Microscopic (TDHF and DC-TDHF) study of heavy-ion fusion and capture reactions with neutron-rich nuclei

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Interfaces between structure and reactions
for rare isotopes and nuclear astrophysics
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Our main goal: microscopic description of fusion reactions involving exotic neutron-rich nuclei

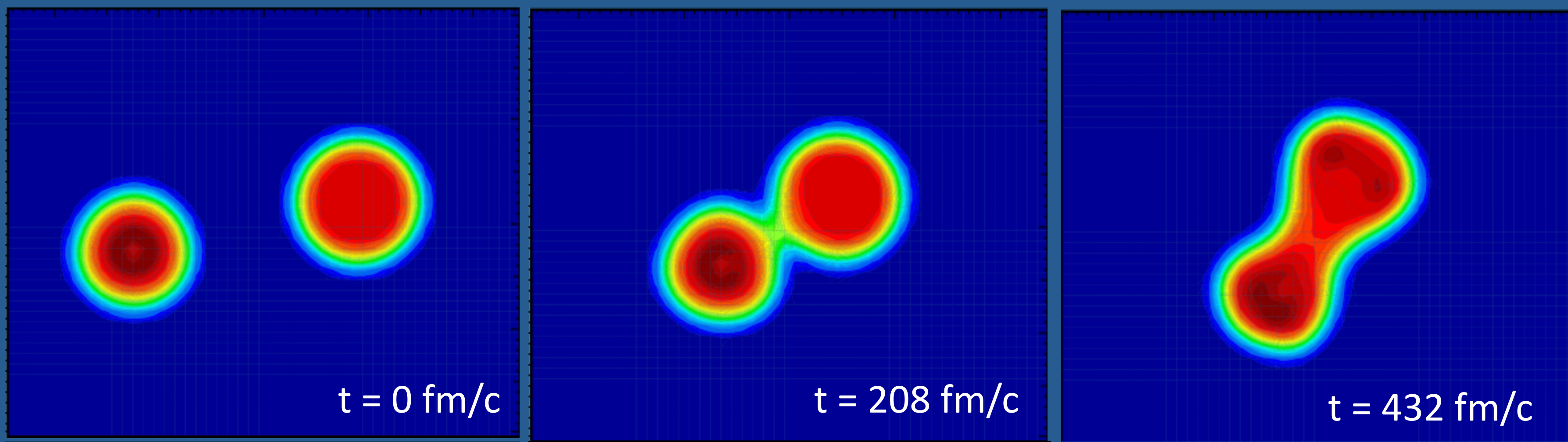
1. Unrestricted TDHF calculations above the fusion barrier
   deep-inelastic and capture / fusion reactions

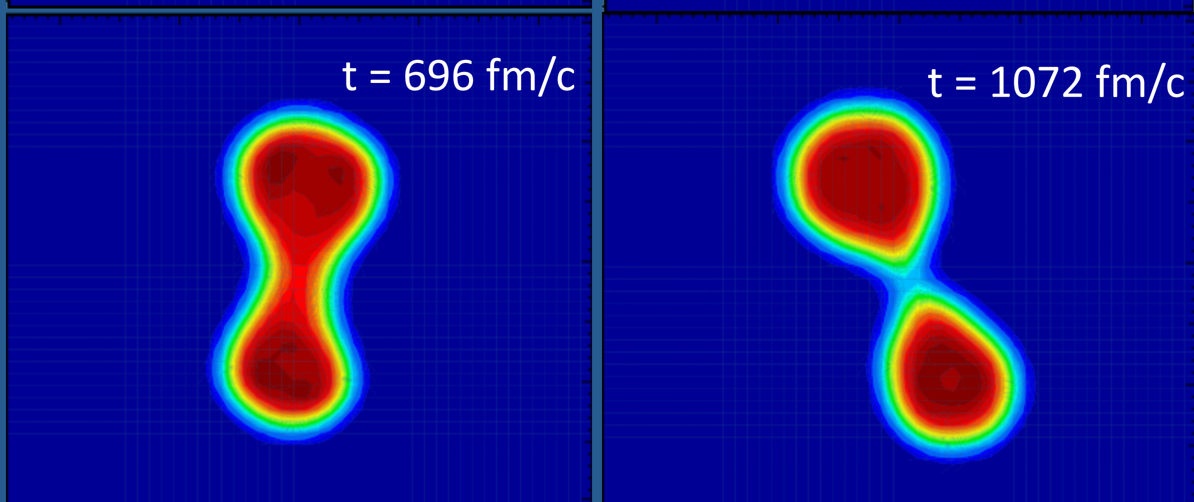
2. Density-constrained TDHF calculations:
   microscopic calculation of heavy-ion potentials
   calculate fusion / capture cross sections
   below and above the barrier

3. Dynamic microscopic study of excitation energy E*(t)

$^{96}Zr + ^{132}Sn$, E_{cm} = 250 MeV, b = 4 fm (deep-inelastic)

TDHF, SLy4 interaction, 3-D lattice (50*42*30 points)





Elapsed time (contact to disintegration) = 984 fm/c
= $3.3 * 10^{-21}$ s

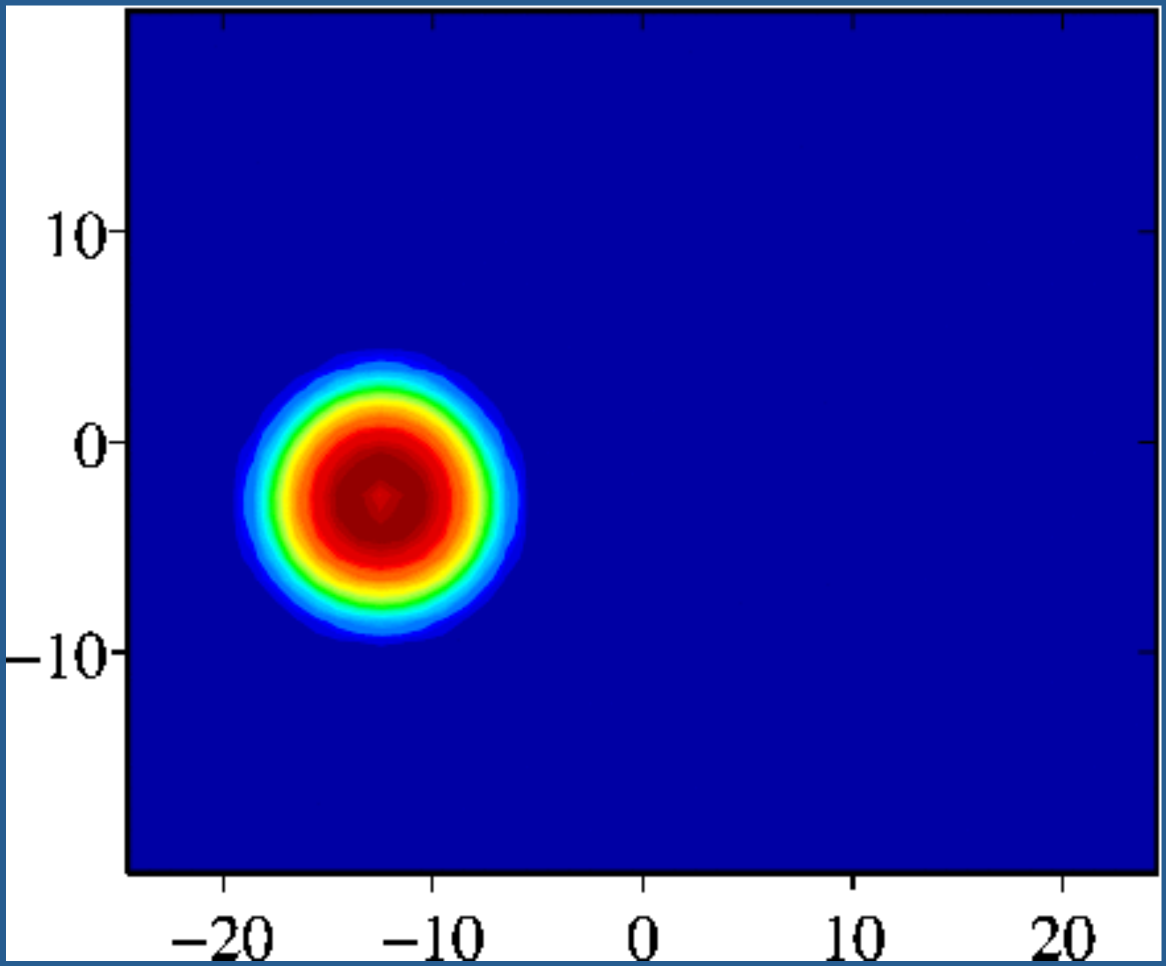
Masses and charges after reaction:

A_1 = 139.5, Z_1 = 54.5

A_2 = 88.5, Z_2 = 35.5

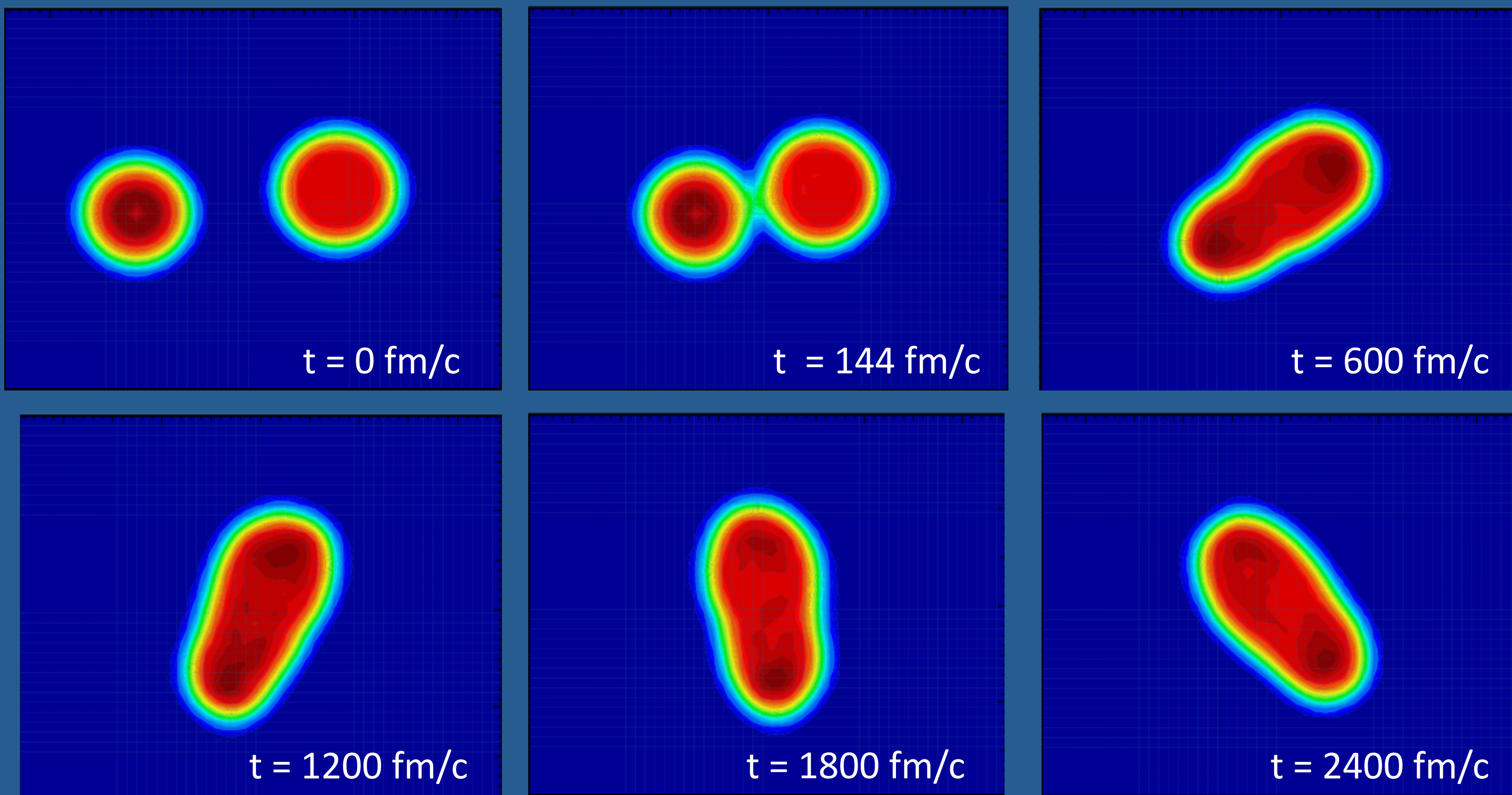
Animation: $^{96}Zr + ^{132}Sn$, E_{cm} =250 MeV, b = 4 fm

