Factorization in high energy nucleus-nucleus collisions

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Outline

1. **Color Glass Condensate**
2. **Factorization in Deep Inelastic Scattering**
3. **Factorization in nucleus-nucleus collisions**
4. **Extensions**

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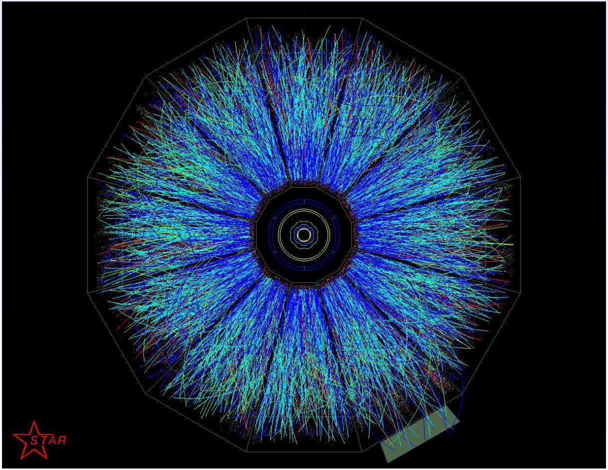
Heavy ion collision



The candid approach...

QCD Lagrangian

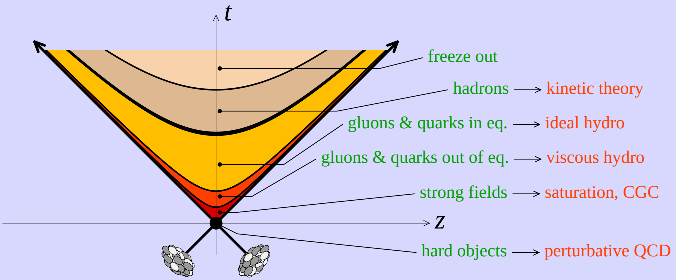
$$\mathcal{L} = -\frac{1}{4}F^2 + \sum_{\text{flavors}} \overline{\psi}_f (i\not{D} - m_f)\psi_f$$

- Free parameters : quark masses m_f, confinement scale Λ_{QCD}

- As my string theory colleagues would put it :

> *"Since you know the Lagrangian,*
> *what is your problem exactly?"*

The multiple facets of QCD in heavy ion collisions



- Except for the production of hard objects (jets, heavy quarks, direct photons) at the impact of the two nuclei, we have to deal with strong interactions in a non-perturbative regime
  NOTE: non-perturbative $\neq$ strongly coupled!!!
- One treats these situations with a range of effective descriptions (CGC, hydrodynamics, kinetic theory) that are more or less closely related to QCD, but always require some QCD input

1 Color Glass Condensate

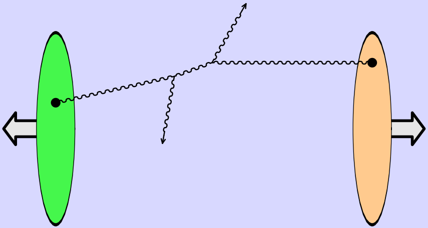
Small-x gluons matter

Gluon saturation

Saturation domain

Color Glass Condensate

Longitudinal momentum fraction in AA collisions

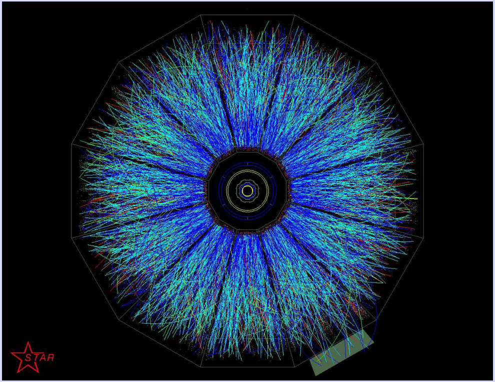


- The partons that are relevant for the process under consideration carry the longitudinal momentum fractions:

$$x_{1,2} = \frac{P_\perp}{\sqrt{s}} e^{\pm Y}$$

  - $P_\perp$: transverse momentum
  - Y : rapidity
  - $\sqrt{s}$: collision energy

