

INT Program INT-11-2d

Interfaces between structure and reactions for rare isotopes and nuclear  
astrophysics

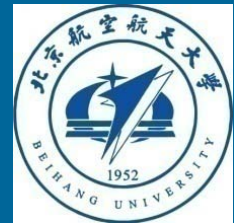
August 8 - September 2, 2011

# Deformed halo and r-process calculation with covariant density-functional theory

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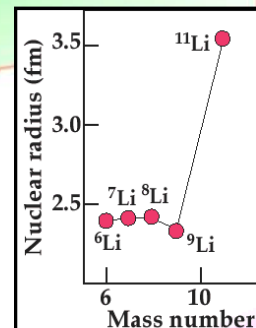
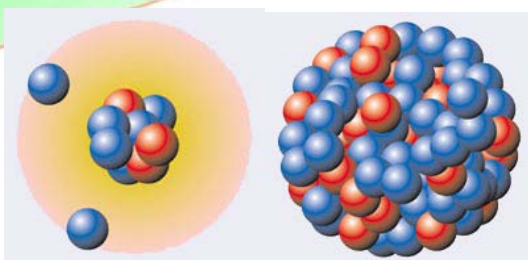
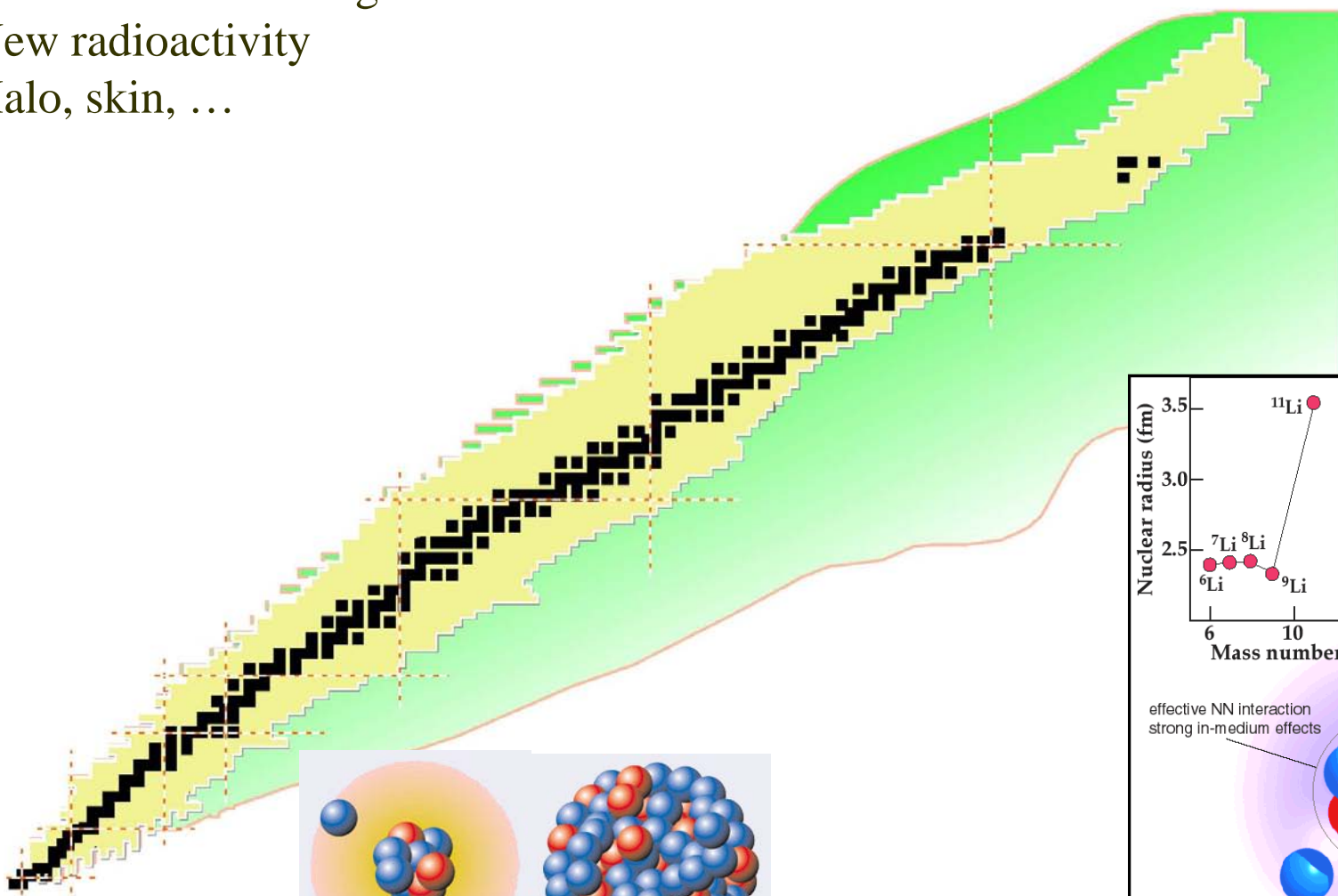
北京航空航天大学物理与核能工程学院  
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# Outline

- Introduction
- Halos in Density Functional Theory: Skyrme HFB / DD  
RHFB / GF Skyrme HFB
- Deformed halos
- R-process calculation with CDFT Mass table
- Summary & Perspectives

# Exotic phenomena in nuclei with extreme N/Z

- Modifications of magic numbers
- New radioactivity
- Halo, skin, ...
- ...



I. Tanihata et al.  
Phys. Rev. Lett. 55, 2676 (1985)

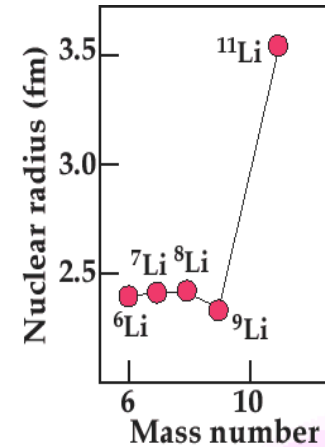
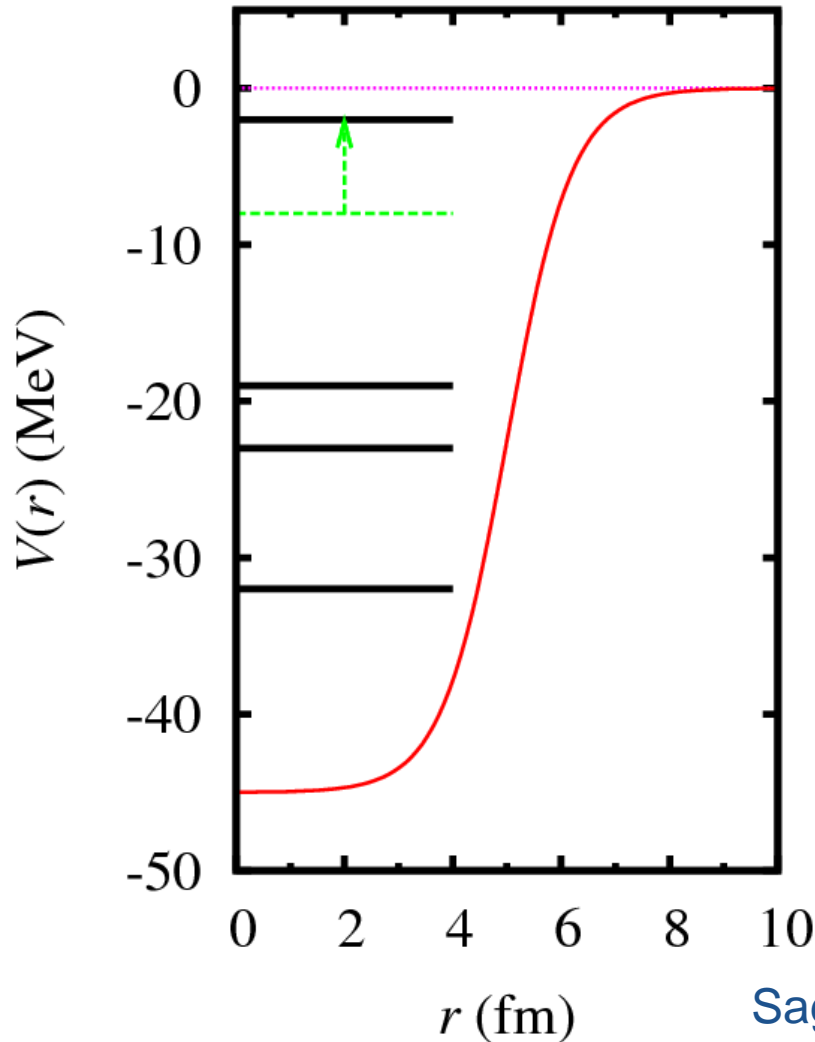
Interaction cross section  
measurements at Bevalac  
(790 MeV/u)

effective NN interaction  
strong in-medium effects

(almost) bare NN interaction  
weak in-medium effects

# Halo in spherical nuclei

## SPL in mean field model



I. Tanihata et al.  
Phys. Rev. Lett. 55, 2676 (1985)

Interaction cross section  
measurements at Bevalac  
(790 MeV/u)

effective NN interaction  
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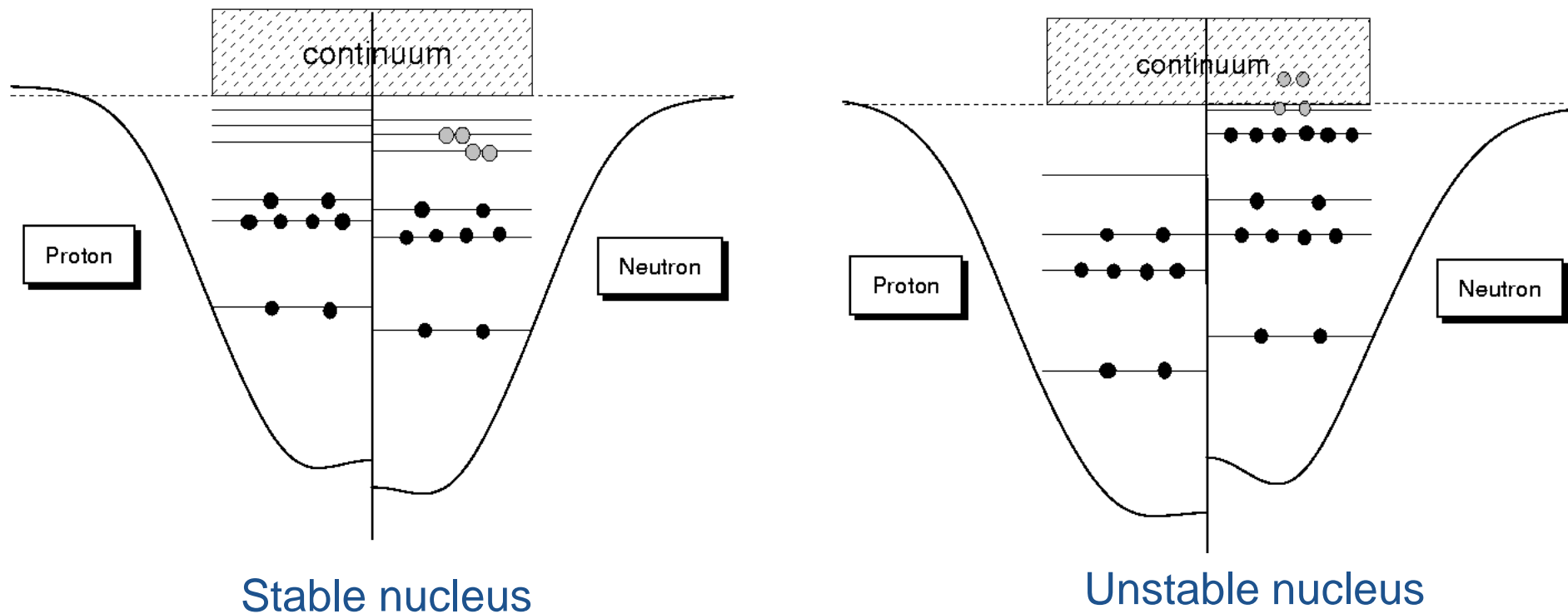
(almost) bare NN interaction  
weak in-medium effects

Sagawa, Phys. Lett. B 286, 7 (1992).  
Zhu et al., Phys. Lett. B 328, 1 (1994).

# Contributions from the continuum

- ❖ Weakly bound; large spatial extension
- ❖ Continuum can not be ignored

## 连续谱



# Contribution of continuum in r-HFB

$$\sum_{\sigma} \int d^3 r' \begin{pmatrix} h(\mathbf{r}\sigma; \mathbf{r}'\sigma') - \lambda & \Delta(\mathbf{r}\sigma; \mathbf{r}'\sigma') \\ -\Delta^*(\mathbf{r}\sigma; \mathbf{r}'\sigma') & -h(\mathbf{r}\sigma; \mathbf{r}'\sigma') + \lambda \end{pmatrix} \begin{pmatrix} U_E(\mathbf{r}'\sigma') \\ V_E(\mathbf{r}'\sigma') \end{pmatrix} = E \begin{pmatrix} U_E(\mathbf{r}\sigma) \\ V_E(\mathbf{r}\sigma) \end{pmatrix}$$

When  $r$  goes to infinity, the potentials are zero

$$-\frac{\hbar^2}{2M} \frac{d^2}{dr^2} U_E(\mathbf{r}\sigma) = (\lambda + E) U_E(\mathbf{r}\sigma)$$

$$-\frac{\hbar^2}{2M} \frac{d^2}{dr^2} V_E(\mathbf{r}\sigma) = (\lambda - E) V_E(\mathbf{r}\sigma)$$

$U$  and  $V$  behave when  $r$  goes to infinity

$$U_E(\mathbf{r}\sigma) \sim \begin{cases} \cos(k_U r + \delta) & \text{for } \lambda + E > 0 \\ \exp(-k'_U r) & \text{for } \lambda + E < 0 \end{cases}$$

$$V_E(\mathbf{r}\sigma) \sim \begin{cases} \cos(k_V r + \delta) & \text{for } \lambda - E > 0 \\ \exp(-k'_V r) & \text{for } \lambda - E < 0 \end{cases}$$

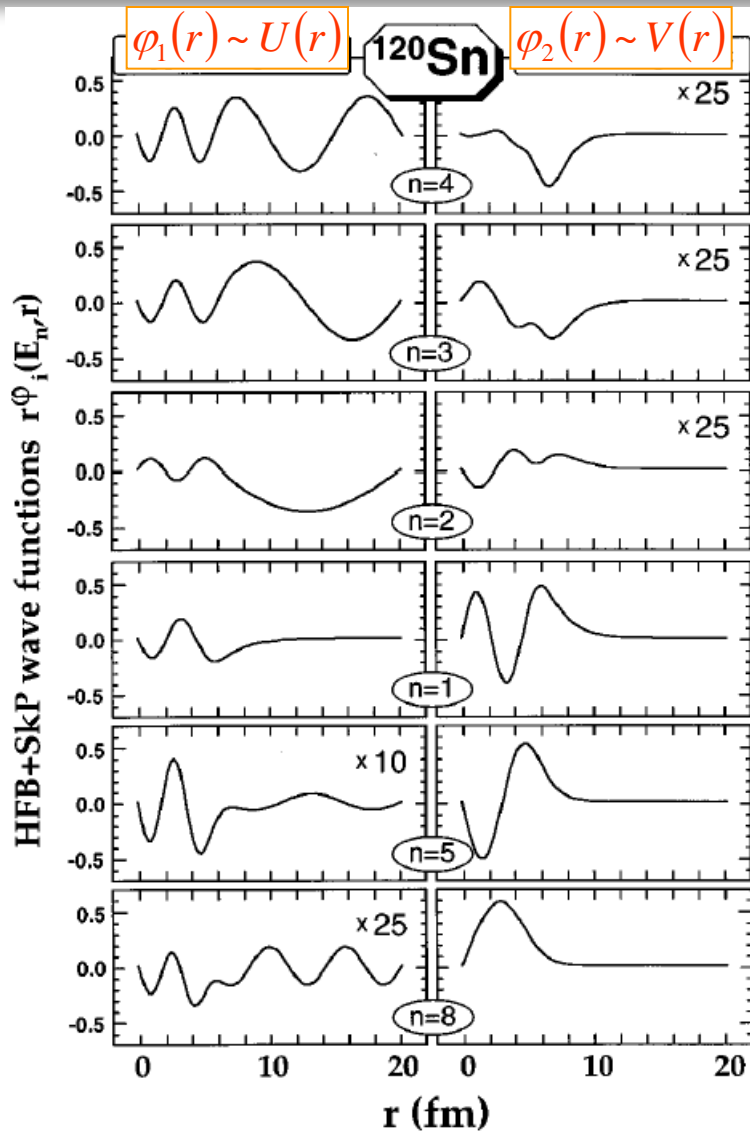
*Continuum contributes automatically and the density is still localized*

Bulgac, 1980 & nucl-th/9907088

Dobaczewski, Flocard & Treiner, NPA422(84)103

Bertsch and Esbensen, Ann. Phys. (N.Y.) **209**, 327 (1991).

