Some Comments on Relativistic Hydrodynamics, Fuzzy Bag Models for the Pressure, and Early Space-Time Evolution of the QCD Matter

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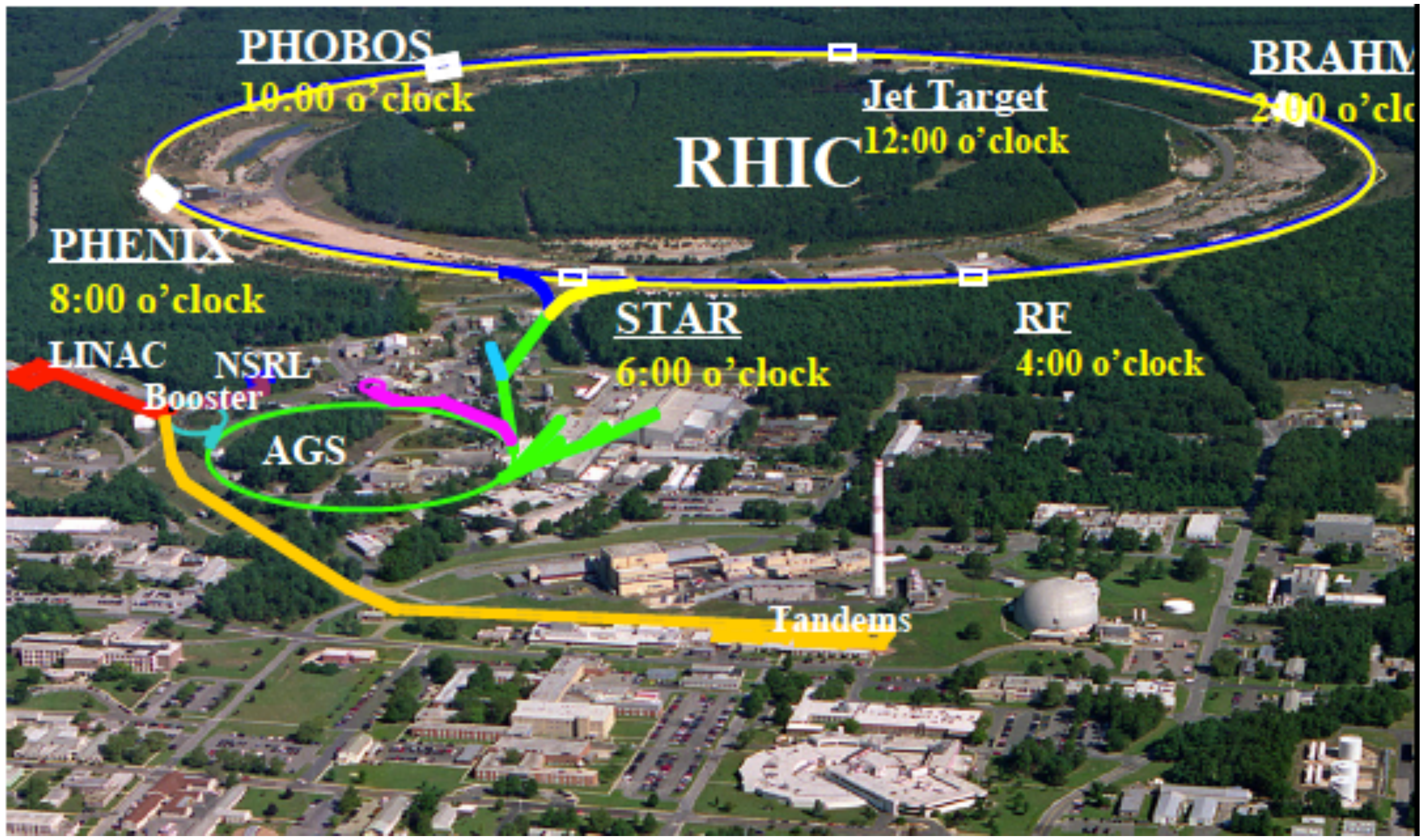
Based on Int.J.Mod.Phys. E20, 2189 (2011)

INT 11-3 Workshop

Motivations

RHIC and LHC

- AA, nuclei on nuclei. $A \sim 60 - 200$ (Cu , Au, Pb)
- pp, protons on protons. Benchmark for “ordinary” QCD
- dA, deuteron on nucleus. QCD in “cold” nuclei
- RHIC @ BNL 200 GeV LHC @ CERN 5500 GeV



