QUARKONIUM FRAGMENTATION FUNCTIONS

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INT11-3: Frontiers in QCD

OVERVIEW

Quarkonium: Bound state of a heavy quark anti-quark pair

$$m_Q \gg \Lambda_{\mathrm{QCD}}$$

$$v \ll 1$$

Charmonium	M	Bottomonium	M
$\eta_c \rightarrow c\bar{c}(n=1, {}^1S_0)$	2.98	$\eta_b \rightarrow b\bar{b}(n=1, {}^1S_0)$	9.39
$J/\psi \rightarrow c\bar{c}(n=1, {}^3S_1)$	3.096	$\Upsilon(1S) \rightarrow b\bar{b}(n=1, {}^3S_1)$	9.46
$\chi_{cJ} \rightarrow c\bar{c}(n=1, {}^3P_J)$	~ 3.5	$\chi_{bJ} \rightarrow b\bar{b}(n=1, {}^3P_J)$	~ 10

$$J = \{0, 1, 2\}$$

NRQCD

Bodwin, Braaten & Lepage; Luke, Manohar & Rothstein; Pineda & Soto

Appropriate EFT for Describing Quarkonium Dynamics: Non-Relativistic QCD

- Remove the heavy quark mass (like HQET) from QCD
- Power counting in relative velocity $v \ll 1$ (not $1/m_Q$)
- $\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_Q} \right) \psi + \chi^\dagger \left(i\partial_0 - \frac{\nabla^2}{2m_Q} \right) \chi$
- Separates the scales: $m_Q, m_Q v, m_Q v^2$

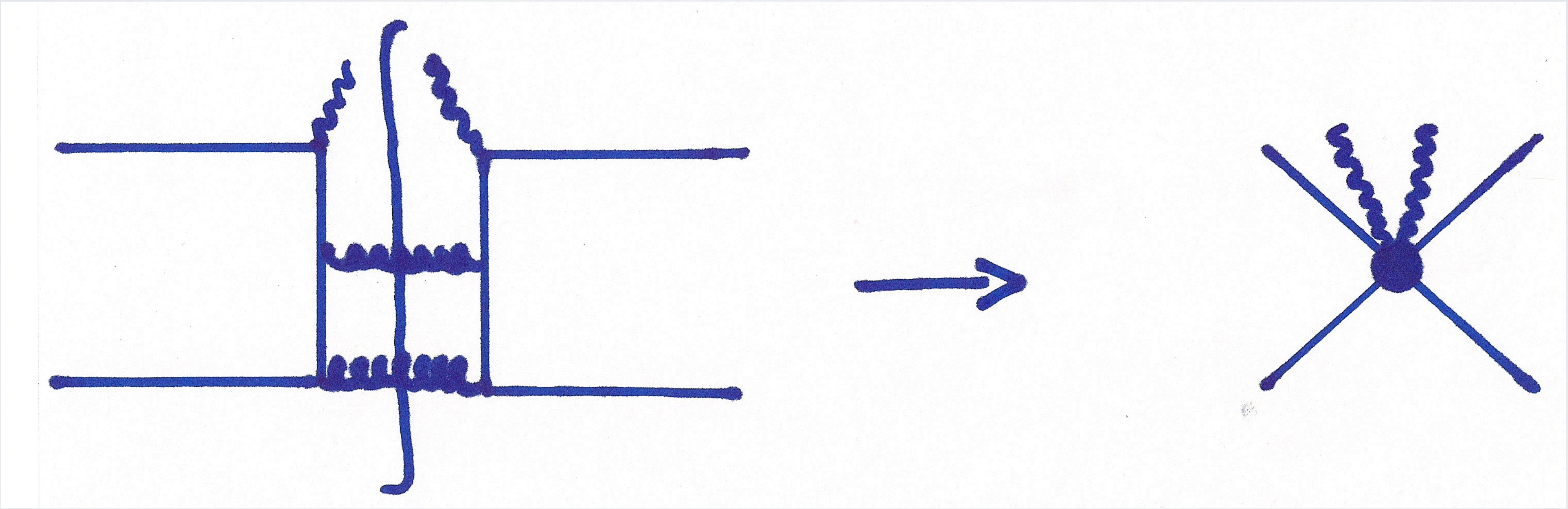
QUARKONIUM DECAY

Bodwin, Braaten & Lepage

Inclusive & Semi-Inclusive Decay Rates can be Calculated via the O.P.E.

E.G. $J/\psi \to \gamma + X$

LO



$$\frac{d\Gamma}{dE_\gamma}(J/\psi \to \gamma + X) = \sum_\beta \frac{d\Gamma}{dE_\gamma}(c\bar{c}[\beta] \to \gamma + X)\langle J/\psi|\mathcal{O}_\alpha|J/\psi\rangle$$

$$\mathcal{O}_1(^3S_1) = \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi \qquad O(v^3)$$

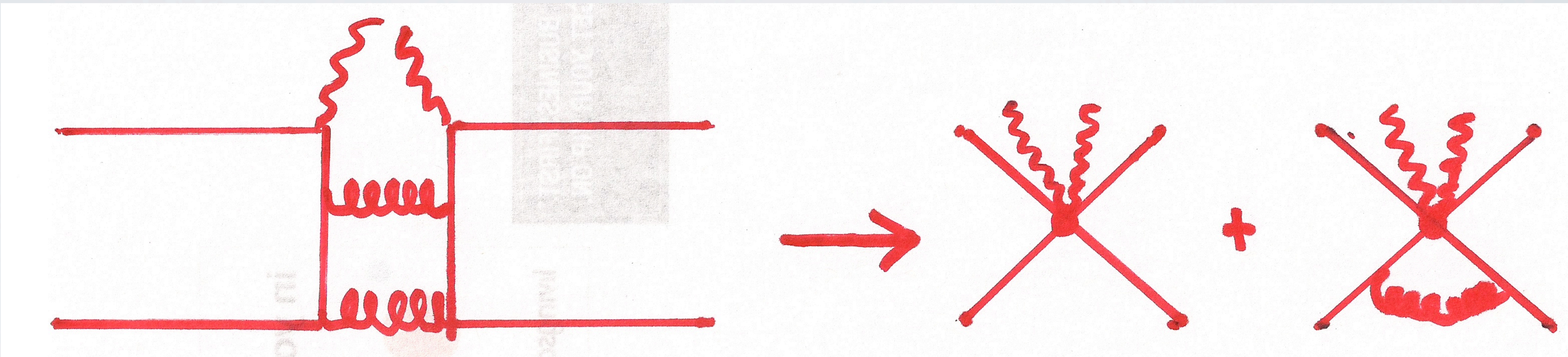
$$\mathcal{O}_8(^3S_1) = \psi^\dagger \boldsymbol{\sigma} T^A \chi \cdot \chi^\dagger T^A \boldsymbol{\sigma} \psi \qquad O(v^7)$$