# Contribution of continuum in r-HFB



## Positive energy States

- $V(r)$  determines the density
- the density is localized even if  $U(r)$  oscillates at large  $r$

## Bound States

# Relativistic Hartree-Bogoliubov theory

- ❖ Quantizing the system;
- ❖ Eliminating the mesonic degrees of freedom;
- ❖ Factorizing the higher order Greens functions;
- ❖ Neglecting retardation effects

H. Kucharek and P. Ring, Z. Phys. A339 (1991) 193.

## RHB Equation

$$\int d\mathbf{r}' \begin{pmatrix} h(\mathbf{r}, \mathbf{r}') - \lambda & \Delta(\mathbf{r}, \mathbf{r}') \\ \Delta(\mathbf{r}, \mathbf{r}') & -h(\mathbf{r}, \mathbf{r}') + \lambda \end{pmatrix} \begin{pmatrix} \psi_U(\mathbf{r}') \\ \psi_V(\mathbf{r}') \end{pmatrix} = E \begin{pmatrix} \psi_U(\mathbf{r}) \\ \psi_V(\mathbf{r}) \end{pmatrix}$$



# Relativistic continuum Hartree Bogoliubov (RCHB) theory

RHB equations:

$$\begin{pmatrix} h - \lambda & \Delta \\ -\Delta^* & -h^* + \lambda \end{pmatrix} \begin{pmatrix} \psi_U \\ \psi_V \end{pmatrix} = E \begin{pmatrix} \psi_U \\ \psi_V \end{pmatrix}$$

$$h(\mathbf{r}) = [\boldsymbol{\alpha} \cdot \mathbf{p} + V(\mathbf{r}) + \beta(M + S(\mathbf{r}))]$$

$$\Delta_{kk'}(\mathbf{r}, \mathbf{r}') = - \int d^3r_1 \int d^3r'_1 \sum_{\tilde{k}\tilde{k}'} V_{kk',\tilde{k}\tilde{k}'}(\mathbf{r}\mathbf{r}'; \mathbf{r}_1\mathbf{r}'_1) \kappa_{\tilde{k}\tilde{k}'}(\mathbf{r}_1, \mathbf{r}'_1)$$

Pairing tensor

$$\kappa_{kk'}(\mathbf{r}, \mathbf{r}') = \langle |a_{k,i} a_{k',i'}| \rangle = \sum_{E_i > 0} \psi_U^{k,i}(\mathbf{r})^* \psi_V^{k',i}(\mathbf{r}')$$

Baryon density

$$\rho(\mathbf{r}, \mathbf{r}') = \sum_{k, E_i > 0} \psi_V^{k,i}(\mathbf{r})^* \psi_V^{k,i}(\mathbf{r}')$$

Pairing force

$$V(\mathbf{r}_1, \mathbf{r}_2) = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{1}{4} [1 - \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2] \left( 1 - \frac{\rho(r)}{\rho_0} \right)$$

# RCHB theory

- ❖ To describe bound states, continuum and the coupling between them, RHB equation must be solved in suitable methods
- ❖ Radial RHB equation in spherical case

( J. Meng, Nucl. Phys. A**635** (1998) 3, and references therein. )

## Relativistic Continuum Hartree-Bogoliubov(RCHB) theory

for spherical nuclei

$$\psi_U^i = \frac{1}{r} \begin{pmatrix} iG_U^{i\kappa}(r)Y_{jm}^l(\theta, \phi) \\ -F_U^{i\kappa}(r)Y_{jm}^{\tilde{l}}(\theta, \phi) \end{pmatrix} \chi_t(t) \quad \psi_V^i = \frac{1}{r} \begin{pmatrix} iG_V^{i\kappa}(r)Y_{jm}^l(\theta, \phi) \\ -F_V^{i\kappa}(r)Y_{jm}^{\tilde{l}}(\theta, \phi) \end{pmatrix} \chi_t(t)$$

$$\begin{cases} \frac{dG_U(r)}{dr} + \frac{\kappa}{r}G_U(r) - (E + \lambda - V(r) + S(r))F_U(r) + r^2\Delta(r)F_V(r) = 0, \\ \frac{dF_U(r)}{dr} - \frac{\kappa}{r}F_U(r) + (E + \lambda - V(r) - S(r))G_U(r) + r^2\Delta(r)G_V(r) = 0, \\ \frac{dG_V(r)}{dr} + \frac{\kappa}{r}G_V(r) + (E - \lambda + V(r) - S(r))F_V(r) + r^2\Delta(r)F_U(r) = 0, \\ \frac{dF_V(r)}{dr} - \frac{\kappa}{r}F_V(r) - (E - \lambda + V(r) + S(r))G_V(r) + r^2\Delta(r)G_U(r) = 0, \end{cases}$$

# Some comments

- ◆ Nucleus has finite volume: the asymptotic RCHB equations for  $r \rightarrow \infty$ :

$$\begin{cases} \frac{d^2 G_U(r)}{dr^2} = -(E + \lambda)(2M + E + \lambda)G_U(r) \\ \frac{d^2 G_V(r)}{dr^2} = -(\lambda - E)(2M + \lambda - E)G_V(r) \end{cases}$$

similar for  $F_U(r)$

similar for  $F_V(r)$

- ◆ Asymptotic solutions:

$$\begin{cases} F_U(r), G_U(r) \sim \begin{cases} \cos(k_U r), \lambda + E > 0 \\ \exp(-k_U' r), \lambda + E < 0 \end{cases} \\ F_V(r), G_V(r) \sim \begin{cases} \cos(k_V r), \lambda - E > 0 \\ \exp(-k_V' r), \lambda - E < 0 \end{cases} \end{cases}$$

if  $E < -\lambda$ , U components is localized, discrete

if  $E > -\lambda$ , U components is non-localized, continuum

# RCHB theory

## Densities

$$\begin{cases} 4\pi r^2 \rho_s(r) &= \sum_i (|G_V^i(r)|^2 - |F_V^i(r)|^2), \\ 4\pi r^2 \rho_v(r) &= \sum_i (|G_V^i(r)|^2 + |F_V^i(r)|^2), \\ 4\pi r^2 \rho_3(r) &= \sum_i \tau_3 (|G_V^i(r)|^2 + |F_V^i(r)|^2), \\ 4\pi r^2 \rho_c(r) &= \sum_i \frac{1}{2} (1 - \tau_3) (|G_V^i(r)|^2 + |F_V^i(r)|^2), \end{cases}$$

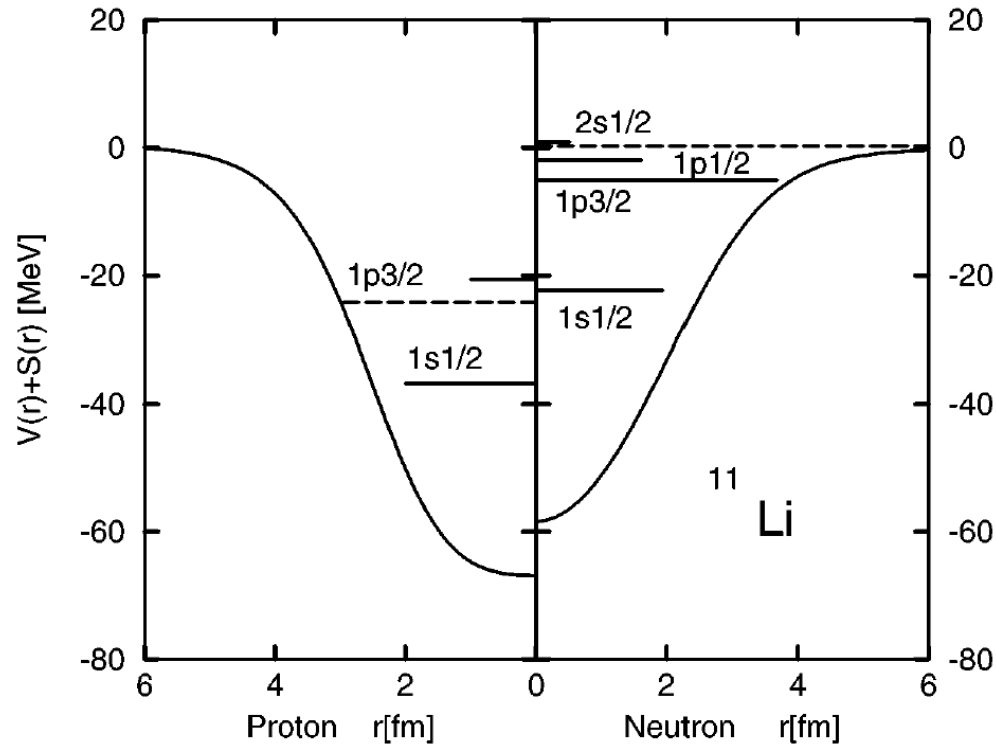
## Total binding energy

$$E = E_{\text{nucleon}} + E_{\sigma} + E_{\omega} + E_{\rho} + E_c + E_{\text{CM}},$$

$$E_{\text{nucleon}} = \sum_i \int dr (\lambda - E^i) [ |G_V^i(r)|^2 + |F_V^i(r)|^2 ] - 2E_{\text{pair}},$$

$$E_{\text{pair}} = -\frac{1}{2} \text{Tr} \Delta \kappa$$

# $^{11}\text{Li}$ : self-consistent RCHB description

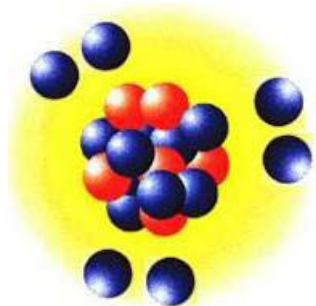
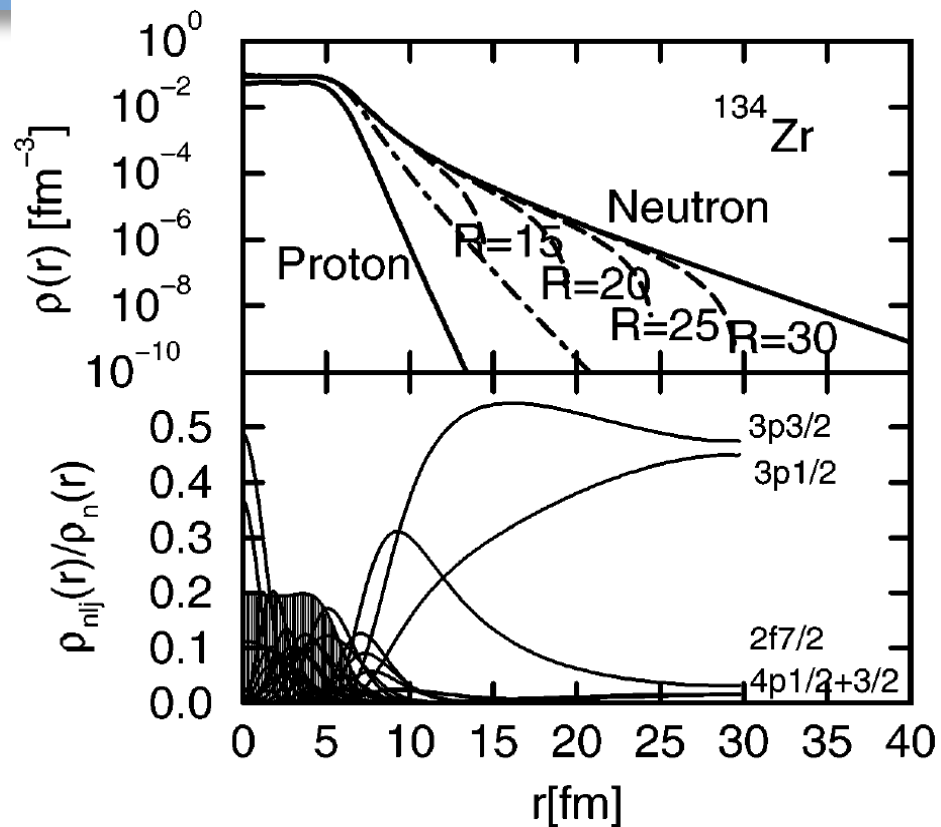
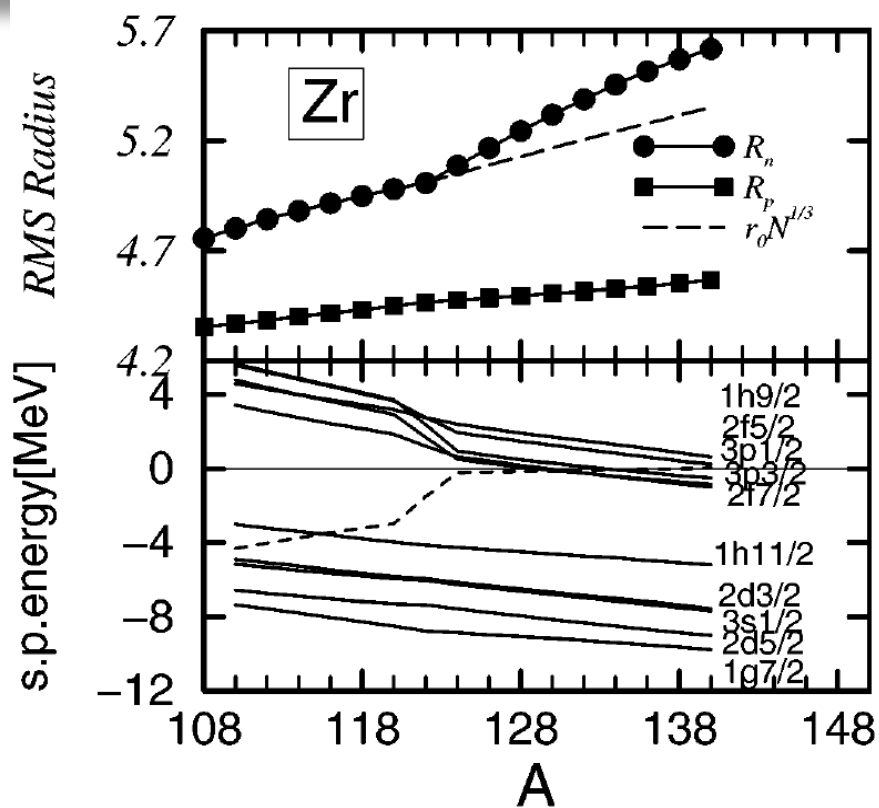


Meng & Ring, PRL77,3963 (96)

Contribution of continuum

Important roles of low- $l$  orbitals close to the threshold

# Giant halo: predictions of RCHB



Halos consisting of up to 6 neutrons

Important roles of low- $l$  orbitals close to the threshold

Meng & Ring, PRL80,460 (1998)

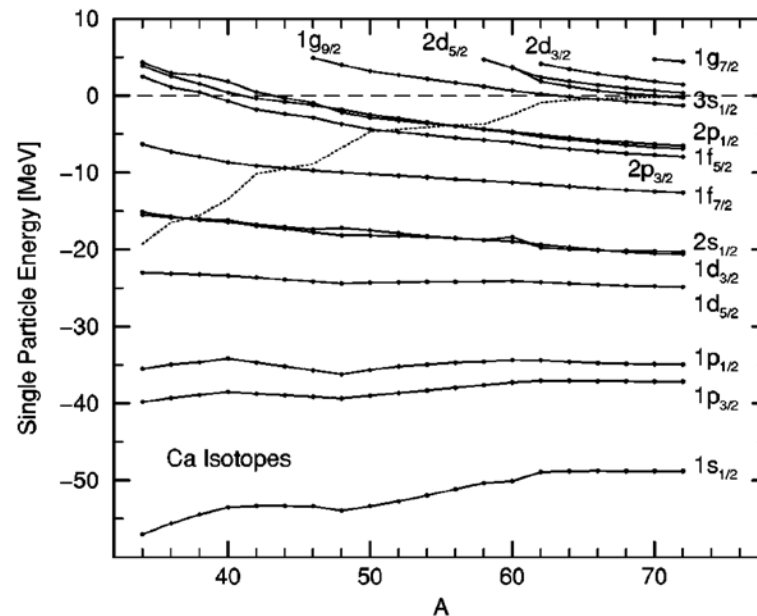
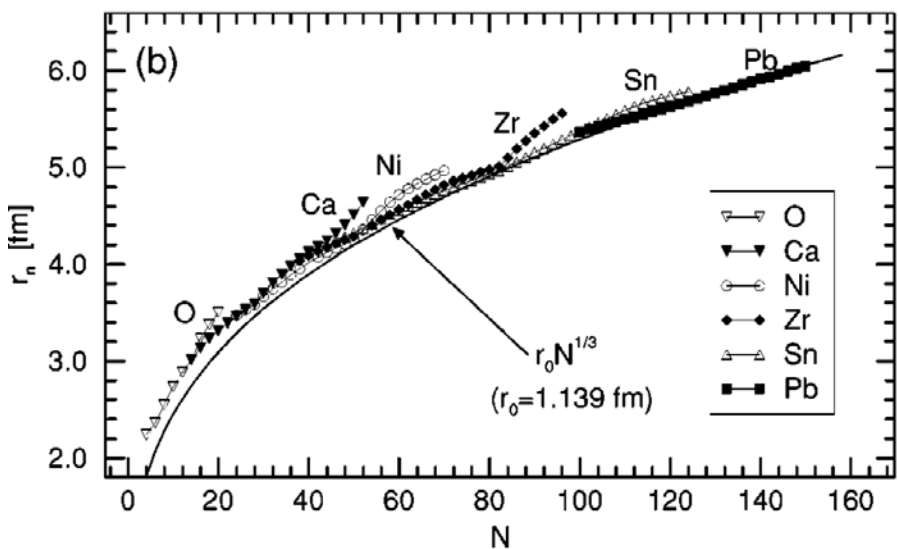
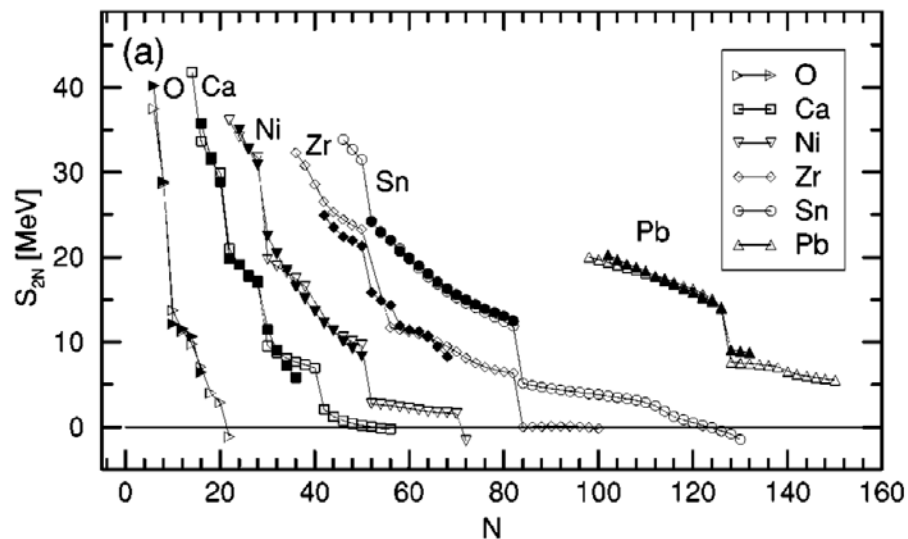
# Prediction of giant halo

Meng, Toki, Zeng, Zhang & Zhou, PRC65,041302R (2002)

Zhang, Meng, Zhou & Zeng, CPL19,312 (2002)

Zhang, Meng & Zhou, SCG33,289 (2003)

