The gluonic field of a heavy quark in a strongly coupled CFT

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Seattle, October 31, 2011.

Main message

By means of the AdS/CFT correspondence we can determine the gluonic field configuration sourced by a heavy quark undergoing arbitrary motion in a non-abelian strongly coupled gauge theory

Based on:

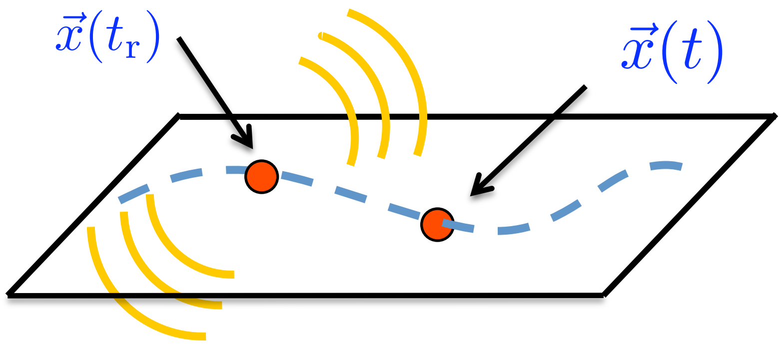
A. Güijosa, J.F. Pedraza and MCh **arXiv:1106.4059**

Plan for the talk

- Motivation
- Ingredients for the computation (stringy)
- The gluonic profile for arbitrary quark motion
- Conclusions

Motivation

When a charge accelerates in vacuum, it produces a propagating disturbance in the associated gauge field



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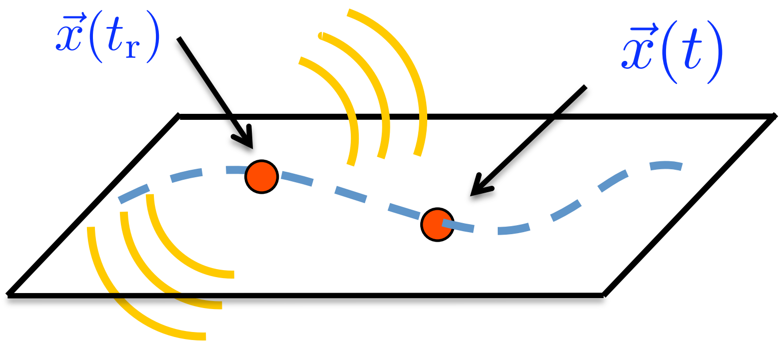
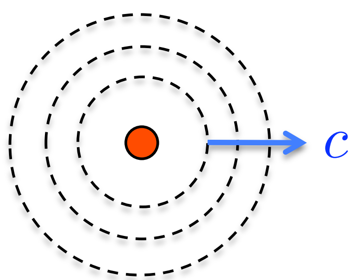


Weak coupling image (in vacuum)

e.g. Charge subject to a kick → the pulse travels at the speed of light and the characteristic width Δr does not change in time and is determined by the duration of the kick.



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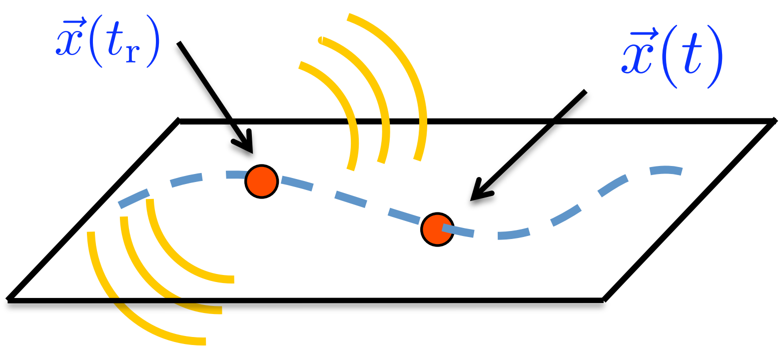


The problem of finding the spacetime profile of this disturbance for an arbitrary charge trajectory was solved long ago for classical EM

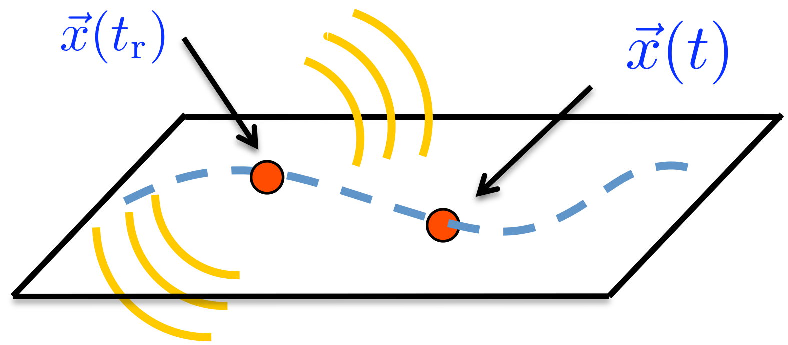
$$\frac{1}{4}F^2 = \frac{e^2}{\left[(x - x(t_r)) \cdot v(t_r)\right]^4}$$

[See Jackson for example]

(Lorentz boost of the static result)

Motivation

When a charge accelerates in vacuum, it produces a propagating disturbance in the associated gauge field



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What happens in the case of a strongly coupled non-abelian gauge theory?

Motivation

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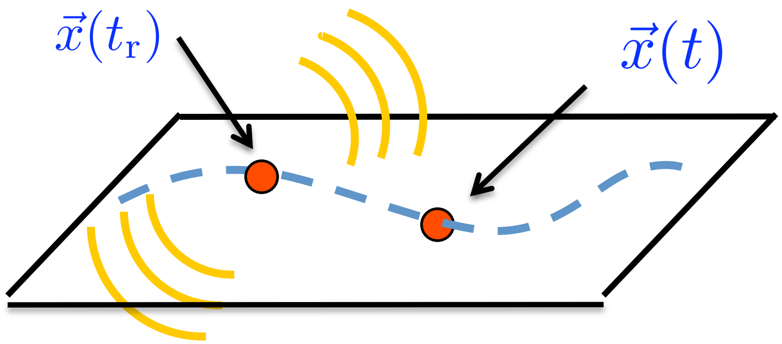

The AdS/CFT correspondence allows us to address this question in a very simple way

Motivation

A more technical motivation ...

Recently, using the AdS/CFT correspondence, the energy density radiated by a heavy quark undergoing arbitrary motion in the vacuum of $\mathcal{N} = 4$ SYM was calculated [Hatta et al., Liu et al].

