Rapidity Renormalization Group
... and its Applications

Jui-Yu Mannie Chiu (CMU)

Physics
Carnegie Mellon University

September 28, 2011/INT-Frontiers in QCD

In Collaboration with:
Ambar Jain, Duff Neill, Ira Rothstein

Ref: ArXiv/1104.0881 and those to appear

Outline

1. Introduction
   - Event Shape, Angularities, and Jet Broadening
2. Soft Collinear Effective Field Theory (SCET)
3. η-Regulator and ν-Renormalization Group
4. Numerics and Data
5. Other Applications
   - Higgs p_T distribution
6. Conclusion

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Event Shapes

- A large class of event shape observables can be written in the form of

$$e(X) = \frac{1}{Q} \sum_{i \in X} \left| p_{i,\perp} \right| f_e(\eta_i),$$

where rapidity $\eta = \frac{1}{2} \log \left(\frac{E+p_{\parallel}}{E-p_{\parallel}} \right)$

  - Thrust
$$\tau = 1 - T \quad = \quad 1 - \frac{1}{Q} \max_{\hat{t}} \left[\sum_i \left| \hat{t} \cdot p_i \right| \right]$$
$$= \quad \frac{1}{\sqrt{s}} \sum_{i \in X} |p_{i,\perp}| \, e^{-|\eta_i|}$$

  - C-paprameter
$$C = \frac{1}{\sqrt{s}} \sum_{i \in X} |p_{i,\perp}| \, \frac{3}{\cosh \eta_i}$$

  - Jet Broadening
$$B_T = \frac{1}{2\sqrt{s}} \sum_{i \in X} |p_{i,\perp}|$$

Angularities

... as a subclass

- Berger, Kucs, Sterman, 03

$$\tau_a = \frac{1}{\sqrt{s}} \sum_{i \in X} E_i (\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$
$$= \frac{1}{\sqrt{s}} \sum_{i \in X} |p_{i,\perp}| \, e^{-|\eta_i|(1-a)}$$

- Infrared safety: $-\infty < a < 2$

- Factorizability: $-\infty < a < 1$? (Hornig, Lee, Ovanesyan)
  SCET$_a$ (Chris' talk)

- For $a \ll 1$, $|p_{soft,\perp}|/|p_{coll.,\perp}| \sim e^{-\eta_{coll.}} \ll 1$
  E.g., a=0, Thrust distribution $\tau = 1 - T$

- For $a \sim 1$, $|p_{soft,\perp}| \sim |p_{coll.,\perp}|$.
  E.g., a=1, jet broadening observable, B.

Factorization of Angularities

- Factorization Theorem in QCD
  - Collins, Soper, Sterman,...
  - Berger, Kucs, Sterman (03)
    Angularities for $a < 1$ been calculated to NLL/LO
- Factorization Theorem in Traditional SCET (SCET I)
  - Bauer, Manohar,Wise, Lee, Sterman, Becher, Schwartz, Fleming, Hoang, Mantry, Stewart, ...
  - Hornig, Lee, Ovanesyan calculated angularities in SCET to NLL/LO for $a < 1$.
- Fail at a=1!
  - Spurious divergences in each sector ($a \geq 1$), yet disappear after summing over sectors as long as $a \leq 2$.
  - Rapidity divergence
- Other cases with divergence in similar nature?

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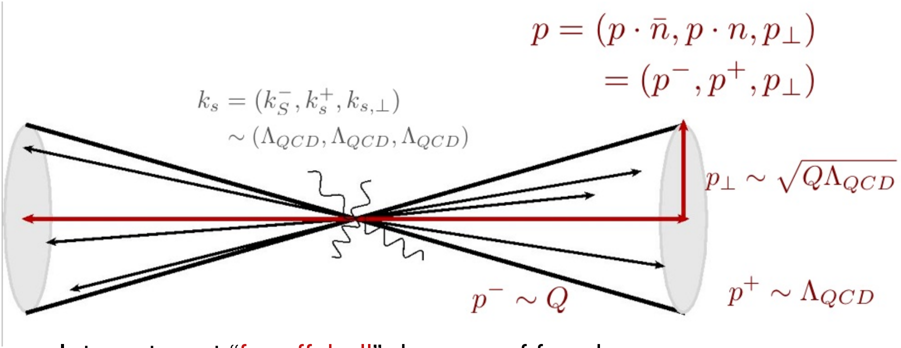
Soft Collinear Effective Theory (SCET I)

(Luke, Bauer, Fleming, Pirjol, Stewart)

- Describe interactions between energetic particles $E \sim Q$.
- Fluctuations, Λ_{QCD} or other low energy scales, about light cone coordinate $n = (1, 0, 0, 1)$.

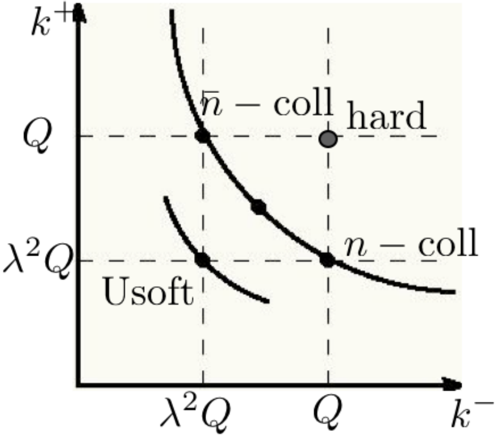


- Integrate out "far offshell" degrees of freedom.
  - soft-collinear decoupling

SCET degrees of freedom (modes)

$$p^\mu = (p^-, p^+, p^\perp); \qquad p^2 = p^+p^- + p_\perp^2$$

- Light Cone Coordinates:
  $n = (1, \vec{n}) \sim (1, 0, 0, 1)$
- power counting parameter
  $\lambda \equiv \frac{\Lambda_{QCD}}{Q}$
- hard modes: $p^2 \sim Q^2$
  integrated out
- n-collinear
  $p^\mu \sim Q(1, \lambda^2, \lambda)$
- $\bar{n}$-collinear
  $p^\mu \sim Q(\lambda^2, 1, \lambda)$
- usoft ($p^2 \sim Q^2\lambda^4$)
  $p^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$



