Reactions and structure aspects of the nuclear many-body dynamics.

Alexander Volya

Florida State University

Nuclear structure and reactions

Questions

- Physics of interplay between structure and reactions
- Overlapping resonances
- Virtual excitations
- Role of continuum, spectroscopic factors, self energy
- Solution methods: adiabatic, sudden, uncorrelated, CDCC
- Convergence of solutions

Outline

- “Simple” example, scattering of a composite object
  - Simple and yet very difficult
  - Role of virtual channels
  - Features of solutions
- Time-dependent continuum shell model

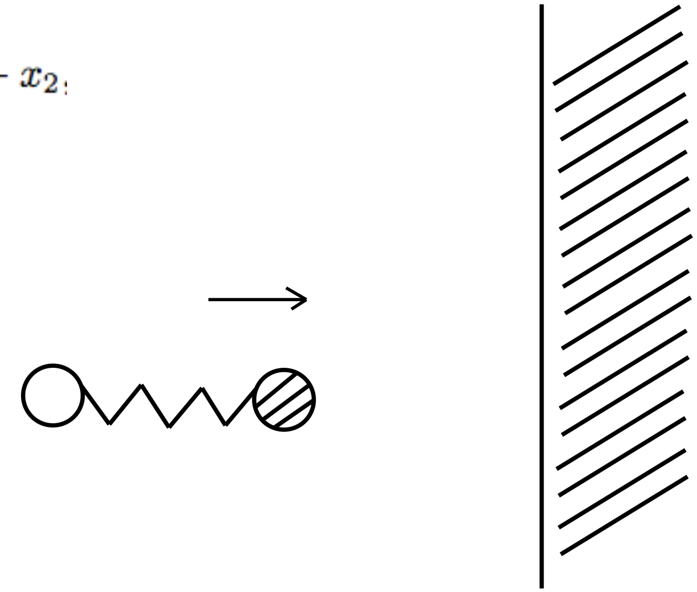
Reflection from the wall

Composite object $X = \frac{m_1 x_1 + m_2 x_2}{M}, \qquad x = x_1 - x_2;$

$$M = m_1 + m_2, \qquad m = \frac{m_1 m_2}{m_1 + m_2}$$

$$h = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + v(x)$$

$$h\psi_n(r) = \epsilon_n \psi_n(r), \qquad n = 0, 1, 2, \ldots$$



Hamiltonian $H = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial X^2} + V(x_1, x_2) + h$

$$V(x_2) = \begin{cases} \infty & \text{when } 0 < x_2 \\ 0 & \text{otherwise} \end{cases}$$

Assume that only one of the particles interacts with the potential!

$$V(x_1, x_2) \rightarrow V(x_2)$$

Approach to solution

$$\Phi(X, x) = \frac{e^{iK_n X}}{\sqrt{|K_n|}}\psi_n(x) + \sum_{m=0}^{\infty} \frac{R_{mn}}{\sqrt{|K_m|}} e^{-iK_m X}\psi_m(x);$$

$$\Phi(X, x) = 0 \text{ at } x_2 = 0.$$

See also A.M. Moro et.al. arXiv:1010.4933

Projection method

Boundary condition $\Phi(\mu_1 x,\, x) = 0$

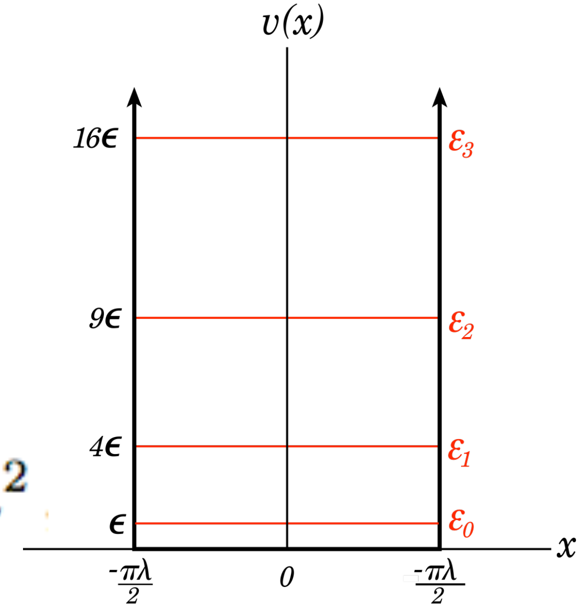
Matrix inversion problem for every energy

$$K_n(E_T) = \frac{1}{\hbar}\sqrt{2M(E_T - \varepsilon_n)}.$$

$$D_{ln}(\varkappa) = \int \psi_l^*(x)\, e^{\varkappa x}\, \psi_n(x)\, dx$$

$$\sum_n \frac{D_{ln}\left[-i\mu_1(K_{n'} + K_n)\right]}{\sqrt{|K_n|}} R_{nn'} = -\frac{\delta_{ln'}}{\sqrt{|K_{n'}|}}$$

HO and Well models

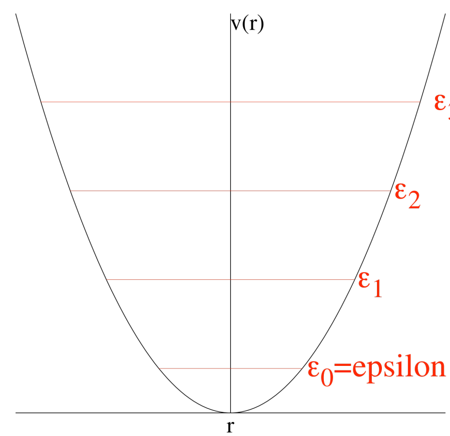


"Well" model

$$v(x) = \begin{cases} 0 & \text{when } |x| < \pi\lambda/2 \\ \infty & \text{otherwise} \end{cases}$$

$$\psi_{n-1}(x) = \sqrt{\frac{2}{\pi}} \sin\left[\left(x + \frac{\pi}{2}\right) n\right], \quad \varepsilon_{n-1} = n^2$$

"HO" model

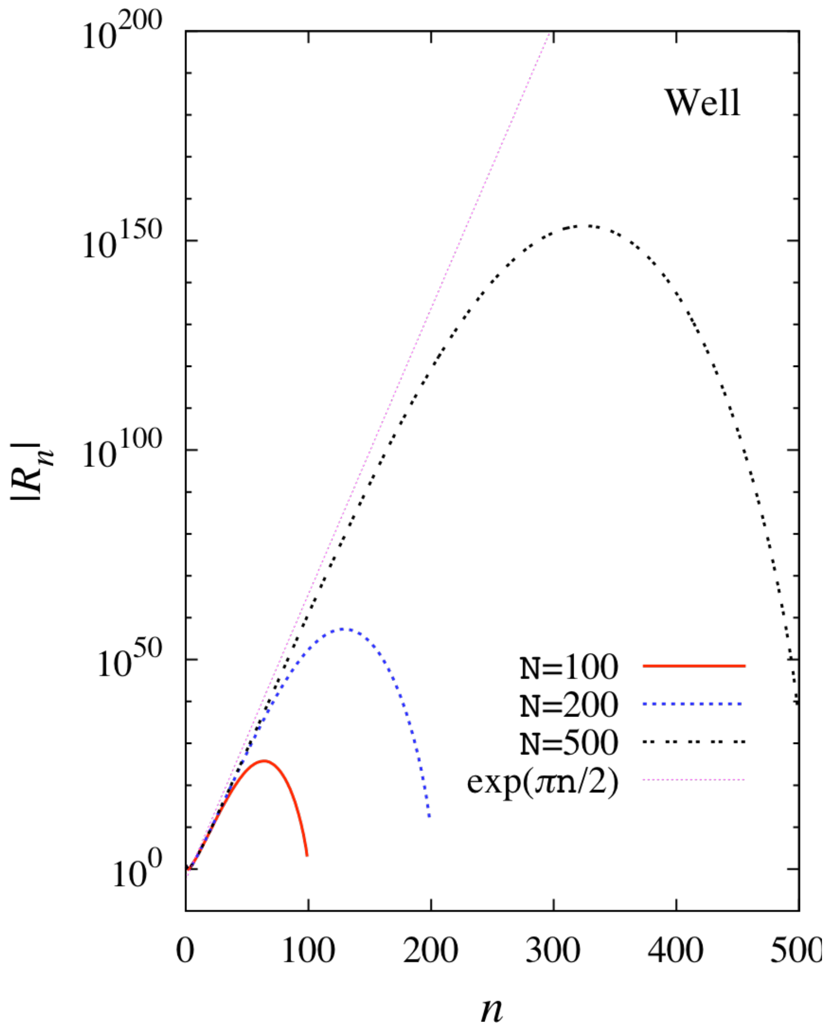


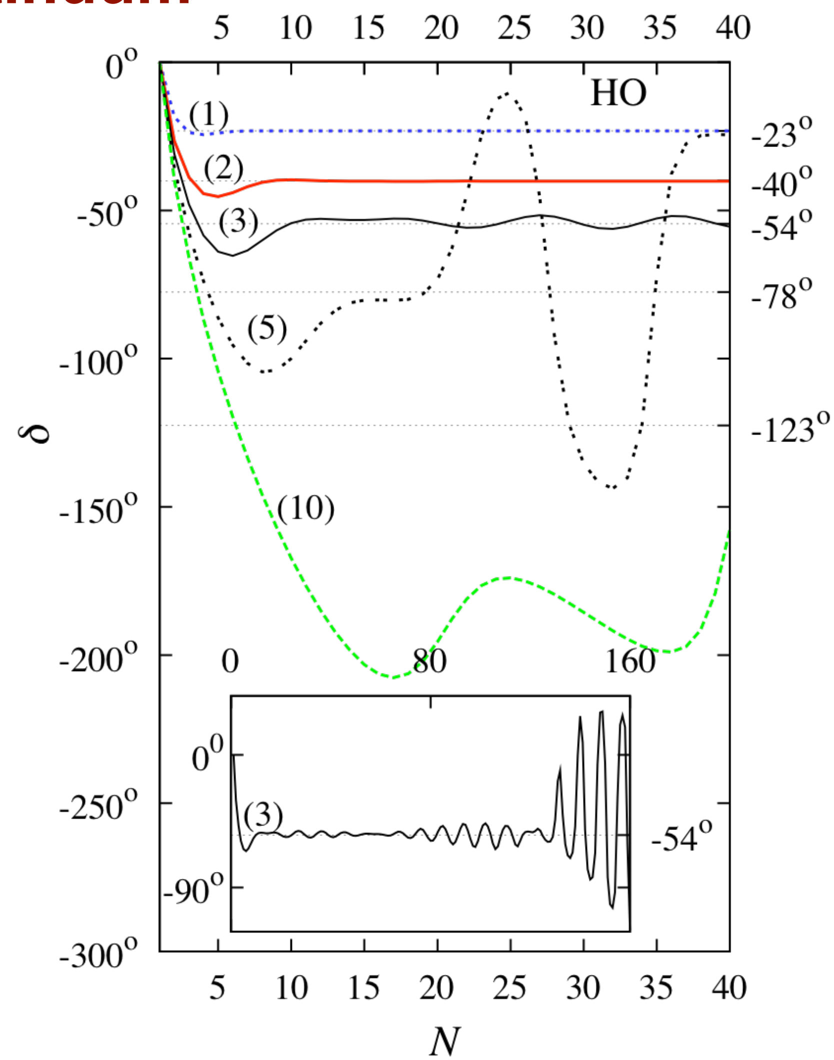
$$v(x) = m\omega^2 x^2/2$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} H_n(x) \exp\left(-\frac{x^2}{2}\right)$$

Two-nucleon system with continuum

finite square well binding



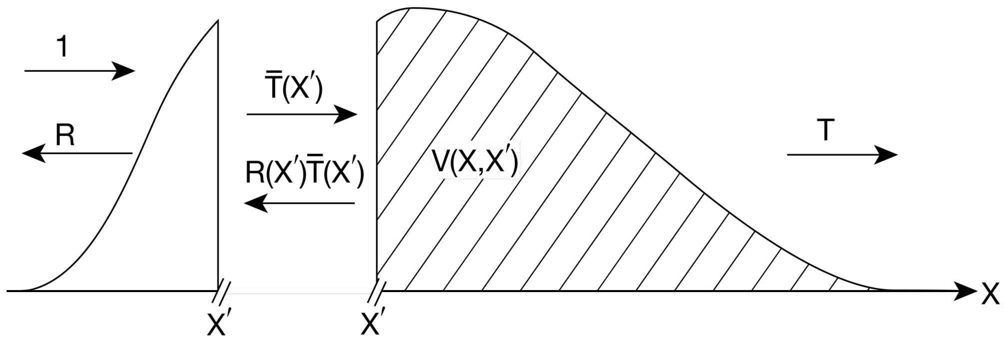


In the exact solution virtual amplitudes are exponentially large.

