# *Medium Induced Collinear Radiation via SCET*

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Frontiers in QCD, INT, Seattle

in collaboration with Hong Liu and Krishna Rajagopal

# Outline

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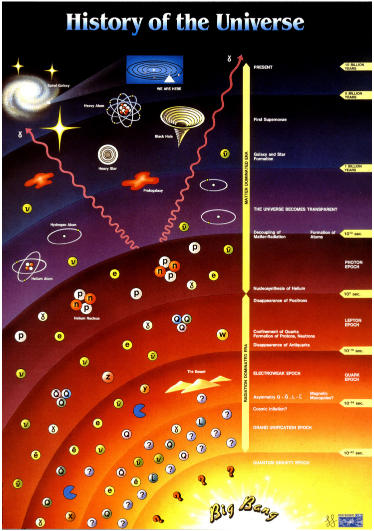
# Thermal History of the Universe and QGP




## Our universe originated in a “Big Bang”

Matter as we know it has not existed forever

First few microseconds after the Big Bang:

- hadrons could not form, quark, antiquarks and gluons deconfined in the quark-gluon plasma (QGP)

- first hadrons formed only when $T = T_{\rm cr} \simeq 170\,{\rm MeV}$

## QGP hidden behind the CMB

To study it we have to recreate on the Earth

# Phases of Quantum ChromoDynamics (QCD)

QCD: $SU(3)_c$ gauge theory of quarks of gluons

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + i\overline{\psi}\gamma^\mu \left(\partial_\mu - ig\frac{\lambda^a}{2} A^a_\mu\right)\psi - m\overline{\psi}\psi$$

Quarks and gluons: color charge, and in the $m \to 0$ limit chiral symmetry

Hadrons: colorless (confinement) and heavy (chiral symmetry breaking) objects

Simplicity of $\mathcal{L}_{\rm QCD}$ hides a wealth of emergent phenomena at the scale $\Lambda_{\rm QCD}$

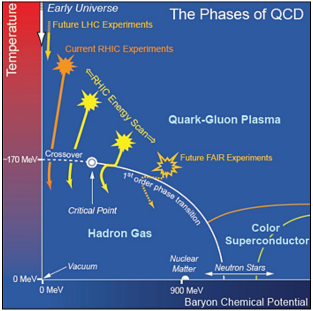
## QCD phase diagram $(\mu_B, T)$

QGP: $T \to \infty$ phase of QCD

- quarks and gluons de-confined
- chiral symmetry is restored

## Lattice QCD

Near $\mu_B \sim 0$ rapid crossover between two phases: the medium properties change quickly and dramatically



# Relativistic Heavy Ion Collisions

QGP recreated today by colliding large nuclei at high energies

## RHIC


RHIC began operation in 2000

p+p and Au+Au collisions at $\sqrt{s} \simeq 200\,\mathrm{GeV} \times A$

## LHC

LHC began operation in 2009

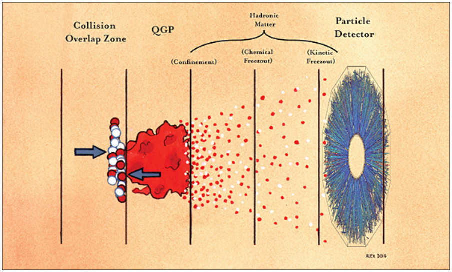
p+p and Pb+Pb collisions at $\sqrt{s} \simeq 5.5\,\mathrm{TeV} \times A$

First heavy-ion collisions at the LHC in November 2010 at $\sqrt{s} \simeq 2.76\,\mathrm{TeV} \times A$
Next run at the LHC scheduled for November 2011 (same c.o.m. energy)

# The Little Bang

Incoming nucleons cannot escape right away into the surrounding vacuum

They rescatter each other, and form a dense and strongly interacting matter




Quick thermalization and large energy density ($e > e_{\rm cr}$) $\Rightarrow$ QGP

Hydrodynamic expansion of the collision fireball

Cooling, when the energy density $e \simeq e_{\rm cr}$ the partons convert to hadrons

We can probe the QGP only indirectly

# Hard Probes for the Quark Gluon Plasma

We will focus on the rare high $p_T$ final state particles ($p_T \geq 2\,\mathrm{GeV}$)

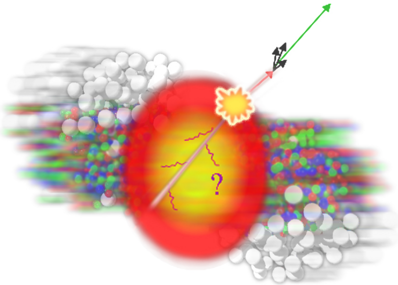
## High $p_T$ particles as probes

From fragmentation of even higher $p_T$ partons

Produced in the collision early stage:

$$p_T \simeq 2\,\mathrm{GeV} \quad \Rightarrow \quad \tau_{\mathrm{f}} \simeq \left(p_T^2\right)^{-1/2} \simeq 0.1\,\mathrm{fm}/c$$

On their evolution they interact with the medium and probe its properties



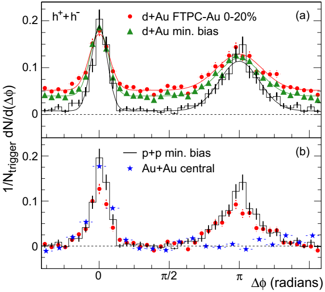

Exploring medium properties by detecting high $p_T$ hadrons outside it:

Jet Emission Tomography (JET)

# Jet Quenching Data - RHIC part 1

## Azimuthal angular correlations

- Trigger on fast particle (say $p_T > 4\,\mathrm{GeV}$)
- look for correlations with other not too soft hadrons (say $p_T > 2\,\mathrm{GeV}$, to remove background from uncorrelated hadrons)




STAR collaboration, arxiv:nucl-ex/0306024

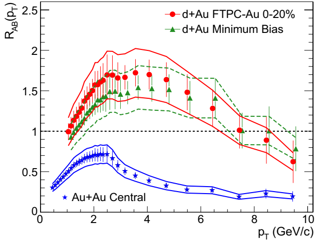
- Partons fragment into a narrow angular cone
- Correlation at small angles, peak at $\Delta\phi = 0$
- In p+p peak at $\Delta\phi = \pi$
- In Au+Au the peak at $\Delta\phi = \pi$ disappears

This is a very pictorial way to convince that jets are quenched.
However it is hard to be quantitative about that...

