Precision Determination of $\alpha_s(m_Z)$ from Thrust Data

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M. Fickinger - UA

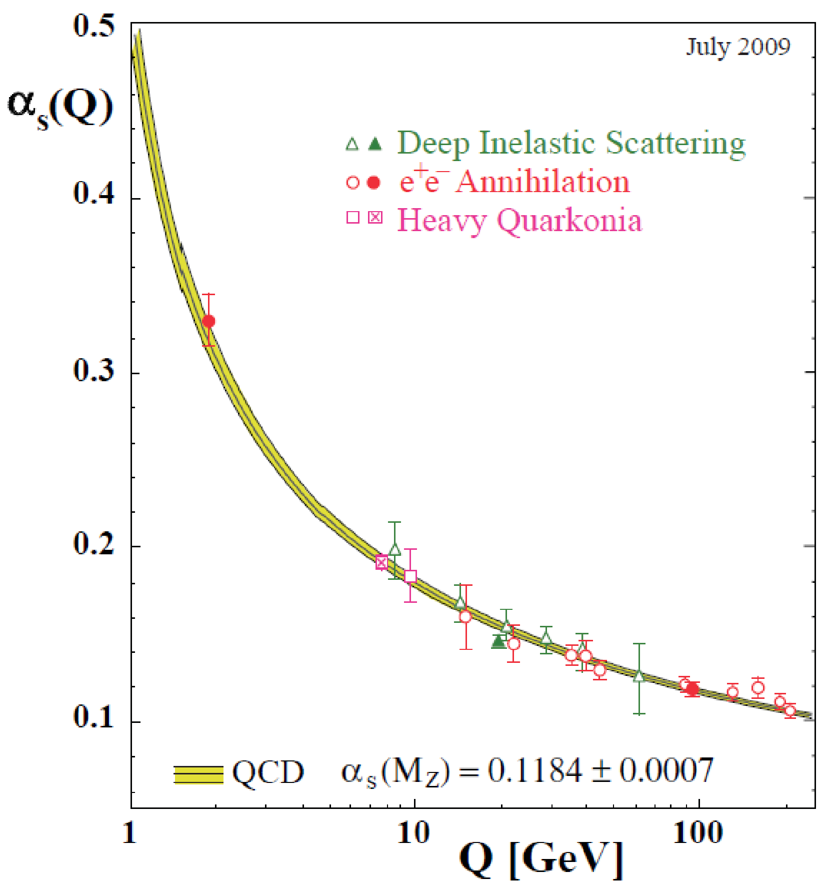
Frontiers in QCD, INT, 6th October 2011

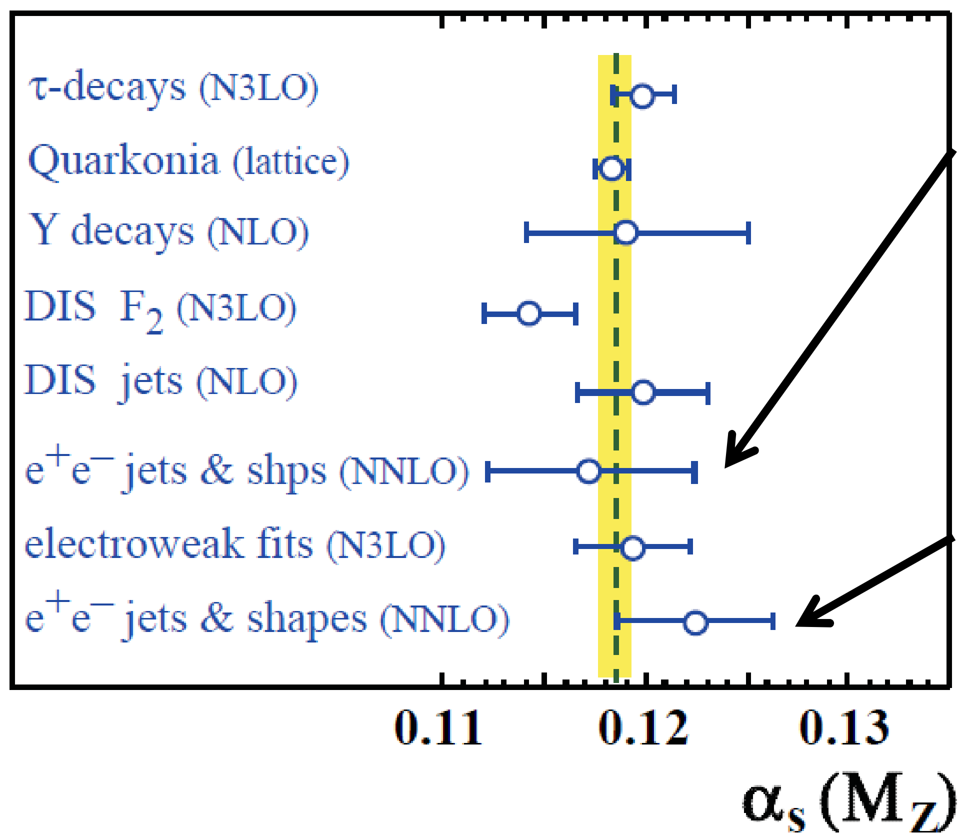
The Strong Coupling Constant α_S

α_S is a key parameter for the analysis of all collider experiments

QCD: $\alpha_S(M_Z)=0.1184(07)$

S. Bethke, Eur. Phys. J. C64 (2009) 689-703

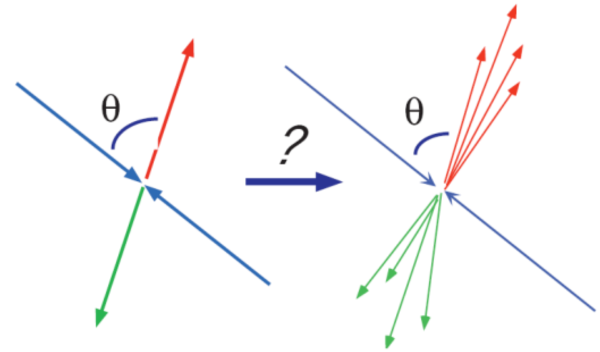




JADE
14-46 GeV
Bethke
et al.

ALEPH
90-206 GeV
Dissertori
et al.

Event shape variable: assigns a number to the shape of an event



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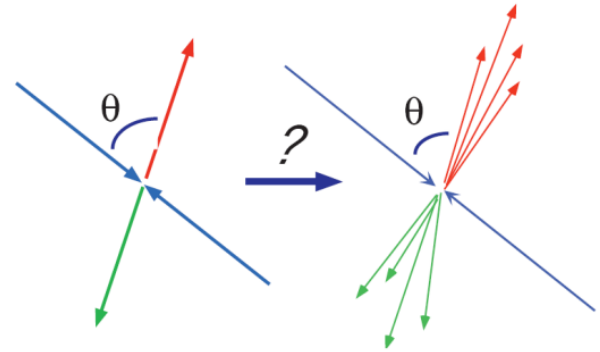
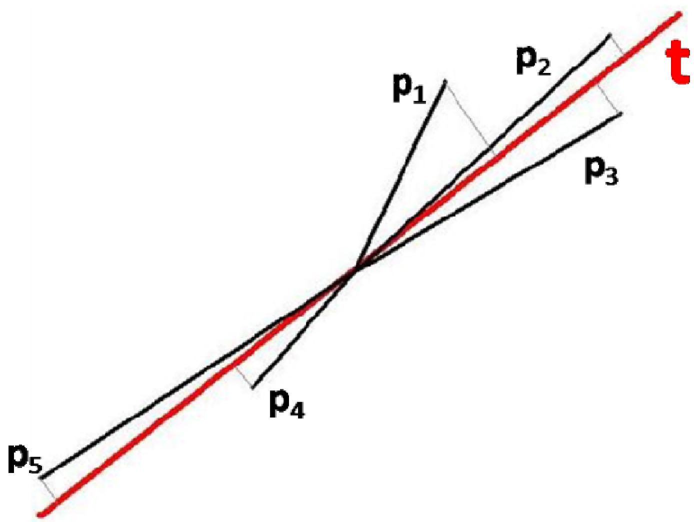


Thrust: $$T \equiv \max_{\hat{t}} \left[\frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|} \right]$$ use $\tau = 1 - T$



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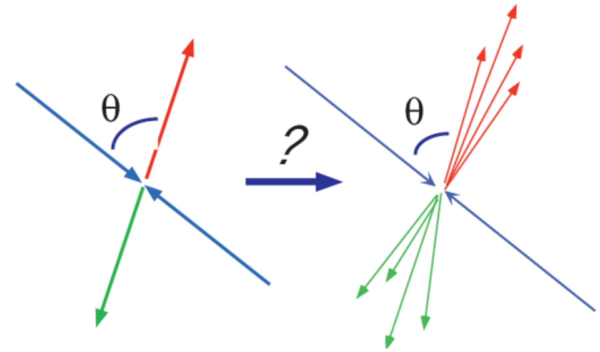

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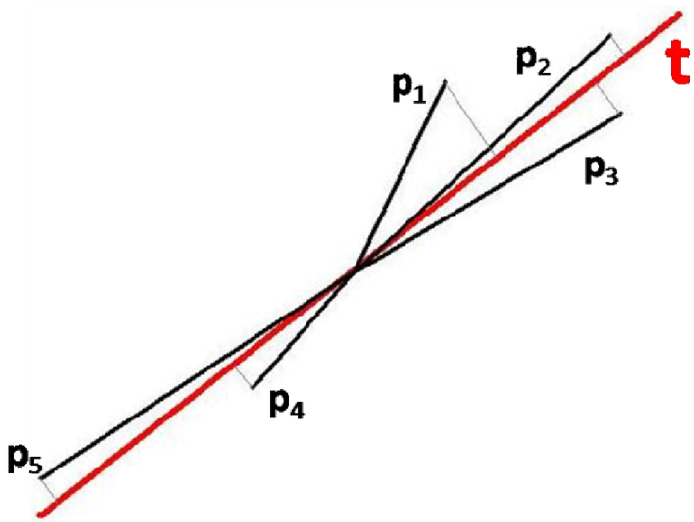
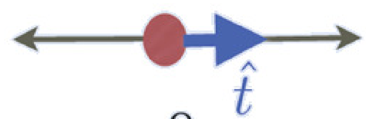
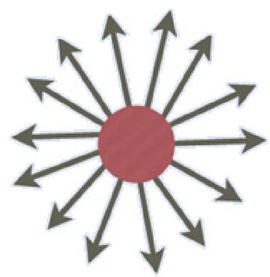


Measures 2-jet likeness



$\tau = 0$

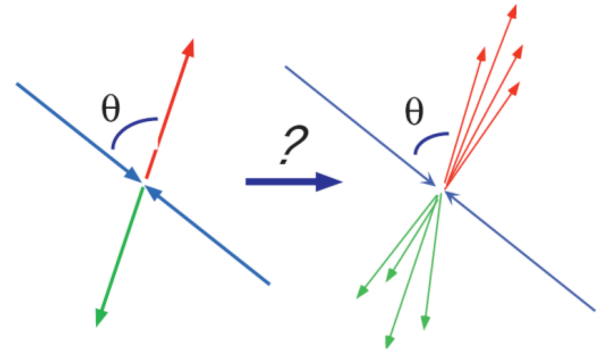
ideal 2-jet event



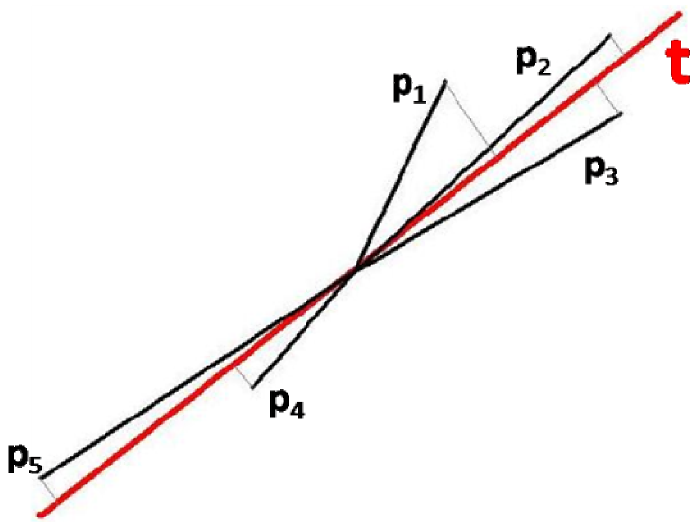
$\tau = \frac{1}{2}$

spherical event

Event shape variable: assigns a number to the shape of an event



Thrust: $T \equiv \max_{\hat{t}} \left[\frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|} \right]$ use $\tau = 1 - T$



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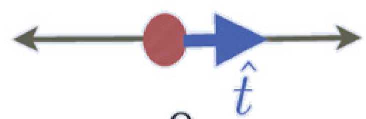
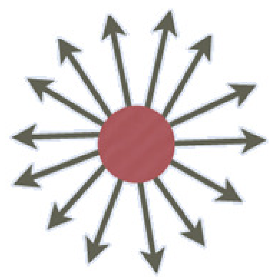


$\tau = 0$ ideal 2-jet event



$\tau = \frac{1}{2}$ spherical event

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_S\left(c_1\delta(\tau) + R_1(\tau > 0)\right) + \ldots$$

→ Observable is very sensitive to α_S !

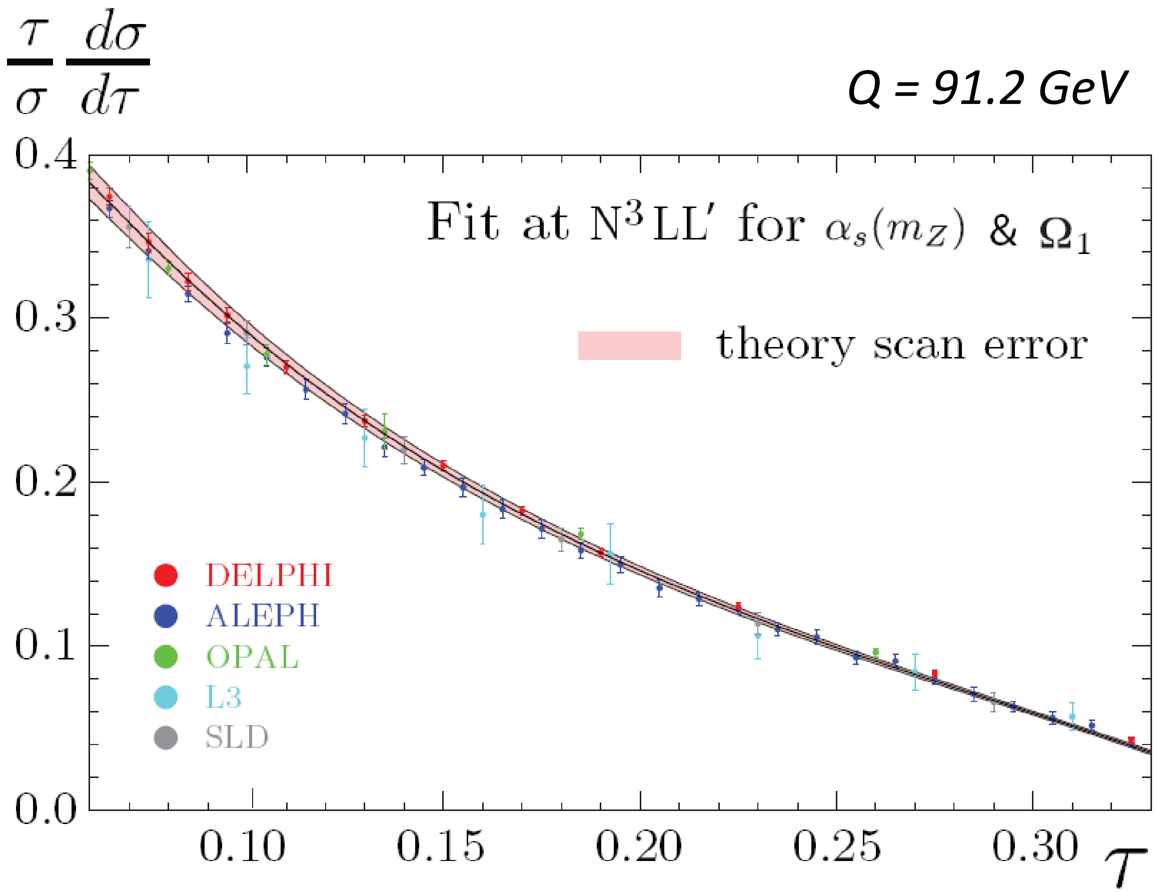
Outline

- Results
- Theory
- Numerical Analysis
- Moment Fits
- Summary

Results

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\mathrm{expt}} \pm (0.0005)_{\mathrm{hadr}} \pm (0.0009)_{\mathrm{pert}}$$

$$\text{with} \quad \frac{\chi^2}{\mathrm{dof}} = 0.91$$



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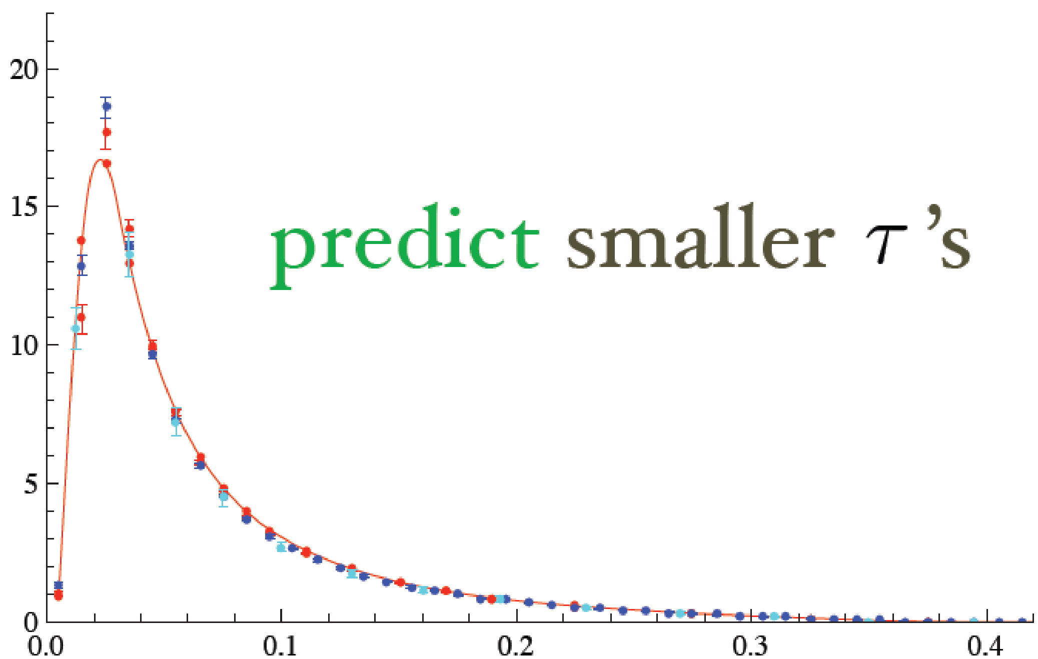
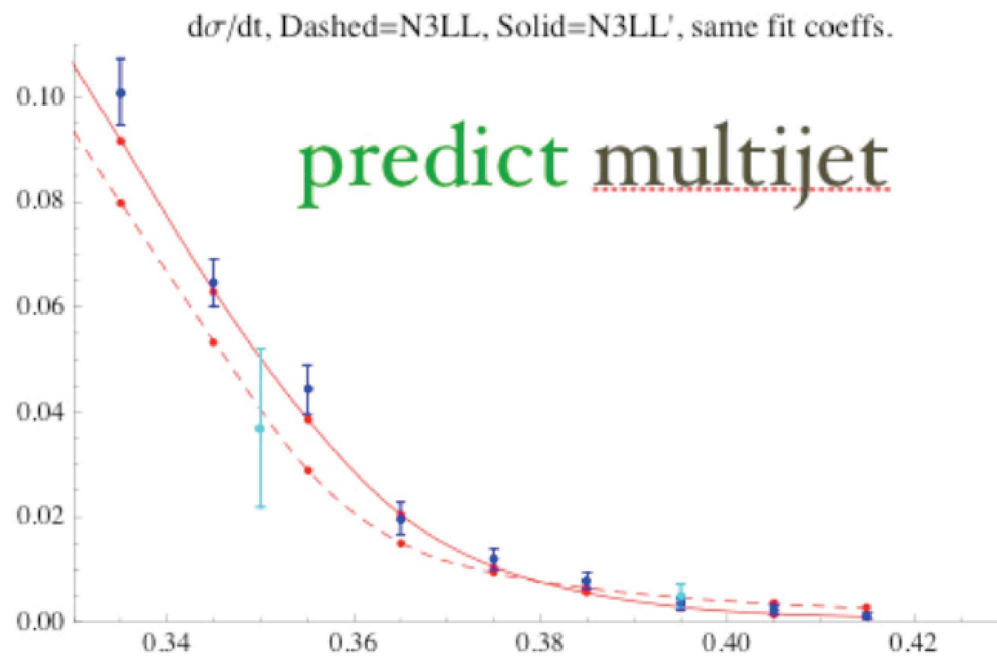