RHIC: in the semi-QGP

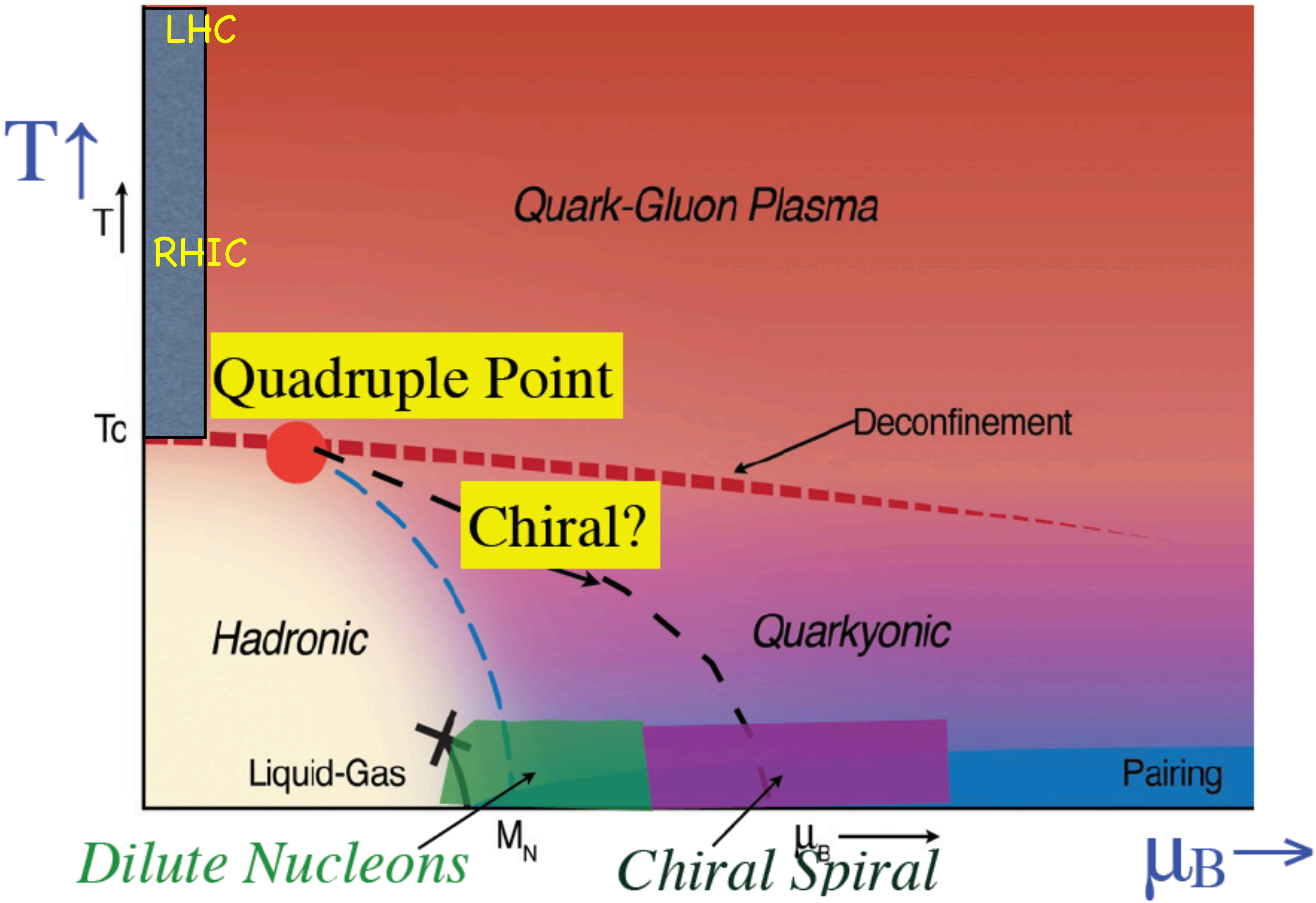
up to ~ 1.5 - 2 Tc

LHC: deep in the complete QGP?