Factorization Theorem for Angularities $\tau_{a<1}$

Bauer, Fleming, Lee, Sterman, 08

<Chis' talk on Tuesday>

- In QCD

$$\frac{d\sigma}{de} = \frac{1}{2Q^2} \sum_X |\mathcal{M}(e^+e^- \to X)|^2 (2\pi)^4 \delta^4(e - e(x))$$

where

$$|\mathcal{M}|(e^+e^- \to X)|^2 = \sum_{i=V,A} L^i_{\mu\nu} \langle 0|j_i^{\mu\dagger}(x)|X\rangle \delta(e - e(X)) \langle X|j_i^\nu(0)|0\rangle$$

and $L^i_{\mu\nu}$ is the lepton tensor, and $j_i^{\mu,\nu}$ are the currents.

- Define operator $\hat{e}$ that returns the value of an event shape for a given final state X, $\hat{e}|X\rangle = e(X)|X\rangle$

$$\frac{d\sigma}{de} = \frac{1}{2Q^2} \int dx e^{iq\cdot x} \sum_{i=V,A} L^i_{\mu\nu} \langle 0|j_i^{\mu\dagger}(x)\delta(e - \hat{e}) j_i^\nu(0)|0\rangle$$

Factorization Theorem for Angularities $\tau_{a<1}$

Bauer, Fleming, Lee, Sterman, 08

- Matching onto SCET

$$j_i^\mu(x) = \sum_n \sum_{\tilde{p}_n, \tilde{p}_{\bar{n}}} C_{n\bar{n}}(\tilde{p}_n, \tilde{p}_{\bar{n}}; \mu) \mathcal{O}_{n,\bar{n}}(x; \tilde{p}_n, \tilde{p}_{\bar{n}}; \mu),$$

in which,

$$\mathcal{O}_{n,\bar{n}}(x; \tilde{p}_n, \tilde{p}_{\bar{n}}; \mu) = e^{i(\tilde{p}_n - \tilde{p}_{\bar{n}})} \bar{\chi}_{n,p_n}(x) Y_n(x) \Gamma_i^\mu \bar{Y}_{\bar{n}}(x) \chi_{\bar{n},p_{\bar{n}}}(x)$$

- Write the event shape distribution in SCET in a factorized form

$$\frac{d\sigma}{de} = \frac{1}{6Q^2} \sum_n |C_{n\bar{n}}(\tilde{p}_n, \tilde{p}_{\bar{n}}; \mu)|^2 \int dx \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) \frac{1}{N_C^2}$$
$$\times \mathrm{Tr}\langle 0 | \chi_{n,Q}(x)_\beta \delta(e_n - \hat{e}_n) \bar{\chi}_{n,Q}(0)_\gamma | 0 \rangle \mathrm{Tr}\langle 0 | \bar{\chi}_{n,-Q}(x)_\alpha \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}) \bar{\chi}_{n,Q}(0)_\delta | 0 \rangle$$
$$\times \mathrm{Tr}\langle 0 | \bar{Y}_{\bar{n}}^\dagger(x) Y_n^\dagger(x) \delta(e_s - \hat{e}_s) Y_n \bar{Y}_{\bar{n}}(0) | 0 \rangle \sum_{i=V,A} L^i \left((\bar{\Gamma}_i^\mu) \right)_{\alpha\beta} \left(\Gamma_{i\mu} \right)_{\gamma\delta}$$

- Final result for differential event shape distribution

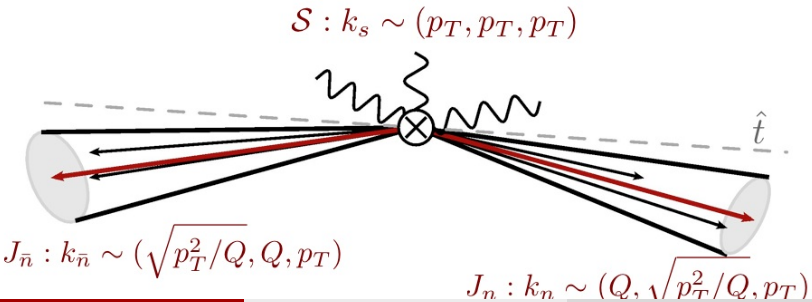
$$\frac{1}{\sigma_0} \frac{d\sigma}{de} = H(s; \mu) \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) J_n(e_n; \mu) J_{\bar{n}}(e_{\bar{n}}; \mu) \mathcal{S}(e_s; \mu)$$

[Not Quite] Back-to-Back Jets in SCET

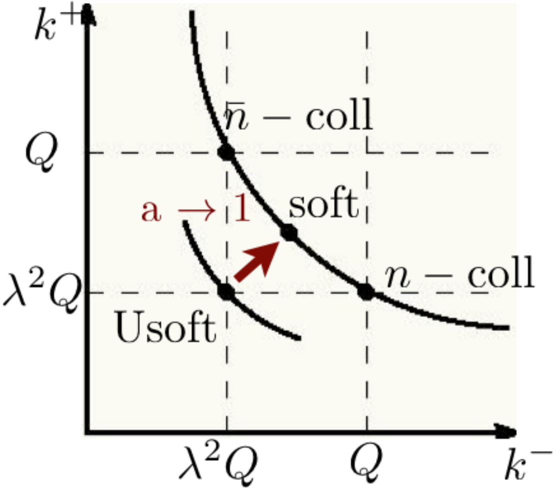
When $a \to 1$

$$\tau_a = \frac{1}{\sqrt{s}} \sum_{i \in X} |p_{i,\perp}| \, e^{-|\eta_i|(1-a)}$$

- For $a \ll 1$, $e_s \sim e_n \sim e_{\bar{n}}$, but $|p_\perp^s| \ll |p_\perp^n| \sim |p_\perp^{\bar{n}}|$ and $|\eta^s| \ll |\eta^n| \sim |\eta^{\bar{n}}|$.
  Soft radiation decoupled from the collinear radiation.
- For $a \to 1$, $\tau_{a=1} = \frac{1}{\sqrt{s}} \sum |p_\perp^i|$ is independent of the rapidity of the rapidity of each sate. For the $e \sim e_s \sim e_n$, we need $|p_\perp^s| \sim |p_\perp^n|$.