Q = 91.2 GeV

Fit to Distribution:

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{expt} \pm (0.0005)_{hadr} \pm (0.0009)_{pert}$$

with $\frac{\chi^2}{dof} = 0.91$

487 data points

$$\Omega_1(\mu_\Delta, R_\Delta) = 0.323 \pm 0.052$$

$\mu_\Delta = R_\Delta = 2GeV$

---

preliminary

Fit to First Moment:

$$\alpha_s(m_Z) = 0.1142 \pm (0.0015)_{expt} \pm (0.0005)_{hadr} \pm (0.0007)_{pert}$$

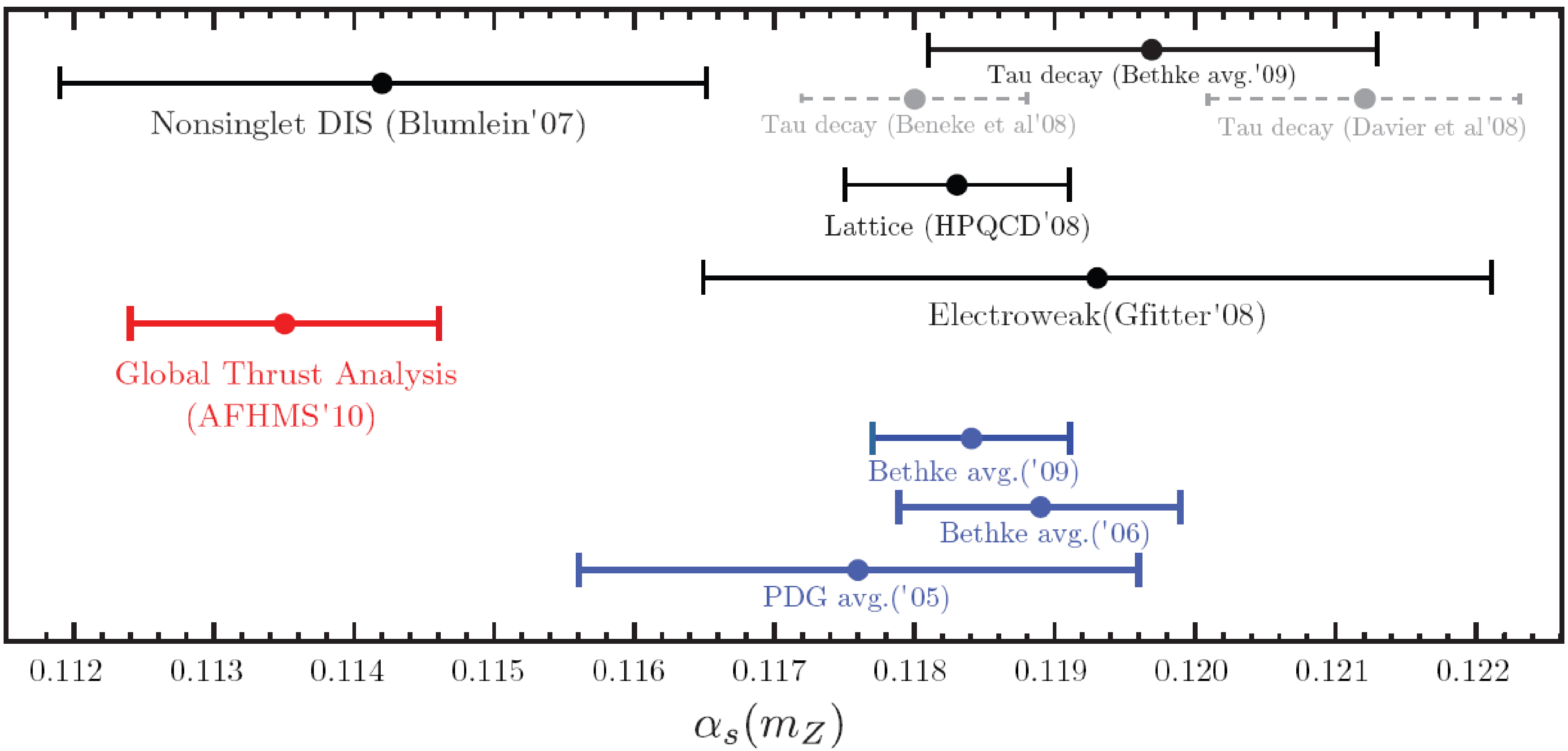
with $\frac{\chi^2}{dof} = 1.10$

34 data points

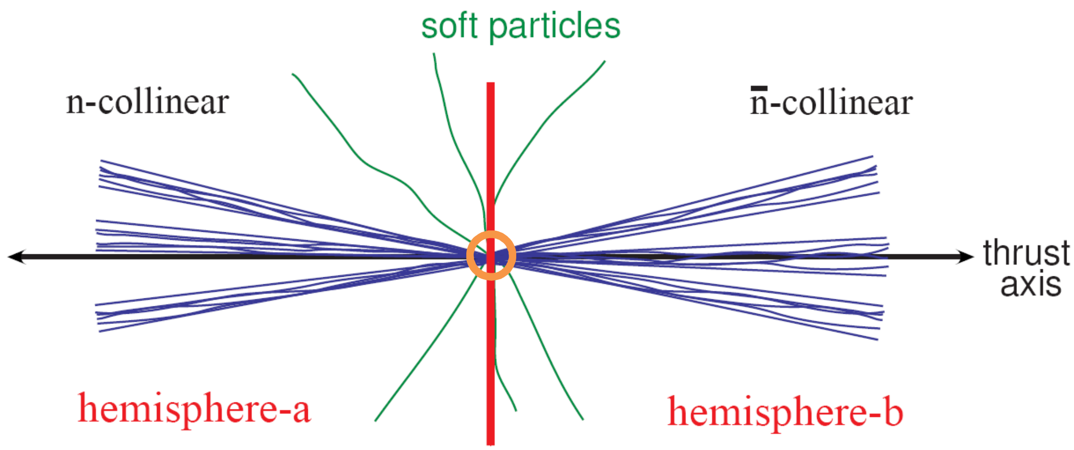
$$\Omega_1(\mu_\Delta, R_\Delta) = 0.388 \pm 0.061$$

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{expt} \pm (0.0005)_{hadr} \pm (0.0009)_{pert}$$

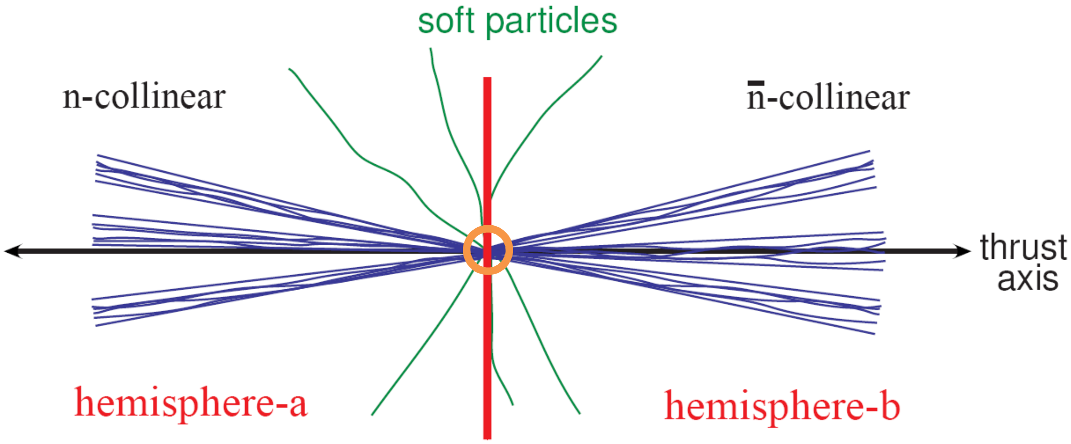
$$\text{with} \quad \frac{\chi^2}{\text{dof}} = 0.91$$



Theory



- hard scale (cm energy)
- jet scale (invariant mass of all energetic particles in one hemisphere)
- soft scale (uniform soft radiation)



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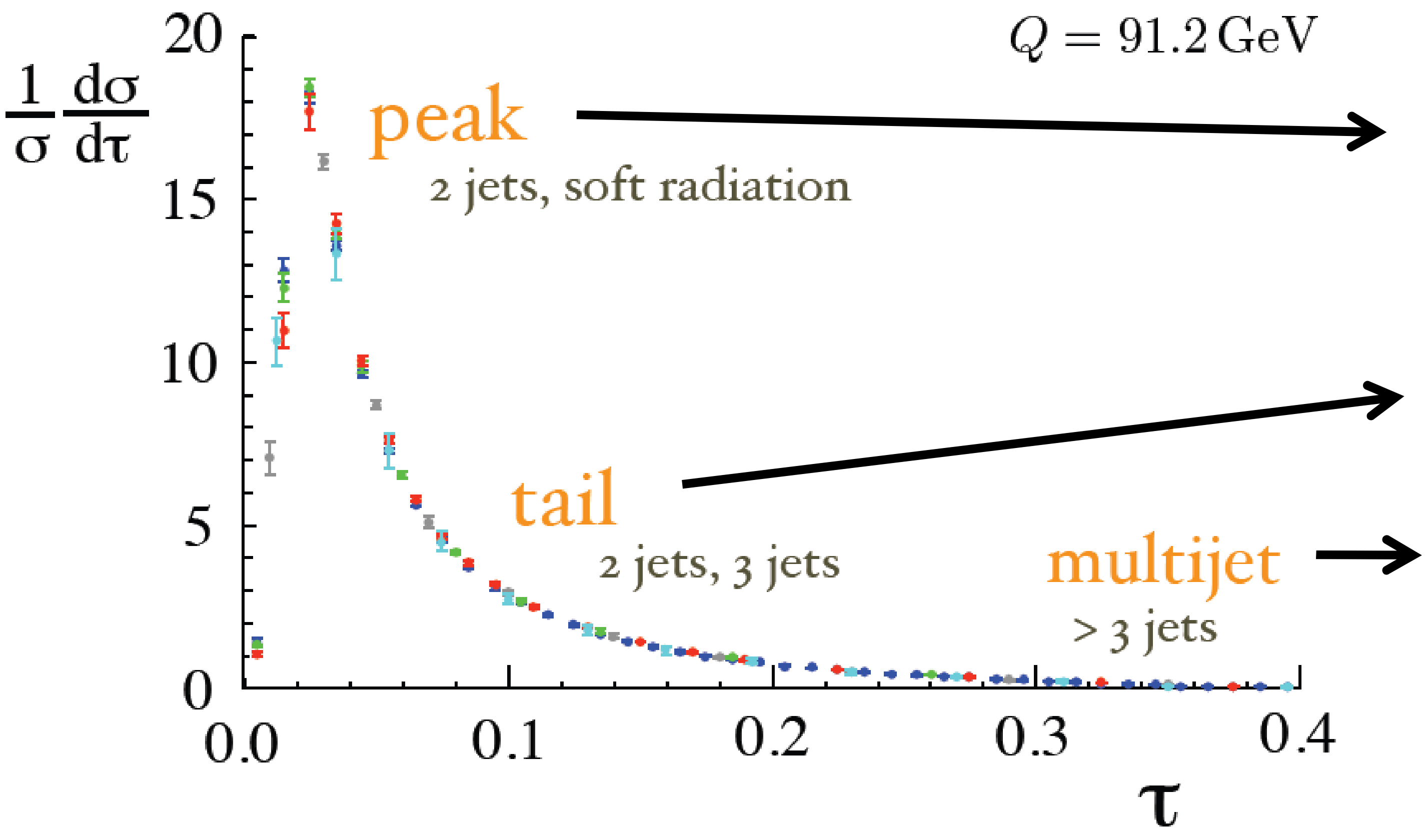


$$Q >> Q\sqrt{\Lambda_{QCD}/Q} >> \Lambda_{QCD}$$

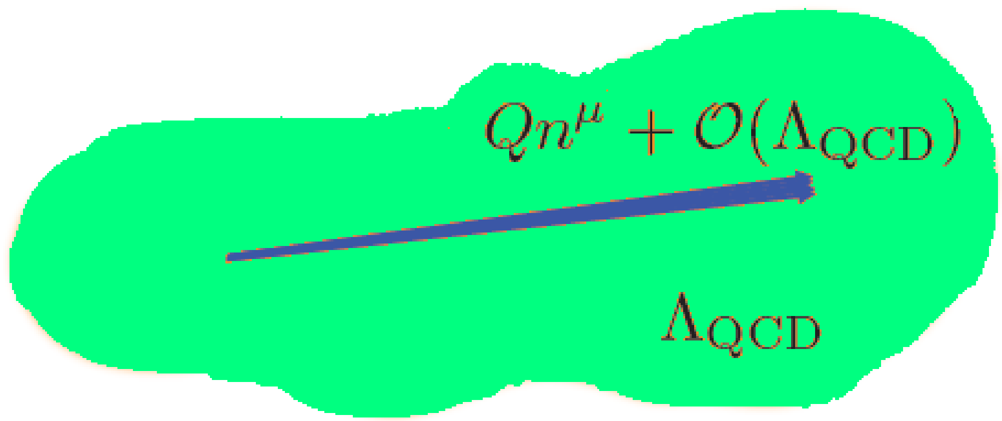
$$Q >> Q\sqrt{\tau} >> Q\tau >> \Lambda_{QCD}$$

$$Q >> \Lambda_{QCD}$$

Soft Collinear Effective Theory (SCET)

Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart

- Describes light-like particles (collinear) interacting with a low energetic background (soft)



- Expansion in $\lambda \approx \sqrt{\frac{\Lambda_{QCD}}{Q}}$
- Power counting:

soft: $p_\mu = (p_+, p_-, p_\perp) \propto Q(\lambda^2, \lambda^2, \lambda^2)$

collinear: $p_\mu = (p_+, p_-, p_\perp) \propto Q(\lambda^2, 1, \lambda)$

Perturbative Series

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_S R_1(\tau) + \alpha_S^2 R_2(\tau) + \alpha_S^3 R_3(\tau) + \dots$$

Perturbative Series

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_S R_1(\tau) + \alpha_S^2 R_2(\tau) + \alpha_S^3 R_3(\tau) + \ldots$$

Problems if: $\alpha_S \gtrsim 1$

coefficients not of *O(1)*

Perturbative Series

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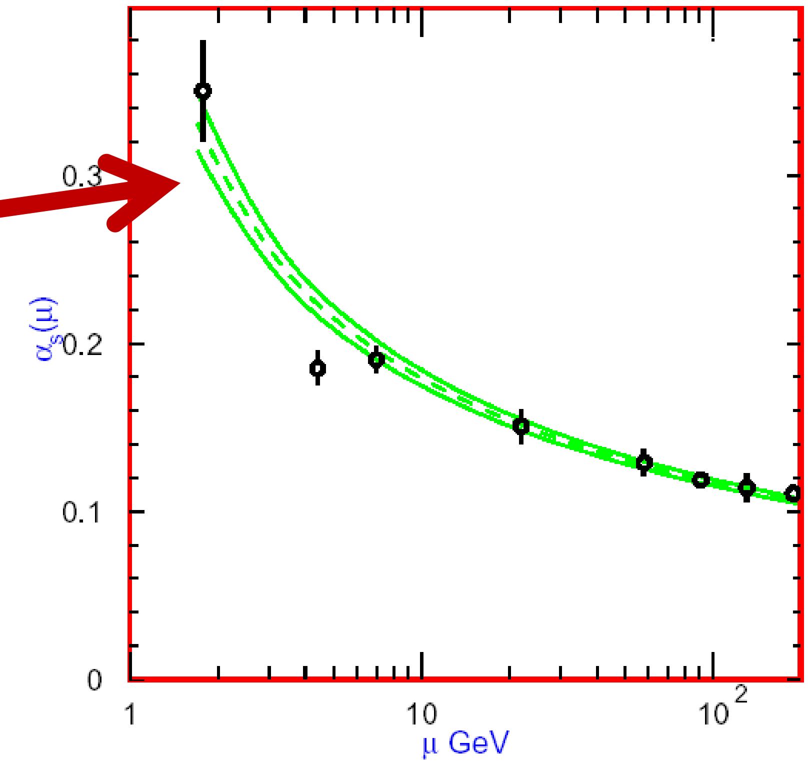
coefficients not of *O(1)*

$$R_i(\tau) \propto \frac{\ln^n(\tau)}{\tau}$$



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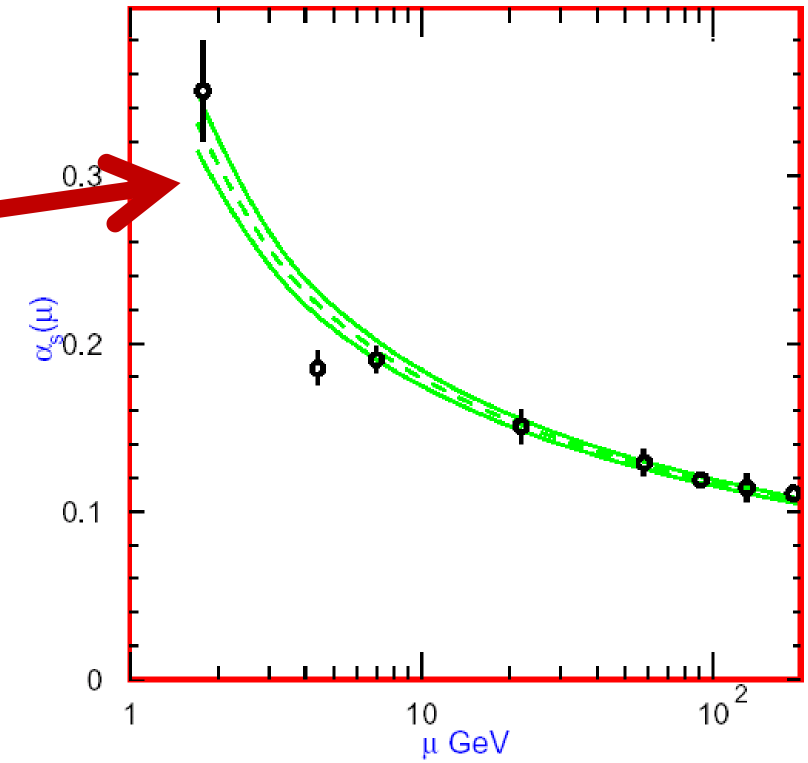

Problems if: $\alpha_S \gtrsim 1$

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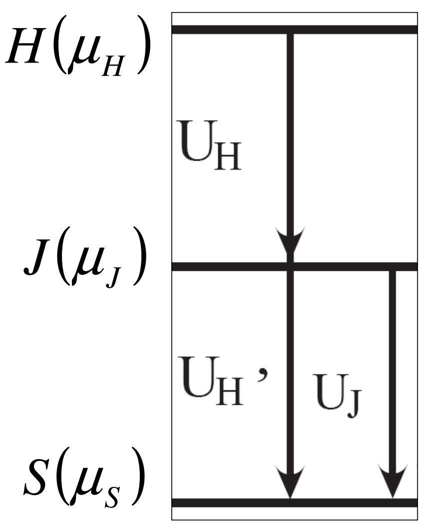
$$R_i(\tau) \propto \frac{\ln^n(\tau)}{\tau}$$