Longitudinal momentum fraction in AA collisions

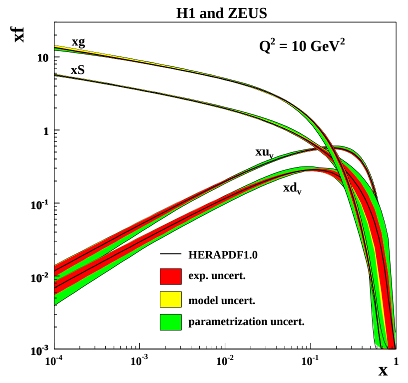


- 99% of the produced particles below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

  ▷ partons at small x are the most important

Growth of the gluon distribution at small x

Parton distributions at small x



- Gluons dominate at any $x \le 10^{-1}$

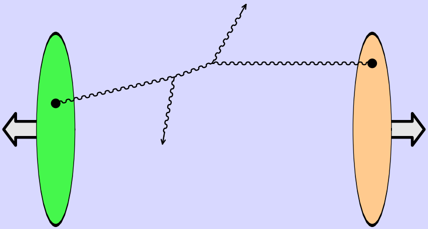
❶ Color Glass Condensate

- Small-x gluons matter
- Gluon saturation
- Saturation domain
- Color Glass Condensate

Multiple scatterings and gluon recombination

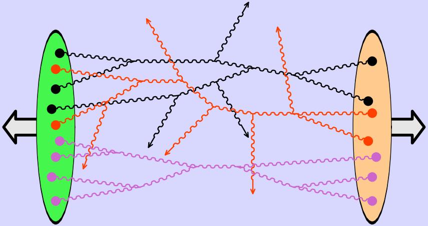
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ large Q^2, no small-x effects
  - ▷ single parton distributions + DGLAP evolution

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial
  - ▷ gluon recombinations are important (saturation)
  - ▷ multi-parton distributions + JIMWLK evolution
  - ▷ new techniques are required (Color Glass Condensate):

$$\mathcal{L} = -\frac{1}{4}F^2 + J \cdot A$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

❶ Color Glass Condensate

- Small-x gluons matter
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- Color Glass Condensate

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

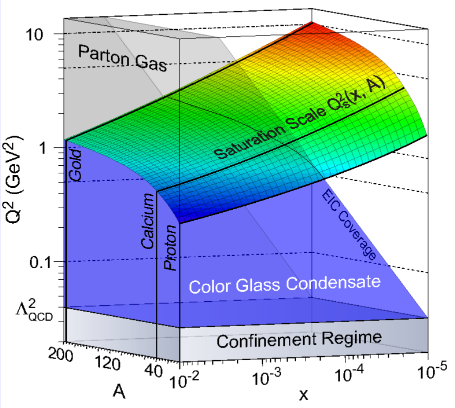
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Saturation domain

Saturation scale as a function of *x* and *A*



❶ Color Glass Condensate

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CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\rho(x^-,\vec{\boldsymbol{x}}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_{\Lambda^+}[\rho]$
- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ must be treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $A_\mu J^\mu$

CGC: renormalization group evolution

Independence w.r.t $\Lambda^+ \to$ evolution equation (JIMWLK) :

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H}\ \ W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int\limits_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta\alpha(\vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta\alpha(\vec{x}_\perp)}$$

where $-\partial_\perp^2\, \alpha(\vec{x}_\perp) = \rho(1/\Lambda^+, \vec{x}_\perp)$

- $\eta(\vec{x}_\perp, \vec{y}_\perp)$ is a non-linear functional of ρ
- Resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- Simplifies into the BFKL equation when the source ρ is small (expand η in powers of ρ)

❷ Factorization in Deep Inelastic Scattering

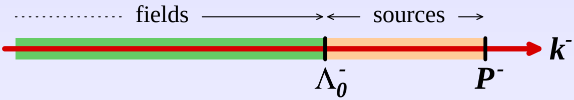
Leading Order

Next to Leading Order

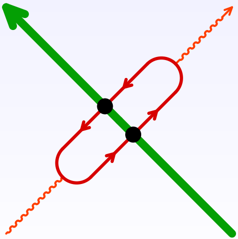
Leading Log resummation

Inclusive DIS at Leading Order

- CGC effective theory with cutoff at the scale Λ_0^- :



- At Leading Order, DIS can be seen as the interaction between the target and a $q\overline{q}$ fluctuation of the virtual photon :