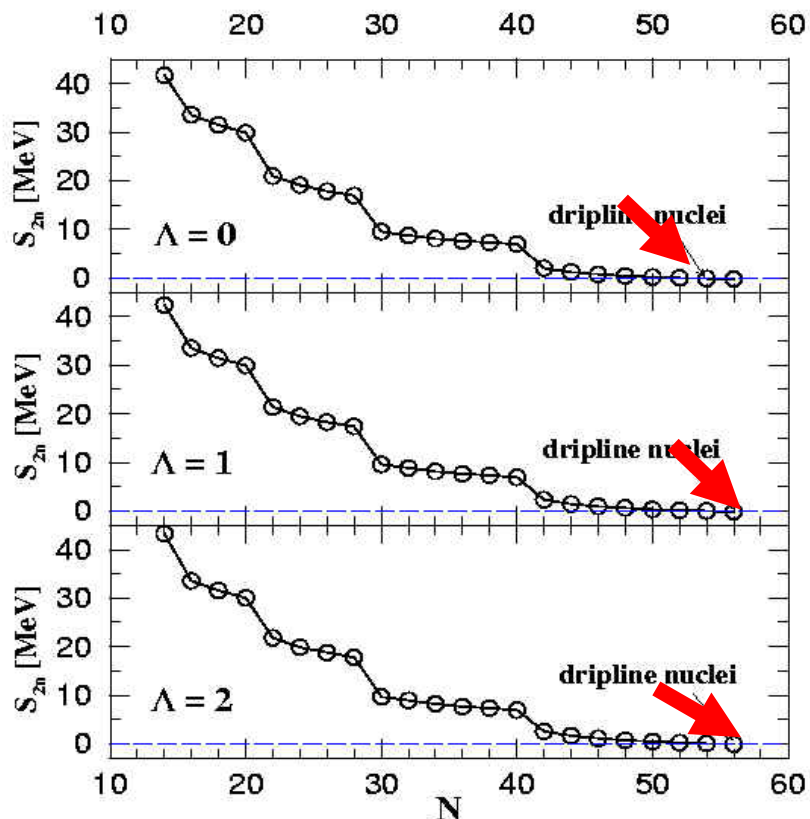
Giant halos in lighter isotopes



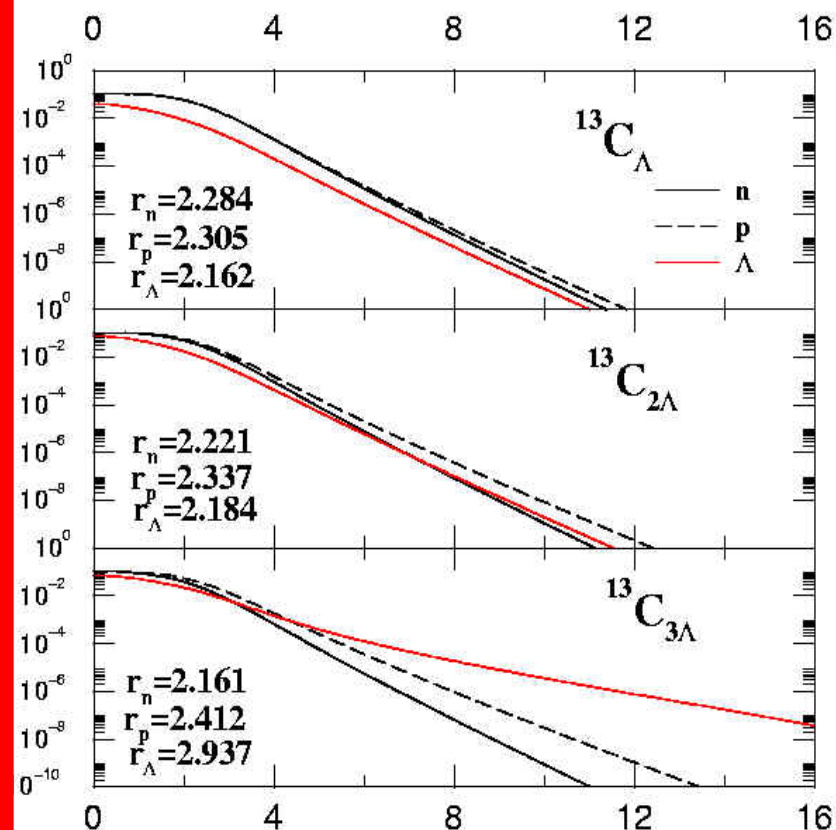
# Exotic Phenomena

Lu & Meng, CPL 19, 1775(2002)  
 Lu, Meng, Zhang & Zhou, EPJ A17,19 (2003)

## Neutron halos in hyper Ca nuclei



## Hyperon halos in $^{13}\text{C}_\Lambda$

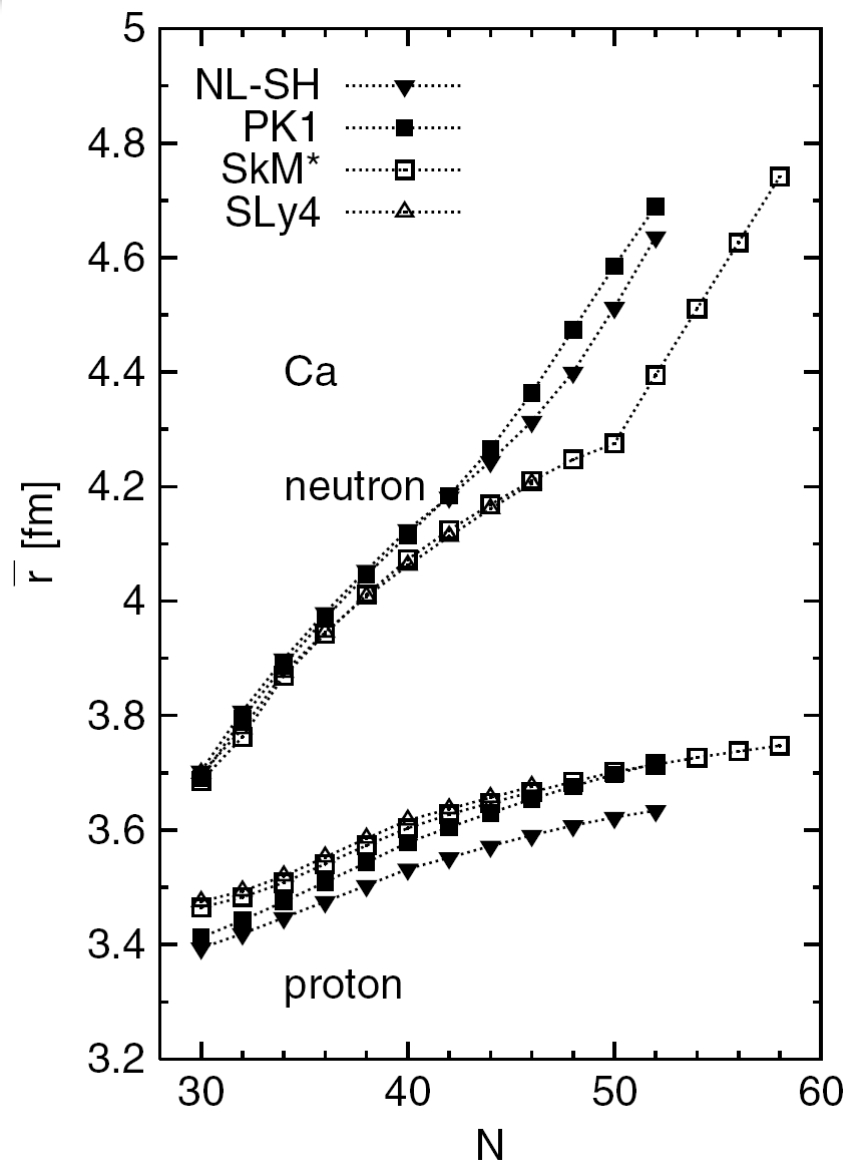
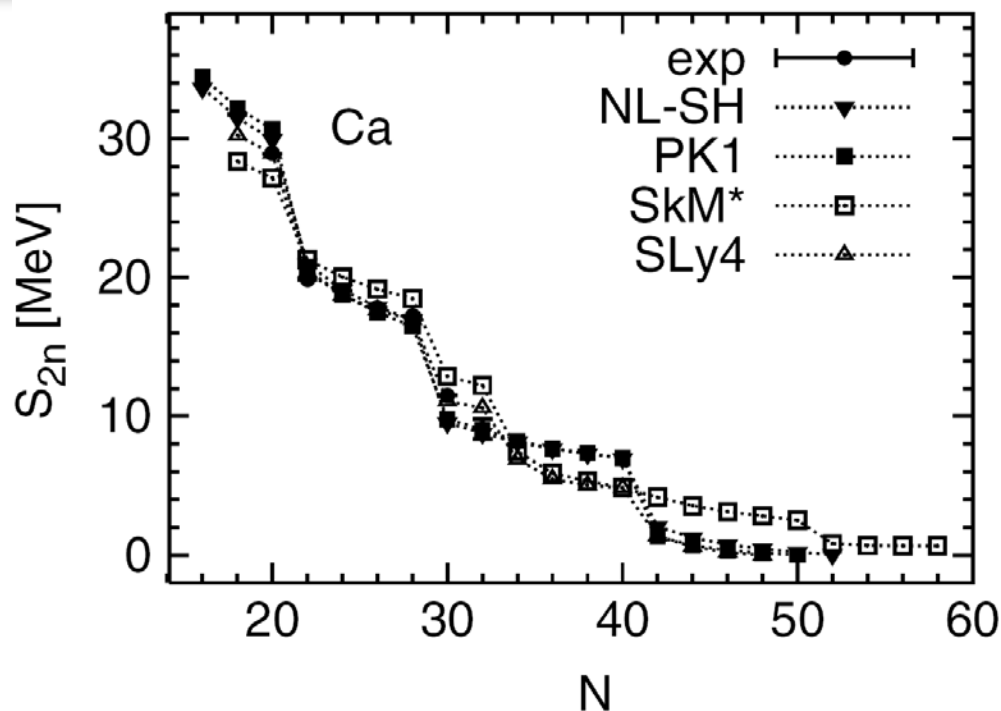




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# Giant halo from Skyrme HFB and RCHB



Giant halos from non-rela. HFB  
 Different predictions for drip line

Terasaki, Zhang, Zhou, & Meng,  
 PRC74 (2006) 054318

# Role of pion and Exchange term: DDRHFB

## ➤ RHFB equation

$$\int d\mathbf{r}' \begin{pmatrix} h(\mathbf{r}, \mathbf{r}') - \lambda & \Delta(\mathbf{r}, \mathbf{r}') \\ \Delta(\mathbf{r}, \mathbf{r}') & -h(\mathbf{r}, \mathbf{r}') + \lambda \end{pmatrix} \begin{pmatrix} \psi_U(\mathbf{r}') \\ \psi_V(\mathbf{r}') \end{pmatrix} = E \begin{pmatrix} \psi_U(\mathbf{r}) \\ \psi_V(\mathbf{r}) \end{pmatrix}$$

## ➤ Single particle Hamiltonian: $h = h^{\text{kin}} + h^{\text{D}} + h^{\text{E}}$

Kinetic energy:  $h^{\text{kin}}(\mathbf{r}, \mathbf{r}') = [\boldsymbol{\alpha} \cdot \mathbf{p} + \beta M] \delta(\mathbf{r}, \mathbf{r}')$ ,

Local potentials:  $h^{\text{D}}(\mathbf{r}, \mathbf{r}') = [\Sigma_T(\mathbf{r})\gamma_5 + \Sigma_0(\mathbf{r}) + \beta\Sigma_S(\mathbf{r})] \delta(\mathbf{r}, \mathbf{r}')$ ,

Non-local Potentials:  $h^{\text{E}}(\mathbf{r}, \mathbf{r}') = \begin{pmatrix} Y_G(\mathbf{r}, \mathbf{r}') & Y_F(\mathbf{r}, \mathbf{r}') \\ X_G(\mathbf{r}, \mathbf{r}') & X_F(\mathbf{r}, \mathbf{r}') \end{pmatrix}$

## ➤ Pairing Force: Gogny D1S

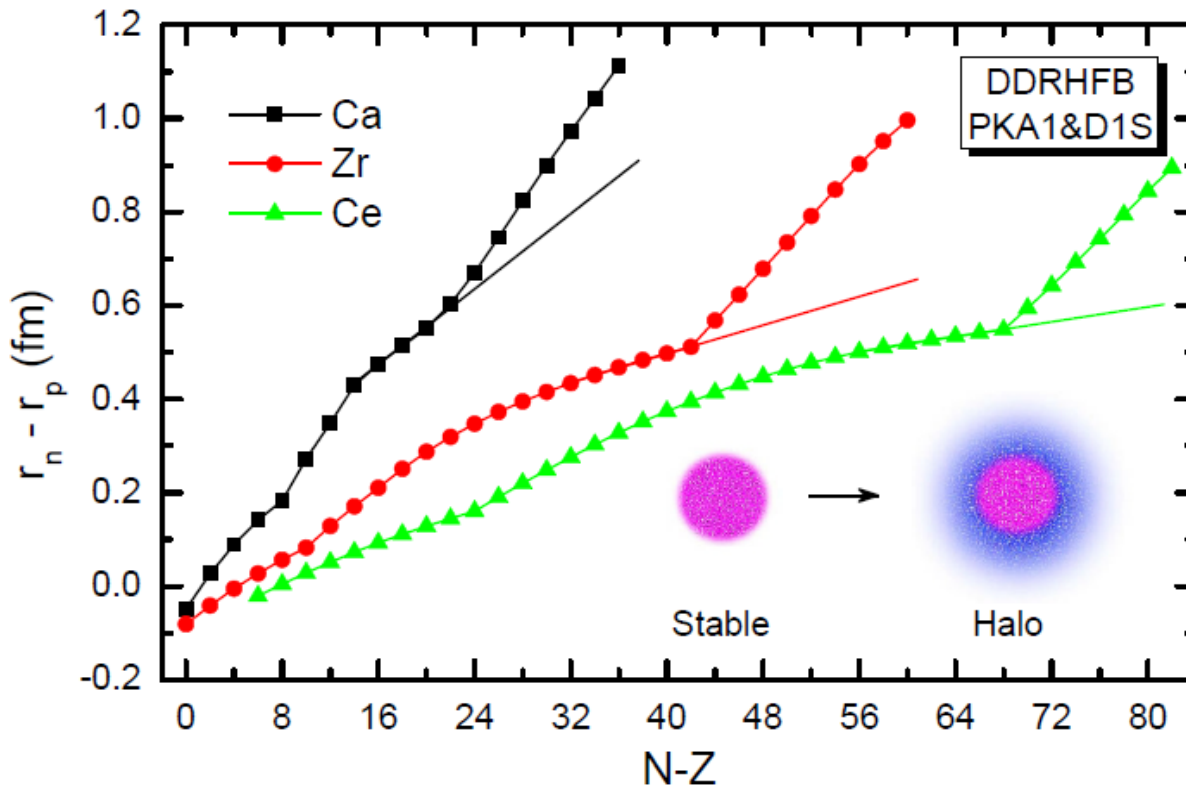
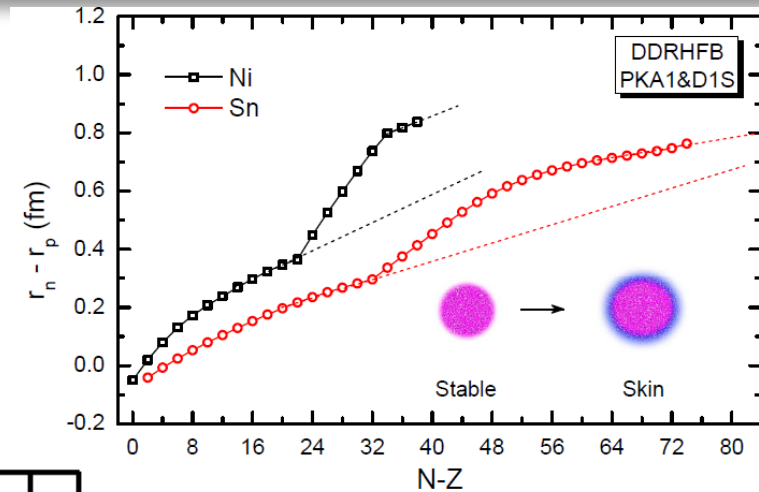
$$V(\mathbf{r}, \mathbf{r}') = \sum_{i=1,2} e^{((r-r')/\mu_i)^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau)$$

## ➤ Dirac Woods-Saxon Basis S.-G. Zhou (2003)

➔ To Solve the integro-differential RHFB equation

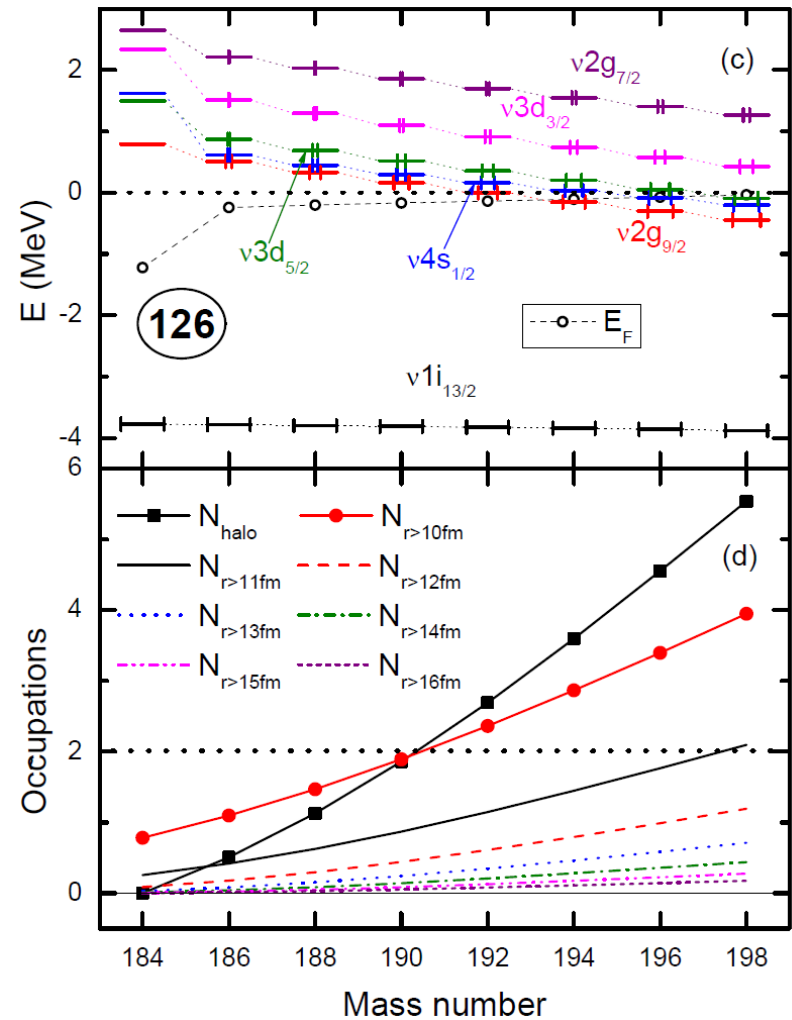
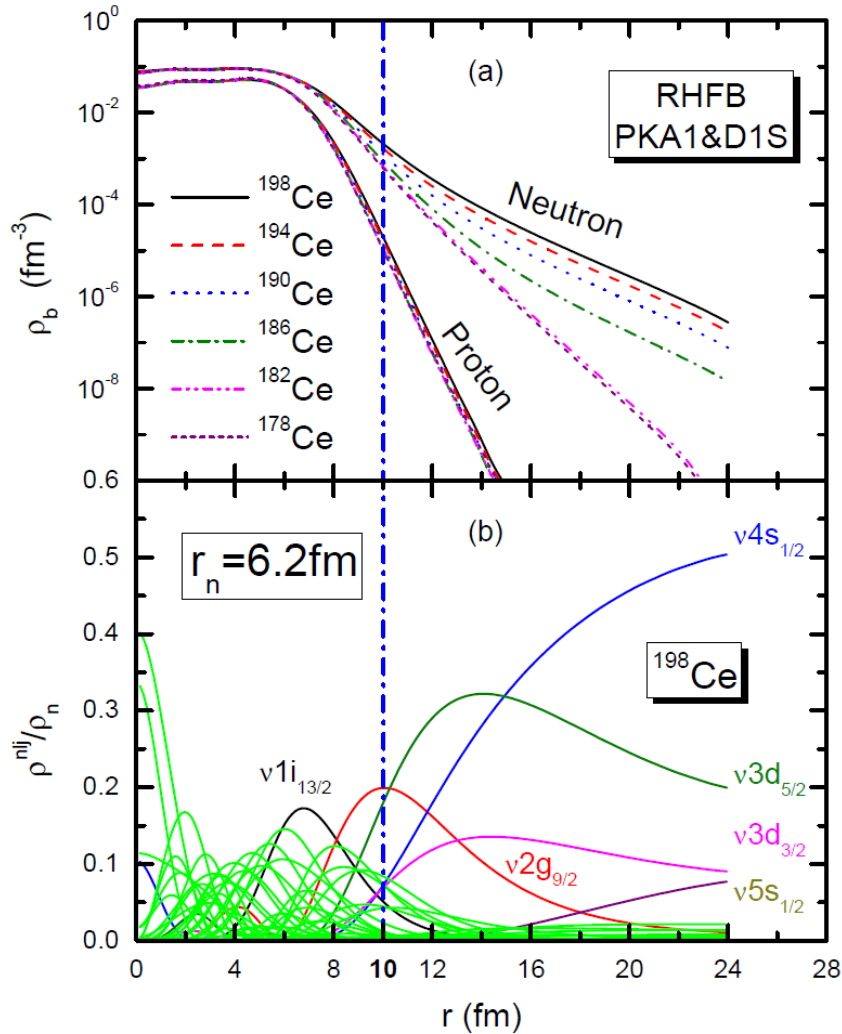
# Halo and giant halo in DDRHFB

