Fusion Reactions at very low energies.

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Interface between structure and reactions for rare isotopes and astrophysics.

- The description is based on the coupled-channels technique.
  It includes couplings to the low-lying 2^+ and 3^- states in projectile and target, and mutual and two-phonon excitations of these states.
- The influence of transfer is also studied,
  - for example, in the fusion of ^{40}Ca+^{48}Ca, ^{12}C+^{13}C, and ^{13}C+^{13}C.
- The analysis of measurements has focused on understanding
  - the hindrance of fusion at extreme subbarrier energies,
  - fusion reactions of interest to astrophysics (e. g. ^{12}C+^{12}C),
  - the constraints on the extrapolation to very low energies.

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Argonne NATIONAL LABORATORY

Coupled equations.

$$(H_n^{(0)} + E_n - E)\phi_n(r) = - \sum_{n' \neq n} \langle n | \delta V_C + \delta V_N | n' \rangle \, \phi_{n'}(r).$$

Nuclear interaction of the form $V_N(r - R_1 - R_2 - \delta R)$, where

$$\delta R = \sum_{\lambda\mu} \alpha_{1\lambda\mu} \, R_1 \, Y^*_{\lambda\mu}(\hat{r}) + \alpha_{2\lambda\mu} \, R_2 \, Y^*_{\lambda\mu}(-\hat{r}).$$

$\alpha_{\lambda\mu}$: dynamic (or static) surface deformation amplitudes.

Rotating frame approximation: $\hat{r} = \hat{z}$,

$$Y_{\lambda\mu}(\hat{r}) = \delta_{\mu,0} \sqrt{\frac{2\lambda + 1}{4\pi}}.$$

Implies that the magnetic quantum number is conserved.
There is only one channel for each state with spin I,
instead of $2I + 1$ or $I + 1$ channels.

Standard approach.

Include Coulomb couplings δV_C to first order,
and nuclear couplings up to second order in δR,

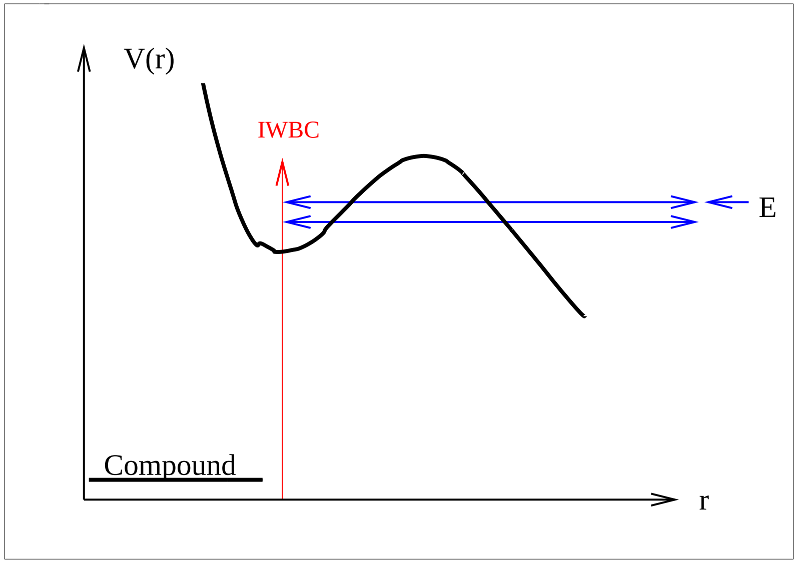
$$\delta V_N = -U'(r)\,\delta R + \frac{1}{2}\,U''(r)\,\Big(\delta R^2 - \langle 0|\delta R^2|0\rangle\Big),$$

where $U(r)$ is the conventional ion-ion potential

$$U(r) = \frac{16\pi\gamma a R_1 R_2}{R_1 + R_2}\,\frac{1}{1+\exp[(r-R_1-R_2)/a]}.$$