The solution based on projection onto restricted Hilbert space (matrix inversion) is not stable. The instability is severe when non-interacting particle is heavy

Variable Phase Method

Dynamic solution



Simple, fast, stable
Complex momentum
Virtual channels

Schrödinger equation

$$\left[\frac{\partial^2}{\partial X^2} + K^2\right]\Psi(X) - V(X)\Psi(X) = 0$$

Truncated potential

$$\Psi(X, X') = T(X')\left[e^{iKX} + e^{2i\delta(X')}\, e^{-iKX}\right]$$

Cauchy boundary condition

$$\Psi(X, X')|_{X'=X} = \Psi(X)$$

$$\left.\frac{\partial\Psi(X, X')}{\partial X}\right|_{X'=X} = \frac{d\Psi(X)}{dX}$$

Phase equation

$$\frac{d\delta(X)}{dX} = \frac{V(X)}{K}\cos^2\left[KX - \delta(X)\right]$$

Amplitude equation

$$\frac{dT(X)}{d(X)} = \frac{V}{2iK}T\left[1 + e^{2i\delta(X)}\, e^{-2iKX}\right]$$

Multichannel Variable Phase Method: Multichannel case

Coupled channels equations

$$\Phi(X,x) = \sum_n \Psi_n(X)\psi_n(x) \qquad \left[\frac{\partial^2}{\partial X^2} + K_n^2\right]\Psi_n(X) - \sum_{n'} V_{nn'}(X)\Psi_{n'}(X) = 0$$

Folded potentials

$$V_{nn'}(X) = \frac{2M}{\hbar^2}\int_{-\infty}^{\infty} \psi_n^*(x)\, V(X,x)\, \psi_{n'}(x) dx$$

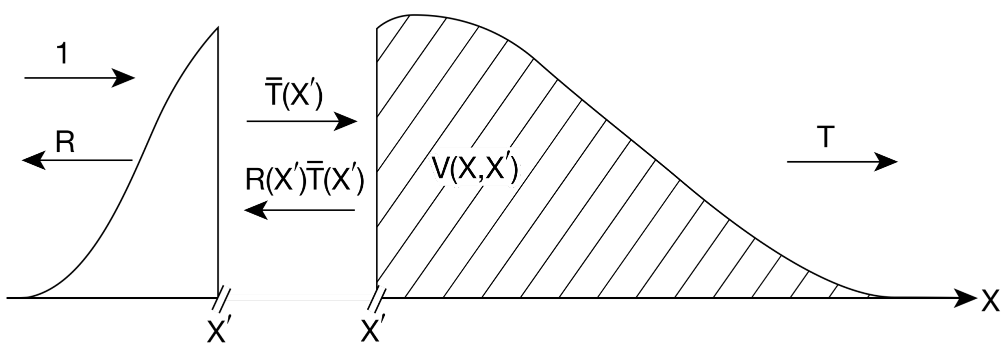
VPM equations

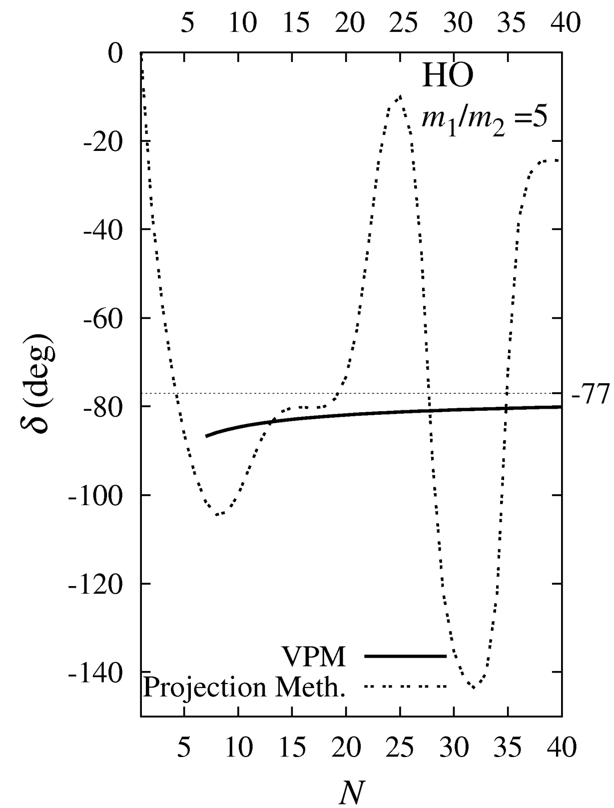
$$\Xi^{\pm}_{nn'}(X) = \frac{e^{\pm i K_n X}}{\sqrt{-2iK_n}}\delta_{nn'}$$

$$\Psi(X,X') = \left[\Xi^+(X) + \Xi^-(X)\, R(X')\right] \overline{T}(X')$$

$$\frac{dT(X)}{dX} = T(X)\,\Xi^-\, V\, \left[\Xi^+ + \Xi^-\, R(X)\right]$$

$$\frac{dR(X)}{dX} = \left[\left(\Xi^+ + R(X)\,\Xi^-\right)\right] V \left[\Xi^+ + \Xi^-\, R(X)\right]$$