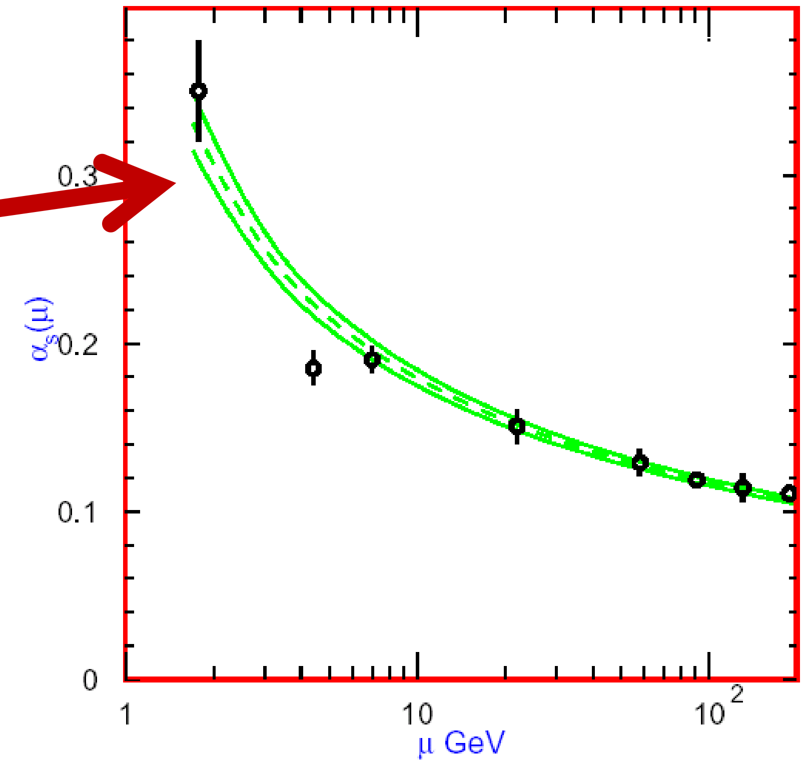
SCET gives factorization:



$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_s}{d\tau} \propto H(\mu_H) \otimes J(\mu_J) \otimes S(\mu_S) \otimes S_{\text{mod}}(\Lambda_{QCD})$$

Perturbative Series

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_s R_1(\tau) + \alpha_s^2 R_2(\tau) + \alpha_s^3 R_3(\tau) + \ldots$$



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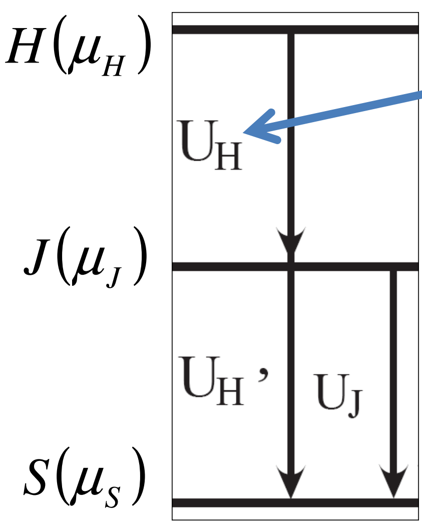


U_i from RGE

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_s}{d\tau} \propto H(\mu_H) \otimes J(\mu_J) \otimes S(\mu_S) \otimes S_{\text{mod}}(\Lambda_{QCD})$$

SCET reorganizes the power series in terms of $(\alpha_S^m \ln^n \tau)$

Factorization Theorem for Thrust

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \tfrac{k}{Q}\right) S_\tau^{\text{mod}}\left(k - 2\overline{\Delta}\right) + O\left(\sigma_0 \, \tfrac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

Methode to simultaneously describe all three regions and multiple Q:

profile functions

Singular partonic:

NNLO matching, including full 3-loop hard function

N^3LL resummation of large logs

Nonsingular partonic:

fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:

nonperturabtive effects treated within field theory

Operator Product Expansion in tail → Ω_1

Interface between power and perturbative corrections:

renormalon subtraction

b-mass effects (~2% effect)

QED effects (~2% effect)

axial anomaly at $O(\alpha_S^2)$ (~1% effect)

Factorization Formula for all Thrust

peak: sum large logs, nonperturbative soft fct.

$Q \gg Q\sqrt{\Lambda_{QCD}/Q} \gg \Lambda_{QCD}$

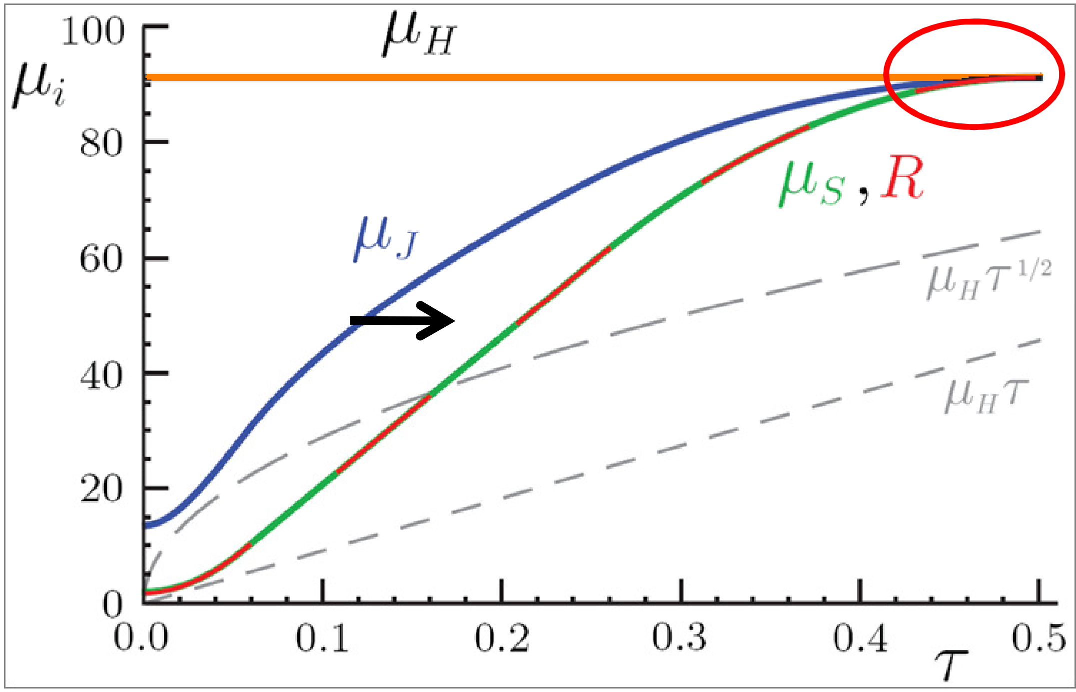
tail: sum large logs, series of nonperturbative power corrections

$Q \gg Q\sqrt{\tau} \gg Q\tau \gg \Lambda_{QCD}$

far tail: fixed order perturbation theory, power corrections

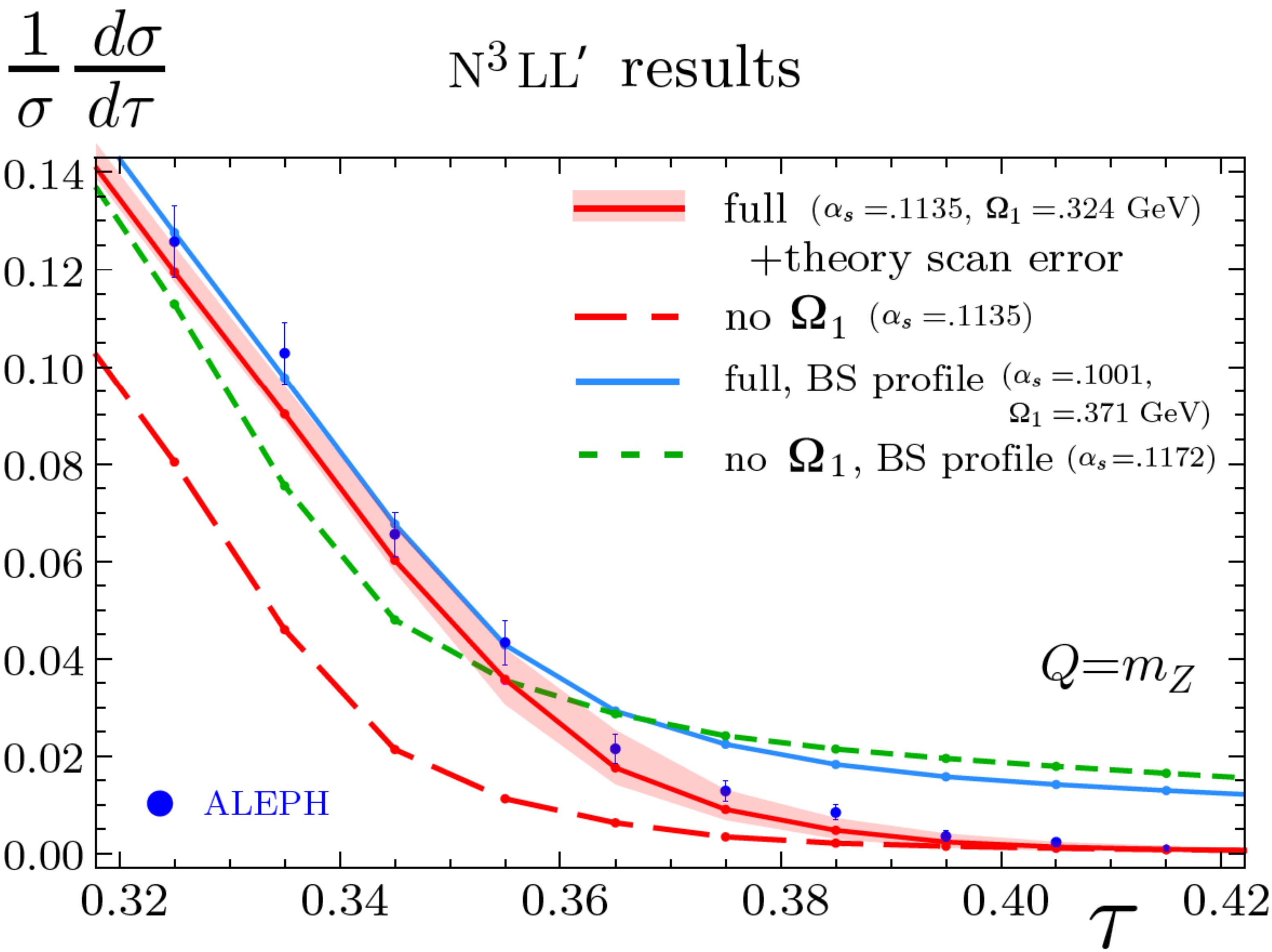
$Q \gg \Lambda_{QCD}$

Profile Functions:



Scales must equal in the far tail: turns of resummation

Far Tail



$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
N^{3}LL′ results
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
full (α_s =.1135, Ω_1 =.324 GeV)
+theory scan error
no Ω_1 (α_s =.1135)
full, BS profile (α_s =.1001,
Ω_1 =.371 GeV)
no Ω_1, BS profile (α_s=.1172)
Q=m_Z
ALEPH
0.32
0.34
0.36
0.38
0.40
0.42
τ

Factorization Theorem for Thrust

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \tfrac{k}{Q}\right) S_\tau^{\mathrm{mod}} \left(k - 2\overline{\Delta}\right) + O\left(\sigma_0 \, \frac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

Methode to simultaneously describe all three regions and multiple Q:

profile functions

Moch, Vermaseren, Vogt
Becher, Neubert
Schwartz; Fleming et al.
Becher, Schwartz; Hoang, Kluth
Dasgupta, Salam

Singular partonic: Baikov et al.

NNLO matching, including full 3-loop hard function

N^3LL resummation of large logs

Nonsingular partonic: Becher, Schwartz

fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:

nonperturabtive effects treated within field theory

Operator Product Expansion in tail → Ω_1

Interface between power and perturbative corrections:

renormalon subtraction

b-mass effects (~2% effect)

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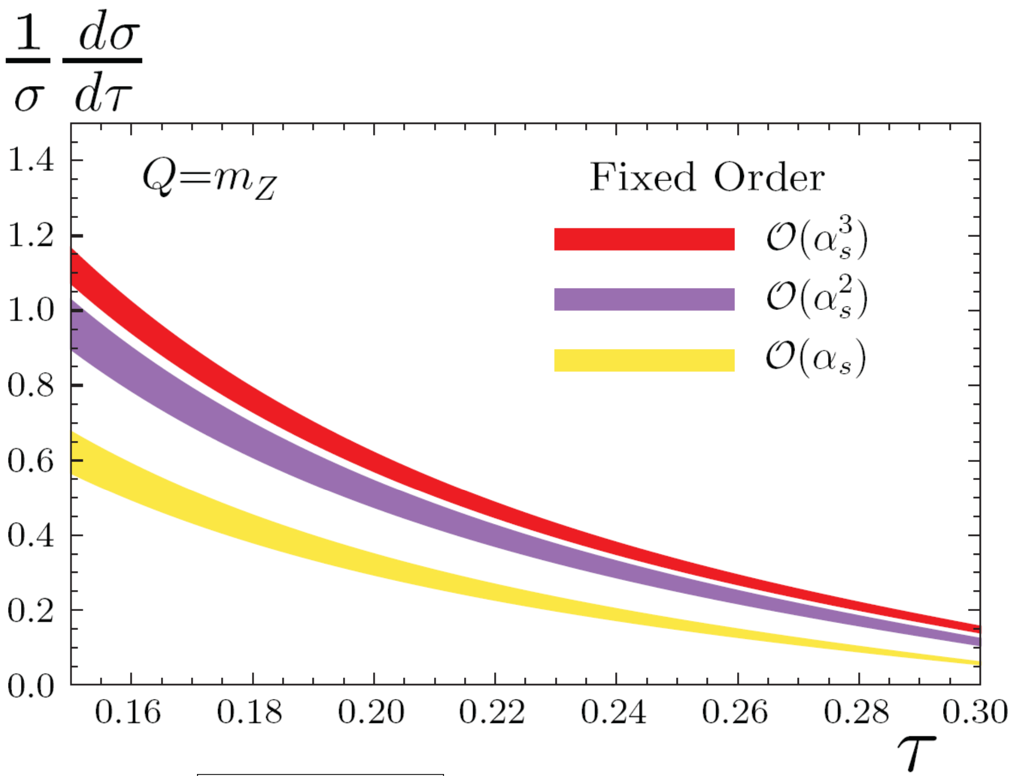
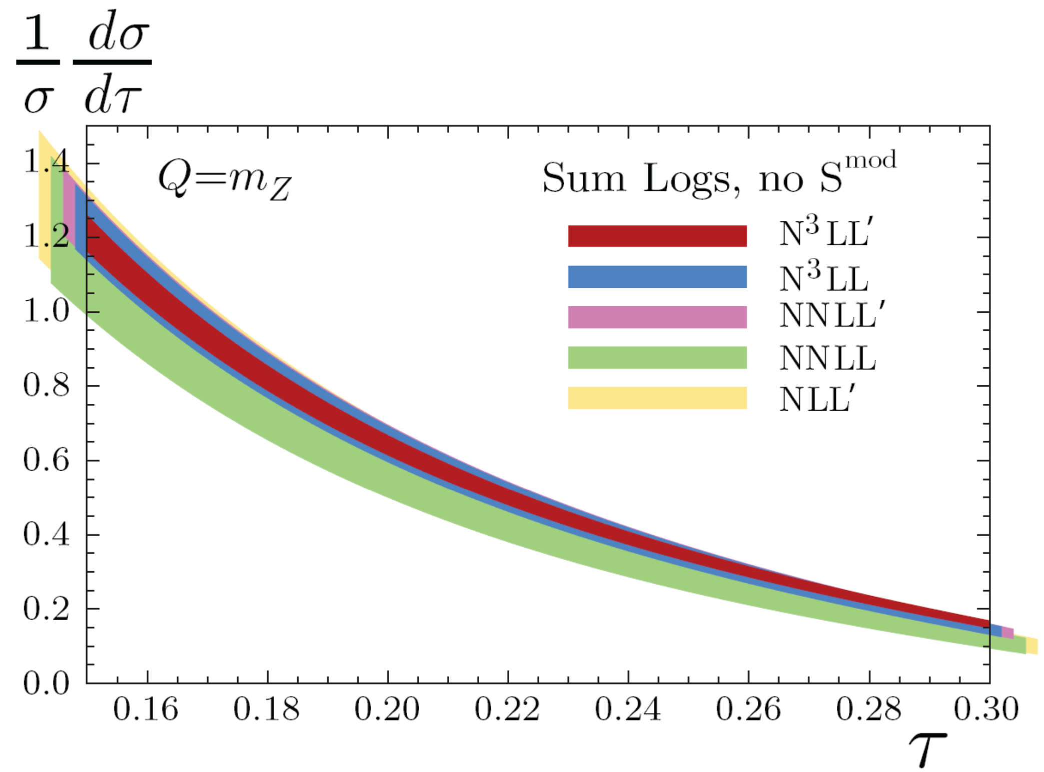
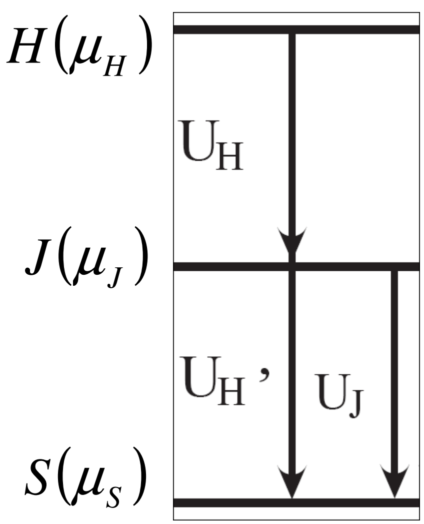
Summing large Logarithms







$$H(\mu_H) \propto \ln^n\left(\frac{\mu_H}{Q}\right)$$

$$J(\mu_J) \propto \ln^n\left(\frac{\mu_J}{Q\sqrt{\tau}}\right)$$

$$S(\mu_S) \propto \ln^n\left(\frac{\mu_S}{Q\tau}\right)$$

$$\ln(U_i) \propto \ln^n\left(\frac{\mu_H}{Q}\right), \ln^n\left(\frac{\mu_J}{Q\sqrt{\tau}}\right), \ln^n\left(\frac{\mu_S}{Q\tau}\right)$$

Setting μ_H, μ_J, μ_S to their natural scales

→ much better convergence!

Factorization Theorem for Thrust

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \tfrac{k}{Q}\right) S_\tau^{\mathrm{mod}}\left(k - 2\overline{\Delta}\right) + O\left(\sigma_0 \frac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

Methode to simultaneously describe all three regions and multiple Q:

profile functions

Singular partonic:

NNLO matching, including full 3-loop hard function

N^3LL resummation of large logs

Nonsingular partonic: *Ellis et al., Catani et al., Gehrmann et al., Weinzierl*

fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:

nonperturabtive effects treated within field theory

Operator Product Expansion in tail → Ω_1

Interface between power and perturbative corrections:

renormalon subtraction

$$\frac{d\hat{\sigma}_{ns}}{d\tau} = \left.\frac{d\sigma}{d\tau}\right|_{fixed-order} - \left.\frac{d\hat{\sigma}_s}{d\tau}\right|_{no-resum.}$$

b-mass effects (~2% effect)

QED effects (~2% effect)

axial anomaly at $O(\alpha_s^2)$ (~1% effect)

Factorization Theorem for Thrust

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Operator Product Expansion in tail → Ω_1

Hoang, Stewart

Ligeti, Stewart, Tackmann

Lee, Sterman

Korchemsky, Sterman

Interface between power and perturbative corrections:

renormalon subtraction

b-mass effects (~2% effect)

QED effects (~2% effect)

axial anomaly at $O(\alpha_S^2)$ (~1% effect)

Nonperturbative Corrections

Soft function from SCET factorization:

$$S_\tau(k,\mu) = \tfrac{1}{N_c}\left\langle 0 \left| Tr\,\overline{Y}_{\bar{n}} Y_n \delta(k - i\partial_\tau) Y_n^+ \overline{Y}_{\bar{n}}^+ \right| 0 \right\rangle$$

$$i\partial_\tau \equiv \theta(i\bar{n}\cdot\partial - in\cdot\partial)\, in\cdot\partial + \theta(in\cdot\partial - i\bar{n}\cdot\partial)\, i\bar{n}\cdot\partial$$

Factorization in perturbative and non-perturbative part:

$$S_\tau(k,\mu) = \int dk' S_\tau^{part}(k-k',\mu) S_\tau^{\mathrm{mod}}(k')$$

Hoang, Stewart
Ligeti, Stewart, Tackmann

complete basis of functions

OPE: $$S_\tau(k,\mu) = S_\tau^{part}(k,\mu) - 2\overline{\Omega}_1 \frac{dS_\tau^{part}}{dk}(k,\mu) + \ldots$$

Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution $\tau \to \tau - 2\Lambda / Q$

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - 2\tfrac{\Lambda}{Q}\right)$$

Manohar, Wise; Webber;
Dokshitzer, Weber; Akhoury, Zakharov;
Nason, Seymour; Korchemsky, Sterman;
Movilla Fernandez, Bethke, Biebel, Kluth

Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution $\tau \to \tau - 2\Lambda/Q$

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - 2\tfrac{\Lambda}{Q}\right) \approx h(\tau) - 2\tfrac{\Lambda}{Q}h'(\tau) \qquad \Lambda/Q \ll 1$$

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$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - 2\tfrac{\Lambda}{Q}\right) \approx h(\tau) - 2\tfrac{\Lambda}{Q}\,h'(\tau) \qquad \Lambda / Q \ll 1$$

h proportional to α_S =>

$$\frac{\delta\alpha_S}{\alpha_S} \approx \frac{h\left(\tau - 2\tfrac{\Lambda}{Q}\right) - h(\tau)}{h(\tau)} \approx -2\frac{\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$

Are non-perturbative effects negligible?