up to ~ 4 Tc

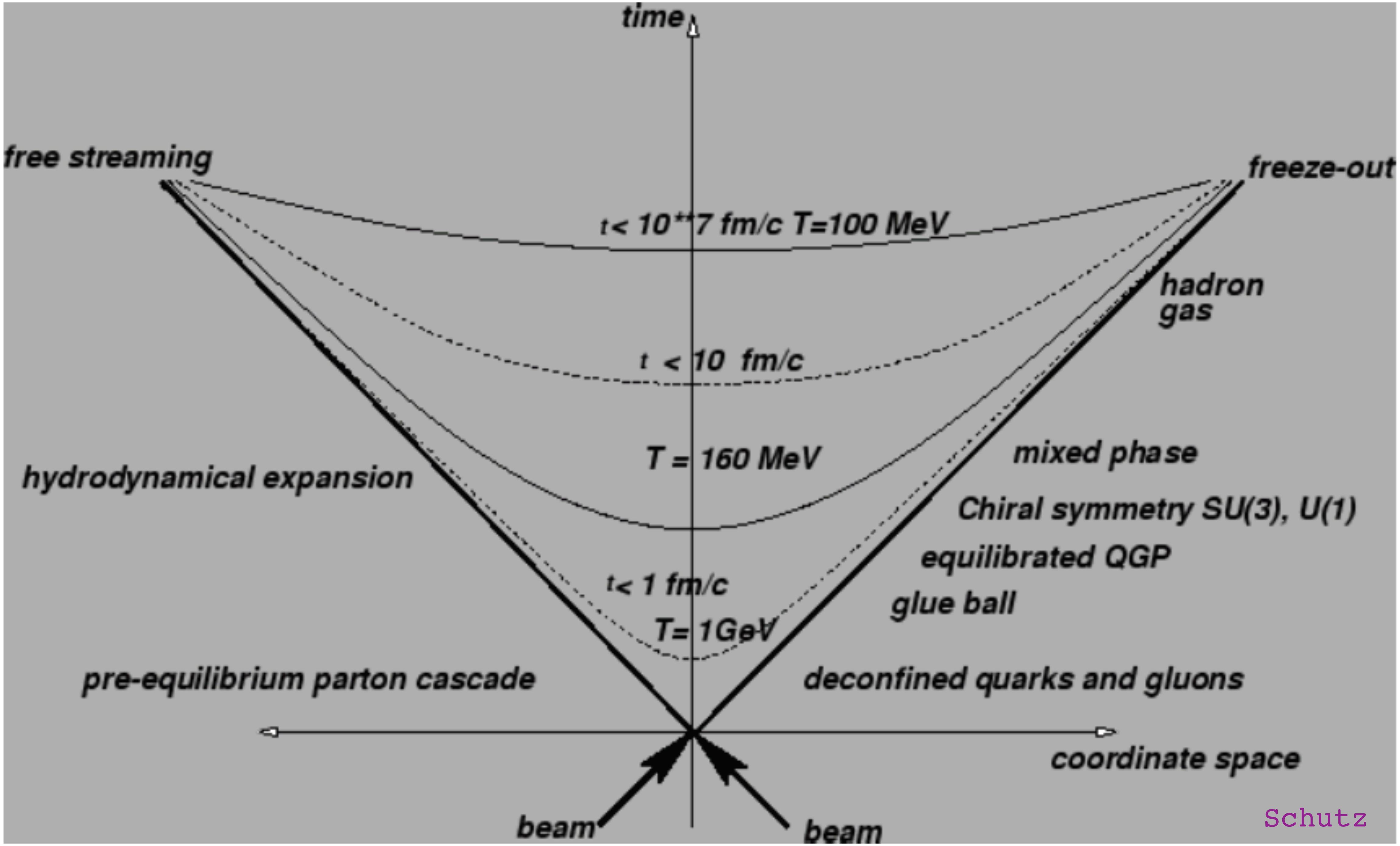
Pisarski

Phase Diagram for QCD 2010



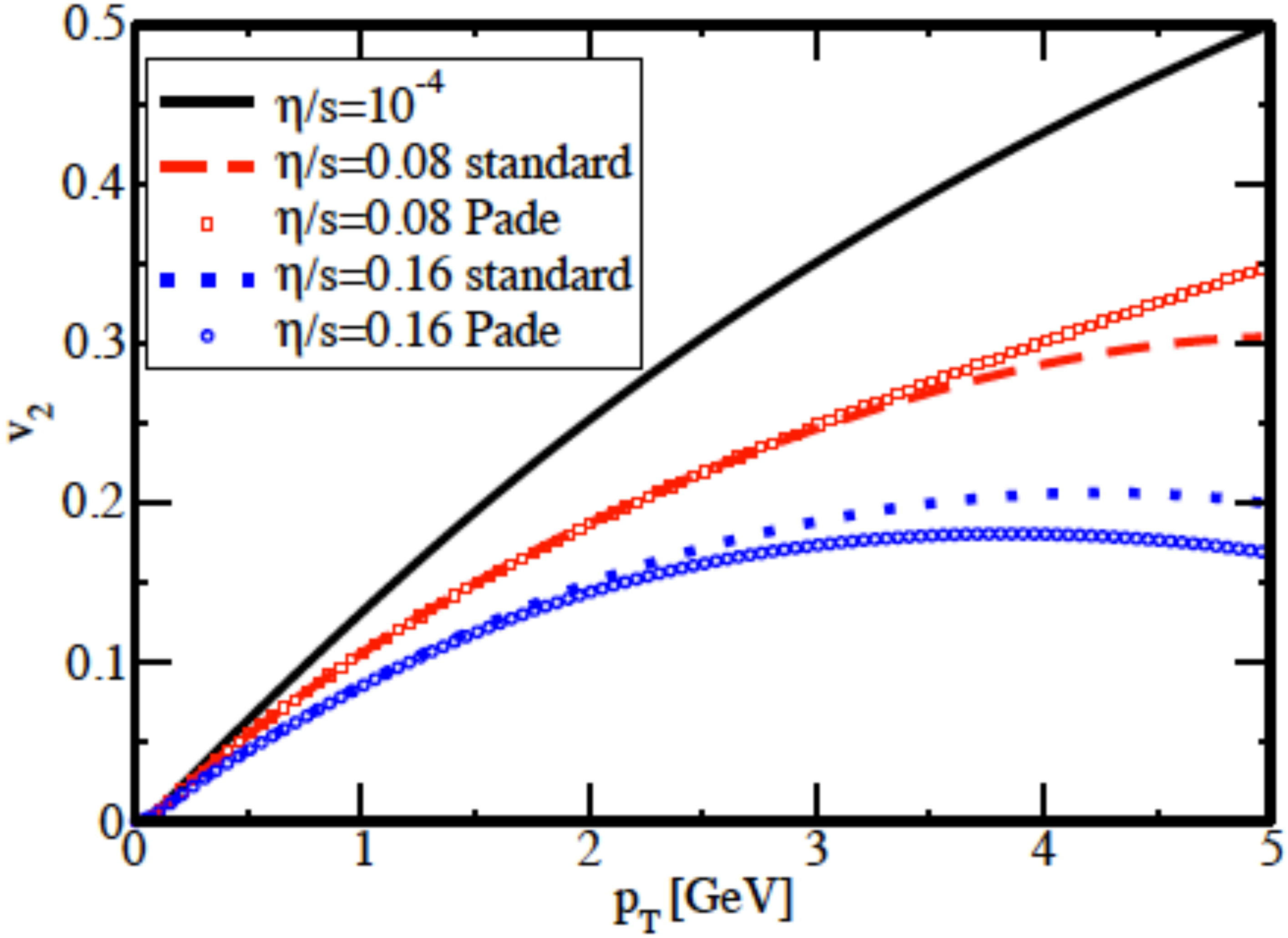
Pisarski

Space-Time Picture of Nucleus-Nucleus Collisions

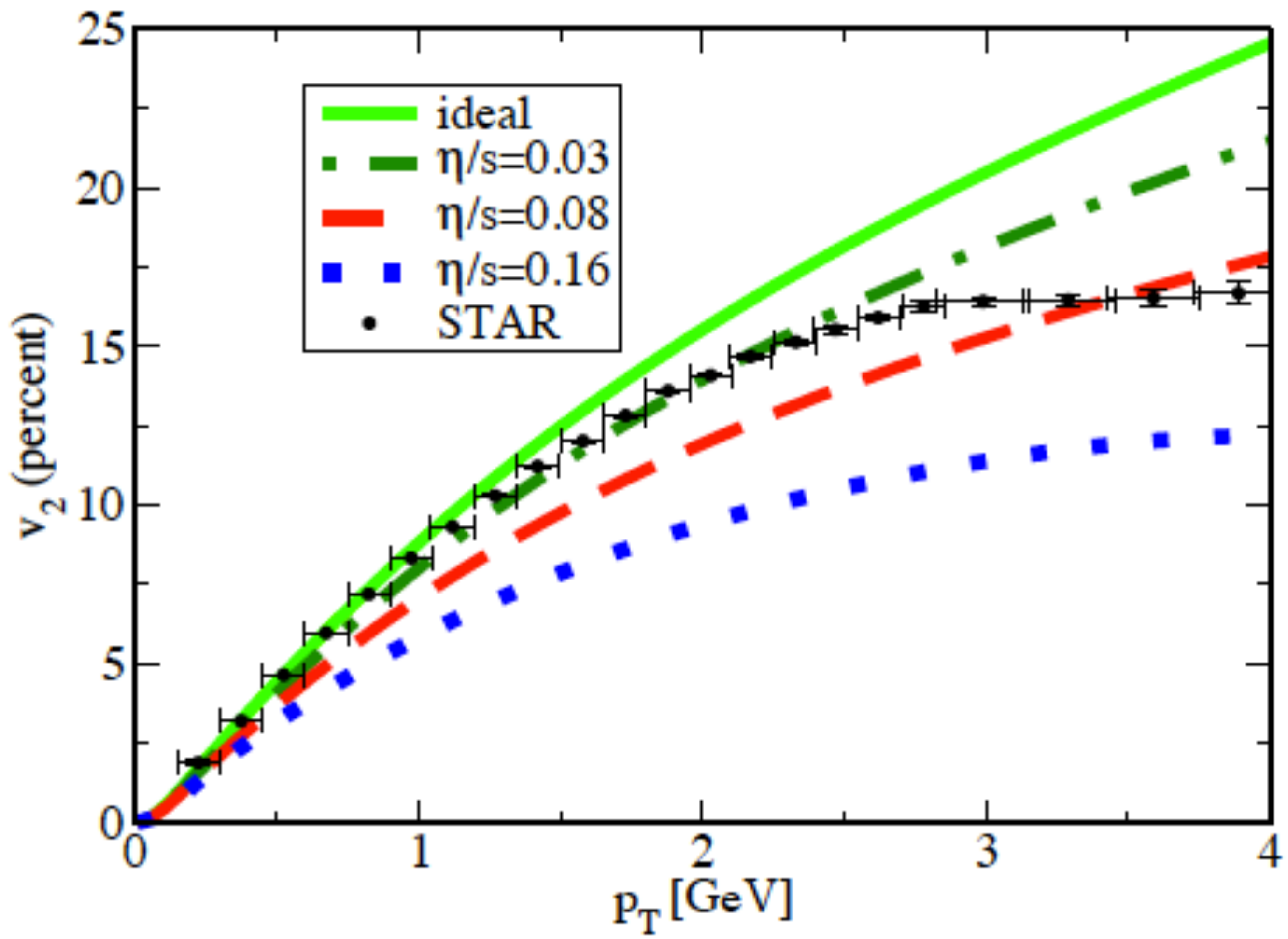


✦ A phase of hydrodynamical expansion

Landau



Luzum, Romatschke



Romatschke-Romatschke

Which parameters matter?

- Initial energy density profile: Glauber or CGC
- Initial value of shear tensor
- Hydrodynamic starting time τ_0
- Second-order coefficients: relaxation time and λ_1
- Ansatz for non-equilibrium particle distribution
- The equation of state
- The freeze-out procedure
- . . .

Which parameters matter?

- Initial energy density profile: Glauber or CGC
- Initial value of shear tensor: $\zeta = 0\,,\ \ \eta/s = const$
- Hydrodynamic starting time τ_0
- Second-order coefficients: relaxation time and λ_1
- Ansatz for non-equilibrium particle distribution
- The equation of state: conformal matter, pQCD, or MIT bag
- The freeze-out procedure
- . . .

Shear and Bulk Viscosities

✦ QGP is the most perfect liquid (relativistic) on earth?