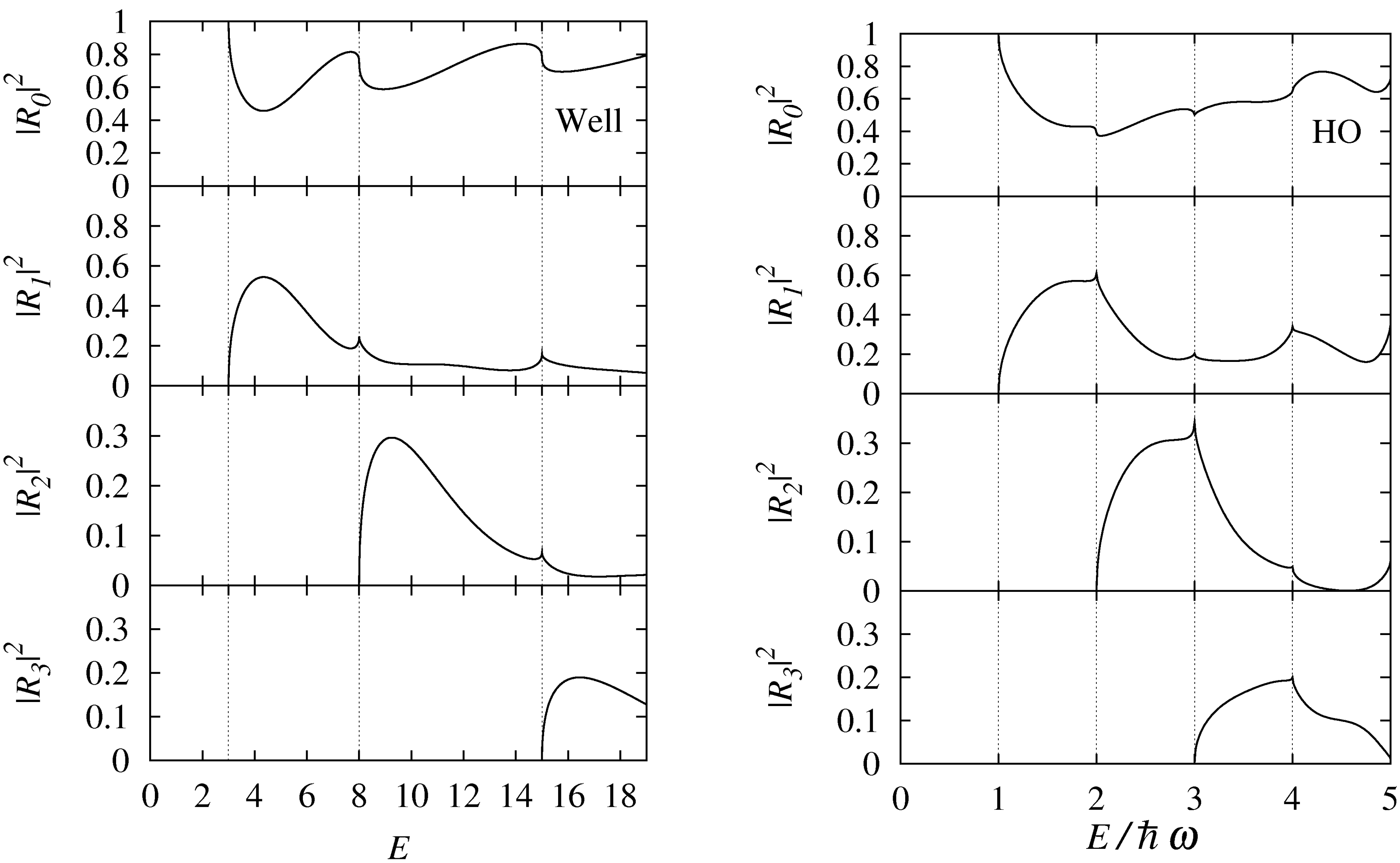




VPM versus Projection method

Results: scattering off an infinite wall

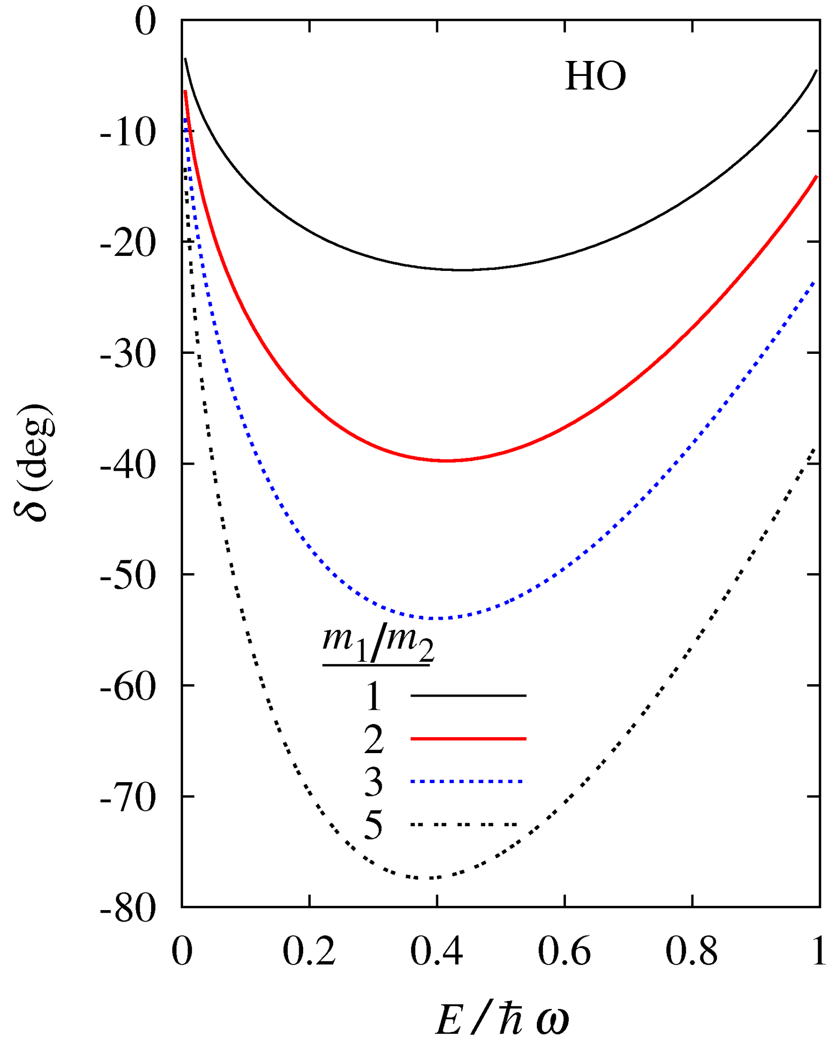
Well | Harmonic oscillator



Results: scattering off an infinite wall

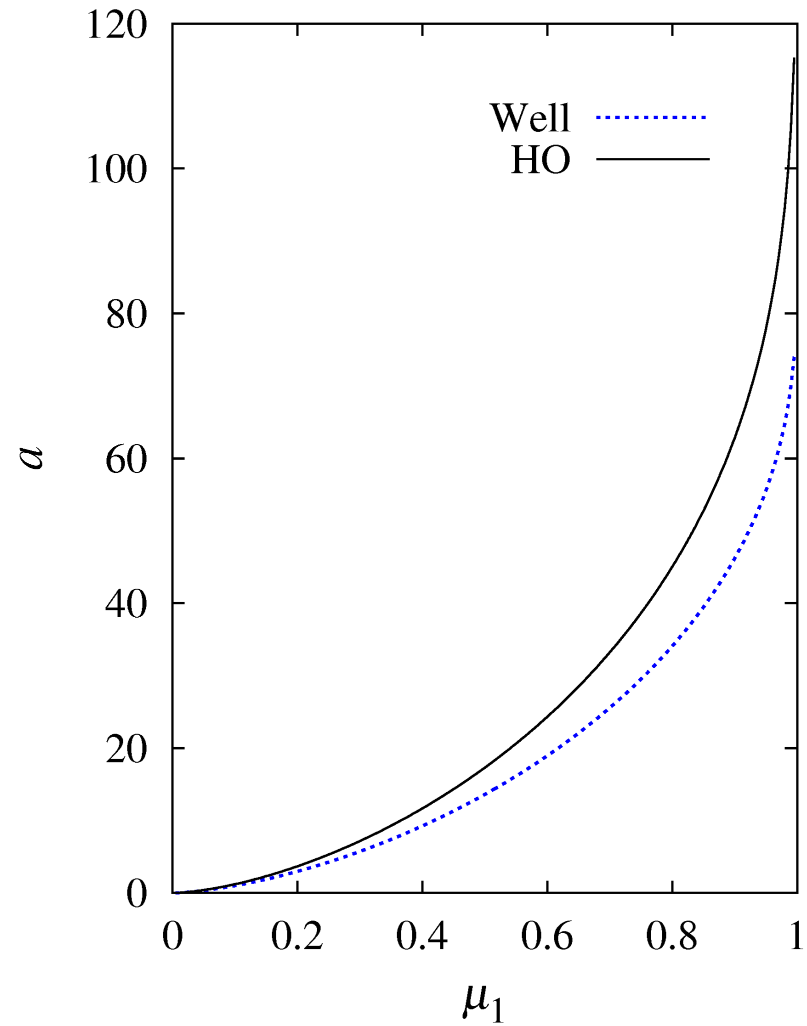
Phase shift

$$e^{2i\delta} = -R_{00}$$

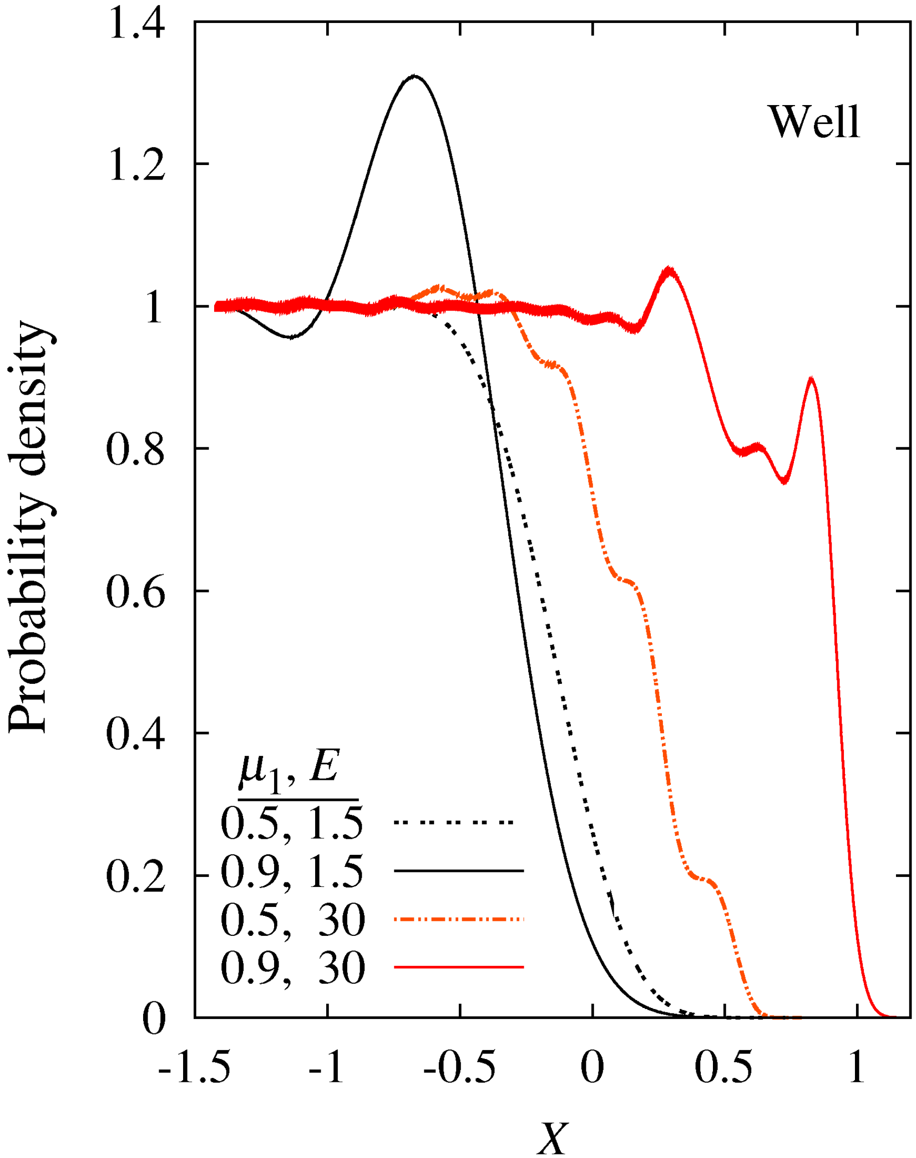


Scattering length

$$\delta = -Ka$$



Center-of-mass penetration probability

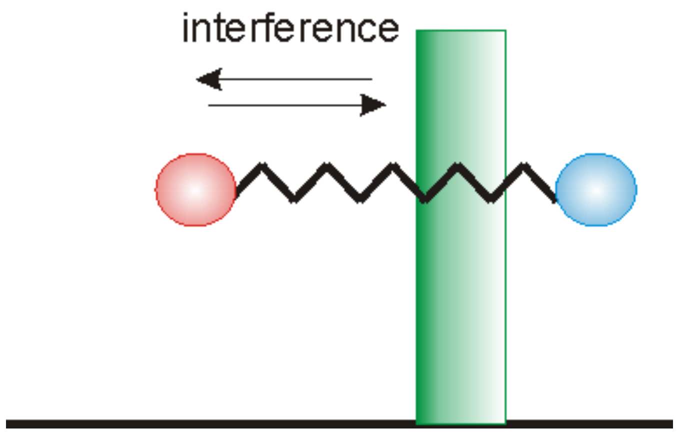


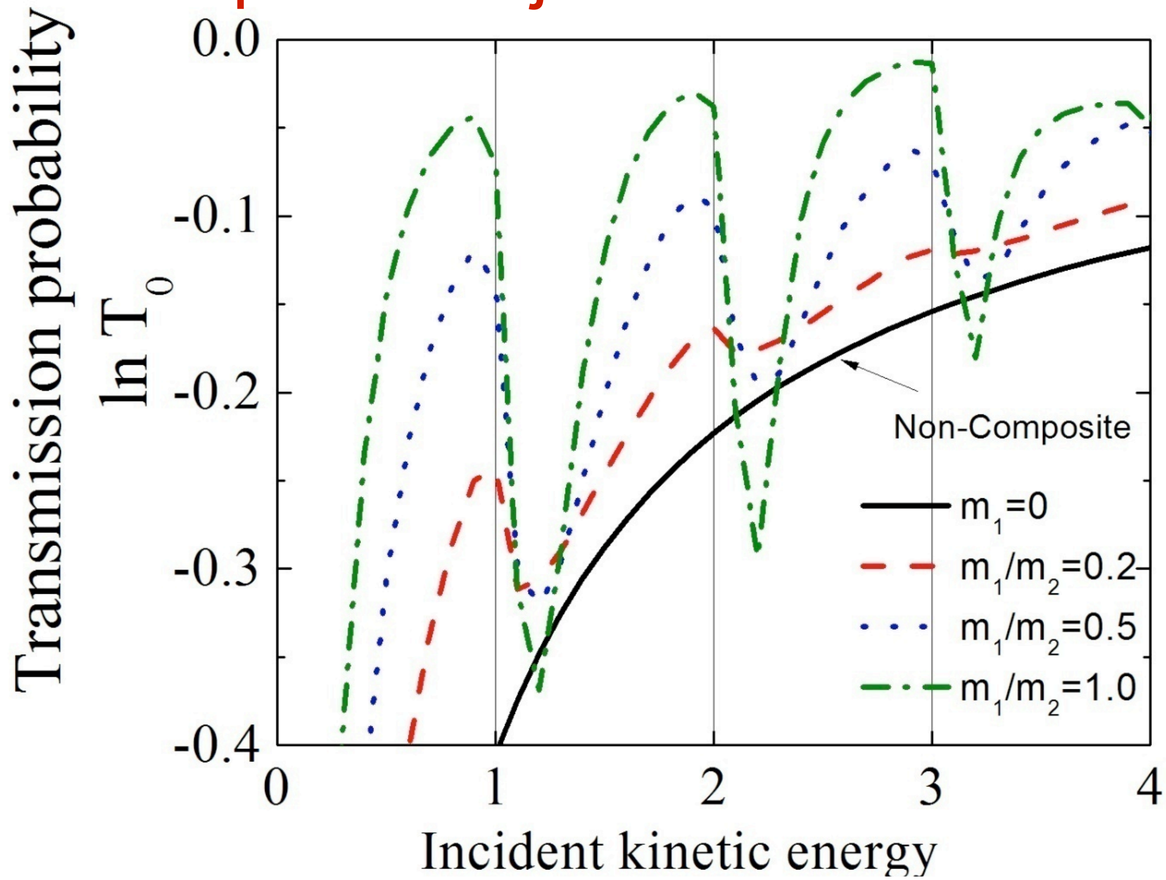