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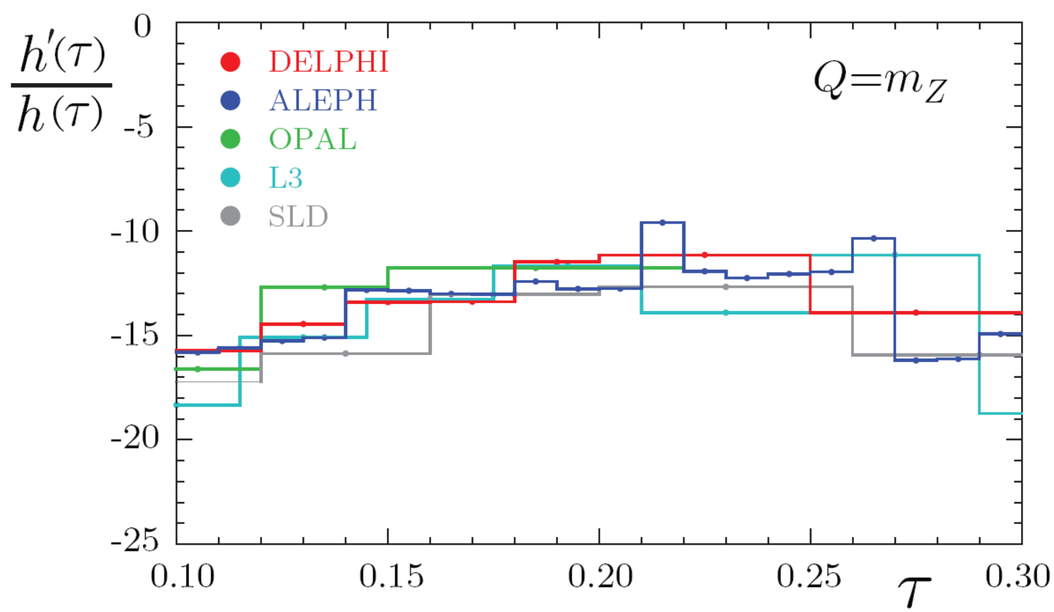
h proportional to α_s =>
$$\frac{\delta\alpha_s}{\alpha_s} \approx \frac{h\left(\tau - 2\tfrac{\Lambda}{Q}\right) - h(\tau)}{h(\tau)} \approx -2\frac{\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$



Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution $\tau \to \tau - 2\Lambda/Q$

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - 2\tfrac{\Lambda}{Q}\right) \approx h(\tau) - 2\tfrac{\Lambda}{Q}h'(\tau) \qquad \Lambda/Q \ll 1$$

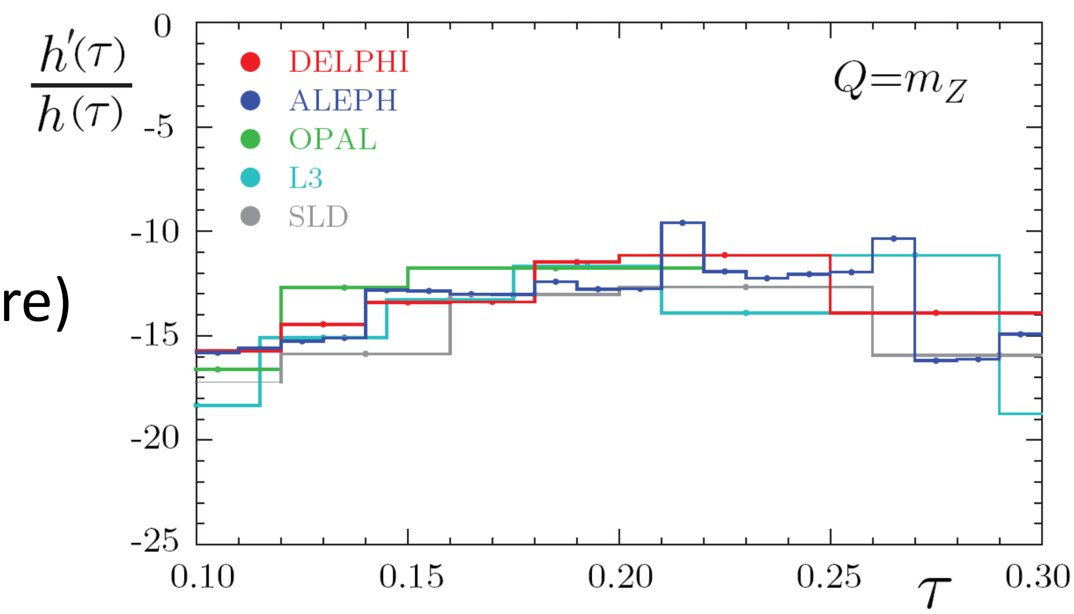
h proportional to α_S =>

$$\frac{\delta\alpha_S}{\alpha_S} \approx \frac{h\left(\tau - 2\tfrac{\Lambda}{Q}\right) - h(\tau)}{h(\tau)} \approx -2\frac{\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$

$\Lambda \approx 0.3\ GeV$ and

$h'/h \approx -14 \pm 4$ in tail region(see figure)

$\Rightarrow \delta\alpha_S/\alpha_S \approx -(9 \pm 3)\%$



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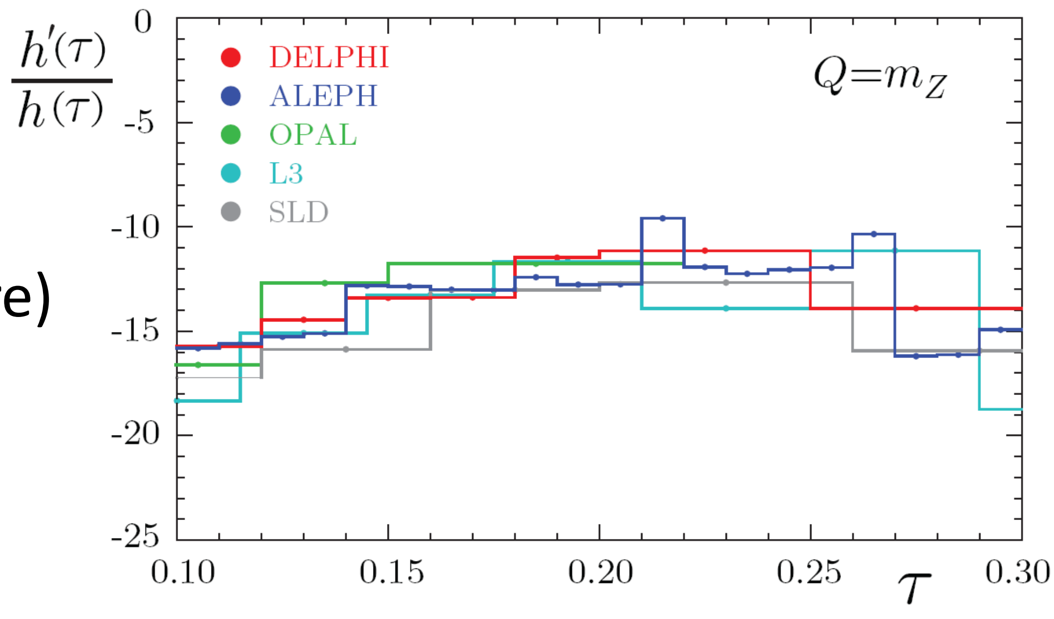
$\Lambda \approx 0.3\ GeV$ and

$h'/h \approx -14\pm4$ in tail region(see figure)

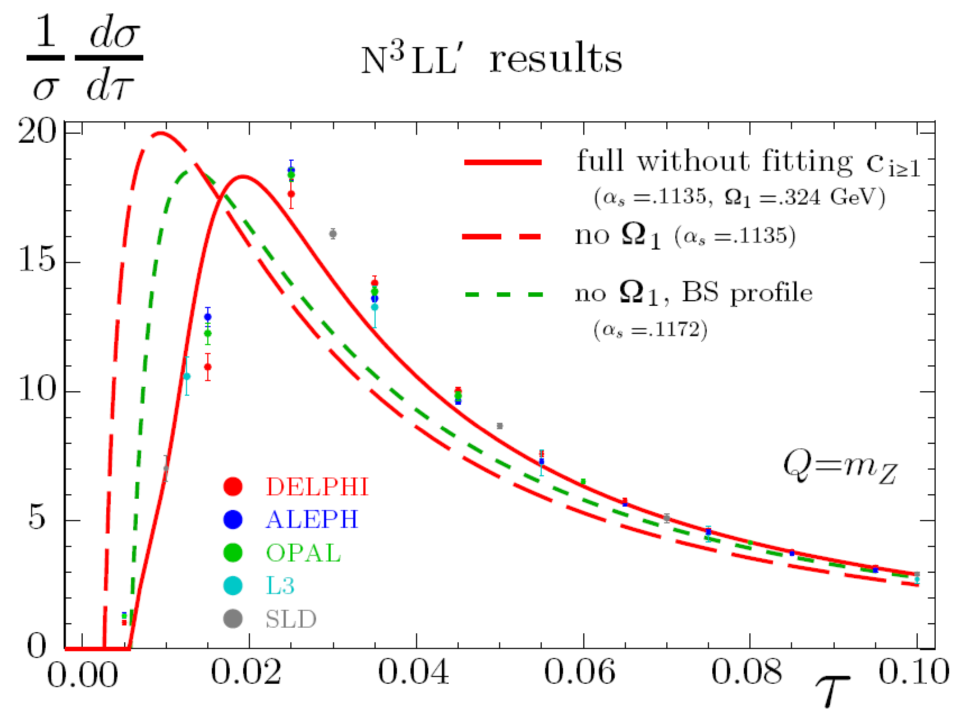
$\Rightarrow \delta\alpha_s/\alpha_s \approx -(9\pm3)\%$

$\Rightarrow$ **NOT** negligible for *2%* analysis



Non-perturbative effects

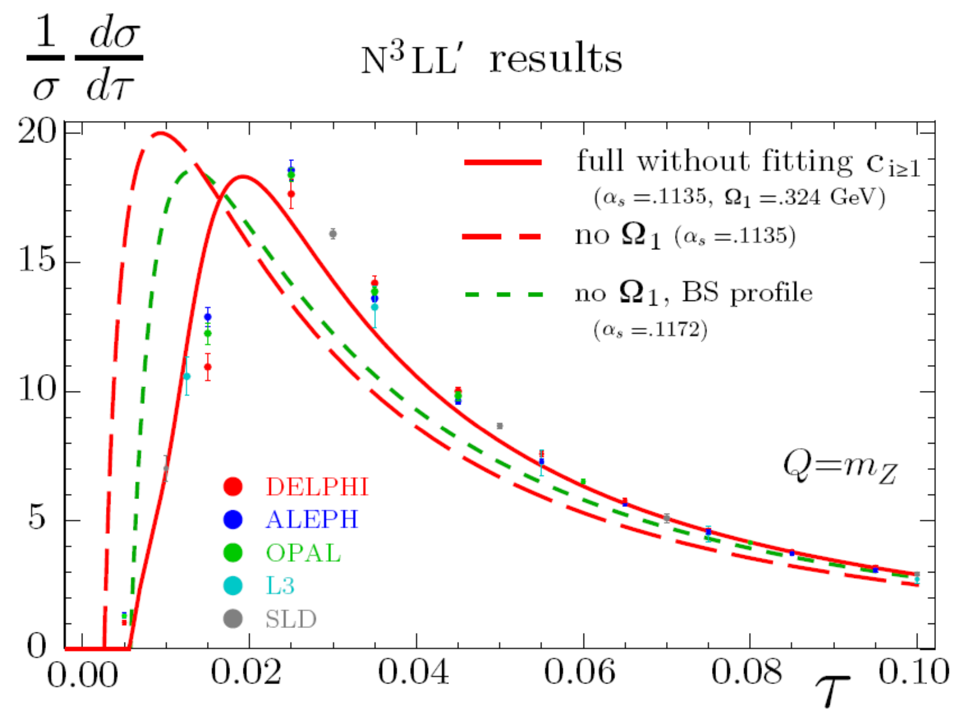
Use tail fit results to predict peak



Non-perturbative effects

Use tail fit results to predict peak

$\Rightarrow$ much better prediction for peak

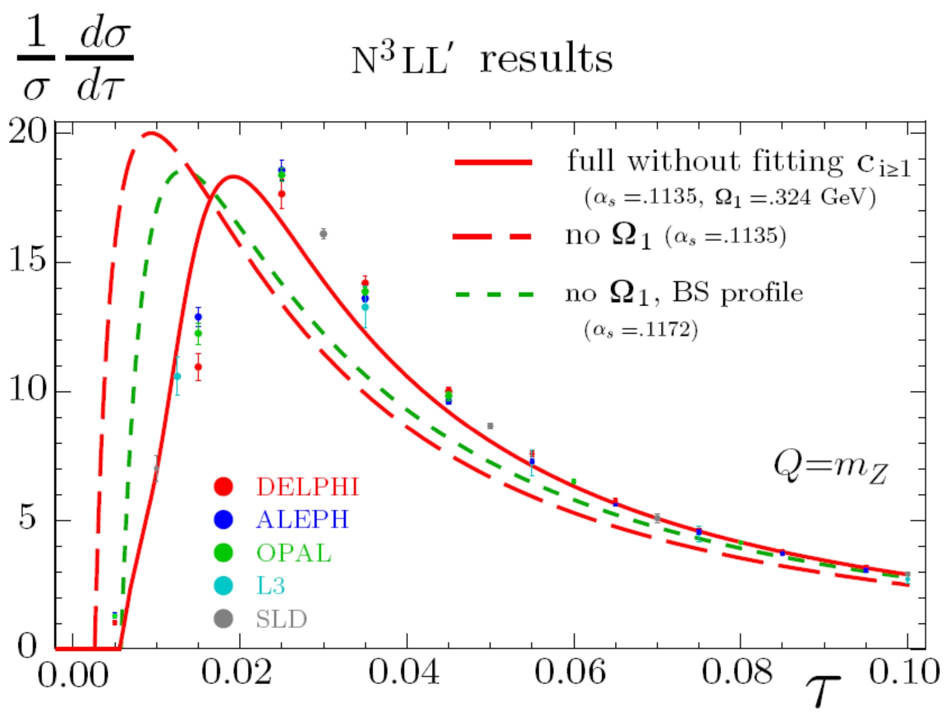


Non-perturbative effects

Use tail fit results to predict peak

$\Rightarrow$ much better prediction for peak

	$\alpha_s(m_Z)\pm$(pert. error)	$\chi^2/$(dof)
N^{3}LL$'$ with $\Omega_1^{\rm Rgap}$	0.1135 ± 0.0009	0.91
N^{3}LL$'$ with $\bar{\Omega}_1^{\overline{\rm MS}}$	0.1146 ± 0.0021	1.00
N^{3}LL$'$ without $S_\tau^{\rm mod}$	0.1241 ± 0.0034	1.26
$\mathcal{O}(\alpha_s^3)$ fixed-order without $S_\tau^{\rm mod}$	0.1295 ± 0.0046	1.12



αS from full analysis is approximately 9% smaller than αS without model, as predicted.

Factorization Theorem for Thrust

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \tfrac{k}{Q}\right) S_\tau^{\mathrm{mod}}\left(k - 2\bar{\Delta}\right) + O\left(\sigma_0 \, \tfrac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

Methode to simultaneously describe all three regions and multiple Q:

profile functions

Singular partonic:

NNLO matching, including full 3-loop hard function

N^3LL resummation of large logs

Nonsingular partonic:

fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:

nonperturabtive effects treated within field theory

Operator Product Expansion in tail → Ω_1

Interface between power and perturbative corrections:

renormalon subtraction

Hoang, Stewart

Hoang, Jain, Scimemi, Stewart

b-mass effects (~2% effect)

QED effects (~2% effect)

axial anomaly at $O(\alpha_s^2)$ (~1% effect)

Renormalon Substraction

$\overline{\text{MS}}$ perturbative series includes fluctuations with arbitrarily small momenta → large unphysical corrections

Both S_τ^{part} and $\overline{\Omega}_1$ suffer from renormalon

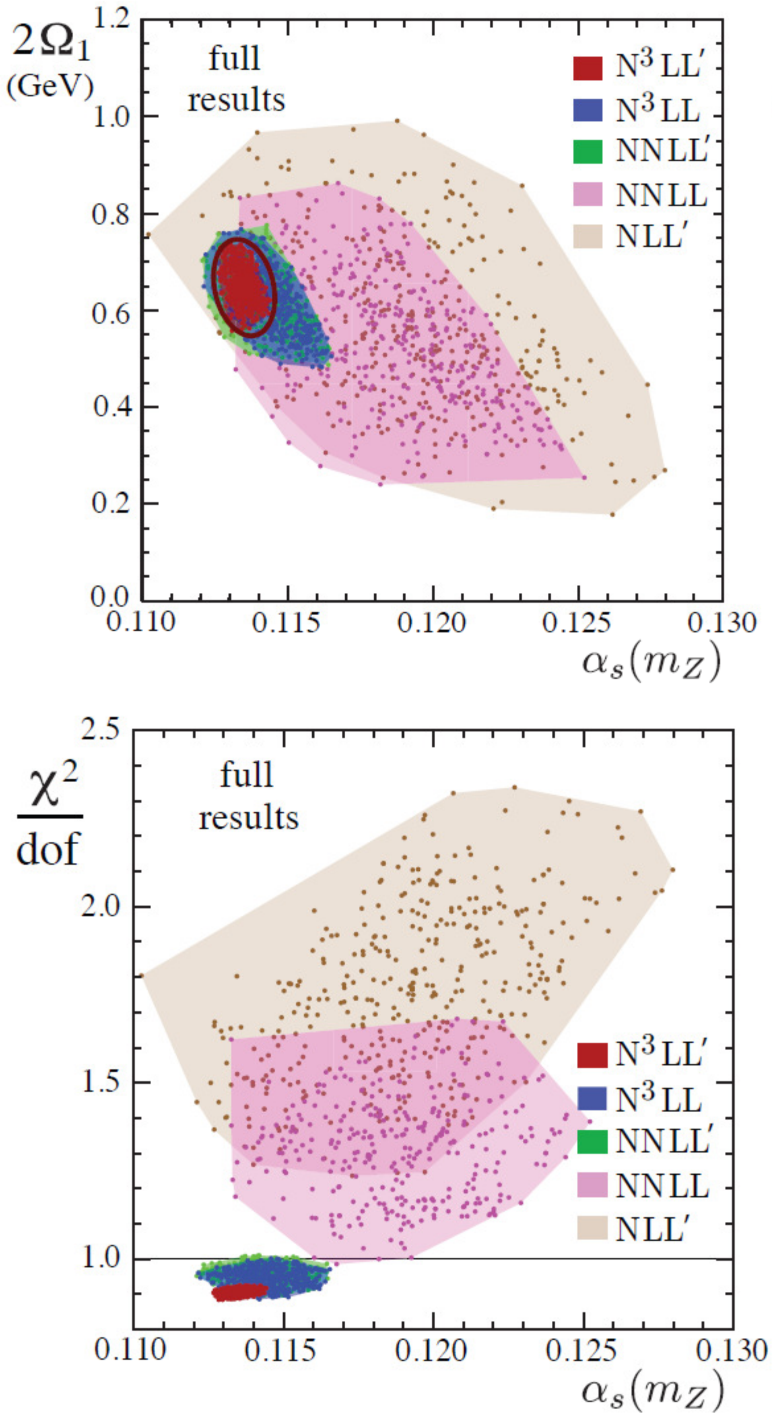
Introduce gap parameter Δ: $S_\tau^{\mathrm{mod}}(k) \rightarrow S_\tau^{\mathrm{mod}}(k-2\Delta)$ Hoang, Stewart

