Amplitudes in $D \neq 4$

Yu-tin Huang

UCLA

(Z.Bern, J. J. Carrasco, W-M Cheng, B. Czech, T. Dennen, D. Gang, H. Ita, E. Koh, S. Lee, A. Lipstein, M. Rozali, W. Siegel)

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$$\lambda^{\text{(Lorentz)}}_{\text{(Little)}}$$

$$P = \lambda\lambda \quad \text{Lorentz} \times \text{Little} - \text{LittleGroup} = D - 1$$

$$D = 3, 4, 6$$

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$$D = 3, 4, 6$$

1. 3-D Chern-Simons matter theory (ABJM)

2. D=6
   - D=6 Maximal SYM
   - $\mathcal{N} = (2, 0)$ Self-Dual Tensor

3D spinor helicity

λ^{α} $\longrightarrow$ SL(2R)

Z_2

Field Content

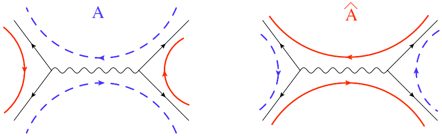
Aharony, Bergman, Jafferis, and Maldacena (ABJM)
Gauge fields:

$$(A)^{a}{}_{b}, \;\; (\hat{A})^{\hat{a}}{}_{\hat{b}} \in \; \mathrm{SU(N)} \times \mathrm{SU(N)}$$

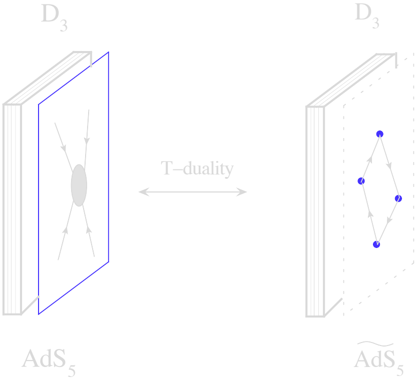
$$\mathcal{L}_{CS} = A \wedge dA + \frac{1}{3} A \wedge A \wedge A$$

Matter fields:

$$(\phi^{I}, \psi^{I})^{a}{}_{\hat{a}}, \;\; (\bar{\phi}_{I}, \bar{\psi}_{I})^{\hat{a}}{}_{b}, \quad I \in \mathrm{SU(4)}$$



- Non-maximal, $\mathcal{N} = 6$
- SCFT $\mathrm{OSp}(6|4)\left\{\begin{array}{l}\mathrm{SO}(6) \sim \mathrm{SU}(4) \text{ R symmetry} \\ \mathrm{Sp}(4) \text{ Conformal symmetry}\end{array}\right.$
- AdS/CFT: IIA String theory $\mathrm{AdS}_4\times$ CP_3/ABJM



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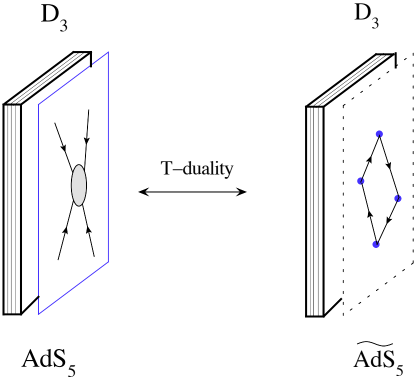

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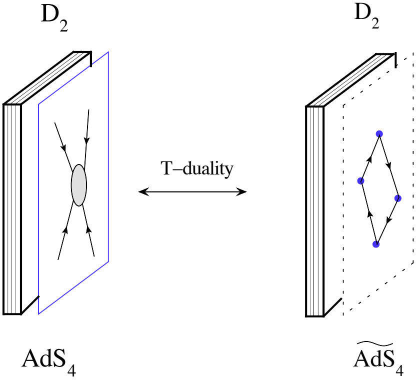

T-duality invariant?