TDHF, SLy4 interaction, 3-D lattice (50*40*30 points)



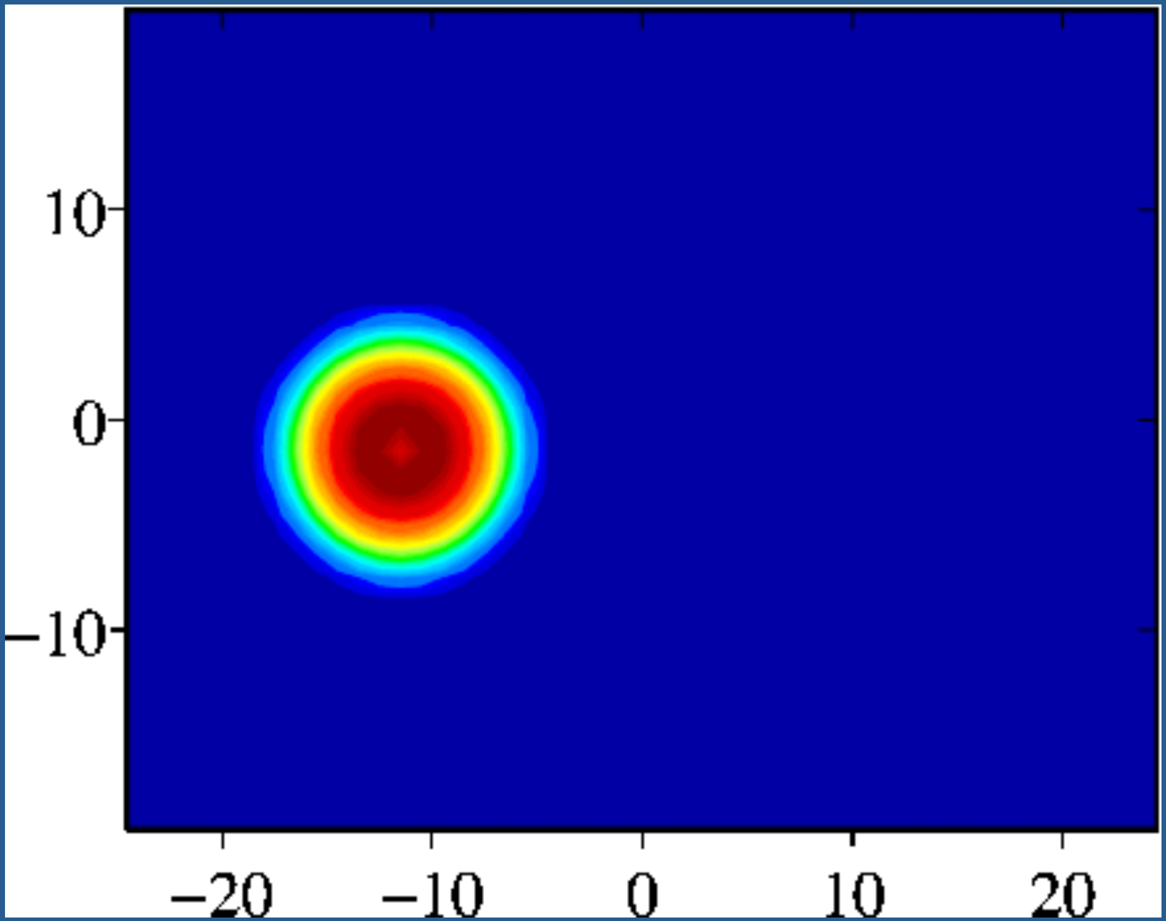
^{96}Zr + ^{132}Sn, E_{cm} = 250 MeV, b = 2 fm (capture)

TDHF, SLy4 interaction, 3-D lattice (50*40*30 points)

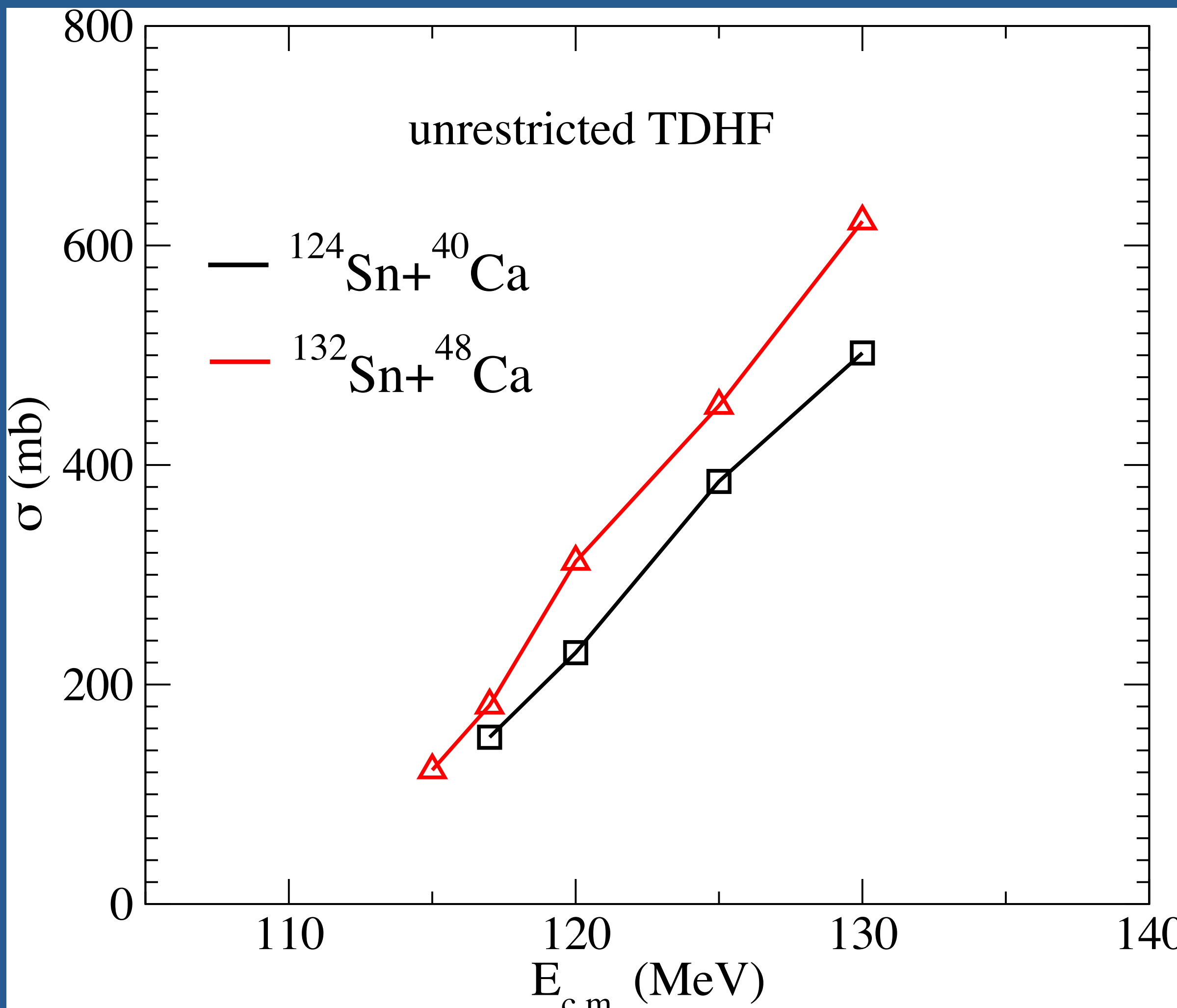


Animation: $^{96}Zr + ^{132}Sn$, E_{cm} =250 MeV, b = 2 fm

TDHF, SLy4 interaction, 3-D lattice (50*40*30 points)



Total cross sections for capture (unrestricted TDHF)



sharp cutoff model

$$\sigma_{capt} = \pi b_{max}^2$$

2. Density-constrained TDHF calculations

- microscopic calculation of heavy-ion potentials
- use with barrier tunneling (Incoming Wave Boundary Condition)
- calculate fusion (capture) cross sections below and above the barrier
- results for ^{132,124}Sn + ^{96}Zr (HRIBF exp.) and for ^{132,124}Sn + ^{48,40}Ca (recent HRIBF exp.)

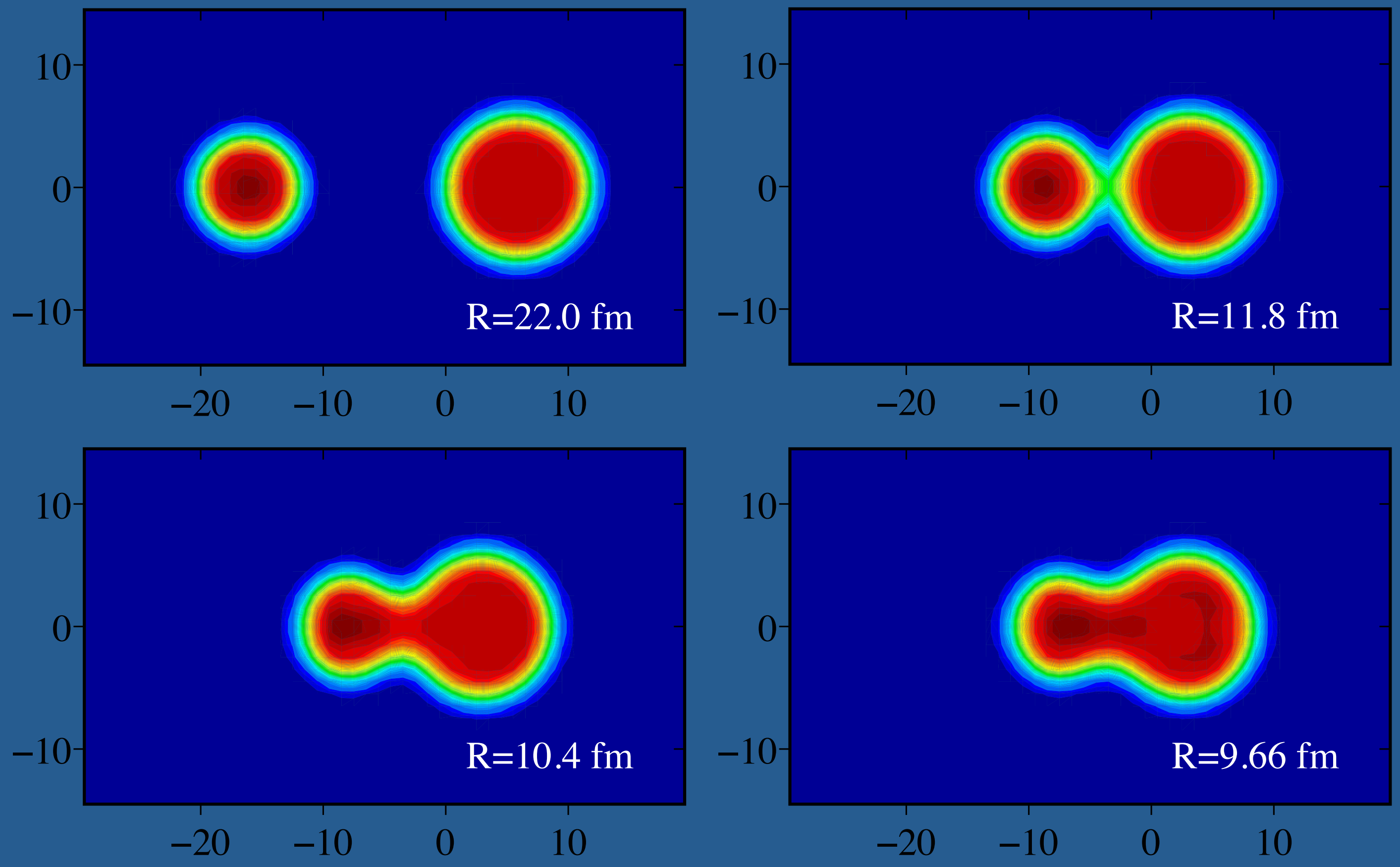
Previous work

Umar & Oberacker, PRC 74, 021601(R) (2006) PRC 74, 061601 (R) (2006)
PRC 76, 014614 (2007) PRC 77, 064605 (2008)
EPJA 39, 243 (2009)
Umar, Maruhn, Itagaki & Oberacker, PRL 104, 212503 (2010)