QUARKONIUM DECAY

Bodwin, Braaten & Lepage

$\chi_{cJ} \to \gamma + X$

LO



$$\frac{d\Gamma}{dE_\gamma}(\chi_{cJ} \to \gamma + X) = \sum_\beta \frac{d\Gamma}{dE_\gamma}(c\bar{c}[\beta] \to \gamma + X)\langle \chi_{cJ}|\mathcal{O}_\alpha|\chi_{cJ}\rangle$$

$$\mathcal{O}_1(^3P_0) = \frac{1}{3}\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\psi \qquad O(v^5)$$

$$\mathcal{O}_8(^3S_1) = \psi^\dagger\boldsymbol{\sigma}T^A\chi\cdot\chi^\dagger T^A\boldsymbol{\sigma}\psi \qquad O(v^5)$$

QUARKONIUM PRODUCTION

Bodwin, Braaten & Lepage

NRQCD Factorization for Production

- Inclusive Production Cross Section

$$\sigma(a+b\to H+X)=\sum_\beta \hat{\sigma}(a+b\to Q\bar{Q}(\beta)+X)\langle 0|\mathcal{O}^H_\beta|0\rangle$$

$$\mathcal{O}^H_n=\chi^\dagger\mathcal{K}_n\psi\left(\sum_X\sum_{m_J}|H+X\rangle\langle H+X|\right)\psi^\dagger\mathcal{K}'_n\chi$$
$$=\chi^\dagger\mathcal{K}_n\psi\left(a^\dagger_H a_H\right)\psi^\dagger\mathcal{K}'_n\chi,$$

"fragmentation" functions

$$\mathcal{O}^H_1(^3S_1)=\chi^\dagger\sigma^i\psi\left(a^\dagger_H a_H\right)\psi^\dagger\sigma^i\chi,$$

$$\mathcal{O}^H_8(^1S_0)=\chi^\dagger T^a\psi\left(a^\dagger_H a_H\right)\psi^\dagger T^a\chi,$$

QUARKONIUM PRODUCTION

Bodwin, Braaten & Lepage

- J/ψ Production at large $p_\perp$ in hadronic collisions

$$\frac{d\sigma}{dp_\perp}(p\bar{p} \to J/\psi(p_\perp) + X) = \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/\bar{p}}(x_2)$$

$$\times \sum_\beta \hat{\sigma}(i\,j \to c\bar{c}(\beta, p_\perp) + X)\langle 0|\mathcal{O}_\beta^{J/\psi}|0\rangle$$

LO $\mathcal{O}_1^H(^3S_1) = \chi^\dagger\sigma^i\psi\left(a_H^\dagger a_H\right)\psi^\dagger\sigma^i\chi$ $\quad$ $\mathcal{O}_8^H(^3S_1) = \chi^\dagger\sigma^i T^a\psi\left(a_H^\dagger a_H\right)\psi^\dagger\sigma^i T^a\chi$

$$\mathcal{O}_8^H(^1S_0) = \chi^\dagger T^a\psi\left(a_H^\dagger a_H\right)\psi^\dagger T^a\chi,$$

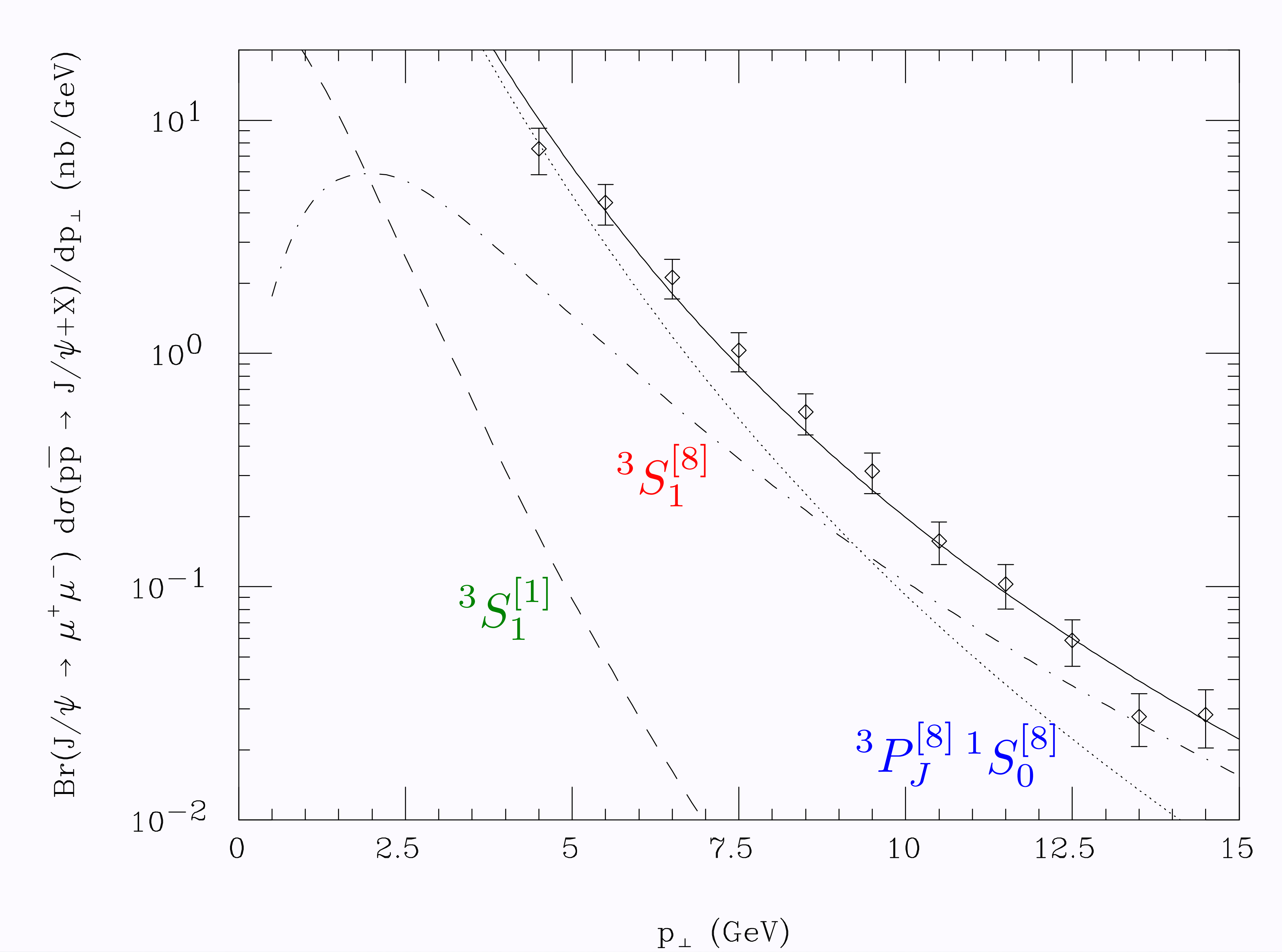
$$\mathcal{O}_8^H(^3P_0) = \frac{1}{3}\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})T^A\psi(a_H^\dagger a_H)\psi^\dagger T^A(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi$$