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The results were rather surprising:

→ The radiation pattern closely resembles that of classical EM and weakly coupled SYM

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A more technical motivation ...

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The results were rather surprising:

→ The radiation pattern closely resembles that of classical EM and weakly coupled SYM

→ The radiation pulse travels at the speed of light and does NOT broaden as it propagates outward

(In a strongly coupled non abelian theory one might expect quanta to propagate slower than speed of light and the energy distribution to be isotropic)

Motivation

More than 10 years ago, Callan and Güijosa, using AdS/CFT calculated $\langle \mathrm{Tr} F^2(x) \rangle$ for an oscillating quark and found that the propagating waves display broadening.

(We will review their calculation later)

Motivation

More than 10 years ago, Callan and Güijosa, using AdS/CFT calculated $\langle \mathrm{Tr} F^2(x) \rangle$ for an oscillating quark and found that the propagating waves display broadening.

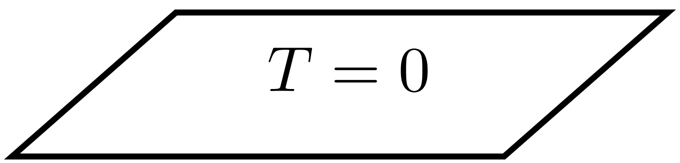
How can it be that the $\langle \mathrm{Tr} F^2(x) \rangle$ profile in this case displays broadening while the pattern of radiation calculated by Hatta et al. does not?

We want to solve this conflict

Ingredients for the computation

Holographic correspondence:

$\mathcal{N}=4$ SYM in Minkowski spacetime $=$ $AdS_5 \times S^5$ (Poincaré patch)



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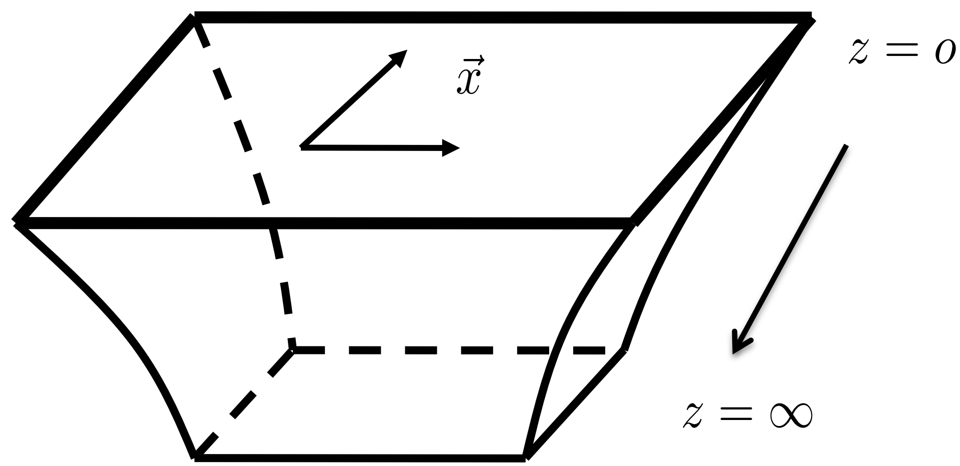


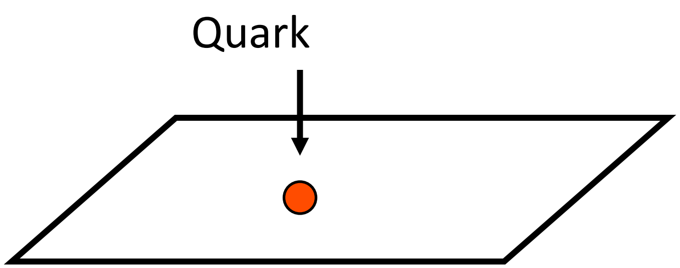
$$\lambda \equiv \frac{L^4}{l_s^4} = g_{\rm YM}^2 N_c$$

$$ds^2 = \frac{L^2}{z^2}\Big[-dt^2 + d\vec{x}^2 + dz^2\Big]$$

Ingredients for the computation

Holographic correspondence:

Probe particle $=$ Probe fundamental string



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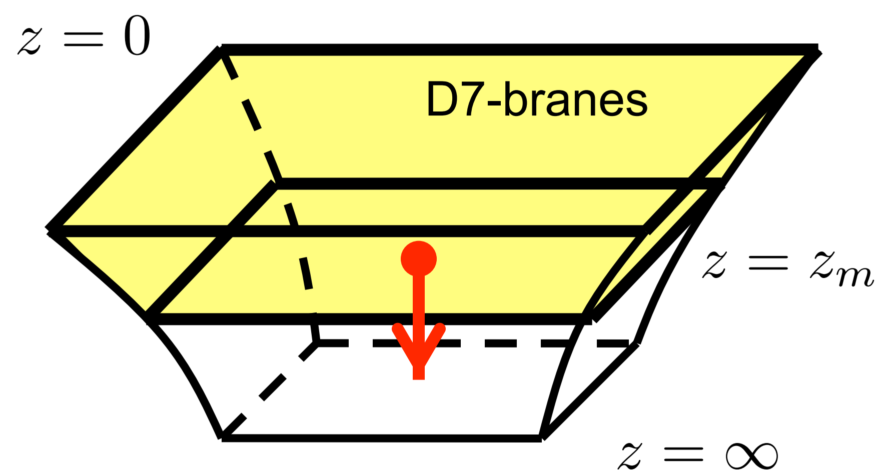

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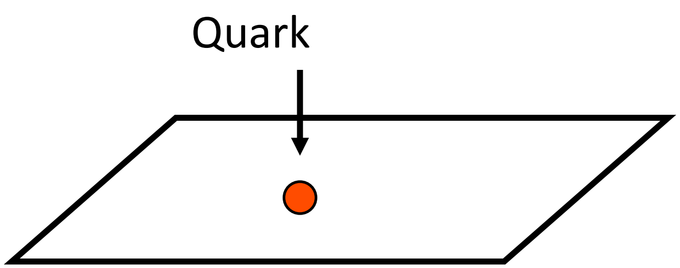