Calculation of heavy-ion potential with DC-TDHF method

$$V(R) = E_{\mathrm{DC}}(R) - E_{\mathrm{A}_1} - E_{\mathrm{A}_2}$$

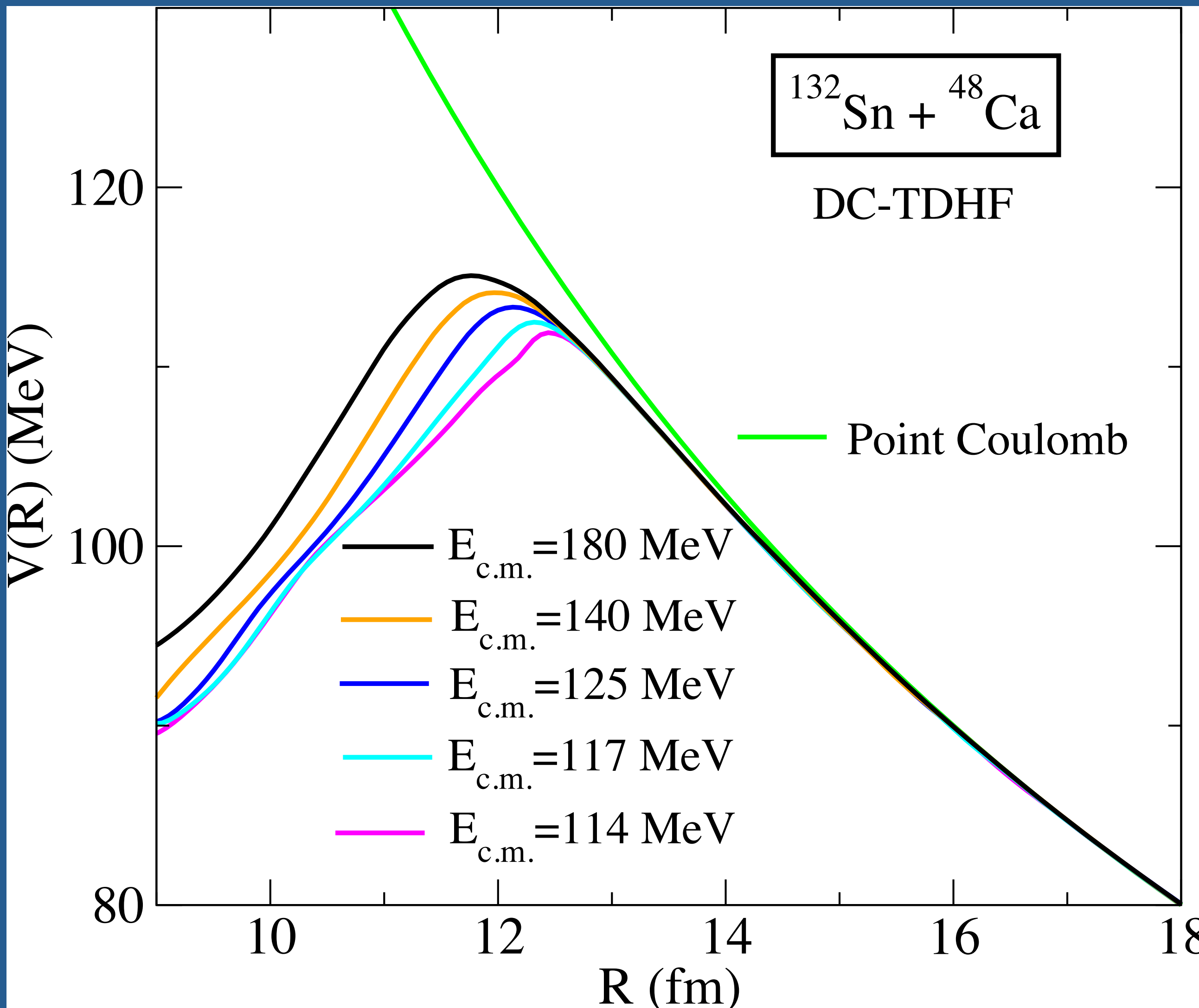
Umar and Oberacker, PRC 74, 021601(R) (2006)



$^{48}Ca + ^{132}Sn$
E_{cm}=140 MeV

V(R) contains dynamical entrance channel effects (neck formation, particle transfer, surface vibrations, giant resonances)

heavy-ion potential for $^{132}Sn+^{48}Ca$: strong E_{cm}-dependence



Dynamical Effective Mass from TDHF

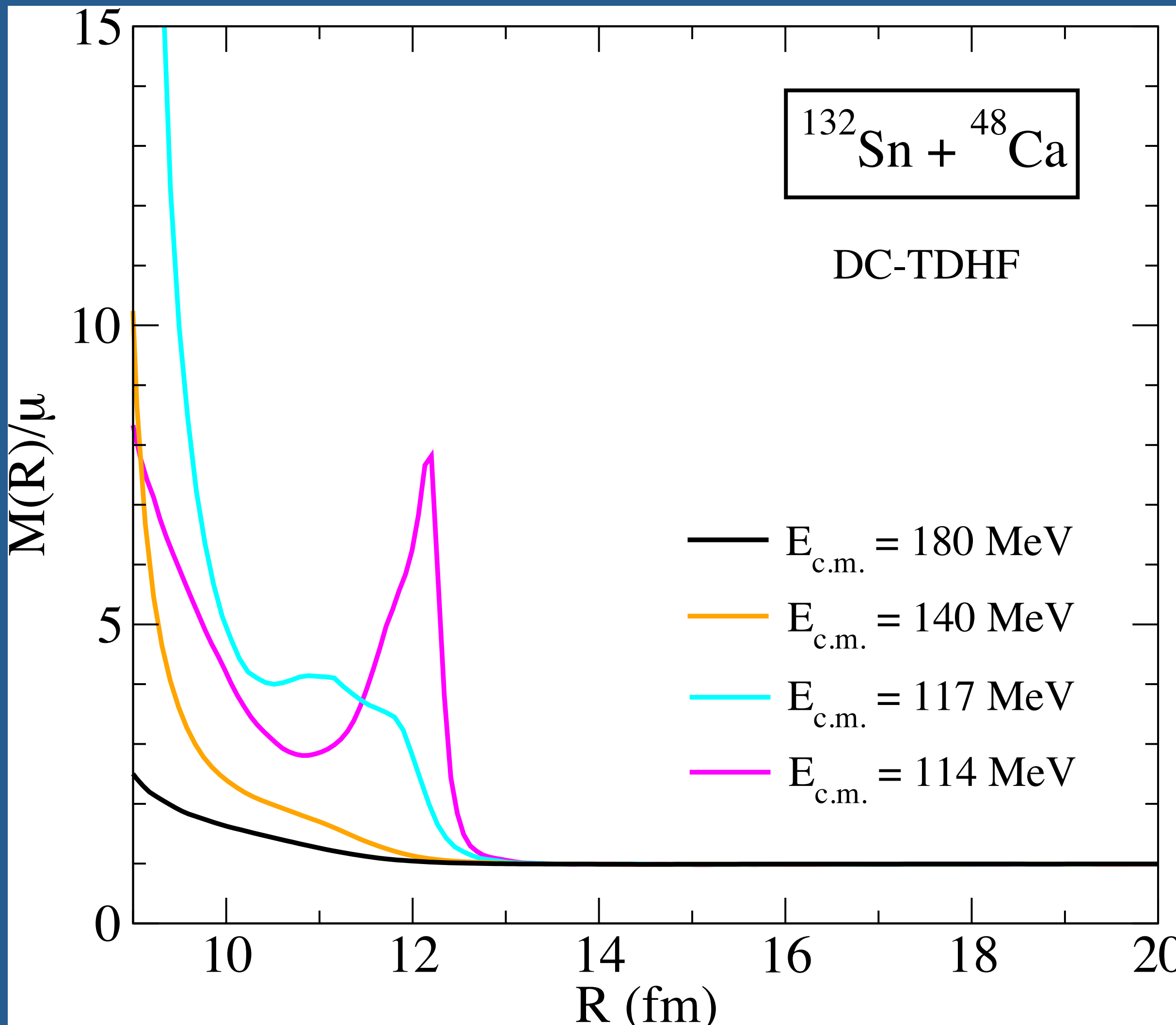
$$E_{\mathrm{c.m.}} = \frac{1}{2} M(R)\dot{R}^2 + V(R)$$

TDHF

DC-TDHF

$$M(R) = \frac{2\left[E_{\mathrm{c.m.}} - V(R)\right]}{\dot{R}^2}$$

mass parameter for $^{132}Sn+^{48}Ca$: strong E_{cm}-dependence



Formalism for coordinate-dependent mass: point transformation to constant mass

$$H(R,P) = \frac{P^2}{2B(R)} + V(R)$$

original Hamiltonian

$$d\bar{R} = \left(\frac{B(R)}{\mu}\right)^{\frac{1}{2}} dR$$

point transformation to new coordinate with constant reduced mass

$$\bar{R} = f(R) \quad \Longleftrightarrow \quad R = f^{-1}(\bar{R})$$

integrate last equation

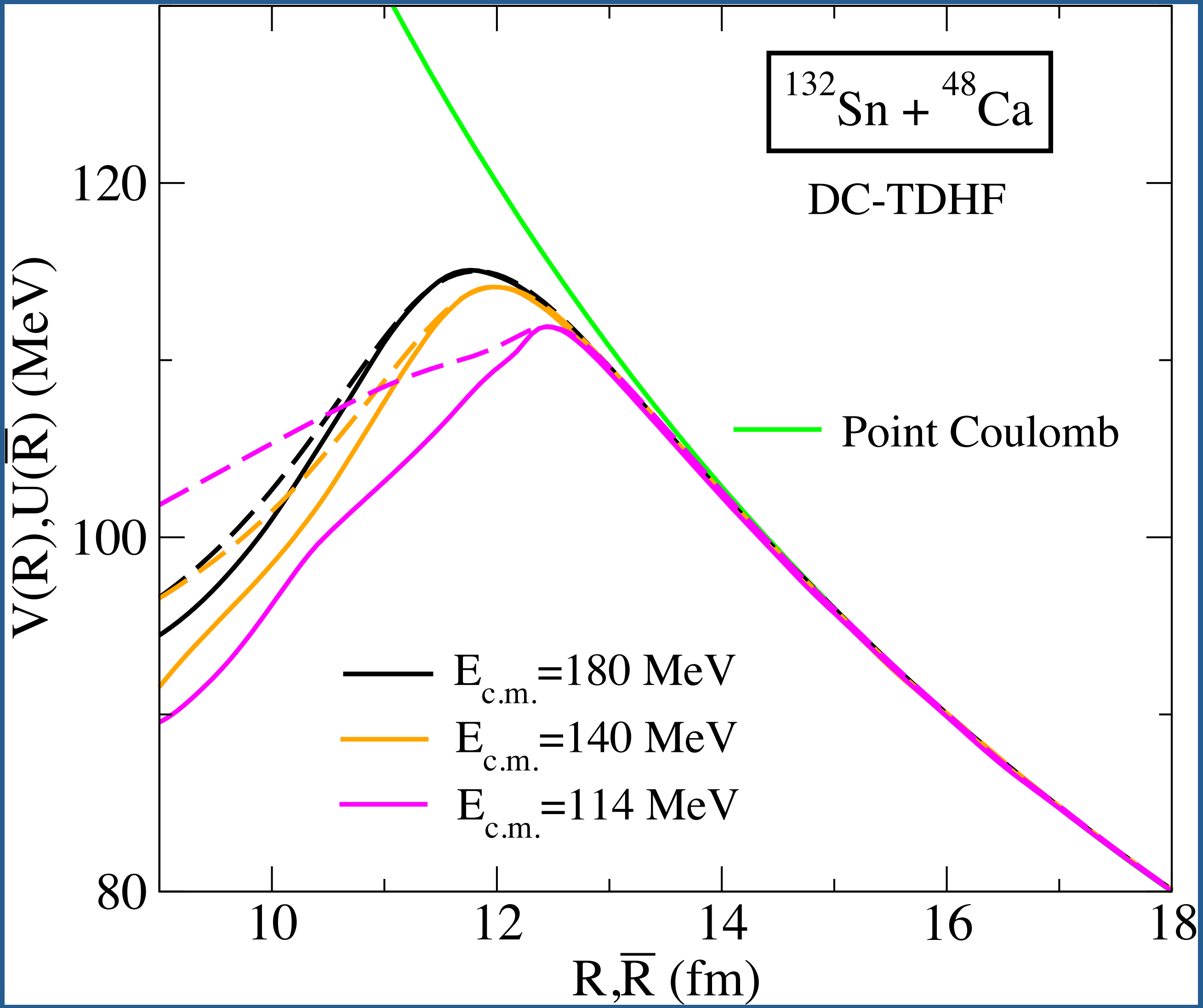
$$V(R) = V(f^{-1}(\bar{R})) \equiv U(\bar{R})$$