$$\mathcal{O}_8^H(^3P_1) = \frac{1}{2}\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma})^i T^A\psi(a_H^\dagger a_H)\chi^\dagger T^A(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma})^i\chi$$

$$\mathcal{O}_8^H(^3P_2) = \chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\sigma^{j)})T^A\psi(a_H^\dagger a_H)\chi^\dagger T^A(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\sigma^{j)})\chi$$

QUARKONIUM PRODUCTION

Cho & Lebovich



QUARKONIUM PRODUCTION

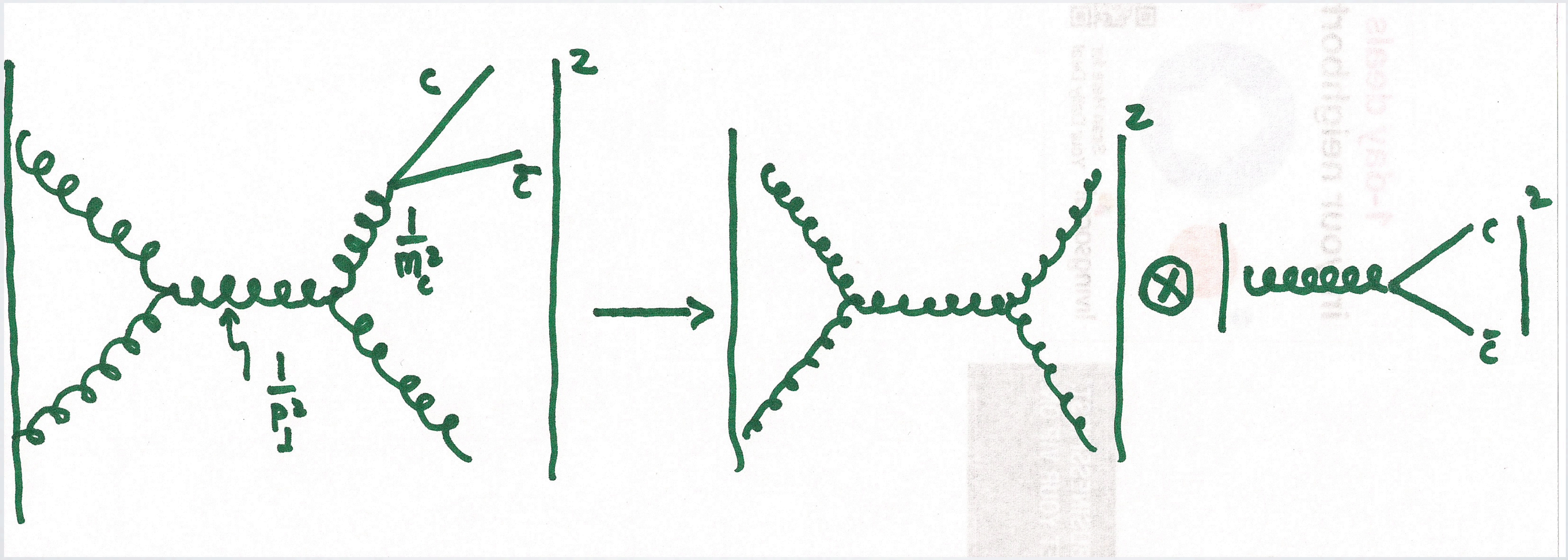
Braaten & Fleming

- J/ψ Production at large $p_\perp$ in hadronic collisions

$$\hat{\sigma}(a+b\to c\bar{c}({}^3S_1^{[8]})+X)\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[8]})|0\rangle \text{ is special}$$

Fragmentation: $$\frac{d\hat{\sigma}}{dp_\perp}(ij\to J/\psi+X)_{\text{octet}}$$

$$\overset{p_\perp\to\infty}{\longrightarrow}\int dz\frac{d\hat{\sigma}}{dp_\perp}(ij\to g(p_\perp/z)+X)D_{g\to J/\psi}(z)$$



QUARKONIUM PRODUCTION

Gluon Fragmentation

- Sum Logarthms: Run from $p_\perp$ to $2m_c$

$$\mu \frac{dD_{g\to\psi_Q}}{d\mu}(z,\mu) = \frac{\alpha_s(\mu)}{\pi} \int_z^1 \frac{dy}{y} P_{gg}(y) D_{g\to\psi_Q}(\frac{z}{y},\mu)$$