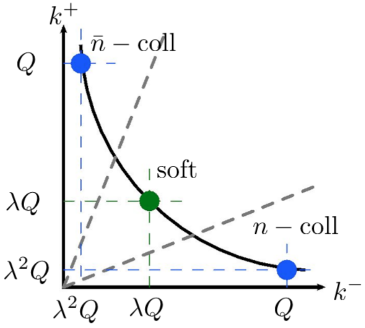


SCET Modes


k^+
$\bar{n} - \text{coll}$
Q
$\text{a} \to 1$
soft
$\lambda^2 Q$
$n - \text{coll}$
Usoft
$\lambda^2 Q$
Q
k^-

For a=1, SCET_{II}

- Jet Broadening Event Shape
  $B_T = \frac{1}{2}\tau_{a=1} = \frac{1}{2}\sum \frac{|\vec{k}_{i,\perp}|}{Q}$
- Demanding $B_T \ll 1$ for dijet events
- Relevant on-shell modes with $|\vec{p}_t| \sim \lambda Q$
- soft modes: $k_s \sim Q(\lambda, \lambda, \lambda)$
  collinear modes: $k_n \sim Q(1, \lambda^2, \lambda)$
  $k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda)$



- Invariant massess of soft and collinear modes are on the same order $\mathcal{O}(Q^2\lambda^2)$
- Rapidity divergence arises as jets go soft or soft radiations go collinear since they are all on the same parabola.

Factorization Theorem for Jet Broadening $B_T = \frac{1}{2}\tau_{a=1}$

Define $e \equiv \tau_{a=1}$ from now on to simplify the notification

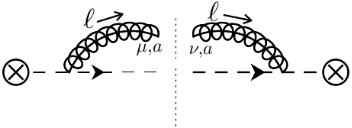
$$\frac{d\sigma}{de} = \sigma_0 H(s) \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s)$$
$$\times \int d\vec{p}_{1t} d\vec{p}_{2t} J_n(Q_-, e_n, \vec{p}_{1t}) J_{\bar{n}}(Q_+, e_{\bar{n}}, \vec{p}_{2t}) \mathcal{S}(e_s, \vec{p}_{1t}, \vec{p}_{2t}),$$

where in covariant guages $(\bar{d} \equiv 2 - 2\epsilon)$

$$J_n = \frac{2\pi\Omega_{\bar{d}}}{N_c} \langle 0|\, \bar{\chi}_n \delta(\hat{P}^- - Q^-)\delta(\hat{e} - e_n)\delta(\hat{P}_\perp + \vec{p}_{1\perp}) \frac{\bar{\not{n}}}{2} \chi_n |\, 0\rangle,$$
$$J_{\bar{n}} = \frac{2\pi\Omega_{\bar{d}}}{N_c} \langle 0|\, \frac{\not{n}}{2} \chi_{\bar{n}} \delta(\hat{P}^+ - Q^+)\delta(\hat{e} - e_{\bar{n}})\delta(\hat{P}_\perp + \vec{p}_{2\perp}) \bar{\chi}_{\bar{n}} |\, 0\rangle,$$
$$\mathcal{S} = p_{1t}^{1-2\epsilon} p_{2t}^{1-2\epsilon} \Omega_{\bar{d}} \int \frac{d\Omega_{12}}{N_c} \times$$
$$\langle 0 \mid S_n^\dagger S_{\bar{n}} \delta(\hat{e} - e_s) \delta^{\bar{d}}(\hat{P}_{n\perp} - \vec{p}_{1\perp}) \delta^{\bar{d}}(\hat{P}_{\bar{n}\perp} - \vec{p}_{2\perp}) S_{\bar{n}}^\dagger S_n |\, 0\rangle.$$

Naive Calculation with Pure Dim-Reg

- Bare jet function:



$$J_n(e_n, p_{i,t} = 0) = \frac{\alpha_s C_F}{\pi} \left(\frac{\mu^2}{Q^2 e_n^2} \right)^{\epsilon} \frac{1}{e_n} \int_0^1 \mathrm{d}z \frac{1 + (1 - z)^2}{z},$$

where $z \equiv l^- / Q$, and l is the momentum of the gluon going across the cut.

- Integral ill-defined as $z \to 0$, the soft region.
- Leftover $\frac{1}{\epsilon}$ divergence multiplies non-zero e_n terms that virtual diagrams, which are always proportional to $\delta(e_n)$, cannot cancel.
- Traditional dim-reg regulating the $\vec{k}_\perp$ part of the real radiation dose not regulate the phase space integral while p_T is fixed.