1. Long, Ring, Giai, and Meng, Physical Review C 81, 024308 (2010)
2. Long, Ring, Meng, Giai, and Bertulani, Physical Review C 81, 031302(R) (2010)



# Halo structures in Cerium isotopes

Halos:  $^{186, 188, 190}\text{Ce}$ ; Giant halos:  $^{192, 194, 196, 198}\text{Ce}$



# Continuum Skyrme HFB with Green's function method

## ❖ Density in discretized and continuum HFB approach

$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') \equiv \langle \Phi_0 | \psi^\dagger(\mathbf{r}'\sigma') \psi(\mathbf{r}\sigma) | \Phi_0 \rangle$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') \equiv \langle \Phi_0 | \psi(\mathbf{r}'\tilde{\sigma}') \psi(\mathbf{r}\sigma) | \Phi_0 \rangle$$



$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \sum_{0 < E_i < |\lambda|} \varphi_2(E_i, \mathbf{r}\sigma) \varphi_2^*(E_i, \mathbf{r}'\sigma') + \int_{|\lambda|}^{\infty} dE \varphi_2(E, \mathbf{r}\sigma) \varphi_2^*(E, \mathbf{r}'\sigma')$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = - \sum_{0 < E_i < |\lambda|} \varphi_2(E_i, \mathbf{r}\sigma) \varphi_1^*(E_i, \mathbf{r}'\sigma') - \int_{|\lambda|}^{\infty} dE \varphi_2(E, \mathbf{r}\sigma) \varphi_1^*(E, \mathbf{r}'\sigma')$$

discretized HFB



continuum HFB



$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \sum_{0 < E_i < E_{\text{cut}}} \varphi_2(E_i, \mathbf{r}\sigma) \varphi_2^*(E_i, \mathbf{r}'\sigma')$$

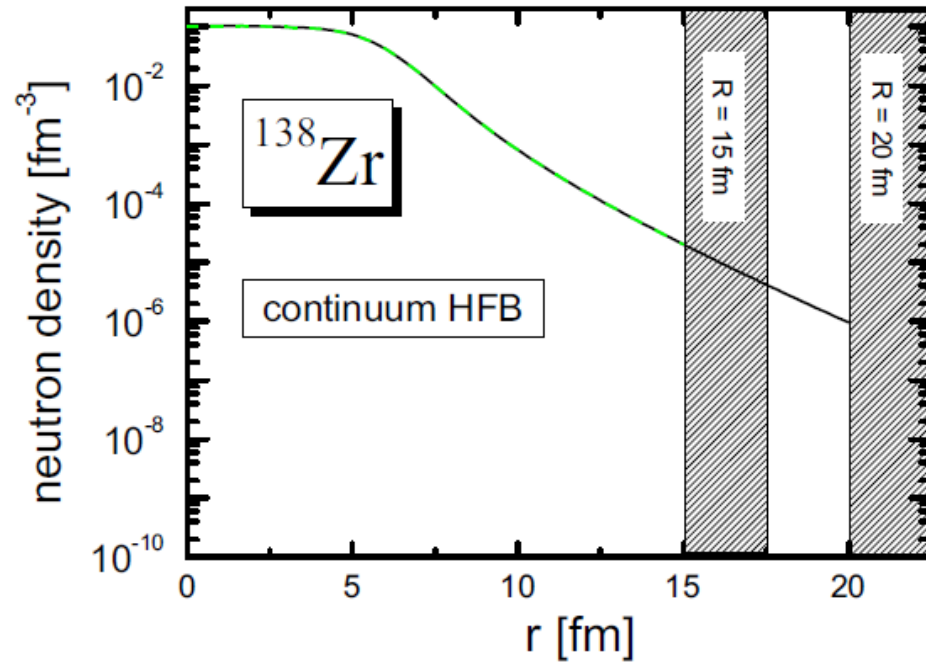
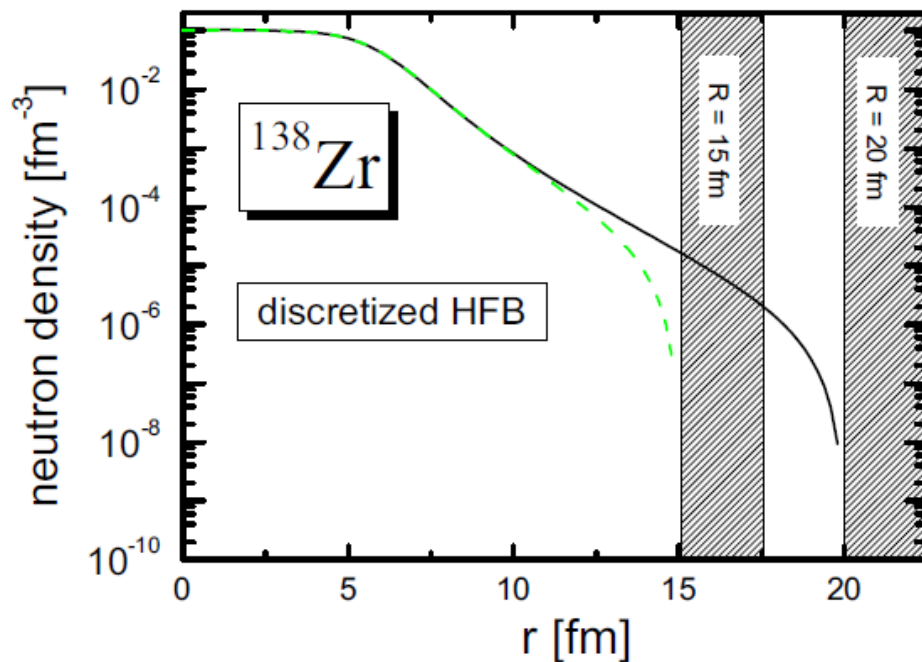
$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \frac{1}{2\pi i} \oint_{C_{E < 0}} dE \varphi_2(E, \mathbf{r}\sigma) \varphi_2^*(E, \mathbf{r}'\sigma')$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = - \sum_{0 < E_i < E_{\text{cut}}} \varphi_2(E_i, \mathbf{r}\sigma) \varphi_1^*(E_i, \mathbf{r}'\sigma')$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = - \frac{1}{2\pi i} \oint_{C_{E < 0}} dE \varphi_2(E, \mathbf{r}\sigma) \varphi_1^*(E, \mathbf{r}'\sigma')$$

# Continuum Skyrme HFB with Green's function method

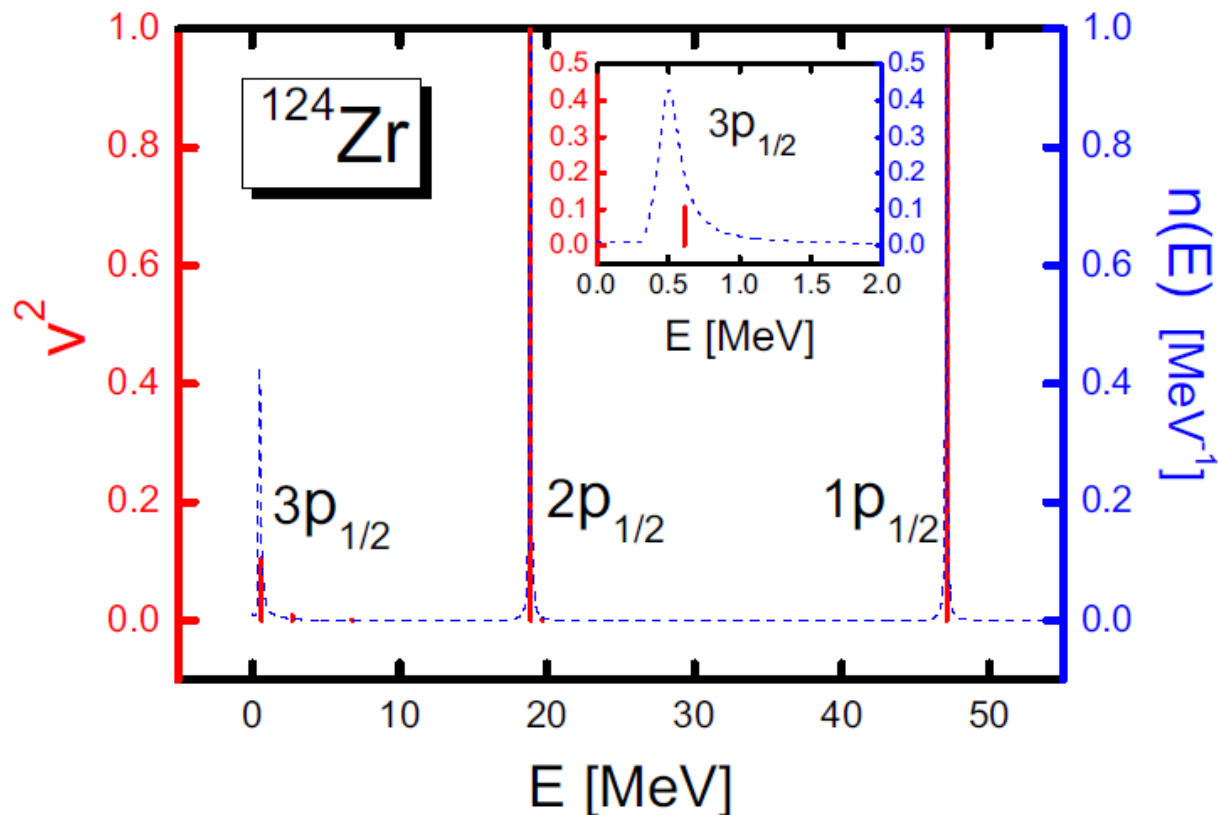
## ❖ Density obtained from discretized and continuum HFB approach



independent on the box size for continuum HFB

# Continuum Skyrme HFB with Green's function method

- ❖  $n(E)$ : occupation number density by continuum HFB cal.
- ❖  $v^2$ : occupation probability by discretized HFB cal.

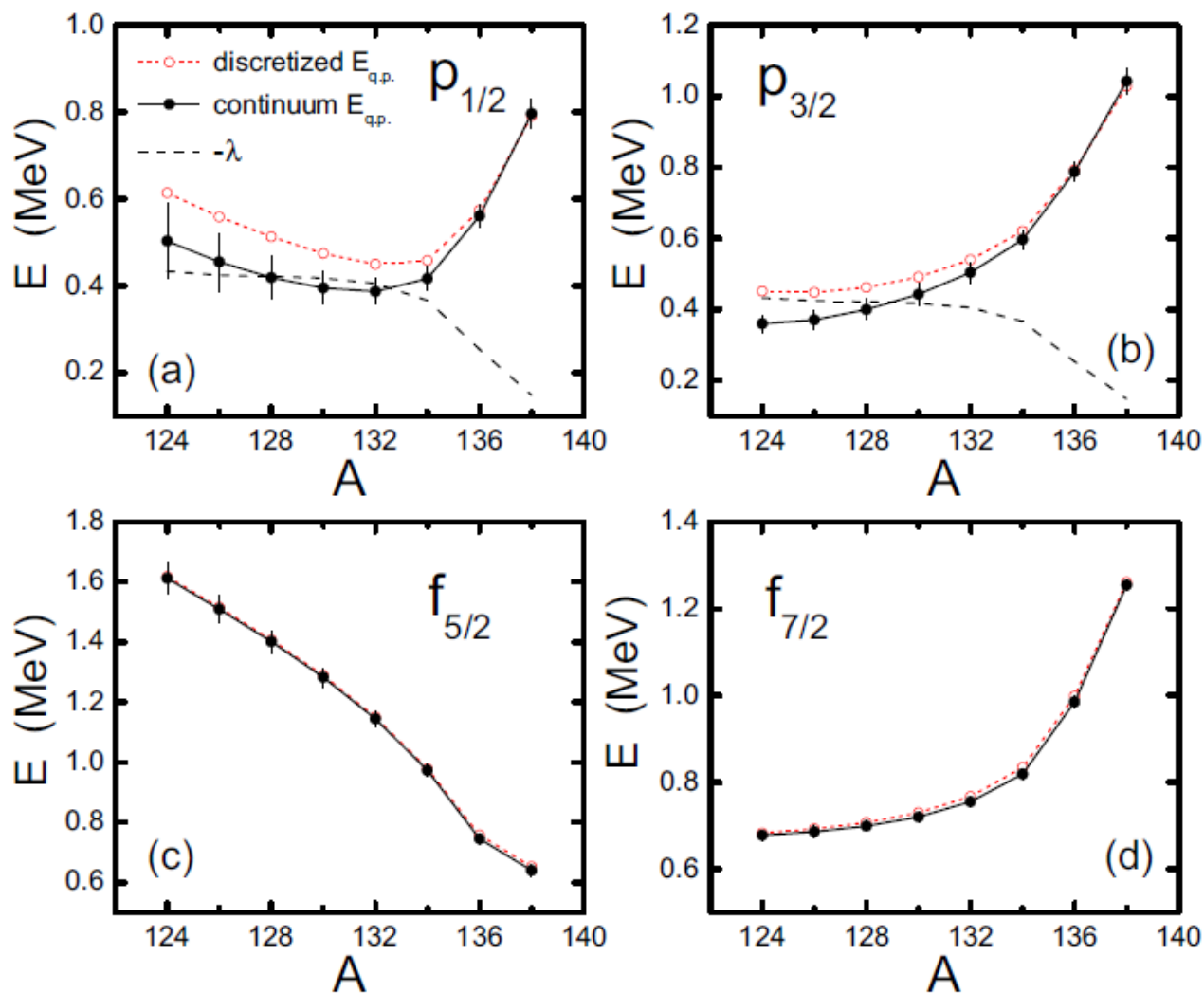


New information: width of q.p. resonance for continuum HFB



# Continuum Skyrme HFB with Green's function method

## ❖ Quasiparticle resonance



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# Deformed RHB in a Woods-Saxon basis

Axially deformed nuclei

$$\beta_{km}^+ = \sum_{(i\kappa)} u_{k,(i\kappa)}^{(m)} a_{i\kappa m}^+ + v_{k,(i\tilde{\kappa})}^{(m)} \tilde{a}_{i\kappa m}$$

$$\begin{pmatrix} U_k^{(m)}(\mathbf{r}\sigma p) \\ V_k^{(m)}(\mathbf{r}\sigma p) \end{pmatrix} = \sum_{i\kappa} \begin{pmatrix} u_{k,(i\kappa)}^{(m)} \varphi_{i\kappa m}(\mathbf{r}\sigma p) \\ v_{k,(i\tilde{\kappa})}^{(m)} \tilde{\varphi}_{i\kappa m}(\mathbf{r}\sigma p) \end{pmatrix}$$

$$\varphi_{i\kappa m}(\mathbf{r}\sigma p) = \frac{1}{r} \begin{pmatrix} iG_{i\kappa}(r)Y_{\kappa m}(\Omega\sigma) \\ -F_{i\kappa}(r)Y_{\kappa m}(\Omega\sigma) \end{pmatrix}$$

$$\sum_{\sigma p} \int d^3r' \begin{pmatrix} h(\mathbf{r}\sigma p; \mathbf{r}'\sigma' p') - \lambda & \Delta(\mathbf{r}\sigma p; \mathbf{r}'\sigma' p') \\ -\Delta^*(\mathbf{r}\sigma p; \mathbf{r}'\sigma' p') & -h(\mathbf{r}\sigma p; \mathbf{r}'\sigma' p') + \lambda \end{pmatrix} \begin{pmatrix} U_E(\mathbf{r}'\sigma' p') \\ V_E(\mathbf{r}'\sigma' p') \end{pmatrix} = E \begin{pmatrix} U_E(\mathbf{r}\sigma p) \\ V_E(\mathbf{r}\sigma p) \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix} = E \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix}$$

$$\mathbf{U} = \begin{pmatrix} u_{k,(i\kappa)}^{(m)} \end{pmatrix}$$

$$\mathbf{V} \equiv \begin{pmatrix} v_{k,(i\tilde{\kappa})}^{(m)} \end{pmatrix}$$