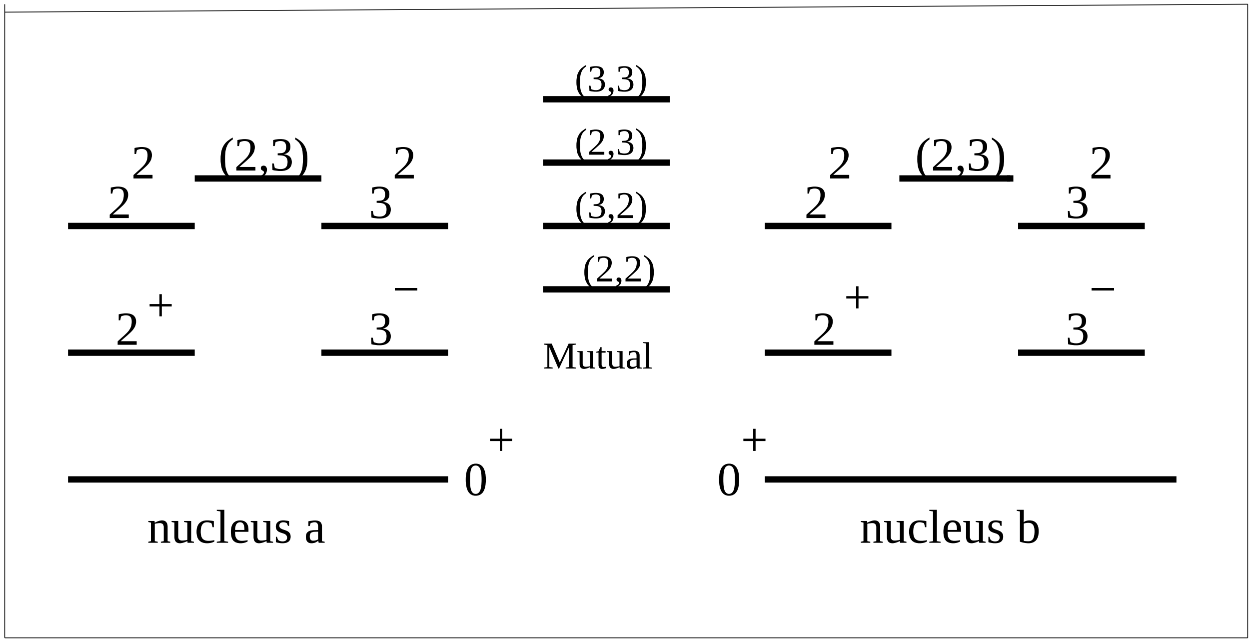
A Proximity type interaction by Broglia and Winther.

Usual scattering boundary
conditions at large value of r.
FUSION is determined by IWBC
(ingoing-wave boundary conditions)
that are imposed for overlapping nuclei.



Avoid imaginary potentials if possible.

Standard two-phonon calculation.



No. of channels: 1 (GS) + 4 (1PH) + 4 (2PH) + 6 (Mutual) = 15 channels.