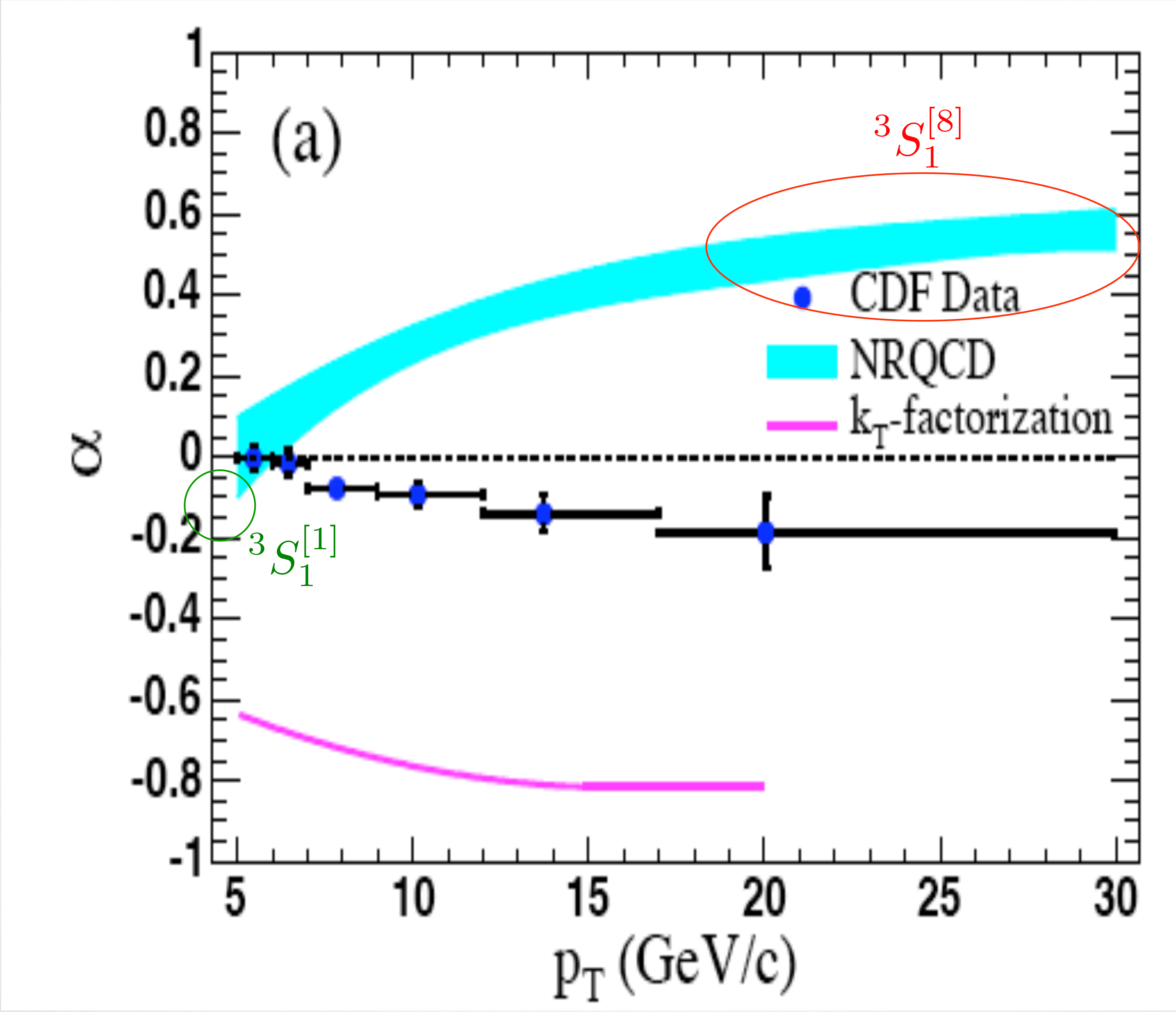
$$P_{gg}(y) = 6\Big[\frac{y}{(1-y)_+} + \frac{1-y}{y} + y(1-y) + \frac{33-2n_f}{36}\delta(1-y)\Big]$$

$$D_{g\to\psi'}(z,\mu) \;=\; \frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)\;\langle 0|\mathcal{O}_8^{\psi'}({}^3S_1)|0\rangle$$

QUARKONIUM PRODUCTION

Cho, & Wise, Beneke & Rothstein

Polarization



OPEN QUESTIONS

1. What about IR behavior?
2. How about summing logarithms in all the other contributions?
3. Are the NRQCD production matrix elements really universal?
4. Will the prediction for polarization change?

Answers from SCET:

(Also from QCD factorization: Kang, Qiu, Sterman)

1. Gauge invariance from Wilson lines.
2. Running of production operators sums logs.
3. No! NRQCD production matrix elements are not universal.
4. Maybe...

SCET APPROACH

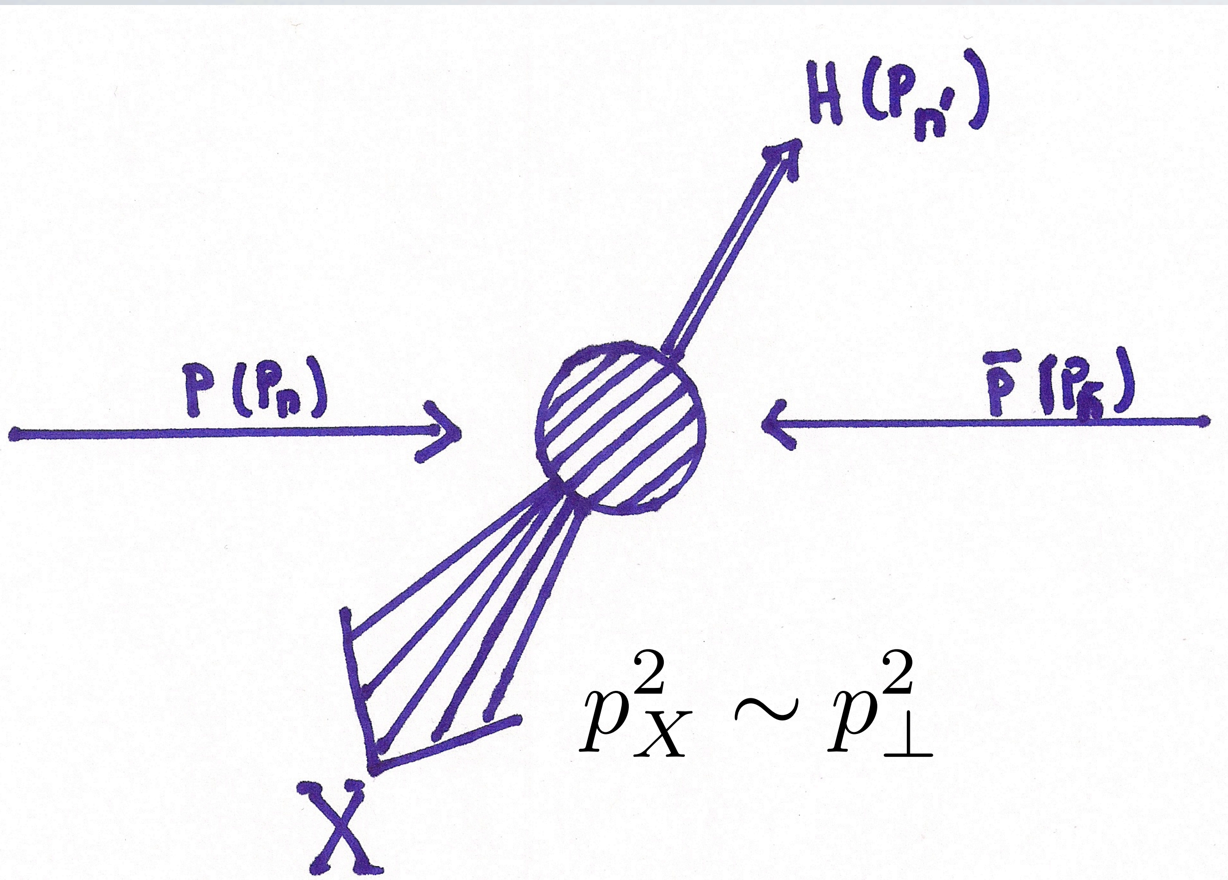
Fleming, Leibovich, Mehen & Rothstein

Quarkonium production at large $p_\perp$ in $p\bar{p}$ collisions:

$$\sqrt{\hat{s}} \sim p_\perp \gg m_Q$$

1. At $\mu \sim p_\perp$ match QCD onto SCET with massive quarks
2. Factor differential cross section
3. Run to $\mu \sim m_Q$
4. Match onto NRQCD