Wall is at X=0

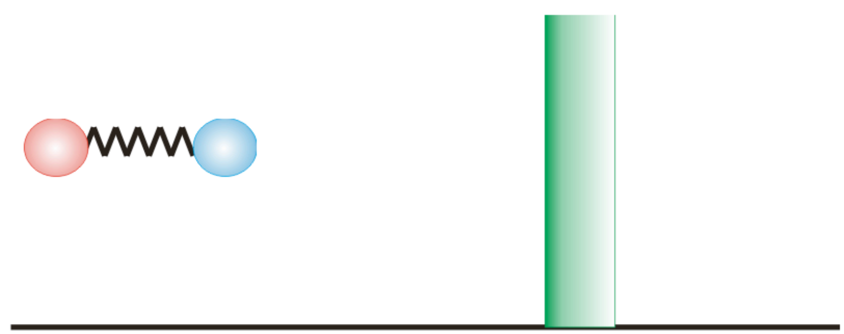
Deep penetration for

- -high energy
- -Massive non interactive particle

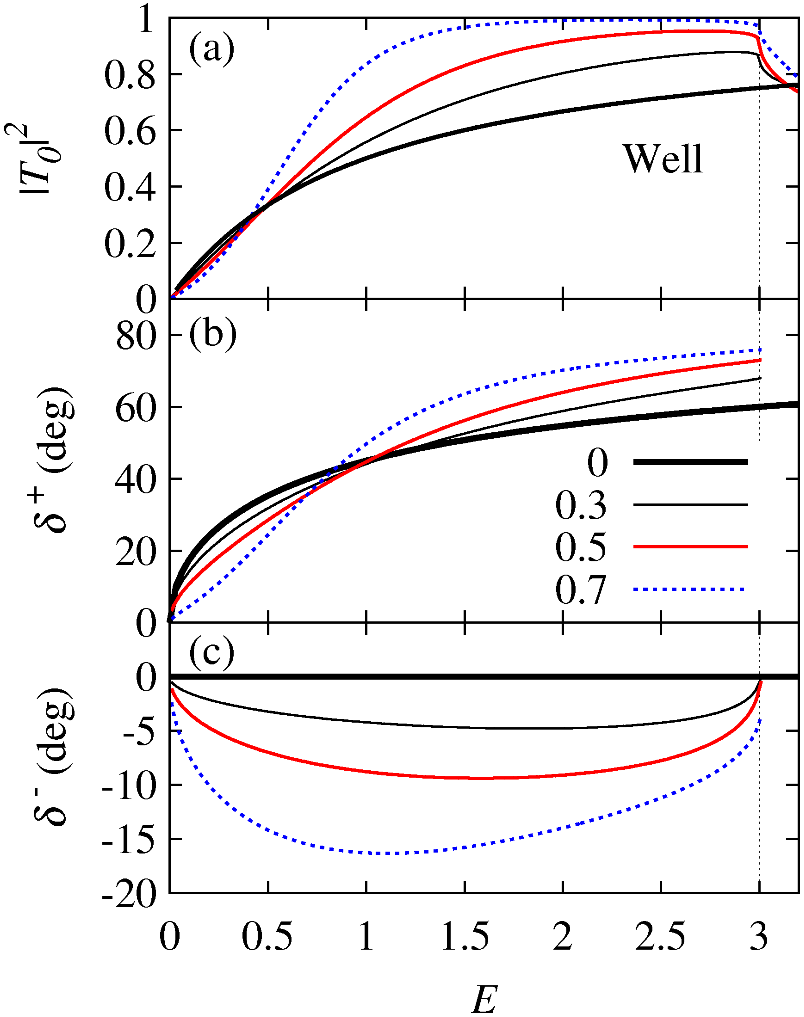
Resonant tunneling of composite objects







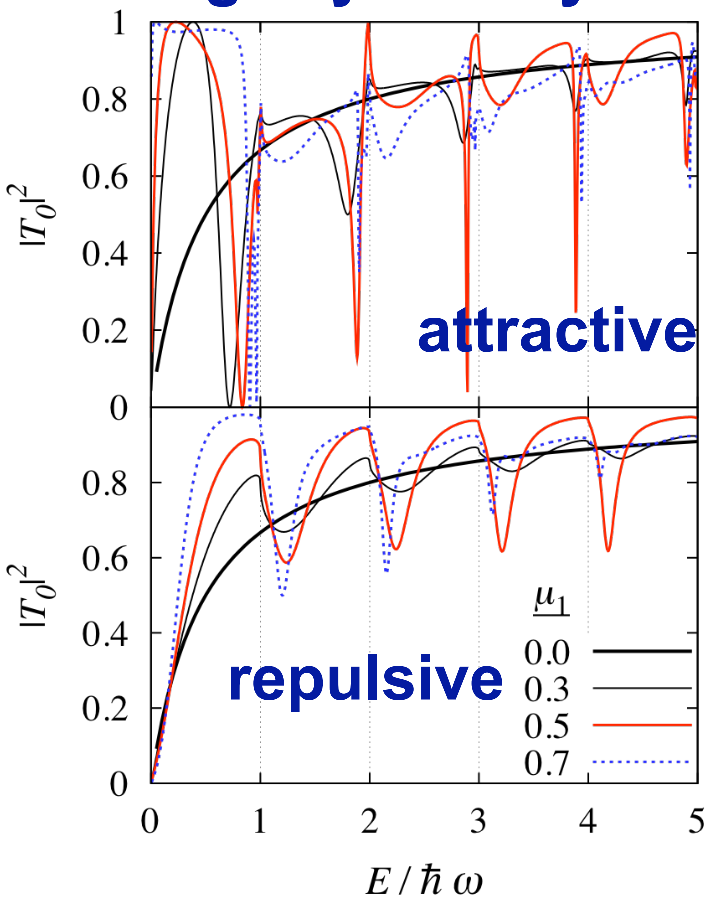
Role of compositeness



$$S^{\pm} = -(R \pm T) = \exp(2i\delta^{\pm})$$

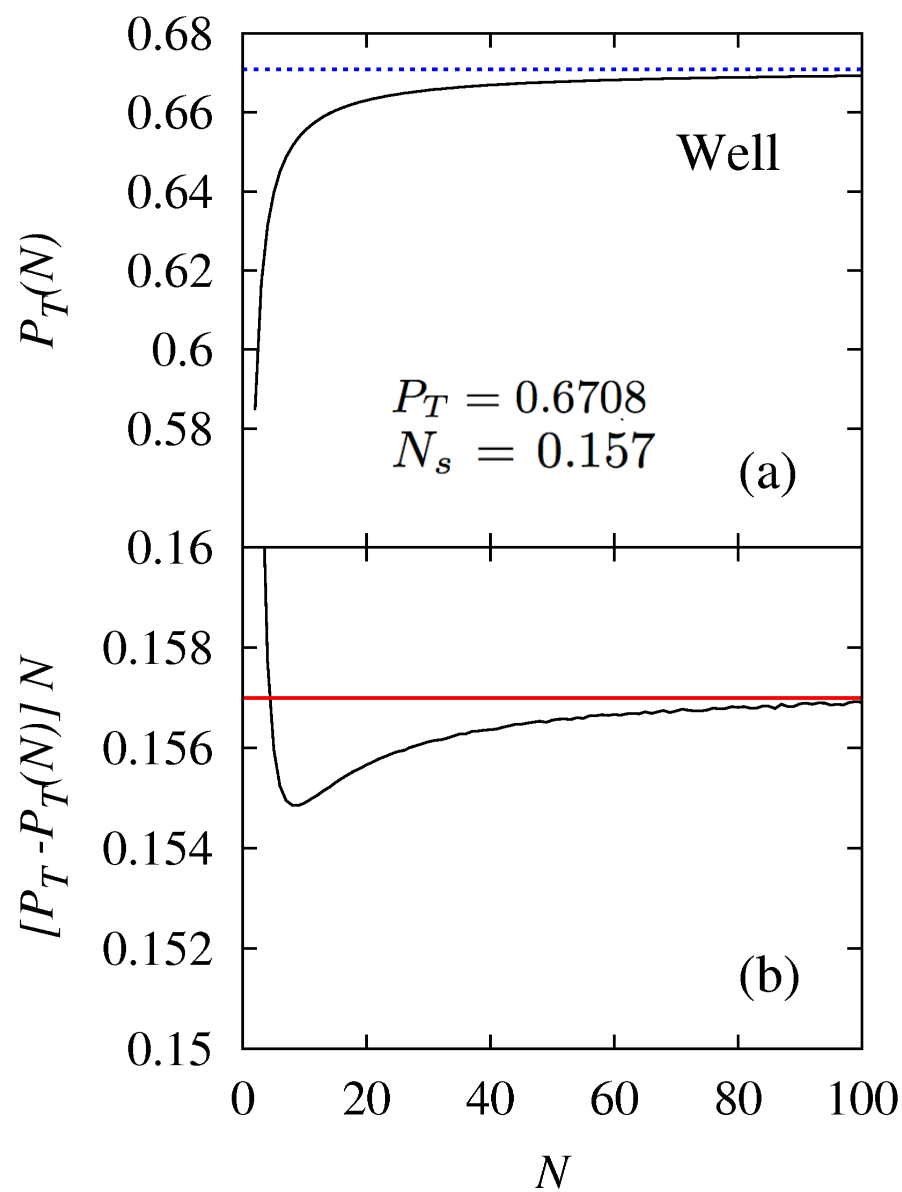
- Cusps at thresholds (unitarity)
- Resonances (enhanced cross section, virtual binding
- Compositeness is important