✦ At RHIC $\eta/s \sim 0.1 \pm 0.1$

✦ In $N = 4$ SU(∞) super YM

$$\eta/s = 1/4\pi \sim 0.08$$

Son et al

✦ Kovtun-Son-Starinets bound

$$\eta/s \geq 1/4\pi$$

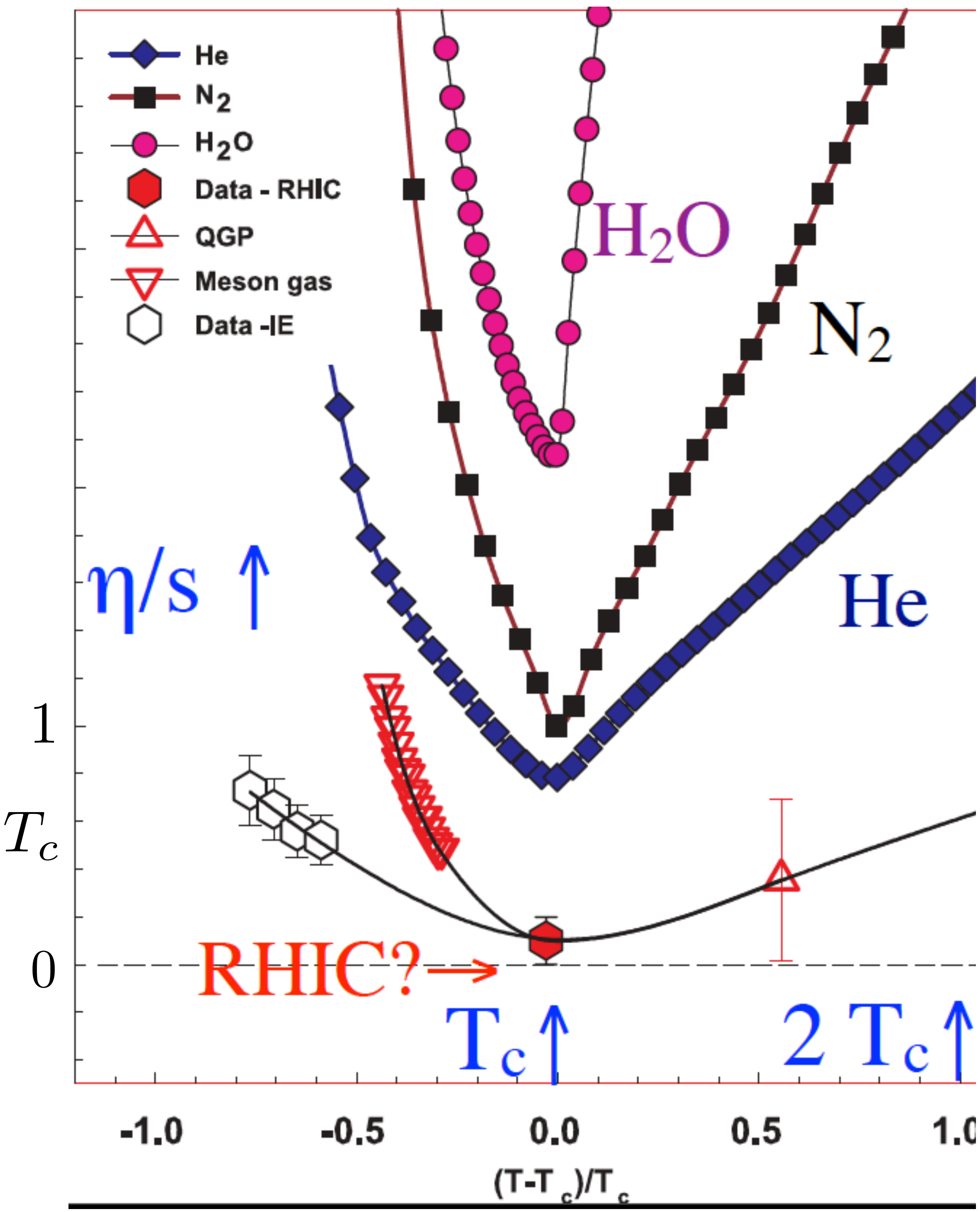
✦ Phenomenological models $T > T_c$

$$\frac{\eta}{s} = A - \frac{B}{T^2}$$

Kapusta, Springer

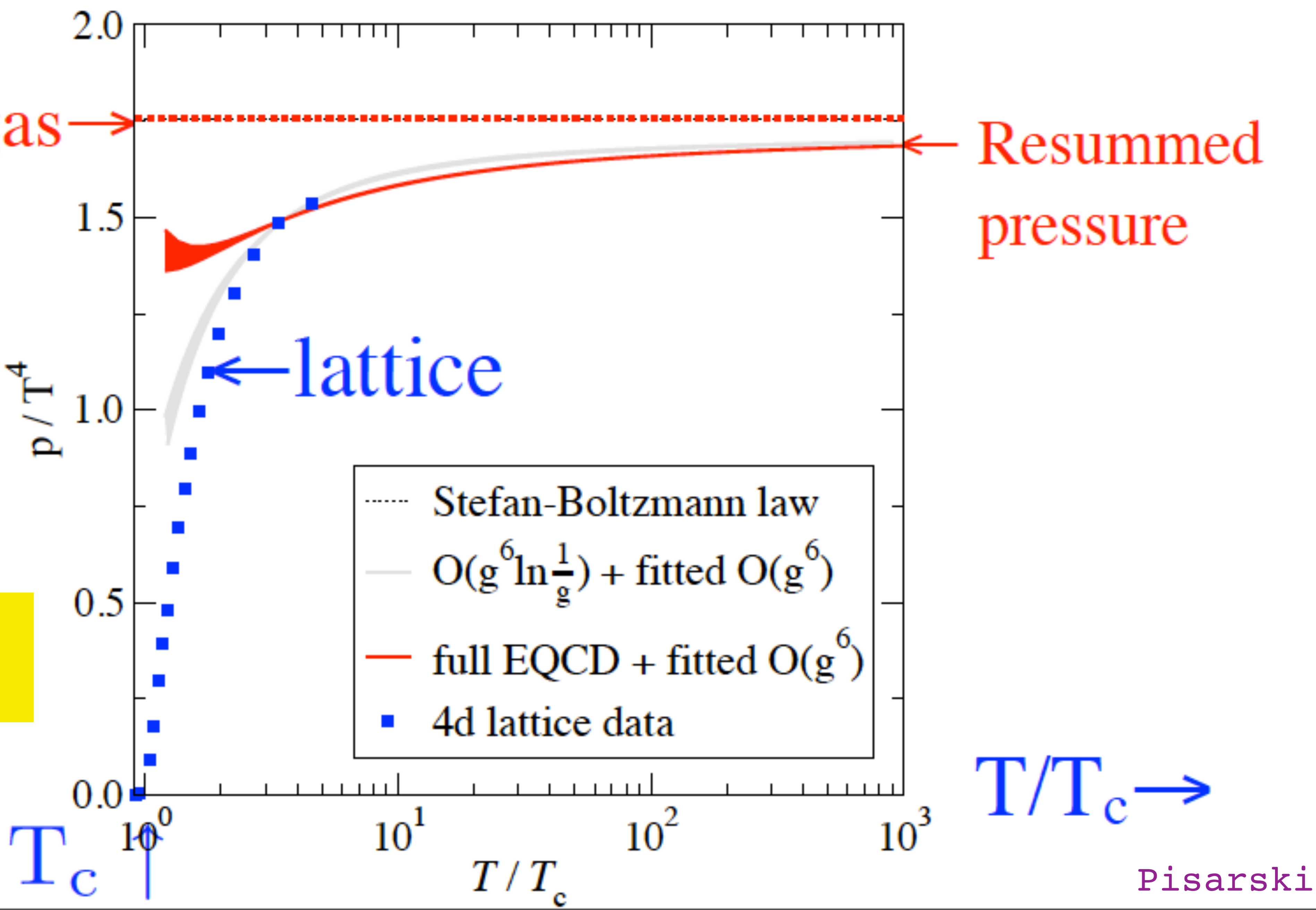
$$\frac{\zeta}{s} = \frac{C}{T^2}$$

Kharzeev, Tuchin



Stocker et al

Equation of State



Pure SU(3) glue (1995)
2+1 flavor QCD (2009)

HOT QCD collaboration

- Phenomenological models for $1.2\,T_c \le T \le 4\,T_c$ (Fuzzy bags)

$$p(T) = a\left(T^4 - \alpha T^2\right) - B$$

Ogilvie et al

- It is mainly semi-QGP such that α_s is not so big

The Model

- admits analytic solutions

  but at the same time

- is physically plausible

1+0 Dimensional Hydrodynamics

- Energy momentum tensor (for relativistic hydrodynamics)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \Pi^{\mu\nu} \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

- Evolution equation (conservation law)

$$D_\mu T^{\mu\nu} = 0$$

- Good approximation for the early time evolution is to

  consider no dependence on the transverse spatial coordinates (x, y)

  change the variables as $\tau = \sqrt{t^2 - z^2}$, $\eta = \mathrm{arctanh}(z/t)$

  impose the boost invariant expansion in the beam-direction (z)

As a result, the problem becomes one-dimensional!