SCET APPROACH



$$p_n \approx \frac{Q}{2} n^\mu; \qquad n^\mu = (1,0,0,1)$$

$$p_{\bar{n}} \approx \frac{Q}{2} \bar{n}^\mu; \qquad \bar{n}^\mu = (1,0,0,-1)$$

$$p_{n'} \approx \frac{p_\perp}{2} n'^\mu; \qquad n'^\mu = (\cosh y, 1, 0, \sinh y)$$

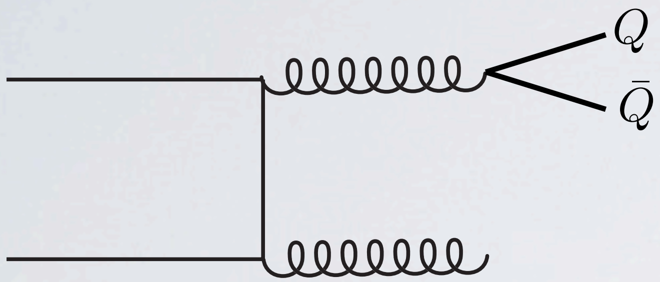
Integrate out X : match differential cross section

$$\frac{d\sigma}{dp_\perp}(p\bar{p} \to H(p_\perp) + X) \to \sum_\beta \int \{d\xi_i\}\, dx_1\, dx_2\, \hat{\sigma}^\beta(\xi_i, x_1, x_2) \times f_{i/p}(x_1) f_{j/\bar{p}}(x_2) D_H^\beta(\xi_i)$$

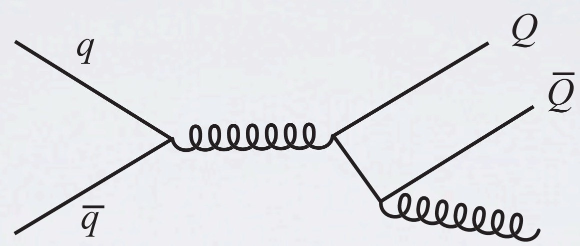
MATCHING

An Example: $q\bar{q} \to Q\bar{Q}$

$\Big|$



$+\cdots+$



$+\cdots+ \Big|^2$

$O(1)$ $\quad$ $O(\lambda^2)$ $\quad$ $\lambda \sim m_Q/p_\perp$

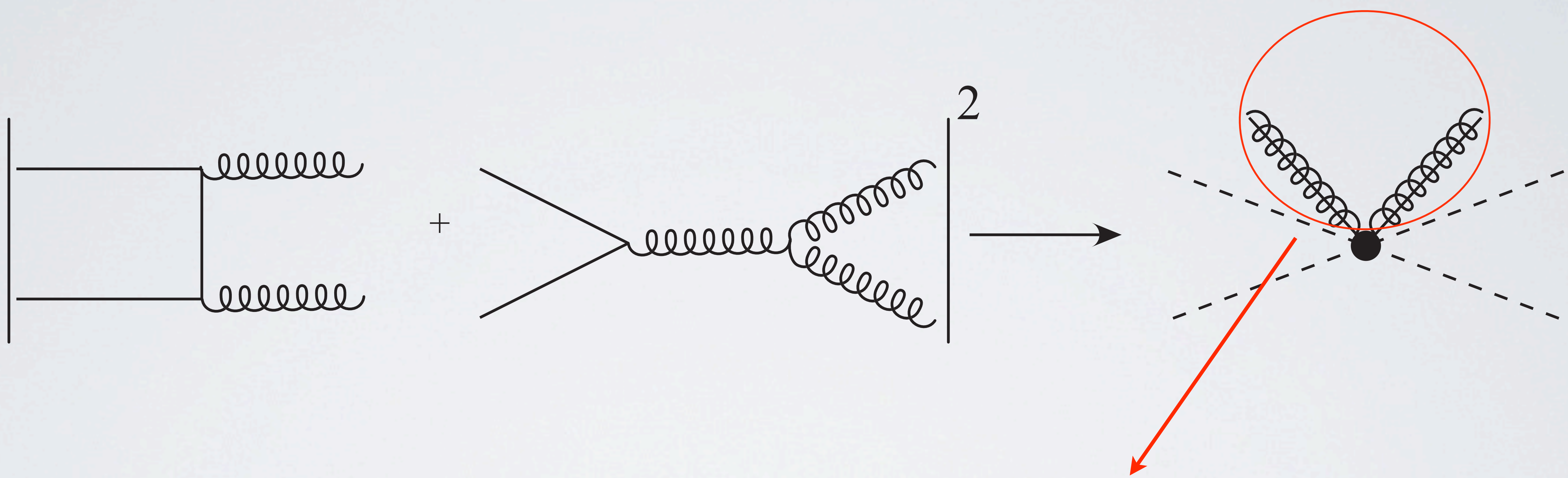
$$\frac{d\sigma}{dp_\perp}(p\bar{p} \to H(p_\perp) + X) \to \sum_\beta \int \{d\xi_i\}\, dx_1\, dx_2\, \hat{\sigma}^\beta(\xi_i, x_1, x_2)$$