Outline

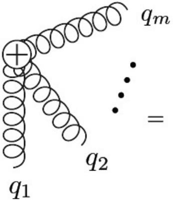
New Regulator and ν-Renomalization Group

- Goal:
  - Multiplicatively Renormalizable
  - In the spirit of dimensional regularization
  - Does not introduce new dimensionful scales in the integrants, and maintains manifest power counting in the effective theory.
- η-regulator

$$W_n = \left[\sum_{\text{perm}} \exp\left(\frac{-g}{\bar{n}\cdot\hat{P}} \left[\frac{|\bar{n}\cdot\hat{P}_g|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_{n,q}(0) \right] \right) \right]$$

$$S_n = \left[\sum_{\text{perm}} \exp\left(\frac{-g}{n\cdot\hat{P}} \left[\frac{|2\hat{P}_g^3|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_{s,q}(0) \right] \right) \right]$$

η-Regulator



$$= \nu^{m\eta} \frac{g^m}{m!} \prod_{i=1}^{m} |\overline{n}.q_i|^{-\eta} \sum_{\text{perms}} \frac{(\overline{n}^{\mu_m} T^{A_m})\ldots(\overline{n}^{\mu_1} T^{A_1})}{[\overline{n}.q_1][\overline{n}.(q_1+q_2)]\ldots[\overline{n}.\sum_{j=1}^{m} q_j]}$$

- Regulates the z-momentum of each gluon coming off Wilson Line.
- Preserves Exponentiation Theorems.
- Preserves modes and their power counting.
- Dim-reg. style evolution equations.
- Does not hide soft functions, as analytic regulators do.
- η contribution spontaneously goes to zero when rapidity divergence is not present

Zero-Bin Subtractions

No zero-bon subtration needed with η-regulator

- Soft-bin contribution is scaleless with Rapidity Ragulator η (as scaleless integral is 0 in pure dim-reg)
- Obtain correct IR- and UV- divergences without 0-bin subtraction
- Soft function is nonetheless non-zero

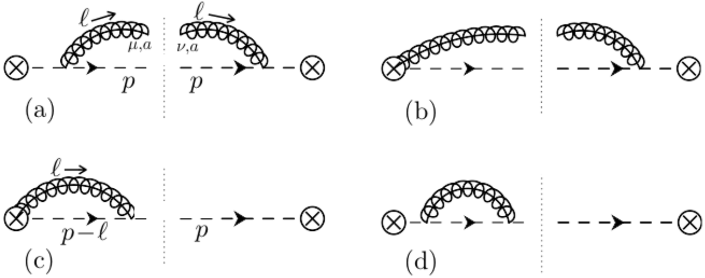
Regulator Comparison

- Analytic regulator

  (QCD: Smirnov, Rakhmetov, 99; Beneke, Feldmann, 04; SCET: JC, Golf, Kelley, Manohar,07; Becher, Neubert, 10;)

  - Hides soft contributions (although still get fix-order matrix element correct after summing different sectors).
  - Each collinear sector has complicated regulator dependent and is meaningless before summing all sectors together...
    $\Rightarrow$breaks factorization
  - Does not exponentiation.
- Δ-regulator JC, Fuhrer, Hoang, Kelley, Manohar, 09
  - Introduces additional scales into integrals
  - Exponentiates after proper zero-bin subtraction
  - No known evolutions equation.
- Off the light cone Collins, Soper, 81
  - Introduces more scales into integrals
  - Proof of exponentiation straightforward
  - Introduces gauge modes that are not appropiate by strict power counting.
  - Understood evolution equation.

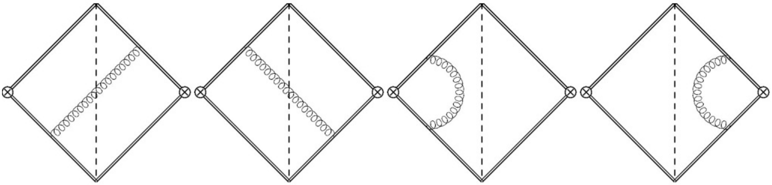
Jet Function Calculation with η-regulator



n-direction jet function with $p_t = 0$ in Laplace space yields

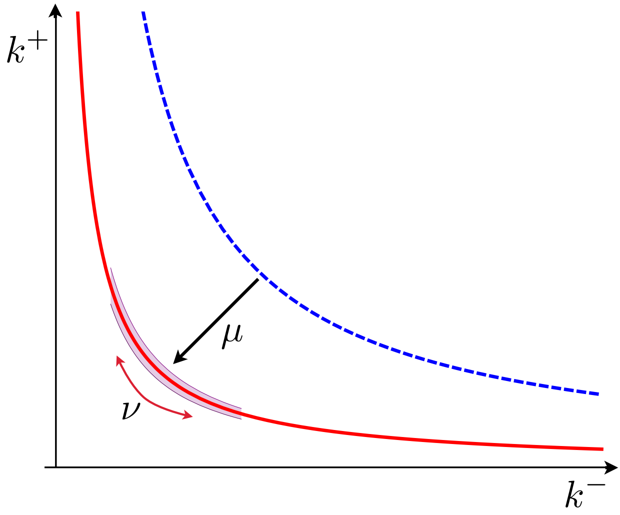
$$J_n \quad \propto \quad \left(\frac{G^2 C_F \Omega_{2-2\epsilon}}{4(2\pi)^{3-2\epsilon}}\right)\left(\frac{\mu\tau}{Q}\right)^{2\epsilon}\left\{\frac{1}{2}\frac{1-\epsilon}{\Gamma(1-\epsilon)}\Gamma(-2\epsilon) + \frac{\Gamma(-\eta)}{\Gamma(1-\epsilon)\Gamma(2-\eta)}\left(\frac{\nu}{Q}\right)^{\eta}\Gamma(-2\epsilon)\right\}$$

Soft Function Calculation with η-regulator



$$S(\tau,\vec{b}_{1t},\vec{b}_{2t}) = \left(\frac{G^2 C_F \Omega_{2-2\epsilon}}{4(2\pi)^{3-2\epsilon}}\right)\left(\frac{\nu\tau}{Q}\right)^{\eta_s}\left(\frac{\mu\tau}{Q}\right)^{2\epsilon}\Gamma(-\eta_s-2\epsilon)\frac{\Gamma(\eta/2)^2}{\Gamma(\eta)}$$
$$\times \left[{}_2F_1\left(\frac{-\eta-2\epsilon}{2},\frac{1-\eta-2\epsilon}{2};1-\epsilon;-\frac{b_1^2Q^2}{\tau^2}\right) + {}_2F_1\left(\frac{-\eta-2\epsilon}{2},\frac{1-\eta-2\epsilon}{2};1-\epsilon;-\frac{b_2^2Q^2}{\tau^2}\right)\right]$$

