Confinement/deconfinement and χsb in gauge theory/string theory correspondence

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Mostly: Nucl.Phys.B 847 297 (2011)[arXiv:1012.2404]

$\Longrightarrow$ Confinement and χsb in gauge theories are strongly coupled phenomena which are difficult to study from first principles

$\Longrightarrow$ I will use gauge theory/string theory correspondence of Maldacena, where the strongly coupled dynamics of certain gauge theories is mapped to essentially classical dynamics of higher dimensional gravitational theories

$\Longrightarrow$ I consider a specific string theory example of gauge gravity correspondence, rather than a phenomenological model of thereof.

$\Longrightarrow$ I will talk about confinement/deconfinements and χsb in $SU(N)$ gauge theory (in the limit $N \to \infty$) with massless adjoint fermions

Outline of the talk:

- Introduction to AdS/CFT correspondence
- Beyond conformal dynamics: from $\mathcal{N} = 4$ SYM to gauge theories with $\beta_{g\ YM} \neq 0$ (Klebanov-Strassler cascading gauge theory)
  - A field-theoretic picture
  - Dual gravitational picture
- Finite temperature confinement/deconfinement phase transition
  - similarities and differences with lattice QCD results
- χsb tachyon in cascading plasma
- Summary and future directions

Basic aspects of AdS/CFT correspondence:

gauge theory		string theory
$\mathcal{N} = 4 \, SU(N) \, \mathrm{SYM}$	$\Longleftrightarrow$	N-units of 5-form flux in type IIB string theory
g^2_{YM}	$\Longleftrightarrow$	g_s

$\Longrightarrow$ Consider the theory in the 't Hooft (planar limit), $N \to \infty$, $g^2_{YM} \to 0$ with Ng^2_{YM} kept fixed. SUGRA is valid $Ng_s \to \infty$. In which case the background geometry is

$$AdS_5 \times S^5$$

$\Longrightarrow$ The main message is that AdS/CFT sets up a framework that could be used in analyzing the dynamics of strongly coupled gauge theories, in particular, it can be a useful model of sQGP

$\Longrightarrow \mathcal{N} = 4$ supersymmetric Yang-Mills theory is conformal. In the absence of chemical potentials for the conserved $U(1)$ charges, temperature is the only scale in the problem. Thus all the thermodynamic potentials are determined by dimensional analysis:

$$\mathcal{F} \propto -T^4 \,, \qquad s \propto T^3 \,, \qquad \mathcal{E} \propto T^4$$

$\Longrightarrow$ There is no finite temperature phase transition in the model. $\mathcal{N} = 4$ SYM is *always* in a deconfined phase for any $T > 0$

$\Longrightarrow$ To make a closer link to realistic systems we need to go beyond the basic AdS/CFT correspondence

$\Longrightarrow$ We would like to perform a bunch of *deformation* steps, starting with $\mathcal{N} = 4$ SYM, and ending with the theory which have a nonzero β-function.

$\Longrightarrow$ Each *deformation* step has a precise string theory dual for type IIB string theory on $AdS_5 \times S^5$

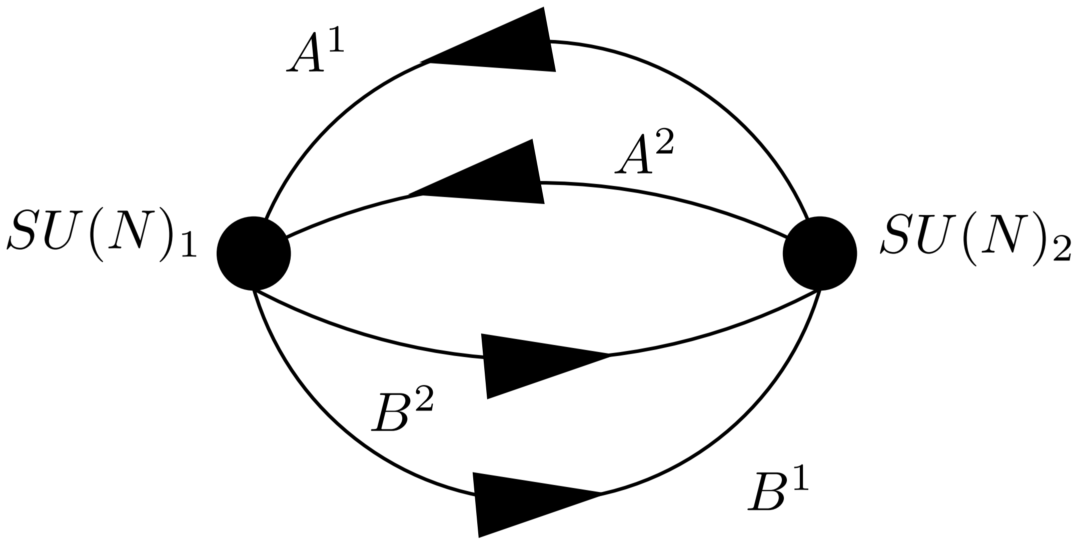
$\Longrightarrow$ At the end of the day we get a theory with a strong coupling scale Λ, and massless chiral fermions. This theory confines in the IR with spontaneous breaking of chiral symmetry.

Klebanov-Strassler model (a QFT story)

$\Longrightarrow$ The staring point again is $\mathcal{N}=4$ SU(N) SYM.