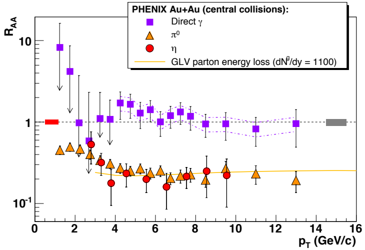
# Jet Quenching Data - RHIC part 2

## Single particle inclusive yield

Nuclear modification factor: $R_{AA} = \frac{1}{N^{AA}_{\rm coll}} \frac{dN^{AA}/dydp_T}{dN^{pp}/dydp_T}$ (No quenching $\rightarrow$ $R_{AA} = 1$)




STAR collaboration, arxiv:nucl-ex/0306024




PHENIX collaboration, arxiv:nucl-ex/0611006

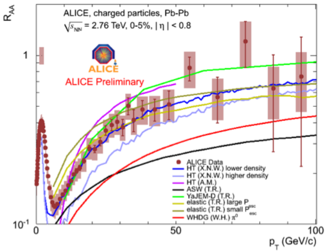
Suppression of high $p_T$ partons in central Au+Au collisions

$R^{AA}$ same for all hadrons, must be parton energy loss.

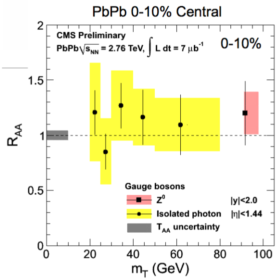
The parton loses energy before hadronization.

# Jet Quenching Data - LHC part 1

$R_{AA}$ for $Pb - Pb$ collisions at $\sqrt{s} \simeq 2.76\,\mathrm{TeV} \times A$




ALICE collaboration, arxiv:1012.1004
Appelshäuser talk at QM2011

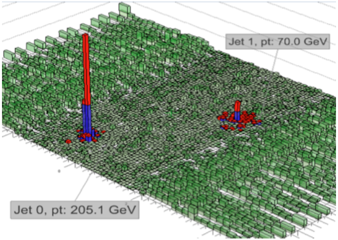



CMS collaboration, arxiv:1102.5435, arxiv:1103.1471
Lee talk at QM2011

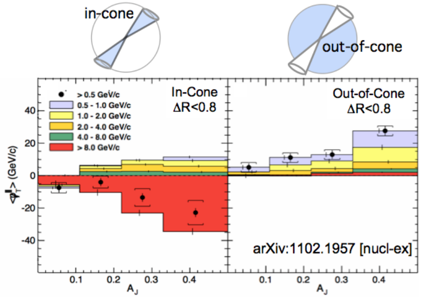
- $R_{AA}$ from 100 MeV to 100 GeV!
- Charged particles and colorless probes (photons, Z bosons)
- Pronounced $p_T$ dependence of $R_{AA}$ at LHC
- Sensitivity to details of the energy loss distribution

# Jet Quenching Data - LHC part 2

Jets in $Pb - Pb$ collisions reconstructed by CMS and ATLAS calorimeters




Jet quenching as a pronounced energy imbalance in central collisions

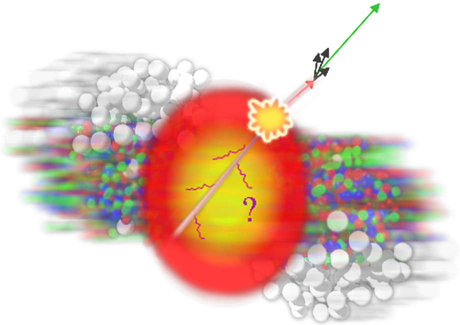



Momentum imbalance recovered when integrating low $p_T$ particles over a wide angular range relative to the away side jet

CMS collaboration, arxiv:1102.1957, Roland talk at QM2011

# Outline

# Jets propagation in dense media



**Hard Probes for the QGP**

Jet Quenching observables make it possible to study how parton fragmentation is affected by the medium

At RHIC: $p_T \simeq$ tens of GeV.

At LHC: $p_T \simeq$ hundreds of GeV.

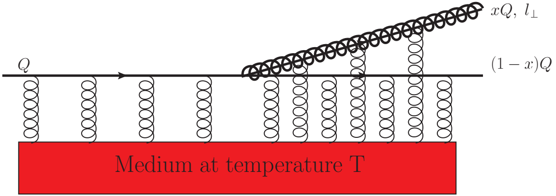
The medium has two main effects on the propagating energetic parton:

- changing direction of its momentum (transverse momentum broadening)
- inducing parton energy loss

# Energy loss in the high energy limit

## Radiative energy loss

High energy limit: QCD analogue of bremsstrahlung dominates




Energetic partons constantly kicked by the medium: all subjects to transverse momentum broadening

## The jet quenching parameter $\hat{q}$

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L}$$

- $\hat{q}$ plays central role in the energy loss calculation
- defined via transverse momentum broadening only

# Effective Field Theory description

## Separation of scales

Particle with energy $Q$ propagating through
a dense medium with characteristic scale $T$

$$\lambda \equiv \frac{T}{Q} \ll 1$$

We can ultimately hope for an Effective Field Theory description:

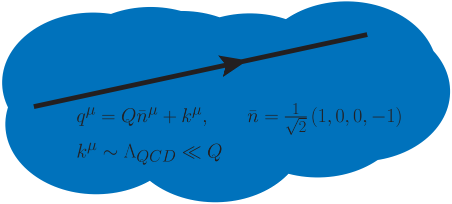
- physics at each scale cleanly separated at leading power
- corrections systematically calculable, order by order in $\lambda$

## Our language: Soft Collinear Effective Theory (SCET)

In the $Q \gg T$ limit natural organization of the modes into kinematic regimes

Other works on SCET applied to parton propagation in dense media:
Idilbi, Majumder, arXiv:0808.1087 Ovanesyan, Vitev, arXiv:1103.1074

# Soft Collinear Effective Theory (SCET)




## SCET

Effective theory of highly energetic approximately massless particles interacting with a soft background

C. Bauer et al. hep-ph/0005275 hep-ph/0011336 hep-ph/0107001 hep-ph/0109045

## SCET degrees of freedom

Introduce fields for infrared degrees of freedom (in operators)

Offshell modes with $q^2 \gg \lambda^2 Q^2$ are integrated out (in coefficients)

| modes | $q^\mu = (q^+, q^-, q_\perp)$ | fields |
|---|---|---|
| collinear | $Q\left(\lambda^2, 1, \lambda\right)$ | $\xi_{\bar{n}},\ A^\mu_{\bar{n}}$ |
| soft | $Q\left(\lambda, \lambda, \lambda\right)$ | $\xi_s,\ A^\mu_s$ |
| ultra-soft | $Q\left(\lambda^2, \lambda^2, \lambda^2\right)$ | $\xi_{us},\ A^\mu_{us}$ |

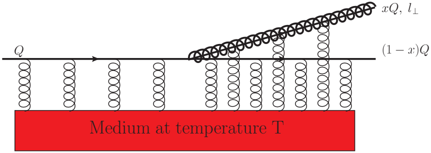
Energetic particle propagating through the medium:
collinear mode of SCET

# Relevance of Glauber modes

Incoming parton, outgoing parton and radiated gluon considered to be collinear.