NO I. Adam, A. Dekel, Y. Oz (09)(10), P. A. Grassi, D. Sorokin, L. Wulff,(09), I. Bakhmatov (10),

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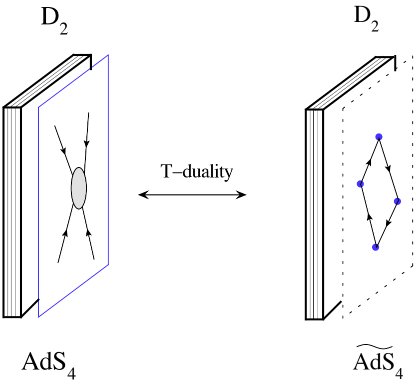

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The On-shell Variables

Three-dimensional kinematics

$$\mathrm{SL}(2,\mathrm{R}): \quad p_i^{(\alpha\beta)} = \lambda_i^\alpha \lambda_i^\beta$$

$$\langle ij \rangle \equiv \lambda_i^\alpha \lambda_{j\alpha}, \quad \langle ij \rangle^2 = -2 p_i \cdot p_j$$

Supersymmetry: $\mathcal{N} = 6, \quad \eta^I,\ I \in SU(3)$

$$\mathcal{A}_n(\lambda_i^\alpha, \eta^I)$$

$$\Phi(\eta) = \phi^4 + \eta^I \psi_I + \frac{1}{2}\epsilon_{IJK}\eta^I\eta^J\phi^K + \frac{1}{3!}\epsilon_{IJK}\eta^I\eta^J\eta^K\psi_4$$

$$\Psi(\eta) = \bar\psi^4 + \eta^I \bar\phi_I + \frac{1}{2}\epsilon_{IJK}\eta^I\eta^J\bar\psi^K + \frac{1}{3!}\epsilon_{IJK}\eta^I\eta^J\eta^K\bar\phi_4$$

General n-point amplitude

- $\mathcal{A}_n$: n=even
- $\mathcal{A}_n$: R-symmetry invariance$\rightarrow$ $\eta^{\frac{3}{2}n}$
- $\mathcal{A}_n$: Supersymmetry invariance.
  $\rightarrow \mathcal{A}_n = \delta^3(P)\delta^3(Q^\alpha)\delta^3(Q_\alpha)f_n(\lambda,\eta)$

$$\mathcal{A}_4 = \frac{\delta^3(P)\delta^3(Q^\alpha)\delta^3(Q_\alpha)}{\langle 12\rangle\langle 23\rangle}, \quad f_4 = \frac{1}{\langle 12\rangle\langle 23\rangle}$$

Is it Dual conformal?

$$p_i = x_i - x_{i+1}, \qquad I[x_i] = \frac{x_i}{x_i^2}$$

$$\rightarrow I[f_4] = \sqrt{x_1^2 x_2^2 x_3^2 x_4^2}\, f_4$$

Explicit computation $\mathcal{A}_4$ $\mathcal{A}_6$ $\rightarrow$ OSp(6|4) Yangian invariance [T. Bargheer, F. Loebert, C. Meneghelli (10)]

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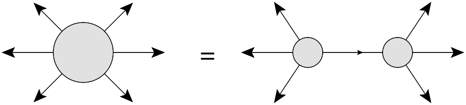

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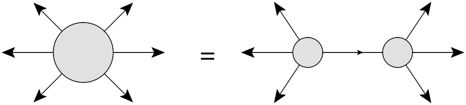

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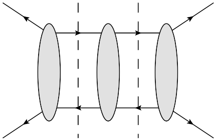

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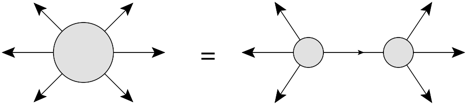

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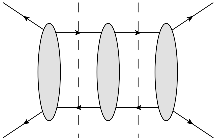

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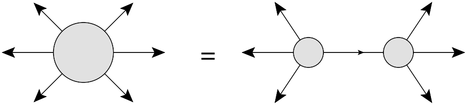