Inclusive DIS at Leading Order

- Forward dipole amplitude at leading order:

$$T_{\mathrm{LO}}(\vec{x}_\perp,\vec{y}_\perp) = 1-\frac{1}{N_c}\,\mathrm{tr}\,(\underbrace{U(\vec{x}_\perp)U^\dagger(\vec{y}_\perp)}_{\text{Wilson lines}})$$

$$U(\vec{x}_\perp) = \mathrm{P}\,\exp i g\int^{1/xP^-} dz^+\,\mathcal{A}^-(z^+,\vec{x}_\perp)$$

$$[\mathcal{D}_\mu,\mathcal{F}^{\mu\nu}] = \delta^{\nu-}\,\rho(x^+,\vec{x}_\perp)$$

▷ at LO, the scattering amplitude on a saturated target is entirely given by classical fields

- Note: the $q\overline{q}$ pair couples only to the sources up to the longitudinal coordinate $z^+ \lesssim (xP^-)^{-1}$. The other sources are too slow to be seen by the probe

❷ Factorization in Deep Inelastic Scattering

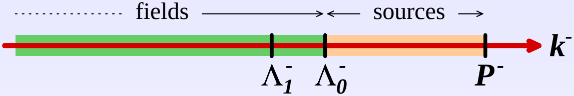
Leading Order

Next to Leading Order

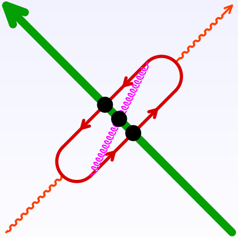
Leading Log resummation

Inclusive DIS at NLO

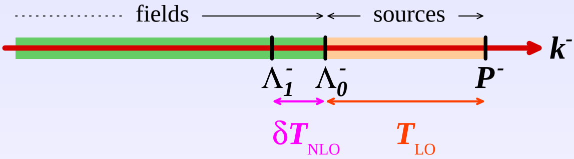
- Consider now quantum corrections to the previous result, restricted to modes with $\Lambda_1^- < k^- < \Lambda_0^-$ (the upper bound prevents double-counting with the sources):



- At NLO, the $q\overline{q}$ dipole must be corrected by a gluon, e.g. :



Inclusive DIS at NLO



- At leading log accuracy, the contribution of the quantum modes in that strip is :

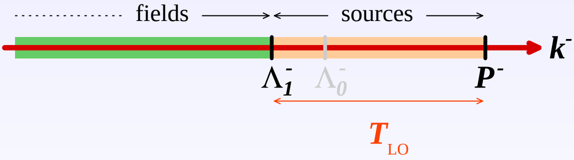
$$\delta T_{\rm NLO}(\vec{x}_\perp,\vec{y}_\perp) = \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\,\mathcal{H}\;T_{\rm LO}(\vec{x}_\perp,\vec{y}_\perp)$$

Inclusive DIS at NLO

- These NLO corrections can be absorbed in the LO result,

$$\left\langle \boldsymbol{T}_{\rm LO} + \delta\boldsymbol{T}_{\rm NLO} \right\rangle_{\Lambda_0^-} = \left\langle \boldsymbol{T}_{\rm LO} \right\rangle_{\Lambda_1^-}$$

provided one defines a new effective theory with a lower cutoff Λ_1^- and an extended distribution of sources $W_{\Lambda_1^-}[\rho]$:



$$W_{\Lambda_1^-} \equiv \left[1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}\right] W_{\Lambda_0^-}$$

(JIMWLK equation for a small change in the cutoff)

❷ Factorization in Deep Inelastic Scattering

Leading Order

Next to Leading Order

Leading Log resummation

Inclusive DIS at Leading Log

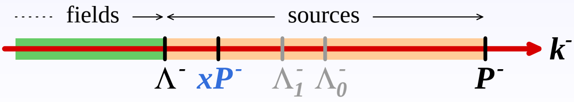
- Iterate the previous process to integrate out all the slow field modes at leading log accuracy:

Inclusive DIS at Leading Log accuracy

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp)$$

$$\sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp) \equiv 2\int d^2\vec{\boldsymbol{X}}_\perp \int [D\rho]\, W_{xP^-}[\rho]\, \boldsymbol{T}_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$$

- One does not need to evolve down to $\Lambda^- \to 0$: the DIS amplitude becomes independent of Λ^- when $\Lambda^- \lesssim xP^-$