Charge symmetry



Convergence of VPM

$$P_T = |T_0|^2 \quad P_T(N) = P_T - \frac{N_s}{N}$$

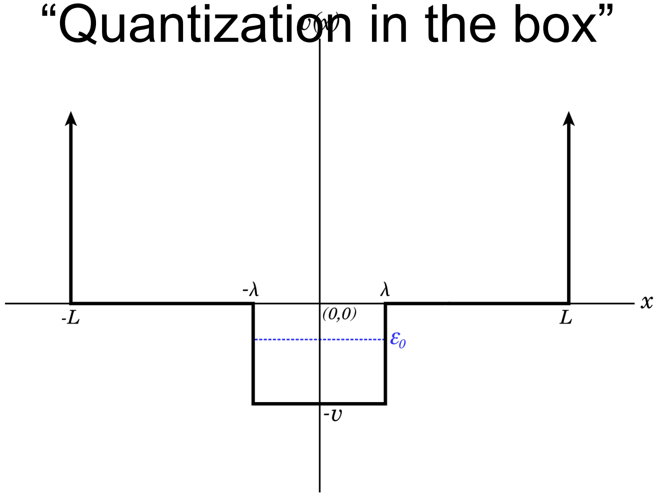


Convergence is power-law $\sim\ 1/|K_N|$

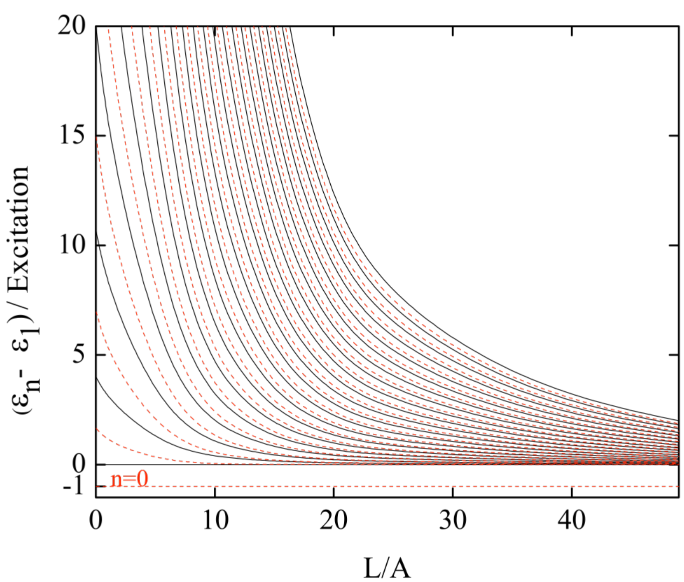
Finite well, breakup and the role of continuum

Intrinsic potential

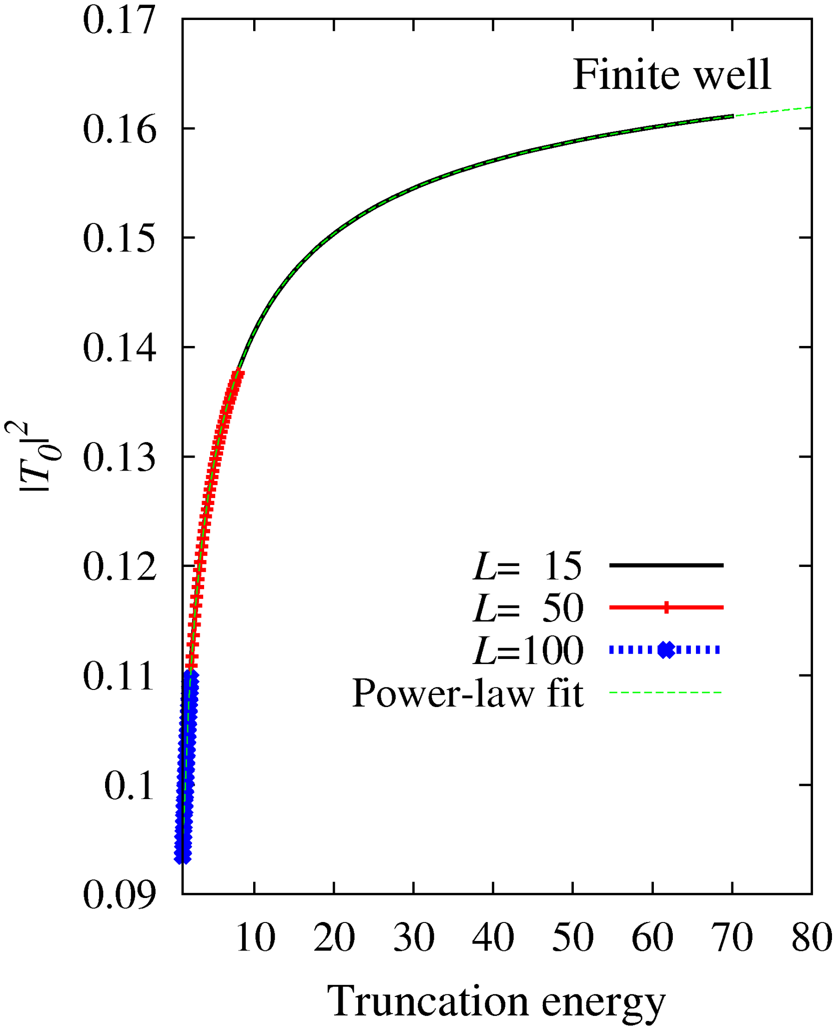
"Quantization in the box"



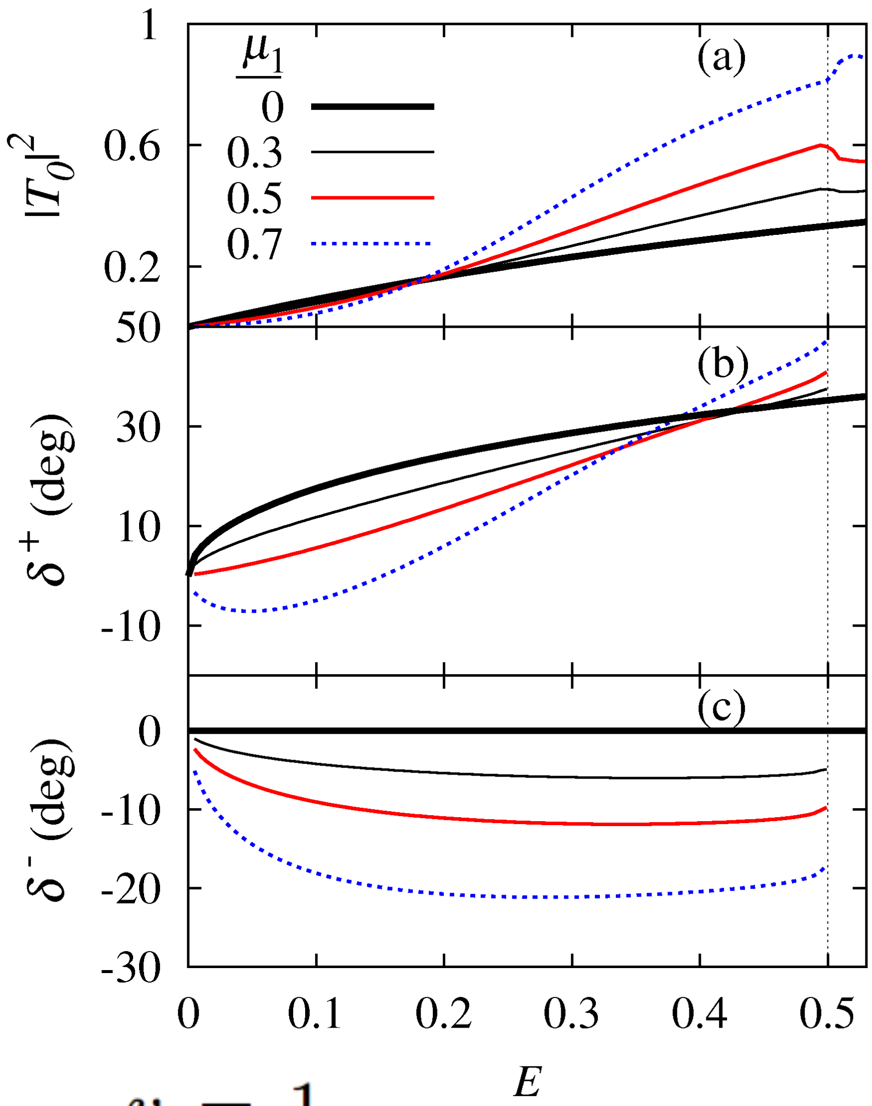
Emergence of "continuum"