# DRHB matrix elements

$$A_{(i\kappa),(i'\kappa')} = \left( h_{(i\kappa),(i'\kappa')}^{(m)} \right) - \lambda \mathbf{I}$$

$$B_{(i\kappa),(i'\tilde{\kappa}')} = \left( \Delta_{(i\kappa),(i'\tilde{\kappa}')}^{(m)} \right)$$

$$C_{(i\tilde{\kappa}),(i'\kappa')} = \left( -\Delta_{(i\tilde{\kappa}),(i'\kappa')}^{(m)} = \Delta_{(i\kappa),(i'\tilde{\kappa}')}^{(m)} \right)$$

$$D_{(i\tilde{\kappa}),(i'\tilde{\kappa}')} = \left( -h_{(i\tilde{\kappa}),(i'\tilde{\kappa}')}^{(m)} \right) + \lambda \mathbf{I}$$

$$V(\mathbf{r}) = \sum_{\lambda} V_{\lambda}(\mathbf{r}) Y_{\lambda}(\Omega) \quad S(\mathbf{r}) = \sum_{\lambda} S_{\lambda}(\mathbf{r}) Y_{\lambda}(\Omega)$$

$$h_{(i\kappa),(i'\kappa')}^{(m)} = \sum_{\lambda} \int dr \{ G_{i\kappa}(\mathbf{r}) G_{i'\kappa'}(\mathbf{r}) [V_{\lambda}(\mathbf{r}) + S_{\lambda}(\mathbf{r})] + F_{i\kappa}(\mathbf{r}) F_{i'\kappa'}(\mathbf{r}) [V_{\lambda}(\mathbf{r}) - S_{\lambda}(\mathbf{r})] \} A(\lambda, \kappa, \kappa', m)$$

$$\Delta(\mathbf{r}, \sigma_1 \sigma_2) = \sum_{\lambda} \sum_{S} \sum_{s} Y_{\lambda}(\Omega) \chi_{S_s}(\sigma_1 \sigma_2) \Delta_{\mu\lambda}^S{}_{;p_1 p_2 M}(\mathbf{r})$$

$$\lambda, \text{ even or odd} \\ \mu = 0, \pm 1$$

$$\Delta_{(i_1 \kappa_1),(i_2 \tilde{\kappa}_2)}^{(m)} = \frac{1}{2} \sum_{\lambda\mu} \sum_{SM_S} \delta_{M_S, -\mu} \sum_{P_1 P_2} \eta_{\lambda\mu; \alpha_1 p_1 \bar{\alpha}_2 p_2}^{SM_S} \int dr R_{i_1 \kappa_1}^{p_1}(\mathbf{r}) R_{i_2 \kappa_2}^{p_2}(\mathbf{r}) \Delta_{\lambda\mu; p_1 p_2}^{SM_S}(\mathbf{r})$$

# Pairing interaction

- ❖ Phenomenological pairing interaction with parameters:  $V_0$ ,  $\rho_0$ ,  $\gamma$ , and the smooth cut off parameters  $E_{\text{cut}}$  and  $\Gamma$

$$V^{\text{pair}} = \frac{1}{4} V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \left( 1 - \frac{\rho(\mathbf{r}_1)}{\rho_0} \right)^\gamma$$

$$s(E_k) = \frac{1}{2} \left( 1 - \frac{E_k - E_{\text{cut}}^{\text{q.p.}}}{\sqrt{(E_k - E_{\text{cut}}^{\text{q.p.}})^2 + (\Gamma_{\text{cut}}^{\text{q.p.}})^2}} \right)$$

Finite range?

Volume or surface?

Microscopic?

PHYSICAL REVIEW C57 (1988) 1229

# How to fix the pairing strength and the pairing window

$^{20}\text{Mg}$ : spherical from DRHBWS calculation

NL3,  $R_{\text{max}} = 20 \text{ fm}$ ,  $\Delta r = 0.1 \text{ fm}$

Zero pairing energy for the neutron

Model	Pairing force	Parameters	$E_{\text{pair}}^{\text{p}}$ (MeV)
SRHBHO	Gogny	D1S	-9.2382
RCHB	Surface $\delta$	$V_0 = 374 \text{ MeV fm}^3$ $\rho_0 = 0.152 \text{ fm}^3$	-9.2387
	Sharp cutoff	$E_{\text{cut}}^{\text{q.p.}} = 60 \text{ MeV}$	
DRHBWS	Surface $\delta$	$V_0 = 380 \text{ MeV fm}^3$ $\rho_0 = 0.152 \text{ fm}^3$	-9.2383
	Smooth cutoff	$E_{\text{cut}}^{\text{q.p.}} = 60 \text{ MeV}$ $\Gamma = 5.65 \text{ MeV}$	

Zhou, Meng, Ring, Zhao  
PRC82(10)011301R

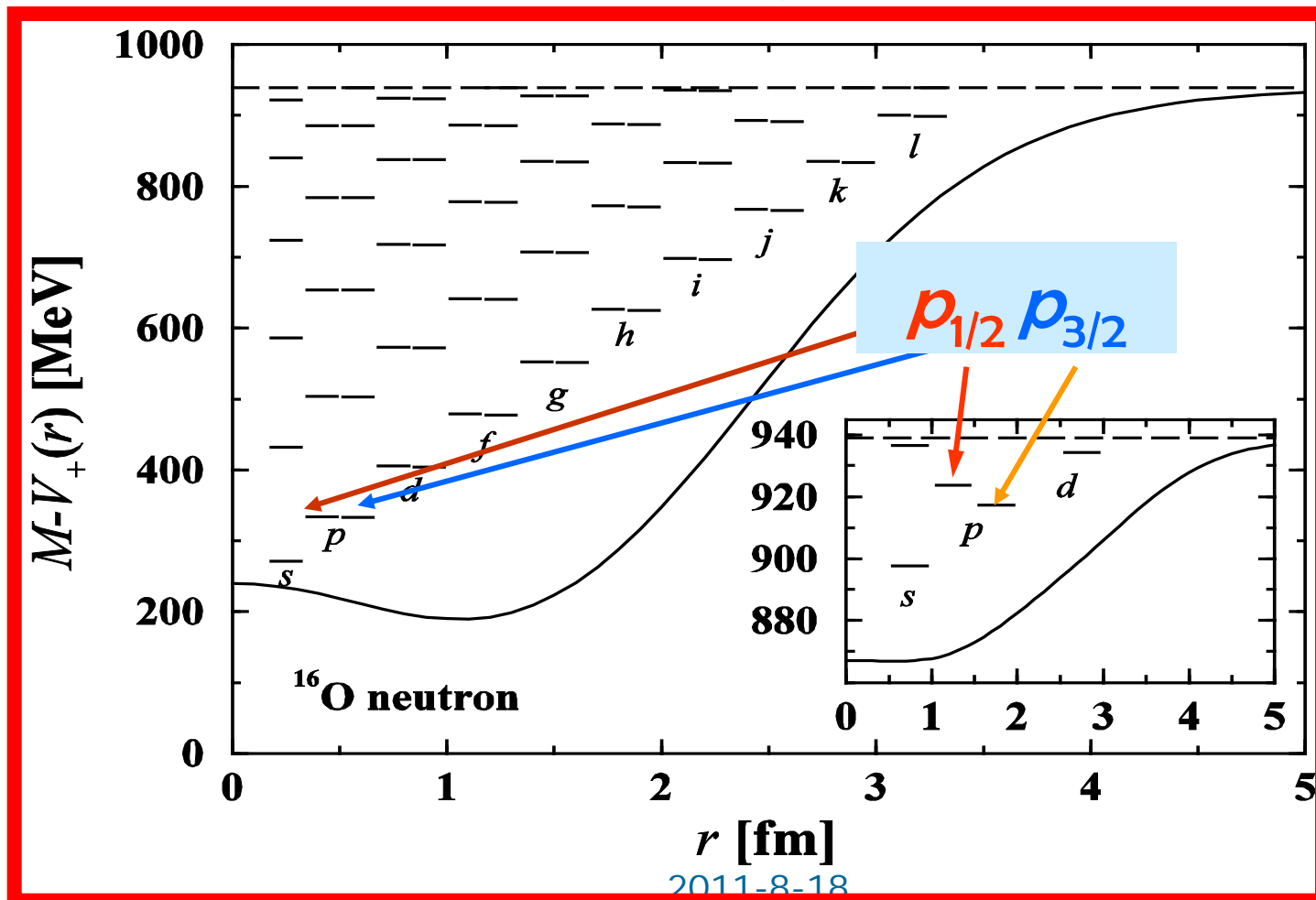
# RMF in a Woods-Saxon basis: progress

Shapes	Model	Schrödinger W-S basis	Dirac W-S basis	
Spherical	Rela. Hartree	SRH SWS	SRH DWS	✓
	Zhou, Meng & Ring, PRC68,034323(03); PRL91, 262501 (03)			
Axially deformed	Rela. Hartree + BCS		DRH DWS	✓
	Zhou, Meng & Ring, AIP Conf. Proc. 865, 90 (06)			
Axially deformed	Rela. Hartree-Bogoliubov		DRHB DWS	✓
	Zhou, Meng, Ring, ISPUN 2007			
Triaxially deformed	Rela. Hartree-Bogoliubov		TRHB DWS	

Woods-Saxon basis might be a reconciler between the HO basis and r space

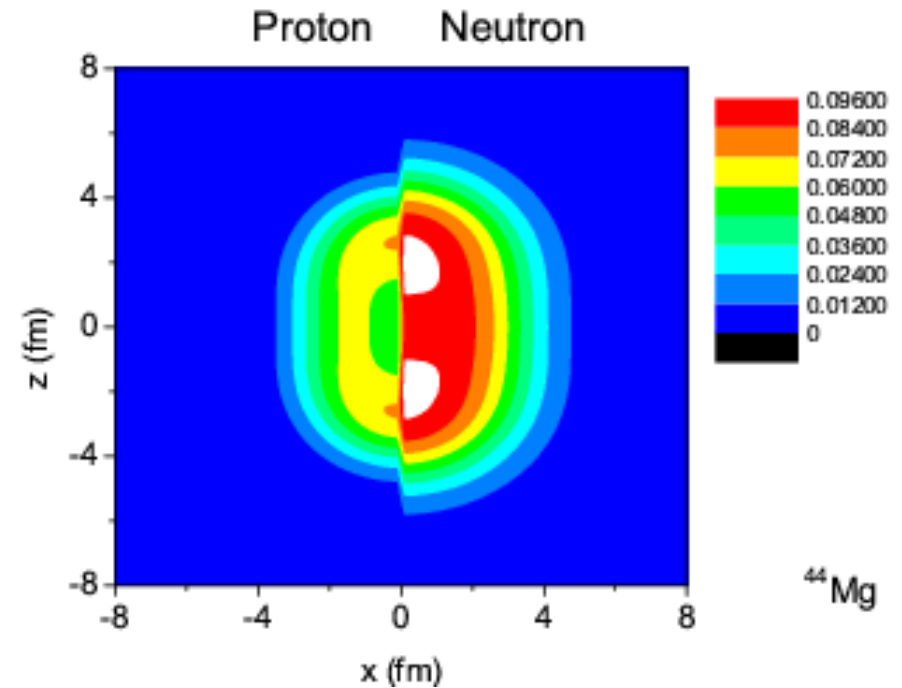
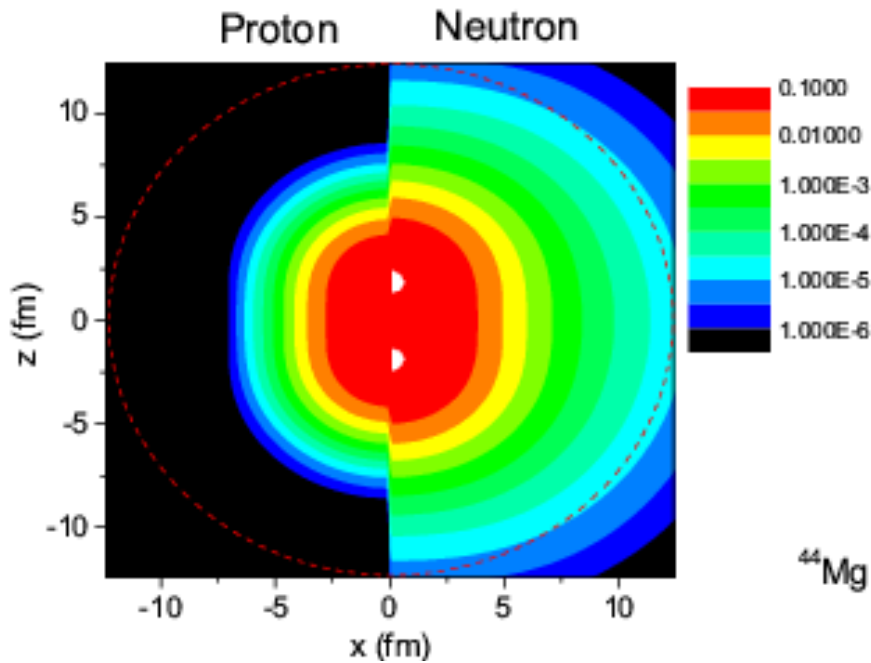
# Origin of the symmetry - Anti-nucleons

Zhou, Meng & Ring, PRL92(03)262501





# $^{44}\text{Mg}$ from DRHBWS



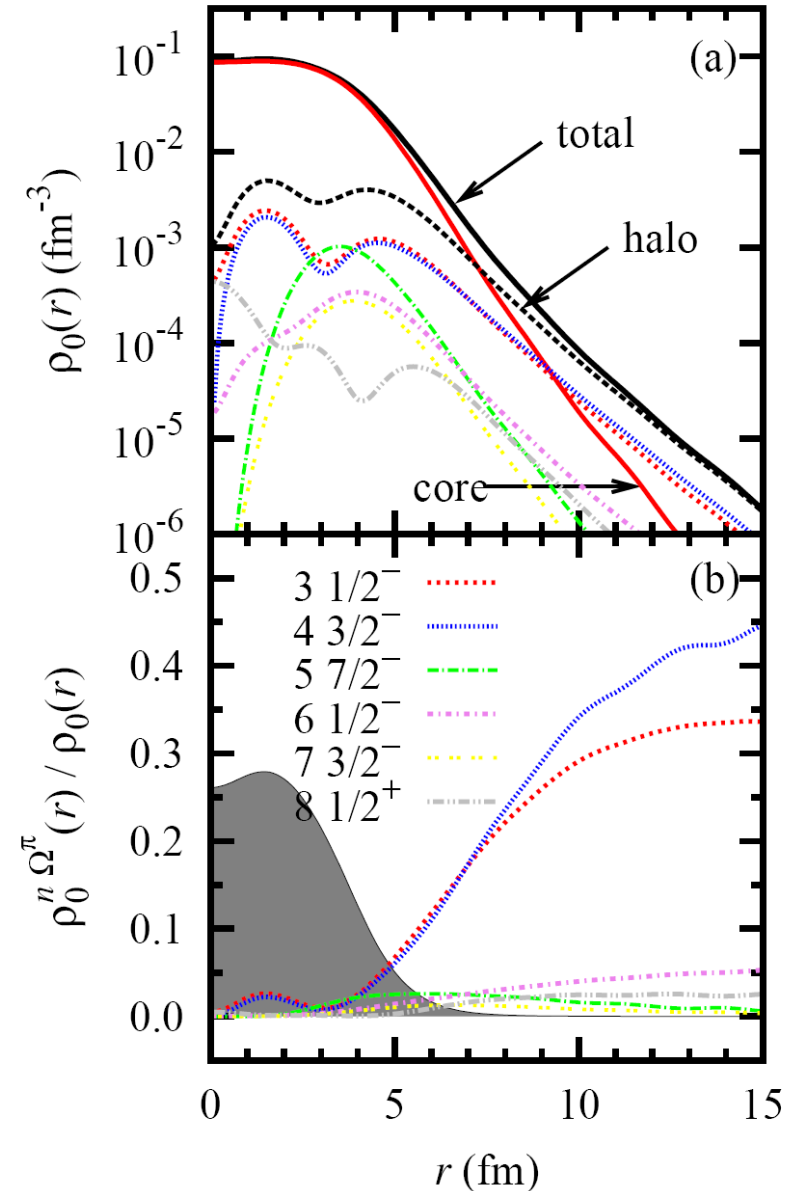
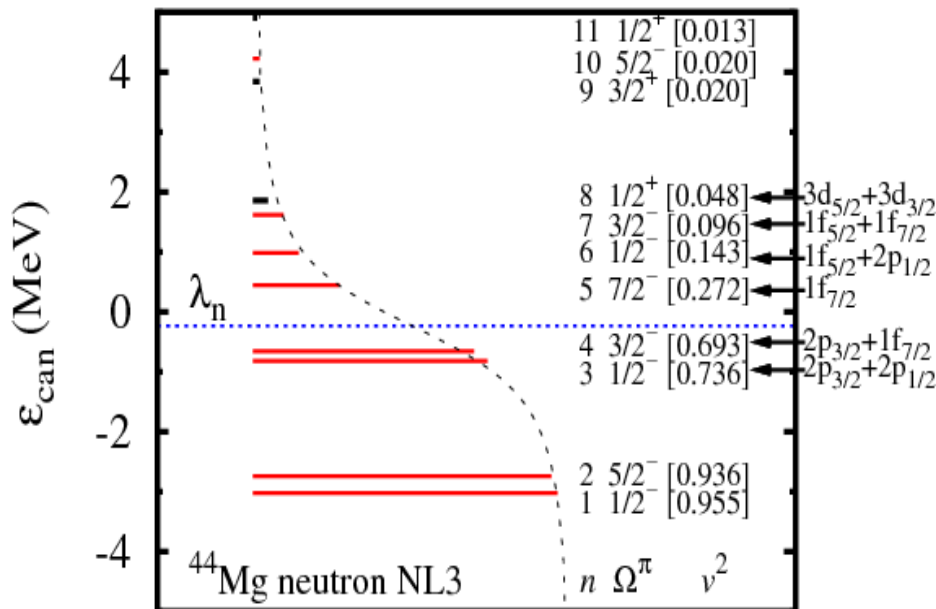
❖ **Prolate deformation**