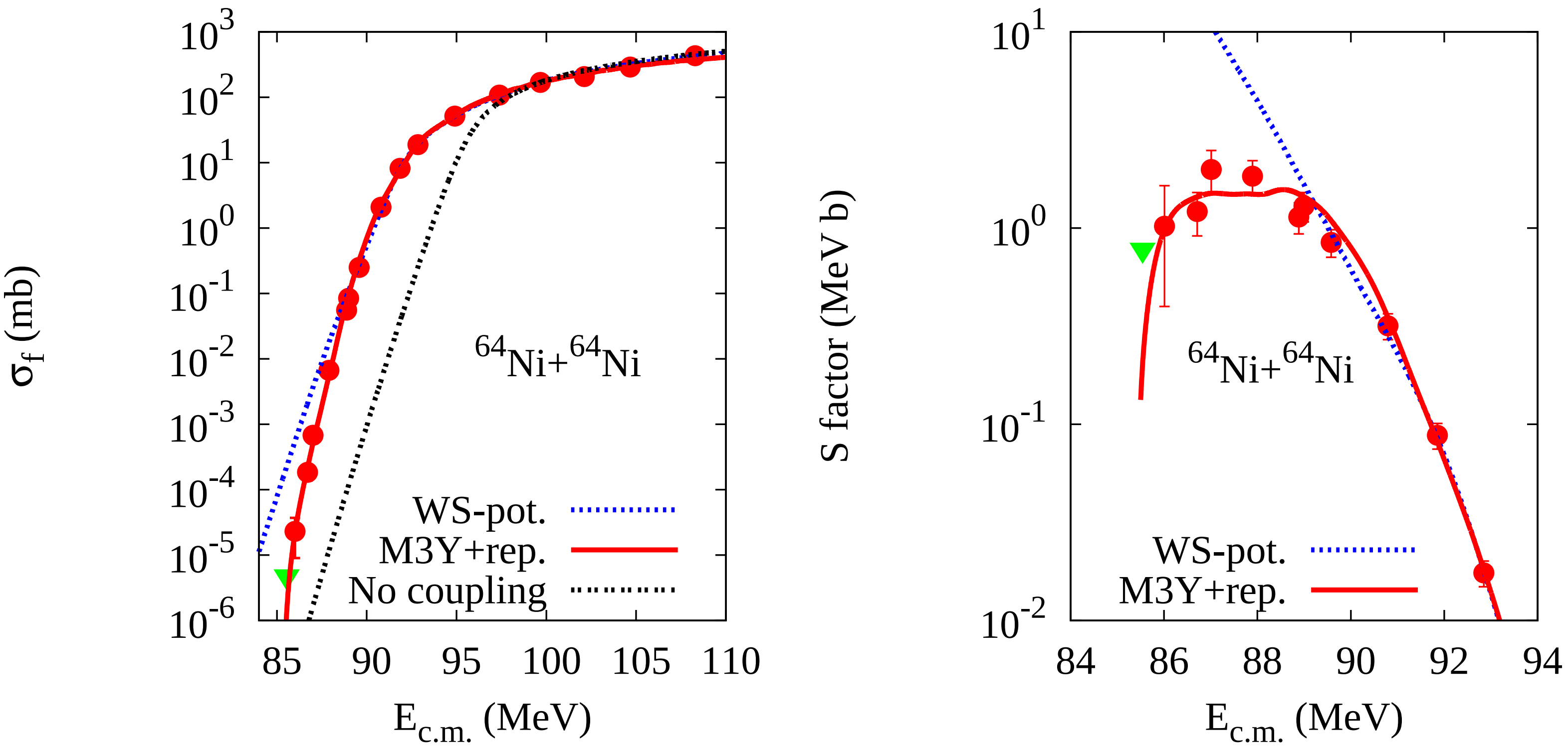
Works quite well for lighter and medium heavy systems, with some exceptions,

- in fusion reactions where transfer plays a role,
- for heavy and soft, or strongly deformed nuclei,
  - become sensitive to high-lying states (multiphonon or high spin states.)

Hindrance of fusion far below the Coulomb barrier.

- Was first recognized at Argonne by Jiang et al., ^{60}Ni+^{89}Y, PRL 89 (2002), ^{64}Ni+^{64}Ni, PRL 93 (2004), ^{64}Ni+^{100}Mo, PRC 71 (2005), ^{28}Si+^{64}Ni, PL B640 (2006).

$$S = E_{c.m.}\ \sigma_f\ exp(2\pi[\eta - \eta_0]).$$



- The data are *hindered* compared to coupled-channels calculations that are based on *a conventional Woods-Saxon potential.*
- Right panel: *the S factor for fusion has a maximum* at low energy.

Theoretical description of the fusion hindrance,

Mişicu and Esbensen, PRL 96, 112701 (2006); PRC 75, 034606 (2007).

Conventional Woods-Saxon potential (WS), $a \approx 0.65$ fm.