Physics of 2-Parameter RG



Renormalization Group Equations

- η-divergences and ν-anomalous dimensions cancels when we sum up the contributions from the jet and soft fuctions.
- Individual J and S are multiplicatively renormalizable.
- η-divergence are absorbed in the renormalization constants, $Z_{J,S}$, such that

$$J_n^{(0)} J_{\bar{n}}^{(0)} S^{(0)} = \left[Z_{J_n}(\mu,\nu) J_n^{\mathrm{R}}(\mu,\nu)\right] \left[Z_{J_{\bar{n}}}(\mu,\nu) J_{\bar{n}}^{\mathrm{R}}(\mu,\nu)\right] \left[Z_S(\mu,\nu) S^{\mathrm{R}}(\mu,\nu)\right],$$

where

$$Z_{J_n}(\mu,\nu) Z_{J_{\bar{n}}}(\mu,\nu) Z_S(\mu,\nu) = Z_H^{-1}(\mu).$$

- E.g. for soft function in Fourier-Laplace space

$$
\begin{aligned}
&Z_S(\tau, b_{1t}, b_{2t}, \mu, \nu) \\
&= \frac{\alpha_s C_F}{2\pi\epsilon^2} + \frac{\alpha_s C_F}{2\pi\epsilon} \ln\left(\frac{\mu^2}{\nu^2}\right) \\
&\quad - \frac{\alpha_s C_F}{\pi} e^{-\epsilon\gamma_E} \frac{\Gamma(-2\epsilon)}{\Gamma(1-\epsilon)} \frac{1}{\eta} \left(\frac{\mu\tau e^{\gamma_E}}{Q}\right)^{2\epsilon} \times \\
&\qquad \left[{}_2F_1\left(-2\epsilon, \frac{1}{2}(1-2\epsilon); 1-\epsilon; -\frac{b_1^2 Q^2}{\tau^2}\right) + b_1 \leftrightarrow b_2 \right]
\end{aligned}
$$

Renormalization Group Equations

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension $(\gamma_J^\nu, \gamma_S^\nu)$:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{S}^{\mathrm{R}}(\mu, \nu) = \gamma_S^\nu \, \mathcal{S}^{\mathrm{R}}(\mu, \nu), \ \nu \frac{\mathrm{d}}{\mathrm{d}\nu} J_n^{\mathrm{R}}(\mu, \nu) = \gamma_J^\nu \, J_n^{\mathrm{R}}(\mu, \nu)$$

  Just like the traditional μ anomalous dimension:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{S}^{\mathrm{R}}(\mu, \nu) = \gamma_S^\mu \, \mathcal{S}^{\mathrm{R}}(\mu, \nu), \text{ and } \mu \frac{\mathrm{d}}{\mathrm{d}\mu} J_n^{\mathrm{R}}(\mu, \nu) = \gamma_J^\mu \, J_n^{\mathrm{R}}(\mu, \nu)$$

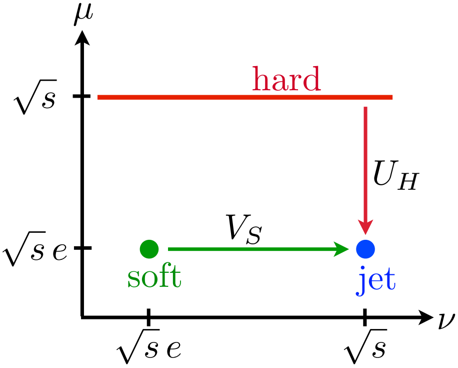
- Since the cross-section is invariant under μ and ν variation, and that the hard function itself is free from rapidity divergence (and therefore $\gamma_H^\nu = 0$), we must have the relations

$$\gamma_H^\mu + \gamma_{J_n}^\mu + \gamma_{J_{\bar{n}}}^\mu + \gamma_S^\mu = 0, \text{ and } \gamma_{J_n}^\nu + \gamma_{J_{\bar{n}}}^\nu + \gamma_S^\nu = 0.$$

  (In some complicated form depending on both e and p_t, will show simple cancelation explicitly in next example for p_T spectrum.)

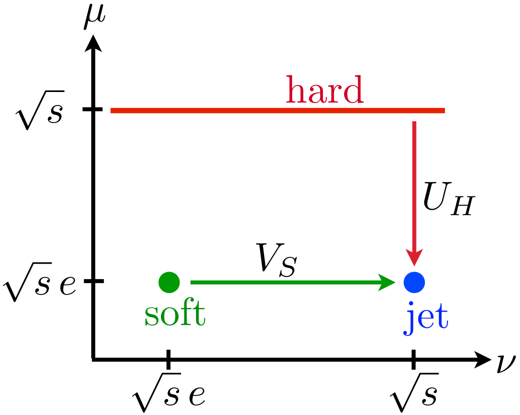
Running Strategy

- Natural scales:
  - hard function: independent of ν, $\mu_H = \sqrt{s}$
  - soft function $(\nu_S, \mu_S) = (\sqrt{s}\, e, \sqrt{s}\, e)$
  - jet functions $(\nu_J, \mu_J) = (\sqrt{s}, \sqrt{s}\, e)$
- 2-Parameter RG
  - UV- and Rapidity-Divergences are indepdendent
  - μ and ν RG are independent (commute)
  - $\partial_\mu \partial_\nu F = \partial_\nu \partial_\mu F$, where F can be jet function $J(\nu, \mu)$ or soft function $S(\nu, \mu)$



Running Strategy

- Natural scales:
  - hard function: independent of ν, $\mu_H = \sqrt{s}$
  - soft function $(\nu_S, \mu_S) = (\sqrt{s}\, e, \sqrt{s}\, e)$
  - jet functions $(\nu_J, \mu_J) = (\sqrt{s}, \sqrt{s}\, e)$
- Running
  - In μ:
    Evolve hard function from high scale $\mu_H = \sqrt{s}$ to common low scale μ_J=μ_S=$\sqrt{s}e$
  - In ν:
    Evolve soft function from $\nu_S = \sqrt{s}e$ to jet scale $\nu_J = \sqrt{s}$