❖ **Large spatial extension in neutron density distribution**

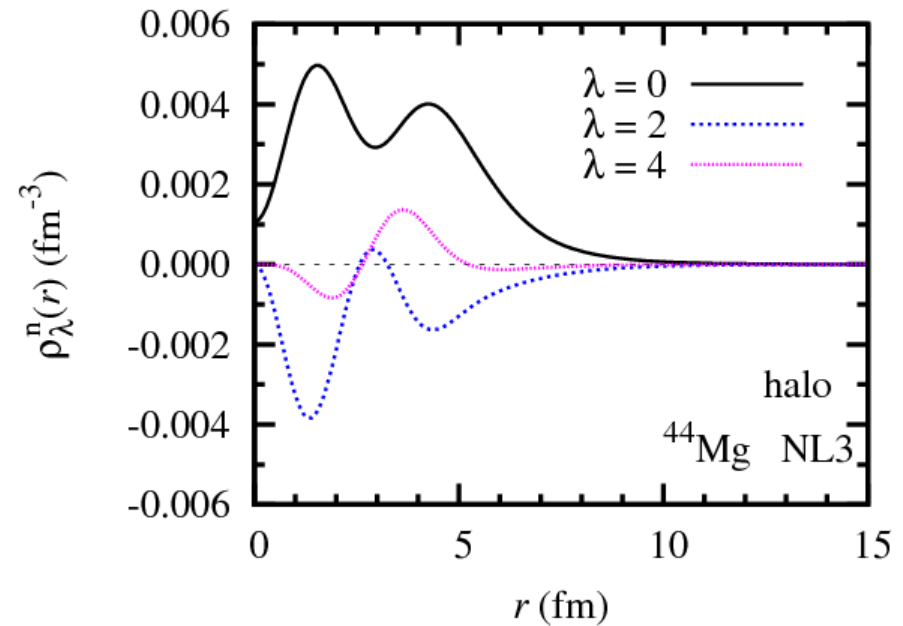
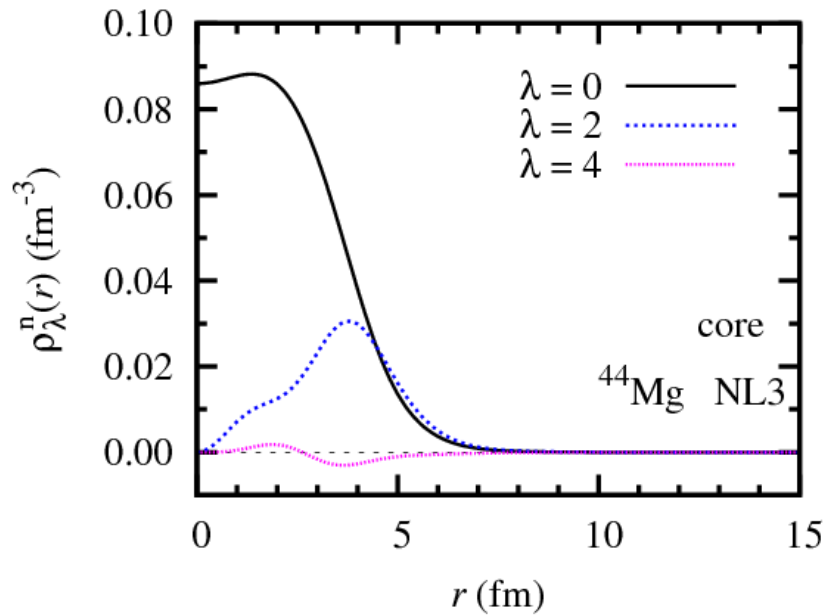
Zhou, Meng, Ring, Zhao  
PRC82(10)011301R

# Decomposition of neut. density distri.

- ❖ The 3rd & 4th states contribute to tail part of neutron density distribution
- ❖ Main component:  $2p_{3/2}$
- ❖  $R_{\text{core}} = 3.72$  fm,  $R_{\text{halo}} = 5.86$  fm

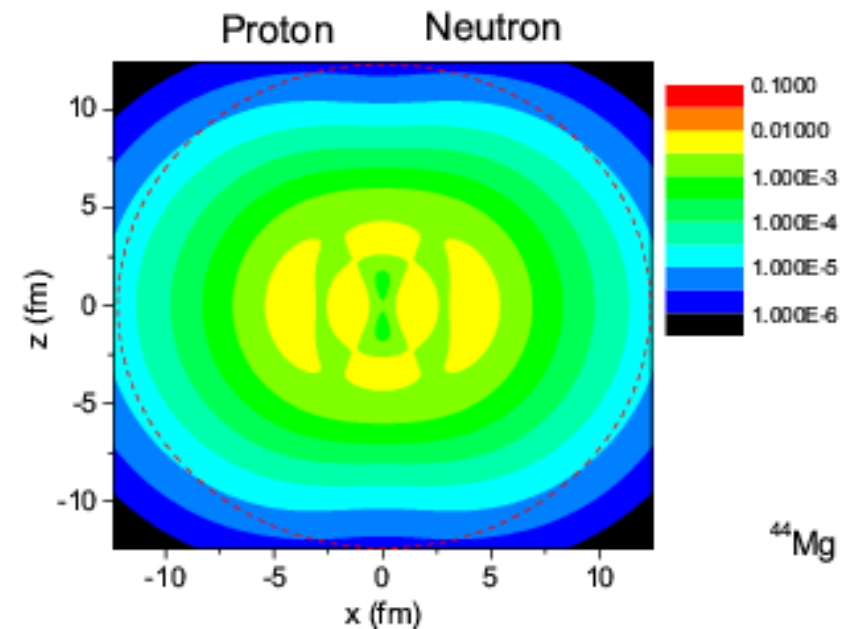
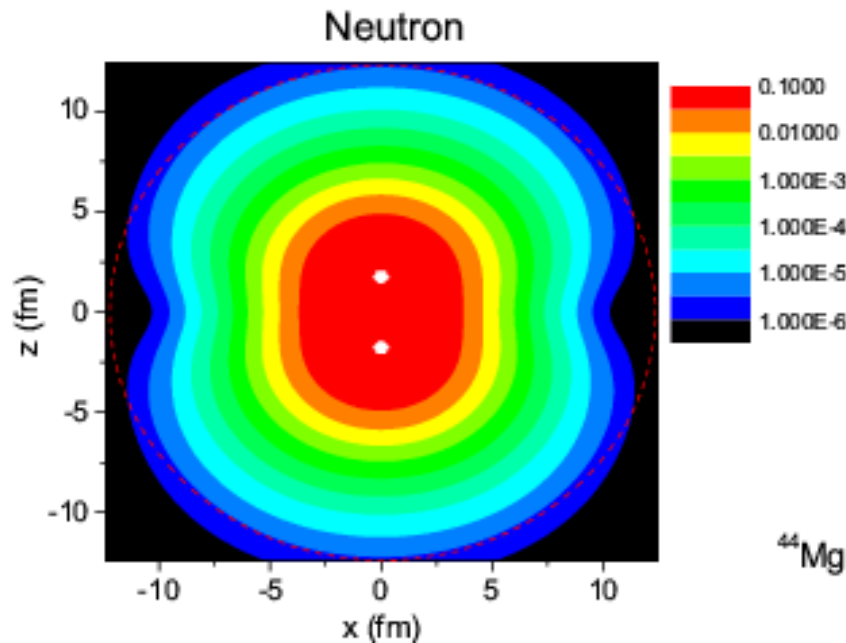
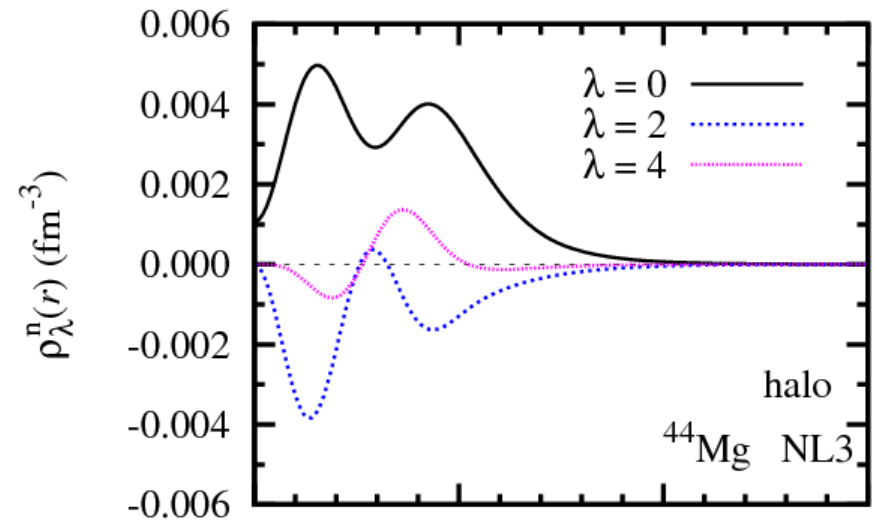
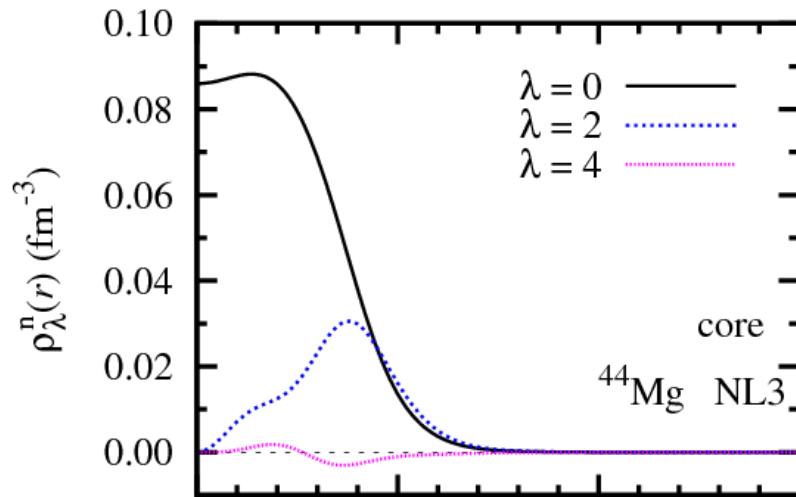


# Density of core & halo



- ❖ **Prolate core, but slightly oblate halo with sizable hexadecapole component !**
- ❖ **Decoupling of deformation betw. core & halo**

# Density of core & halo



# Outline

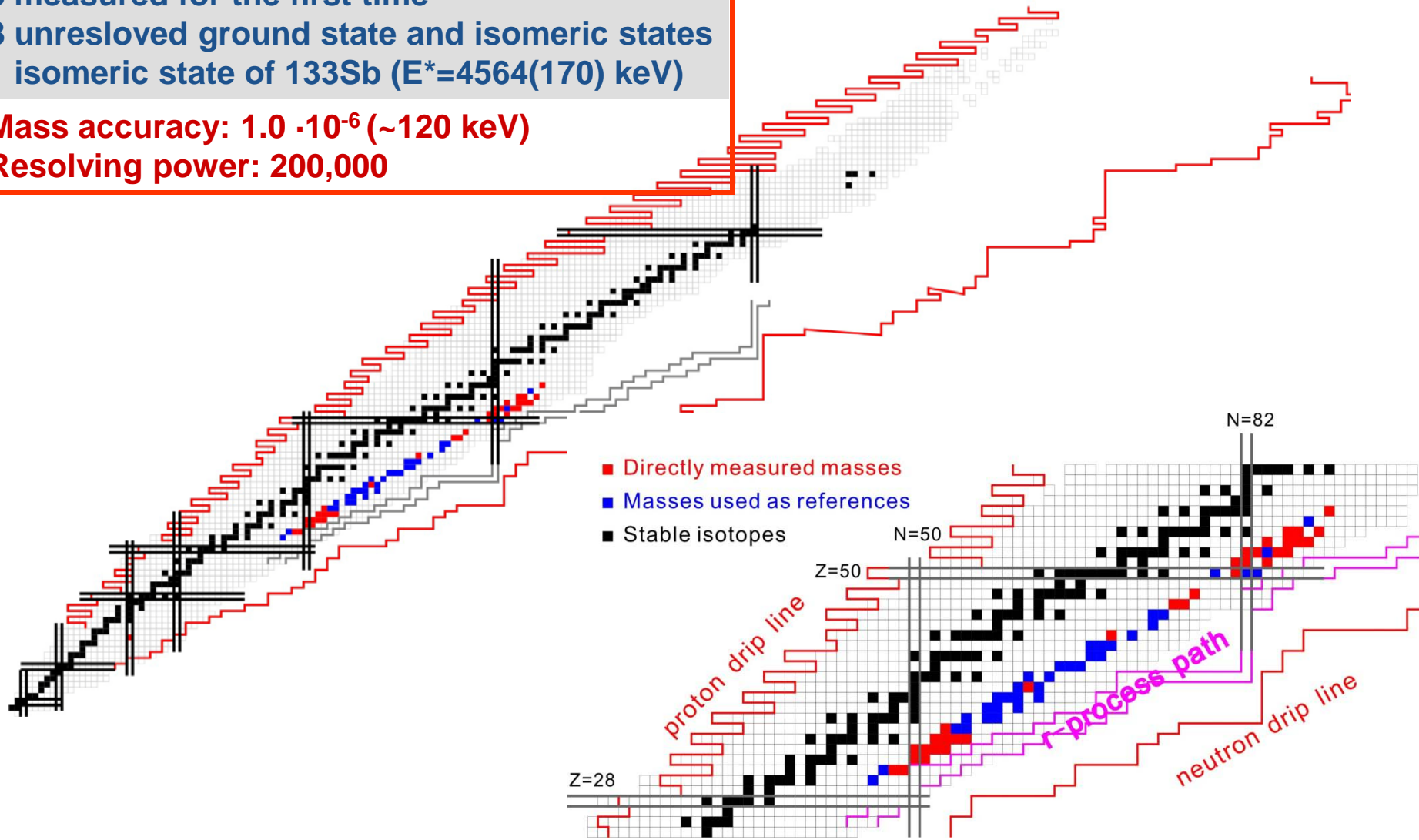
- Introduction
- Halos in Density Functional Theory: Skyrme HFB / DD  
RHFB / GF Skyrme HFB
- Deformed halos
- R-process calculation with CDFT Mass table
- Summary & Perspectives

# Short-Lived Neutron-Rich Nuclei with the Novel Large-Scale Isochronous Mass Spectrometry at the FRS-ESR Facility Sun et al. NPA 812: 1-12 , 2008

71 nuclides covered  
27 nuclides were measured  
8 measured for the first time  
8 unresolved ground state and isomeric states  
1 isomeric state of  $^{133}\text{Sb}$  ( $E^*=4564(170)$  keV)

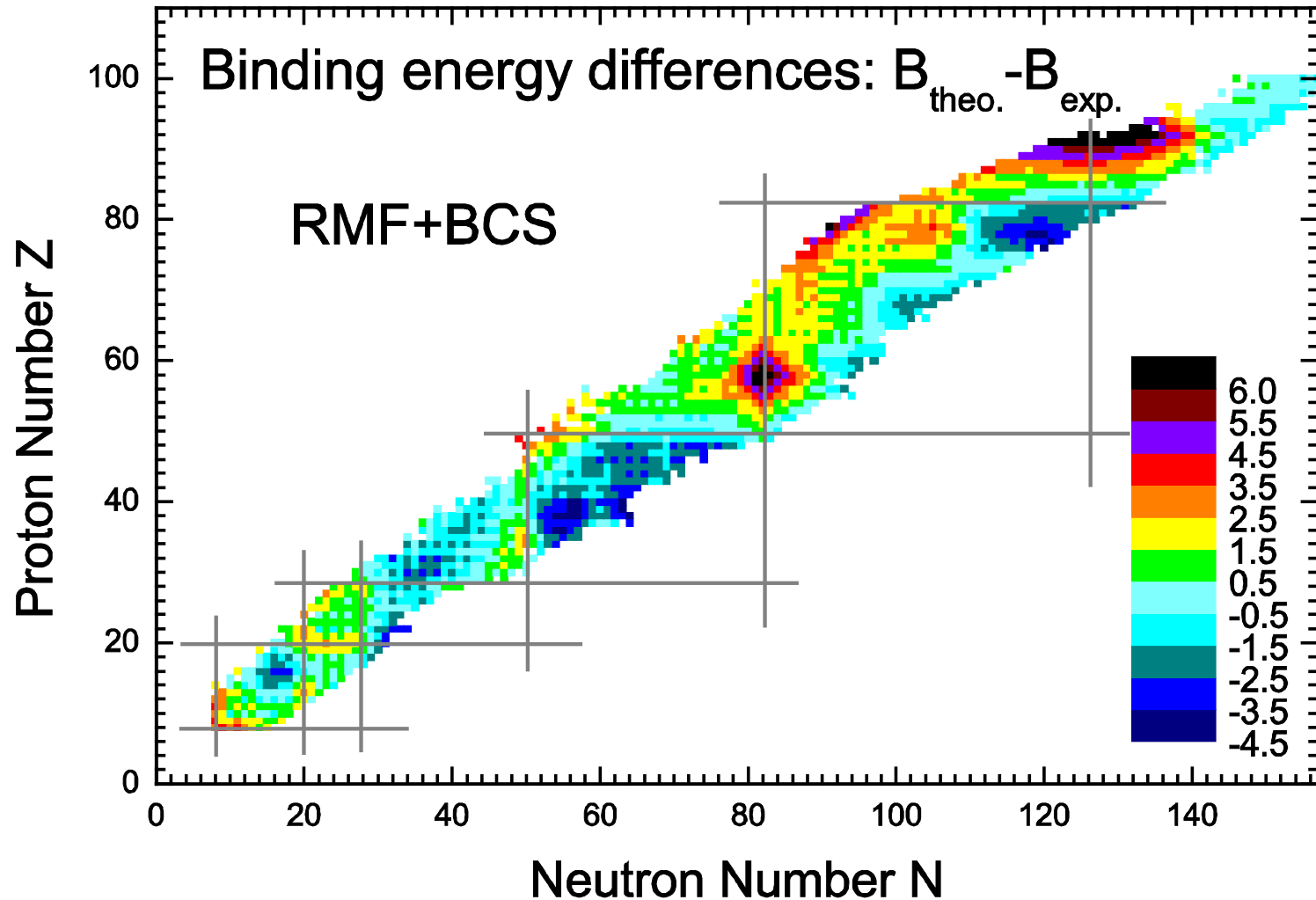
Mass accuracy:  $1.0 \cdot 10^{-6}$  (~120 keV)