transformed heavy-ion potential, corresponding to constant reduced mass

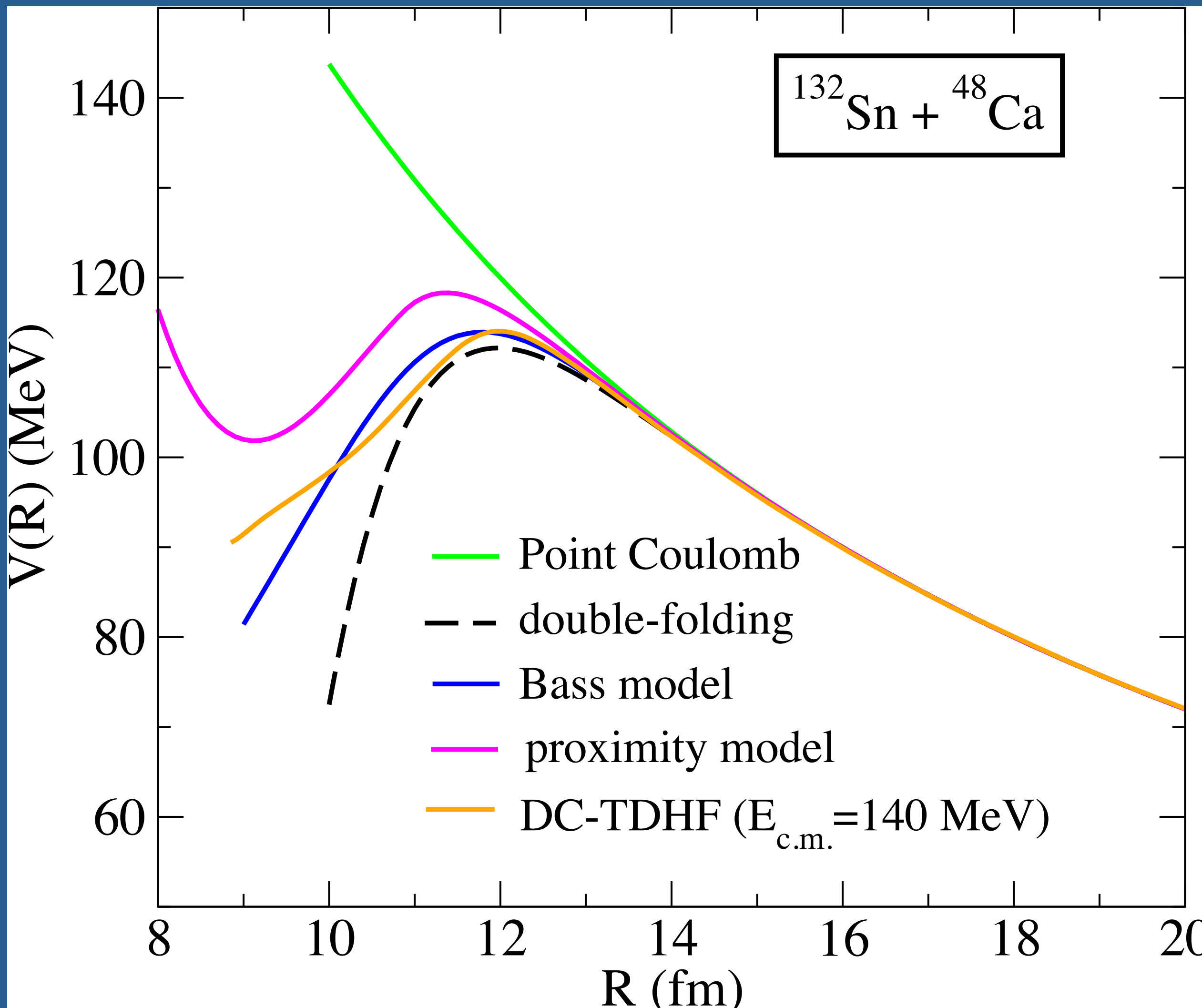
$$H(\bar{R},\bar{P}) = \frac{\bar{P}^2}{2\mu} + U(\bar{R})$$

transformed Hamiltonian

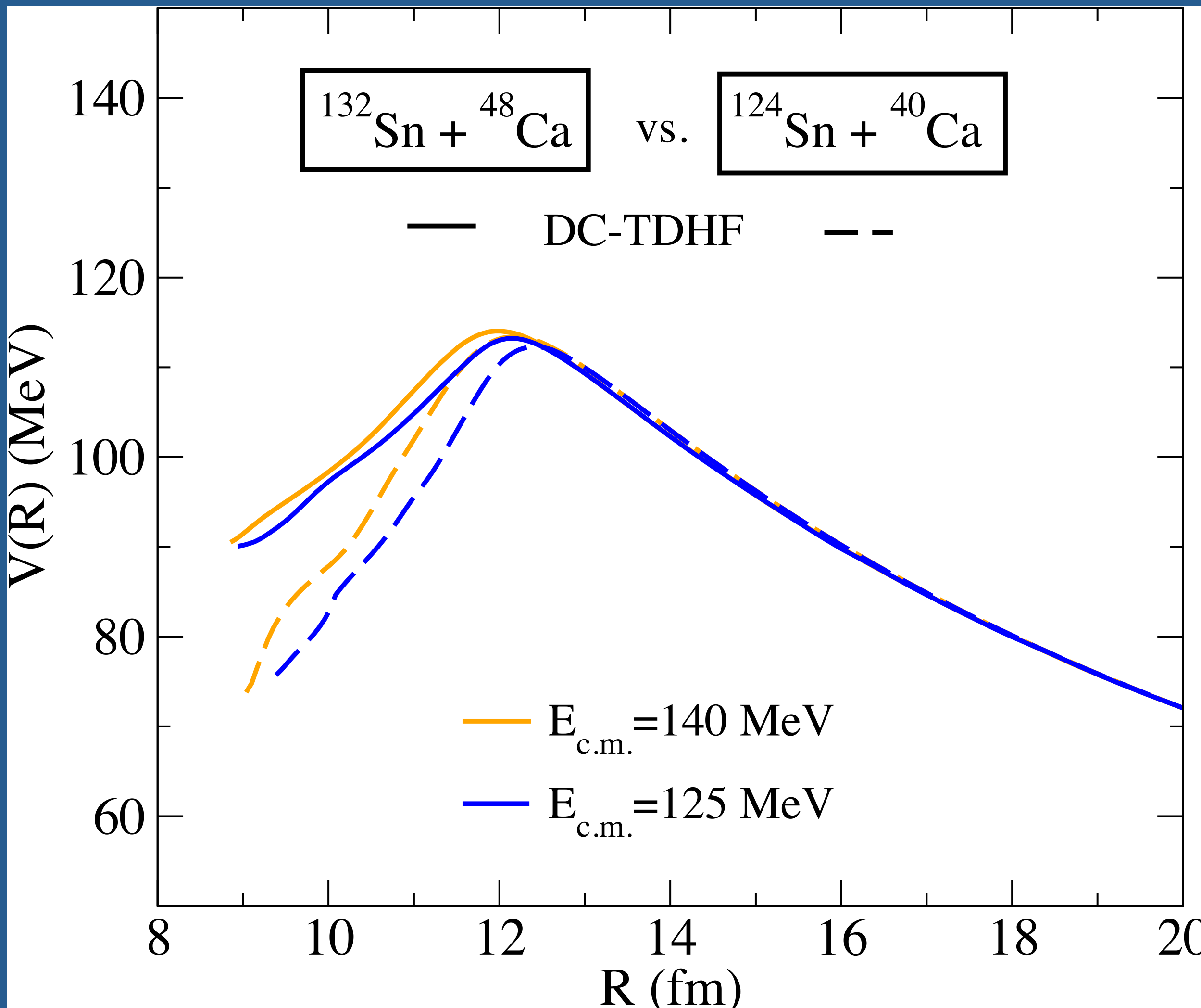
transformed heavy-ion potential for $^{132}Sn+^{48}Ca$:



Comparison with phenomenological models



Comparison: neutron-rich vs. stable system



Fusion / capture cross section for two spherical nuclei

$$\sigma_f(E_{\rm c.m.}) = \frac{\pi\hbar^2}{2\mu E_{\rm c.m.}} \sum_{\ell=0}^{\infty} (2\ell+1) T_\ell(E_{\rm c.m.})$$

total fusion cross section

Schrödinger equation for transformed radial coordinate

$$\left[\frac{-\hbar^2}{2\mu} \frac{d^2}{d\bar{R}^2} + \frac{\hbar^2 \ell(\ell+1)}{2\mu \bar{R}^2} + U(\bar{R}) - E_{c.m.} \right] \phi_\ell(\bar{R}) = 0$$

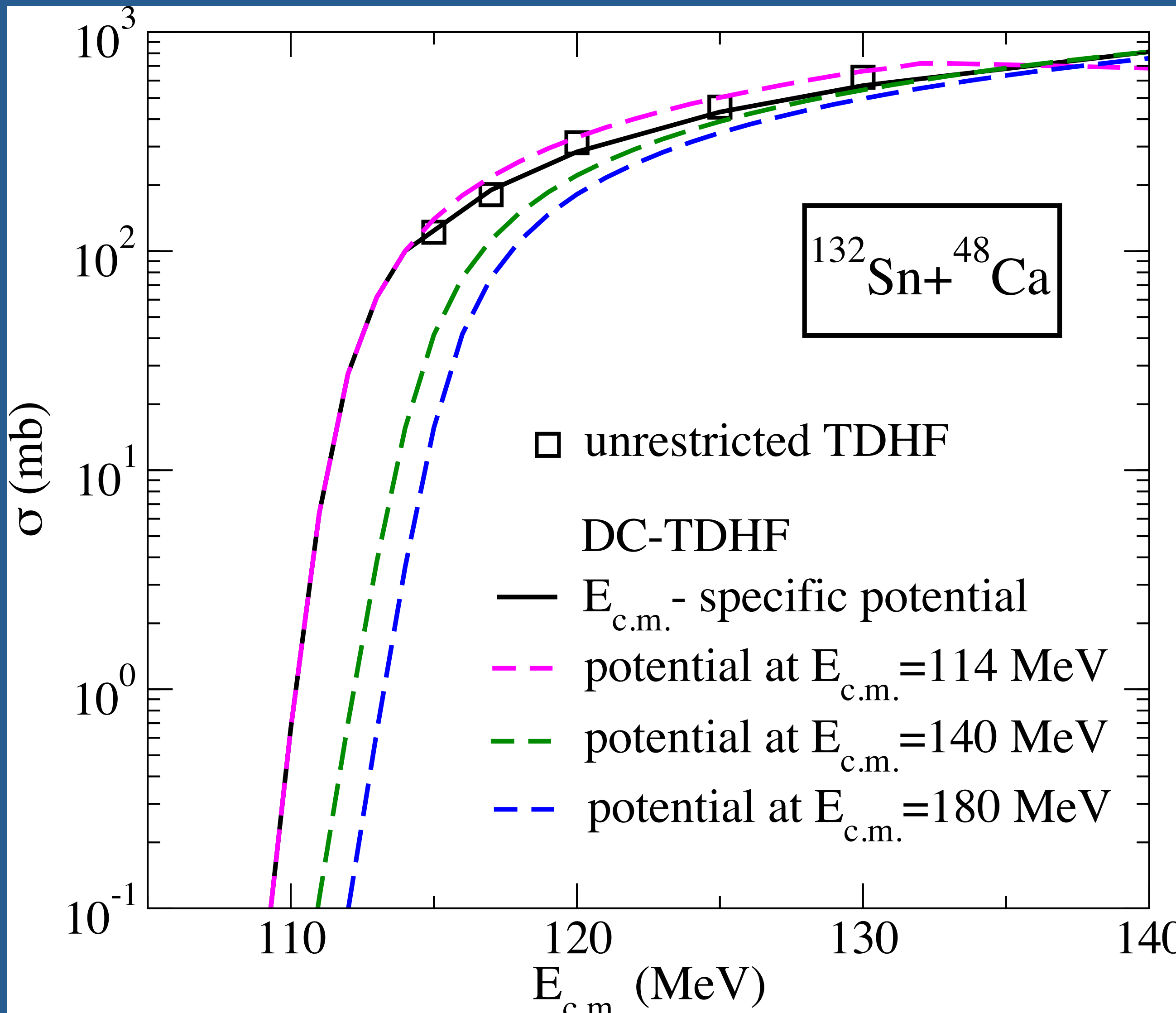
↑ reduced mass

↑ transformed potential

solve Schrödinger equation numerically, with Incoming Wave Boundary Condition (IWBC)