- Consider a $\mathbb{Z}_2$ orbifold of above SYM:

$$\mathcal{N}=4 \qquad \to \qquad \mathcal{N}=2$$



	$SU(N)_1 \times SU(N)_2$
A^i :	$(\bar{N}, N)$
B^i :	$(N, \bar{N})$

$$\mathcal{W}_{\mathcal{N}=2} = g_1 \;\mathrm{Tr}\,\Phi_1 \left[A^1 B^1 + A^2 B^2\right] + g_2 \;\mathrm{Tr}\,\Phi_2 \left[B^1 A^1 + B^2 A^2\right]$$

Note: $\beta_i = 0 \Longrightarrow g_1, g_2$ are exactly marginal couplings

- Turn on the mass term that breaks SUSY $\mathcal{N}=2 \rightarrow \mathcal{N}=1$

$$\mathcal{W}_{\mathcal{N}=2} \rightarrow \mathcal{W}_{\mathcal{N}=1} = \mathcal{W}_{\mathcal{N}=2} + m \,\mathrm{Tr}\left(\Phi_1^2 - \Phi_2^2\right)$$

$\Longrightarrow$ Integrating out the massive fields we find

$$\mathcal{W}_{eff} = \lambda \,\mathrm{Tr}\, A^i B^j A^k B^\ell \epsilon^{ik} \epsilon^{j\ell}$$

$\Longrightarrow$ Klebanov and Witten argued that at energy scales $\ll m$ the theory flows to a strongly interactive superconformal field theory; the coupling λ is exactly marginal, and thus the fields A^i, B^j develop large anomalous dimensions

$$[A^i]^{UV} = 1 \quad \rightarrow \quad [A^i]^{IR} = \frac{3}{4} \qquad \Longrightarrow \gamma_{A^i} = -\frac{1}{4}$$

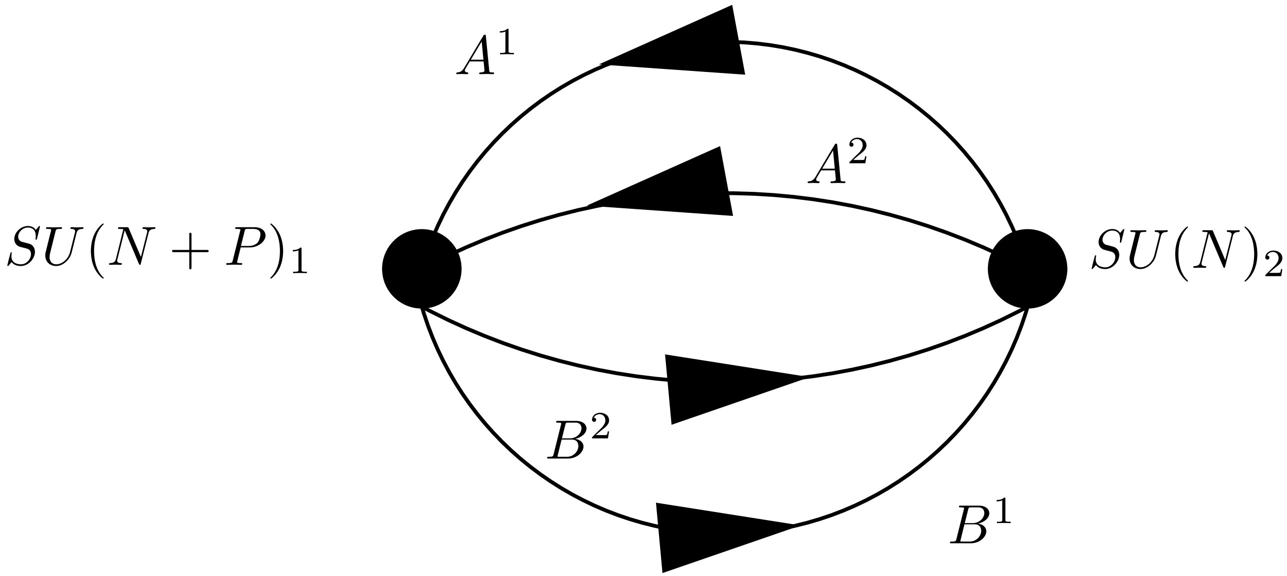
$$[B^i]^{UV} = 1 \quad \rightarrow \quad [B^i]^{IR} = \frac{3}{4} \qquad \Longrightarrow \gamma_{B^i} = -\frac{1}{4}$$

$\Longrightarrow$ From the exact NSVZ gauge β-functions (accounting for the anomalous dim of fields) we find

$$\beta_i = 0$$

- Consider a discrete deformation

$$SU(N)_1 \;\to\; SU(N+P)_1\,, \qquad P \ll N$$



$$\beta_1 \sim 3(N+P) - 2N(1-\gamma_{A^i}-\gamma_{B^j}) = 3P + \mathcal{O}(P^3/N^2)$$

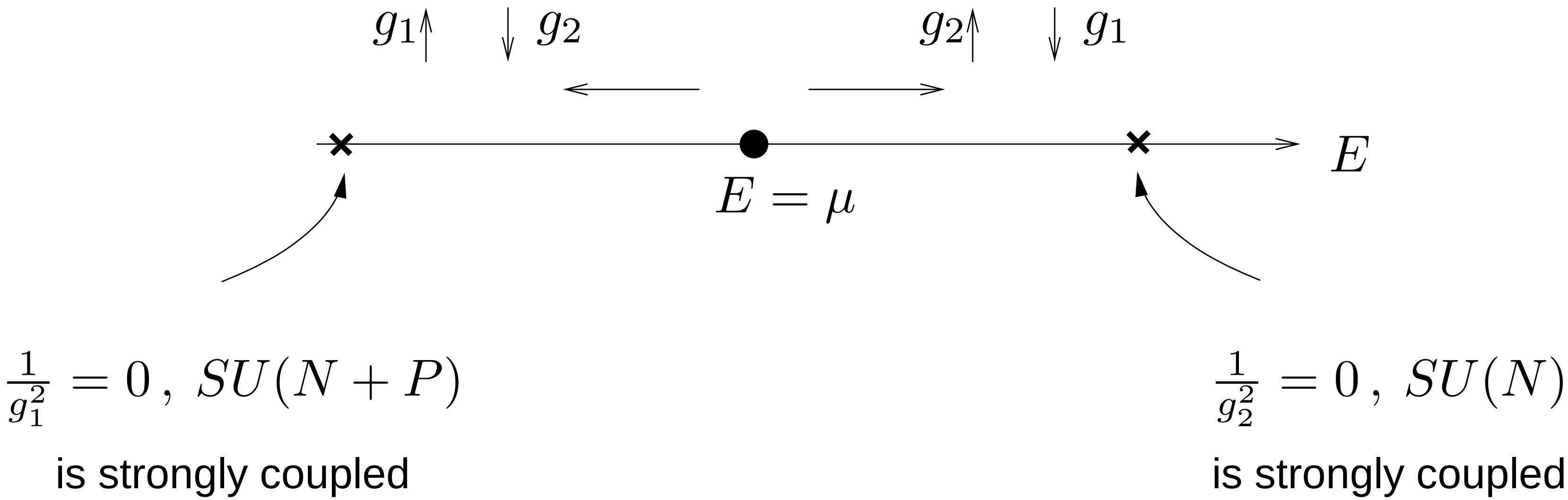
$$\beta_2 \sim 3N - 2(N+P)(1-\gamma_{A^i}-\gamma_{B^j}) = -3P + \mathcal{O}(P^3/N^2)$$