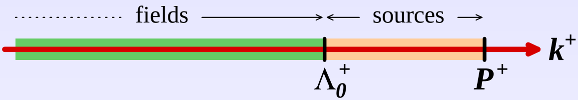


❸ Factorization in nucleus-nucleus collisions

- Power counting
- Leading Order
- Next to Leading Order
- Initial state factorization

Power counting

- CGC effective theory with cutoff at the scale Λ_0^+ :



$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{\rm YM}} + \int \underbrace{\left(J_1^\mu + J_2^\mu\right)}_{\text{fast partons}} A_\mu$$

- Expansion in g^2 in the saturated regime:

$$\frac{dN_1}{dy d^2\vec{\boldsymbol{p}}_\perp} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

❸ Factorization in nucleus-nucleus collisions

Power counting

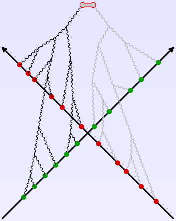
Leading Order

Next to Leading Order

Initial state factorization

Leading Order

$$\left.\frac{dN_1}{dy\,d^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} = \sum_{\rm trees}$$



Inclusive gluon spectrum at LO :

$$\left.\frac{dN_1}{dy\,d^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} \propto \int d^4x d^4y\; e^{ip\cdot(x-y)} \cdots \mathcal{A}^\mu(x)\mathcal{A}^\nu(y)$$

$$\underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J_1^\nu + J_2^\nu}_{\rm Yang-Mills\ \ equation} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

❸ Factorization in nucleus-nucleus collisions

Power counting

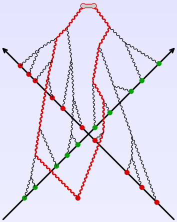
Leading Order

Next to Leading Order

Initial state factorization

Next to Leading Order [FG, Venugopalan (2006)]

$$\left.\frac{dN_1}{dy\,d^2\vec{p}_\perp}\right|_{\rm NLO} = \sum_{\rm trees}$$



Inclusive gluon spectrum at NLO :

$$\left.\frac{dN_1}{dy\,d^2\vec{p}_\perp}\right|_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}\in\Sigma}\int_{k}\left[a_k\,\mathbb{T}\right]_{u}\left[a_k^*\,\mathbb{T}\right]_{v} + \int_{\vec{u}\in\Sigma}\left[\alpha\,\mathbb{T}\right]_{u}\right]\left.\frac{dN_1}{dy\,d^2\vec{p}_\perp}\right|_{\rm LO}$$

$\Sigma =$ initial Cauchy surface , $\mathbb{T}\sim\delta/\delta\mathcal{A}_{\rm init}$

- does not include virtual quarks loops
- a_k and α are calculable analytically

Shift operator $\mathbb{T}$ – Definition

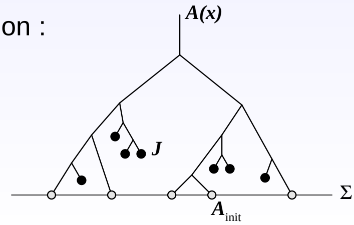
Equations of motion for a field and a small perturbation

$$\begin{aligned} \square\mathcal{A} + V'(\mathcal{A}) &= J \\ \left[\square + V''(\mathcal{A})\right]\; a &= 0 \end{aligned}$$

- Linear relationship between $\mathcal{A}$ and a :

$$a(x) = \int\limits_{\vec{u}\in\Sigma} \left[a\,\mathbb{T}\right]_{u}\;\; \mathcal{A}(x)$$

- Diagrammatic interpretation :



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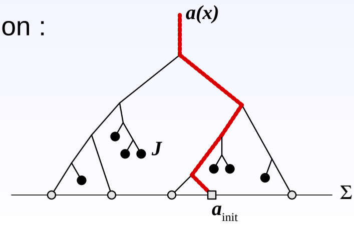


Shift operator $\mathbb{T}$ – Main properties

- $\mathbb{T}$ acts as a 1st order linear differential operator :

$$\begin{aligned}\mathbb{T}(AB) &= (\mathbb{T}A)B + A(\mathbb{T}B)\\ \mathbb{T}(F(A)) &= F'(A)(\mathbb{T}A)\end{aligned}$$