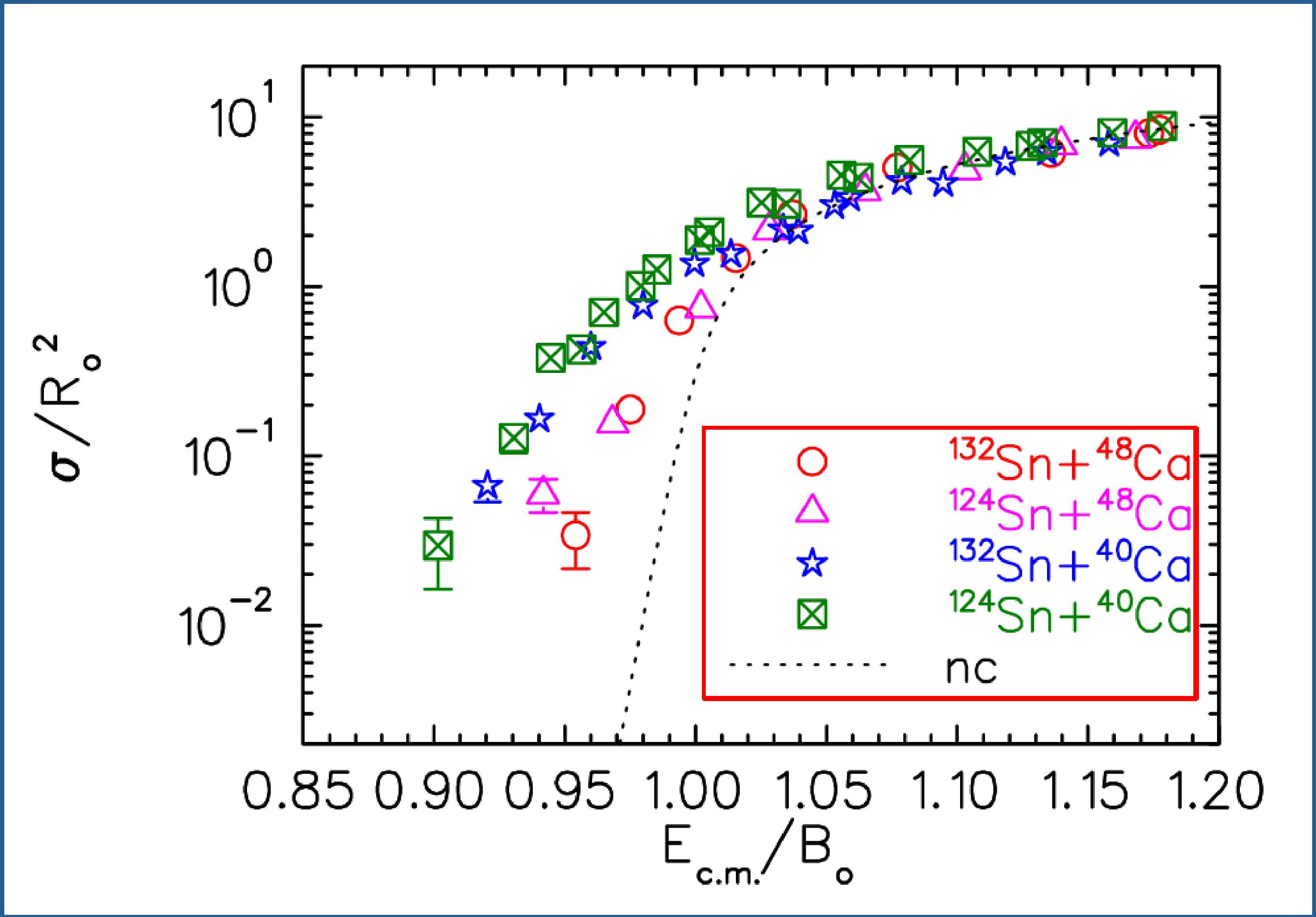
$$\longrightarrow T_\ell(E_{\rm c.m.})$$

Total fusion cross section for $^{132}Sn+^{48}Ca$



Exp. fusion cross sections (HRIBF 2011)

J.F. Liang, Proceedings of Fusion 11 conference, St. Malo (France)



Scaling used for exp. data

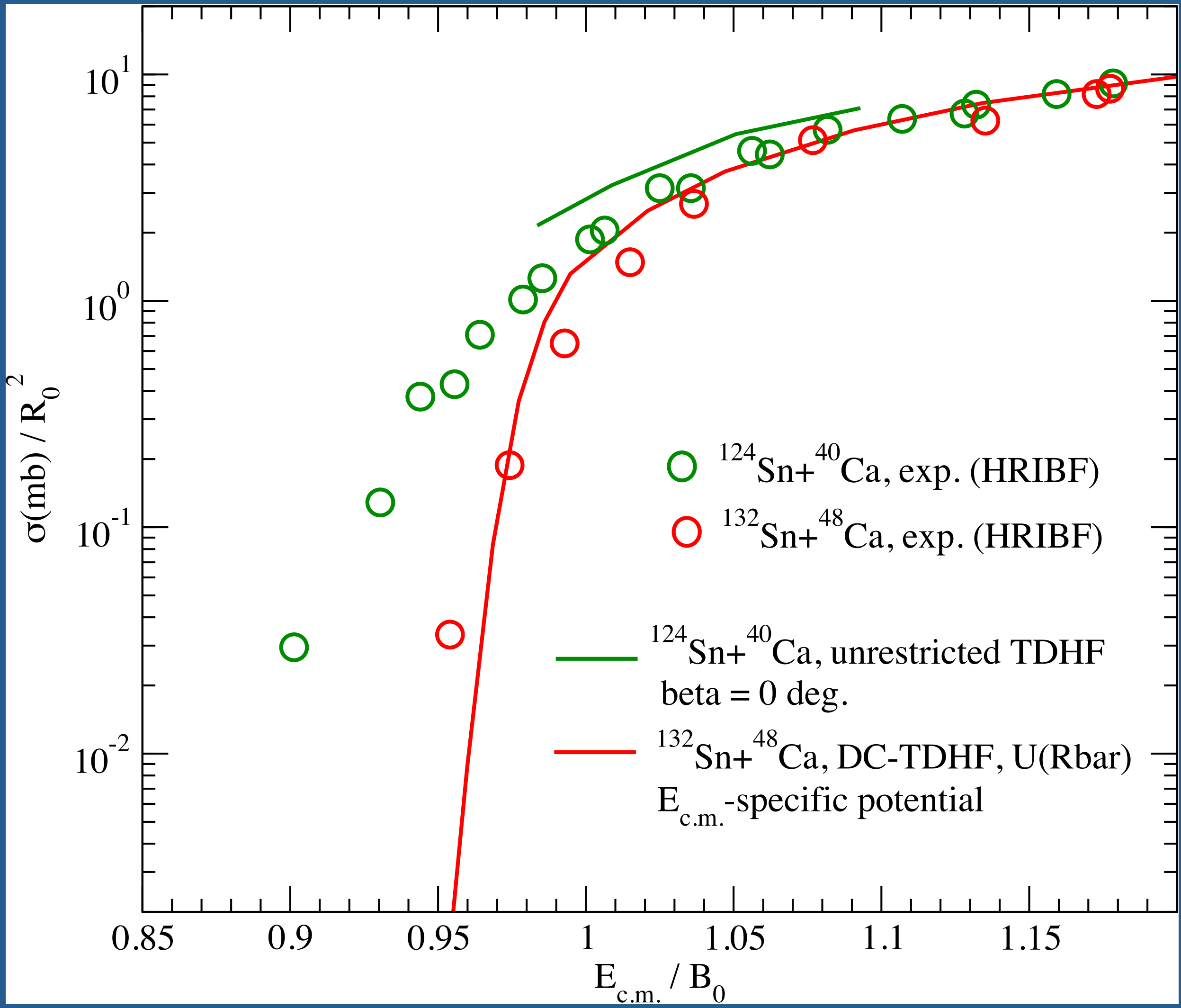
Introduce 2 scaling parameters

$$R_0 = A_1^{1/3} + A_2^{1/3} \qquad B_0 = Z_1 Z_2 / R_0$$

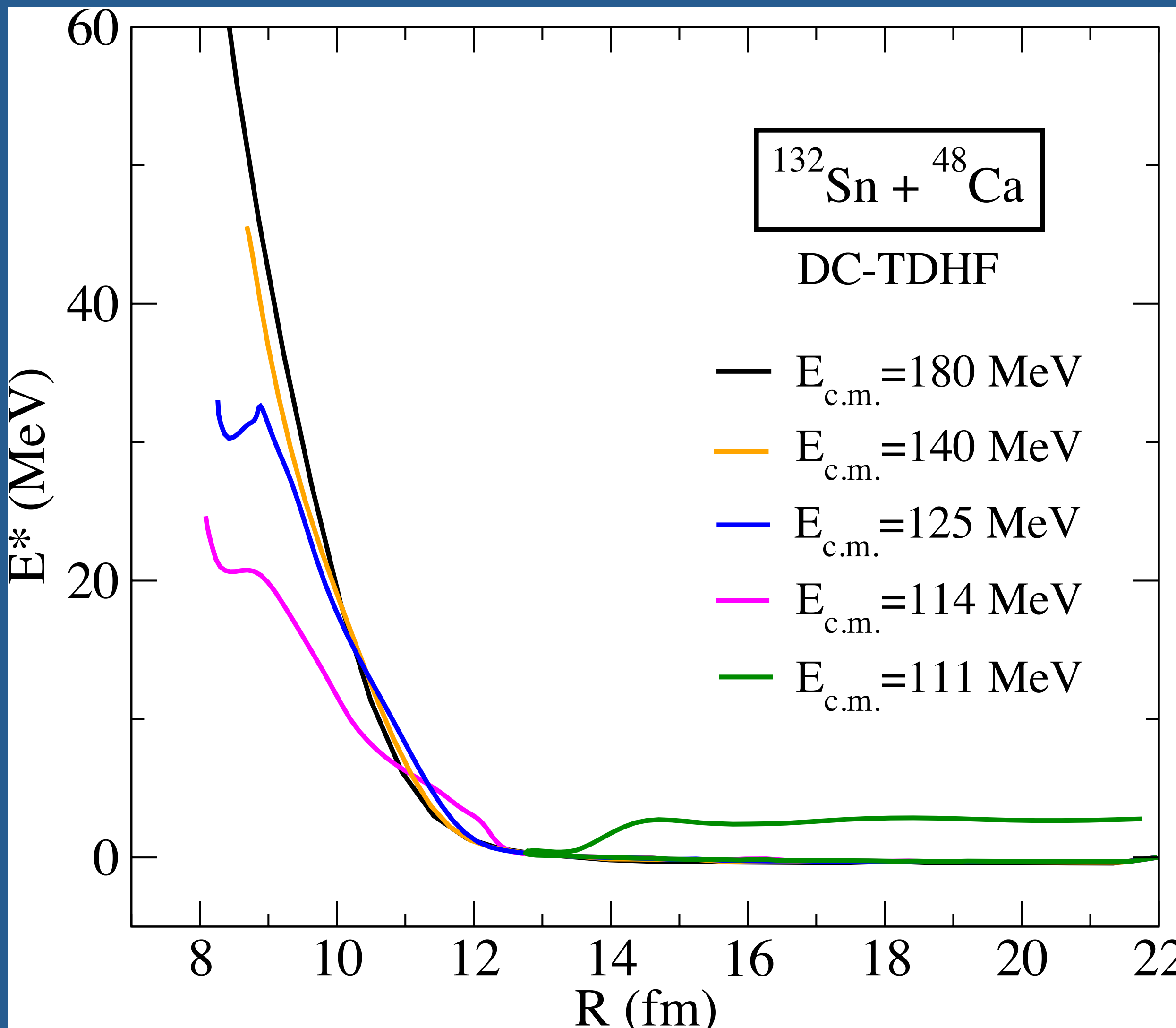
Scaling of fusion cross section $\sigma \rightarrow \sigma / R_0^2$

Scaling of center-of-mass energy $E_{c.m.} \rightarrow E_{c.m.} / B_0$

Fusion cross sections: theory vs. experiment



Excitation energy E* (DC-TDHF)