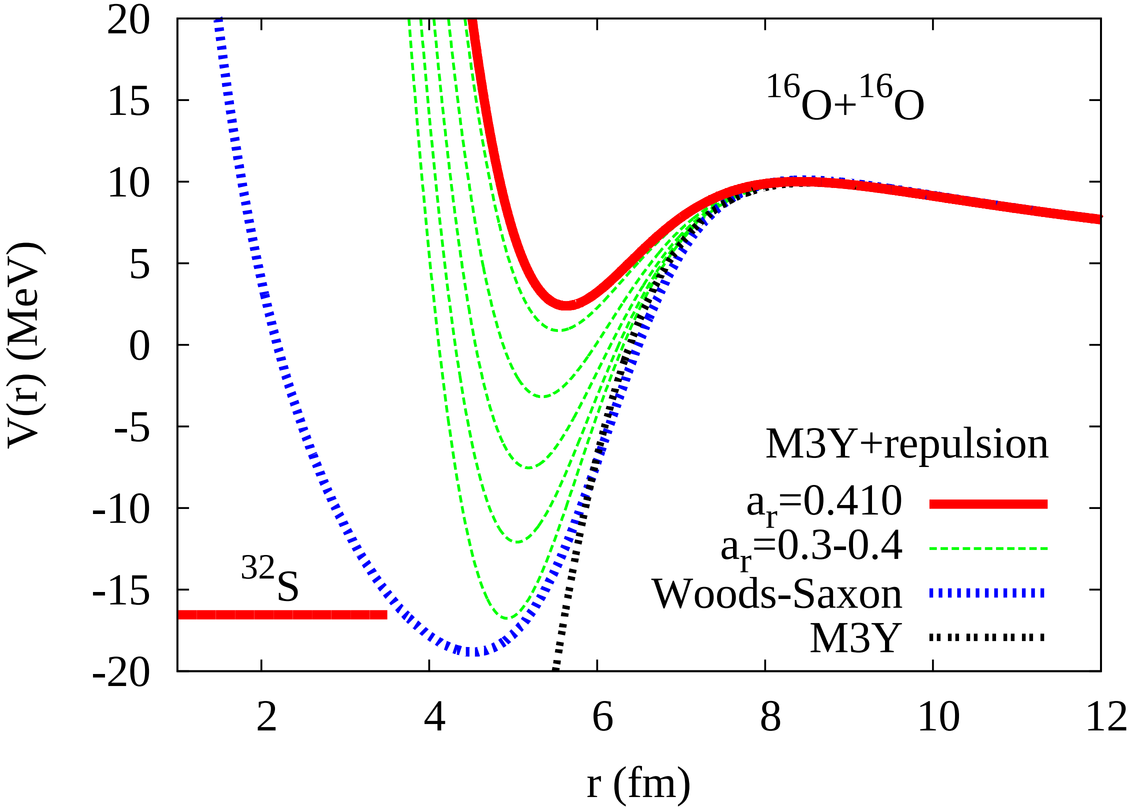
M3Y double-folding potential: $U(\mathbf{r}) = \int d\mathbf{r}_1 d\mathbf{r}_2 \; \rho_1(\mathbf{r}_1) \; \rho_2(\mathbf{r}_2) \times v_{NN}(\mathbf{r}+\mathbf{r}_2-\mathbf{r}_1)$. v_{NN} = M3Y effective NN interaction.

M3Y+repulsion potential:

Supplement with a repulsive term, $v_{NN}^{\mathrm{rep}} = v_r \delta(\mathbf{r}+\mathbf{r_2}-\mathbf{r_1})$.

Use $\rho(r)$ with adjustable diffuseness a_r.

v_r is calibrated to give the nuclear incompressibility K = 234 MeV.