$$\times f_{q/p}(x_1) f_{\bar{q}/\bar{p}}(x_2) D_H^\beta(\xi_i)$$

$$D_{g\to H}(z) \qquad D_{Q\bar{Q},Q\bar{Q}\to H}(u, v, z)$$

MATCHING

$O(1)$

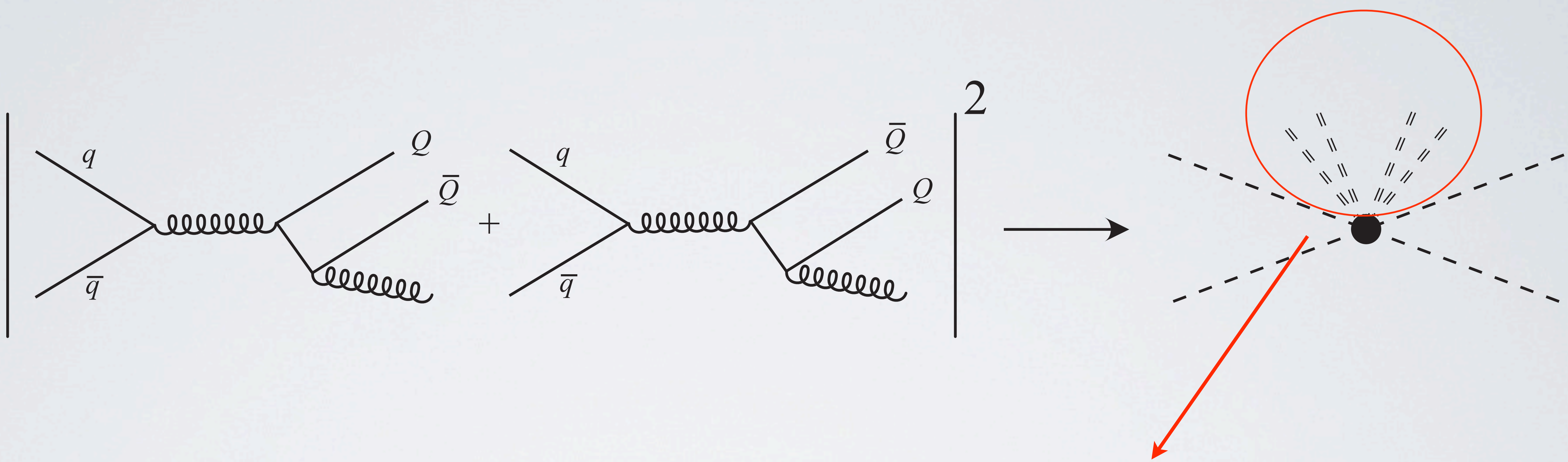


$$\langle 0|\mathrm{Tr}\Big\{(B^{a\nu}_{n',\omega_1'})\mathcal{P}_H^\dagger(p_\perp,y)\mathcal{P}_H(p_\perp,y)(B^{a\rho}_{n',\omega_2'})\Big\}|0\rangle$$

$$= -\frac{\omega_+^{\prime 2}}{2}\int_0^1 \frac{dz}{z}\delta(\omega_-')\delta(\omega_+' - \frac{2\bar{n}'\cdot p}{z})D_{H/g}(z)$$

MATCHING

$O(\lambda^2)$

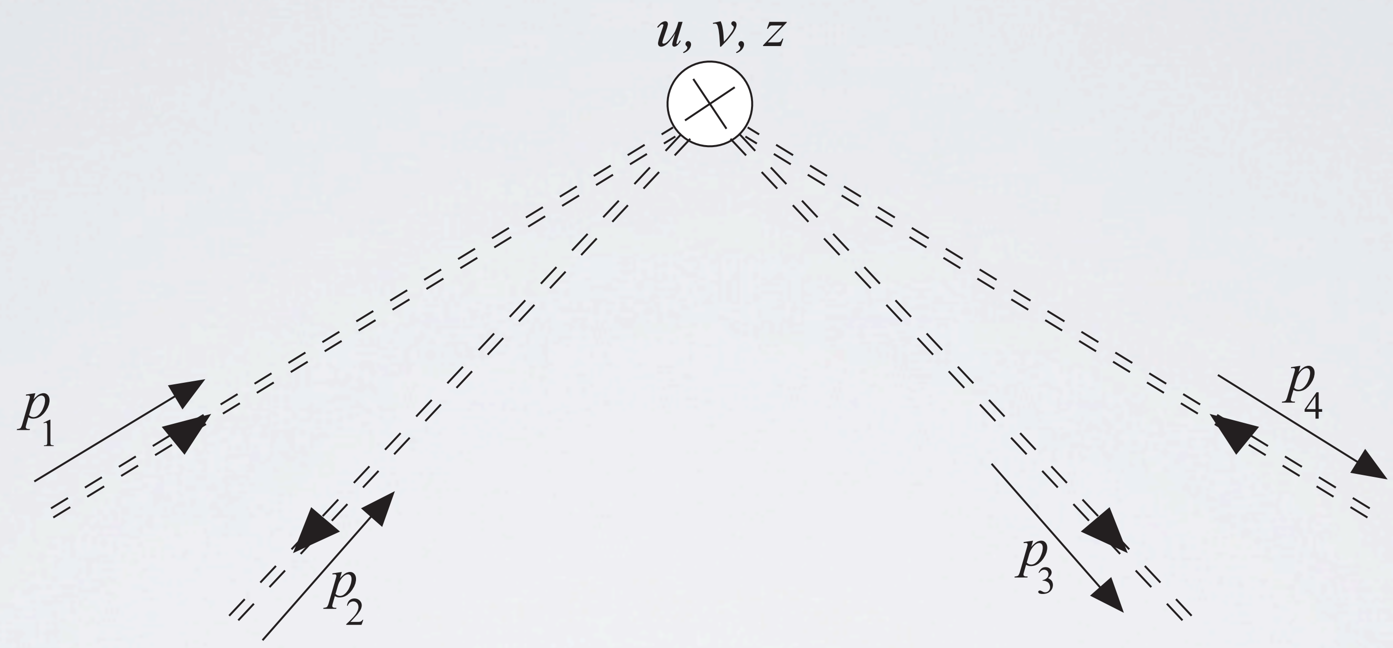


$$\langle 0|(\bar{\chi}_{n',\omega_2'}\tilde{\Gamma}^a\chi_{n',\omega_1'})\mathcal{P}_H^\dagger(p_\perp,y)\mathcal{P}_H(p_\perp,y)(\bar{\chi}_{n',\omega_4'}\tilde{\Gamma}^a\chi_{n',\omega_3'})|0\rangle$$

$$= -16\,\delta(\omega_1'-\omega_2'+\omega_3'-\omega_4')\int dz\,du\,dv\,\delta(\omega_1'-\omega_2'-\frac{\bar{n}'\cdot p}{z})$$

$$\times\delta(\omega_2'-\frac{\bar{n}'\cdot p}{2z}(2v-1)+\frac{\bar{n}'\cdot p}{2z})\delta(\omega_4'-\frac{\bar{n}'\cdot p}{2z}(2u-1)-\frac{\bar{n}'\cdot p}{2z})D_{Q\bar{Q}\to H}(u,v,z)$$