TDHF / DC-TDHF for heavy systems with fission competition
Comparison of the reactions $^{132,124}Sn + {}^{96}Zr$

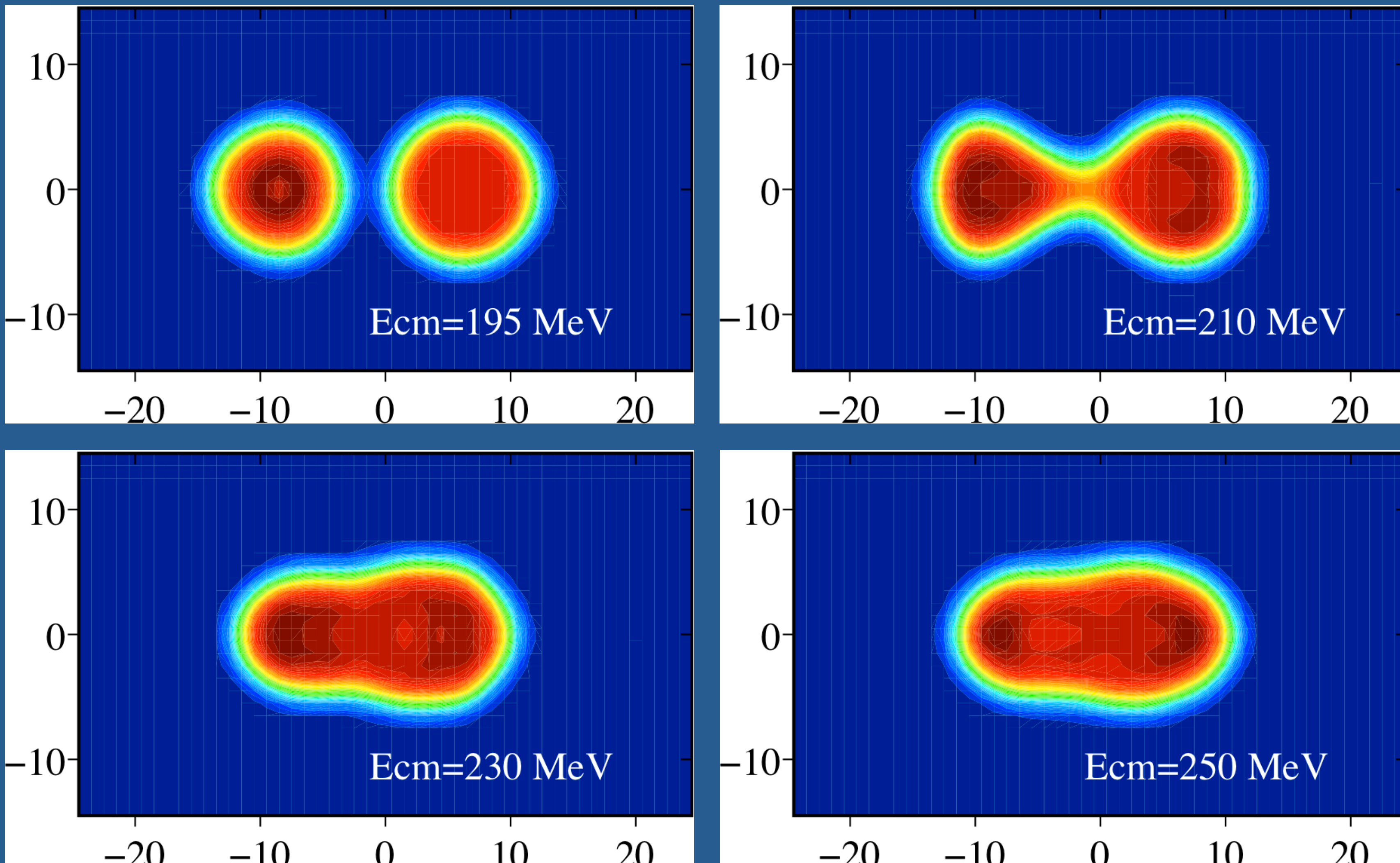
Unrestricted TDHF above the barrier:
- capture cross sections

DC-TDHF (below and above barrier):
- interaction barriers (nuclear surfaces touch)
- microscopic heavy-ion potentials, fusion barriers
- deep-inelastic and capture cross sections
- investigate E* at capture point, compare to $E^* = E_{cm} + Q_{gg}$

^{132}Sn + ^{96}Zr, central collision, mass densities at R_{min}

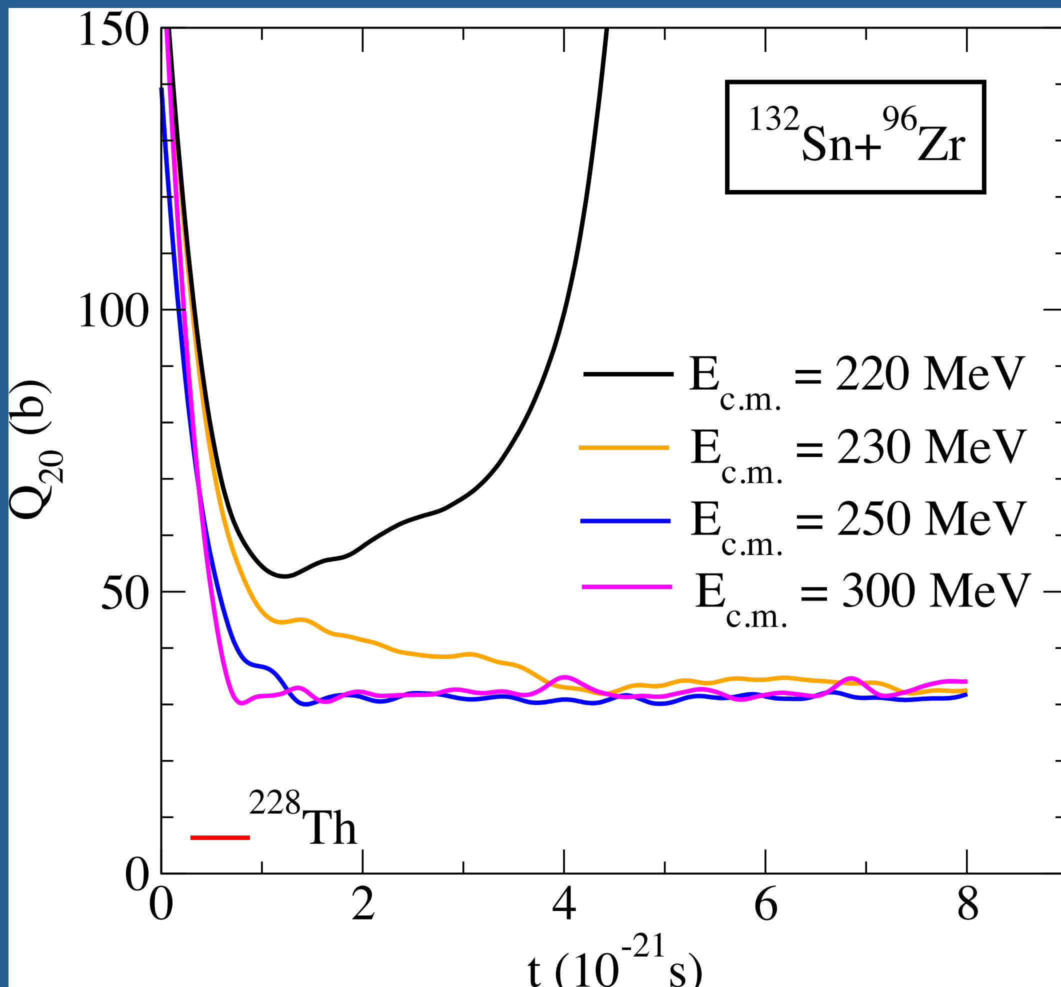
Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)

Interaction barrier



Central collision, dynamic quadrupole moment $Q_{20}(t)$

Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)



Interaction barriers and heavy-ion potential barriers, dependence on isospin $T_Z=½(Z-N)$

interaction barrier and position

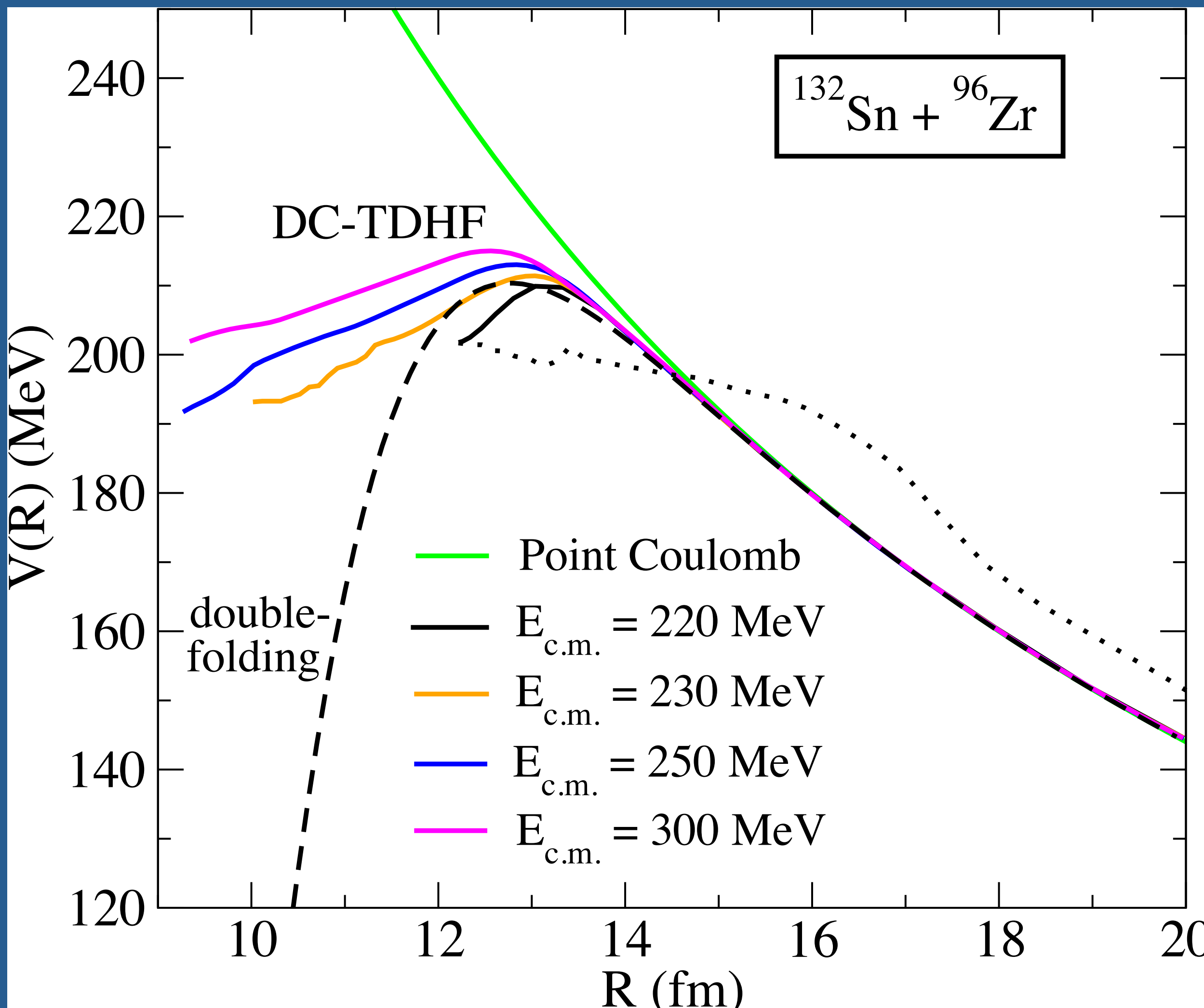
heavy-ion potential barrier and position

Reaction	V_I [MeV]	R_I [fm]	V_F [MeV]	R_F [fm]
$^{132}Sn+^{96}Zr$	195	14.77	211.4 (at E_{cm}=230 MeV) 215.0 (at E_{cm}=300 MeV)	13.03 12.56
$^{124}Sn+^{96}Zr$	204	14.05	210.6 (at E_{cm}=230 MeV) 213.7 (at E_{cm}=300 MeV)	13.06 12.59

Conclusion: interaction barriers differ substantially,
but heavy-ion potential barriers are fairly similar