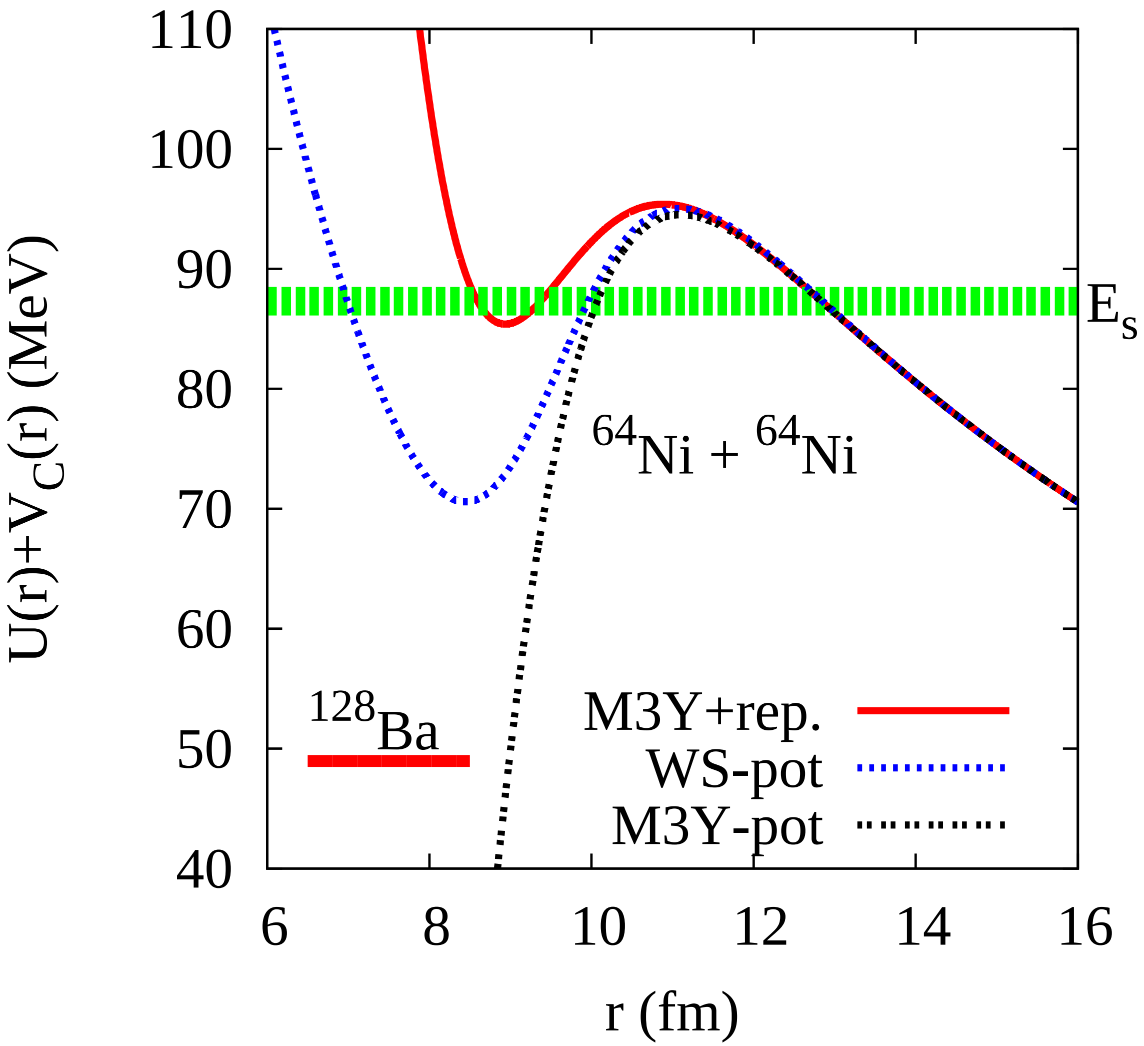
Adjustable parameters: R and a_r. Can produce a shallow pocket and a thicker barrier.

Hindrance of fusion far below the Coulomb barrier.

The hindrance sets in near E_s.