Bjorken Solution

- Energy momentum tensor in (τ, x, y, η) coordinates

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi + \frac{1}{2}\Phi & 0 & 0 \\ 0 & 0 & \Pi + \frac{1}{2}\Phi & 0 \\ 0 & 0 & 0 & \Pi - \Phi \end{pmatrix}$$

- At first order $\Phi = \frac{4\eta}{3\tau}$ and $\Pi = -\frac{\zeta}{\tau}$
- For perfect fluid $\eta = \zeta = 0$
  and the equation of state $p(T) = aT^4 - B$
  Bjorken found

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}}$$

At early proper times, it defines a base line on top of which dissipative and QCD effects have to established

Perfect Fluid and QCD effects

- Now, for $\eta = \zeta = 0$ and $p(T) = a(T^4 - \alpha T^2) - B$

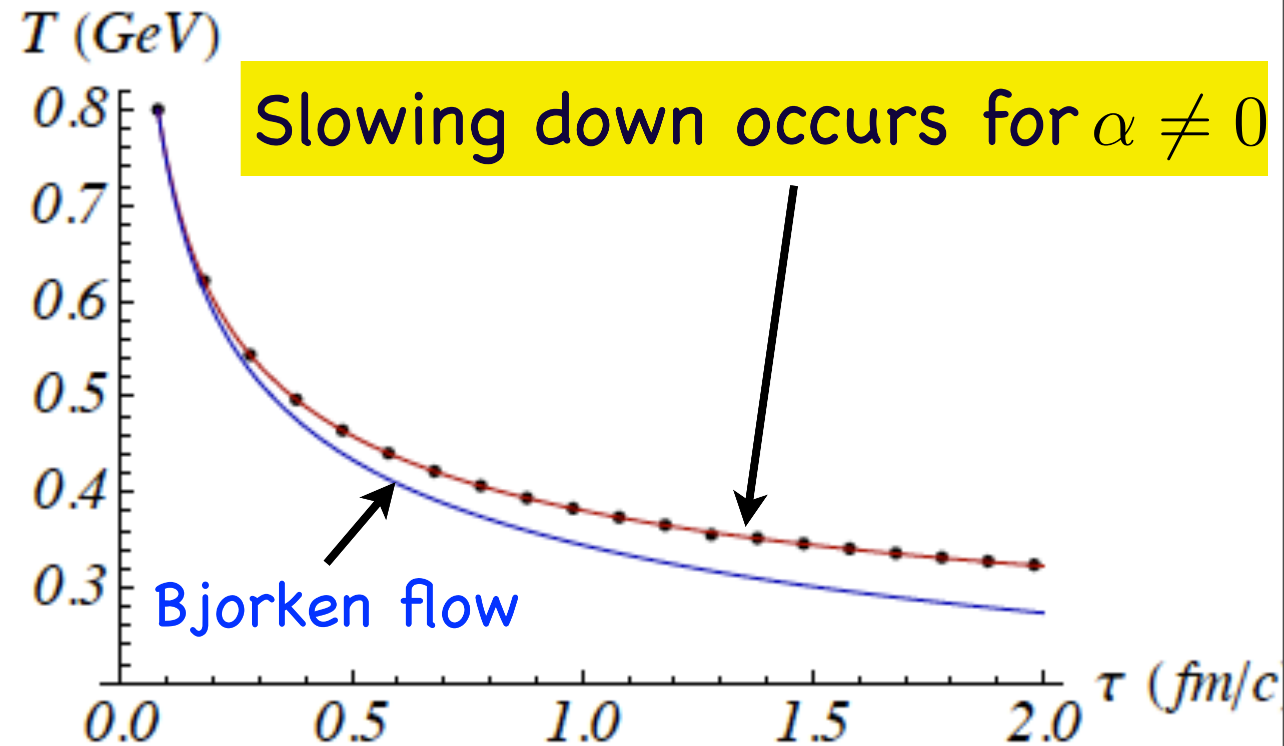
$$T(\tau) = \begin{cases} \left(\frac{c}{\tau}\right)^{\frac{1}{3}}\left[\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{\alpha^3}{54}\left(\frac{\tau}{c}\right)^2}\right)^{\frac{1}{3}} + \left(\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{\alpha^3}{54}\left(\frac{\tau}{c}\right)^2}\right)^{\frac{1}{3}}\right] & \text{if } \tau \le \sqrt{\frac{54}{\alpha^3}}c, \\ \sqrt{\frac{2}{3}\alpha}\,\cos\left[\frac{1}{3}\arccos\left(\sqrt{\frac{54}{\alpha^3}}\frac{c}{\tau}\right)\right] & \text{if } \tau \ge \sqrt{\frac{54}{\alpha^3}}c \end{cases}$$

with $c = \tau_0 T_0 (T_0^2 - 0.5\alpha)$

- For $T_0 = 0.8\,\mathrm{GeV}$, $\tau_0 = 0.08\,\mathrm{fm/c}$
- For small τ

$$T(\tau) \approx \left(\frac{c}{\tau}\right)^{\frac{1}{3}} + \frac{\alpha}{6}\left(\frac{\tau}{c}\right)^{\frac{1}{3}}$$