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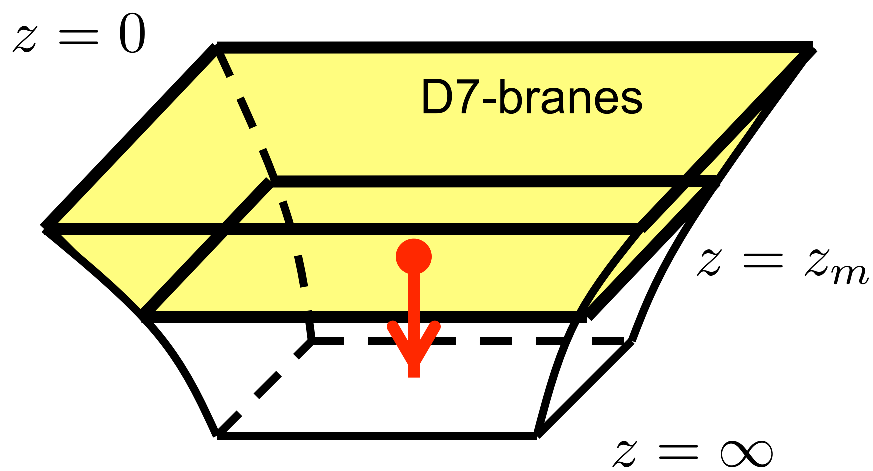


Mass of the quark: $m = \frac{\sqrt{\lambda}}{2\pi z_m}$

To be more precise:

• The string endpoint represents the quark, while the rest of the string codifies the profile of the gluonic field

Ingredients for the computation

Infinitely massive quark:

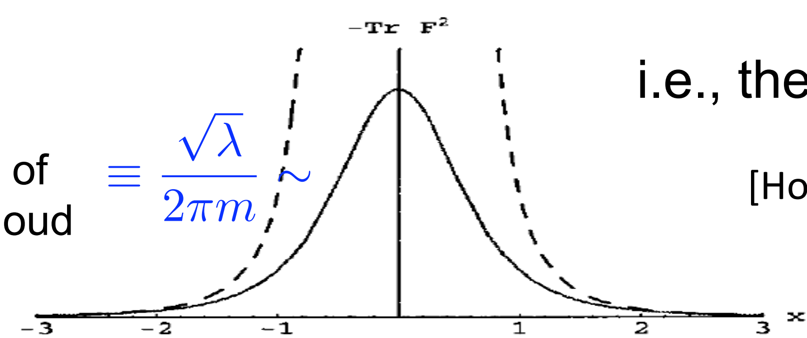
$$\langle \mathrm{Tr} F^2(\vec{x},t)\rangle = \frac{\sqrt{\lambda}}{16\pi^2|\vec{x}|^4}$$

[Danielsson, Kruczenski, Keski-Vakkuri]

i.e., behaves as a pointlike charge

Finite mass quark:

$$\langle \mathrm{Tr} F^2(\vec{x},t)\rangle = \frac{\sqrt{\lambda}}{128\pi^2}\left[-\left(\frac{2\pi m}{\sqrt{\lambda}}\right)^4 + \frac{7}{4|\vec{x}|^4}\left(\frac{2\pi m|\vec{x}|}{\sqrt{\lambda}}\right)^6 + \ldots\right] \qquad |\vec{x}| < \frac{\sqrt{\lambda}}{2\pi m}$$



width of the cloud $\equiv \frac{\sqrt{\lambda}}{2\pi m} \sim$

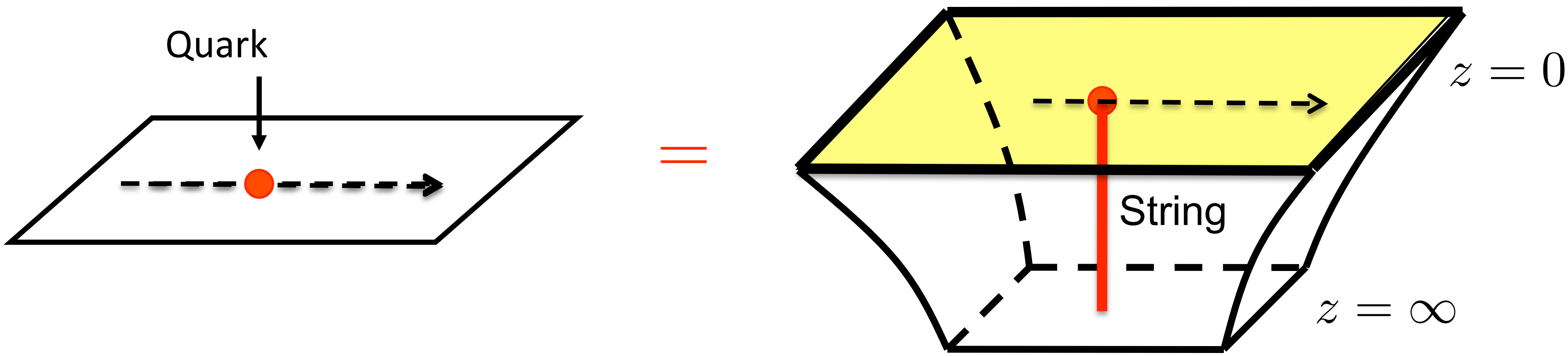
i.e., the field does not diverge at the source

[Hovdebo, Kruczenski, Mateos, Myers, Winters]

Ingredients for the computation

Holographic correspondence:

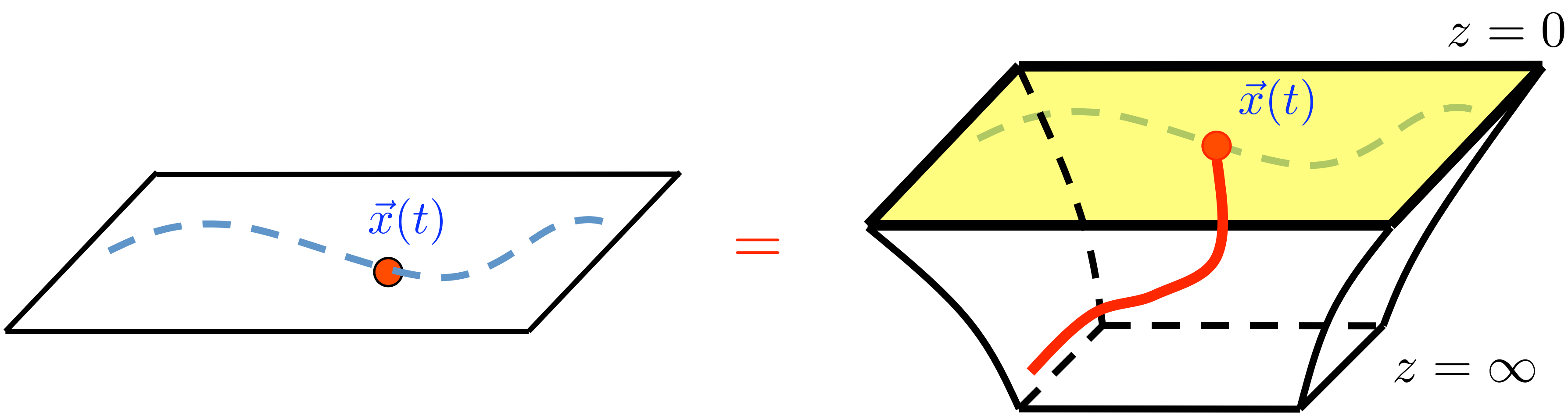
Quark moving with constant v $=$ Vertical string moving with constant v



