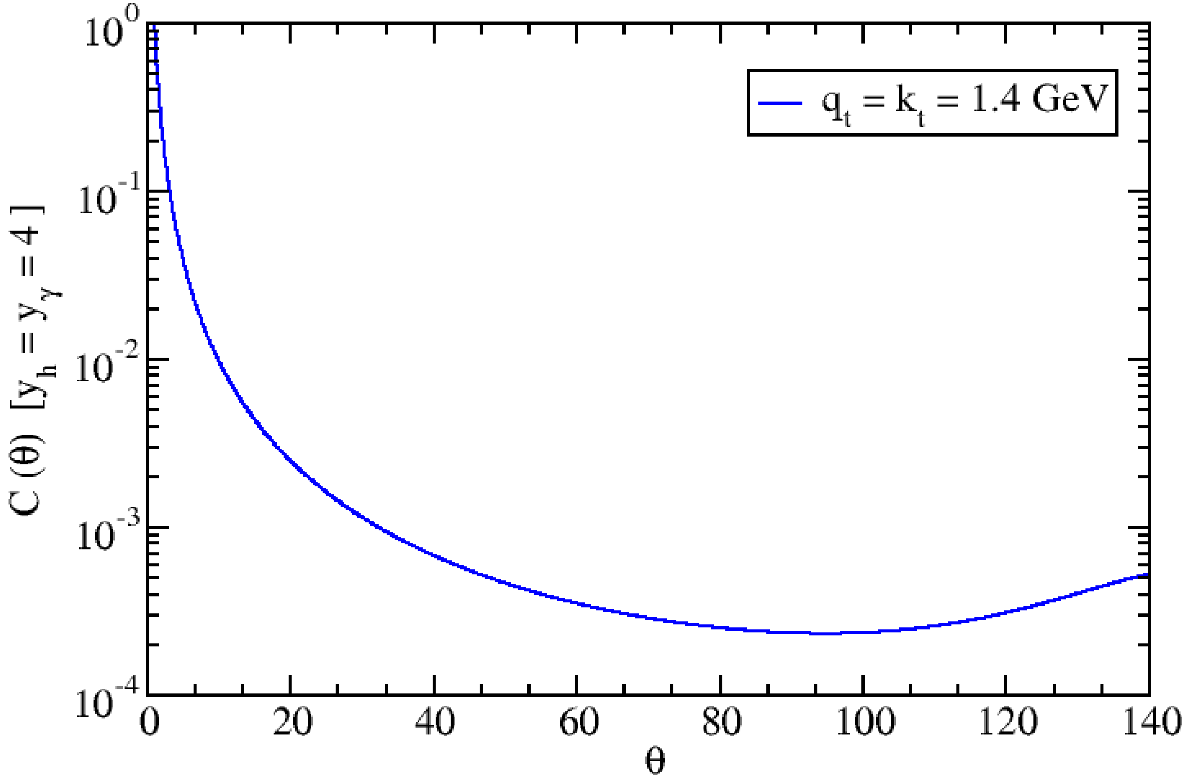
$$p_t >> Q_s$$

Photon-Hadron correlations:dA



$$\mathbf{C}(\theta) \equiv \frac{\mathbf{d}\sigma}{\mathbf{dq_t^2\, dk_t^2\, dy_h\, dy_\gamma\, d}\theta} \Bigg/ \int \mathbf{d}\theta \frac{\mathbf{d}\sigma}{\mathbf{dq_t^2\, dk_t^2\, dy_h\, dy_\gamma\, d}\theta}$$

Photon-Hadron correlations:dA



$$\mathbf{C}(\theta) \equiv \frac{\mathbf{d}\sigma}{\mathbf{dq_t^2\, dk_t^2\, dy_h\, dy_\gamma\, d}\theta} \Bigg/ \int \mathbf{d}\theta \frac{\mathbf{d}\sigma}{\mathbf{dq_t^2\, dk_t^2\, dy_h\, dy_\gamma\, d}\theta}$$