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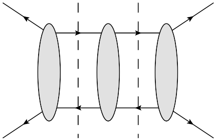

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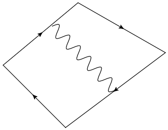
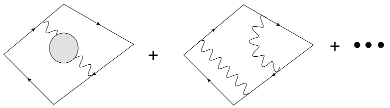
ABJM Wilson-loops

1-loop:



$\langle W_4^1 \rangle = 0$

2-loop:



$$\langle W_4^2 \rangle = 1 - \left(\frac{N}{K}\right)^2 \left[\frac{(-\mu'^2 x_{13}^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{(-\mu'^2 x_{24}^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{1}{2}\log^2\left(\frac{-x_{13}^2}{-x_{24}^2}\right) + const + \mathcal{O}(\epsilon)\right]$$

N=4 SYM

$$\langle W_4^1 \rangle = 1 + \left(\frac{g^2 N}{8\pi^2}\right)^2 \left[\frac{(-\mu'^2 x_{13}^2)^{-\epsilon}}{(\epsilon)^2} + \frac{(-\mu'^2 x_{24}^2)^{-\epsilon}}{(\epsilon)^2} - \frac{1}{2}\log^2\left(\frac{-x_{13}^2}{-x_{24}^2}\right) + const + \mathcal{O}(\epsilon)\right]$$

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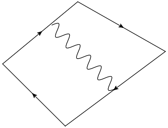

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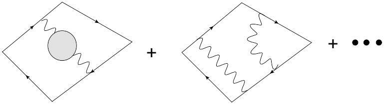
2-loop:



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Amplitude(W-M Chen, Y. Huang [11])

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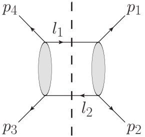
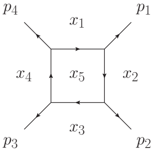


$= 0 \quad ?$

$$I = \frac{2x_{51}^2 \epsilon_{\mu\nu\rho} x_{21}^{\mu} x_{31}^{\nu} x_{41}^{\rho} + 2x_{31}^2 \epsilon_{\mu\nu\rho} x_{51}^{\mu} x_{21}^{\nu} x_{41}^{\rho}}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}$$



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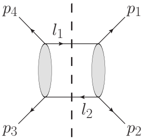

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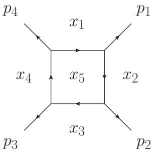

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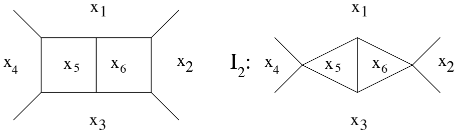
2-loop:

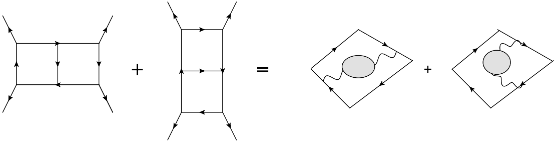
I1: I2:

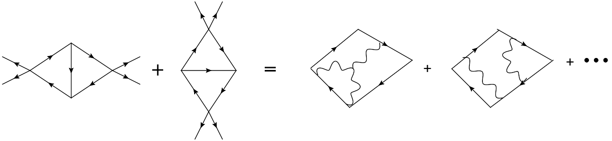


$$I_1 = \frac{4\xi_s}{x_{51}^2 x_{53}^2 x_{54}^2 x_{56}^2 x_{61}^2 x_{62}^2 x_{63}^2 x_{24}^2}$$

$$I_2 = \frac{x_{13}^4}{x_{51}^2 x_{53}^2 x_{56}^2 x_{61}^2 x_{63}^2}$$

$$\xi_s \equiv \epsilon_{\mu\nu\rho}(x_{51}^2 x_{21}^{\mu} x_{31}^{\nu} x_{41}^{\rho} + x_{31}^2 x_{51}^{\mu} x_{21}^{\nu} x_{41}^{\rho}) \epsilon_{\gamma\sigma\eta}(x_{61}^2 x_{21}^{\gamma} x_{31}^{\sigma} x_{41}^{\eta} + x_{31}^2 x_{61}^{\gamma} x_{21}^{\sigma} x_{41}^{\eta})$$