Ingredients for the computation

Holographic correspondence:

Accelerated quark $=$ Accelerated string



For the accelerating quark, the string trails behind its endpoint

i.e., the quark has a 'tail', and it is this tail that is responsible for the damping effect

(This same mechanism has been seen to be responsible for drag force in thermal plasma) [Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser; Casalderrey, Teaney]

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We want to calculate $\langle \mathrm{Tr} F^2(x) \rangle$ for an arbitrary trajectory

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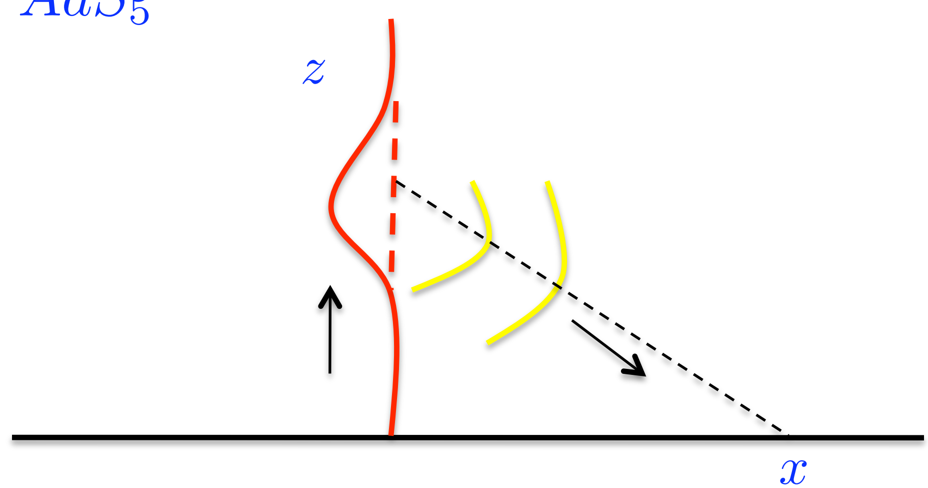
And according to GKPW recipe

$$\langle \mathrm{Tr} F^2(x) \rangle = -\lim_{z \to 0} \left(\frac{1}{z^3} \partial_z \phi(x,z) \right)$$

i.e. We need to calculate the dilaton field $\phi(x,z)$ sourced by the string dual to the quark, and and pick out the $\mathcal{O}(z^4)$ term in its expansion near the boundary of AdS_5



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Quick note: the calculation of the energy density requires the determination of the gravitational waves emitted by the string and involves an integral over all points of the source.

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Danielsson, Keski-Vakkuri and Kruczenski showed us the way ...

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Some of the steps

→ Solve the linearized eom for the dilaton

$$\partial_m \left(\sqrt{-G_{\mathrm{E}}} G_{\mathrm{E}}^{mn} \partial_n \phi \right) = J(x) \; ; \quad J(x) \propto \sqrt{-g_{\mathrm{E}}} \delta(\vec{x} - X(t,z))$$

↑ embedding functions

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→ Using Greens' function methods

$$\phi(x,z) = \frac{1}{16\pi^2\alpha'}\int dt'dz'\sqrt{g}\frac{d}{dU}\left(\frac{2U^2-1}{\sqrt{1-U^2}}\theta(1-|U|)\right)$$

where $$U = 1 - \frac{(t-t')^2 + (\vec{x}-\vec{x}')^2 + (z-z')^2}{2zz'}$$

is the invariant distance between the observation (unprimed) and the source (primed) point.

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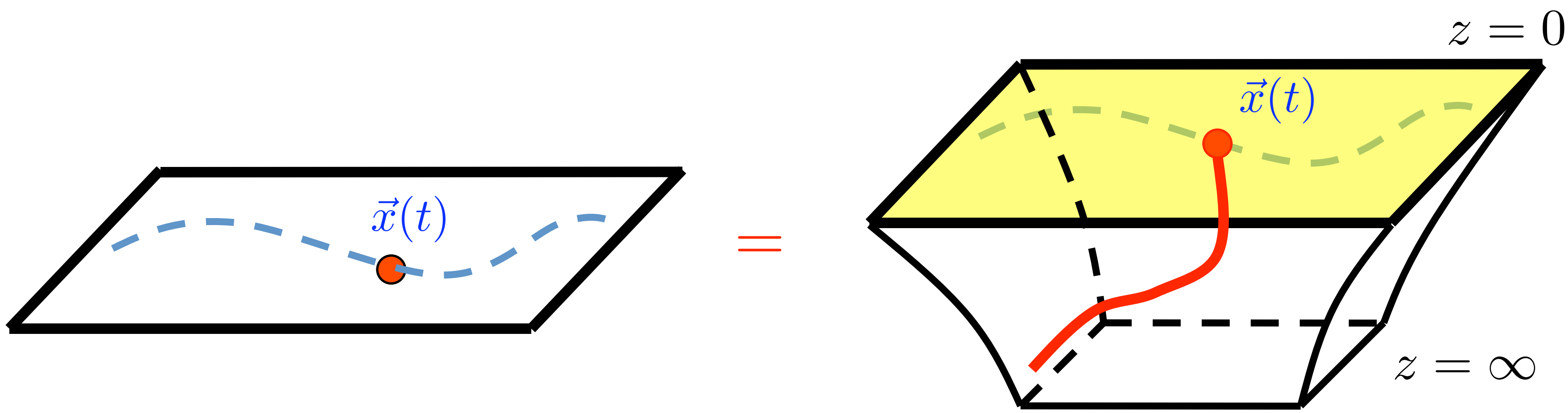
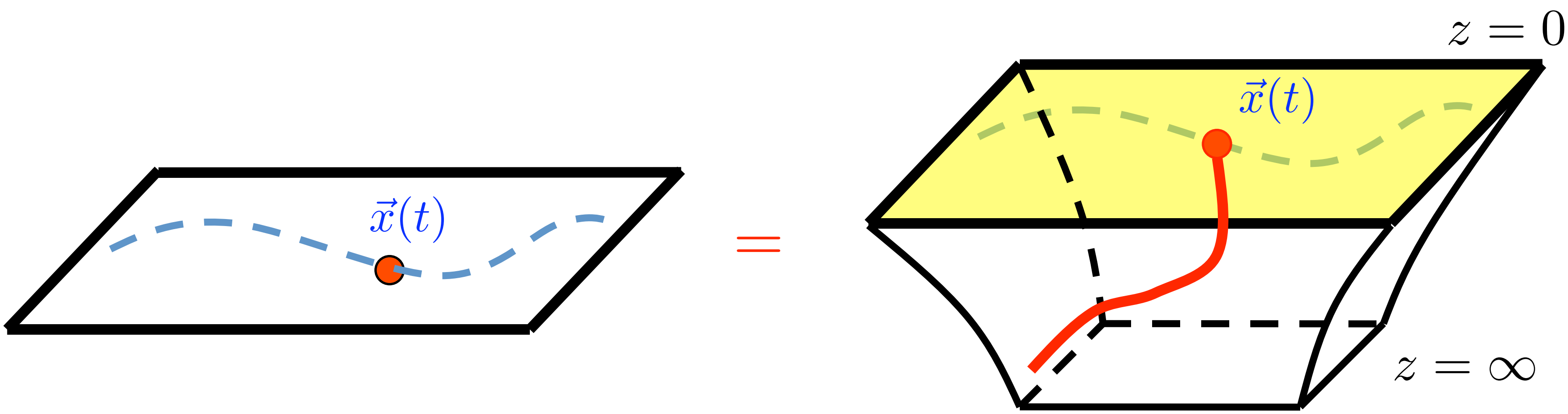
Then, all we need is the string embedding!