$D_{Q\bar{Q}\to H}(u, v, z)$



$$\bar{n}'\cdot p_1 = u\,\bar{n}'\cdot p_{Q\bar{Q}}$$

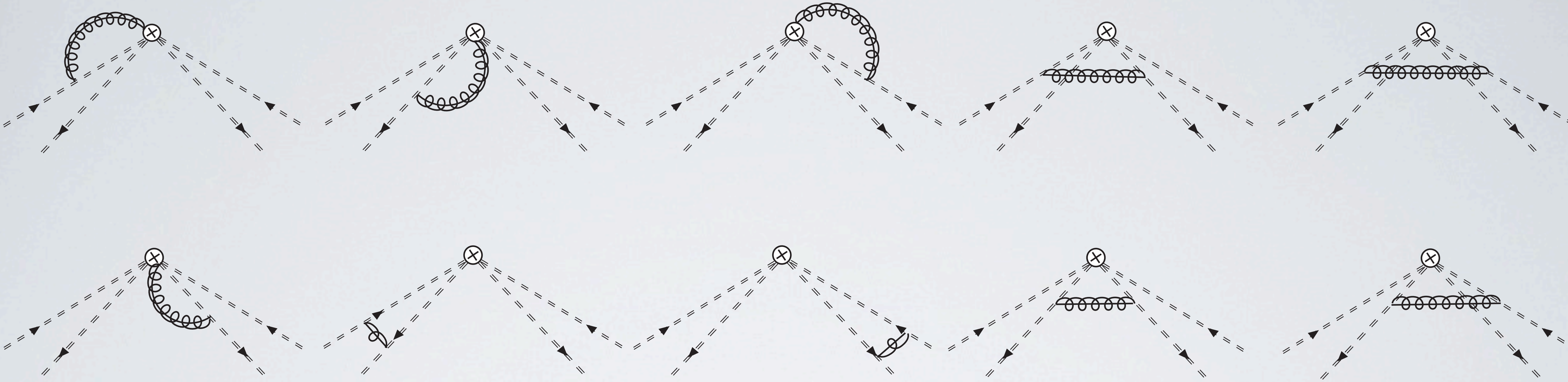
$$\bar{n}'\cdot p_3 = (1-v)\,\bar{n}'\cdot p_{Q\bar{Q}}$$

$$\bar{n}'\cdot p_2 = (1-u)\,\bar{n}'\cdot p_{Q\bar{Q}}$$

$$\bar{n}'\cdot p_4 = v\,\bar{n}'\cdot p_{Q\bar{Q}}$$

$$\bar{n}'\cdot p_H = z\bar{n}'\cdot p_{Q\bar{Q}}$$

RUNNING



Efremov-Radyushkin-Brodsky-Lepage Evolution in u, v

$$\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u,v,z;\mu) = \int_0^1 dw V(u,w;\mu) D^{[1]}_{Q\bar{Q}\to H}(w,v,z;\mu)$$

DGLAP Evolution in z

$$\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u,v,z;\mu) = \int_0^1 dx P_{Q\bar{Q}[1]\to Q\bar{Q}[8]}(x;\mu) D^{[8]}_{Q\bar{Q}\to H}(w,v,z/x;\mu)$$

MATCHING NRQCD

$$D_{g\to H}(z)$$

$$\longrightarrow \sum_i C_i(z,m_Q)\langle 0 \mid (\psi^\dagger S_{n'})\tilde{\Gamma}_i^b(S_{n'}^\dagger\chi)\mathcal{P}_H^\dagger(p_\perp,y)\mathcal{P}_H(p_\perp,y)(\chi^\dagger S_{n'})\tilde{\Gamma}_i^b(S_{n'}^\dagger\psi) \mid 0\rangle$$

LO

$$C\frac{\alpha_s}{m_Q^2}\delta(1-z)\langle 0 \mid (\chi^\dagger S_{n'})T^A\sigma^i(S_{n'}^\dagger\psi)\mathcal{P}_H^\dagger(p_\perp,y)\mathcal{P}_H(p_\perp,y)(\psi^\dagger S_{n'})T^A\sigma^i(S_{n'}^\dagger\psi) \mid 0\rangle$$

$$\chi^\dagger T^A\sigma^i\psi\mathcal{P}_H^\dagger(p_\perp,y)\mathcal{P}_H(p_\perp,y)\psi^\dagger T^A\sigma^i\chi$$

$$D^{[1,8]}_{Q\bar{Q}\to H}(u,v,z)$$

$$\longrightarrow \sum_i C_i(u,v,z,m_Q)\langle 0 \mid (\psi^\dagger S_{n'})\tilde{\Gamma}_i^b(S_{n'}^\dagger\chi)\mathcal{P}_H^\dagger(p_\perp,y)\mathcal{P}_H(p_\perp,y)(\chi^\dagger S_{n'})\tilde{\Gamma}_i^b(S_{n'}^\dagger\psi) \mid 0\rangle$$

Octet Matrix Elements Depend on Quarkonium Direction: Less Universal than Supposed

POLARIZATION FIX? MAYBE!

$$\frac{d\sigma}{dp_\perp}(p\bar{p} \to J/\psi(p_\perp) + X)$$

${}^3S_1^{[1]}$ $\alpha \to 0$

${}^3S_1^{[8]}$ $\alpha \to 1$

But $$\frac{\frac{d\sigma}{dp_\perp}({}^3S_1^{[1]})}{\frac{d\sigma}{dp_\perp}({}^3S_1^{[8]})} \sim \lambda^4$$ Extra Suppression

SCET Matching Coefficient at High Scale For a $Q\bar{Q}$ Configuration that results in a color-singlet ${}^3S_1$ $Q\bar{Q}$ pair

Expansion in $\alpha_s(p_\perp)$ and $\lambda \sim \dfrac{m_Q}{p_\perp}$

0

$$(C^{(1)}(\alpha_s^3) + C^{(2)}(\alpha_s^4) + \dots)\, D^{[1](LO)}_{Q\bar{Q}\to H}(u, v, z)$$

$$(\tilde{C}^{(1)}(\alpha_s^3) + \dots)\, D^{[1](NLO)}_{Q\bar{Q}\to H}(u, v, z)$$

POLARIZATION FIX? MAYBE!