From the β-functions:

$$\frac{4\pi}{g_1^2(\mu)} + \frac{4\pi}{g_2^2(\mu)} = \text{const}$$

$$\frac{4\pi}{g_1^2(\mu)} - \frac{4\pi}{g_2^2(\mu)} \sim P \, \ln \frac{\mu}{\Lambda}$$

where Λ is the strong coupling scale of the theory



What is the effective description of the theory past the Landau poles?

$\Longrightarrow$ Using Seiberg duality for $\mathcal{N}=1$ SUSY gauge theory, the extension of the model past the Landau poles results in self-similarity cascade (Klebanov and Strassler):

$$N \to N(\mu) \sim 2P^2 \ \ln \frac{\mu}{\Lambda}$$

$$\mathrm{UV}: \qquad N \to N+P\,, \qquad\qquad\qquad\qquad \mathrm{IR}: \qquad N \to N-P$$

$\Longrightarrow$ If N is a multiple of P, the theory in the deep infrared is $\mathcal{N}=1$ $SU(P)$ SYM; this theory confines with the spontaneous chiral $U(1)_R$ symmetry breaking

Klebanov-Strassler model (a supergravity story)

It is possible to derive an effective 5d action from string theory dual to KS model:

$$S_5 = \frac{108}{16\pi G_5}\int_{\mathcal{M}_5} d^5\xi\sqrt{-g}\,\Omega_1\Omega_2^2\Omega_3^2\left\{R_{10} - \frac{1}{2}\left(\nabla\Phi\right)^2 - \frac{1}{2}e^{-\Phi}\left(\frac{(h_1-h_3)^2}{2\Omega_1^2\Omega_2^2\Omega_3^2}\right.\right.$$

$$\left.+\frac{1}{\Omega_3^4}\left(\nabla h_1\right)^2+\frac{1}{\Omega_2^4}\left(\nabla h_3\right)^2\right)-\frac{1}{2}e^{\Phi}\left(\frac{2}{\Omega_2^2\Omega_3^2}\left(\nabla h_2\right)^2+\frac{1}{\Omega_1^2\Omega_2^4}\left(h_2-\frac{P}{9}\right)^2+\frac{1}{\Omega_1^2\Omega_3^4}h_2^2\right)$$

$$\left.-\frac{1}{2\Omega_1^2\Omega_2^4\Omega_3^4}\left(4\Omega_0+h_2\left(h_3-h_1\right)+\frac{1}{9}Ph_1\right)^2\right\},$$

where:

$$R_{10} = R_5+\left(\frac{1}{2\Omega_1^2}+\frac{2}{\Omega_2^2}+\frac{2}{\Omega_3^2}-\frac{\Omega_2^2}{4\Omega_1^2\Omega_3^2}-\frac{\Omega_3^2}{4\Omega_1^2\Omega_2^2}-\frac{\Omega_1^2}{\Omega_2^2\Omega_3^2}\right)-2\Box\ln\left(\Omega_1\Omega_2^2\Omega_3^2\right)$$

$$-\left\{\left(\nabla\ln\Omega_1\right)^2+2\left(\nabla\ln\Omega_2\right)^2+2\left(\nabla\ln\Omega_3\right)^2+\left(\nabla\ln\left(\Omega_1\Omega_2^2\Omega_3^2\right)\right)^2\right\},$$

$\Longrightarrow$ We have “Einstein-Hilbert” gravity in 5d with 7 scalar fields:

$$\{g_{\mu\nu}, \Omega_1, \Omega_2, \Omega_3, \Phi, h_1, h_2, h_3\}$$

$\Longrightarrow$ Apriori, on a field theory side it is difficult to identify the operators of the gauge theory which are important at strong coupling. Given a dual holographic picture we can readily identify these operators using AdS/CFT dictionary:

$$\{T_{\mu\nu}\,, \mathcal{O}_4^{1,2}\,, \mathcal{O}_6\,, \mathcal{O}_8\}\ +\{\mathcal{O}_3^{1,2}\,, \mathcal{O}_7\}$$

where I separated operators that have vanishing VEV in chirally symmetric states:

$$\mathcal{O}_3^i \propto \langle\lambda\lambda\rangle^{1,2}\,, \qquad \mathcal{O}_7 \propto \langle FF\lambda\lambda\rangle$$

$\Longrightarrow$ Euclidean gravitational solutions in this 5-dim theory of gravity coupled to various scalars fields with compactified time-direction describe **confined equilibrium states** of the cascading plasma. As usual,

$$t_E \sim t_E + \frac{1}{T_{plasma}}$$

$\Longrightarrow$ Black holes with translationary invariant horizon describes **deconfined equilibrium states** of the cascading gauge theory plasma, with:

$$T_{plasma} \qquad \Longleftrightarrow \qquad T_{Hawking}$$

$$s_{plasma} \qquad \Longleftrightarrow \qquad s_{Bekenstein-Hawking}$$

$$\mathcal{E}_{plasma} \qquad \Longleftrightarrow \qquad \text{Black hole mass density}$$

$$\mathcal{F}_{plasma} \qquad \Longleftrightarrow \qquad \text{Black hole gravitational action}$$