Ingredients for the computation



Extremizing the NG action we determine the string embedding $X^{\mu}(\tau, z)$

Ingredients for the computation



Extremizing the NG action we determine the string embedding $X^\mu(\tau, z)$

There is an analytic solution for an arbitrary time-like quark trajectory $\vec{x}(t)$ with purely outgoing waves as a boundary condition

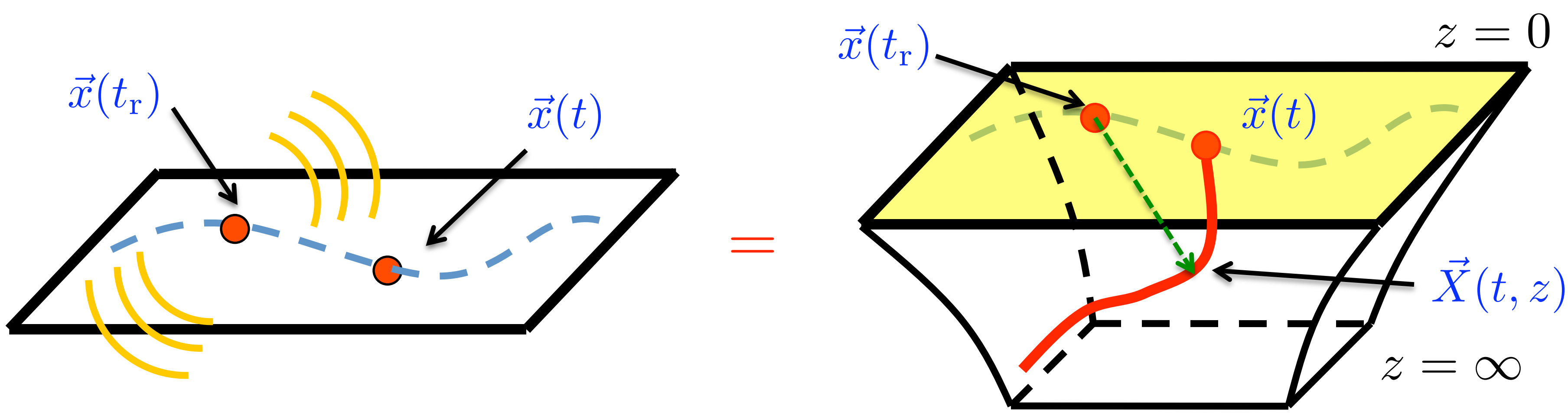
$$X^\mu(\tau, z) = z\partial_\tau x^\mu(\tau) + x^\mu(\tau)$$

[Mikhailov]

Proper time ↑

Ingredients for the computation

Geometrical interpretation of the solution:



The behavior at time t of the string segment located at radial position z is completely determined by the behavior of the string endpoint at a retarded time t_r

Ingredients for the computation

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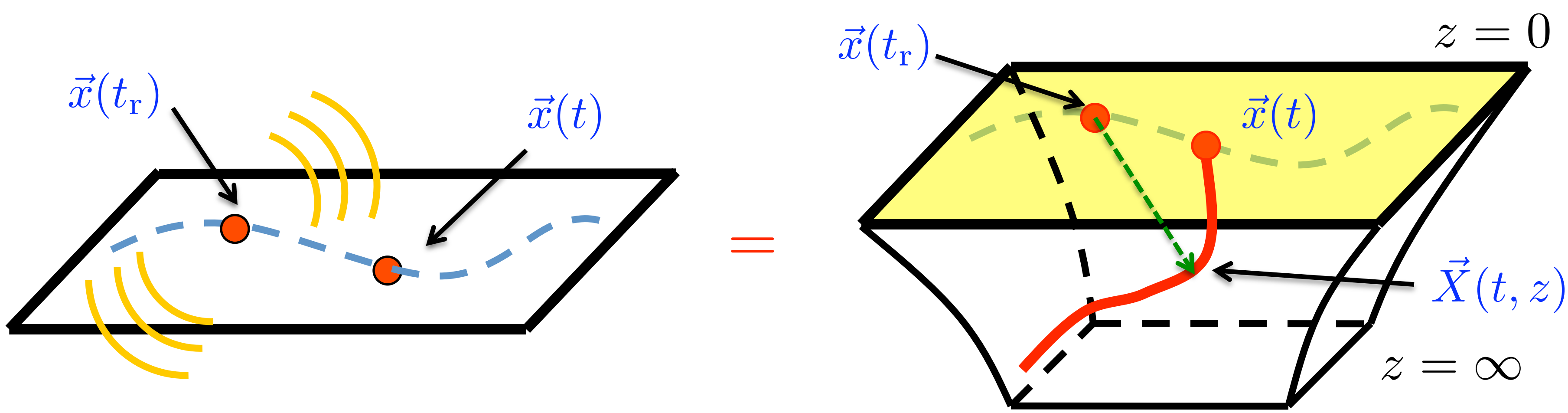

Finally, with the string embedding at hand (for an arbitrary trajectory), we can determine the resulting dilaton profile, and using the GKPW recipe obtain the expectation value of the gluonic field generated by the quark.