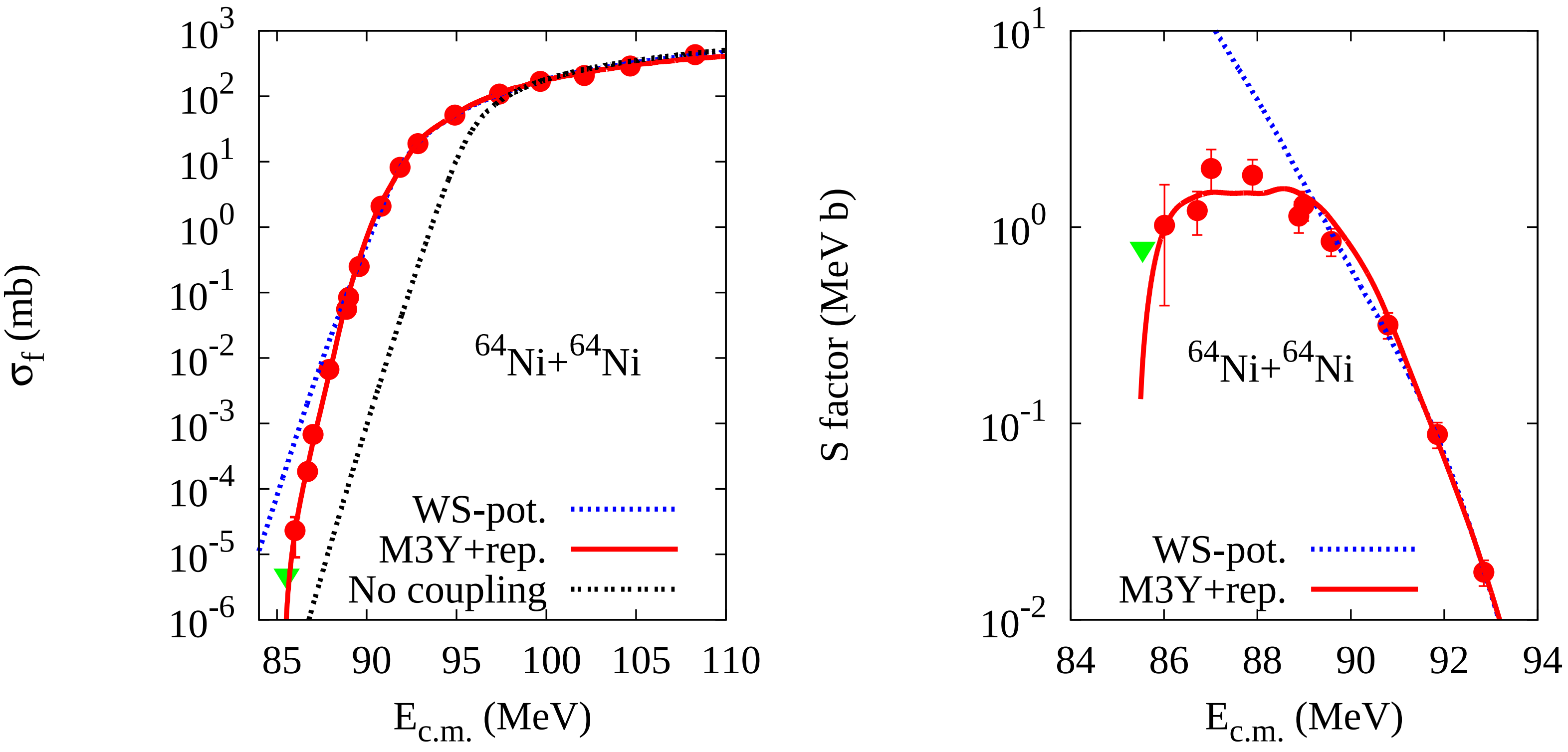
Adjust a_r so pocket is slightly deeper: a_r = 0.403 fm.

Fusion cross section goes to zero for $E_{cm} \leq E_{pocket}$.