and $\Delta = \overline{\Delta}(R,\mu_S) + \delta(R,\mu_S)$

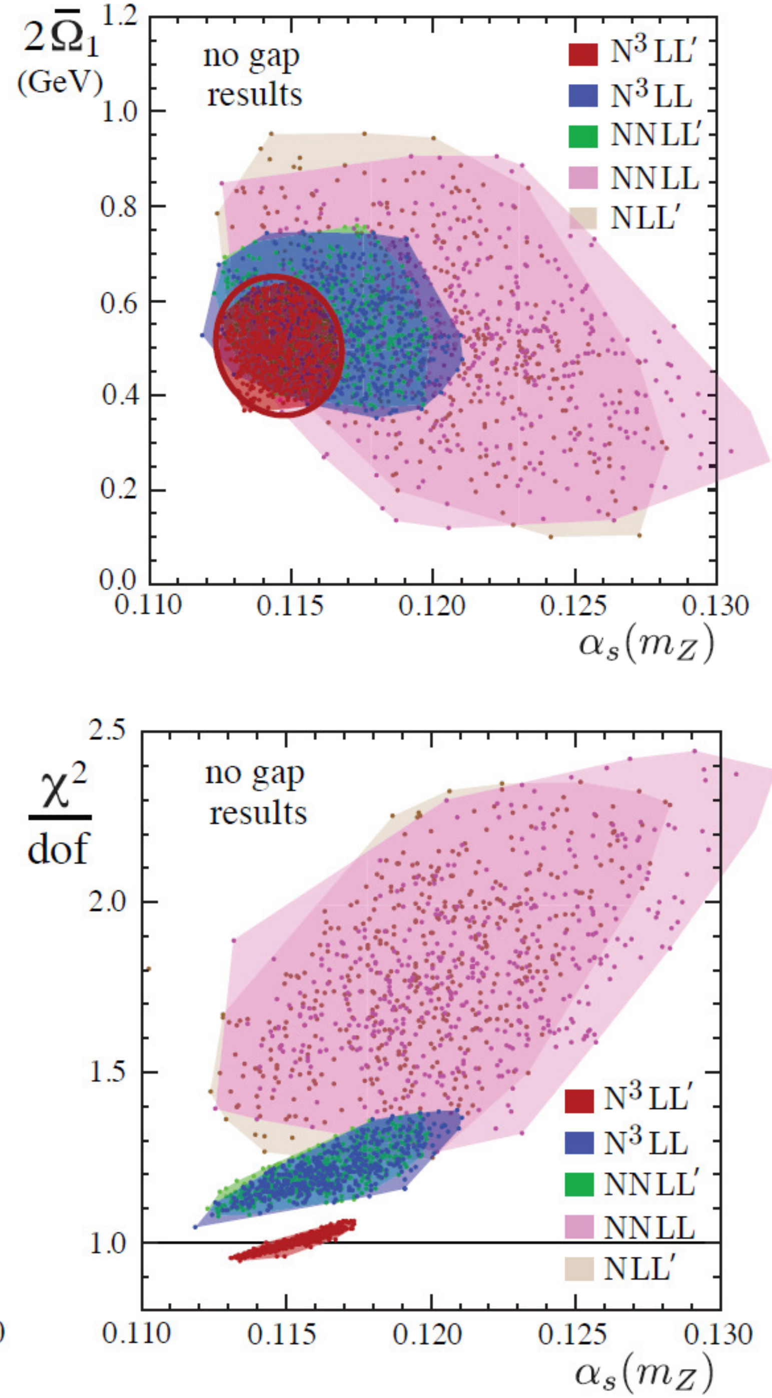
→ renormalon free soft function:

$$S_\tau(k,\mu) = \int dk' \left(e^{-2\delta \frac{\partial}{\partial k}} S_\tau^{part}(k-k',\mu) \right) S_\tau^{\mathrm{mod}}(k'-2\overline{\Delta})$$

renormalon substracted



renormalon **NOT** substracted



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→ large unphysical corrections

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Fleming, Hoang, Mantry, Stewart

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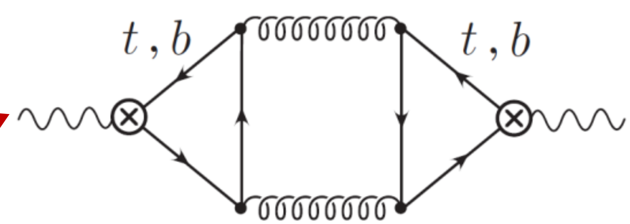
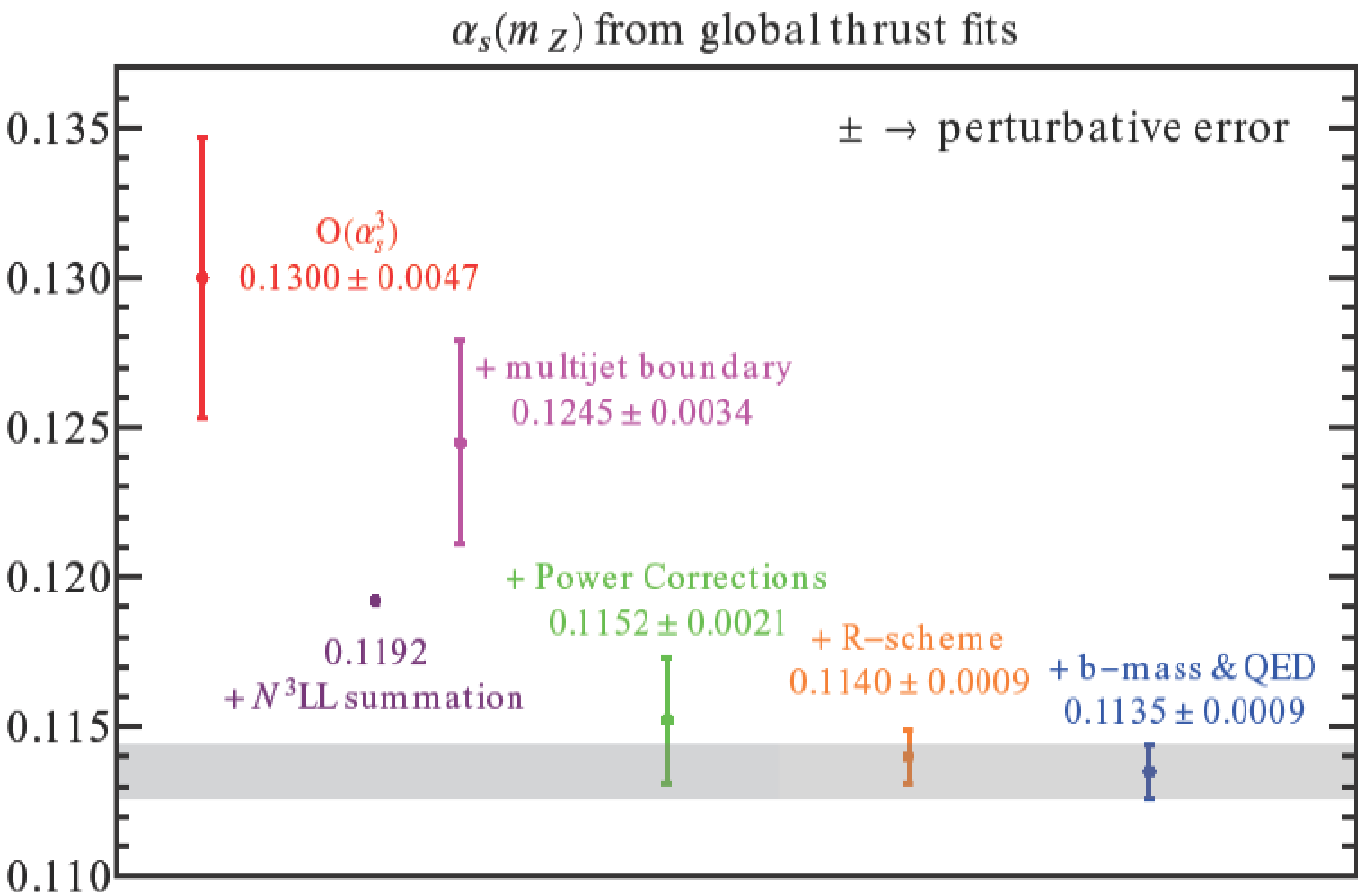


Kniehl, Kuhn

Hagiwara, Kuruma, Yamada



$\alpha_s(m_Z)$ from global thrust fits
± → perturbative error
0.135
0.130
0.125
0.120
0.115
0.110
$O(\alpha_s^3)$
0.1300 ± 0.0047
+ multijet boundary
0.1245 ± 0.0034
0.1192
+ N^3LL summation
+ Power Corrections
0.1152 ± 0.0021
+ R–scheme
0.1140 ± 0.0009
+ b–mass & QED
0.1135 ± 0.0009

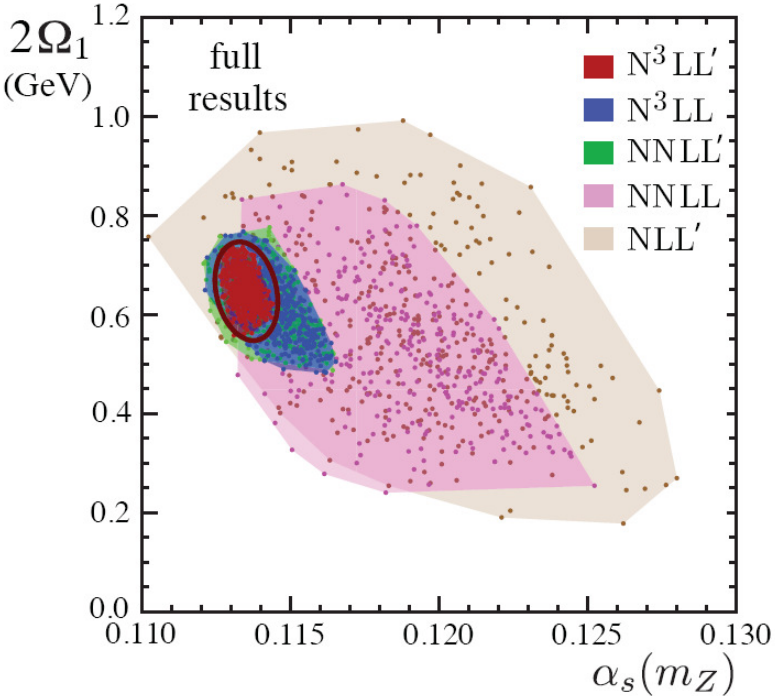
Comparison with similar analysis

	sum logs	power corrections	data	$\alpha_S(M_Z)$
Dissertori et al.	no	Monte Carlo (MC)	ALEPH	0.1240(34)
Dissertori et al.	NLL	Monte Carlo	ALEPH	0.1224(39)
Becher, Schwartz	N3LL	uncertainty from MC	ALEPH, OPAL	0.1172(21)
Davison, Webber	NLL	effective coupling model	Most of data	0.1164(28)
Bethke et al.	NLL	Monte Carlo	JADE	0.1172(51)

Becher, Schwartz:

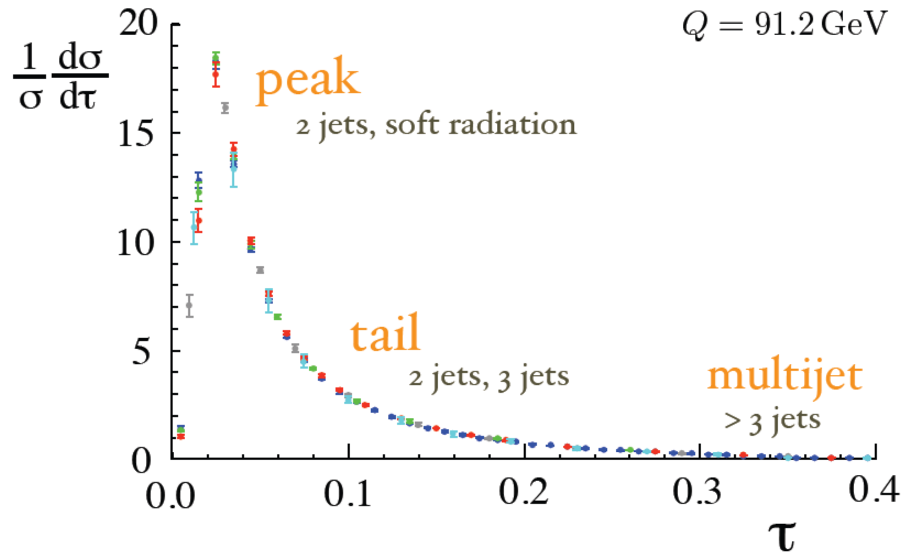
- no nonperturbative soft function
- different profile function
- different way of calculating binned cross section

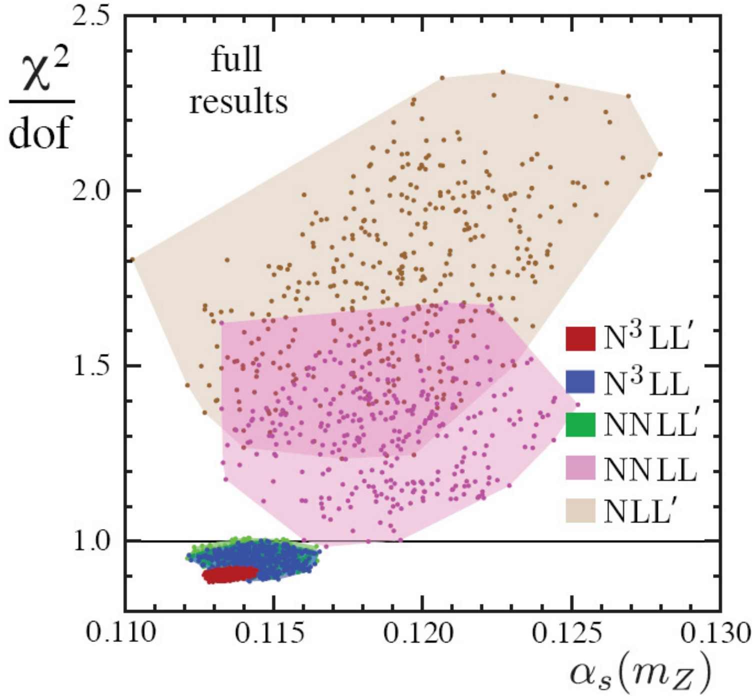
Numerical Analysis



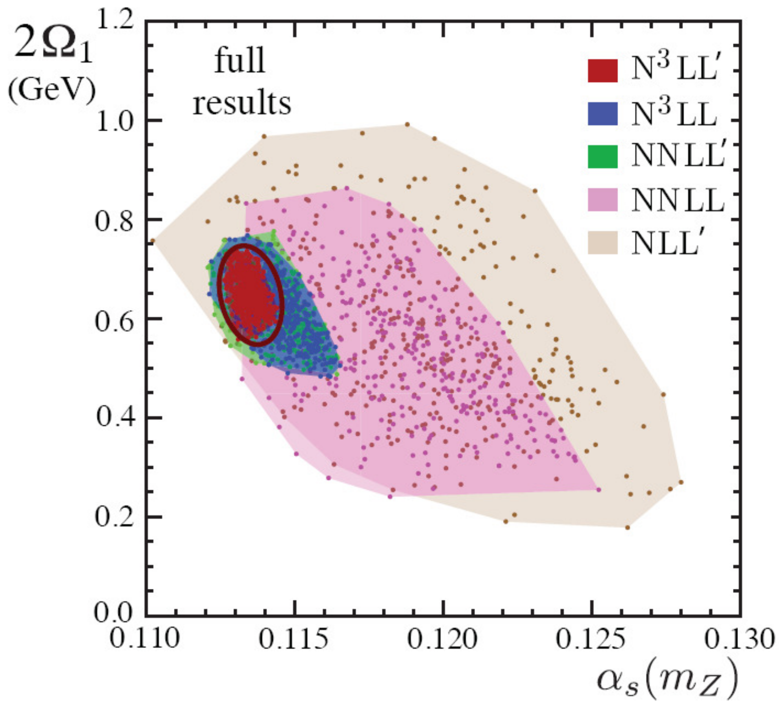
Two parameter fit in tail region (factorization formula valid for all tau)

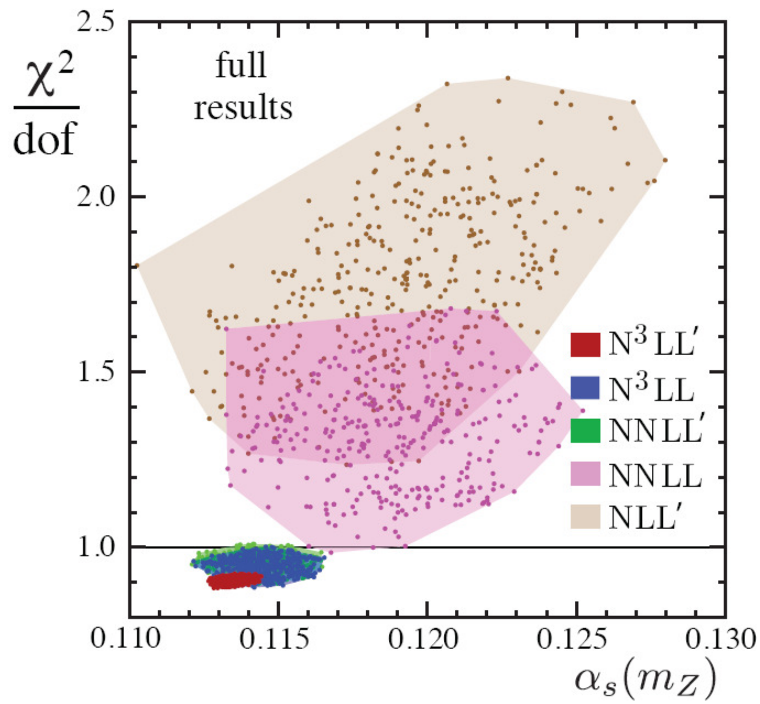
Ω_1 is the 1st nonperturbative power correction





	Experiment	Values of Q
LEP	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
	DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
	OPAL	{91.0, 133.0, 161.0, 172.0, 177.0, 183.0, 189.0, 197.0}
	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLAC	SLD	{91.2}
DESY	TASSO	{(14.0), (22.0), 35.0, 44.0}
	JADE	{35.0, 44.0}
KEK	AMY	{55.2}





Order Counting

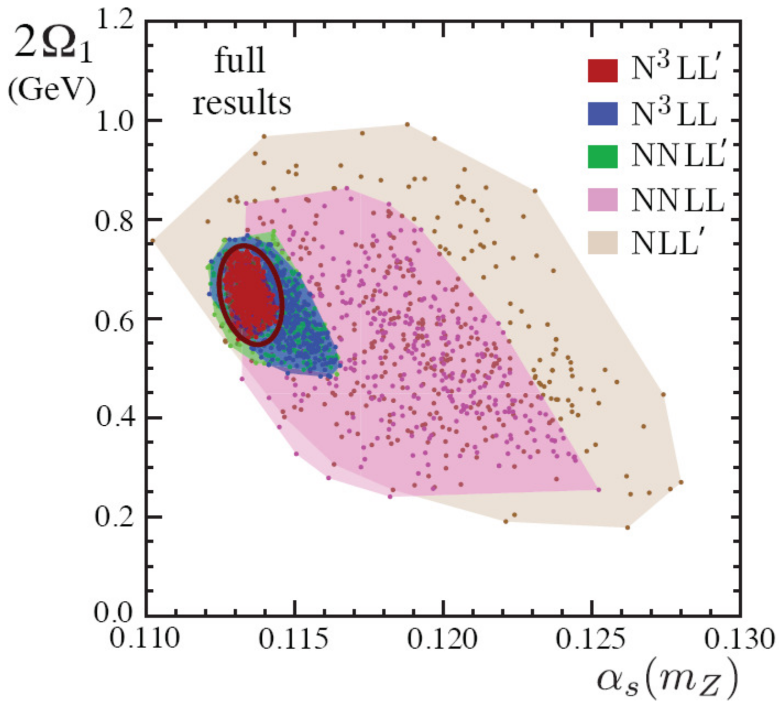
$$\ln\left(\tfrac{d\sigma}{dy}\right) \sim \left(\alpha_s \ln\right)^k \ln + \left(\alpha_s \ln\right)^k + \alpha_s\left(\alpha_s \ln\right)^k + \alpha_s^2\left(\alpha_s \ln\right)^k + \ldots$$

