QUARKONIUM FRAGMENTATION FUNCTIONS

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INT11-3: Frontiers in QCD

OVERVIEW

Quarkonium: Bound state of a heavy quark anti-quark pair

 $m_Q \gg \Lambda_{\text{\tiny QCD}}$

 $v \ll 1$

Charmonium *M* Bottomonium *M* $J/\psi \to c\bar{c}(n=1,{}^{3}S_{1})$ 3.096 $\eta_c \to c\bar{c}(n=1,{}^1S_0)$ 2.98 $\eta_b \to b\bar{b}(n=1,{}^1S_0)$ 9.39 $\chi_{cJ} \rightarrow c\bar{c}(n=1,{}^3P_J) \sim 3.5 \left[\begin{array}{c} \chi_{bJ} \rightarrow b\bar{b}(n=1,{}^3P_J) \end{array} \right]$ *J* = *{*0*,* 1*,* 2*}* ∼ 3*.*5 $\Upsilon(1S) \to b\overline{b}(n=1, \frac{3}{5}S_1)9.46$ $~\sim 10$

NRQCD

Bodwin, Braaten & Lepage; Luke, Manohar & Rothstein; Pineda & Soto

Appropriate EFT for Describing Quarkonium Dynamics: Non-Relativistic QCD

• Remove the heavy quark mass (like HQET) from QCD • Power counting in relative velocity $v \ll 1$ (not $1/m_Q$) \bullet $\mathcal{L} = \psi^{\dagger}$! $i\partial_0 +$ $\sqrt{2}$ 2m*^Q* " $\psi + \chi^\dagger$ $\left(i\partial_0 - \frac{\nabla^2}{2m}\right)$ 2m*^Q* $\overline{ }$ χ

•Separates the scales: *mQ, mQv,mQv*²

QUARKONIUM DECAY

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Bodwin, Braaten & Lepage

Inclusive & Semi-Inclusive Decay Rates can be Calculated via the O.P.E.

ARKONIUM DECAY

Bodwin, Braaten & Lepage

 $\chi_{cJ} \rightarrow \gamma + X$

QUARKONIUM PRODUCTION in terms of the operator a† ^H that creates the quarkonium H in the out state. A sum over the angular-momentum quantum numbers m^J is implicit in a† ^Ha^H . The factors ^Kⁿ and ^K! 6/23

Bodwin, Braaten & Lepage in the operator are products of a color matrix (either the unit matrix or T^a), a spin matrix

NRQCD Factorization for Production (ANNQUD FaCLOI IZALION TOI FFOQUCLION)

•Inclusive Production Cross Section • Inclusive Production Cross Section

$$
\sigma(a+b \to H + X) = \sum_{\beta} \hat{\sigma}(a+b \to Q\bar{Q}(\beta) + X)\langle 0|\mathcal{O}_{\beta}^{H}|0\rangle
$$

$$
\mathcal{O}_{n}^{H} = \chi^{\dagger}\mathcal{K}_{n}\psi\left(\sum_{X} \sum_{m_{J}}|H + X\rangle\langle H + X|\right)\psi^{\dagger}\mathcal{K}'_{n}\chi
$$

$$
= \chi^{\dagger}\mathcal{K}_{n}\psi\left(a_{H}^{\dagger}a_{H}\right)\psi^{\dagger}\mathcal{K}'_{n}\chi,
$$

"fragmentation" functions

$$
\mathcal{O}_{1}^{H}({}^{3}S_{1}) = \chi^{\dagger}\sigma^{i}\psi\left(a_{H}^{\dagger}a_{H}\right)\psi^{\dagger}\sigma^{i}\chi,
$$

$$
\mathcal{O}_{8}^{H}({}^{1}S_{0}) = \chi^{\dagger}T^{a}\psi\left(a_{H}^{\dagger}a_{H}\right)\psi^{\dagger}T^{a}\chi,
$$

QUARKONIUM PRODUCTION \sim \sim \sim ⁿ is invariant under color and spatial rotations.³ We assume (either the unit matrix or σⁱ), and a polynomial in the covariant derivative D and other covariant derivative D and other covariant derivative D LUANNUIVII FRUDUCII 7/23

Bodwin, Braaten & Lepage n will be evaluated in the matter in the quarkonium rest frame; or the quarkonium rest frame; or the contract frame; or the contract of \sim 100 μ m \sim 1