Solution to the μ-RGE for the hard function

$$H(s,\mu) = U_H(s;\mu_H,\mu) H(s;\mu_H)$$

with

$$U_H(s;\mu_H,\mu) = \left| e^{K_H(\mu_H,\mu)} \left(\frac{-s - i0}{\mu_H^2} \right)^{\eta_H(\mu_H,\mu)} \right|^2$$

Solution to the ν-RGE for the soft function at one-loop

$$S^{\mathrm{R}}(\tau, b_{1t}, b_{2t}, \mu, \nu) = \left(\frac{\mu\, e^{\gamma_E} (\tau + \sqrt{b_1^2 Q^2 + \tau^2})}{2Q} \right)^{\zeta} \left(\frac{\mu\, e^{\gamma_E} (\tau + \sqrt{b_2^2 Q^2 + \tau^2})}{2Q} \right)^{\zeta} \times S^{\mathrm{R}}(\tau, b_{1t}, b_{2t}, \mu, \nu_S)$$

in which $\zeta = -2 \frac{\alpha_s}{\pi} C_F \ln \frac{\mu^2}{\nu^2}$.

NLO singular cross-section

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = \frac{1}{e}\frac{\alpha_s(\mu)C_F}{\pi}\left(-3 - 4\log\frac{e}{2}\right)$$

Resummed cross-section up to NLL

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} \quad = H(Q,\mu)\frac{4e^{2\gamma_E\zeta}}{\Gamma(-2\zeta)}\frac{1}{e}\left(\frac{\mu}{eQ}\right)^{2\zeta}\Big(1 - {}_2F_1(1,1;1-\zeta;-1)\Big)^2$$

To compare with standard Total Jet Broadening observable B_T, recall

$$e = 2B_T$$

Comparison with literature...

QCD calculations with resummation in the literature:

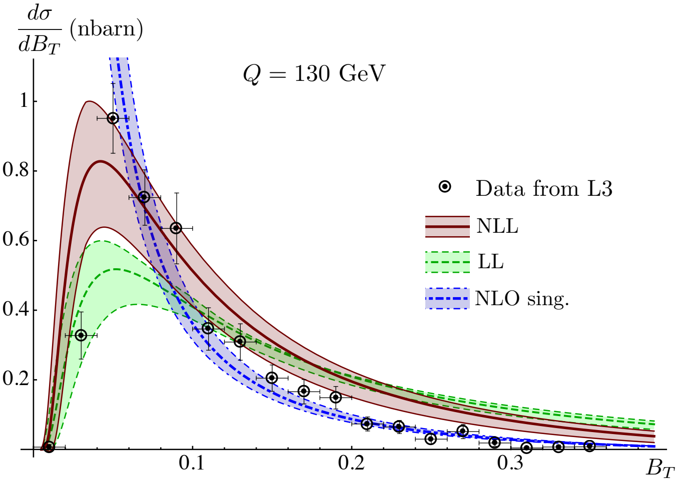
1. Catani, Turnock and Webber (1992) $\Rightarrow$ Correct up to NLL. Agree with our NLL result.
2. Dokshitzer, Lucenti, Marchesini and Salam (1998)
   $\Rightarrow$ Corrections to our previous result in [1] at NLL' or NNLL.
   $\Rightarrow$ Agree with updated result in this talk

SCET resummation for B_T after our letter...

3. Becher, Bell and Neubert (2011)
   $\Rightarrow$ Claim to agree with [2] for all logs at α_S^2 order.
   $\Rightarrow$ Factorization broken by regulator

Outline

Comparison with data



Outline

Other Applications?

Rapidity divergences do not only appear when observing Jet Broadening...

- p_T spectrum for Higgs/Drell-Yan Production
- TMD-PDF, Generalized Parton Distribution
- Electroweak corrections to high energy process at the LHC
- ...
- In general, processes or observables involving collinear and soft mode with similar transverse momentum, invariant mass, or off-shellness.

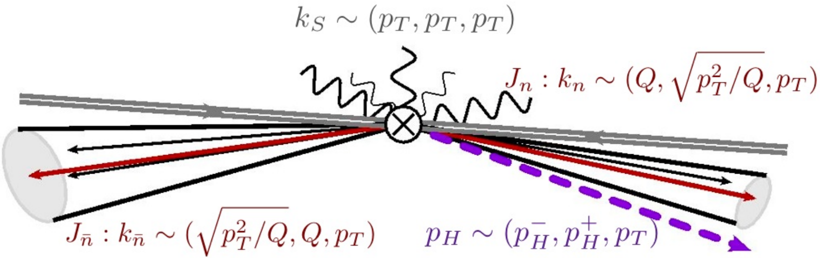
p_T Resummation in SCET

- Idilbi, Ji, Yuan (2005)
  - Calculation using SCET, no factorization theorem derived
- Mantry and Petriello (2009, 2010)
  - Factorization theorem derived in SCET
  - Keep residual momentum, and thus power suppressed terms for each sector to be well regularized.
- Becher and Neubert (2010)
  - Absence of soft function
  - Analytic regulator break factorization

When observing p_T (or related observables)

- When measuring thurst distribution or ,
  $|p_\perp^s| \ll |p_\perp^n| \sim |p_\perp^{\bar{n}}|$ and $|\eta^s| \ll |\eta^n| \sim |\eta^{\bar{n}}|$.
  Ultra-Soft radiation decoupled from the collinear radiation.

- When observing transverse momentum (p_T) distribution or jet broadening event shape (B_T), $k_{S,\perp} \sim k_{n,\perp} \sim k_{\bar{n},\perp}$, both soft and collinear radiation contribute to the transverse momentum at the same order.