Radiation vertex already present in SCET

What about momentum broadening?
Are the SCET d.o.f. enough?



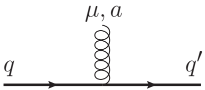

Momentum broadening in the high energy limit dominated by interactions between the energetic collinear parton and Glauber modes from the medium

- Glauber modes: $p = (\lambda^2, \lambda^2, \lambda)Q$

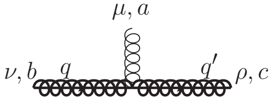
Idilbi, Majumder, arXiv:0808.1087

Glaubers not d.o.f. of SCET, need to extend SCET to include them

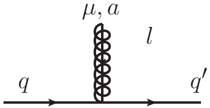
# SCET + Glauber Feynman Rules




$$= igt^a_F \overline{n}_\mu \not{n}$$




$$= -2\, ig\, (t^a_G)_{bc}\, \overline{n}^\mu \times \Big[ g^{\nu\rho} Q + n^\nu (q'_\perp - q_\perp)^\rho - n^\rho (q'_\perp - q_\perp)^\nu - \frac{\alpha-1}{2\alpha} \left( n^\rho q^\nu + n^\nu q'^\rho \right) \Big]$$




$$= i\, g\, t^a_F \left[ \bar{n}_\mu - \frac{1}{2} \frac{\not{q}_\perp + \not{l}_\perp}{n\cdot(q+l)} \frac{\not{q}_\perp}{n\cdot q} n_\mu + \gamma^\perp_\mu \frac{\not{q}_\perp}{2\, n\cdot q} + \frac{\not{q}_\perp + \not{l}_\perp}{2\, n\cdot(q+l)} \gamma^\perp_\mu \right] \not{n}$$

We are ready to compute Feynman diagrams!

# Outline

# First step: Transverse Momentum Broadening

Formulation of the momentum broadening
in the language of Soft Collinear Effective Theory (SCET)

**Our focus**

Non-radiative $k_\perp$ broadening in the high energy limit

- easiest case to handle
- natural context in which $\hat{q}$ arises

**"Semi-controlled" calculation**

Radiation artificially turned off, other than that controlled in the $Q \gg T$ limit

# Set-up of the problem

## Energy scales

Hard parton propagating through some form of QCD matter

Initial four momentum: $q_0 \equiv (q_0^+, q_0^-, q_{0\perp}) = (0, Q, 0)$

We assume $Q \gg T$, we have a small dimensionless ratio $\lambda \equiv \frac{T}{Q} \ll 1$

Example: QGP in equilibrium at temperature $T$
(our analysis would apply to other forms of matter)

## Goal: compute $P(k_\perp)$ and $\hat{q}$

$P(k_\perp)$: probability distribution for the hard parton to acquire transverse momentum $k_\perp$ after traversing the medium

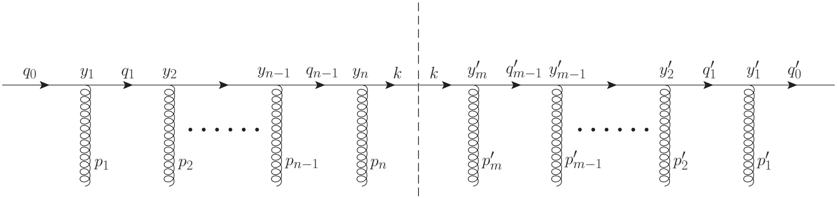
From $P(k_\perp)$ it is straightforward to obtain $\hat{q}$

$$\hat{q} = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \, P(k_\perp) \qquad \int \frac{d^2 k_\perp}{(2\pi)^2} P(k_\perp) = 1$$

# Forward scattering amplitude

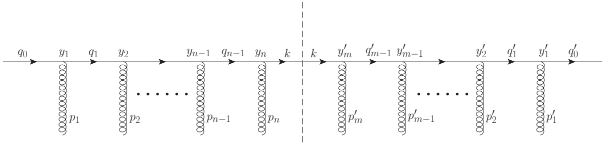
## Strategy

- Compute $2\,\mathrm{Im}\;M_{\alpha\alpha}$ by cutting the appropriate diagrams
- Use the unitarity relation to identify $\sum_\beta |M_{\beta\alpha}|^2$
- Read off $|M_{\beta\alpha}|^2$, and evaluate $P(k_\perp)$ for $k_\perp \neq 0$
- The normalization condition $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$ fixes $P(0)$




$$2\,\mathrm{Im}\,M_{\alpha\alpha} = \sum_{m=1,n=1}^{\infty} \mathcal{A}_{mn} = \sum_{m=1,n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$$

# Forward scattering amplitude evaluation




$$
\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp} = \frac{1}{\sqrt{2}QL^3}\int \frac{dk^+dk^-}{(2\pi)^2}\prod_{i=1}^{n-1}\frac{d^4q_i}{(2\pi)^4}\prod_{j=1}^{m-1}\frac{d^4q'_j}{(2\pi)^4}
$$

$$
\times\, \bar{\xi}_{\bar{n}}(q'_0)\prod_{j=m-1}^{1}\left[(-ig)A^+(-p'_j)\,\not{n}\,\frac{-iQ}{2Qq'^{\,+}_j - q'^{\,2}_{j\,\perp} - i\epsilon}\,\not{\bar{n}}\right](-ig)A^+(-p'_m)\not{n}
$$

$$
\times\, 2\pi Q\delta\left(2k^+Q - k_\perp^2\right)\not{\bar{n}}\; igA^+(p_n)\not{n}\prod_{i=1}^{n-1}\left[\frac{iQ}{2Qq_i^+ - q^2_{i\,\perp} + i\epsilon}\,\not{\bar{n}}\; igA^+(p_i)\not{n}\right]\xi_{\bar{n}}(q_0)
$$

(Analysis is analogous if the hard parton is a collinear gluon.)

$Q \to \infty$ limit: amplitude evaluated for $Q \gg k_\perp^2 L \sim \hat{q}L^2$.