• J/ψ Production at large p_\perp in hadronic collisions t and α is obtained in the quarkonium rest frame; or α in the quarkonium rest frame; or α $\frac{1}{\sqrt{2}}$ γ γ γ γ γ γ and γ and γ the γ the γ of γ γ γ γ γ γ γ • J/ψ Production at large p_{\perp} in hadronic

$$
\frac{d\sigma}{dp_{\perp}}(p\overline{p} \to J/\psi(p_{\perp}) + X) = \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/\overline{p}}(x_2)
$$

$$
\times \sum_{\beta} \hat{\sigma}(i j \to c\bar{c}(\beta, p_{\perp}) + X) \langle 0 | \mathcal{O}_{\beta}^{J/\psi} | 0 \rangle
$$

LO $\mathcal{O}_1^H({}^3S_1) = \chi^{\dagger} \sigma^i \psi$ $\int a_H^{\dagger} a_H$ $\mathcal{O}_1^H({}^3S_1) = \chi^{\dagger} \sigma^i \psi \left(a_H^{\dagger} a_H \right) \psi^{\dagger} \sigma^i$ $\chi \qquad \qquad {\cal O}_8^H({}^3S_1) = \chi^\dagger \sigma^i T^a \psi \left(a_H^\dagger a_H^{} \right)$ $\mathcal{O}_8^H({}^1S_0) = \chi^\dagger T^a \psi \left(a_H^\dagger a_H^{} \right)$ $\mathcal{O}_8^H({}^1S_0) = \chi^\dagger T^a \psi \left(a_H^\dagger a_H \right) \psi^\dagger T^a \chi,$ $\mathcal{O}_8^H({}^3P_1) = \frac{1}{2}\chi^{\dagger}(-\frac{\imath}{2}\stackrel{\leftrightarrow}{\mathbf{D}}\times\boldsymbol{\sigma})^i T^A \psi(a_H^{\dagger}a_H) \chi^{\dagger} T^A (-\frac{\imath}{2}\stackrel{\leftrightarrow}{\mathbf{D}}\times\boldsymbol{\sigma})^i \chi$ polarized production, a† $\mathcal{O}_8^H({}^3P_2) = \chi^{\dagger}(-\frac{i}{2}\ \overrightarrow{D}~^{(i}\sigma^{j)})T^A\psi(a_H^{\dagger}a_H)\chi^{\dagger}T^A(-\frac{i}{2}\ \overrightarrow{D}~^{(i}\sigma^{j)})\chi$ $\left(a_{H}^{\dagger}a_{H}\right)^{2}$ $v^{\dagger} \sigma^i$ $\int \psi^{\dagger} T^a \chi,$ $\left(a_{H}^{\dagger}a_{H}\right)$ $\Big)\, \psi^\dagger \sigma^i T^a$ χ $\begin{array}{ccc} \n\cdot & i & \leftrightarrow & \n\cdot & -1 & \cdot & + & -1 & \n\end{array}$ \ddot{i} $-\frac{\imath}{2}$ $\stackrel{\leftrightarrow}{\mathbf{D}}$ $\mathcal{O}_8^H({}^3P_1) = \frac{1}{2}\chi^{\dagger}(-\frac{i}{2}\stackrel{\leftrightarrow}{\mathbf{D}}\times\boldsymbol{\sigma})^i T^A \psi(a_H^{\dagger}a_H) \chi^{\dagger} T^A(-\frac{i}{2}\stackrel{\leftrightarrow}{\mathbf{D}}\times\boldsymbol{\sigma})^i \chi$ $(a_H^{\dagger} a_H)$ $\cdot \big) \chi^\dagger T^A ($ $-\frac{v}{2}$ D⁽ⁱ⁾ $\mathcal{O}_8^H({}^3P_2) = \chi^\dagger(-\frac{i}{2}\stackrel{\leftrightarrow}{\mathbf{D}}({}^i\sigma^j)) T^A \psi(a_H^\dagger a_H) \chi^\dagger T^A(-\frac{i}{2}\stackrel{\leftrightarrow}{\mathbf{D}}({}^i\sigma^j)) \chi$ 2 $\chi^{\dagger}(-\frac{i}{2})$ 2 \leftrightarrow $(\mathbf{D}\!\times\! \boldsymbol{\sigma})^i T^A \psi(a_I^{\dagger})$ *^H*a*H*)χ†^T *^A*([−] ⁱ 2 \leftrightarrow $\mathbf{\widetilde{D}}\!\times\! \boldsymbol{\sigma})^i\chi$ $\mathcal{O}_8^H({}^3P_0) = \frac{1}{3}$ 3 $\chi^{\dagger}(-\frac{i}{2})$ 2 $\stackrel{\leftrightarrow}{\mathbf{D}}\cdot\boldsymbol{\sigma})T^{A}\psi(a_{H}^{\dagger}a_{H})\psi^{\dagger}T^{A}(-\frac{i}{2})$ 2 \leftrightarrow $\mathbf{D}\cdot{\boldsymbol{\sigma}}\big)\chi$ 2 ↔ D (*ⁱ* σ*^j*))T *^A*ψ(a† *^H*a*H*)χ†^T *^A*([−] ⁱ 2 $\stackrel{\leftrightarrow}{\bf D}{}^{(i} \sigma^{j)}) \chi$