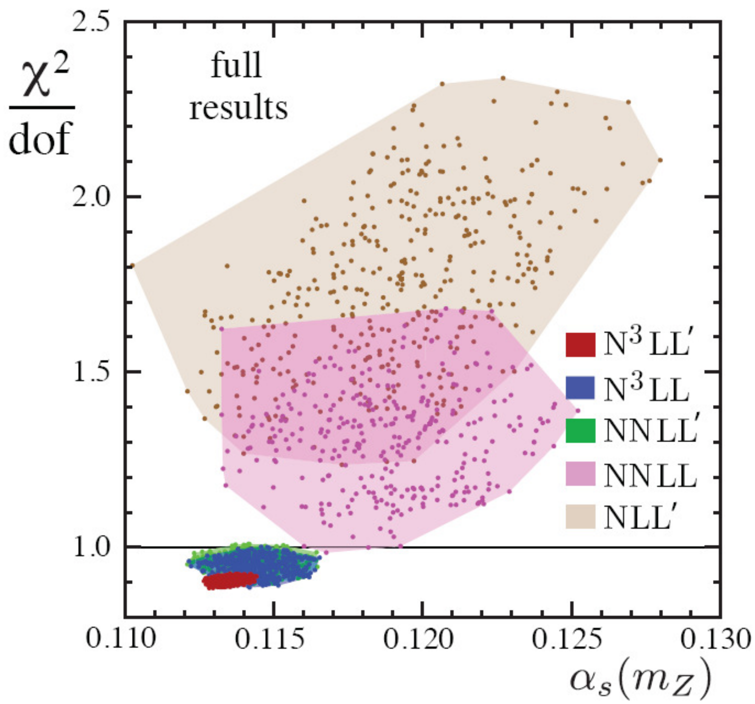
LL NLL NNLL N^3LL

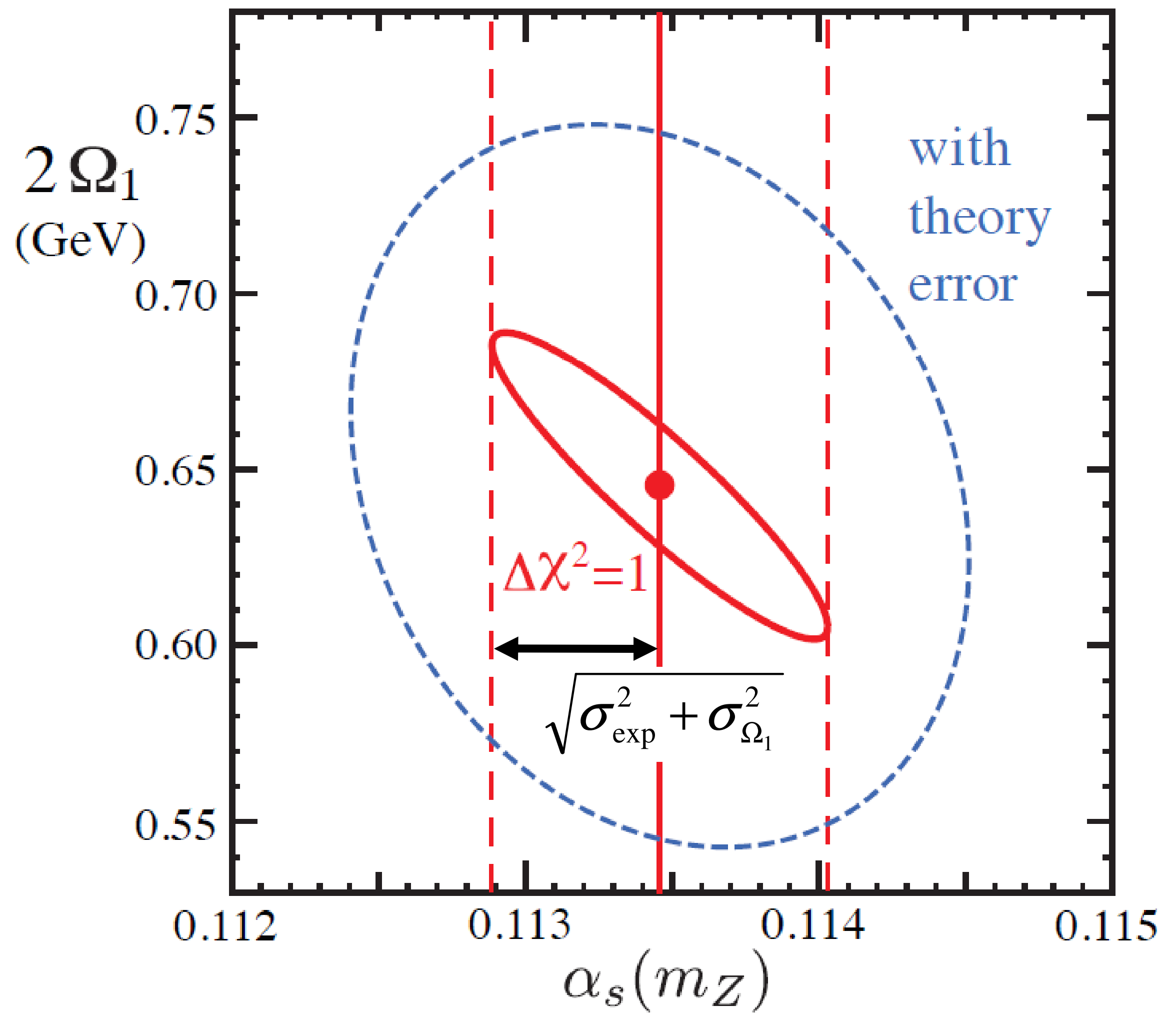
	cusp	non-cusp	matching	alphas
LL	1	-	tree	1
NLL	2	1	tree	2
NNLL	3	2	1	3
N^3LL	4^{pade}	3	2	4
LL'	1	-	tree	1
NLL'	2	1	1	2
NNLL'	3	2	2	3
N^3LL'	4^{pade}	3	3	4

Primed counting is better if fixed order results are important

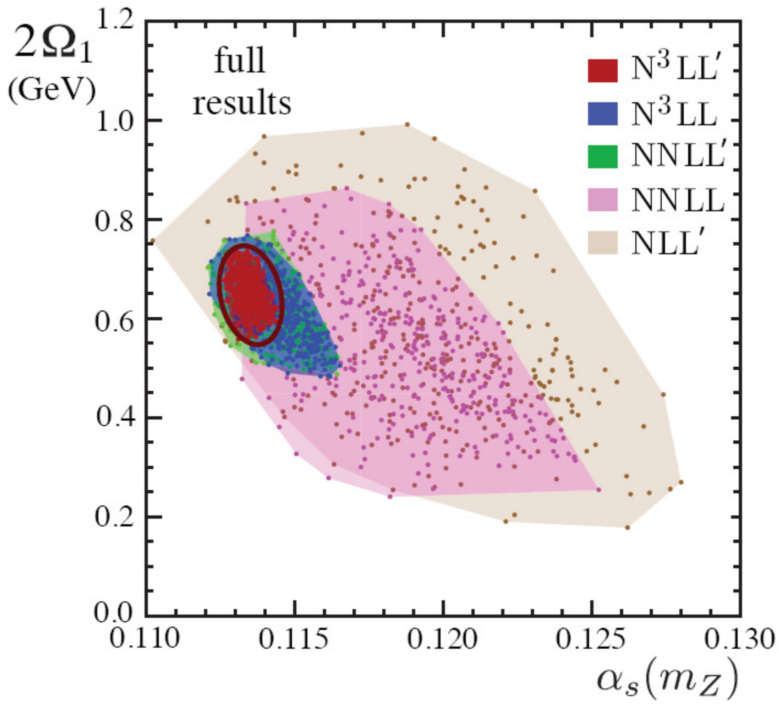
Experimental and Hadronic Uncertainty

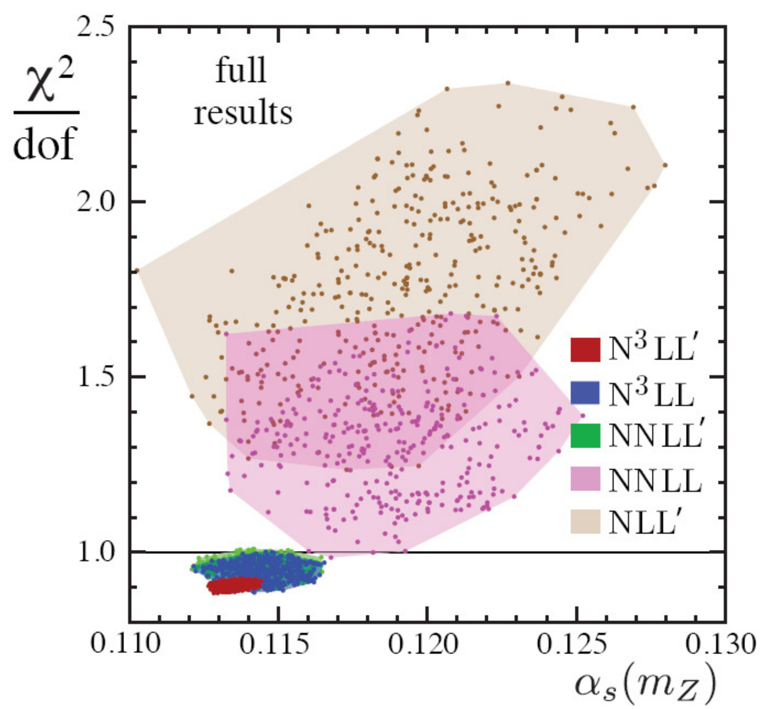






Perturbative Uncertainty

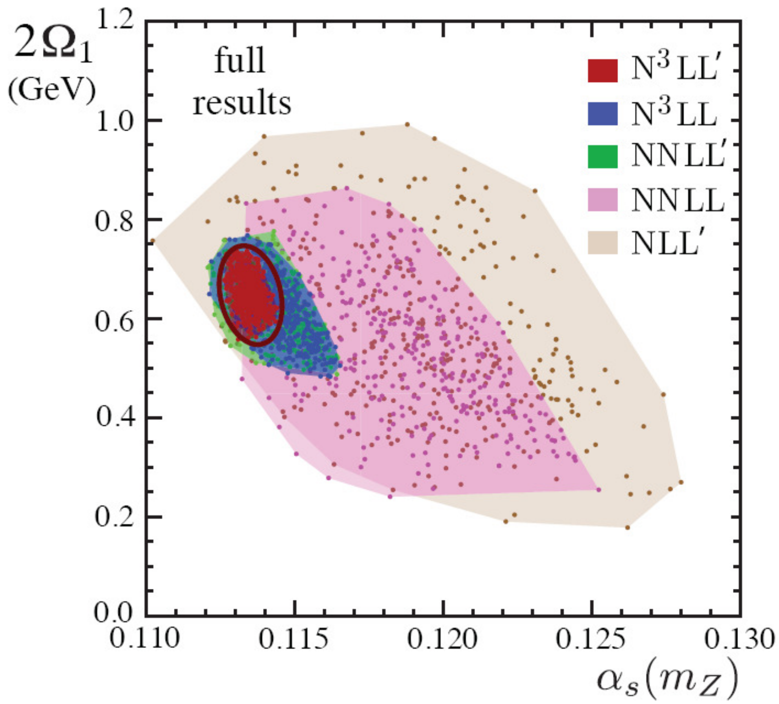


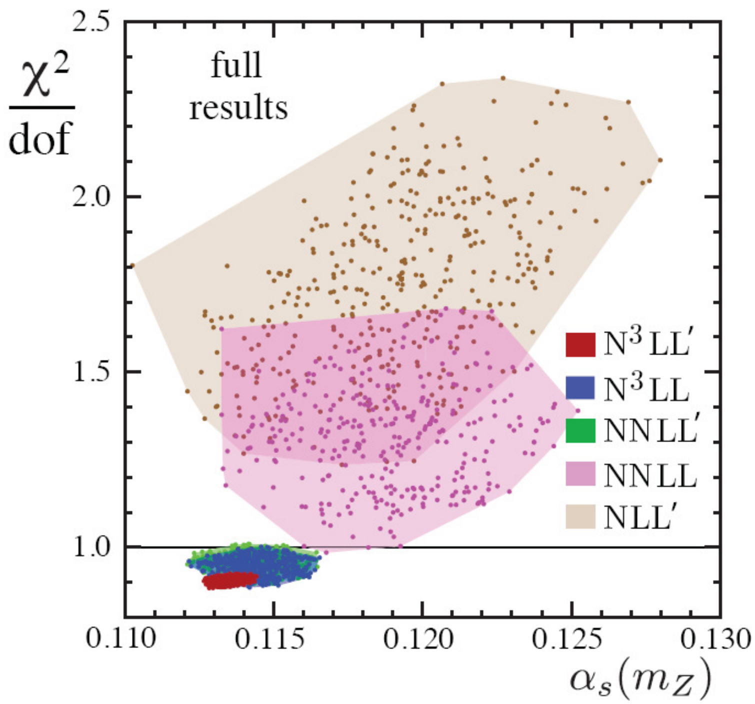


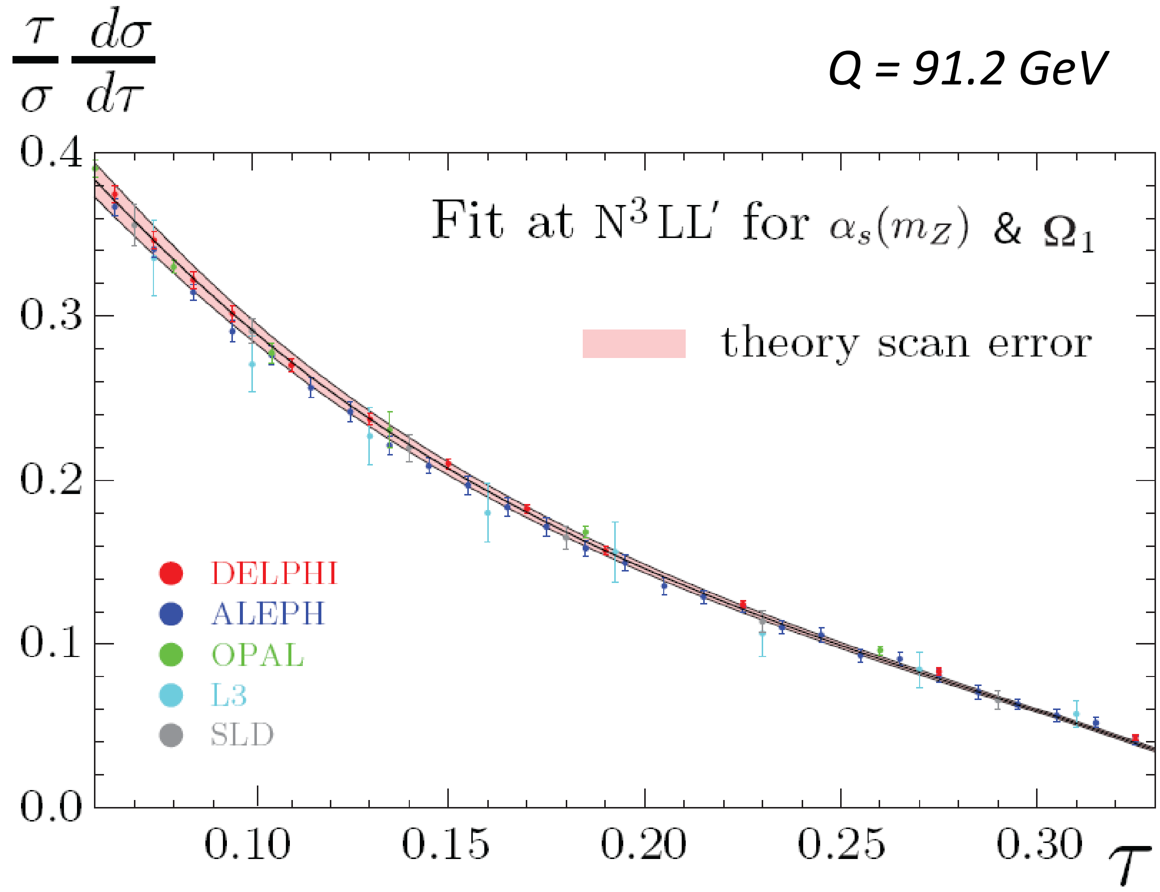
12 theory parameters:

- 6 parameters for the variation of the renormalization scales
- 3 parameters related to the statistical uncertainties of numerical fixed order calculations
- 3 parameters for Padé approximants of unknown constants

1. Flat random scan over theory parameters
2. Fitting for each parameter set
3. Range of best fits → perturbative uncertainty

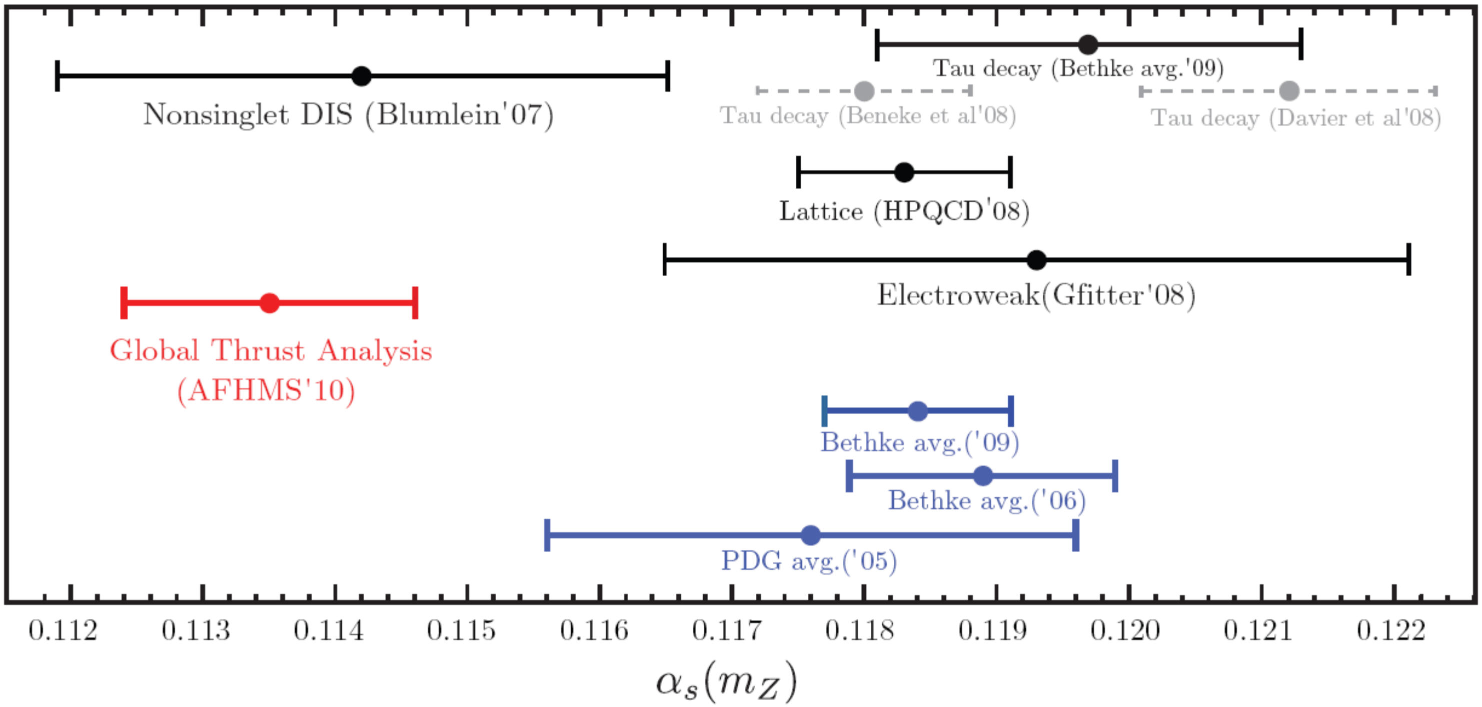






$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$$

with $\frac{\chi^2}{\text{dof}} = 0.91$



same analysis for other event shape variables: **Heavy Jet Mass**

AHMS+Schwartz

fits to **moment data**

AFHMS

Moment Fits

n^{th} moment: $$M_n = \int_0^{\tau_{\max}} d\tau\tau^n \int_0^{Q\tau} dp \frac{d\hat{\sigma}}{d\tau}(\tau - \tfrac{p}{Q}) S_\tau^{\text{mod}}(p) + O\left(\tfrac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

purely perturbative

$$M_n = \sum_{k=0}^{n} \binom{n}{k} \hat{M}_k \left(\frac{2}{Q}\right)^{n-k} \Omega_{n-k} + O\left(\tfrac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

where $\Omega_i = \int dp \left(\frac{p}{2}\right)^i S_\tau^{\text{mod}}(p)$ is defined as for the distribution