# Result for $P(k_\perp)$

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp), \qquad \mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr} \left[ W^\dagger_\mathcal{R}[x_\perp] \, W_\mathcal{R}[0] \right] \right\rangle$$

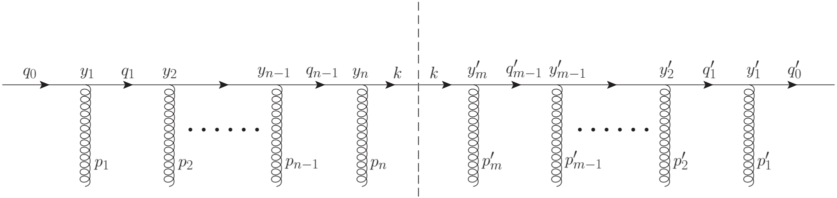
for a collinear particle in the $SU(N)$ representation $\mathcal{R}$, with dimension $d\,(\mathcal{R})$

- $P(k_\perp)$ is a soft function, it depends only on the medium property
- Transverse momentum broadening without radiation:
  field theoretically well-defined property of the medium
- $\hat{q} = \frac{1}{L} \int \frac{d^2k_\perp}{(2\pi)^2} \, k_\perp^2 \, P(k_\perp)$

FDE, Liu, Rajagopal, arXiv:1006.1367 [hep-ph]

$P(k_\perp)$ derived with other techniques: Casalderrey-Solana and Salgado, arXiv:0712.3443 [hep-ph]
Liang, Wang and Zhou, arXiv:0801.0434 [hep-ph]

# Glaubers from the medium




## $A_\mu$ as a background field

(i) hard parton propagating in a specific field configuration $A_\mu(p)$

(ii) average over field configurations: $\left\langle \mathrm{Tr}\left[ W^\dagger_{\mathcal{R}}[x_\perp]\, W_{\mathcal{R}}[0] \right] \right\rangle$

Nature of the medium (strongly coupled? weakly coupled?) affects only (ii)

Same framework for the gluon radiation calculation

# $\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM

At RHIC physics of the QGP at scales $\sim T$ is not weakly coupled

Insights can be obtained by calculating $\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM by using gauge/gravity duality

Liu, Rajagopal, Wiedemann, hep-ph/0605178

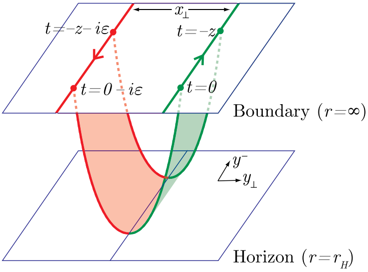
LRW evaluated $\mathcal{W}_{\mathcal{R}}(x_\perp)$ with the standard, i.e. wrong, operator ordering

## Standard AdS/CFT evaluation

- $\mathcal{N} = 4$ $SU(N_c)$ gauge theory, large $N_c$ and $g^2_{YM} N_c$ limit;
- Gravity dual: 4+1dimensional AdS Schwarzschild black hole with Hawking temperature $T$;
- $\langle W(\mathcal{C}) \rangle = \exp\left[i\left\{S(\mathcal{C}) - S_0\right\}\right]$.

($S(\mathcal{C})$ is the action of an extremized string worldsheet, bounded by the Wilson lines along the contour $\mathcal{C}$ located at the 3+1 dimensional boundary, "hanging" into the AdS black hole spacetime. $S_0$ is twice the action of a disconnected world sheet hanging straight down from one Wilson line to the horizon.)

# $\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM revisited




## Old result unchanged!

LRW is the only world sheet which touches the horizon

Subtlety resolved

Result unchanged

FDE, Liu, Rajagopal, arXiv:1006.1367 [hep-ph]

$$\mathcal{W}_\mathcal{A}(x_\perp) = \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}L^- x_\perp^2\right] \quad \Rightarrow \quad \begin{cases} P_\mathcal{A}(k_\perp) = \frac{4\sqrt{2}\pi}{\hat{q}L^-}\exp\left[-\frac{\sqrt{2}k_\perp^2}{\hat{q}L^-}\right] \\ \hat{q}_\mathcal{A} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{g^2 N_c}\,T^3 \end{cases}$$

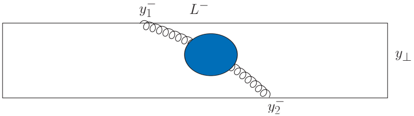
$P(k_\perp)$ at strong coupling is a Gaussian $\Rightarrow$ diffusion in $k_\perp$ space

$\hat{q}$ in the same ballpark as the values of $\hat{q}$ inferred from RHIC data

# $\hat{q}$ in weakly coupled QCD plasma

QCD plasma at high enough temperature that physics at scale $\sim T$ is weakly coupled

Arnold, Xiao, arXiv:0810.1026 [hep-ph]
Caron-Huot, arXiv:0811.1603 [hep-ph]




At lowest order in $g$: $P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) - \delta^2(k_\perp)\int d^2q_\perp P_>(q_\perp)$

Non divergent anywhere and correctly normalized

Broadening probability: $P_>(k_\perp) = \sqrt{2}g^2 C_{\mathcal{R}} L \int \frac{dk^-}{2\pi}\, k_\perp^2\, G^>_{\mu\nu}(0, k^-, k_\perp)\, \bar{n}^\mu \bar{n}^\nu$

$P(k_\perp)$ very different than at strong coupling, falls off slowly $\sim k_\perp^{-4}$ at large $k_\perp$

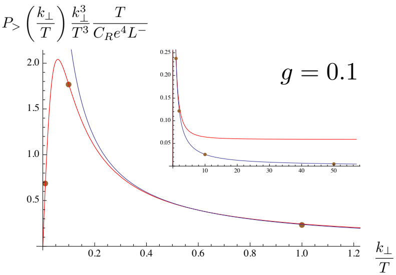
$\hat{q}_{\mathcal{R}} \propto g^4 N_c^2$, and UV log divergent

Probability of picking up large $k_\perp$
much larger than in a strongly coupled plasma,
even though the mean $k_\perp$ picked up is smaller

# $\hat{q}$ in weakly coupled QCD plasma - results

"Abelian" contribution (gluon propagator from quark loops)

- Full expression of the gluon propagator, with no approximations
- IR ($k_\perp \ll T$) well described by HTL
- IR and UV smoothly overlap



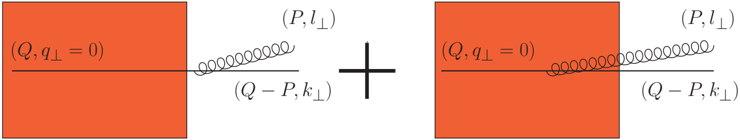

Add the non-abelian contribution — work in progress.....
(FDE, C. Lee, M. Lekaveckas, H. Liu, K. Rajagopal)