The gluonic profile

- Infinitely massive quark

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Reminder:

$$\phi(x,z) = \frac{1}{16\pi^2\alpha'}\int dt'dz'\sqrt{g}\,\frac{d}{dU}\left(\frac{2U^2-1}{\sqrt{1-U^2}}\theta(1-|U|)\right)$$

→ Contributions from all points along the string.

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→ Contributions from all points along the string.

However through a “convenient” change of variables this integral can be written as a total derivative and therefore the final result is a surface term.

When all dust settles, the observed $\langle \mathrm{Tr} F^2(x)\rangle$ depends only on the behavior of the (lower) string endpoint.

The gluonic profile

- Infinitely massive quark

Skipping the details of the calculation, the final result is

$$\langle \mathrm{Tr} F^2(x) \rangle = \frac{\sqrt{\lambda}}{16\pi^2} \frac{1}{\Big[(x - x(\tau_0)) \cdot v(\tau_0) \Big]^4}$$

[MCh, Güijosa, Pedraza]

Such that $(x - x(\tau_0))^2 = 0$, i.e. signals propagate purely along null intervals.

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The gluonic field is simply the boosted version of the Coulombic profile set up by a static point like quark, at the appropriate retarded time.

The gluonic profile

Some remarks about the result:

$$\langle \mathrm{Tr} F^2(x) \rangle = \frac{\sqrt{\lambda}}{16\pi^2} \frac{1}{\Big[(x - x(\tau_0)) \cdot v(\tau_0) \Big]^4}$$

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- Is identical to the classical EM result $\frac{1}{4}F^2$ for a point like electric charge replacing $e^2 \to \sqrt{\lambda}/8\pi^2$

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- The net profile propagates at the speed of light and there is no broadening

The gluonic profile

• Finite mass quark

Again, skipping the details, the final result is

$$\langle \mathrm{Tr} F^2(x) \rangle = \frac{\sqrt{\lambda}}{32\pi^2} H(x, v, a, j, f, \dot{f}, \ddot{f})$$

[MCh, Güijosa, Pedraza]

$$f = \gamma(\vec{F} \cdot \vec{v}, \vec{F})$$

where all dynamical quantities are understood to be evaluated at the retarded time τ_0 defined by $(x - x(\tau_0))^2 = -z_m^2$

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• Signals no longer travel at the speed of light, but rather propagate along a timelike interval.

• The presence of higher derivative terms are due to the fact that the quark is no longer pointlike (the gluonic cloud is deformed).

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• Signals no longer travel at the speed of light, but rather propagate along a timelike interval.

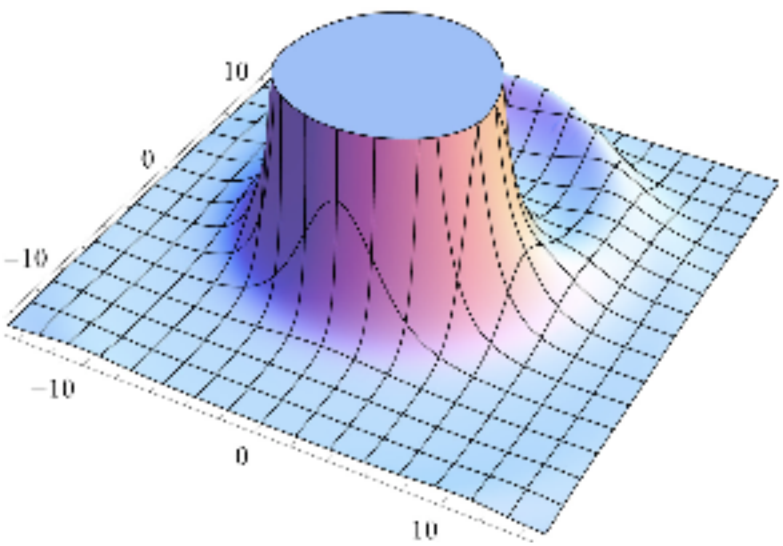
• In fact, the external force replaces a dependence on a infinite number of higher derivative terms which indicates nonlocality.

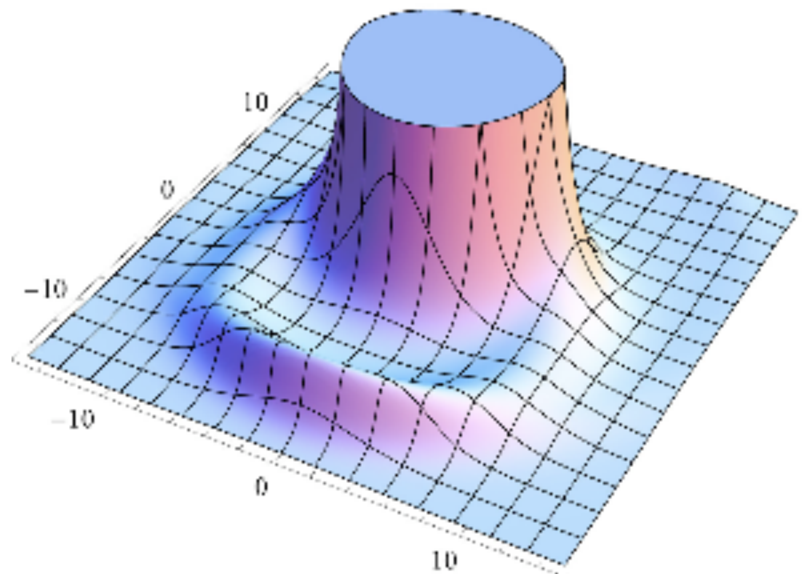
The gluonic profile

Applications

- Harmonic motion

The gluonic profile of a finite mass quark undergoing harmonic motion in one direction (two snapshots)