JUARKONIUM PRODUCTIO 8/23

Cho & Lebovich

 p_{\perp} (GeV)

QUARKONIUM PRODUCTION

Braaten & Fleming

• J/ψ Production at large p_{\perp} in hadronic collisions

 $\hat{\sigma}(a+b\rightarrow c\bar{c})$ ${}^{3}S_{1}^{[8]}) + X \rangle \langle 0| \mathcal{O}^{J/\psi} ({}^{3}S_{1}^{[8]})|0 \rangle$ is special

Fragmentation:

$$
\frac{d\hat{\sigma}}{dp_{\perp}}(ij \to J/\psi + X)_{\text{octet}}
$$

$$
\stackrel{p_{\perp} \to \infty}{\longrightarrow} \int dz \frac{d\hat{\sigma}}{dp_{\perp}} (ij \to g(p_{\perp}/z) + X) D_{g \to J/\psi}(z)
$$

QUARKONIUM PRODUCTION **AB → ψQX\$** II INA DL $A = \frac{1}{2}$ z \mathcal{L} $\sqrt{2}$ The gluon fragmentation function function function $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$ is readily $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$, $\mathcal{L}^{\mathcal{L}}$, \overline{a} \bigcup $ARKON$ = \mathbf{r} $\overline{}$ IMI Y∐ Pt SOL $\overline{1}$ $\begin{array}{c} \hline \end{array}$ $|CTT|$ (2.18) $f(x)$ is and the leading-order short-distance process is and the rate is $f(x)$ OUTIMURIS ENGLISH 10/23

Gluon Fragmentation $\overline{\mathcal{Q}}$ $\overline{}$ the cc¯ pair in the ψ" can make a double E1 transition with amplitude of order v² from the

 L eading log $\mathcal C$ corrections to this result may be summed up using the Altarelli-Parisi may be summed up using the $\mathcal C$ •Sum Logarthms: Run from $p_$ ⊥ to $2m_c$ \overline{D} m \bullet Sum Logarthms: Run from p_1 to $2m_c$

$$
\mu \frac{dD_{g \to \psi_Q}}{d\mu}(z,\mu) = \frac{\alpha_s(\mu)}{\pi} \int_z^1 \frac{dy}{y} P_{gg}(y) D_{g \to \psi_Q}(\frac{z}{y},\mu)
$$

$$
P_{gg}(y) = 6\left[\frac{y}{(1-y)_+} + \frac{1-y}{y} + y(1-y) + \frac{33-2n_f}{36}\delta(1-y)\right]
$$

$$
D_{g \to \psi'}(z,\mu) = \frac{\pi \alpha_s(2m_c)}{24m_c^3} \delta(1-z) \langle 0 | \mathcal{O}_8^{\psi'}(^3S_1) | 0 \rangle
$$

SURPRODUCTION QUARKONIUM PRODUCTION

Cho, & Wise, Beneke & Rothstein

Polarization

OPEN QUESTIONS

1. What about IR behavior?

2. How about summing logarithms in all the other contributions? 3. Are the NRQCD production matrix elements really universal? 4. Will the prediction for polarization change?

Answers from SCET:

(Also from QCD factorization: Kang, Qiu, Sterman)

1. Gauge invariance from Wilson lines. 2. Running of production operators sums logs. 3. No! NRQCD production matrix elements are not universal. 4. Maybe...