Higgs p_T distribution in SCET$_{II}$

with η-regulator and ν-RG

$$\frac{d\sigma}{dQ^2 dp_T^2 dy} \propto \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \delta(\vec{p}_T^H + \vec{p}_{n,t} + \vec{p}_{\bar{n},t} + \vec{p}_{s,t}) g^{\perp}_{\alpha\sigma} g^{\perp}_{\beta\omega}$$

$$\times \; H(m_H^2, \mu) \mathcal{S}\left(\vec{p}_{s,t}; \mu_s, \mu; \nu_s, \nu\right)$$

$$\times \; \mathcal{F}^{\alpha\beta}_{n;g}\left(x_1, \vec{p}_{n,t}; \mu_B, \mu; \nu_B, \nu\right)$$

$$\times \; \mathcal{F}^{\sigma\omega}_{\bar{n};g}\left(x_2, \vec{p}_{\bar{n},t}; \mu_B, \mu; \nu_B, \nu\right)$$

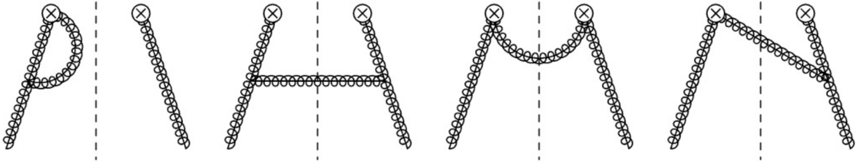
$$f_{g/p}\left(\frac{\omega_a}{P^-}, \mu\right) = -\sum_{\text{spins}} \theta(\omega_a)\omega_a \langle p_n | B^{c\mu}_{n\perp}(0) \delta\left(\frac{\omega_a}{P^-} - \bar{\mathcal{P}}_n\right) B^{c}_{n\perp\mu}(0) | p_n \rangle$$

$$\mathcal{F}^{\alpha\beta}_{g}\left(\frac{\omega_a}{P^-}, \vec{p}_t, \mu\right) = -\sum_{\text{spins}} \int d^2\vec{b}_t e^{-i\vec{b}_t\cdot\vec{p}_t} \theta(\omega_a) \langle p_n | B^{c\alpha}_{n\perp}(\vec{b}_t) \delta\left(\frac{\omega_a}{P^-} - z\right) B^{c\beta}_{n\perp}(0) | p_n \rangle$$

$$= \sum_i \frac{1}{z} \int_z^1 \frac{dz'}{z'} \int d^2\vec{b}_t e^{-i\vec{b}_t\cdot\vec{p}_t} I^{\alpha\beta}_{gi}\left(\frac{\omega_a}{z' P^-}, \vec{b}_t, \mu\right) f_i(z', \mu)$$

- divergent/ill-defined integral by pure dim-reg.

Beam Function Calculation with η-regulator



Total beam function in n-direction including real and virtual yields

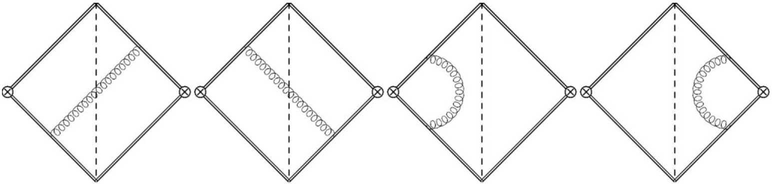
$$\mathcal{F}^{\mu\nu}_{g\leftarrow g}(z,\vec{p}_t) \propto \Gamma(1+\epsilon)\frac{\mu^{2\epsilon}}{(p_t^2)^{(1+\epsilon)}}\Big[g_t^{\mu\nu}\frac{\delta(1-z)}{\eta}\Big(\frac{\nu}{\omega_a}\Big)^{\eta} + p_{gg}\left(\frac{1}{z}\right) g_t^{\mu\nu} + 4\frac{(1-z)}{z^2}\Big(\Big(\frac{p_t^\mu p_t^\nu}{p_t^2}+\frac{1}{2}g_t^{\mu\nu}\Big)+2\epsilon\Big(\frac{p_t^\mu p_t^\nu}{p_t^2}-\frac{1}{2\epsilon}g^{\mu\nu}_{-2\epsilon}\Big)\Big)\Big]$$

- splitting function

$$p_{gg}(z) = \frac{1+(1-z)^4+z^4}{[1-z]_+ z}$$

- $(\mu_B,\nu_B) = (p_t,\omega)$

Soft Function Calculation with η-regulator



Soft

$$\mathcal{S}(\vec{p}_t) \propto \Gamma\left(1+\epsilon+\frac{\eta}{2}\right)\Gamma\left(\frac{\eta}{2}\right)\left(\frac{\mu^{2\epsilon}}{(p_t^2)^{(1+\epsilon)}}\right)\left(\frac{\nu^{\eta}}{(p_t^2)^{\eta/2}}\right)$$

- $(\mu_s, \nu_s) = (p_t, p_t)$

μ and ν RG similar to the previous case

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension $(\gamma_B^\nu, \gamma_S^\nu)$:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{S}^{\mathrm{R}}(\mu,\nu) = \gamma_S^\nu \mathcal{S}^{\mathrm{R}}(\mu,\nu),\ \nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{B}_n^{\mathrm{R}}(\mu,\nu) = \gamma_B^\nu \mathcal{B}_n^{\mathrm{R}}(\mu,\nu)$$

  Just like the traditional μ anomalous dimension:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{S}^{\mathrm{R}}(\mu,\nu) = \gamma_S^\mu \mathcal{S}^{\mathrm{R}}(\mu,\nu), \text{ and } \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{B}_n^{\mathrm{R}}(\mu,\nu) = \gamma_B^\mu \mathcal{B}_n^{\mathrm{R}}(\mu,\nu)$$