Resolving power: 200,000



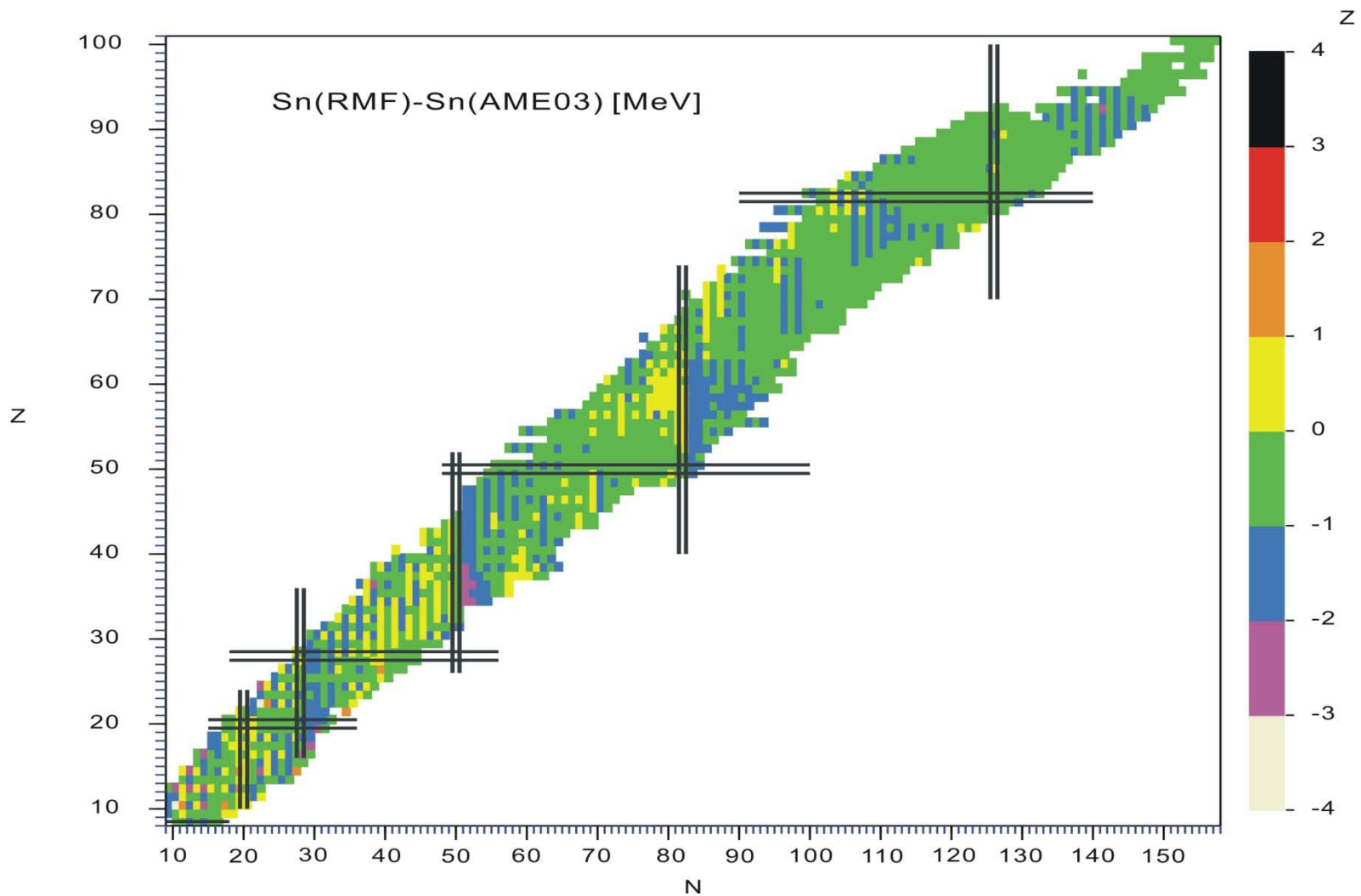
# First RMF mass table in 2005: Theo. vs. Exp.

rms deviation of nuclear masses : 2.1 MeV



# Nuclear single neutron separation energy: Theo. vs. Exp.

rms deviation of  $S_n$ : 0.654 MeV

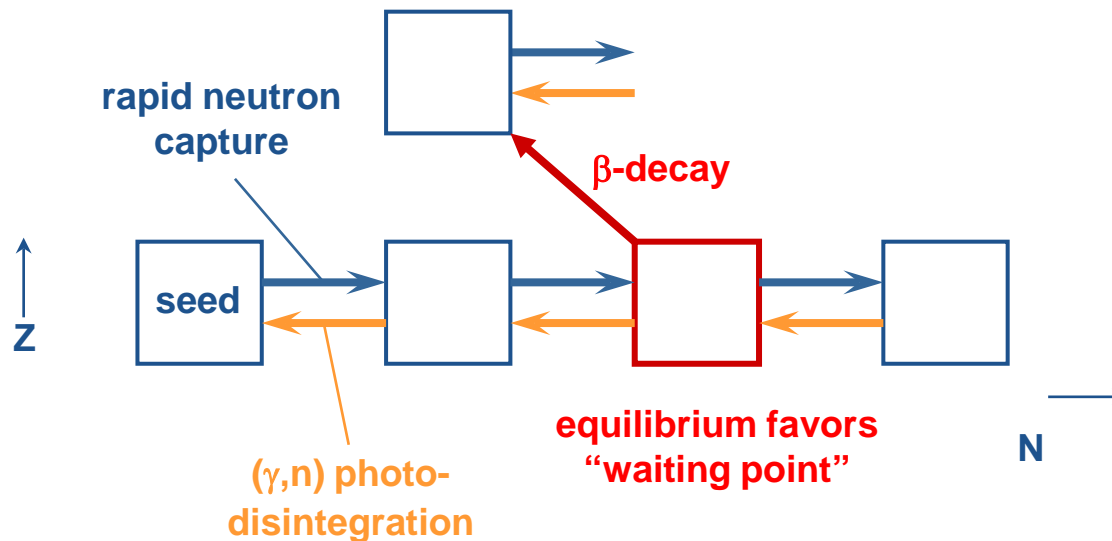




# Classical r-process calculation

Assume:

- $(n,\gamma) \leftrightarrow (\gamma,n)$  equilibrium within isotopic chain, and
- elemental distribution of neighboring z-chain is determined by the  $\beta$ -decays
- neglect the effect of fission
- constant  $T_9$ , multi r-process components with  $n_n=10^{20-27}$ .



The nucleus with maximum abundance in each isotopic chain has smaller neutron capture rate and must wait for the longer time to continue via  $\beta$ -decay

# Classical r-process calculation

## Astrophysical conditions:

$T_9=1.5$ ,  
 16-component fit with  $n_n=10^{20}-3*10^{27} \text{ cm}^{-3}$ ,  
 which fulfill the following equations:

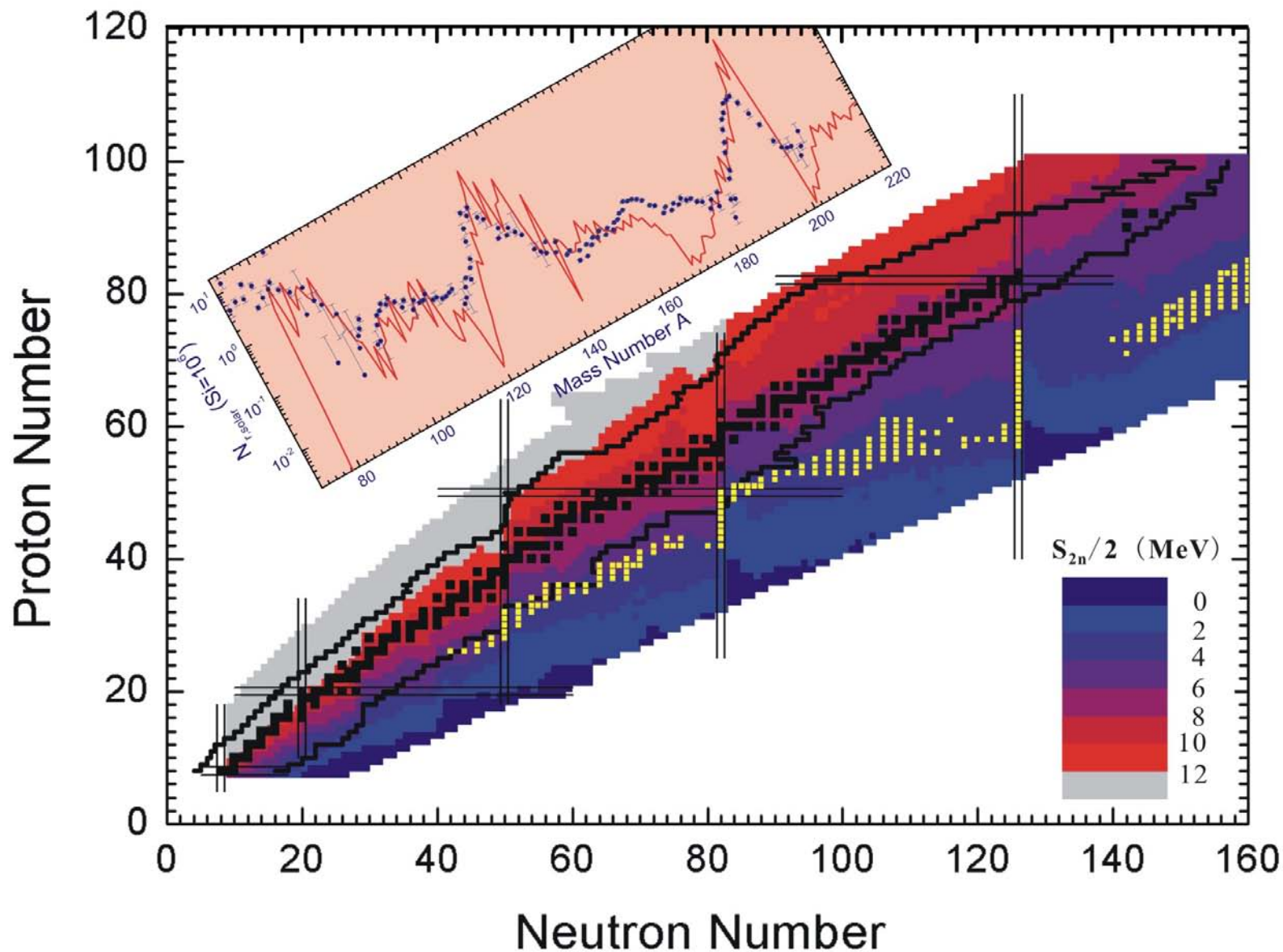
$$\omega(n_n) = n_n^a, \tau(n_n) = bn_n^c$$

where  $\omega$  and  $\tau$  are respectively the weight and neutron irradiation time, and  $a$ ,  $b$ ,  $c$  are the alterable parameters, which will be determined by least-square fit to the solar r-process abundance.

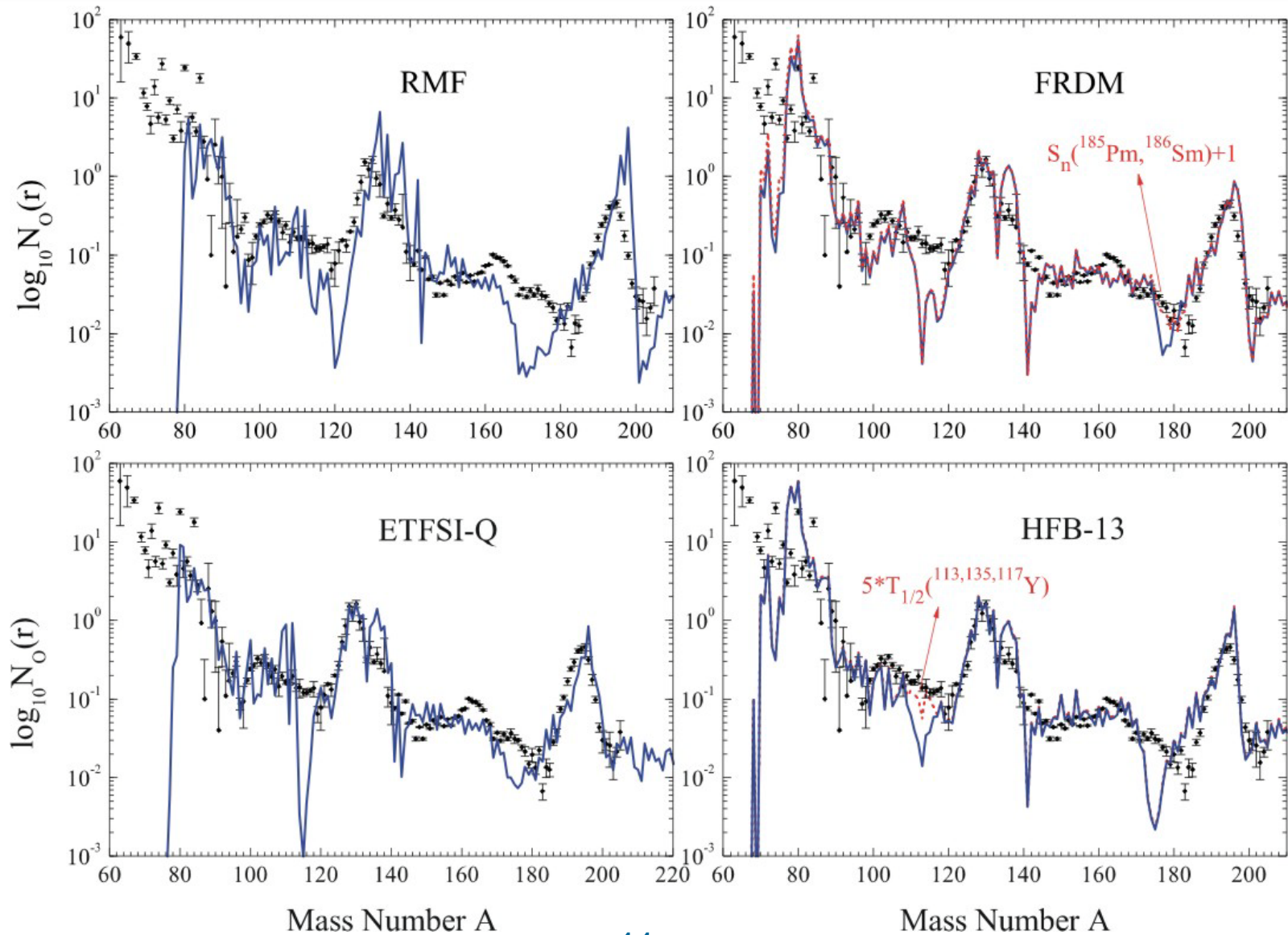


**Good approximation for astrophysical environment studies !**

nuclear inputs:  $S_n$ (RMF),  $T_{1/2}$ ( $\beta$ -decay),  $P_{1n}$ ,  $P_{2n}$ ,  $P_{3n}$  (FRDM),  
astrophysical parameters:  $T_9=1.5$ ,  $n_n=10^{20-28}$ ,  $\omega$ ,  $\tau$  (least-square fit),



# Nuclear Mass Model dependence



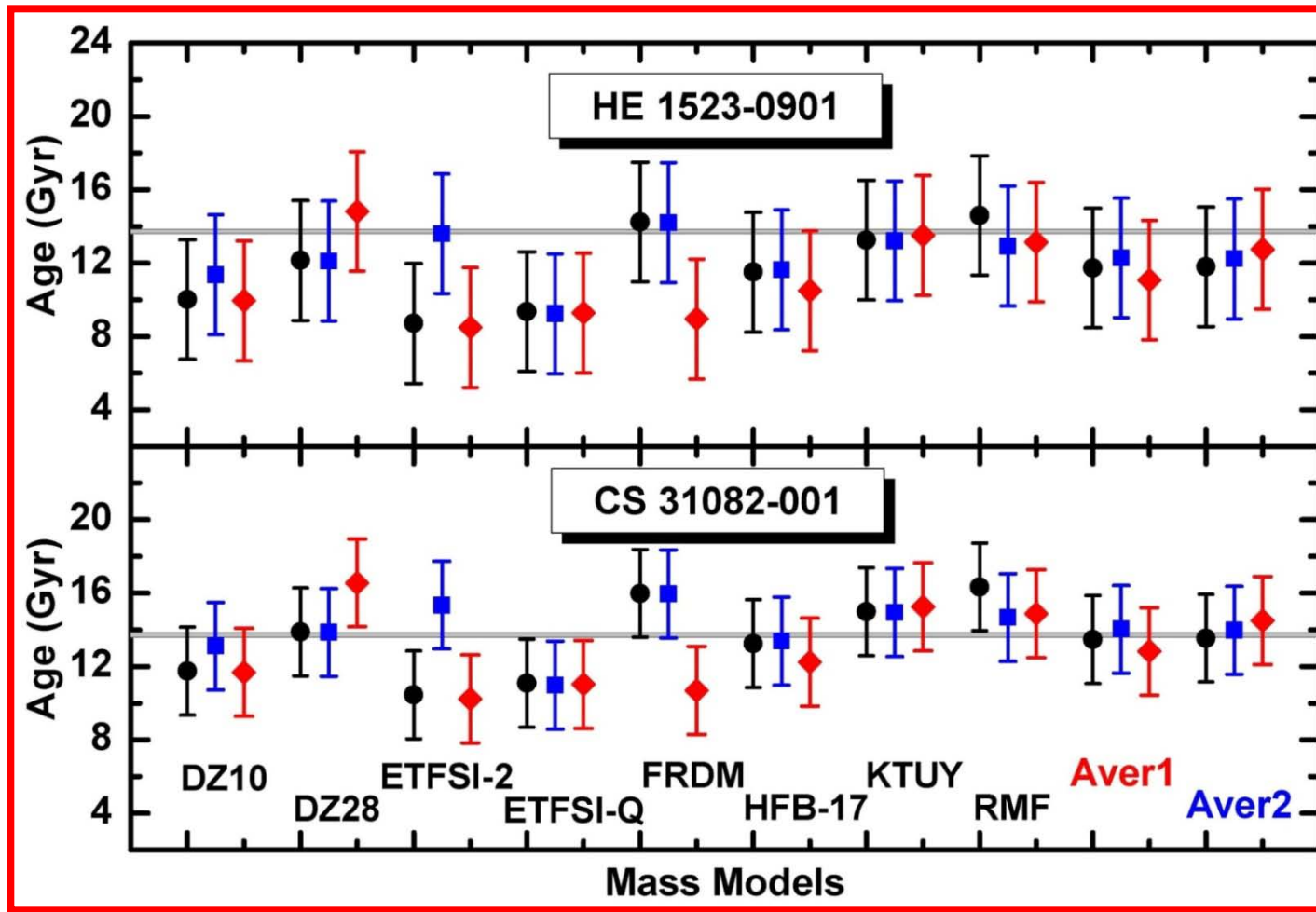
# Th/U chronometer

- The age of the universe is one of the most important physical quantities in cosmology.
- The metal-poor star is formed at the early stage of the universe, so its age provides constraint to the age of the universe.
- The age of metal-poor star:

$$\frac{Th}{U}_{\text{present}} = \frac{Th}{U}_{\text{initial}} e^{-(\lambda_{Th} - \lambda_U)t}$$

- **Present abundances: astronomical observations.**
  - R. Cayrel, et al., *Nature* 409, 691 (2001).
  - J.J. Cowan, et al., *ApJ* 572, 861 (2002).
  - A. Frebel, et al., *ApJ* 660, 117 (2007).
- **Initial abundances: r-process calculations (Th, U are r-only nuclei).**
- **The classical r-process model is usually employed in r-process calculations.**
  - P.A. Seeger, et al., *ApJS* 11, 121 (1965).
  - K.-L. Kratz, et al., *ApJ* 403, 216 (1993).