# Outline

# Medium induced collinear gluon radiation

Incoming collinear quark radiates a gluon collinear in the same direction

Effective theory valid for any collinear gluon energy

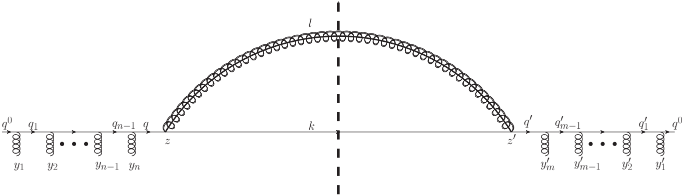



Total amplitude: vacuum + medium emission

$$\mathcal{M} = \mathcal{M}_v + \mathcal{M}_m \quad \Rightarrow \quad |\mathcal{M}|^2 = |\mathcal{M}_v|^2 + |\mathcal{M}_m|^2 + 2\,\mathrm{Re}\,(\mathcal{M}_m \mathcal{M}_v^*)$$

We integrate over the final quark kinematical variables,
matrix element squared will only depend on $l_\perp$ and $P$

# Vacuum emission




$$|\mathcal{M}_v|^2 = 4\, g^2\, C_F \int d^2 x_\perp \exp\left[i\, l_\perp \cdot x_\perp\right] \mathcal{Y}[x_\perp, l_\perp, P]$$

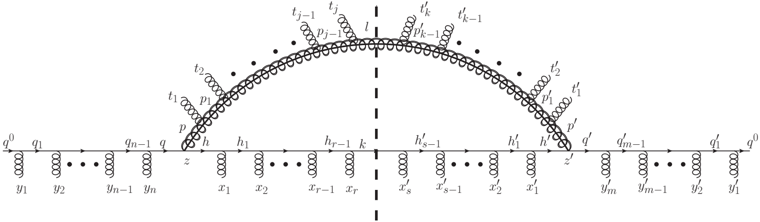
where: $\mathcal{Y}[x_\perp, l_\perp, P] \equiv$

$$\mathcal{W}_F(x_\perp) \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{\exp[i\, k_\perp \cdot x_\perp]}{\left(\frac{k_\perp^2}{Q-P} + \frac{l_\perp^2}{P}\right)^2} \left[\frac{1}{2}\frac{k_\perp^2}{(Q-P)^2} + \frac{1}{2}\frac{(l_\perp + k_\perp)^2}{Q^2} + \frac{l_\perp^2}{P^2} - \frac{l_\perp \cdot k_\perp}{P(Q-P)} - \frac{l_\perp \cdot (l_\perp + k_\perp)}{QP}\right]$$

Check $P \ll Q$ limit: $\mathcal{Y}[x_\perp, l_\perp, P] \simeq \delta^2(x_\perp) \frac{1}{l_\perp^2}$ $\Rightarrow$ $|\mathcal{M}_v|^2 = \frac{4\, g^2\, C_F}{l_\perp^2}$

Wiedemann, hep-ph/0005129, hep-ph/0008241

# Medium emission




- Incoming quark described by a Wilson line
- Outgoing quark and radiated gluon described by 2-dimensional path integrals in the transverse space
- Glauber fields both in the fundamental and the adjoint
- Emission vertex has $q_\perp \Rightarrow$ transverse derivatives acting on Glauber fields

$$
\begin{aligned}
|\mathcal{M}_m|^2 = & 2\, g^2 \int d^4z\, d^4z' \int dy'^+ d^2y'_\perp\, dy^+ d^2y_\perp \int dx'^+ d^2x'_\perp\, dx^+ d^2x_\perp \int dt'^+ dt^+ \\
& \int \prod_{i=1}^{k} dt_i'^- d^2t'_{i\,\perp} \prod_{i=1}^{j} dt_i^- d^2t_{i\,\perp} \int \prod_{i=1}^{m} dy_i'^- \prod_{i=1}^{s} dx_i'^- d^2x'_{i\,\perp} \prod_{i=1}^{r} dx_i^- d^2x_{i\,\perp} \prod_{i=1}^{n} dy_i^- \\
& \int \frac{d^4q}{(2\pi)^4} \frac{d^4q'}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} \int \frac{d^4h}{(2\pi)^4} \frac{d^4h'}{(2\pi)^4} \\
& \exp\Big[ i\, q_0 \cdot (y'_1 - y_1) + i\, q' \cdot (z' - y'_m) + i\, p' \cdot (t'_1 - z') + i\, h' \cdot (x'_1 - z') + \\
& i\, l \cdot (t_j - t'_k) - i\, k \cdot (x'_s - x_r) - i\, q \cdot (z - y_n) - i\, h \cdot (x_1 - z) - i\, p \cdot (t_1 - z) \Big] \\
& \frac{1}{N_c} \mathrm{Tr} \Bigg[ \prod_{i=1}^{m} (-ig) A_F^+(y'^+, y_i'^-, y'_\perp)\, t_F^a \prod_{i=1}^{s} (-ig) A_F^+(x'^+, x_i'^-, x'_\perp) \\
& \prod_{i=r}^{1} ig A_F^+(x^+, x_i^-, x_\perp)\, t_F^b \prod_{i=n}^{1} ig A_F^+(y^+, y_i^-, y_\perp) \Bigg] \\
& \prod_{i=1}^{m-1} \theta(y_{i+1}'^- - y_i'^-) \prod_{i=1}^{s-1} \theta(x_{i+1}'^- - x_i'^-) K\left(x'_{i+1}; x'_i\right) \prod_{i=1}^{r-1} \theta(x_{i+1}^- - x_i^-) K\left(x_{i+1}; x_i\right) \prod_{i=1}^{n-1} \theta(y_{i+1}^- - y_i^-) \\
& 2S_Q^*(q')\, 2S_{Q-P}^*(h')\, 2S_P^*(p')\, 2S_{Q-P}(h)\, 2S_Q(q)\, 2S_P(p)\, (Q-P)\, \Delta(q_\perp, \ldots) \\
& \Bigg[ \prod_{i=1}^{k-1} \theta(t_{i+1}'^- - t_i'^-) \prod_{i=1}^{k} (-ig) A_G^+(t'^+, t_i'^-, t'_{i\,\perp}) \prod_{i=1}^{k-1} K\left(t'_{i+1}; t'_i\right) \\
& \prod_{i=1}^{j-1} \theta(t_{i+1}^- - t_i^-) \prod_{i=j}^{1} ig A_G^+(t^+, t_i^-, t_{i\,\perp}) \prod_{i=1}^{j-1} K\left(t_{i+1}; t_i\right) \Bigg]_{ab}
\end{aligned}
$$

# Medium emission

## Key question

What soft function controls collinear radiation?

namely

What medium property is "measured" by parton energy loss?