SCET APPROACH

Fleming, Leibovich, Mehen & Rothstein

Quarkonium production at large *p*⊥ in *pp* collisions: $\hat{s} \sim p_{\perp} \gg m_Q$

µ ∼ *p*_⊥ match QCD onto SCET with massive quarks 2. Factor differential cross section

3. Run to $\mu \sim m_Q$

4. Match onto NRQCD

SCET APPROACH \overline{a} 2 ⁿ¯^µ; ¯n^µ = (1, ⁰, ⁰, [−]1), (4)

ate out $X \cdot r$ a t ⊥ h difform \overline{P} ⊥ e−^y , 1 ≈ p[⊥] (1 + Integrate out X : match differential cross section

$$
\frac{d\sigma}{dp_{\perp}}(p\bar{p} \to H(p_{\perp}) + X) \to \sum_{\beta} \int \{d\xi_i\} dx_1 dx_2 \hat{\sigma}^{\beta}(\xi_i, x_1, x_2)
$$

$$
\times f_{i/p}(x_1) f_{j/\bar{p}}(x_2) D_H^{\beta}(\xi_i)
$$

MATCHING

An Example: $q\bar{q} \rightarrow QQ$

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Γεγονότα

$$
\frac{d\sigma}{dp_{\perp}}(p\bar{p} \to H(p_{\perp}) + X) \to \sum_{\beta} \int \{d\xi_i\} dx_1 dx_2 \hat{\sigma}^{\beta}(\xi_i, x_1, x_2)
$$

$$
\times f_{q/p}(x_1) f_{\bar{q}/\bar{p}}(x_2) D_H^{\beta}(\xi_i)
$$

$$
D_{g \to H}(z) \qquad D_{Q\bar{Q}, Q\bar{Q} \to H}(u, v, z)
$$

MATCHING Γ 0 \bigcap " ¹

MATCHING

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 $D_{Q\bar{Q}\rightarrow H}(u, v, z)$

p 1 *p* 2 *p* 3 *p* 4 *u, v, z*

 $\mathbf{F}^{\mathcal{A}}$ for $\mathbf{F}^{\mathcal{A}}$ $\bar{n}'\!\cdot\! p_1 = u\,\bar{n}'\!\cdot\! p_{Q\bar{Q}}$ $\bar{n}'\!\cdot\! p_3 = (1-v)\,\bar{n}'\!\cdot\! p_{Q\bar{Q}}$

 $\bar{n}'\!\cdot\! p_2 = (1-u)\,\bar{n}'\!\cdot\! p_{Q\bar{Q}}$ $\bar{n}'\!\cdot\! p_4 = v\,\bar{n}'\!\cdot\! p_Q\bar{Q}$

p

$$
\bar{n}'\!\cdot\! p_H\overset{\bigtriangleup}{\underset{\bigtriangleup}{\bigtriangleup}} z\bar{n}'\!\cdot\! p_Q\bar{Q}
$$

RUNNING

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Efremov-Radyushkin-Brodsky-Lepage Evolution in u, v

$$
\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u,v,z;\mu) = \int_0^1 dw V(u,w;\mu) D^{[1]}_{Q\bar{Q}\to H}(w,v,z;\mu)
$$

DGLAP Evolution in *z*

$$
\mu^2 \frac{d}{d\mu^2} D_{Q\bar{Q}\to H}^{[1]}(u,v,z;\mu) = \int_0^1 dx P_{Q\bar{Q}[1]\to Q\bar{Q}[8]}(x;\mu) D_{Q\bar{Q}\to H}^{[8]}(w,v,z/x;\mu)
$$

MATCHING NRQCD

 \Box $C \frac{\alpha_s}{2} \delta(1-z) \langle 0 \mid (\chi^\dagger S_{n'}$ $D_g \rightarrow H(z)$ $\overline{\bigcap}$ $m_{\bm{Q}}^2$ $\delta(1-z)\langle 0 \mid (\chi^\dagger S_{n'})T^A\sigma^i (S^\dagger_{n'}\psi)\mathcal{P}^\dagger_H(p_\perp,y)\mathcal{P}_H(p_\perp,y)(\psi^\dagger S_{n'})T^A\sigma^i (S^\dagger_{n'}\psi) \mid 0 \rangle$ $\chi^\dagger T^A \sigma^i \psi \mathcal{P}_H^\dagger(p_\perp,y) \mathcal{P}_H(p_\perp,y) \psi^\dagger T^A \sigma^i \chi$ $\sqrt{}$ *i* $C_i(z, m_Q) \langle 0 \mid (\psi^\dagger S_{n'}) \tilde{\Gamma}^b_i(S_{n'}^\dagger \chi) \mathcal{P}^\dagger_H(p_\perp, y) \mathcal{P}_H(p_\perp, y) (\chi^\dagger S_{n'}) \tilde{\Gamma}^b_i(S_{n'}^\dagger \psi) \mid 0 \rangle$