- Note $\alpha \approx 0.09\,\mathrm{GeV}^2$



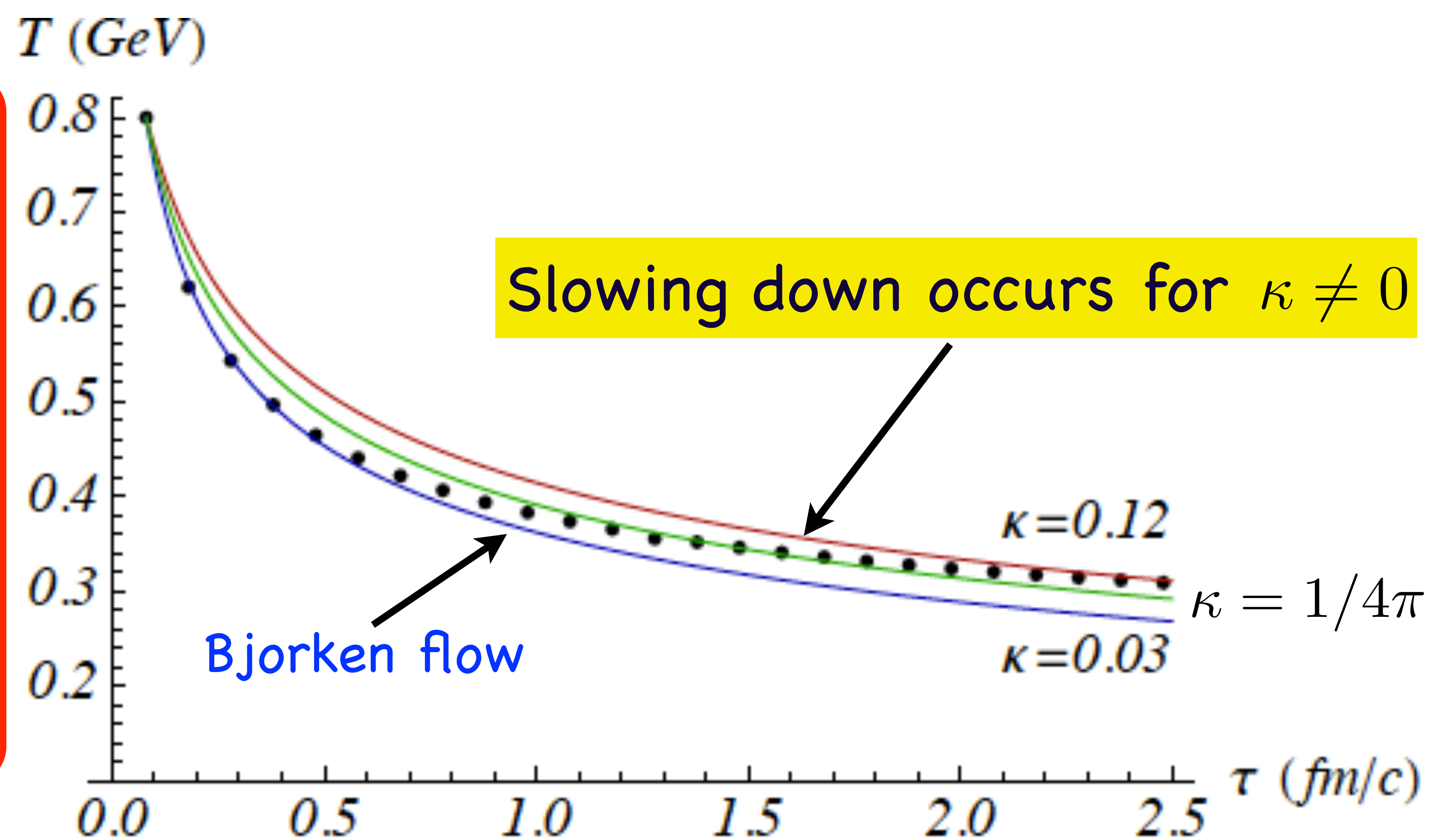
Viscous Fluid

- Now, for $\eta/s = \kappa$ and $p(T) = aT^4 - B$

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}} + \frac{2\kappa}{3\tau_0}\left[\left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}} - \frac{\tau_0}{\tau}\right],$$

Kouno et al

In the interval of primary interest
$\eta/s < 0.2$
the effect of α-correction is as important as that of shear viscosity !



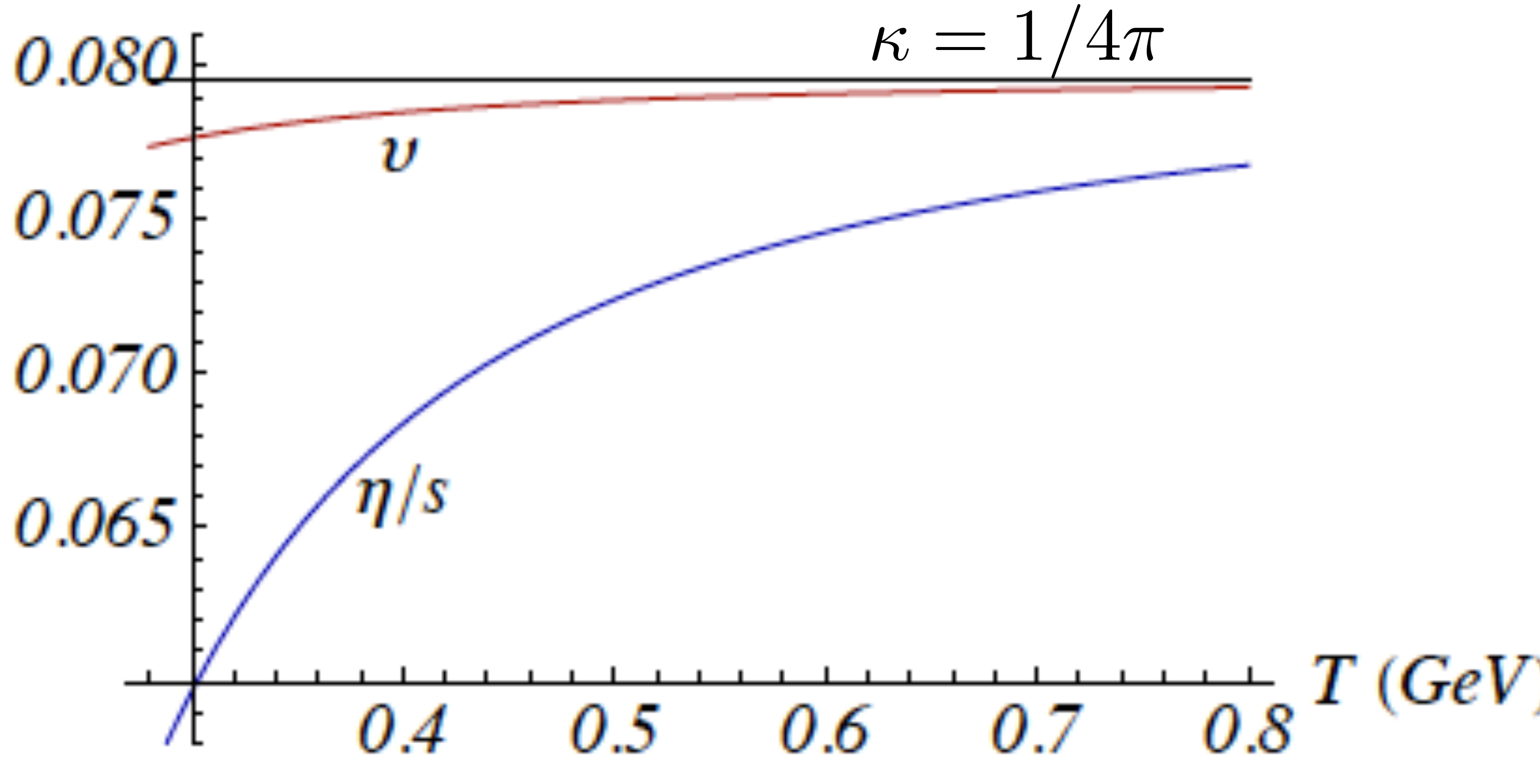
Attempt of Synthesis

- Now, $\eta/s = \kappa\left(1 - k\dfrac{\alpha}{T^2}\right)$, $\zeta/s = m\dfrac{\alpha}{T^2}$, $p(T) = a\left(T^4 - \alpha T^2\right) - B$

- At first order, one can combine the viscosities as

$$v = \frac{\eta}{s} + \frac{3\zeta}{4s}$$

Surprizingly, the difference between v and conjectured bound value $\kappa = 1/4\pi$ is very small: cancelation !