- $\mathbb{T}$ generates shifts of the initial field on Σ in any functional of the classical field :

$$\exp\left[\int\limits_{\vec{\boldsymbol{u}}\in\Sigma}\left[a\,\mathbb{T}\right]_{\boldsymbol{u}}\right]\;F[\mathcal{A}_{\rm init}] = F[\mathcal{A}_{\rm init}+a]$$

❸ Factorization in nucleus-nucleus collisions

- Power counting
- Leading Order
- Next to Leading Order
- Initial state factorization

Leading Logs [FG, Lappi, Venugopalan (2008)]

Logs of Λ^+ and Λ^-

$$\frac{1}{2}\underset{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma}{\int\int_{\boldsymbol{k}}} \left[a_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{u}} \left[a^*_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{v}} + \underset{\vec{\boldsymbol{u}}\in\Sigma}{\int} \left[\boldsymbol{\alpha}\,\mathbb{T}\right]_{\boldsymbol{u}} =$$

$$= \ln\left(\Lambda^+\right)\,\mathcal{H}_1 + \ln\left(\Lambda^-\right)\,\mathcal{H}_2 + \text{terms w/o logs}$$

$$\mathcal{H}_{1,2} = \text{JIMWLK Hamiltonian}$$

- Roughly speaking, the mapping is:

$$\left[a_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{u}} \longrightarrow \int d^2\vec{\boldsymbol{x}}_\perp\,\frac{\boldsymbol{u}^i_\perp-\boldsymbol{x}^i_\perp}{(\boldsymbol{u}_\perp-\boldsymbol{x}_\perp)^2}\,\left[\Omega(\boldsymbol{x}_\perp)-\Omega(\boldsymbol{u}_\perp)\right]_{ab}\,\nabla^b_{\boldsymbol{x}}$$

- No mixing between the logs of Λ^+ and Λ^-
- Ensures the factorizability of these logs into JIMWLK-evolved distributions $W[\rho_{1,2}]$

Factorization of the Leading Logs of 1/x

- One can factorize all the powers of $\alpha_s \log(1/x_{1,2})$

Single inclusive gluon spectrum at Leading Log accuracy

$$\left\langle \frac{dN_1}{dyd^2\vec{\boldsymbol{p}}_\perp} \right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\, W_1\big[\rho_1\big]\, W_2\big[\rho_2\big]\, \underbrace{\frac{dN_1[\rho_{1,2}]}{dyd^2\vec{\boldsymbol{p}}_\perp}}_{\text{for fixed }\rho_{1,2}}$$

- The factor $dN_1/dyd^2\vec{\boldsymbol{p}}_\perp$ under the integral does not depend on y: the rapidity dependence comes entirely from the distributions $W_{1,2}$
- This factorization establishes a link to other reactions (such as DIS) in the saturated regime

Multi-gluon correlations at Leading Log

- The previous factorization can be extended to multi-particle inclusive spectra :

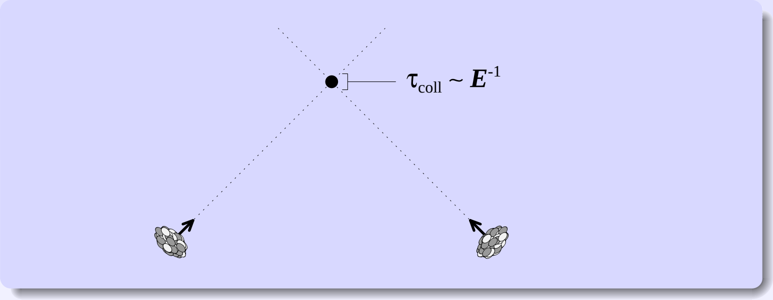
$$\left\langle \frac{dN_n}{dy_1 d^2\vec{\boldsymbol{p}}_{1\perp}\cdots dy_n d^2\vec{\boldsymbol{p}}_{n\perp}} \right\rangle_{\rm LLog} =$$
$$= \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]\; \frac{dN_1\big[\rho_{1,2}\big]}{dy_1 d^2\vec{\boldsymbol{p}}_{1\perp}} \cdots \frac{dN_1\big[\rho_{1,2}\big]}{dy_n d^2\vec{\boldsymbol{p}}_{n\perp}}$$

- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W\big[\rho_{1,2}\big]$

  $\triangleright$ they are a property of the pre-collision initial state

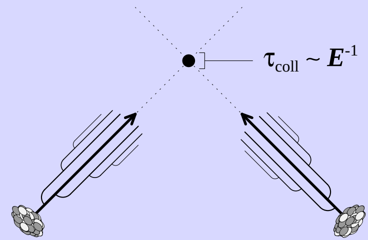
- This formula predicts long range ($\Delta y \sim \alpha_s^{-1}$) correlations in rapidity

Why factorization works: causality



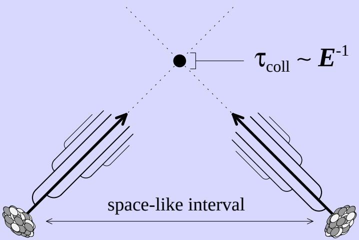
- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  $\triangleright$ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  $\triangleright$ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

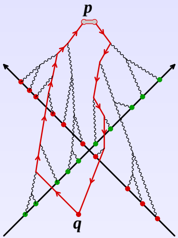
❹ Extensions

Quark production

Exclusive processes

Quark spectrum at LO [FG, Kajantie, Lappi (2005)]

$$\left.\frac{dN_Q}{dy\,d^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} = \sum_{\rm trees}$$



Inclusive quark spectrum at LO :

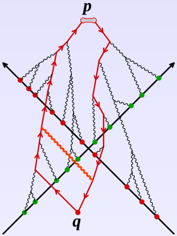
$$\left.\frac{dN_Q}{dy\,d^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} \propto \int d^4x\,d^4y\; e^{ip\cdot(x-y)} \cdots \psi_{\boldsymbol{q}}^\dagger(x)\,\psi_{\boldsymbol{q}}(y)$$

$$\underbrace{[i\partial\!\!\!/ - gA\!\!\!/ - m]\;\psi_{\boldsymbol{q}}(x) = 0}_{\text{Dirac equation}} \quad , \quad \lim_{t\to-\infty}\psi_{\boldsymbol{q}}(t,\vec{\boldsymbol{x}}) = v(\boldsymbol{q})\,e^{iq\cdot x}$$

- Note : for the quarks, LO = 1-loop, NLO = 2-loops

Quark spectrum at NLO [FG, Laidet (in progress)]

$$\left.\frac{dN_Q}{dyd^2\vec{p}_\perp}\right|_{\rm NLO} = \sum_{\rm trees}$$



Inclusive quark spectrum at NLO :

$$\left.\frac{dN_Q}{dyd^2\vec{p}_\perp}\right|_{\rm NLO} = \Bigg[\frac{1}{2}\int\!\!\int_{\vec{u},\vec{v}\in\Sigma}\int_{\boldsymbol{k}}\left[a_{\boldsymbol{k}}\,\mathbb{T}+b_{\boldsymbol{k}}\,\mathbb{T}_\psi\right]_{\boldsymbol{u}}\left[a^*_{\boldsymbol{k}}\,\mathbb{T}+b^\dagger_{\boldsymbol{k}}\,\mathbb{T}_\psi\right]_{\boldsymbol{v}} + \int_{\vec{u}\in\Sigma}\left[\alpha\,\mathbb{T}+\beta\,\mathbb{T}_\psi\right]_{\boldsymbol{u}}\Bigg]\left.\frac{dN_Q}{dyd^2\vec{p}_\perp}\right|_{\rm LO}$$

$$\mathbb{T}_\psi \sim \delta/\delta\psi_{\rm init}$$

Quark spectrum – Leading Logs

- Reminder :

$$\left[a_{\boldsymbol{k}}\,\mathbb{T}\right]_{\boldsymbol{u}} \longrightarrow \int d^2\vec{\boldsymbol{x}}_\perp\,\frac{\boldsymbol{u}^i_\perp-\boldsymbol{x}^i_\perp}{(\boldsymbol{u}_\perp-\boldsymbol{x}_\perp)^2}\,\left[\Omega(\boldsymbol{x}_\perp)-\Omega(\boldsymbol{u}_\perp)\right]_{ab}\,\left[\nabla^b_{\boldsymbol{x}}\right]_A$$

where $[\cdots]_A$ means that the derivative hits only the Wilson lines contained in color fields