ural de la propieta de la propieta
La propieta de la pr $D_{\overline{O}}^{[1,8]}$ _{$\rightarrow H$} (u, v, z) $m_{\alpha\beta}$ $\sum_{i=1}^{n}$ contribution in the power contribution in the power contribution in the power counting. In the power counting. In the power counting $\sum_{i=1}^{n}$ counting. In the power countries of the power countries of the \overline{Q} \overline{Q} \rightarrow *H* (u, v, z) \overline{a} \sum *i* $C_i(u, v, z, m_Q) \langle 0 \mid (\psi^{\dagger} S_{n'}) \tilde{\Gamma}_i^b (S_{n'}^{\dagger} \chi) \mathcal{P}_H^{\dagger}(p_{\perp}, y) \mathcal{P}_H(p_{\perp}, y) (\chi^{\dagger} S_{n'}) \tilde{\Gamma}_i^b (S_{n'}^{\dagger} \psi) \mid 0 \rangle$

Octet Matrix Elements Depend on Quarkonium Direction: Less Universal than Supposed \mathcal{G} and the incoming matter we keep only those field bilinears for the incomendation of the incoming \mathcal{G} directions n and $\overline{}$ with that have non-vanishing overlap with the initial hadrons h **QUEL FRUIA LICE** $\overline{}$. At this order the matching coefficient is proportional experimental is proportional experimental experimental experimental experimental experimental experimental experimental experimental experimental experi

POLARIZATION FIX? MAYBE!

$$
\frac{d\sigma}{dp_{\perp}}(p\overline{p} \to J/\psi(p_{\perp}) + X)
$$

${}^{3}S_{1}^{[8]}$ $\alpha \to 1$ ${}^{3}S_{1}^{[1]}$ $\alpha \to 0$ But $\frac{d\sigma}{dp_{\perp}}({}^3S_1^{[1]}$ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\frac{d\sigma}{dp_{\perp}}({}^3S_1^{[8]}$ $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ $\sim \lambda^4$ Extra Suppression

SCET Matching Coefficient at High Scale For a QQ Configuration that results in a color-singlet ³S₁ QQ pair

Expansion in
$$
\alpha_s(p_\perp)
$$
 and $\lambda \sim \frac{m_Q}{p_\perp}$

\n
$$
(C)^{(1)}(\alpha_s^3) + C^{(2)}(\alpha_s^4) + \dots) D^{[1](LO)}_{Q\bar{Q}\to H}(u, v, z)
$$
\n
$$
(\tilde{C}^{(1)}(\alpha_s^3) + \dots) D^{[1](NLO)}_{Q\bar{Q}\to H}(u, v, z)
$$

POLARIZATION FIX? MAYBE!

However

 $D_{\overline{O}}^{[8](LO)}$ \overline{Q} \overline{Q} \rightarrow *H* (u, v, z)

Has non-zero *LO* in $\alpha_s (\propto \alpha_s^3)$ matching coefficient

Generate color-singlet through running

$$
\mu^2 \frac{d}{d\mu^2} D_{Q\bar{Q}\to H}^{[8]}(u, v, z; \mu) = \int_0^1 dx P_{Q\bar{Q}[8]\to Q\bar{Q}[1]}(x; \mu) D_{Q\bar{Q}\to H}^{[1]}(w, v, z/x; \mu)
$$

Will degrade the polarization... But is it enough?!?!?!

POLARIZATION FIX? MAYBE! *² +q ^c p /2 ² p /2 ^c -q ¹ -q ^c ¹ p /2 +q cp /2 ² -q ^c p /2 ¹ -q ^c p /2* **h** *1 d c c <i>c c c c c c c c c c* \neg in the fewnman diagrams represent the fragmentation of a heavy \neg

 \mathbf{F} and polarization (upper panel) and polarization (lower panel) as a function of pT . The pT . Th Still a factor of \sim 5 smaller than Octet Fragmentation

CONCLUSIONS

•Rigorous treatment of momentum region between *p*⊥ and m_Q is crucial for an accurate description of Quarkonium production at high *p*⊥

• May resolve the polarization problem, but this has to wait on a complete analysis