ABJM: $\mathcal{A}_4^{2-loop} =$

$$\frac{1}{16\pi^2}\left(\frac{N}{K}\right)^2 \mathcal{A}_4^{tree}\left[-\frac{(-s/\tilde{\mu}^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{(-t/\tilde{\mu}^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{1}{2}\log^2\left(\frac{-s}{-t}\right) + a + \mathcal{O}(\epsilon)\right]$$

$\mathcal{N} = 4 \quad \mathcal{A}_4^{1-loop} =$

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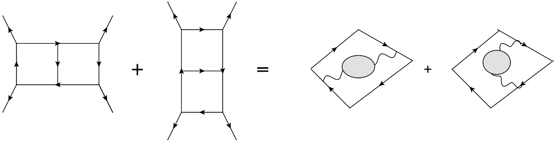

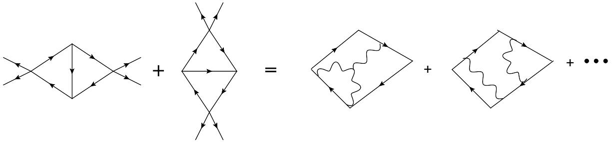

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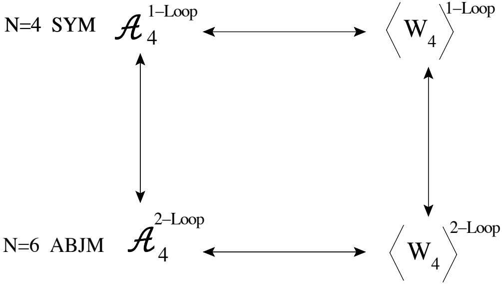
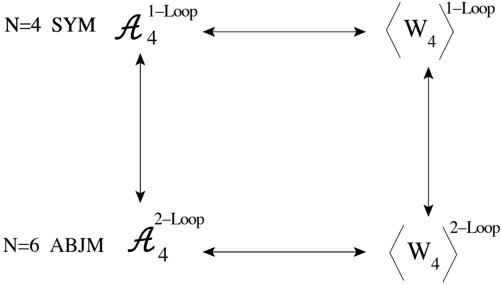


(1) Higher loops ? (BDS)

(2) Higher points ? (N^nMHV)



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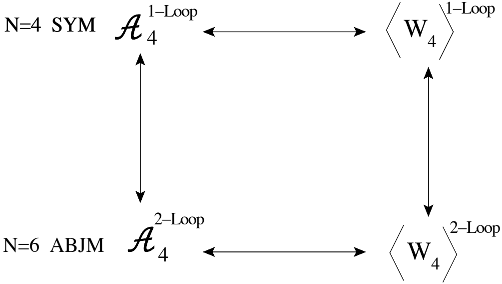

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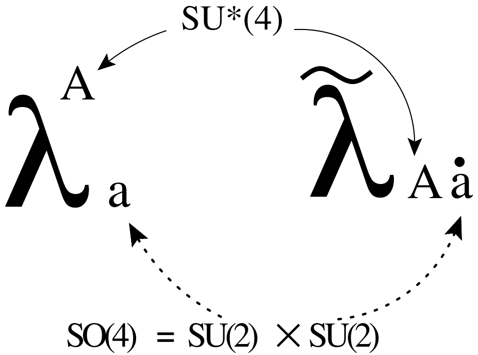
SU*(4)
$\lambda^{A}{}_{a}$
$\tilde{\lambda}_{A\dot{a}}$
SO(4) = SU(2) × SU(2)