# Ages of metal-poor stars



Age (HE 1523-0901) =  $11.8 \pm 3.7$  Gyr    Age (CS 31082-001) =  $13.5 \pm 2.9$  Gyr  
 Z. Niu et al., PRC 80 065806 (2009)

APS » Journals » Physics » Synopses » Calibrating the cosmic clock

## Calibrating the cosmic clock



### Influence of nuclear physics inputs and astrophysical conditions on the Th/U chronometer

Zhongming Niu (牛中明), Baohua Sun (孙保华), and Jie Meng (孟杰)

Phys. Rev. C 80, 065806 (Published December 22, 2009)

• [Cosmology](#) • [Nuclear Physics](#)

Knowing when nucleosynthesis—the formation of new nuclei from existing nuclei—occurred in astrophysical sites can be crucial to our understanding of cosmology. One method to pin the process down in time is to compare the current abundance ratio of thorium to uranium (both of which have lifetimes of the order of the age of the universe) with calculations of this ratio at the time at which the nucleosynthesis that formed these elements took place. The assumption is that the nucleosynthesis itself happens over a time scale that is short compared to the time since it occurred.

In a paper published in *Physical Review C*, Zhongming Niu of Peking University and Baohua Sun and Jie Meng of Beihang University, both in China, present a study of the uncertainties in the calculation of the initial ratio of thorium to uranium. In particular, they focus on the importance of the models used to determine the nuclear masses and the nucleosynthesis processes themselves. Utilizing the abundances of uranium and thorium in the sun to restrict the models, Niu *et al.* are able to minimize the impact of the uncertainties. They find that the error due to the nuclear input alone is about 1.6–2.2 billion years (for reference, the age of the universe is about 14.6 billion years). This estimate is lower, but not significantly so, than the observational uncertainties. In addition, they determine when nucleosynthesis occurred for three stars. New observations for other elements in stars and improved mass models could make a major impact on the thorium-uranium chronometer, and in general, studies of this kind help us learn what parts of the universe were undergoing the extreme conditions needed for nucleosynthesis to occur, and when. — *William Gibbs*

### Coming Soon in *Physics*

- Symmetry breaking in bilayer graphene
- Discovery of a neutron halo in  $^{22}\text{C}$

### Now in Focus

#### Chemistry Drives Convection

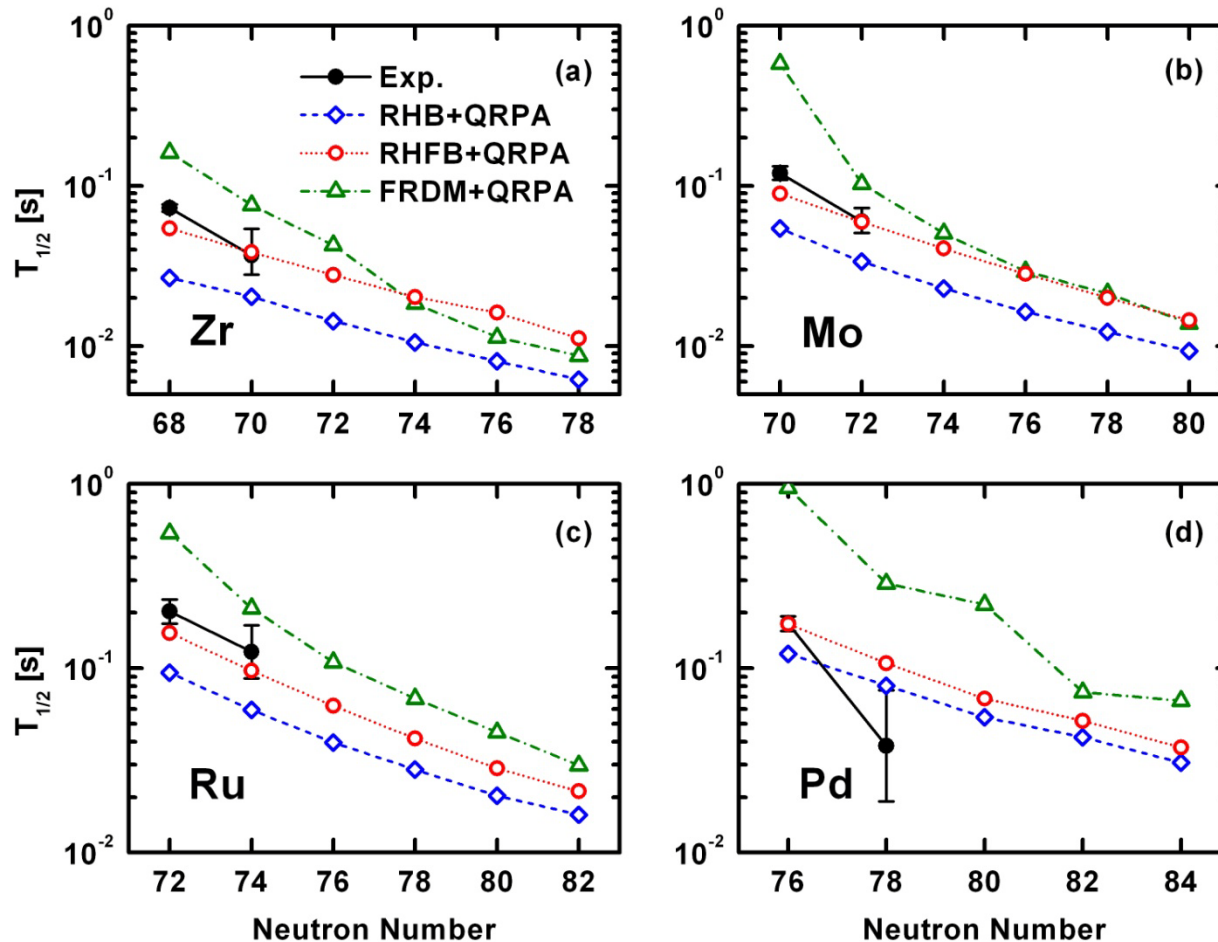
January 29, 2010

A combination of simulations and experiments explicitly demonstrates that common chemical reactions can drive convection flows in fluids.

### Feedback

Let us know what you think of *Physics*. Please email [physics@aps.org](mailto:physics@aps.org) with your comments, ideas, or suggestions for topics.

# Future: $\beta$ -decay half-lives from CDFT



- **RHFB+QRPA:** well reproduces the experimental data
- **RHB+QRPA:** underestimates the half-lives of  $^{108,110}\text{Zr}$ ,  $^{112,114}\text{Mo}$ , and  $^{116,118}\text{Ru}$ .
- **FRDM+QRPA:** systematically overestimates the nuclear half-lives.

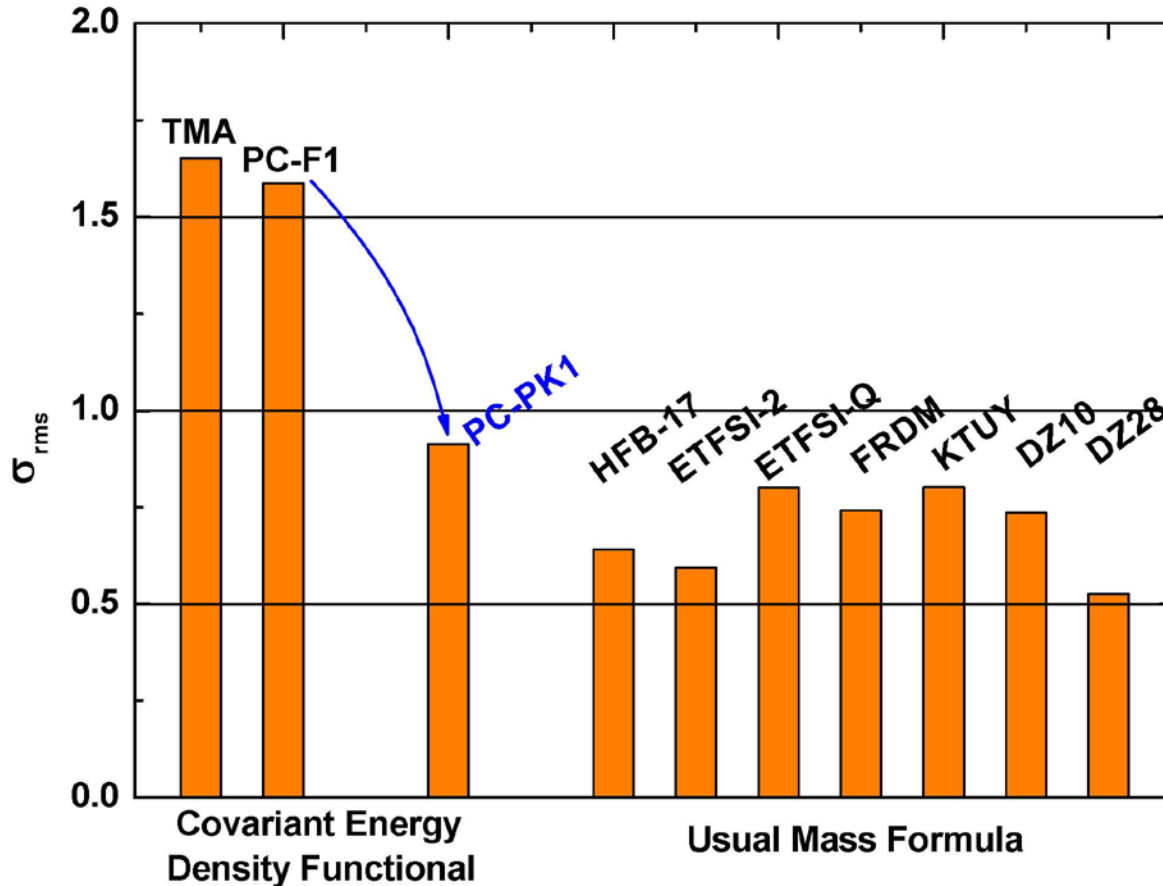


# Observed neutron-richest e-e nuclei with $26 \leq Z \leq 100$

rms mass deviations:

$$\sigma_{\text{rms}} = \sqrt{\frac{\sum_{i=1}^N (M_i^{\text{cal}} - M_i^{\text{exp}})^2}{N}}$$

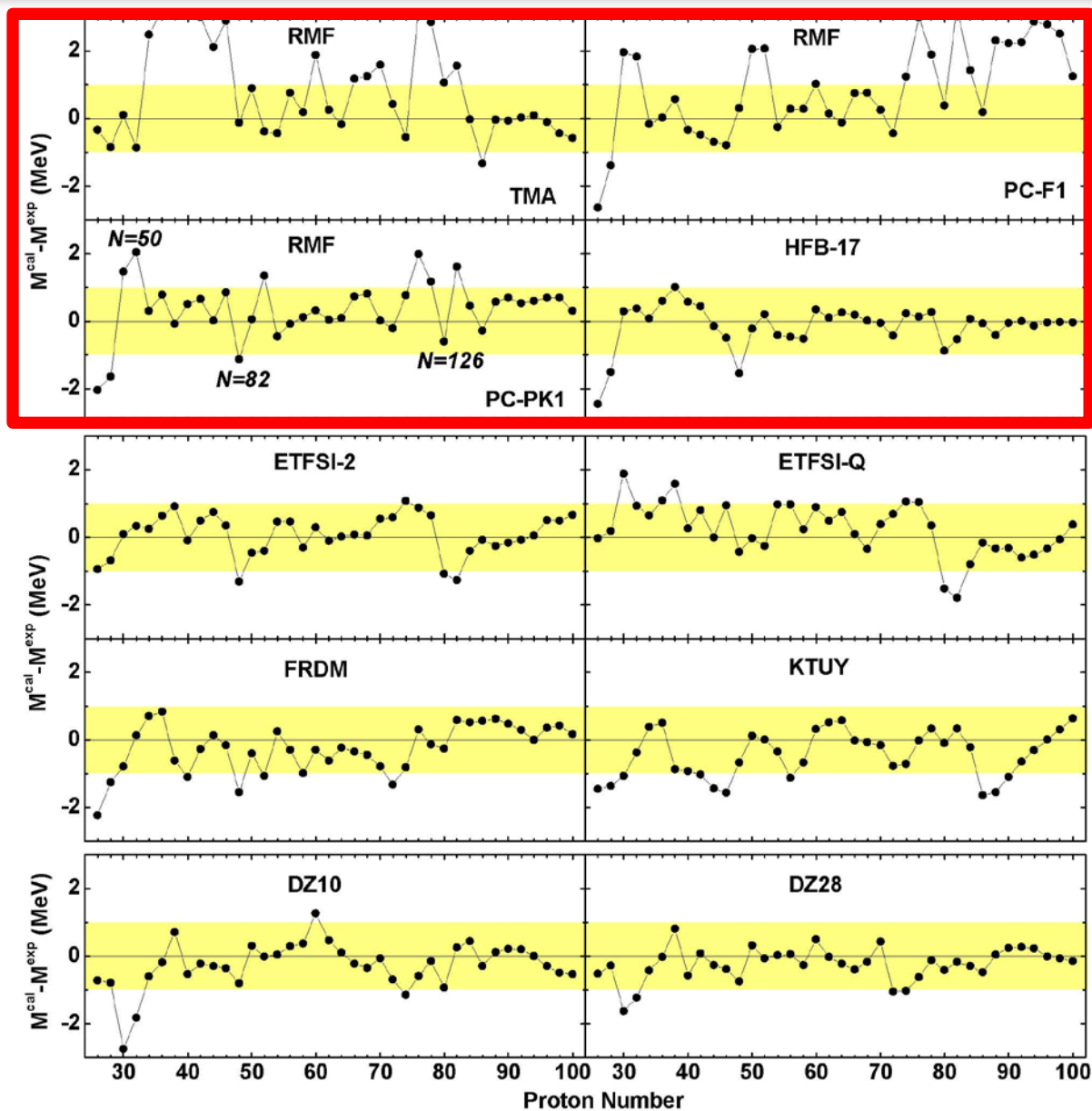
- PC-PK1 improves the description remarkably .
- Similar accuracy for the others



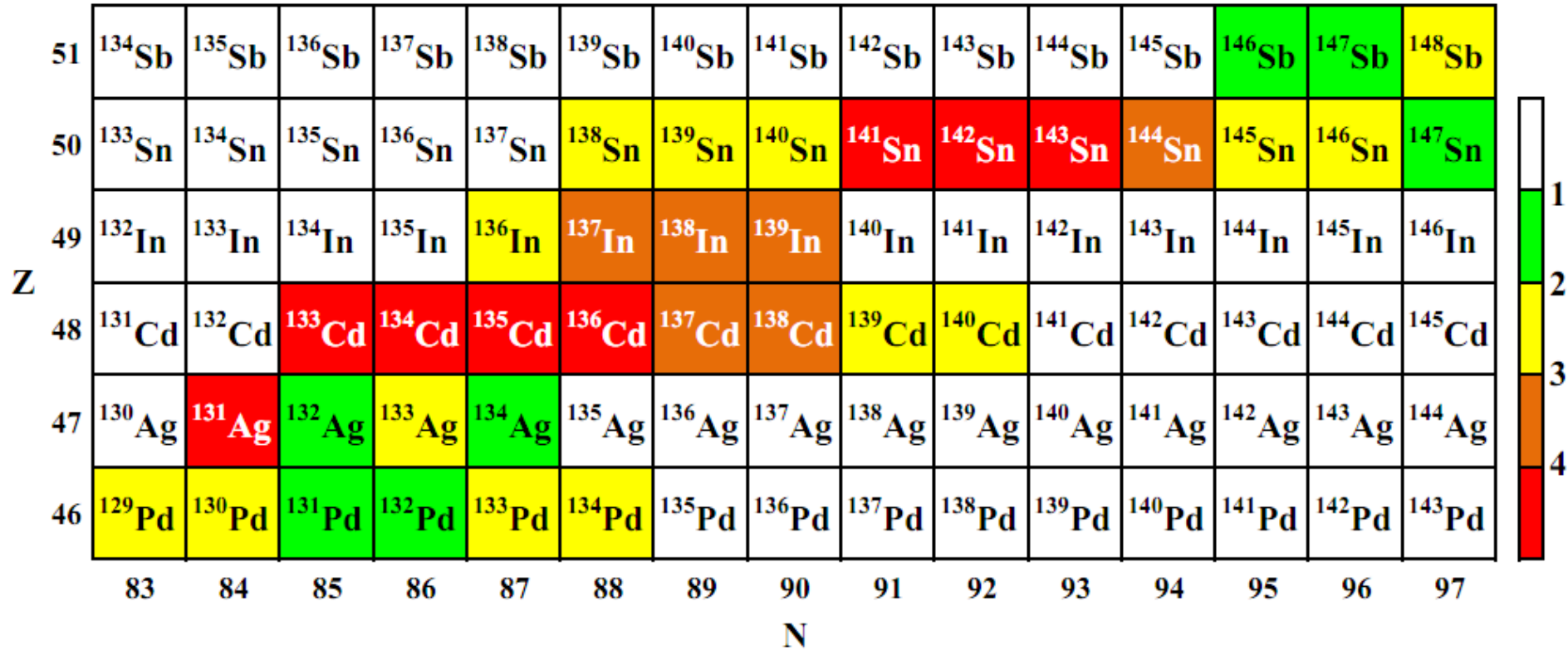
PC-PK1: RRC 82, 054319

TMA: PTP 113, 785.  
 PC-F1: PRC 65, 044308  
 HFB-17: PRL 102, 152503.  
 ETFSI-2: AIP 529, 287.  
 ETFSI-Q: PLB 387, 455.  
 KTUY: PTP 113, 305.  
 DZ28: PRC 52, R23.

# Observed neutron-richest e-e nuclei with $26 \leq Z \leq 100$



# Key nucleus in r-process: $S_n(A \sim 140)$ change by 1 MeV



Mass model: HFB-17

S. Goriely, N. Chamel, and J. M. Pearson, *PRL* 102, 152503.

$\beta$ -decay properties: FRDM+QRPA

P. Moller, B. Pfeiffer, and K.-L. Kratz, *PRC* 67, 055802.

$$F = \sum_{A=90}^{209} \left| \lg(Y_{A,\text{baseline}}) - \lg(Y_A) \right|$$

- $S_n$  of Ag, Cd, Sn have great influence on the r-process abundances
- $^{131}\text{Ag}$ ,  $^{133,134}\text{Cd}$ ,  $^{141,142}\text{Sn}$ :  $F > 4$  using both HFB-17 and FRDM

# Summary and Perspectives

## □ Recent development

RH or RHB in WS basis	Time-odd (constrained) Triaxial RMF
Resonance in RMF+ACCC	DDRHF + RPA
DDRHF+QRPA	3D AMP+GCM
Tilted Cranking RMF	<b>Bohr H based on CDFT</b> ...

## □ Recent application

- (Deformed) Exotic phenomena: halos
- CDFT mass formula and r-process calculation
- Symmetry: Spin and pseudo-spin symmetry
- Low-lying nuclear spectra
- Magnetic rotation & chirality
- Quantum shape transitions
- Spin-Dipole Resonances
- CKM matrix  $|V_{ud}| \dots$

## □ Open issues

- (Q)RPA for excited states
- 3D Cranking RMF for chirality
- Projected RMF:H-K isomers | shape transition | NP pairing
- Deformed RHF & RHFB & (Q)RPA



**Thank You!**