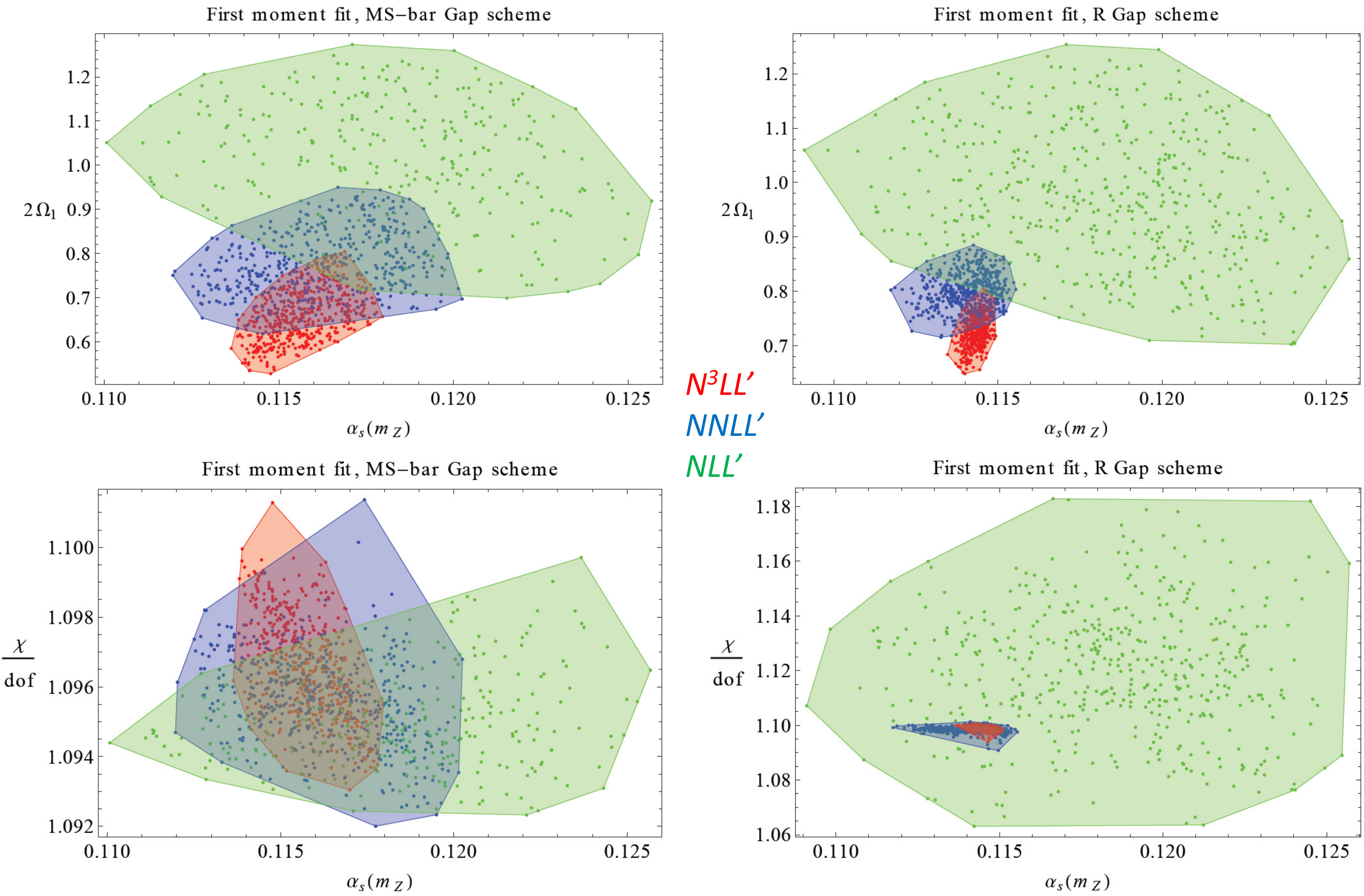
Higher sensitivity for higher moments with primed moments:

$$M'_n = \hat{M}'_n + \left(\frac{2}{Q}\right)^n \Omega'_n + O\left(\tfrac{\alpha_S \Lambda_{QCD}}{Q}\right)$$

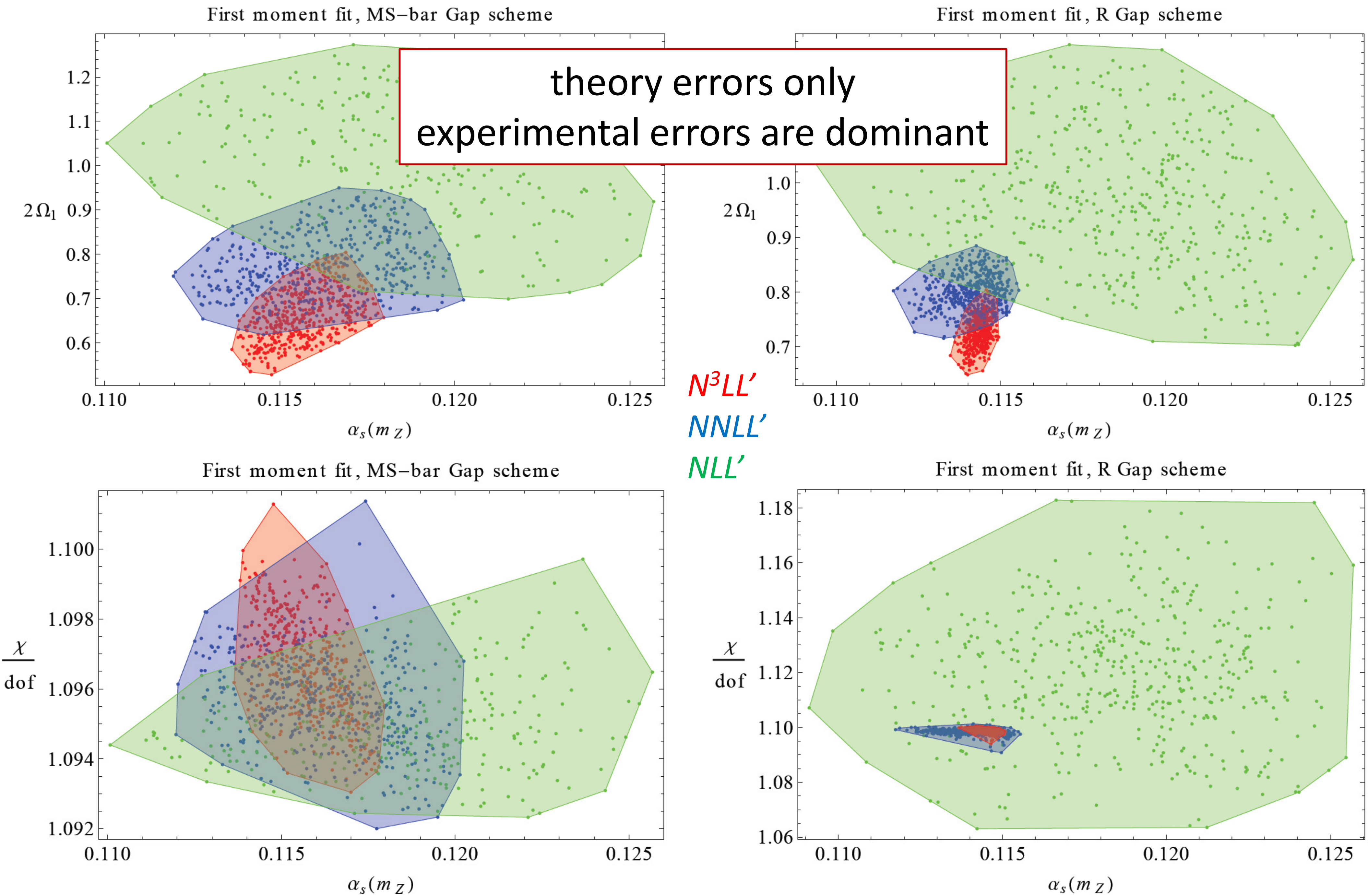
$$\Omega'_1 = \Omega_1$$
$$\Omega'_2 = \Omega_2 - \Omega_1^2$$
$$\Omega'_3 = \Omega_3 - 3\Omega_2\Omega_1 + 2\Omega_1^3$$

appear to also be $1/Q^n$!!!!

Global fits to M_1 data

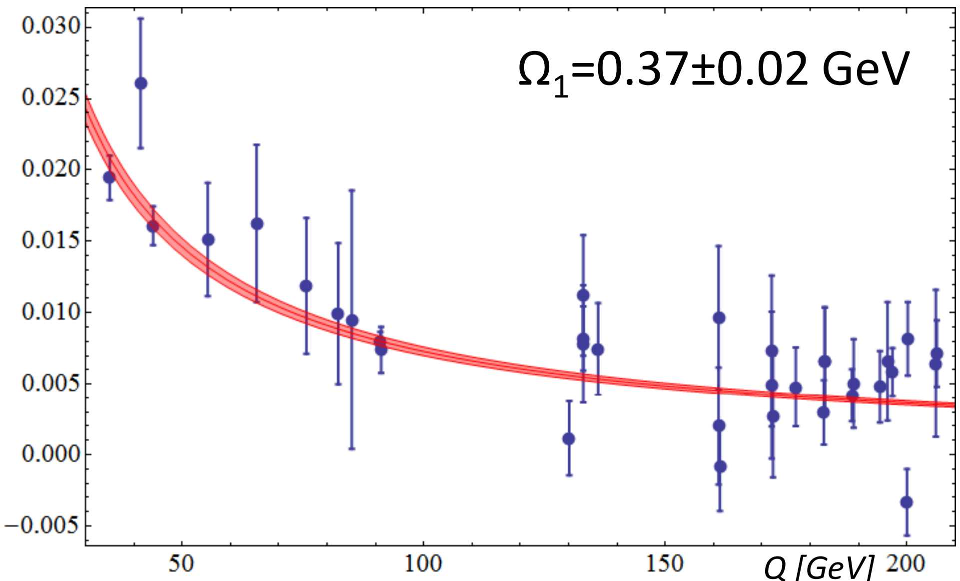


Global fits to M_1 data

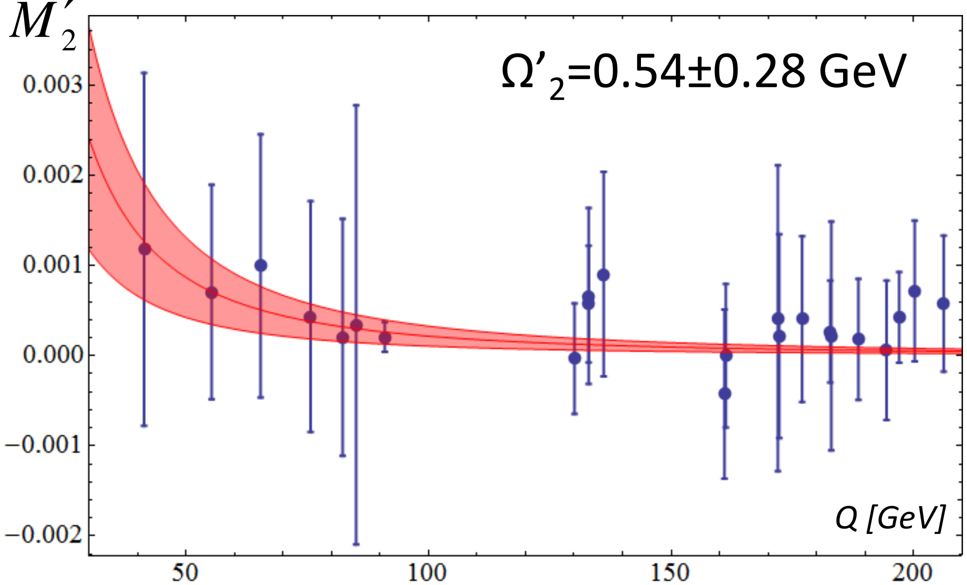


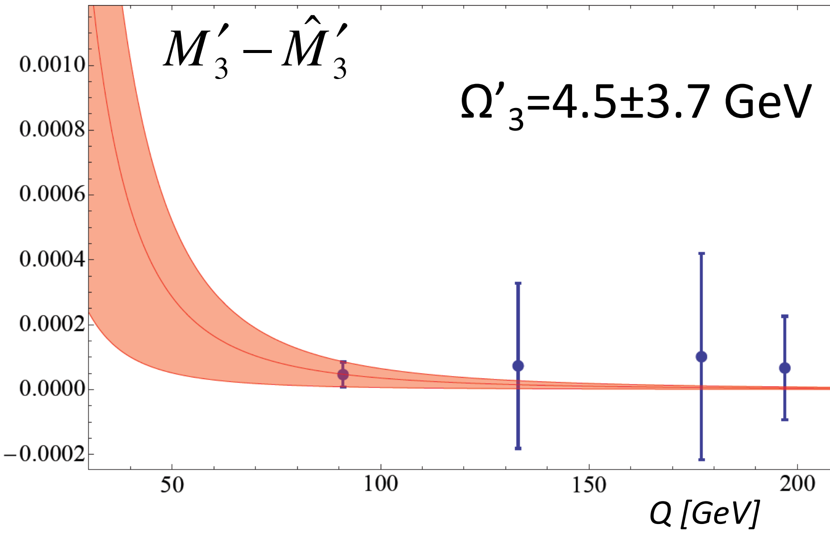
Fits for $(2/Q)^i\Omega_i$ at $\alpha_s(m_Z)=0.114$, $\overline{\text{MS}}$ scheme:
cancelation for primed moments
→ no new information from higher moment data

$M_1 - \hat{M}_1$

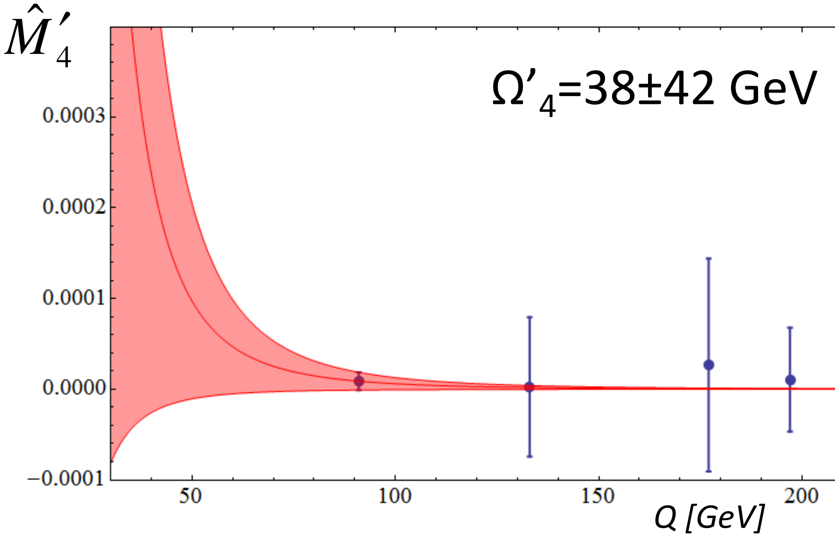


$M'_2 - \hat{M}'_2$

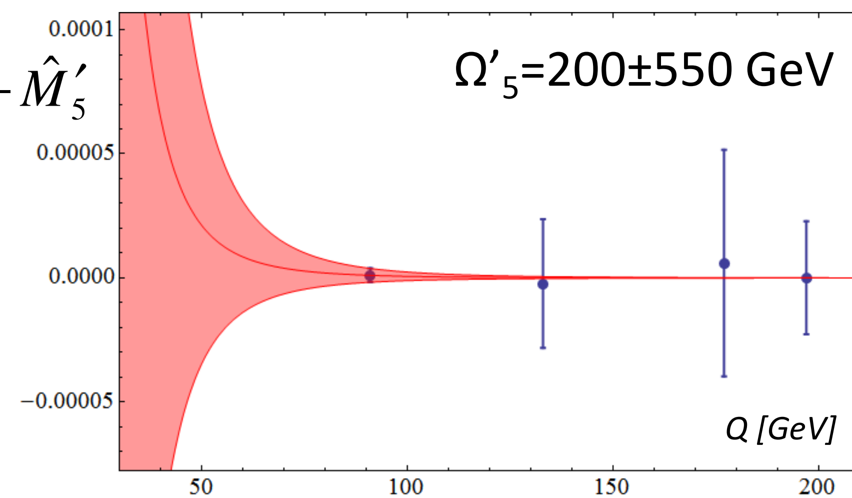




$M'_4 - \hat{M}'_4$



$M'_5 - \hat{M}'_5$

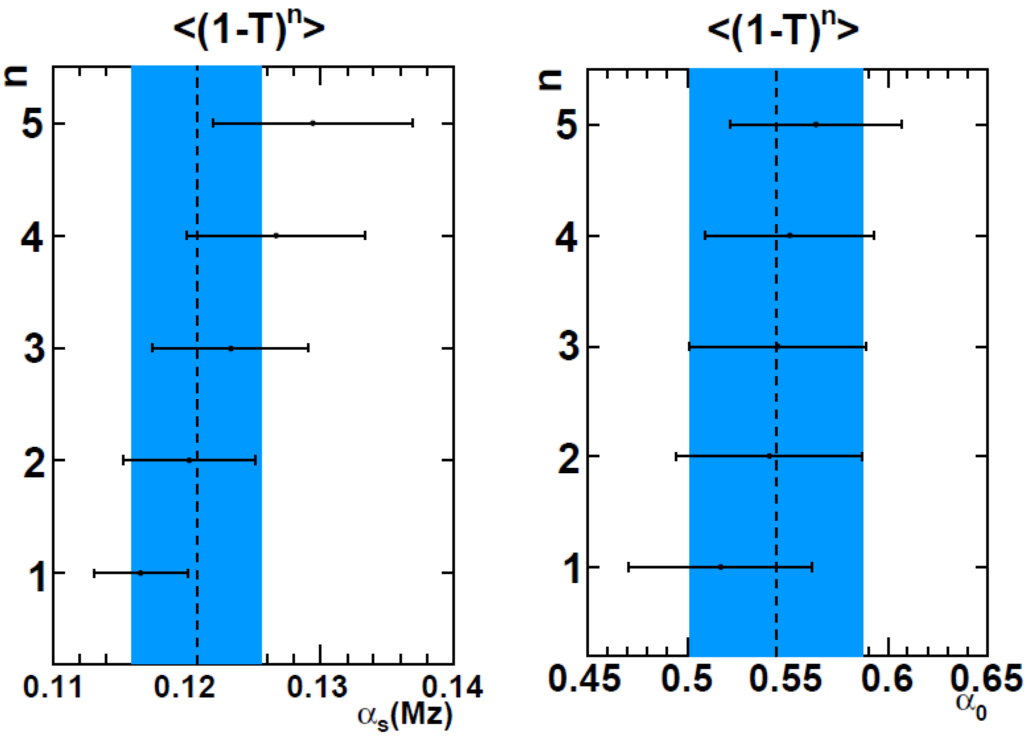


Comparison to Gehrmann

$$M_n^{Gehrmann} = \sum_{k=0}^{n} \binom{n}{k} \hat{M}_k (2P)^{n-k}$$

no $\Omega'_{i>1}$!

$$M_n = \sum_{k=0}^{n} \binom{n}{k} \hat{M}_k \left(\frac{2}{Q}\right)^{n-k} \Omega_{n-k}$$



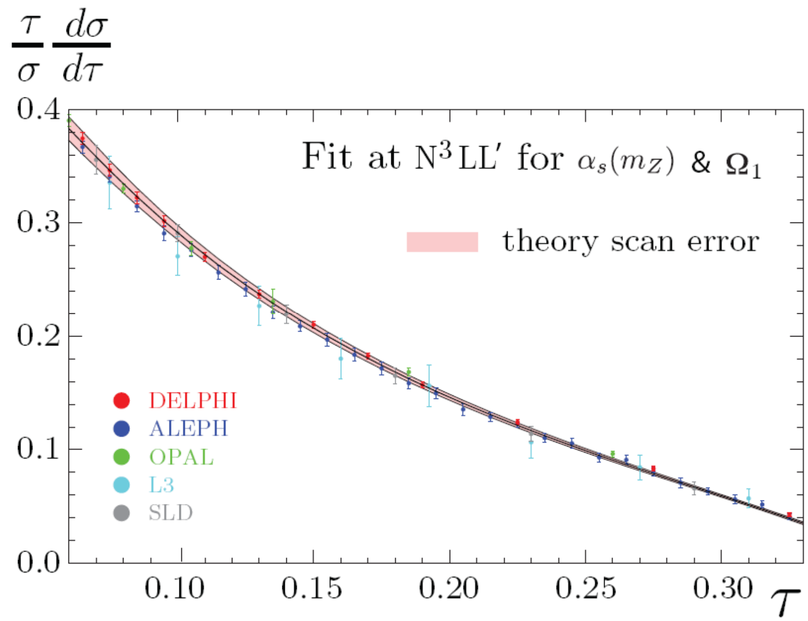
→ fits to the higher moments are all sensitive to the same $\hat{M}_1$

with dispersive model for power corrections:

$$P = \frac{4C_F}{\pi^2} \cdot M \cdot \left\{ \alpha_0 - \left(\alpha_S(\mu_R) + \frac{\beta_0}{\pi} \alpha_S^2(\mu_R) \left(\ln \frac{\mu_R}{\mu_I} + 1 + \frac{K}{2\beta_0} \right) + O(\alpha_S^3) \right) \right\} \cdot \frac{\mu_I}{Q}$$