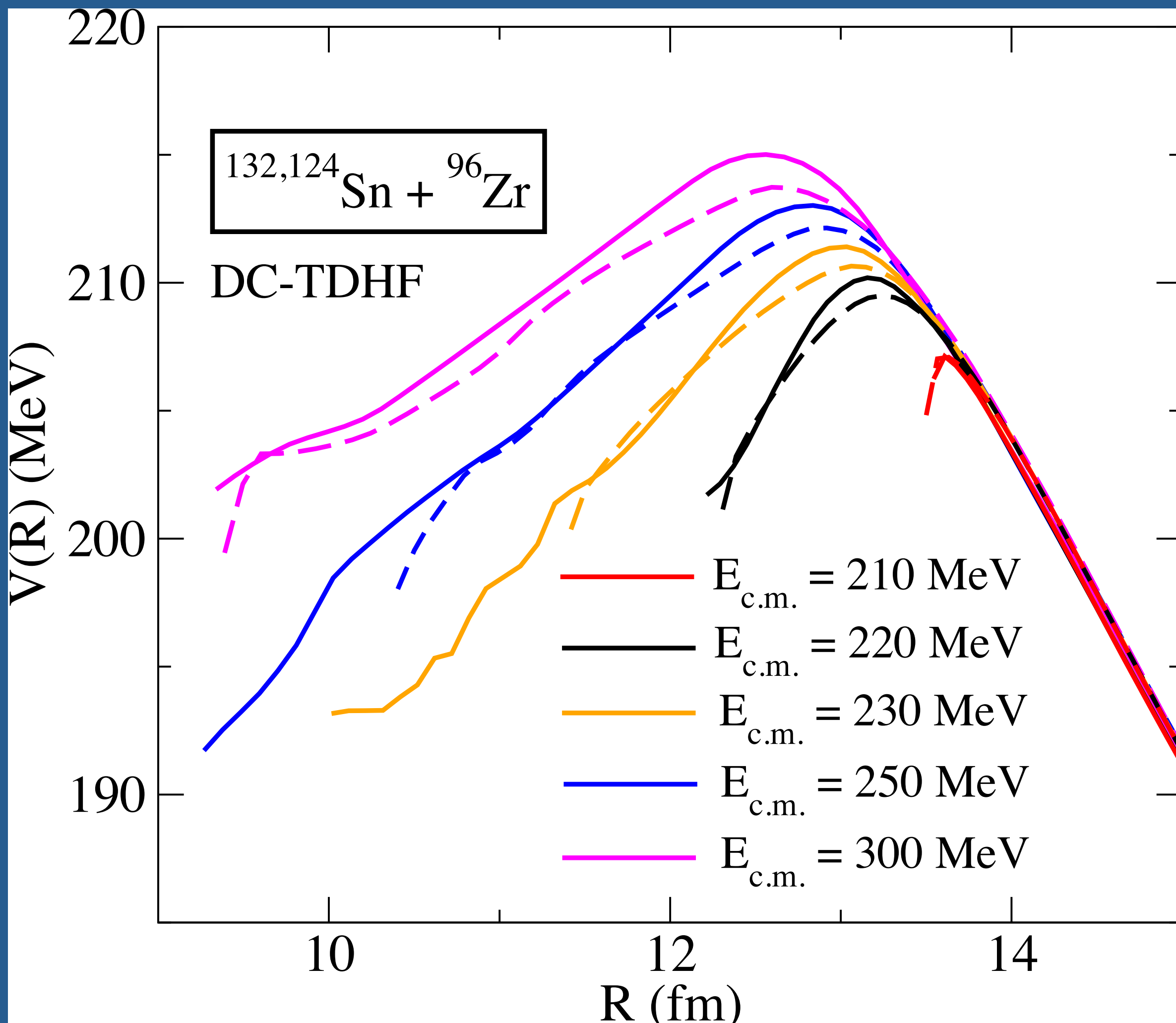
Heavy neutron-rich system: strong E_{cm}-dependence

Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)

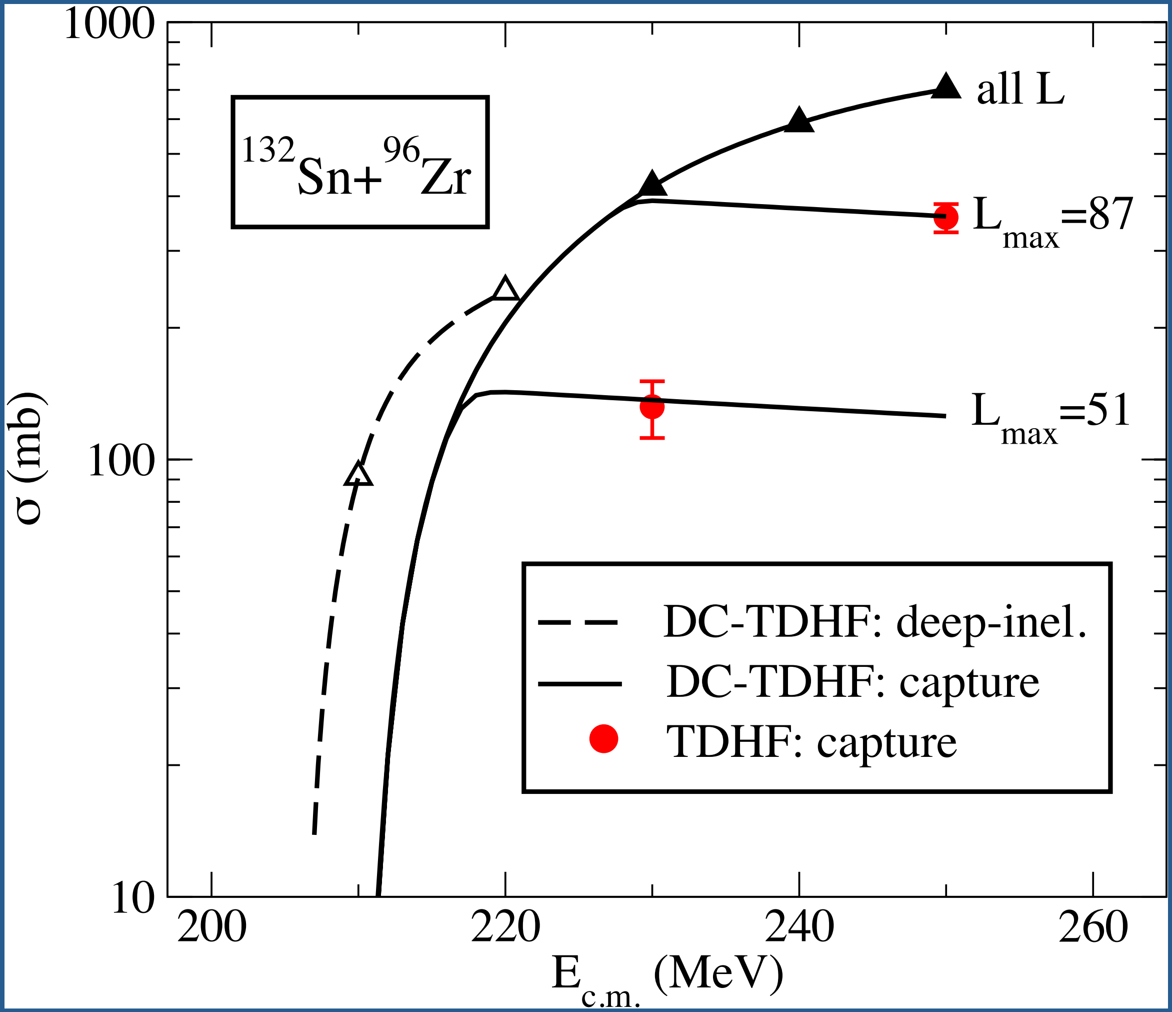


$^{132}Sn+^{96}Zr$ (solid lines) vs. $^{124}Sn+^{96}Zr$ (dashed lines)

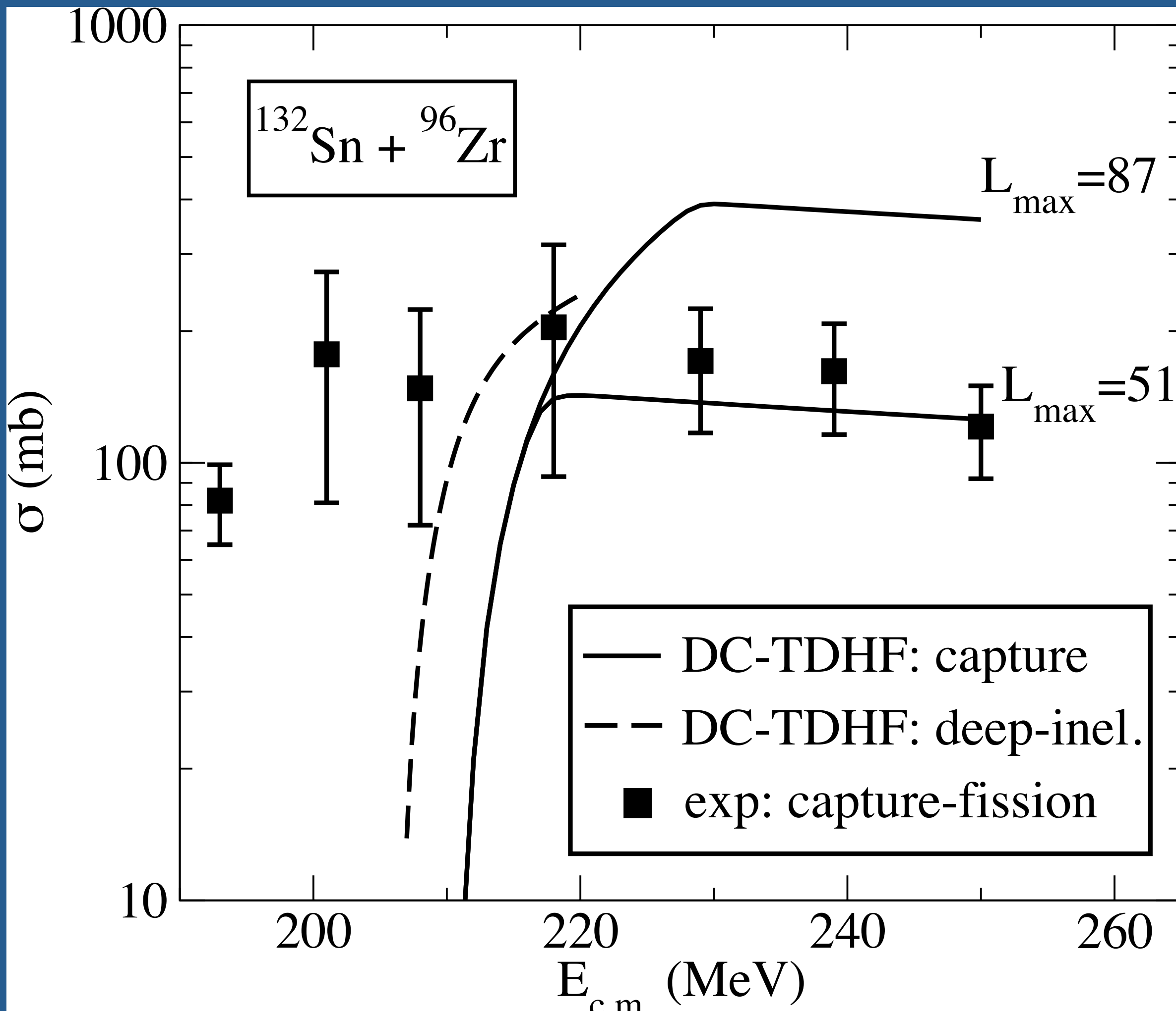
Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)