- In impact parameter space

$$\gamma_S^\nu = 2\frac{\alpha_s}{\pi} C_A \ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right) \quad \text{and} \quad \gamma_B^\nu = -\frac{\alpha_s}{\pi} C_A \ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)$$

$$\gamma_S^\mu = -2\frac{\alpha_s}{\pi} C_A \ln\left(\frac{\mu^2}{\nu^2}\right) \quad \text{and} \quad \gamma_{B_n}^\mu = -\frac{\alpha_s}{\pi} C_A \ln\left(\frac{\nu^2}{\omega_n^2}\right)$$

- Since the cross-section is invariant under μ and ν variation, and that the hard function itself is free from rapidity divergence (and therefore $\gamma_H^\nu = 0$), we must have the relations

$$\gamma_H^\mu + \gamma_{B_n}^\mu + \gamma_{B_{\bar{n}}}^\mu + \gamma_S^\mu = 0, \text{ and } \gamma_{B_n}^\nu + \gamma_{B_{\bar{n}}}^\nu + \gamma_S^\nu = 0.$$

- In impact parameter space

$$\begin{aligned}
\gamma_H^\mu &= 2\frac{\alpha_s}{\pi} C_A \ln\left(\frac{\mu^2}{\omega_n \omega_{\bar{n}}}\right) \\
&= 2\frac{\alpha_s}{\pi} C_A \ln\left(\frac{\mu^2}{\nu^2}\right) + \frac{\alpha_s}{\pi} C_A \ln\left(\frac{\nu^2}{\omega_n^2}\right) + \frac{\alpha_s}{\pi} C_A \ln\left(\frac{\nu^2}{\omega_{\bar{n}}^2}\right) \\
0 &= 2\frac{\alpha_s}{\pi} C_A \ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right) - 2\frac{\alpha_s}{\pi} C_A \ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)
\end{aligned}$$

- Recover CSS formula by μ and ν RG

Solution to the μ-RGE for the hard function

$$H(s,\mu) = U_H(M_H;\mu_H,\mu)H(M_H;\mu_H)$$

with

$$U_H(M_H;\mu_H,\mu) = \left| e^{K_H(\mu_H,\mu)} \left(\frac{-M_H^2 - i0}{\mu_H^2} \right)^{\eta_H(\mu_H,\mu)} \right|^2$$

Solution to the ν-RGE for the soft function

$$\mathcal{S}(\mu,\nu) = V_s\left(\mu, \frac{\nu}{\nu_s}\right) \otimes \mathcal{S}(\mu,\nu_S)$$

with

$$V_s(p_t;\omega_s,\mu,\nu) = e^{-2\gamma_E\omega_s} \frac{\Gamma(1-\omega_s)}{\Gamma(1+\omega_s)} \left[\frac{\omega_s}{\mu} \left[\frac{1}{(\frac{p_t}{\mu})^{1-\omega_s}} \right]_+ + \delta(p_t) \right]$$

and $\omega_s\left(\mu, \frac{\nu}{\nu_s}\right) = 2\Gamma_{cusp}[\alpha_s(\mu)] \log\frac{\nu}{\nu_s}$.

Resummed cross-section up to NLL

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_T} = U_H(M_H;\mu_H,\mu = p_T)\, V_s(p_t;\omega_s,\mu = p_T,\nu = M_H)$$

Comparison with literature...

QCD calculations with resummation in the literature:

1. Collins, Soper, and Sterman (1985)

SCET resummation for p_T distribution

2. Gao, Li, Liu (2005)
3. Idilbi, Ji, Yuan (2005)
   - SCET-like calculation, no factorization theorem derived
   - log hidden in phase space
4. Mantry, Petriello
   - Factorization theorem derived in SCET
   - Keep residual momentum, and thus power suppressed terms for each sector to be well regularized.
   - log hidden in phase space
5. Becher, Bell and Neubert (2010)
   - Absence of soft function
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Outline

Conclusion

- When measuring transverse momentum related observables...
  - Soft contributions are important
  - Uncanceled divergences remain in each sector, rapidity divergence.
  - New kind of logarithms to resum, yet related to the cups angle (the high scale).
- There are other cases with rapidity divergence such as, higgs p_T distribution, and electroweak corrections to LHC processes.
- Rapidity RG making use of the η-regulator provides controllable form to divergences, and a way to resum the log systematically.

Jet Broadening Resummation in [3]

Bechera, Bell and Neubert also attempted to resum the rapidity logs for the jet broadening...

$$\frac{1}{\sigma_0}\frac{d\sigma}{db_T} = H(Q^2,\mu)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\frac{1}{b_T}\left(\frac{b_T}{\mu}\right)^{2\eta} I^2(\eta)$$

where

$$\begin{aligned} I(\eta) &= \frac{4^\eta}{1+\eta}\,{}_2F_1(\eta, 1+\eta, 2+\eta, -1) \\ &= 1+\eta^2\left[\frac{\pi^2}{12}-\log^2 2\right]+\mathcal{O}\left(\eta^3\right) \end{aligned}$$

and

$$\eta \equiv \frac{\alpha_s(\mu)}{\pi}C_F\log\frac{Q^2}{\mu^2} \sim \frac{\alpha_s(\mu)}{\pi}C_F\log\frac{1}{b_T^2}$$

- Not claimed to be correct when we did NLL resum.
- Reproduced by RRG when properly convolving in $\vec{p}_T$

back

Comparison in Fixed order Expanded Result

$$\frac{b_T}{\sigma_0}\frac{d\sigma}{db_T} = \frac{\alpha_s(Q)}{2\pi}A(b_T) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B(b_T)$$

$$A^{\rm NLL}(b_T) = C_F\left(-8L-6\right),$$

$$B^{\rm NLL}(b_T) = C_F^2\left[32L^3 + 72L^2 + \left(92 - \frac{40\pi^2}{3} - 64\ln^2 2\right)L\right]$$

$$+ C_F C_A\left[\frac{88}{3}L^2 + \left(\frac{4\pi^2}{3} - \frac{70}{9}\right)L\right] + C_F T_F n_f\left(-\frac{32}{3}L^2 + \frac{8}{9}L\right)$$

back