$\Longrightarrow$ Spectrum of physical excitation in deconfined gauge theory plasma corresponds to spectrum of black-hole quasinormal modes

$\Longrightarrow$ Comments on confinement/deconfinement transition in $N \to \infty$ gauge theories:

- In the deconfined phase the free energy density and the entropy density

$$\mathcal{F}_{deconfined} \propto \mathcal{O}\left(N^2\right) , \qquad s_{deconfined} \propto \mathcal{O}\left(N^2\right)$$

- In the confined phase the free energy density

$$\mathcal{F}_{confined} \propto \mathcal{O}\left(N^0\right) , \qquad s_{confined} \propto \mathcal{O}\left(N^0\right)$$

- Since

$$\lim_{N\to\infty} \left.\frac{\mathcal{F}}{N^2}\right|_{deconfined} \neq 0\,, \qquad \text{or} \qquad \lim_{N\to\infty} \left.\frac{\mathcal{F}}{sT}\right|_{deconfined} \neq 0$$

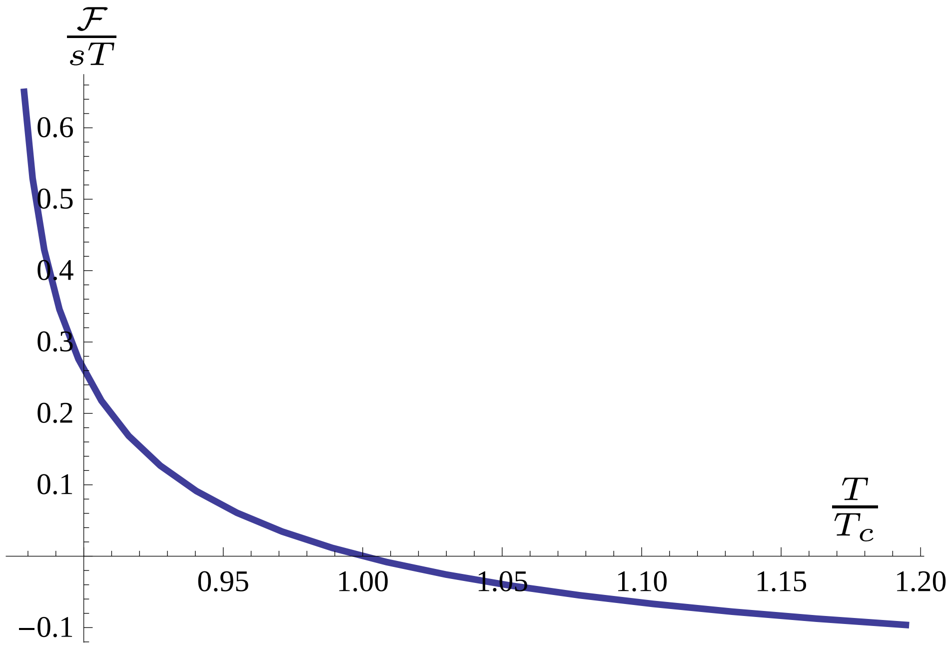
and

$$\lim_{N\to\infty} \left.\frac{\mathcal{F}}{N^2}\right|_{confined} = 0\,,$$

the confined phase of plasma is thermodynamically favourable once

$$\frac{\mathcal{F}}{sT} > 0\,, \qquad \text{provided} \qquad s \sim \mathcal{O}\left(N^2\right)$$

$\Longrightarrow$ Confinement/deconfinement phase transition in cascading plasma (with unbroken chiral symmetry)



- T_C is the critical temperature

$$T_c = 0.6141111(3)\Lambda$$

- The phase transition is of the first-order, between the **deconfined chirally symmetric** phase and the **confined phase with broken chiral symmetry**

$\Longrightarrow$ How close is the cascading plasma thermodynamics to that of the QCD?

Recall the lattice data for the QCD:

- for the energy density

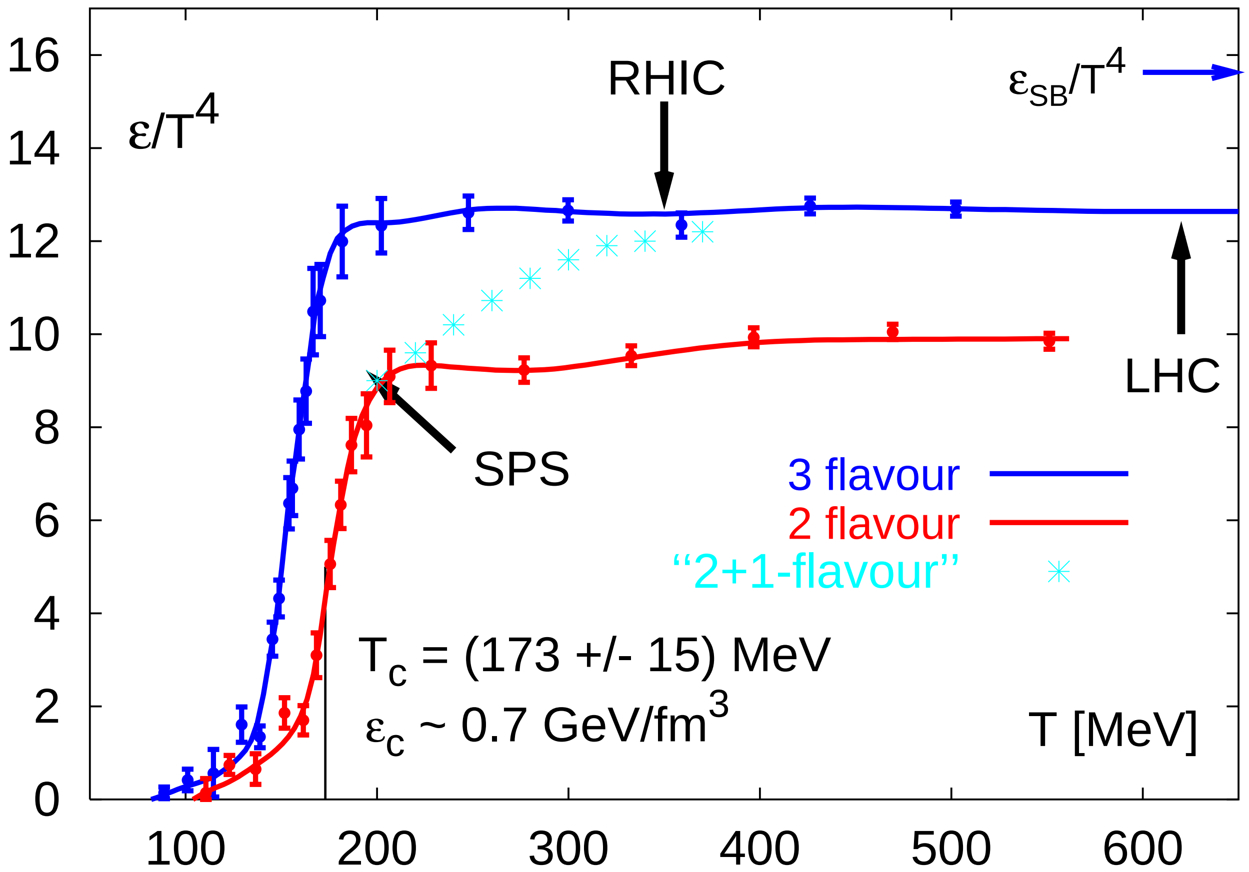


Figure 1: QCD thermodynamics from lattice; F.Karsch and E.Laermann, hep-lat/0305025.

- for the pressure:

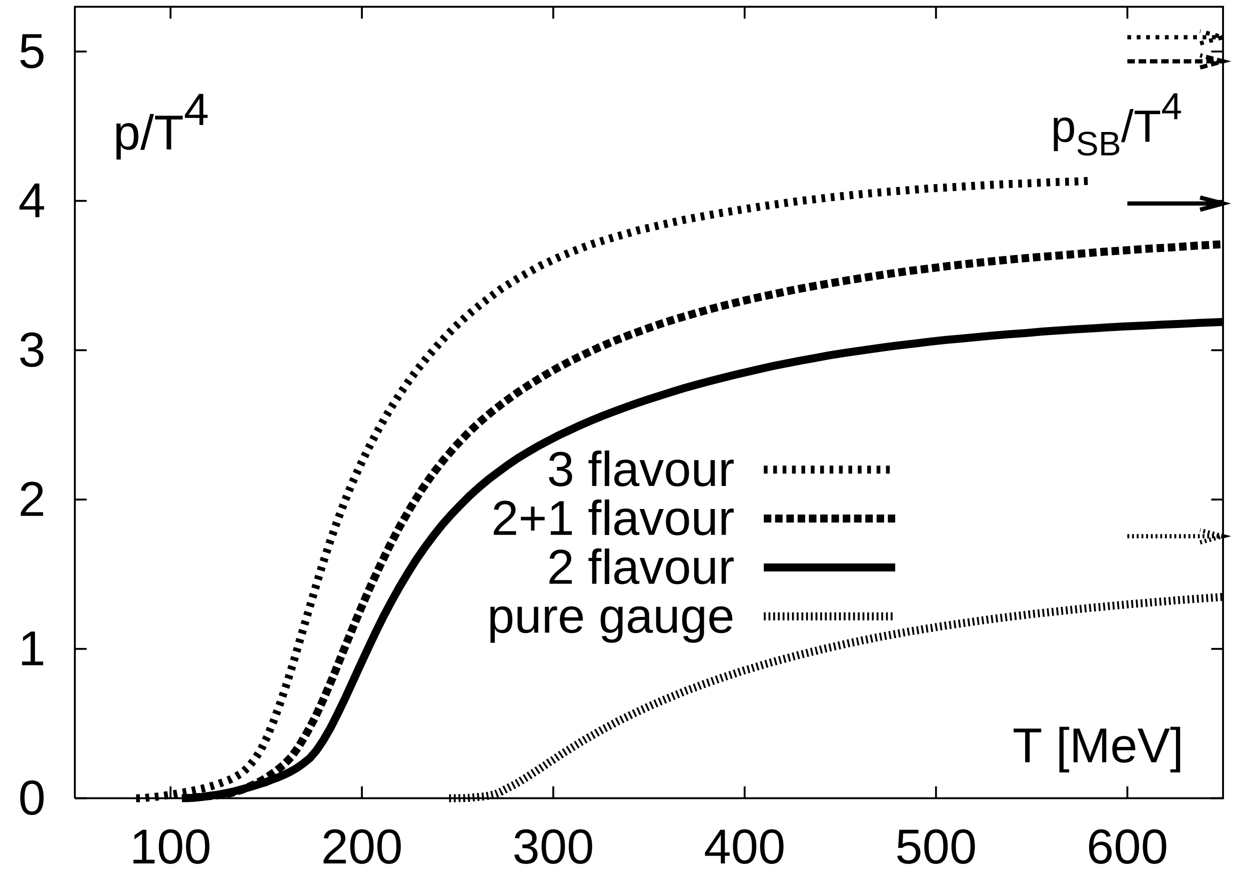


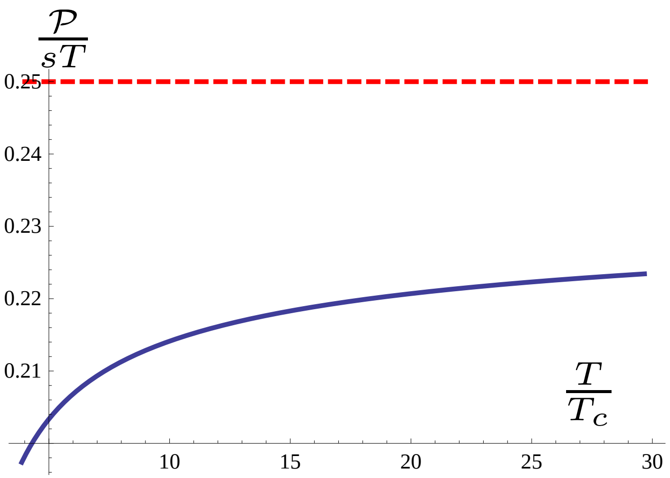
Figure 2: QCD thermodynamics from lattice; F.Karsch and E.Laermann, hep-lat/0305025.