→ The over all pattern decays very fast: $\propto \frac{1}{|\vec{x}|^4}$

i.e. no radiation falloff

The gluonic profile

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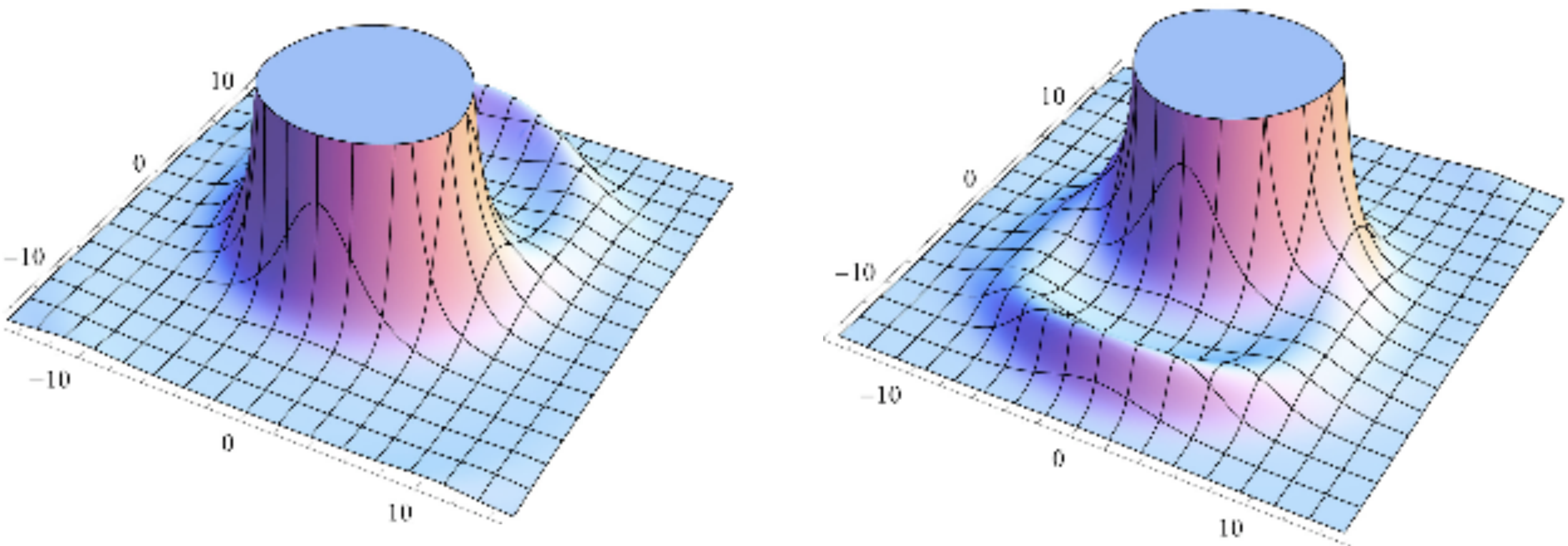

Let us now make contact with the results of Callan and Güijosa

The gluonic profile

Assumptions for their calculation:

→ Small oscillation amplitude $A \ll 1/\omega$

and thereby treated the string dynamics in a linearized approximation which implies $z \ll 1/\omega$

→ Through the UV/IR this translates into $|\vec{x}| \ll 1/\omega$

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Using our general formula and taking this considerations into account

$$\langle \mathrm{Tr} F^2(x) \rangle = \frac{\sqrt{\lambda}}{16\pi^2 |\vec{x}|^4} + \frac{\sqrt{\lambda}(\vec{A}\cdot\vec{x})}{4\pi^2|\vec{x}|^6} e^{-i\omega(t-|\vec{x}|)}(1 - i\omega|\vec{x}|) + \mathcal{O}(A^2)$$

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$$\langle \mathrm{Tr} F^2(x) \rangle = \underbrace{\frac{\sqrt{\lambda}}{16\pi^2 |\vec{x}|^4}}_{\text{static contribution}} + \underbrace{\frac{\sqrt{\lambda}(\vec{A}\cdot\vec{x})}{4\pi^2|\vec{x}|^6} e^{-i\omega(t-|\vec{x}|)}(1 - i\omega|\vec{x}|)}_{\text{Dynamical contribution linear in } A} + \mathcal{O}(A^2)$$

Contains a single time delay corresponding to propagation strictly at the speed of light

The gluonic profile

Their result (leading dynamical contribution)

$$\langle \mathrm{Tr} F^2(x) \rangle = \frac{\sqrt{\lambda}(\vec{A}\cdot\vec{x})}{32\pi^2}\int_0^\infty dz' z'^2 (1 - i\omega z') f(\sqrt{z'^2 + |\vec{x}|^2}) e^{-i\omega(t - \sqrt{z'^2+|\vec{x}|^2} - z')}$$

with $$f(u) = \frac{105}{u^9} - \frac{57 i\omega}{u^8} - \frac{12\omega^2}{u^7} + \frac{i\omega^3}{u^6}$$

[Callan and Güijosa]

The gluonic profile

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[Callan and Güijosa]

Under the corresponding assumptions, and after integration by parts, this can be rewritten as

$$\langle \mathrm{Tr\ F}^2(x) \rangle = \frac{\sqrt{\lambda}(\vec{A}\cdot\vec{x})}{4\pi^2|\vec{x}|^6} e^{-i\omega(t-|\vec{x}|)}(1 - i\omega|\vec{x}| + \ldots +) + \mathcal{O}(\omega|\vec{x}|^2, A^2)$$

i.e. There is a single time delay (NO broadening) and therefore NO conflict with the results of Hatta et al.

The gluonic profile

Their result (leading dynamical contribution)

$$\langle \mathrm{Tr} F^2(x) \rangle = \frac{\sqrt{\lambda}(\vec{A}\cdot\vec{x})}{32\pi^2}\int_0^\infty dz' z'^2 (1 - i\omega z') f(\sqrt{z'^2 + |\vec{x}|^2}) e^{-i\omega(t - \sqrt{z'^2+|\vec{x}|^2} - z')}$$

with $$f(u) = \frac{105}{u^9} - \frac{57 i\omega}{u^8} - \frac{12\omega^2}{u^7} + \frac{i\omega^3}{u^6}$$

[Callan and Güijosa]

Under the corresponding assumptions, and after integration by parts, this can be rewritten as

$$\langle \mathrm{Tr}\ \mathrm{F}^2(x) \rangle = \frac{\sqrt{\lambda}(\vec{A}\cdot\vec{x})}{4\pi^2|\vec{x}|^6} e^{-i\omega(t-|\vec{x}|)}(1 - i\omega|\vec{x}| + \ldots +) + \mathcal{O}(\omega|\vec{x}|^2, A^2)$$