Summary

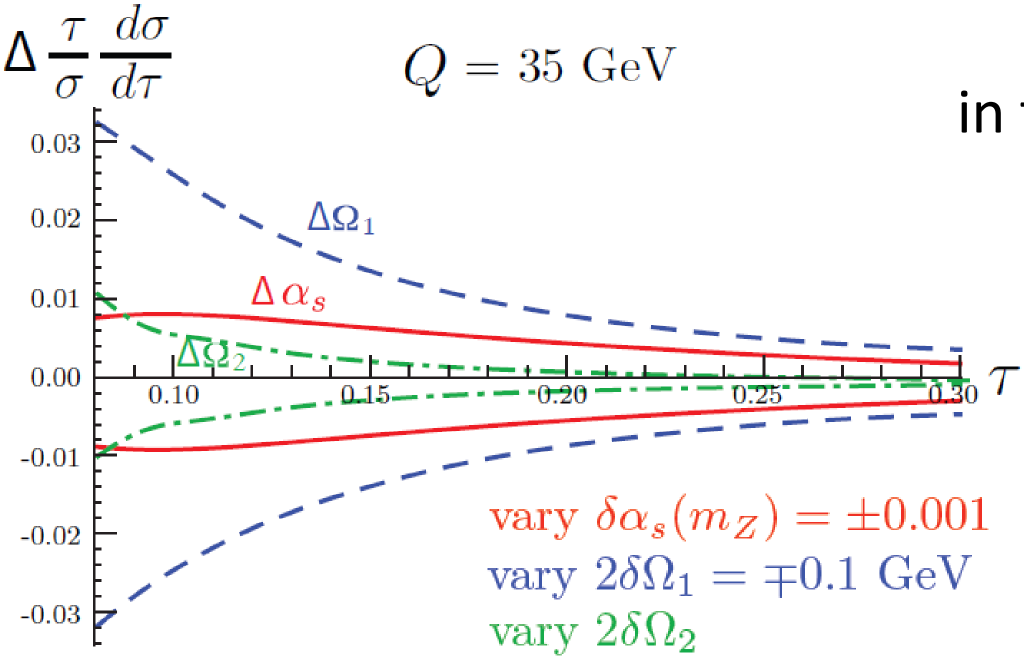
- *α_S = 0.1135 ± (0.0002)$_{expt}$ ± (0.0005)$_{hadr}$ ± (0.0009)$_{pert}$*



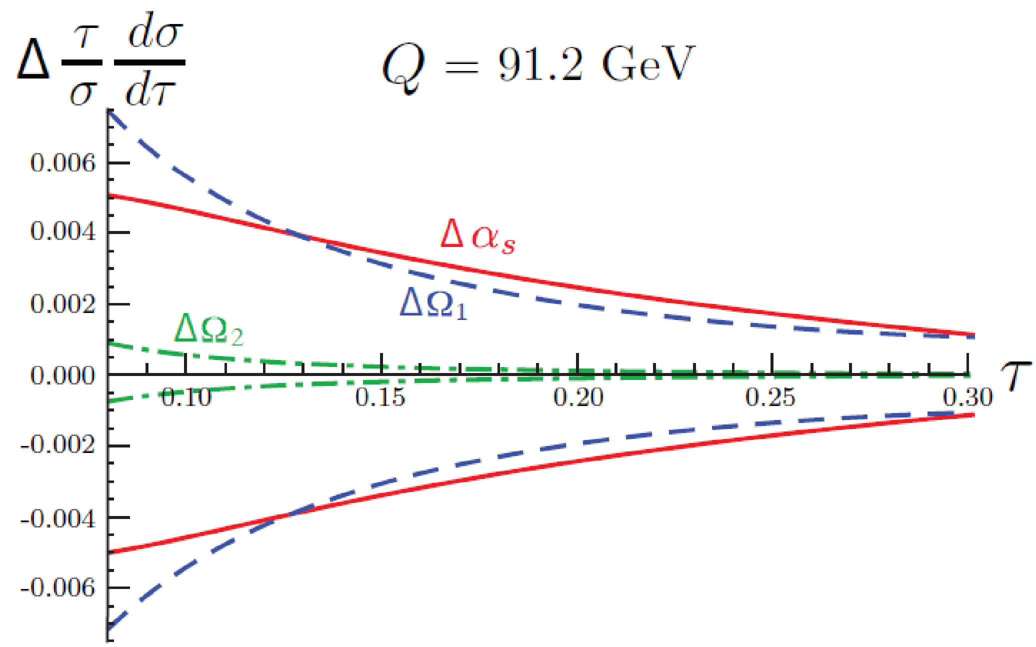
- SCET can improve convergence and therefore precision
- SCET allows to include non-perturbative effects in a systematic manner
- Profile functions allow to combine several kinematic regions

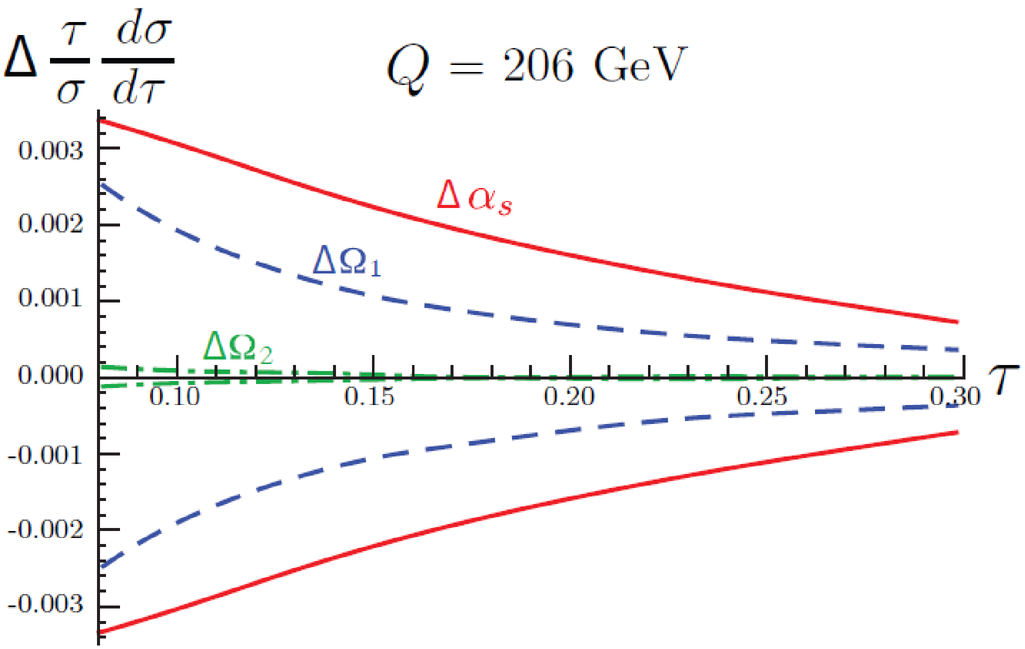
Backup Slides

Why do we need a global fit?



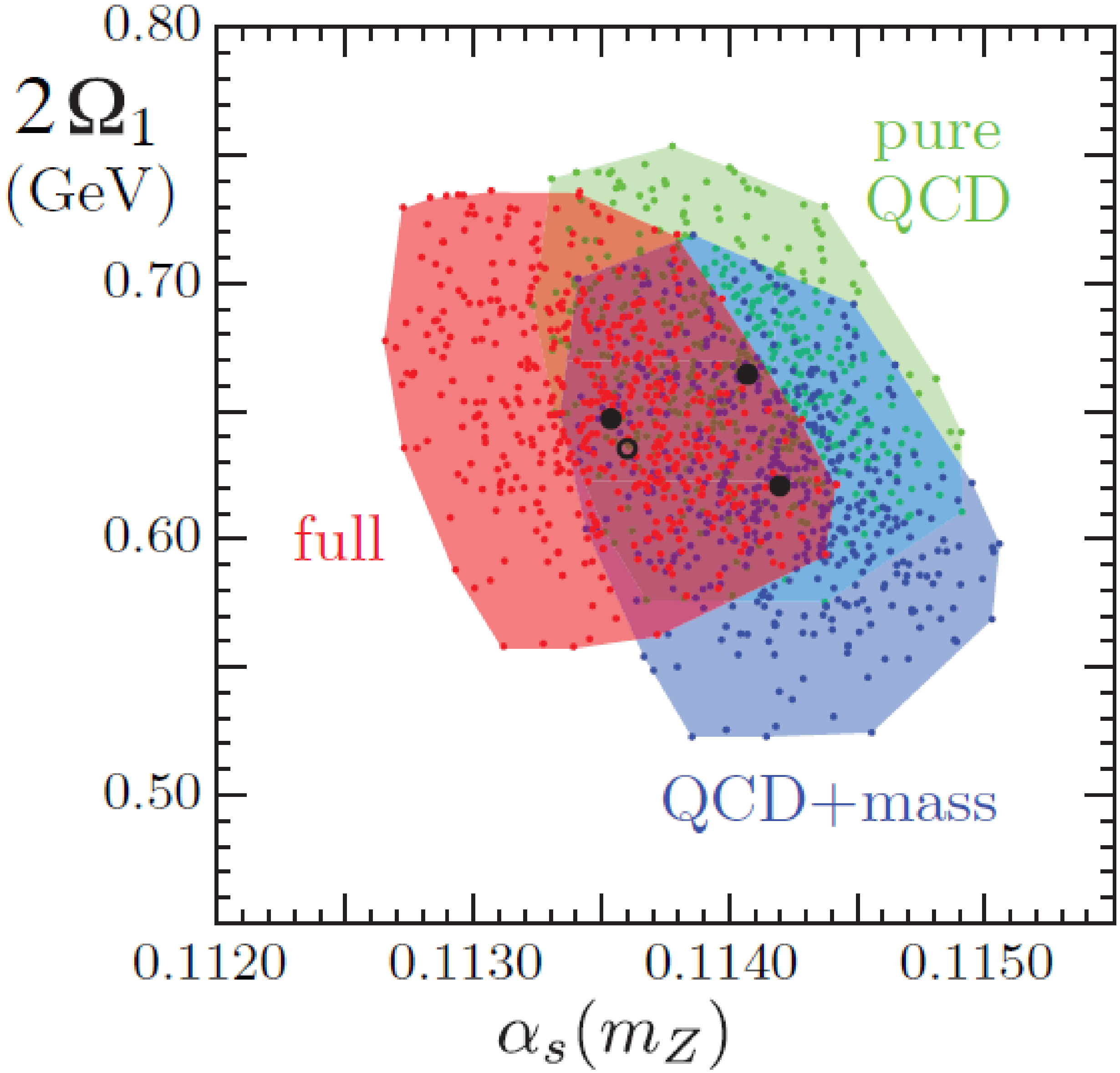
in the tail $\alpha_s(m_Z)$ and Ω_1 are degenerated for a single Q





degeneracy is lifted by simultaneously fitting multiple Q

QED and b-mass effects



b-mass:

horizontal shift of thrust distribution towards larger τ

→ lower Ω_1

QED:

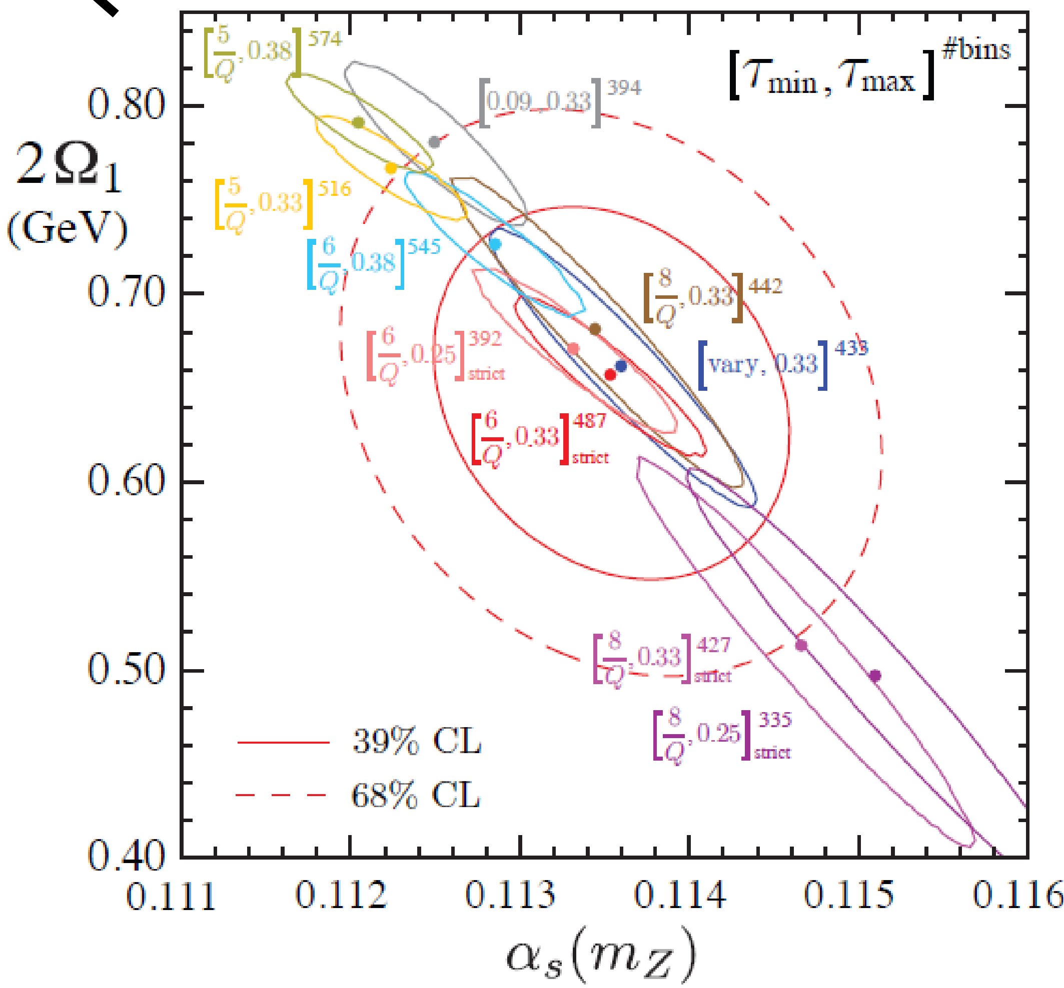
effective increase of coupling strength

→ lower α_s

Cut on Dataset

Ω_2 effects increase

τ_{min}



decrease of $\Delta\tau$

→ increase of statistical uncertainty

τ_{max}

missing $\alpha_S\Lambda_{QCD}/Q$ effects become important

Theory Parameters

parameter	default value	range of values	
μ_0	$2\,\mathrm{GeV}$	1.5 to 2.5 GeV	profile function (variation of renormalization scales)
n_1	5	2 to 8	
t_2	0.25	0.20 to 0.30	
e_J	0	-1,0,1	
e_H	1	0.5 to 2.0	
n_s	0	-1,0,1	
s_2	-39.1	-36.6 to -41.6	
Γ_3^{cusp}	1553.06	-1553.06 to $+4569.18$	Padé approximats
j_3	0	-3000 to $+3000$	
s_3	0	-500 to $+500$	
ϵ_2	0	-1,0,1	non-singular stat. uncertainty
ϵ_3	0	-1,0,1	

Include b-mass effects in Factorization Thm: $(\sim 2\%$ effect$)$

$$\frac{\Delta\, d\hat{\sigma}^b}{d\tau} = \frac{d\hat{\sigma}^{NNLL}_{\text{massive}}}{d\tau} - \frac{d\hat{\sigma}^{NNLL}_{\text{massless}}}{d\tau}$$

- at this order it effects only the jet function and τ limits
- use SCET massive fact. thm — Fleming, Hoang,Mantry,Stewart
- charm quarks are much smaller effect

Include QED effects in Factorization Thm: $(\sim 2\%$ effect$)$

- count $\alpha \sim \alpha_s^2$, include only final state radiation
- include $\mathcal{O}(\alpha_s^2\, \alpha)$ corrections to QCD β-function
- include one-loop QED corrections to H_Q, J_τ, S_τ

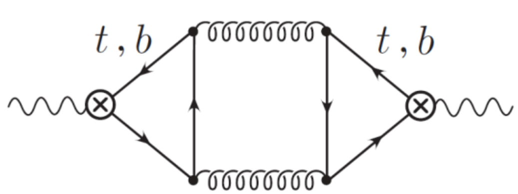
Include axial anomaly contribution $(\sim 1\%$ effect$)$

- affects H_Q^{ua}, H_Q^{da} at $\mathcal{O}(\alpha_s^2)$

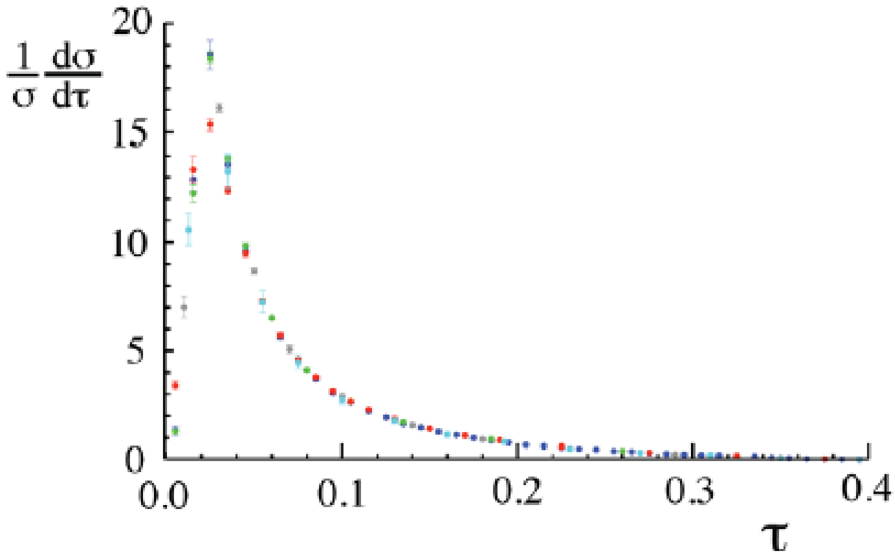
$$H_Q^a = H_Q^v + H^{\text{singlet}} \qquad f^{da}(\tau,1) = f^v(\tau,1) + \frac{\alpha_s^2}{4\pi^2} f_{\text{singlet}}\Big(\tau, \frac{Q^2}{4\, m_t^2}\Big)$$

- due to large top-bottom mass splitting

Kniehl, Kuhn
Hagiwara, Kuruma, Yamada



Calculating binned cross-sections



At this level of precission, it is important to calculate theory results for the entire bin (even though they are quite fine)

How should we calculate the bins?

Difference of cumulants

$$\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))$$

[all classic resummation analyses]

[Becher & Schwartz] [Chien & Schwartz]

Integrating the differential distribution

$$\int_{\tau_1}^{\tau_2} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau'))$$

[AFHMS] [AHMS+Schwartz]

The two procedures look the same, but actually they are not

$$\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) =$$

$$\underbrace{\int_{\tau_1}^{\tau_2} \mathrm{d}\tau' \frac{\mathrm{d}\sigma}{\mathrm{d}\tau'}(\tau', \mu_i(\tau_2))}+\underbrace{\Sigma(\tau_1, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))}$$

$$\simeq \int_{\tau_1}^{\tau_2} \mathrm{d}\tau' \frac{\mathrm{d}\sigma}{\mathrm{d}\tau'}(\tau', \mu_i(\tau')) + (\tau_2 - \tau_1) \frac{\mathrm{d}\mu_i(\tau_1)}{\mathrm{d}\tau_1} \int_0^{\tau_1} \mathrm{d}\tau' \frac{\mathrm{d}\sigma}{\mathrm{d}\tau'}(\tau', \mu_i(\tau_1))$$

Enhanced uncertainty from the peak !