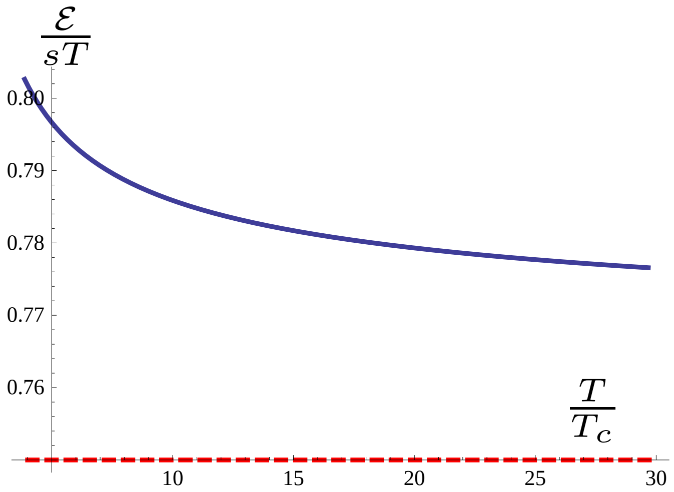
In the cascading plasma, similar to QCD,

$$\frac{\mathcal{P}}{T^4} \propto (1-\delta) \ , \qquad \frac{\mathcal{E}}{T^4} \propto \left(1+\frac{1}{3}\delta\right) \ , \qquad \delta \equiv \frac{1}{2\ln\frac{T}{\Lambda}} \ll 1$$

$\Longrightarrow$ The energy density approaches the conformal plateau in the UV asymptotically 3 times as fast as the free energy

$\Longrightarrow$ The difference with the QCD is that this the UV plateau is approached much more slowly here:





The pressure $\mathcal{P}$ and the energy density $\mathcal{E}$, divided by sT, as a function of $\frac{T}{T_c}$. T_c is the temperature for the deconfinement phase transition in the cascading plasma.

$\Longrightarrow$ Even though in the UV the cascading theory is quite different from the QCD, in the IR it is simply $\mathcal{N} = 1$ SYM

$\Longrightarrow$ Is the deconfined chirally symmetric phase of the cascading plasma perturbatively stable?

$\Longrightarrow$ To answer this question:

- we look at linearized χsb fluctuations $\propto e^{-i\omega t+i\vec{k}\cdot\vec{x}}$ about chirally symmetric thermal state. Suppose that these fluctuations have a dispersion relation

$$\mathfrak{w}=\mathfrak{w}(\mathfrak{q}^2)\,,\qquad \mathfrak{w}\equiv\frac{\omega}{2\pi T}\,,\qquad \mathfrak{q}=\frac{|\vec{k}|}{2\pi T}$$

- These χsb fluctuations are unstable, provided

$$\mathrm{Im}\,(\mathfrak{w})>0\qquad \text{for}\qquad \mathrm{Im}\,(\mathfrak{q})=0$$

- Using the holographic duality, one can precisely map these fluctuations into quasinormal modes of the 5d black hole solution, describing the deconfined chirally symmetric equilibrium phase of the cascading plasma

$$S_{\chi \mathrm{SB}}\Big[\delta f, \delta k_1, \delta k_2\Big] = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} \mathrm{vol}_{\mathcal{M}_5}\, h^{5/4} f_2^{1/2} f_3^2 \Big\{\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5\Big\},$$

$$\mathcal{L}_1 = -\frac{(\delta f)^2}{f_3^2}\left(-\frac{P^2 e^{\Phi}}{2 f_2 h^{3/2} f_3^2} - \frac{(\nabla K)^2}{8 f_3^2 h P^2 e^{\Phi}} - \frac{K^2}{2 f_2 h^{5/2} f_3^4}\right),$$

$$\mathcal{L}_2 = -\frac{9 f_3^2 - 24 f_2 f_3 + 4 f_2^2}{f_2 h^{1/2} f_3^4}\,(\delta f)^2 + 2\Box\frac{(\delta f)^2}{f_3^2} - \left(\nabla\frac{(\delta f)^2}{f_3^2}\right)^2$$
$$-2\nabla\left(\ln h^{1/4} f_3^{1/2}\right)\nabla\left(\frac{(\delta f)^2}{f_3^2}\right) + 2\nabla\left(\ln f_2^{1/2} h^{5/4} f_3^2\right)\nabla\left(\frac{(\delta f)^2}{f_3^2}\right),$$

$$\mathcal{L}_3 = -\frac{1}{2P^2 e^{\Phi}}\Bigg(\frac{9}{2 f_2 h^{3/2} f_3^2}\,(\delta k_1)^2$$
$$+\frac{1}{2 h f_3^4}\left(2(\nabla K)^2\,(\delta f)^2 + f_3^2\,(\nabla\delta k_1)^2 + 4 f_3 \delta f\, \nabla K \nabla \delta k_1\right)\Bigg),$$

$$\mathcal{L}_4 = \frac{P^2 e^{\Phi}}{2}\left(\frac{2}{9 h f_3^2}\,(\nabla\delta k_2)^2 + \frac{2}{f_2 h^{3/2} f_3^4}\Big(3\,(\delta f)^2 + 4 f_3\,\delta f \delta k_2 + f_3^3\,(\delta k_2)^2\Big)\right),$$

$$\mathcal{L}_5 = \frac{K}{f_2 h^{5/2} f_3^6}\,\Big(f_3^2\,\delta k_1 \delta k_2 - K\,(\delta f)^2\Big)\,.$$