## $P \ll Q$ limit correctly reproduced

$$|\mathcal{M}_m|^2 = \frac{1}{P^2} g^2 \int dz'^{-} dz^{-} \int d^2 t'_\perp d^2 t_\perp \exp\left[-i\, l_\perp \cdot (t_\perp - t'_\perp)\right] W_A^{ab}(z'^{-}, z^{-})\,[0_\perp]$$
$$\left[\frac{\partial}{\partial \bar{y}_\perp} G\left(\bar{y}_\perp = 0_\perp, z'^{-}; t'_\perp, L^{-};\right) \frac{\partial}{\partial y_\perp} G\left(t_\perp, L^{-}; y_\perp = 0_\perp, z^{-}\right)\right]_{ab}$$

Wiedemann, hep-ph/0005129, hep-ph/0008241

- For $P \ll Q$ the soft function has only quantities in the adjoint representation
- Only in this limit parton energy loss "measures" $\hat{q}$

# Outline

# Summary and future directions

Transverse Momentum Broadening within SCET $\surd$

- Glaubers responsible for $k_\perp$ broadening in the absence of radiation
- $P(k_\perp)$ and $\hat{q}$ governed by a field theoretically well-defined property of the medium

Medium Induced Collinear Radiation within SCET (in progress)

- radiated gluon collinear to the incoming quark, any fraction of its energy

## Future directions

- complete the collinear gluon spectrum, evaluate the new soft functions
- include $\lambda$ power corrections

and also...

- introduce soft modes ($p \sim Q(\lambda, \lambda, \lambda)$) allowing for large angle radiation
- allow for emission in any collinear direction

(FDE, C. Lee, H. Liu, K. Rajagopal)

# Backup slides

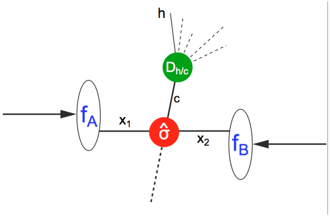
BACKUP
SLIDES

# Hard processes in heavy ion collisions

Factorization assumed between the perturbative hard part and the non-perturbative fragmentation (FF) and parton distribution functions (PDF).

The PDF are assumed to be universal (known from DIS).

Hard scattering cross sections computed in pQCD.



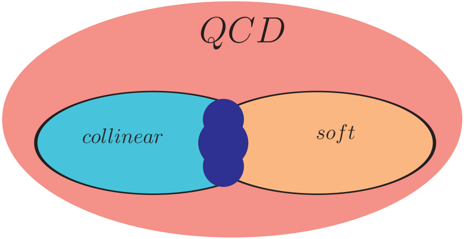

$$\sigma^{AB\to h} = f_A(x_1, Q^2) \otimes f_B(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{c\to h}(z, Q^2)$$

## Medium modified fragmentation function

FF modified when the fragmentation takes place in the medium.

Measuring these modifications: characterization of the medium properties.

# More about SCET




## SCET Lagrangian

$$\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_{\mathrm{c}} + \mathcal{L}_{\mathrm{s}} + \mathcal{L}_{\mathrm{c,s}}$$

Collinear sector (QCD in boosted frame) and soft sector (QCD) coupled through a single term.

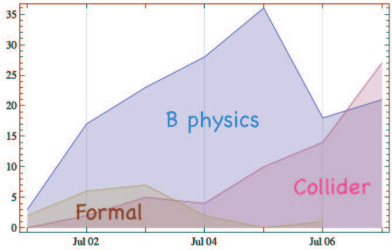
## What is SCET good for?

Factorization: obtained from field redefinition and simple algebraic manipulations (decouple soft from collinear in the Lagrangian).

Summation of logarithms at the edges of phase space: obtained from Renormalization Group Equations (RGEs).

Systematically incorporate power corrections in $\lambda$.

# SCET applications




## B physics

Understand many new processes

Power corrections for better precisions

Improve perturbative results by proper resummation of logarithms

## Collider physics

Factorization easier to understand

Perturbative calculation by standard EFT steps: sequences of matching and running

Processes with several scales easily understood

SCET gives operator definitions of all non-perturbative quantities

SCET is a systematic expansion order by order.

from C. Bauer, talk at SCET08

# Glauber gluons

Transverse momentum in excess of their longitudinal momentum.
They cannot be be thought of as being on the mass-shell.

## Glauber gluons in SCET

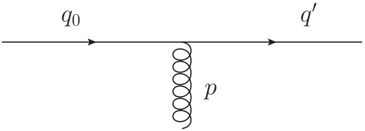
Attempt to prove Glauber factorization in Drell-Yan (flaws in the argument...).
Liu and Ma, arxiv:0802.2973 [hep-ph].

Explicit shown that Glaubers need to be included for a certain class of processes.
Bauer et all., arxiv:1010.1027 [hep-ph].

They might be integrated out of the effective theory, leading to a potential between pairs of collinear fields in opposite directions.
Stewart and Rothstein, in progress, talk at SCET2010.

Used to describe jet broadening in dense QCD medium.
Idilbi and Majumder, PRD80(2009); D'Eramo, Liu and K. Rajagopal, arxiv:1006.1367 [hep-ph].

# $k_\perp$ broadening in the high energy limit




$$q_0 = (0, Q, 0)$$

$$q' = q_0 + p$$

## Soft gluon: $p = (\lambda, \lambda, \lambda)Q$

Final state $Q(\lambda, 1, \lambda)$ not collinear.
Kicked off-shell by ${q'}^2 \sim \lambda Q^2$ .
Process suppressed by $\alpha_s(\sqrt{TQ})$.
Subsequent radiation induced.

## Glauber gluon: $p = (\lambda^2, \lambda^2, \lambda)Q$

Final state $Q(\lambda^2, 1, \lambda)$ is collinear
Further Glaubers keep the parton collinear.
Not induced radiation.
Interaction vertex: $\alpha_s(T)$

## Relevance of Glauber gluons

Both processes yield $k_\perp$ broadening of order $\lambda Q \sim T$, soft suppressed by $\alpha_s(\sqrt{TQ})$.
Glauber gluons responsible for momentum broadening in the absence of radiation.
All processes (including radiation) must be included before comparing to data.

# SCET + Glauber Effective Lagrangian I

## Effective Lagrangian derivation for collinear quarks

Start from the QCD Lagrangian and keep only the relevant d.o.f.

Integrate out $\xi_n(x)$ by using its equations of motion

$$\mathcal{L}_{QCD} = \bar{\xi} i \not{D} \xi \quad \Rightarrow \quad \mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i \not{n} \,(\bar{n}\cdot D)\, \xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\, i \not{D}_{\perp} \frac{1}{2\, in\cdot D}\, i\not{D}_{\perp}\, \not{n}\, \xi_{\bar{n}}$$

Restrict to interactions with Glauber gluons only in $D_\mu \equiv \partial_\mu - igA_\mu$, which can only change the perpendicular momentum $q_\perp$ of the collinear quark field.