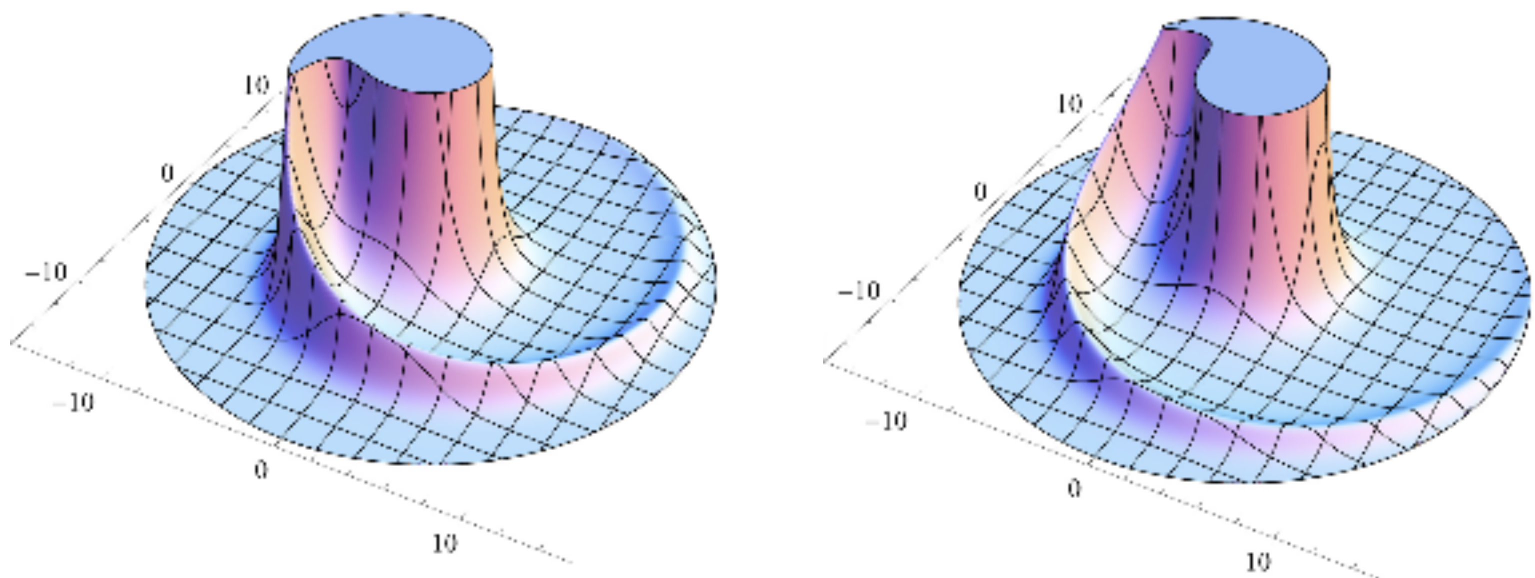
All it was needed is a “convenient” choice of worldsheet parameterization to solve the integral ...

The gluonic profile

Applications

- Circular motion

The gluonic profile of a finite mass quark undergoing circular motion



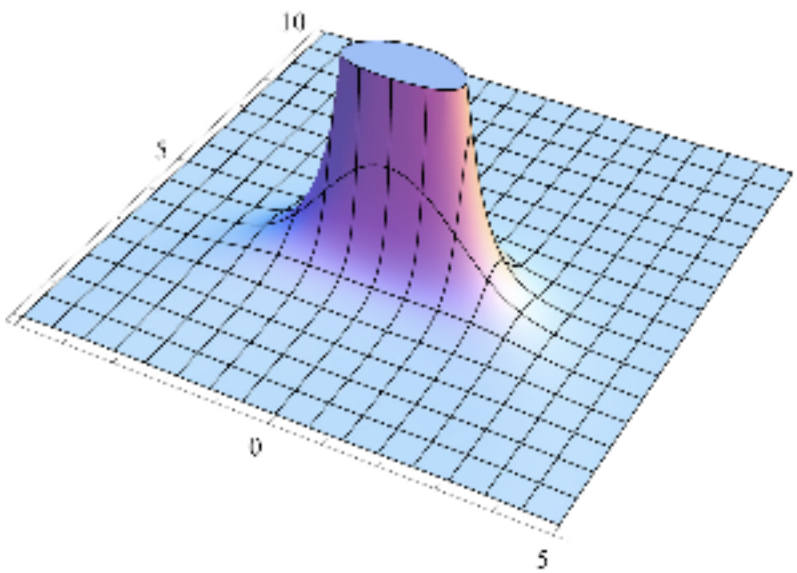
As the mass approaches infinity, the pattern reduces to the expected synchrotron form (Liu et al.)

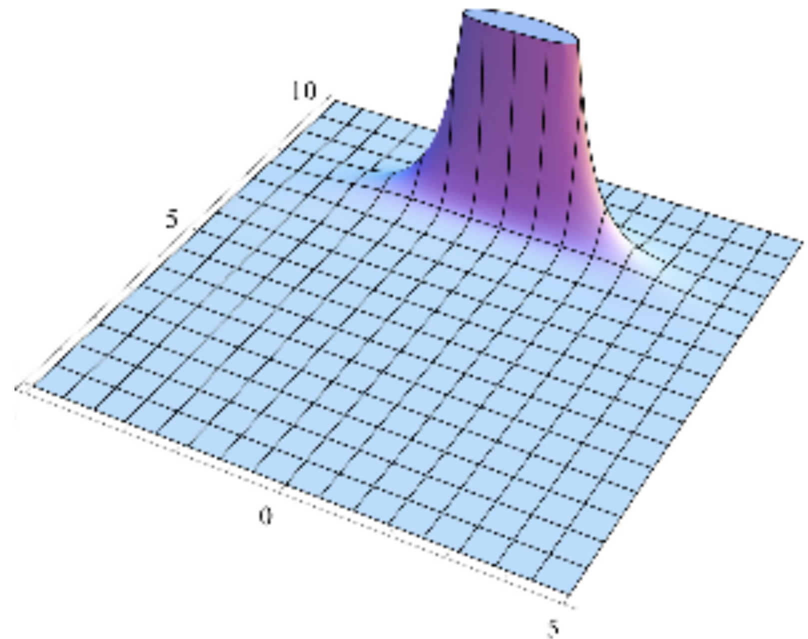
The gluonic profile

Applications

- Constant proper acceleration

The gluonic profile of a finite mass quark moving with constant proper acceleration.





We see the expected Lorentz contraction in the longitudinal direction as the quark increases its velocity.

Conclusions

• We have determined the gluonic field configuration sourced by a heavy quark undergoing arbitrary motion in $\mathcal{N} = 4$ SYM at strong coupling.

• Signals propagate without temporal broadening just as was found for the energy density (radiation pattern).

• The form of our result depends crucially on the choice of the boundary conditions for the string embedding (outgoing).

• There is no real conflict between Callan and Güijosa's result and that of Hatta et al. (or ours).

• It would be interesting to inquire to what extent the surprising pattern of unbroadened propagation obtained here (and in Hatta's work) is present in other setups (using AdS/CFT).

Thank you