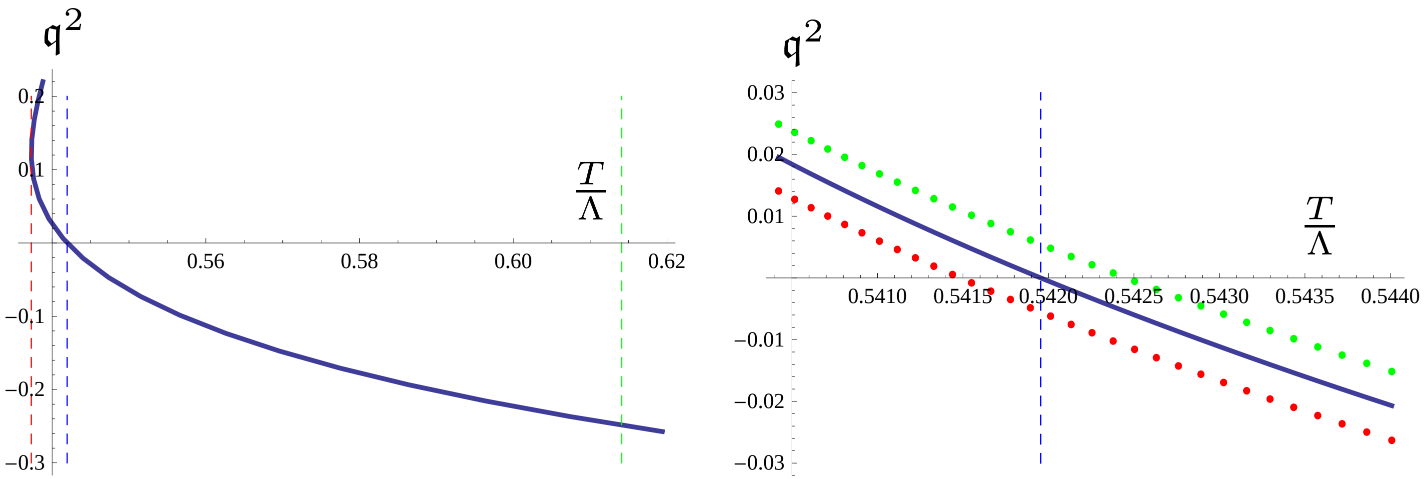
$\Longrightarrow$ The fluctuations $\{\delta k_1, \delta k_2, \delta f\}$ are dual to χsb operators $\mathcal{O}_3^{1,2}, \mathcal{O}_7$

$$\delta f = e^{-i\omega t + ikx_3} F\,, \qquad \delta k_1 = e^{-i\omega t + ikx_3} \mathcal{K}_1\,, \qquad \delta k_2 = e^{-i\omega t + ikx_3} \mathcal{K}_2\,,$$

where $\{F, \mathcal{K}_1, \mathcal{K}_2\}$ are functions of the radial coordinate only

$\Longrightarrow$ Imposing the incoming-wave boundary conditions are the horizon, and the normalizability of the fluctuations wave-functions in the UV we find dispersion relation

$$\mathfrak{w} = \mathfrak{w}(\mathfrak{q}^2)$$



$\Longrightarrow$ The left plot represents the dispersion relation of the chiral fluctuations at the threshold of instability, *i.e.,* , with $\mathfrak{w}(\mathfrak{q}^2) = 0$. The blue dashed vertical lines represent the onset of instability: $T = T_{\chi\mathrm{SB}}$, such that $\left(i\mathfrak{w} = 0, \mathfrak{q}^2 = 0\right)$. The vertical green dashed line represents the confinement/deconfinement critical temperature T_c,

$$T_{\chi\mathrm{SB}} = 0.882503(0)T_c$$

$\Longrightarrow$ On the right plot: the green dots indicate quasinormal modes with $\left(\mathfrak{w} = -i0.01, \mathfrak{q}^2\right)$ as a function of $\frac{T}{\Lambda}$ — these fluctuations are stable. The red dots indicate quasinormal modes with $\left(\mathfrak{w} = i0.01, \mathfrak{q}^2\right)$ as a function of $\frac{T}{\Lambda}$ — these fluctuations are genuine tachyons whenever $\mathfrak{q}^2 > 0$.

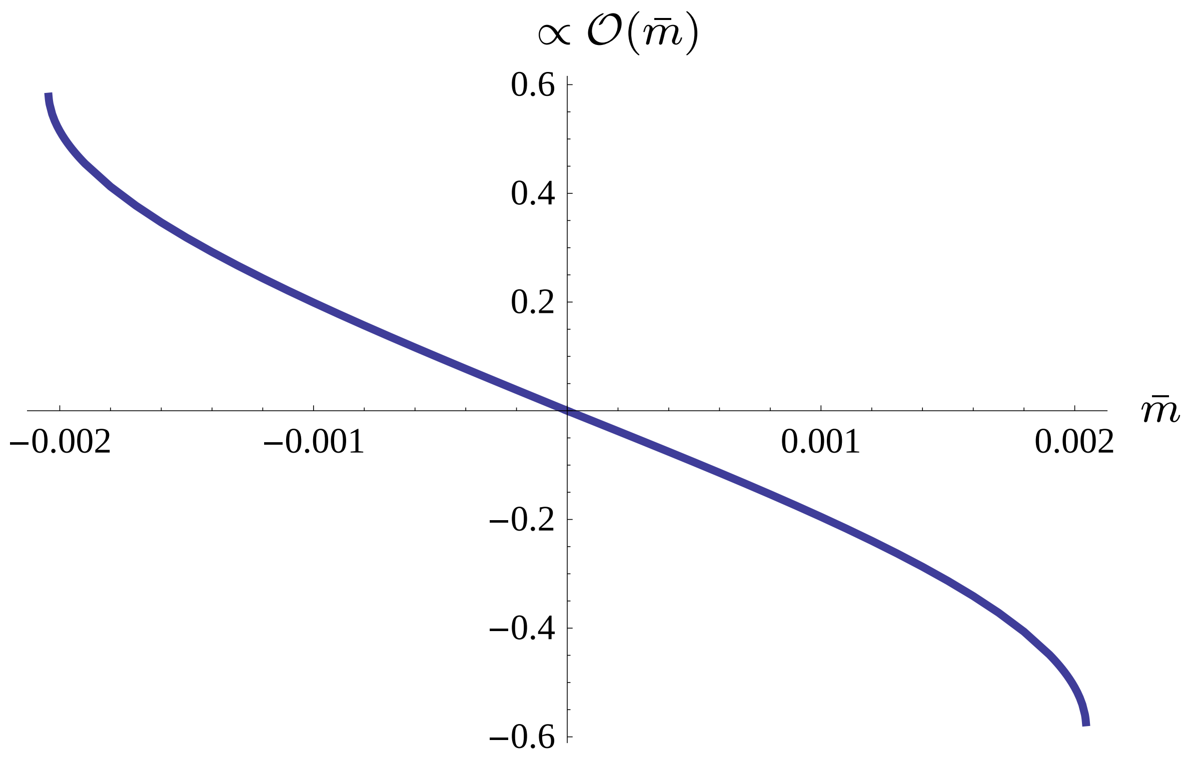
$\Longrightarrow$ If chiral tachyons condense with zero momentum in the new ground state, there must exit homogeneous and isotropic deconfined phase of the cascading plasma, with spontaneous broken chiral symmetry for $T < T_{\chi\mathrm{SB}}$.