Remove "large" phases from $\xi_{\bar{n}}(x)$: $\xi_{\bar{n}}(x) = e^{-iQx^+} \sum_{q_\perp} e^{iq_\perp \cdot x_\perp} \xi_{\bar{n},q_\perp}(x)$

Power counting in $\lambda$: $\xi_{\bar{n}}(x) \sim \lambda, \quad i\partial_\mu \xi_{\bar{n},q_\perp}(x) \sim \lambda^2 \xi_{\bar{n},q_\perp}(x), \quad A^+ \sim \lambda^2$.

At the leading order in $\lambda$:

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp}\, \bar{\xi}_{\bar{n},q'_\perp} \left[ i\bar{n}\cdot D + \frac{q_\perp^2}{2Q} \right] \not{n} \xi_{\bar{n},q_\perp}$$

Idilbi, Majumder, Phys.Rev.D80:054022,2009. [arXiv:0808.1087]

# SCET + Glauber Effective Lagrangian II

## Goal

Derive an effective Lagrangian to describe a theory of collinear partons (quarks or gluons) and Glauber gluons.

Top-down EFT: start from QCD Lagrangian, keep only relevant d.o.f.

## EFT fields

Light-cone unit vectors: $\bar{n} \equiv \frac{1}{\sqrt{2}}(1, 0, 0, -1)\,, \qquad n \equiv \frac{1}{\sqrt{2}}(1, 0, 0, 1).$

Quark field decomposition:

$$\xi(x) = \xi_{\bar{n}}(x) + \xi_n(x), \qquad \xi_{\bar{n}}(x) \equiv \frac{\not{\bar{n}}\not{n}}{2}\xi(x), \qquad \xi_n(x) \equiv \frac{\not{n}\not{\bar{n}}}{2}\xi(x).$$

Collinear quark field: "large" component $\xi_{\bar{n}}(x)$,
the "small" component $\xi_n(x)$ is integrated out.

Collinear gluon field: $A^\mu_{\bar{n}}(x)$.

Glauber gluon field: $A^\mu_G(x)$ (background field).

# Optical theorem

## Field theory tools

Use the optical theorem to relate $P(k_\perp)$ to a matrix element that we can calculate using the Feynman rules we have just derived.

## Unitarity of the $S$-matrix

Probability amplitude for the process $\alpha \to \beta$: $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$.

The $S$-matrix is unitary: $\sum_\beta |S_{\beta\alpha}|^2 = 1 \Rightarrow 2\,\mathrm{Im}\; M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$.

Cubic box of sides $L$. Periodic BC $\Rightarrow \mathbf{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$.

With radiation turned off $\beta$ differs from $\alpha$ only on $k_\perp$: $\sum_\beta = L^2 \int \frac{d^2 k_\perp}{(2\pi)^2}$.

## Probability distribution $P(k_\perp)$

We identify: $P(k_\perp) = L^2 \begin{cases} |M_{\beta\alpha}|^2 & \beta \neq \alpha \\ 1 - 2\mathrm{Im}\, M_{\alpha\alpha} + |M_{\alpha\alpha}|^2 & \beta = \alpha \end{cases}$

Unitarity of S-matrix $\quad \leftrightarrow \quad$ $P(k_\perp)$ is normalized.

# Forward scattering amplitude evaluation II

## Three simple steps

- average over the color indices;
- some Dirac algebra;
- gluon fields in the coordinate space $A^{+}(p_i) = \int d^4 y_i e^{ip_i y_i} A^{+}(y_i)$.

$$\frac{d^2 \mathcal{A}_{mn}}{d^2 k_{\perp}} = \frac{2^{n+m}}{\sqrt{2} L^3 N_c} \int \prod_{i=1}^{n} d^4 y_i \prod_{j=1}^{m} d^4 y'_j \, e^{-iq_0 \cdot (y_1 - y'_1)} \, \mathrm{Tr} \left[ \prod_{j=m}^{1} (-ig) A^{+}(y'_j) \prod_{i=1}^{n} igA^{+}(y_i) \right]$$
$$\times \, g(y_n - y'_m, k_{\perp}) \prod_{j=1}^{m-1} f^{*}(y'_j - y'_{j+1}) \prod_{i=1}^{n-1} f(y_i - y_{i+1})$$

$$f(z) \equiv \int \frac{d^4 q}{(2\pi)^4} \frac{iQ}{2Qq^{+} - q_{\perp}^2 + i\epsilon} e^{iq \cdot z} = \delta(z^{+}) \theta(-z^{-}) \frac{iQ}{4\pi z^{-}} e^{-i \frac{Q}{2z^{-}} z_{\perp}^2},$$

$$g(z, k_{\perp}) \equiv \int \frac{dk^{+} dk^{-}}{(2\pi)^2} 2\pi Q \delta \left( 2k^{+} Q - k_{\perp}^2 \right) e^{ik \cdot z} = \frac{1}{2} \delta(z^{+}) e^{-ik_{\perp} \cdot z_{\perp} + i \frac{k_{\perp}^2}{2Q} z^{-}} \quad .$$

# The $Q \to \infty$ limit and its physical significance

## $Q \to \infty$ for $f(z)$ and $g(z, k_\perp)$

So far not used the $Q \to \infty$ limit (although used in setting up the problem). In this limit both $f(z)$ and $g(z, k_\perp)$ simplify.

- $Q \gg p_\perp^2 z^- \quad \Rightarrow \quad f(z) \approx \frac{1}{2}\delta(z^+)\,\theta(-z^-)\,\delta^2(z_\perp)$
- $Q \gg k_\perp^2 z^- \quad \Rightarrow \quad g(z, k_\perp) \approx \frac{1}{2}\delta(z^+)\,e^{-ik_\perp \cdot z_\perp}$

Criterion for $g(z, k_\perp)$ stronger, we require: $Q \gg k_\perp^2 L \sim \hat{q} L^2$.

## Physical significance of the $Q \to \infty$ limit: $Q \gg k_\perp^2 L \sim \hat{q} L^2$

The propagators of the internal quarks are $f(z) \propto \delta^2(z_\perp)$.

It requires $L$ is short enough that the hard parton trajectory in position space remains well-approximated as a straight line, even though it picks up transverse momentum.