Capture and deep-inelastic cross sections
Unrestricted TDHF and DC-TDHF



Comparison of exp. capture-fission cross section to theor. capture cross section

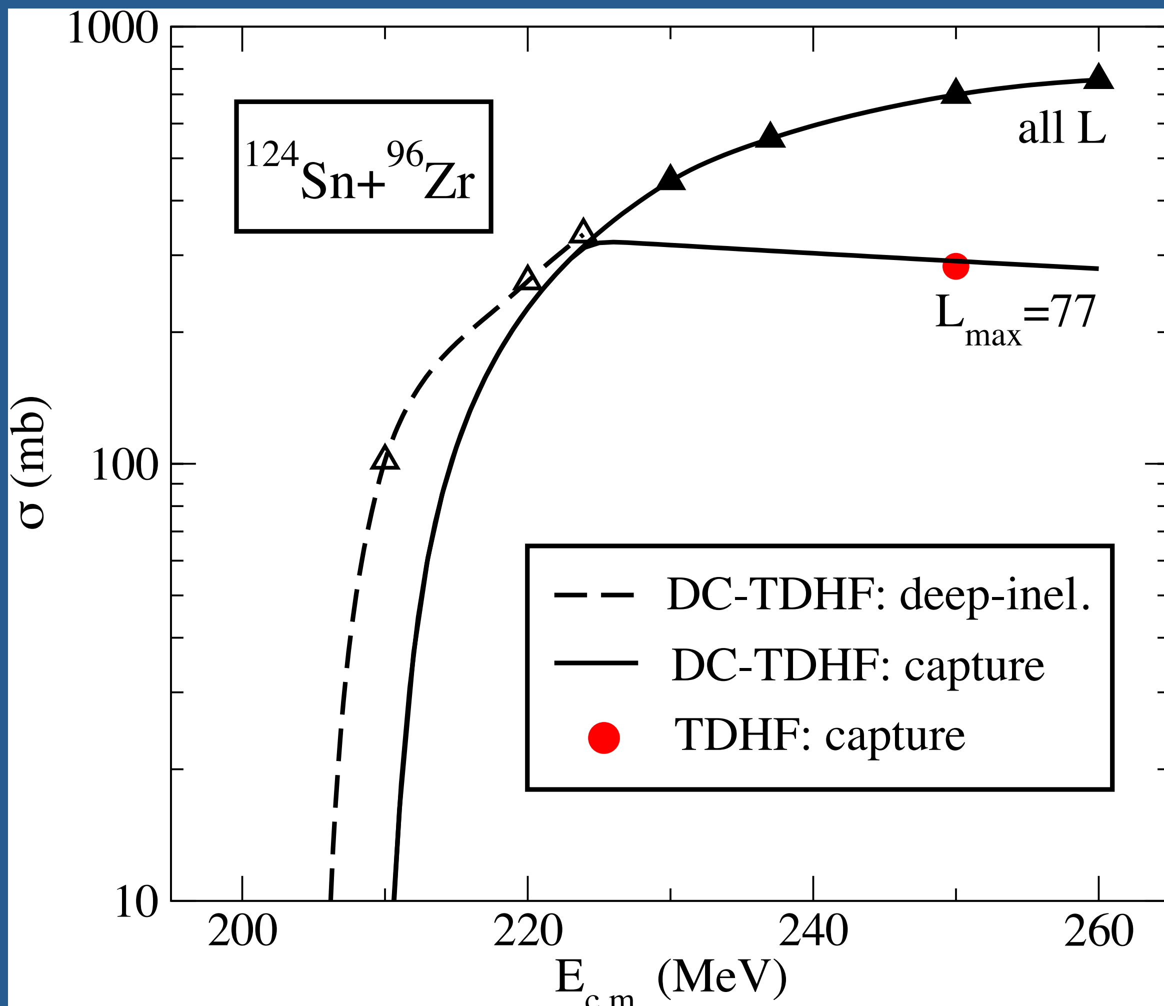


Theory:
Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)

theor. interpretation of low-energy data:
quasi-elastic and deep-inelastic rather than capture-fission

exp. data (HRIBF):
Vinodkumar et al., PRC 78, 054608 (2008)

Capture and deep-inelastic cross sections
Unrestricted TDHF and DC-TDHF



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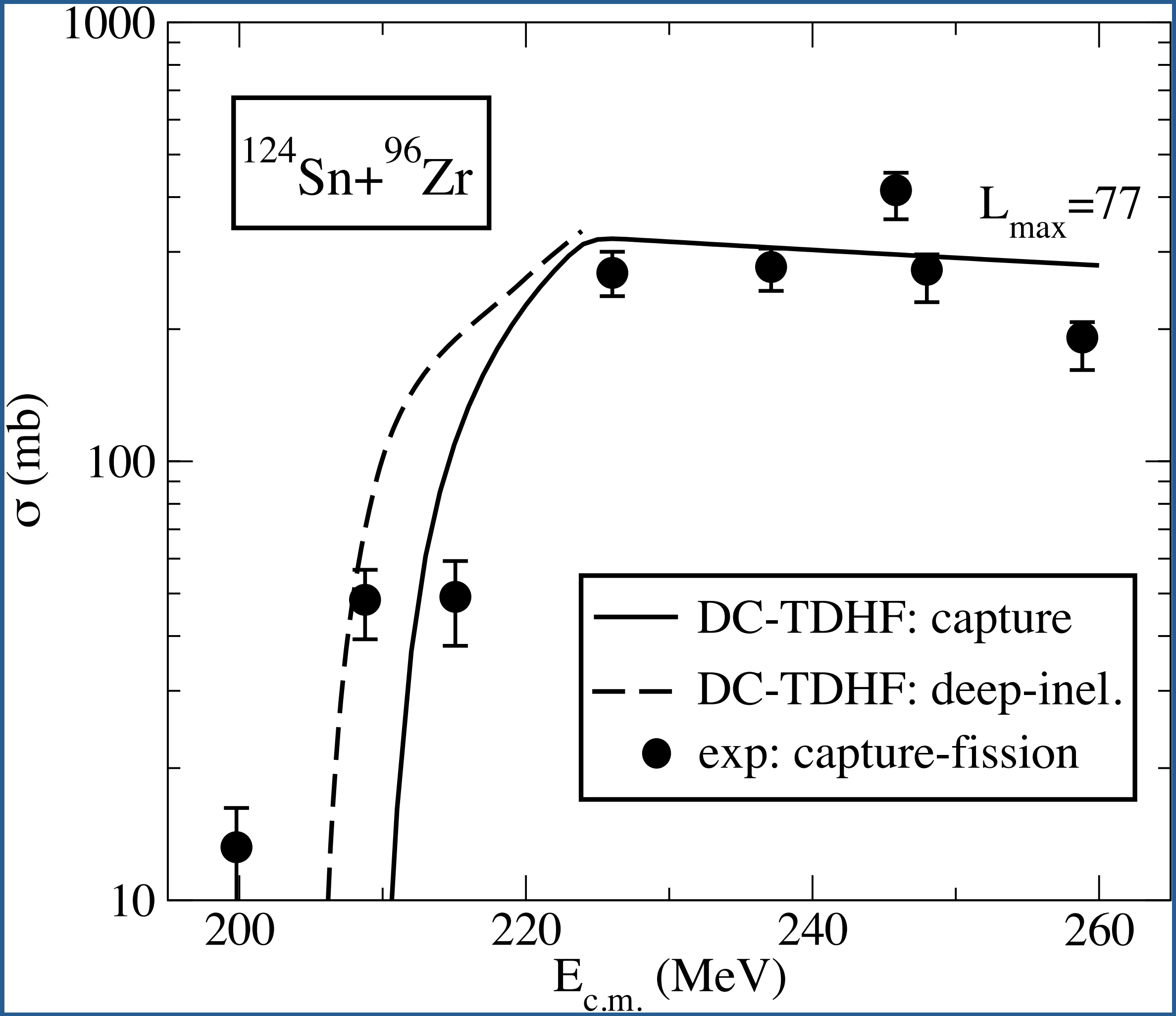

Theory:
Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)

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Vinodkumar et al., PRC 78, 054608 (2008)

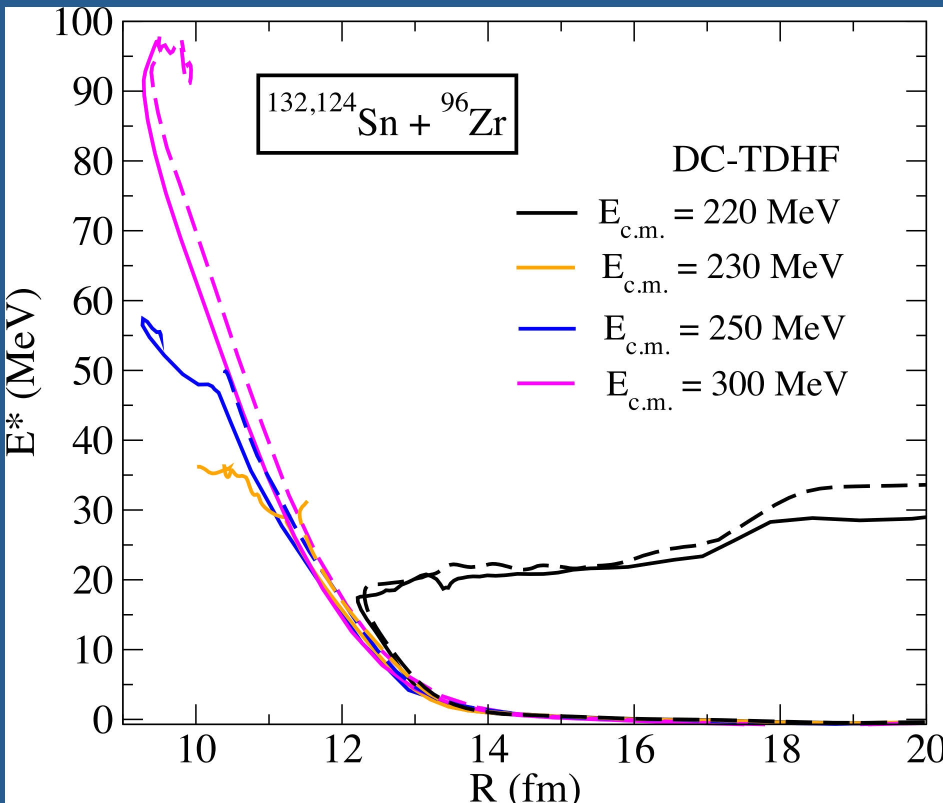
Excitation energy E* (DC-TDHF)

Oberacker, Umar, Maruhn & Reinhard, PRC 82, 034603 (2010)

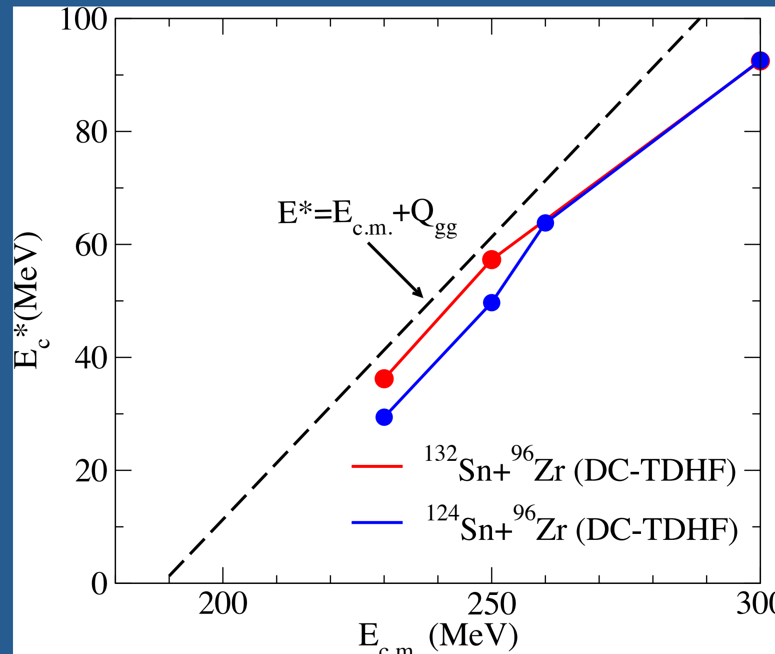
E* vs. internuclear distance R

solid lines: $^{132}Sn + ^{96}Zr$

dashed lines: $^{124}Sn + ^{96}Zr$



E* at capture point vs. E_{cm}



Conclusions

- Microscopic description of heavy-ion reactions (DC-TDHF)
  only input: Skyrme N-N interaction, no adjustable parameters
  - ➞ heavy-ion interaction potential V(R), cross sections for capture and deep-inelastic reactions
  - ➞ dynamic excitation energy E*(t)

- applied to reactions of ^{132,124}Sn + ^{96}Zr, ^{132,124}Sn + ^{48,40}Ca (HRIBF)
  V(R) shows strong E_{cm} dependence, significant changes in width of potential barrier

- investigated E* at capture point, compared to $E^* = E_{cm} + Q_{gg}$

<u>In Future:</u>

- systematic study of heavy-ion potential barriers vs. T_z=(Z-N)/2
- excitation energy division between the two fragments

Open Problems and Challenges

- Improved energy-density functionals
  parameter fits should include deformed and neutron-rich nuclei

- Include pairing (TDHF does not, densities may be unrealistic)
  a) very challenging: TDHFB code for 2 nuclei on 3-D lattice
     (exists only for 1 nucleus in time-dep. external field)
  b) simplification: start from TDHFB equation, derive formalism
     for TDHF with time-dep. BCS-pairing amplitudes $u_k(t)$
  c) easy: use frozen BCS pairing amplitudes of projectile and target