$\mathcal{N}=(1,1)$ SYM [J. J. Carrasco, T. Dennen, Y. Huang, H. Ita, W.Siegel, (10)(11)]

$$\mathcal{A}_4 = \delta^6\left(\sum_{i=1}^{4} p_i\right)\delta^4\left(\sum_{i=1}^{4} q_i^A\right)\delta^4\left(\sum_{i=1}^{4}\tilde{q}_{iA}\right)\frac{i}{st}$$

$$f_4 = \frac{i}{st} = \frac{i}{x_{13}^2 x_{24}^2}$$

$$I[f_4] = x_1^2 x_2^2 x_3^2 x_4^2 f_4$$

$$\rightarrow \quad I[f_n] = \prod_i^n x_i^2 f_n$$

$$\rightarrow I[I_n^L] = \left(x_{\ell_1}^2 \cdots x_{\ell_L}^2\right)^4 \prod_i^n x_i^2 I_n^L$$

$\mathcal{N} = (1,1)$ SYM [J. J. Carrasco, T. Dennen, Y. Huang, H. Ita, W.Siegel, (10)(11)]

$$\mathcal{A}_4 = \delta^6\left(\sum_{i=1}^{4} p_i\right)\delta^4\left(\sum_{i=1}^{4} q_i^A\right)\delta^4\left(\sum_{i=1}^{4} \tilde{q}_{iA}\right)\frac{i}{st}$$

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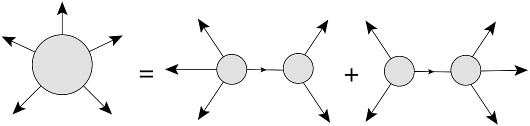

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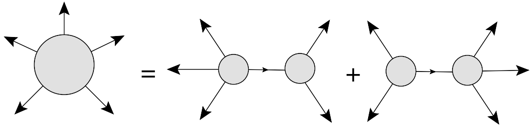

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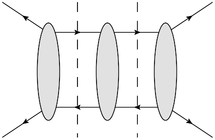

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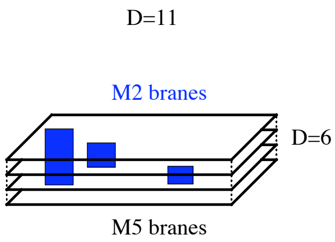

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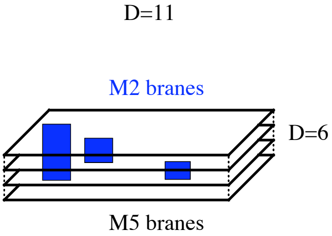

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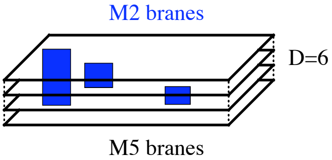

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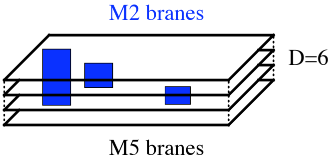

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3-point kinematics

4D: $s_{ij} = \langle ij\rangle[ji] = 0 \rightarrow \begin{array}{l} [ij]=0, \ \langle ij\rangle \neq 0 \\ \langle ij\rangle = 0, \ [ij]\neq 0 \end{array}$

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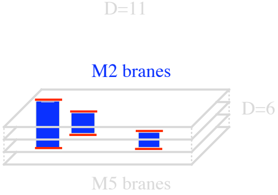
The search for $\mathcal{A}_3$(I)

- $\mathcal{N} = (2,0),(1,0)$ SUSY → No pure tensor-interaction Wrong degrees of freedom?