# Forward scattering amplitude evaluation III

## Summing over all the diagrams

Summing over $m$ and $n$ and taking the $\langle \ldots \rangle$ at the end of the calculation

$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{\sqrt{2}}{L^3 N_c} \int dy^+ dy_\perp dy'_\perp \, e^{-ik_\perp \cdot (y_\perp - y'_\perp)} \left\langle \mathrm{Tr}\left[ \left( W_F^\dagger[y^+, y'_\perp] - 1 \right) \left( W_F[y^+, y_\perp] - 1 \right) \right] \right\rangle$$

where we have introduce the fundamental Wilson line along the lightcone

$$W_F\left[y^+, y_\perp\right] \equiv P\left\{ \exp\left[ ig \int_0^{L^-} dy^- \, A^+(y^+, y^-, y_\perp) \right] \right\}$$

## Cleaning up the result

The medium is translation invariant:
result independent on $y^+$ and depends only on $x_\perp = y_\perp - y'_\perp$.

The incident flux is $1/L^3$, so $t/L$ particles going through the box in time $t$.
Divide the result by $t/L$ to obtain $P(k_\perp)$ for a single particle.

# Forward scattering amplitude evaluation IV

## Final form for the forward scattering amplitude

$$\sum_{m=1,n=1}^{\infty} \frac{d^2\mathcal{A}_{nm}}{d^2k_\perp} = \frac{1}{N_c} \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr} \left[ \left( W_F^\dagger[0, x_\perp] - 1 \right) \, \left( W_F[0,0] - 1 \right) \right] \right\rangle$$

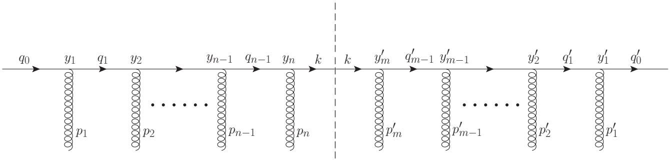
## $|M_{\beta\alpha}|^2$ from the unitarity relation

The unitarity relation $\int \frac{d^2k_\perp}{(2\pi)^2} \sum_{n=1,m=1}^{\infty} \frac{d^2\mathcal{A}_{nm}}{d^2k_\perp} = 2\,\mathrm{Im}\, M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$

allows us to identify

$$|M_{\beta\alpha}|^2 = \frac{1}{L^2\,N_c} \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr} \left[ \left( W_F^\dagger[0, x_\perp] - 1 \right) \, \left( W_F[0,0] - 1 \right) \right] \right\rangle$$

# A few comments on $\frac{d^2 \mathcal{A}_{mn}}{d^2 k_\perp}$




## Gluon momenta

Gluon momenta $p_i$ and $p'_j$ fixed by four-momentum convervation at each vertex
$p_i = q_i - q_{i-1}$ $(i = 1, \ldots, n-1)$; $p_n = k - q_{n-1}$; $p'_j = q'_j - q'_{j-1}$ $(j = 1, \cdots, m-1)$.

$n + m$ gluon field insertions, but only $n + m - 1$ independent momentum integrations

## The cut momentum: $k_\perp$ not integrated over

The cut momentum $k$ is the four-momentum of the hard parton in the final state.

For forward scattering amplitude: $q_0 = q'_0 \Rightarrow k_\perp = \sum_{i=1}^{n} p_{i\perp} = \sum_{i=1}^{m} p'_{i\perp}$

$p_{i\perp}$'s and $p'_{i\perp}$'s are of order $\lambda Q = T$ , $k_\perp$ may turn out to be larger.

Typical value of $k_\perp^2$ is $\hat{q}L$, in particular $k_\perp^2$ grows with $L$.

# Operator ordering for the $\hat{q}$ evaluation

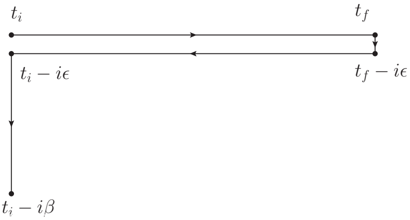
## $\hat{q}$ from light-like Wilson lines

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{\sqrt{2}}{L^-} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \int d^2 x_\perp \, e^{i k_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

$\mathcal{W}_\mathcal{R}(x_\perp)$: different operator ordering than a standard Wilson loop ($A^+ = (A^+)^a t^a$).

## Standard Wilson loop

$(A^+)^a$ time ordered, $t^a$ path ordered.




## Wilson lines in $\mathcal{W}_\mathcal{R}(x_\perp)$

$(A^+)^a$ path ordered, $t^a$ path ordered.

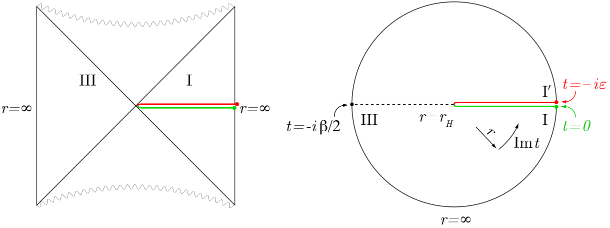
$\mathcal{W}_\mathcal{R}(x_\perp)$ should be described using the Schwinger-Keldysh contour

- one of the light-like Wilson line on the $\mathrm{Im}\, t = 0$ segment
- the other light-like Wilson line on the $\mathrm{Im}\, t = -i\epsilon$ segment

# Taking the order into account

Our procedure to take the order into account: specific example of the more general Lorentzian AdS/CFT. (Skenderis, van Rees, JHEP 0905:085,2009. [arXiv:0812.2909].)

Construct the bulk geometry for the $\mathrm{Im}\, t = -i\epsilon$ segment of the Schwinger-Keldysh contour.




Any string world sheet connecting the Wilson lines at $\mathrm{Im}\, t = 0$ and $\mathrm{Im}\, t = -i\epsilon$, as in our case, must touch the horizon.

# RHIC data and strong coupling result

High $p_T$ suppression entirely due to parton energy loss.

$$d\sigma_{med}^{AA \to h\, rest} = \sum_f d\sigma_{vac}^{AA \to f\, X} \otimes P_f(\Delta E, L, \hat{q}) \otimes D_{f \to h}^{vac}(z)$$

High $p_T$ limit: properties of the medium enter $P_f$ only through $\hat{q}$.

## RHIC data fit

Introduce: $\hat{q} = 2\, K\, e^{3/4}$

More stable on $K$ rather than $\hat{q}$.

Fitting RHIC data: $K = 4.1 \pm 0.6$.

At RHIC temperature regime:
$e \sim (9 - 11)\, T^4$.

Therefore we get: $\hat{q} \sim 4.5\, \mathrm{GeV}^2/\mathrm{fm}$

## Strong coupling result

We rewrite the result:
$\hat{q} = 57 \sqrt{\alpha_{SYM} \frac{N_c}{3}}\; T^3$

By comparing with the energy density we get a good match for $\alpha_{SYM} \sim 0.66$ and $N_c = 3$.

Extraction of $\hat{q}$ from LHC data should be under better control, since the separation of scale will be more quantitatively reliable.