Convergence of the method



Low energy

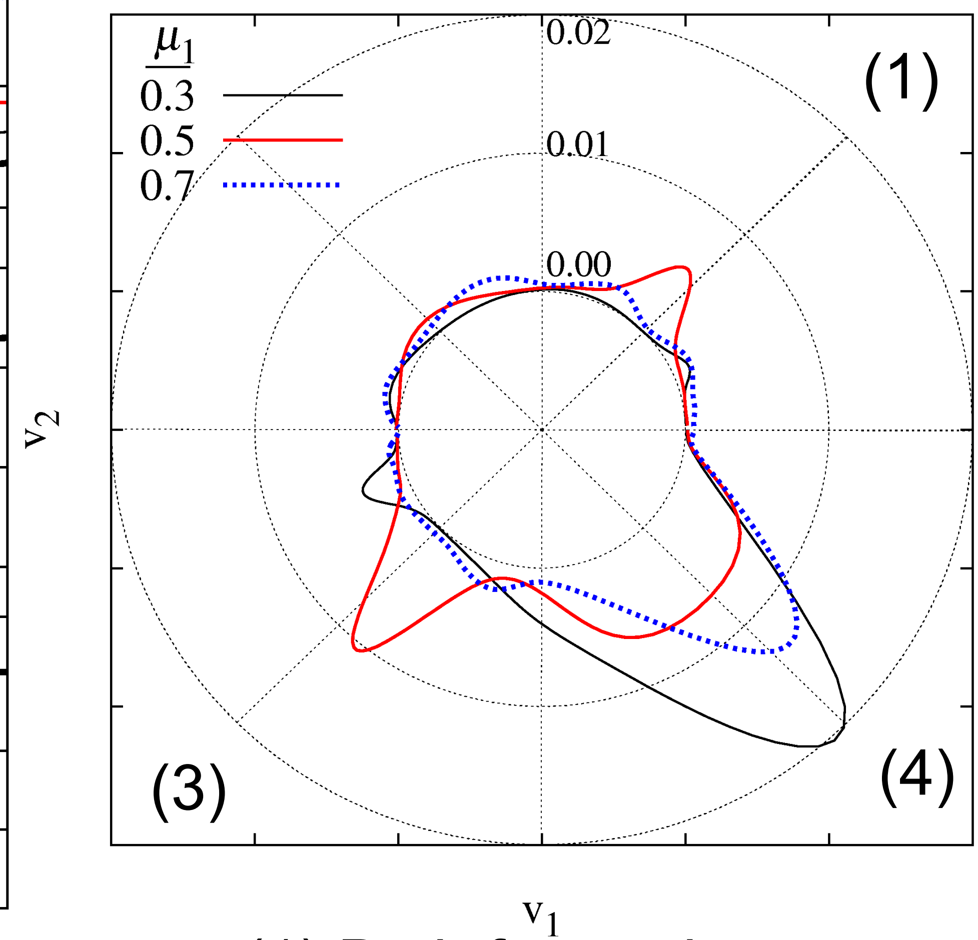


$$v = 1$$

$$\mu_1 = 0.3,\ 0.5,\ \text{and}\ 0.7$$

Bound state at 0.495

Breakup probability distribution

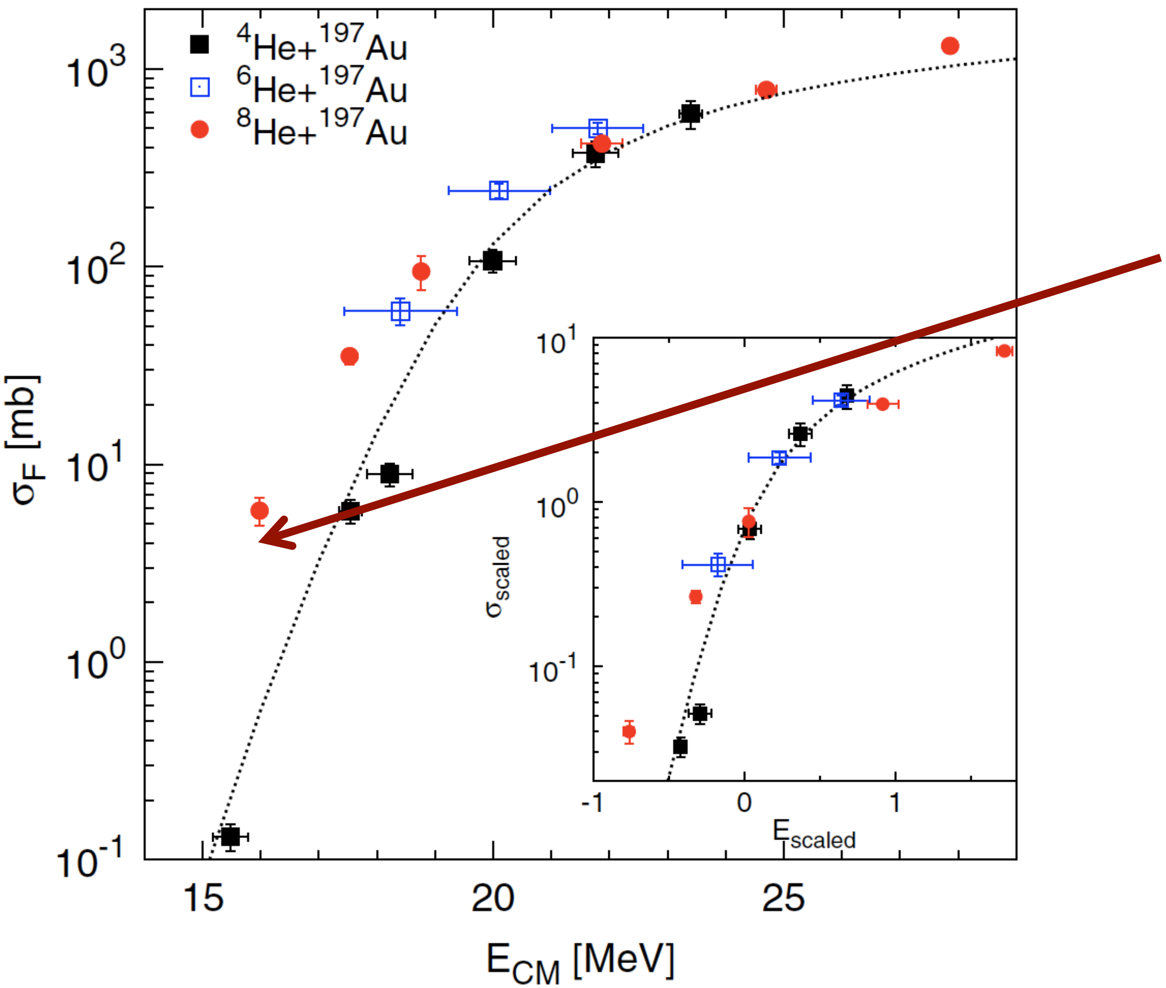


(1) Both forward

(3) Both reflect

(4) Non-interacting forward interacting reflects.

Enhanced tunneling probability for composite objects



Enhancement of tunneling probability.

A. Lemasson, et.al. PRL 103, 232701 (2009)

The "simple" model; lessons learned

Technical aspects

- Projection versus dynamic methods.
- Numerical problems of large and small
- Low rate of convergence (power law)

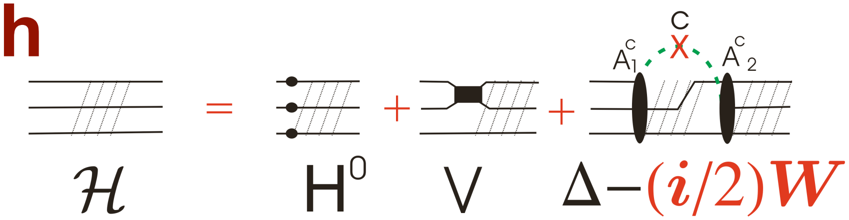
Physics

- Scattering is shaped by virtual channels and (virtual) continuum
- Phenomena: resonances, cusps, infinite scattering length, charge asymmetry
- Realistic cases and models

Reference: N. Ahsan and A Volya, *Quantum tunneling and scattering of a composite object: revisited and reassessed*. arXiv:1010.3973 [nucl-th]
N. Ahsan and A.Volya, in *changing facets of nuclear structure* World Scientific (2008)

Feshbach Projection approach

Intrinsic effective Hamiltonian



Hilbert space is separated into intrinsic P $|1\rangle$) and external Q-subspaces ($|c;E\rangle$)

The Hamiltonian in P is:

$$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

Channel-vector:

$$|A^c(E)\rangle = P_{\mathcal{P}}H|c;E\rangle$$

Self-energy:

$$\Delta(E) = \frac{1}{2\pi}\int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E-E'}$$

Irreversible decay to the excluded space:

$$W(E) = \sum_{c(\mathrm{open})} |A^c(E)\rangle\langle A^c(E)|$$

Scattering Matrix and Reactions

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left(\frac{1}{E-\mathcal{H}(E)}\right) |A^{c'}(E)\rangle$$

Cross section:

$$\sigma = \frac{\pi}{k'^2}\sum_{cc'} \frac{(2J+1)}{(2s'+1)(2I'+1)} |\mathbf{T}_{cc'}|^2$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*,
North-Holland Publishing, Amsterdam 1969

Time-dependent continuum shell model

A. Volya, Phys. Rev. C 79, 044308 (2009).

Approach

- Feshbach projection formulation
- Time-dependent propagator
- Dyson’s equation

Features

- Numerical stability
- Exact unitarity
- Complex structure-reaction components
- Practical applications

Time dependent propagator

$$G(E) = \frac{1}{E - H} = -i \int_0^{\infty} dt \; \exp(iEt)\exp(-iHt)$$

- Scale Hamiltonian so that eigenvalues are in [-1 1]
- Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2 - \delta_{n0}) J_n(t) T_n(H)$$

- Chebyshev polynomial $T_n[\cos(\theta)] = \cos(n\theta)$
- Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

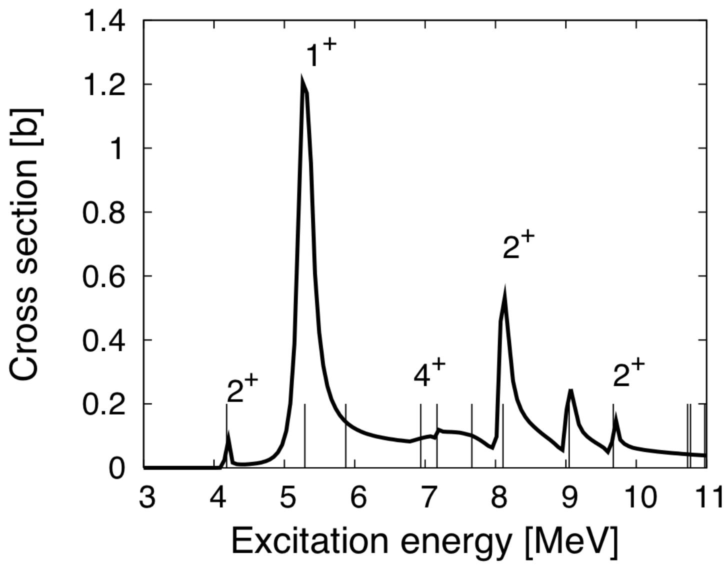
$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle \quad |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

$$\langle\lambda'|T_{n+m}(H)|\lambda\rangle = 2\langle\lambda'_m|\lambda_n\rangle - \langle\lambda'|\lambda_{n-m}\rangle, \quad n \geqslant m$$

- Use FFT to find return to energy representation

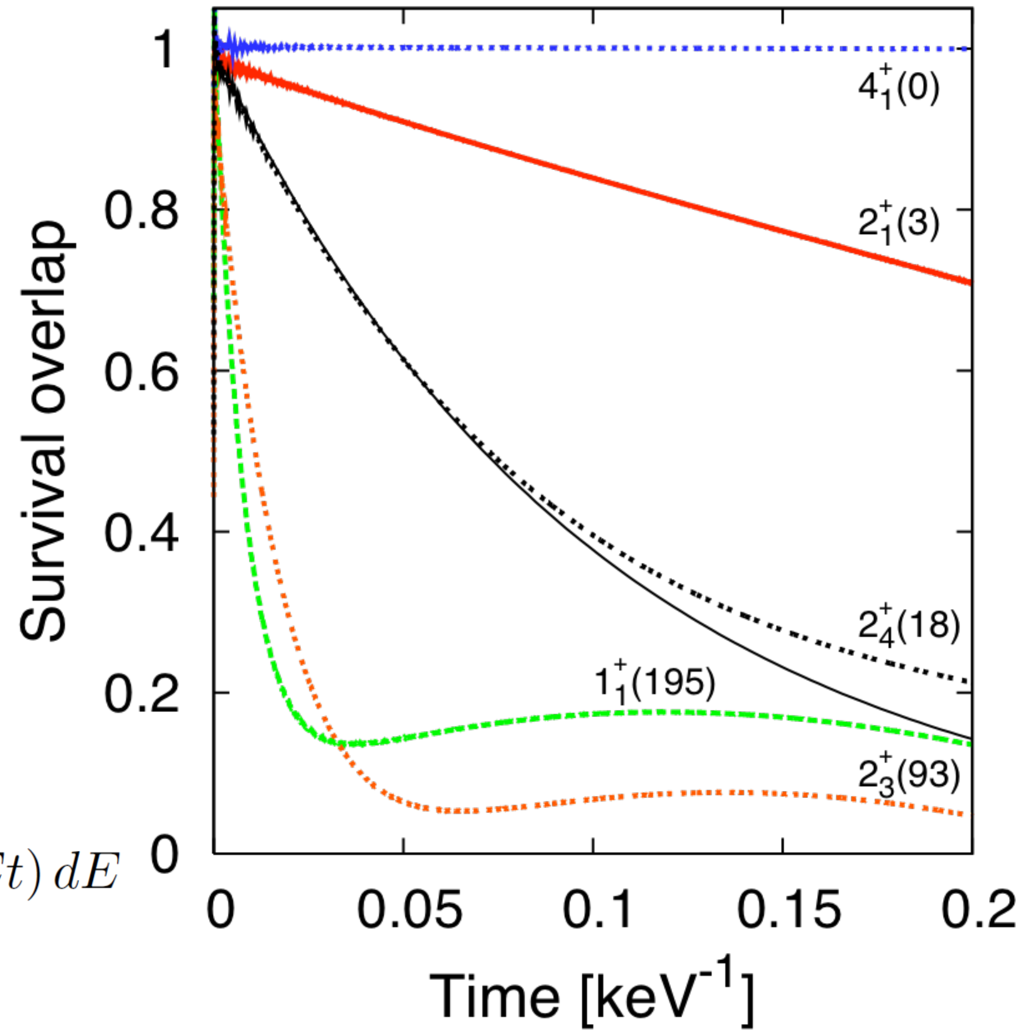
T. Ikegami and S. Iwata, J. of Comp. Chem. **23** (2002) 310-318