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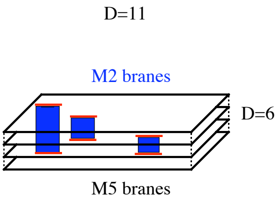

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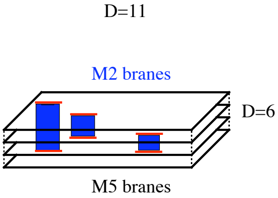

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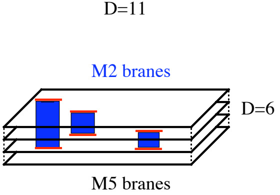

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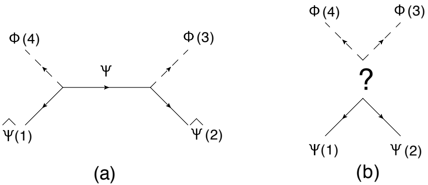
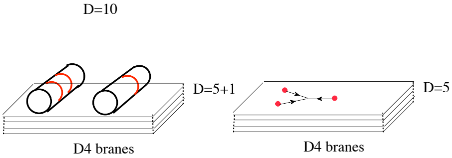


The search for $\mathcal{A}_3$(II)

Lets go to D=5: SU(4)→USp(2,2)



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- The only pure $B_{\mu\nu}$ interaction, requires one leg massless
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- BCFW recursion $\mathcal{A}_4 = \frac{\delta^4(Q)\delta^4(\hat{Q})}{st} \rightarrow f^{a[12}f_a^{3]4} = 0$(Jacobi)
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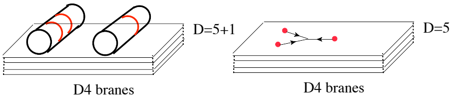

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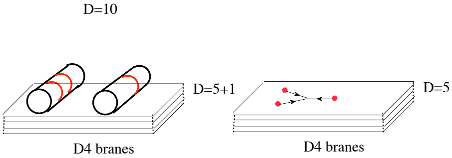

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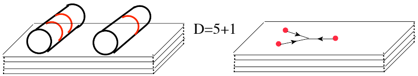

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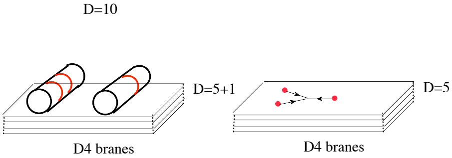

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Conclusion

- Dual superconformal symmetry of ABJM
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- Dual conformal symmetry of non-conformal theory (6D SYM)
- Gone where no action has gone before. First $\mathcal{A}_3, \mathcal{A}_4$ for M-theory.

Conclusion

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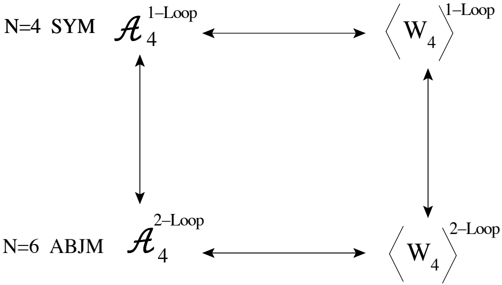
- 

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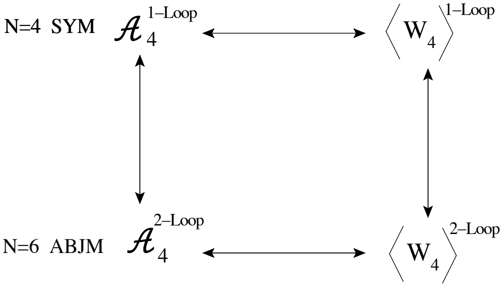
- 

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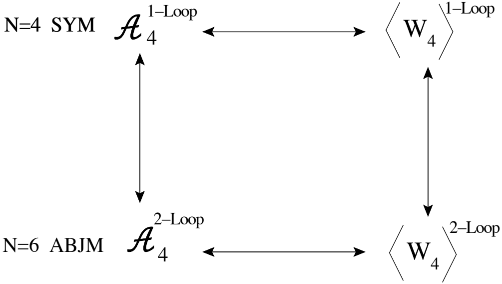
- 

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