Here $\kappa = 1/4\pi$, $k = 1/4$, $m = 0.024$

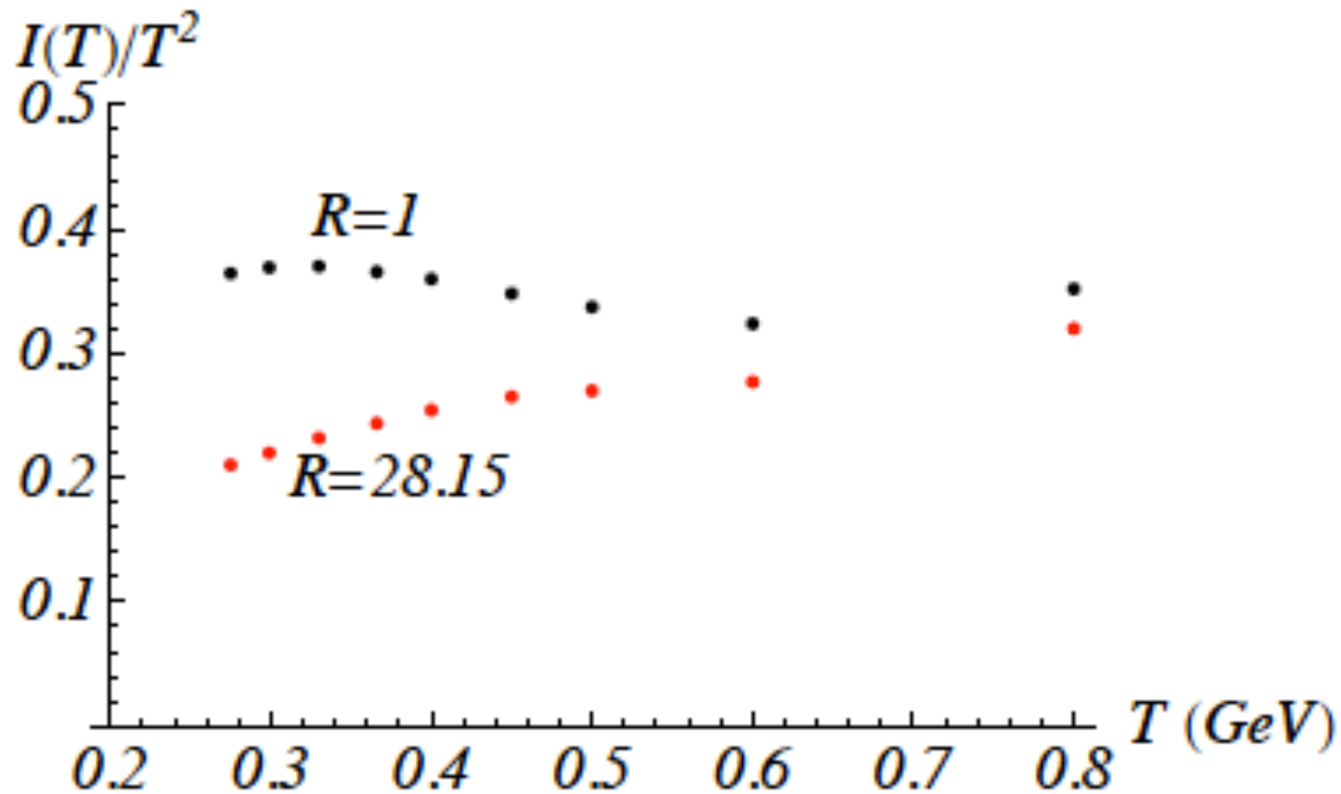
- Approximate solution

$$T(\tau) \approx T_{\rm v} + \frac{\alpha}{6}\left[\frac{1}{T_{\rm v}} - \frac{2\kappa}{3c}(1+6\delta)\left(\frac{c}{\tau}\right)^{\frac{1}{3}}\left(\ln(\tau T_{\rm v}) - \frac{2\kappa}{3\tau T_{\rm v}}\right)\right], \quad T_{\rm v}(\tau) = \left(\frac{c}{\tau}\right)^{\frac{1}{3}} - \frac{2\kappa}{3\tau}$$

The Light Quarks Make the Life Difficult

$$I(T) = \varepsilon - 3p$$

$$R = m_s^{phys}/m_{ud}$$



2+1 flavor QCD (2009)

Wuppertal-Budapest collaboration

- parameterization (Fizzy bag) for $1.2\,T_c \leq T \leq 4\,T_c$

$$p(T) = a\Big(T^4 + \gamma T^3 - \alpha T^2 + \beta T\Big) - B$$

- or even simpler

$$p(T) = a\Big(T^4 - \alpha T^2 + \beta T\Big) - B$$

Still Solvable for Perfect Fluid

$$T(\tau) = -2\gamma + f^{\frac{1}{3}}\left[\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{(\alpha+24\gamma^2)^3}{54f^2}}\right)^{\frac{1}{3}} + \left(\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{(\alpha+24\gamma^2)^3}{54f^2}}\right)^{\frac{1}{3}}\right]$$

$$f = c\tau^{-1} - 16\gamma^3 - \alpha\gamma - \beta$$

$$c = \tau_0(T_0^3 + 6\gamma T_0^2 - \tfrac{1}{2}\alpha T_0 + \beta)$$

- ✦ Our findings stand such a modification of the equation of state

Conclusions

✦ Can our findings stand the test of a 3+1 hydrodynamical code?

✦ But many interesting things to do also in one-dimension

- ✓ second order hydrodynamics
  (issue of uncertainty about the relaxation coefficients)
- ✓ effects of finite baryon chemical potential
  (equation of state, viscosities)
- ✓ analytical solutions for lower temperatures
  (if any for $T < 0.3\,\mathrm{GeV}$)