Time evolution of decaying states



Time evolution of several SM states in ^{24}O. The absolute value of the survival overlap is shown $|\langle\alpha|\mathcal{U}(t)|\alpha\rangle|$

$$\mathcal{U}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \mathcal{G}(E)\ \exp\left(-iEt\right) dE$$



For an isolated narrow resonance

$$|\langle\alpha|\exp(-i\mathcal{E}_\alpha t)|\alpha\rangle| = \exp(-\Gamma_\alpha t/2)$$

Unitarity and flux conservation

Take: $\mathbf{W} = \mathbf{a}\mathbf{a}^\dagger$

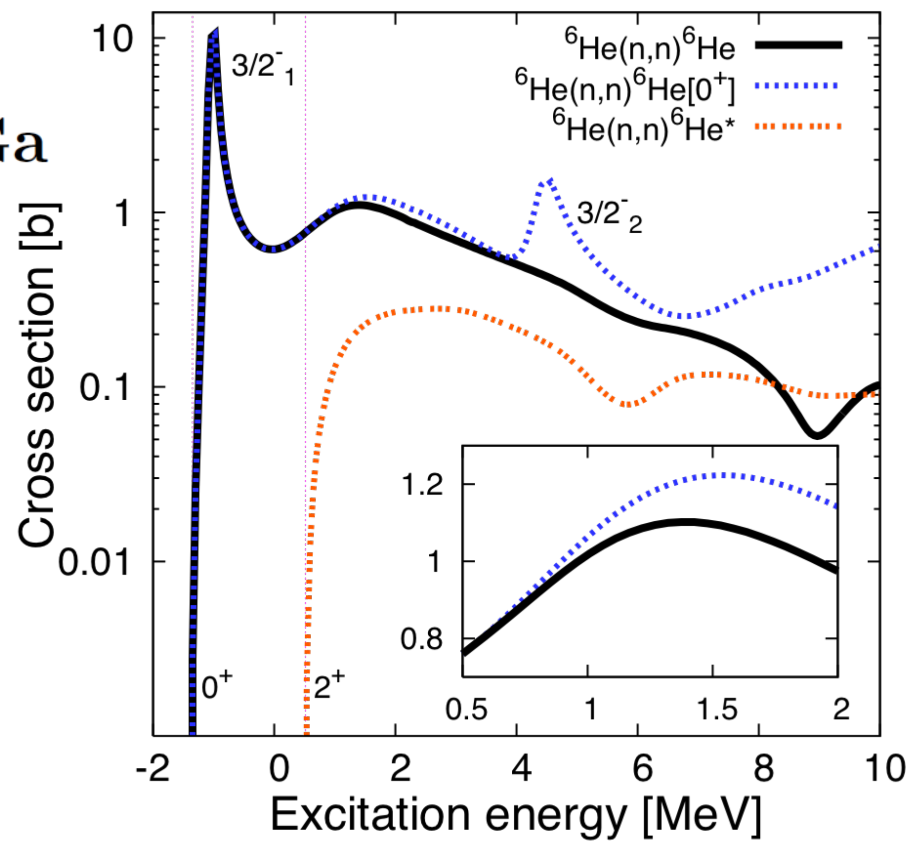
Exact relation:

$$\mathbf{S} = \frac{\mathbf{1} - i/2\,\mathbf{K}}{\mathbf{1} + i/2\,\mathbf{K}} \qquad \mathbf{K} = \mathbf{a}^\dagger \mathbf{G} \mathbf{a}$$

$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{S}^\dagger \mathbf{S} = \mathbf{1}$$

- Cross section has a cusp when inelastic channels open
- The cross section is reduced due to loss of flux
- The p-wave discontinuity $E^{3/2}$

Figure: ^{6}He(n,n) cross section
- Solid curve-full cross section
- Dashed (blue) only g.s. channel
- Dotted (red) inelastic channel



The role of self-energy

Energy-dependent contribution from virtual excitation to continuum, the self-energy.

Figure: $^{23}O(n,n)^{23}O$ Effect of self-energy term (red curve). Shaded areas show experimental values with uncertainties.

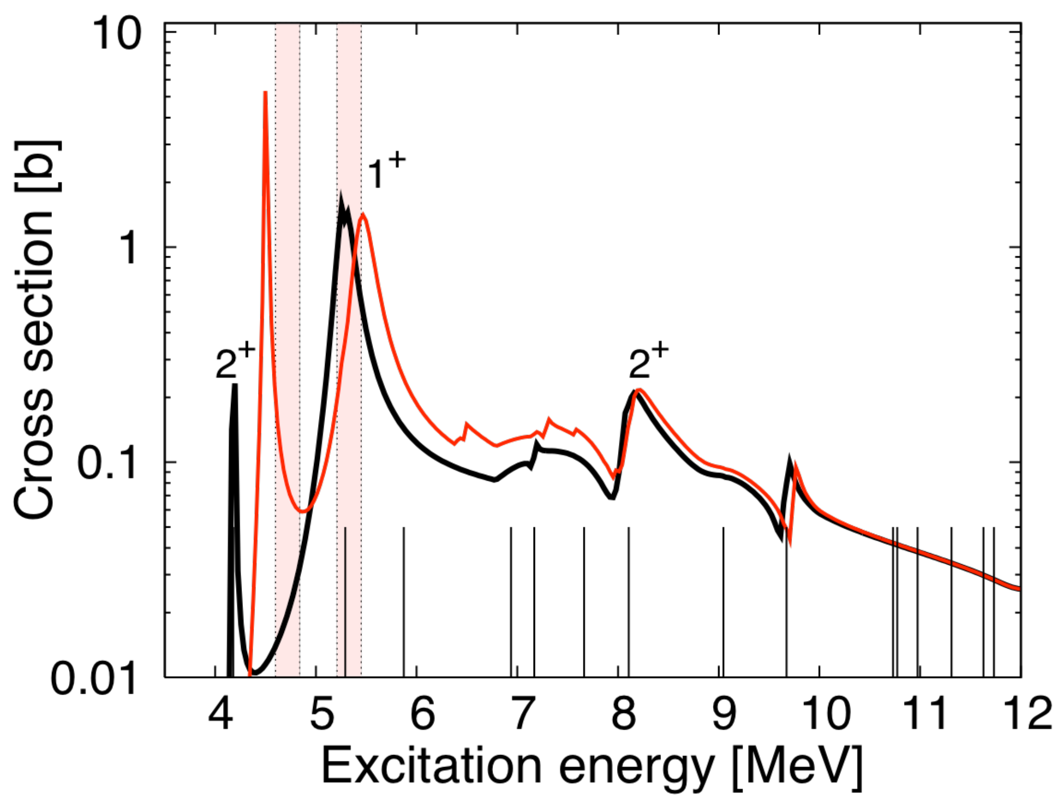
In channel space

$$\mathbf{\Delta}_{cc'} = \delta_{cc'}\Delta_c(E)$$

$$\Delta_c(E) = \frac{1}{2\pi} \fint dE' \frac{|a^c(E')|^2}{E - E'}$$

Near-threshold form

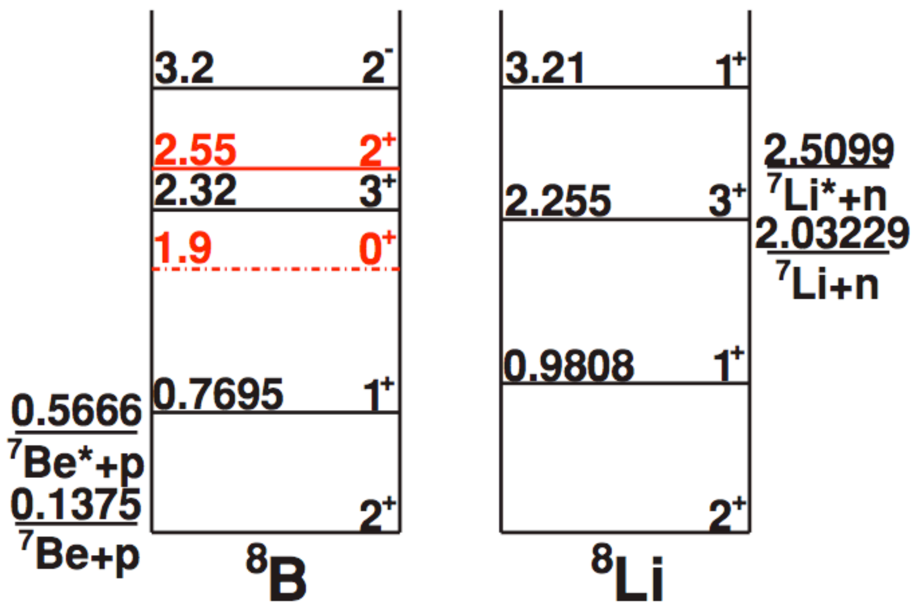
$$\Delta_c(\epsilon) = \frac{\varkappa^2}{2}\Theta(-\epsilon)\epsilon^l\sqrt{-\epsilon}$$

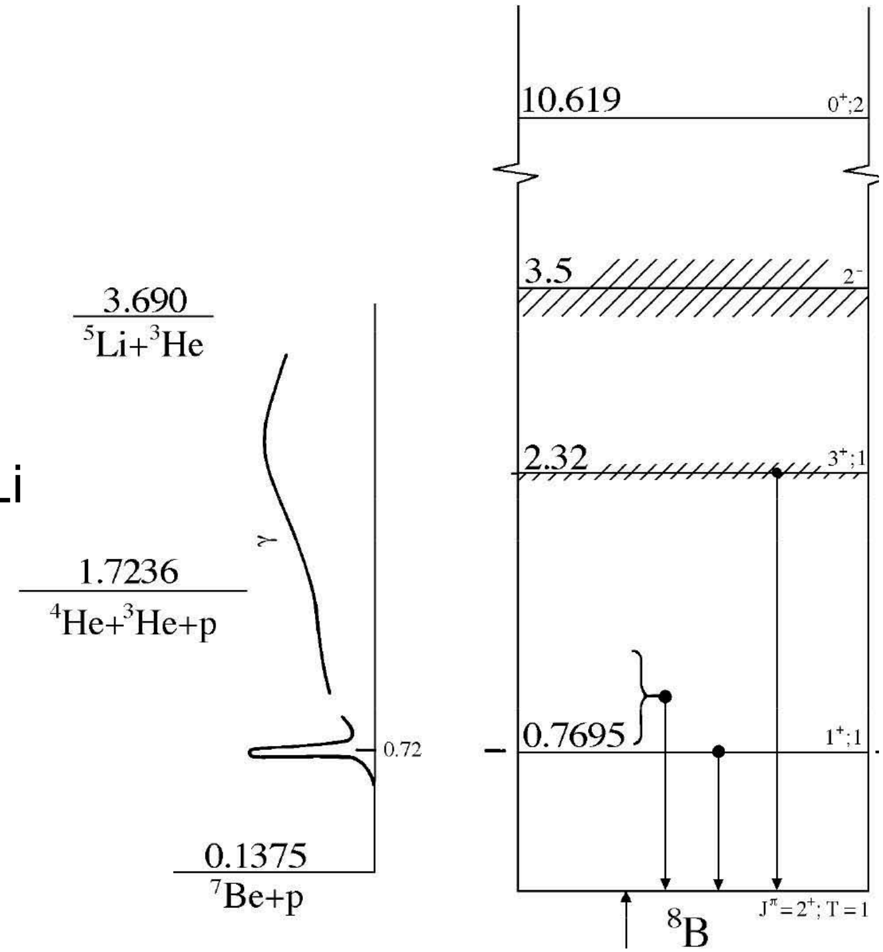


Experimental data from:
C. Hoffman, et.al. Phys. Lett. **B672**, 17 (2009)