- Conjecture :

$$\left[b_{\boldsymbol{k}}\,\mathbb{T}_\psi\right]_{\boldsymbol{u}} \longrightarrow \int d^2\vec{\boldsymbol{x}}_\perp\,\frac{\boldsymbol{u}^i_\perp-\boldsymbol{x}^i_\perp}{(\boldsymbol{u}_\perp-\boldsymbol{x}_\perp)^2}\,\left[\Omega(\boldsymbol{x}_\perp)-\Omega(\boldsymbol{u}_\perp)\right]_{ab}\,\left[\nabla^b_{\boldsymbol{x}}\right]_\psi$$

- Then: $\left[\nabla^b_{\boldsymbol{x}}\right]_A+\left[\nabla^b_{\boldsymbol{x}}\right]_\psi=\nabla^b_{\boldsymbol{x}}$, with no restriction on where the derivative acts (color field or spinor)

  ▷ we would recover the JIMWLK Hamiltonian

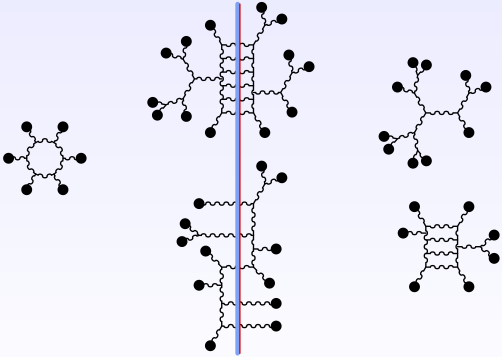
④ Extensions

Quark production

Exclusive processes

Exclusive processes

- Main difficulty: the vacuum graphs are complex when the fields are coupled to an external source, and they do not cancel in exclusive quantities



Exclusive processes – Generating functional

- Consider a function $z(\vec{\boldsymbol{p}})$, and define the functional

$$F[z] \equiv \sum_n \frac{1}{n!} \int \left[d\Phi_n\right] z(\vec{\boldsymbol{p}}_1)\cdots z(\vec{\boldsymbol{p}}_n) \; \left|\left\langle \vec{\boldsymbol{p}}_1 \cdots \vec{\boldsymbol{p}}_{n\,\mathrm{out}} \middle| 0_{\mathrm{in}}\right\rangle\right|^2$$

- Any physical quantity can be obtained from $F[z]$
  - Single inclusive spectrum :

$$\frac{dN_1}{dyd^2\vec{\boldsymbol{p}}_\perp} = \left.\frac{\delta F[z]}{\delta z(\vec{\boldsymbol{p}})}\right|_{z=1}$$

  - Differential probability for producing exactly one gluon :

$$\frac{dP_1}{dyd^2\vec{\boldsymbol{p}}_\perp} = \left.\frac{\delta F[z]}{\delta z(\vec{\boldsymbol{p}})}\right|_{z=0}$$

$\triangleright$ differ only by the point where the derivative is evaluated

Exclusive processes – Generating functional at LO

Leading Order

$$\left.\frac{\delta \log F[z]}{\delta z(\vec{\boldsymbol{p}})}\right|_{\rm LO} \propto \int d^4x d^4y\; e^{ip\cdot(x-y)} \cdots \mathcal{A}_+(x)\mathcal{A}_-(y)$$

- $\mathcal{A}_+$ and $\mathcal{A}_-$ are solutions of the Yang-Mills eqs.
- But non-retarded z-dependent boundary condition
- When applied to P_1, we get

$$\left.\frac{dP_1}{dy d^2\vec{\boldsymbol{p}}_\perp}\right|_{\rm LO} = F[0] \times \int d^4x d^4y\; e^{ip\cdot(x-y)} \cdots \mathcal{A}_+(x)\mathcal{A}_-(y)\Big|_{z=0}$$

- $F[0]$ = probability of producing nothing
  = survival probability
- Main problem: for inclusive quantities, the NLO calculation could be arranged nicely because all the fields are retarded. Not true here...

Summary

- Collisions of hadrons/nuclei at high energy require some knowledge about the multigluon Fock states of the projectiles. An effective description of these states is provided by the color glass condensate
- The LO is given by classical fields
- Higher orders contains logs of the energy
- For inclusive gluonic quantities, the leading logs can be factorized into two factors that describe the color content of the projectiles
- Most likely, this is also true for the inclusive quark spectrum
- Exclusive quantities are much more complicated...