However

$$D^{[8](LO)}_{Q\bar{Q}\to H}(u,v,z)$$

Has non-zero *LO* in α_s ($\propto \alpha_s^3$) matching coefficient

Generate color-singlet through running

$$\mu^2 \frac{d}{d\mu^2} D^{[8]}_{Q\bar{Q}\to H}(u,v,z;\mu) = \int_0^1 dx P_{Q\bar{Q}[8]\to Q\bar{Q}[1]}(x;\mu) D^{[1]}_{Q\bar{Q}\to H}(w,v,z/x;\mu)$$

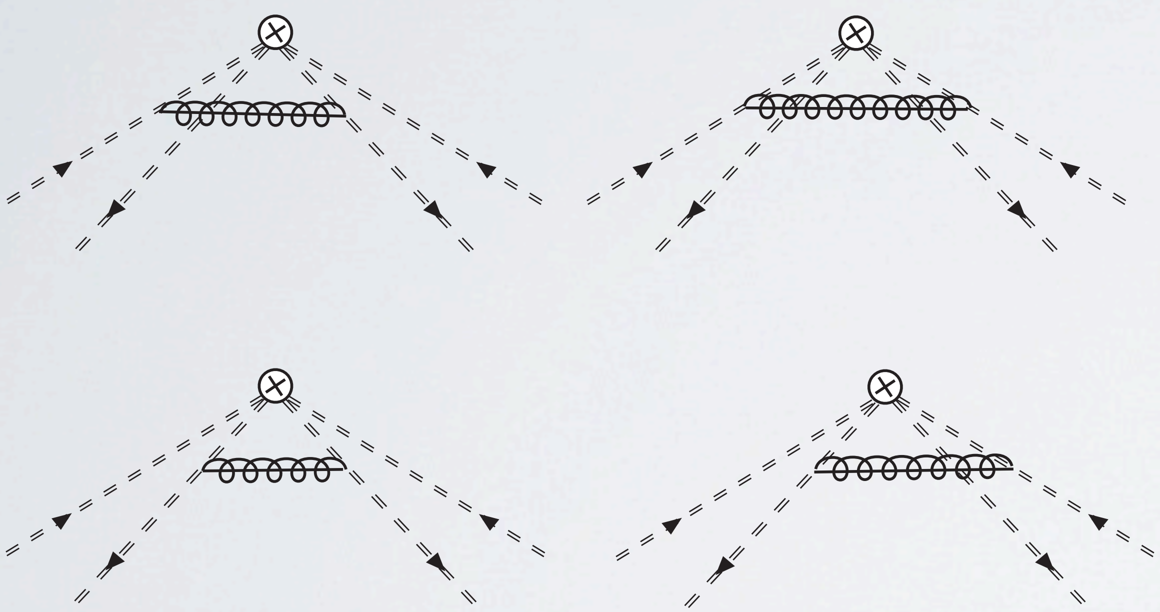
Will degrade the polarization... But is it enough?!?!?!

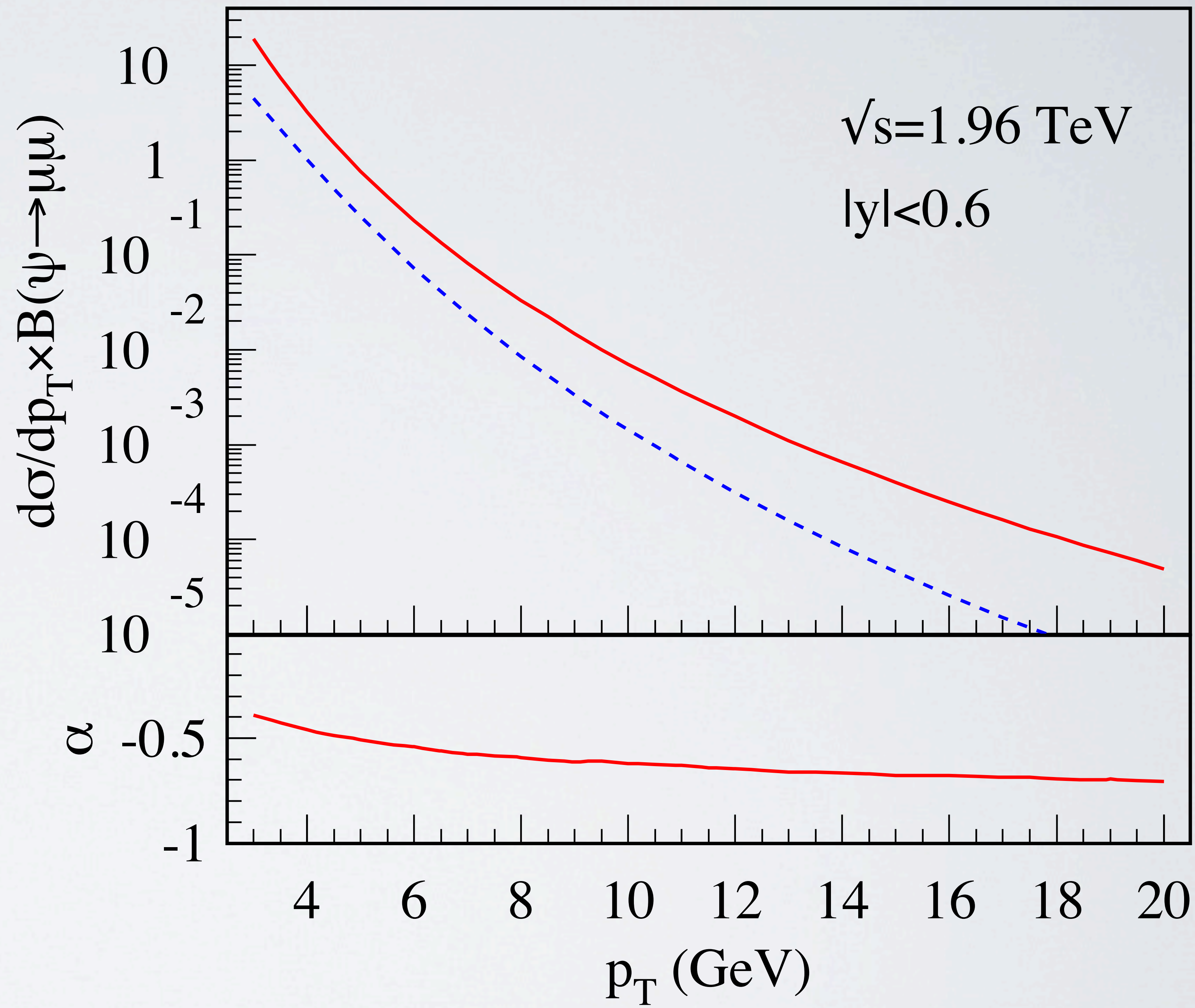
POLARIZATION FIX? MAYBE!

Kang, Qiu, Sterman

One-Loop Estimate

$Q\bar{Q}(8) \rightarrow Q\bar{Q}(1)$ Mixing





Still a factor of ~ 5 smaller than Octet Fragmentation

CONCLUSIONS

- Rigorous treatment of momentum region between $p_\perp$ and m_Q is crucial for an accurate description of Quarkonium production at high $p_\perp$
- May resolve the polarization problem, but this has to wait on a complete analysis