States in ^8B

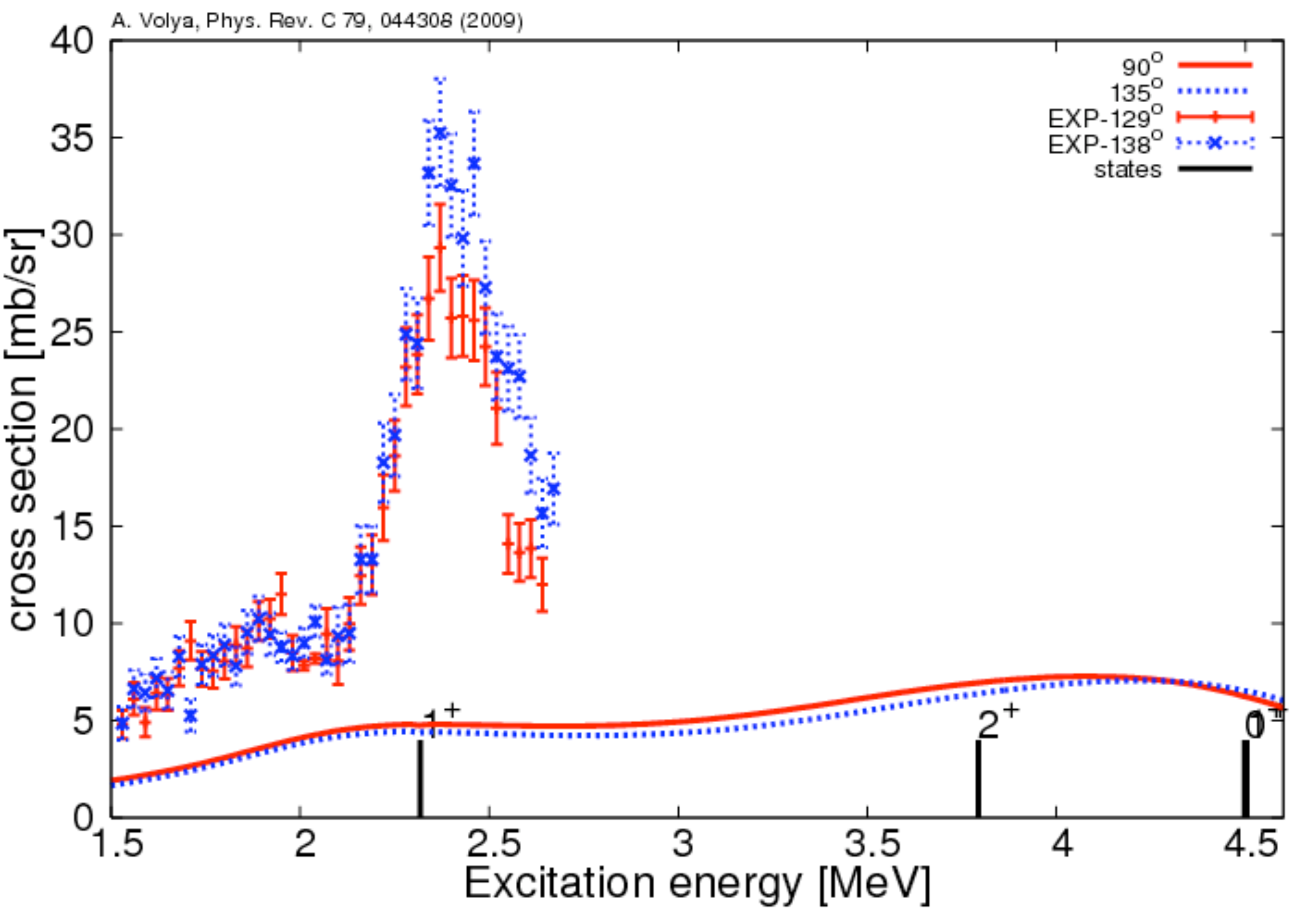
- Ab-initio and no core theoretical models predict low-lying 2^+, 0^+, and 1^+ states
- Recoil-Corrected CSM suggests low-lying states
- Traditional SM mixed results
- These states were not seen in ^8B and in ^8Li



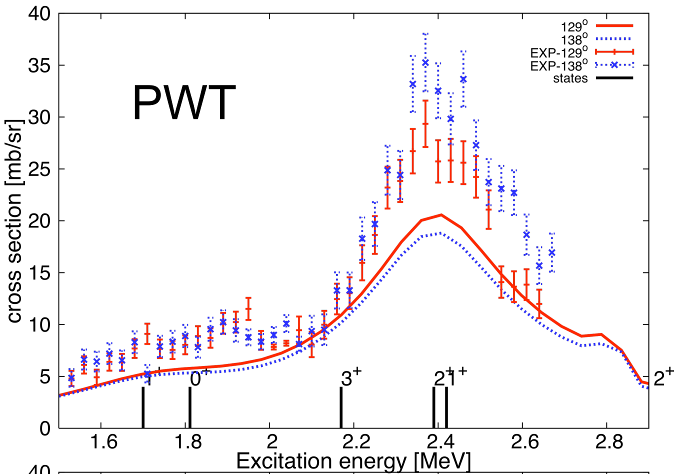


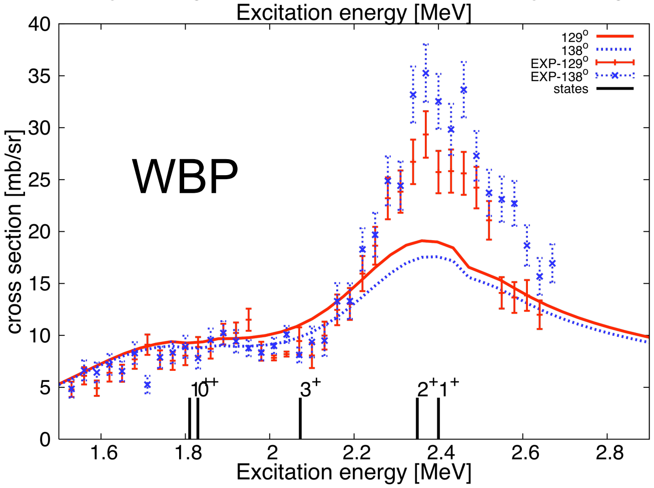
Resonances and their positions inelastic $^7Be(p,p')^7Be$ reaction in TDCSM

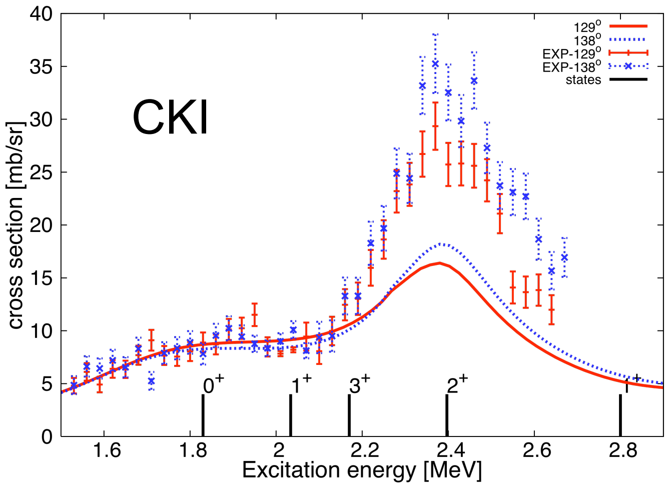
CKI+WS Hamiltonian



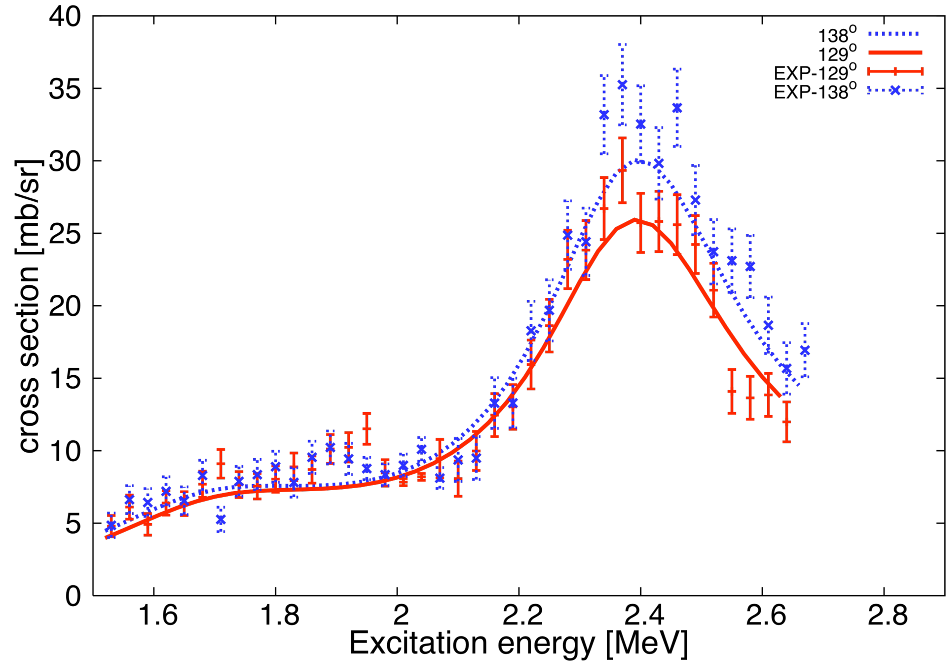
See animation at www.volya.net







R-matrix fit and TDCSM for $^7Be(p,p)^7Be$



Channel Amplitudes from TDCSM and final best fit

	J^π	$p_{1/2}$, I=3/2	$p_{3/2}$, I=3/2	$p_{1/2}$, I=1/2	$p_{3/2}$, I=1/2
FIT	2^+	-0.293	0.293		0.534
CKI	2^+	-0.168	0.164		0.521
FIT	1^+	-0.821	-0.612	0.375	0.175
CKI	1^+	-0.840	-0.617	0.332	0.178

Acknowledgements:

Thanks to: N. Ahsan, G. Rogachev, D. Robson, C. Hoffman, S. Tabor, I. Wiedenhöver, V. Zelevinsky
Funding support: Department of Energy DE-FG02-92ER40750

Recent publication:

•A. Volya, Phys. Rev. C 79, 044308 (2009).
•N. Ahsan and A Volya, *Quantum tunneling and scattering of a composite object: revisited and reassessed*. arXiv:1010.3973 [nucl-th]
•J. Mitchell et. al. Phys. Rev. C **82,** 011601 (2010)