$\Longrightarrow$ I will argue now that such homogeneous and isotropic phase does not exist. Specifically, for $T < T_{\chi\mathrm{SB}}$

- we turn the (gaugino) fermion mass $\bar{m} \equiv \frac{M}{T}$, explicitly breaking the chiral symmetry;
- the chiral condensates $T^{-3}\langle\lambda\lambda\rangle \equiv \mathcal{O}(\bar{m})$
- we can compute now

$$\lim_{\bar{m}\to 0} \mathcal{O}(\bar{m})$$

$\Longrightarrow$ Once again, holographic correspondence allows us to compute above expectation value, without any approximation!



$\Longrightarrow$ Homogeneous and isotropic deconfined phase with spontaneously broken chiral symmetry does not exist.

$\Longrightarrow$ It appears chiral tachyons must condense with finite momentum, resulting in some inhomogeneous phase.

Summary

- I considered a cascading gauge theory, which is in the same universality class in the IR as $\mathcal{N} = 1\ SU(M)$ SYM, in the planar limit, and for (infinitely) large 't Hooft coupling.
- Using holography, I argued, without any further approximations, that this theory undergoes a first order confinement phase transition (with spontaneous broken chiral symmetry) at T_c
- Below T_c, the metastable chirally symmetric deconfined phase in this theory becomes perturbatively unstable at

$$T_{\chi \mathrm{SB}} = 0.882503(0) T_c$$

- Chiral symmetry breaking tachyons at $T < T_{\chi \mathrm{SB}}$ do not condense at zero momentum —- indication of the inhomogeneous ground state?
- Amusingly, (S.Katz lattice QCD computations (lecture in St. Goat, 2011))

$$\frac{151}{175} = 0.862857$$

Need to study the tachyons of the cascading plasma further!

$\Longrightarrow$ This is not to say that we can not see a thermodynamic plateau in holography:

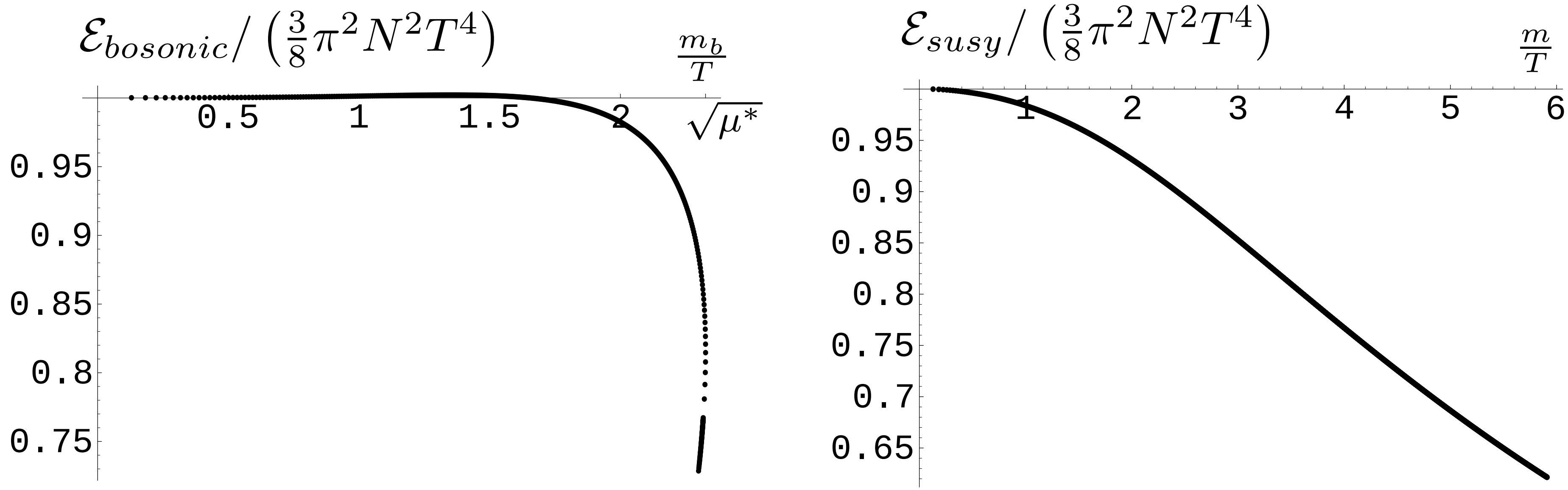


Figure 3: Equation of state of the mass deformed $\mathcal{N}=4$ gauge theory plasma. At $T\sim m$ the deviation from the conformal thermodynamics is $\sim 2\%$. For the ideal gas approximation the deviation is about 40%. (S.Deakin, P.Kerner, J.Liu, AB, hep-th/0701142.)

$\Longrightarrow \mathcal{N}=2^*$ model appears to share a 'thermodynamic plateau' with QCD, but there is not confinement in this model.