Jiang et al., ^{64}Ni+^{64}Ni, PRL 93 (2004).

$$S = E_{c.m.}\ \sigma_f\ exp(2\pi[\eta - \eta_0]).$$



- The data are *hindered* compared to coupled-channels calculations that are based on *a conventional Woods-Saxon potential.*
- Strong hindrance (right panel):
  - the S factor for fusion has a maximum at low energy.

Does fusion hindrance occur in systems with positive Q values?

Important issue for extrapolations in astrophysics.

- Confirmed in new experiments at Argonne:

  ^{28}Si+^{30}Si, PRC 78, 17601 (2008),

  ^{27}Al+^{45}Sc, PRC 81, 24611 (2010).

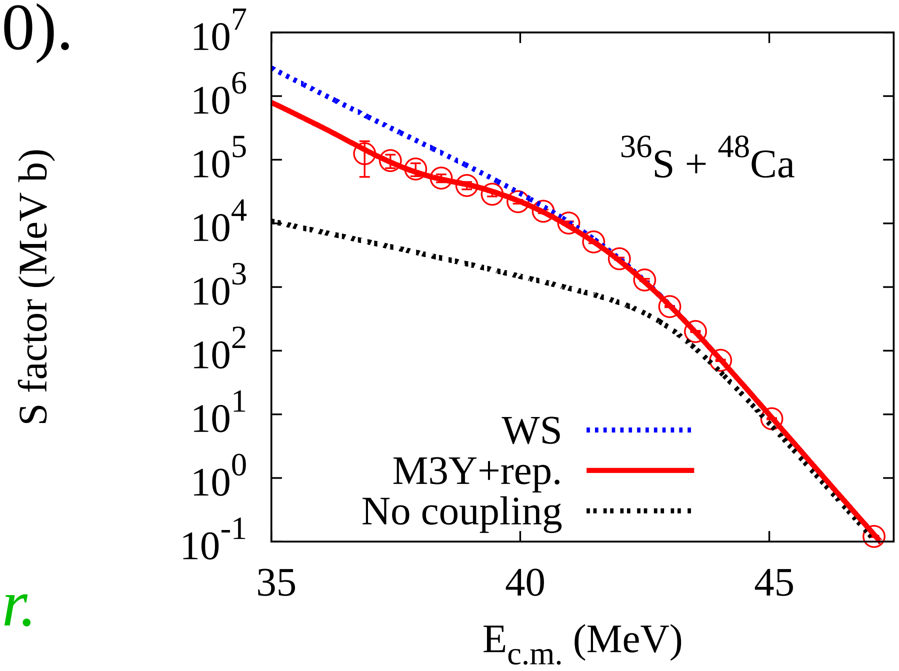
  Almost a maximum S factor.

- ^{36}S+^{48}Ca, Q_{gg} = +7.6 MeV.

  Stefanini et al.,

  PRC 78, 044607 (2008).

  Hindrance does occur but not always with a maximum S factor.



- Systematic study of fusion with calcium isotopes:

  ^{48}Ca+^{48}Ca, Stefanini et al., PLB 679, 95 (2009),

  ^{40}Ca+^{48}Ca, Jiang et al. (incl. Esbensen), PRC 82 (2010),

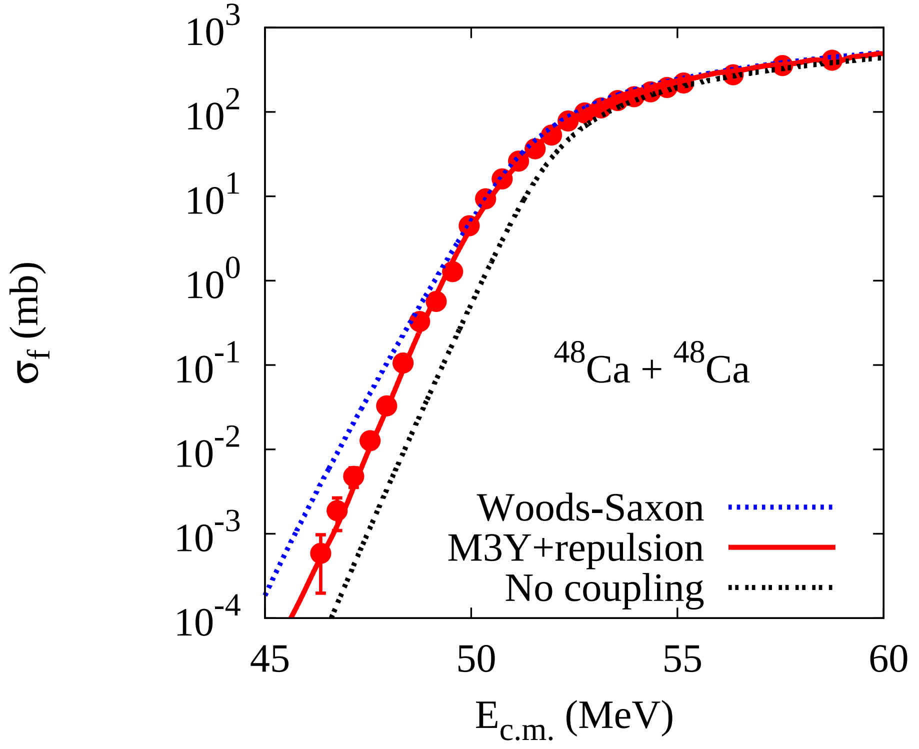
  ^{40}Ca+^{40}Ca, Stefanini et al. (incl. Esbensen, unpublished.)

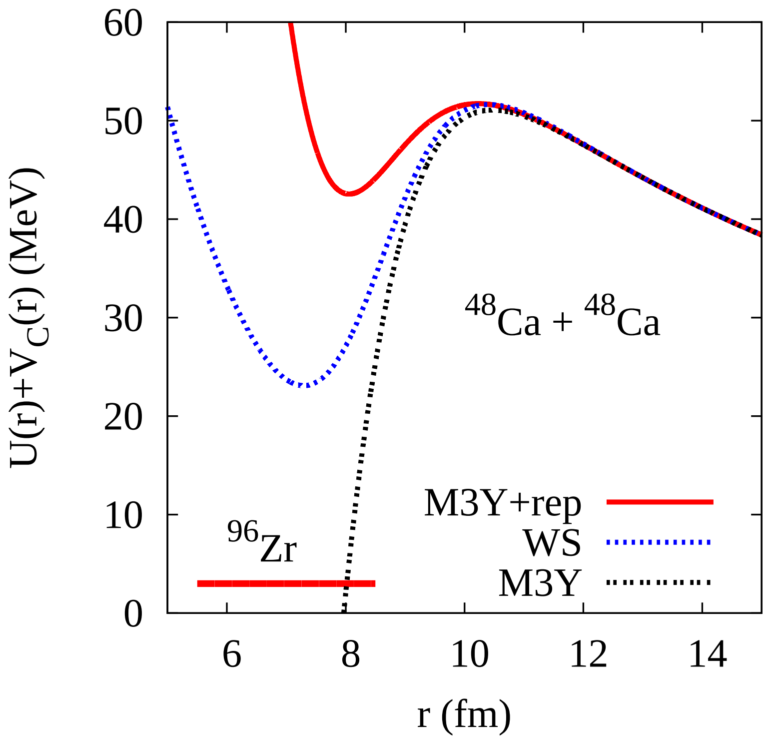
  \- Previously studied by Aljuwair et al. (84-Exp), and Esbensen et al. (89-Theo.)

Analysis of ^{48}Ca+^{48}Ca fusion experiment,

Esbensen, Jiang, and Stefanini, PRC 82, 054621 (2010).

Coupling to the low-lying 2^+, 3^-, 5^- states, and mutual excitations.





Fusion is a sensitive probe of the surface of nuclei.

Best fit density of ^{48}Ca has the rms-radius = 3.56 fm.

Compare to point-protons: 3.39 fm, point-nucleons: 3.53 fm.

Extracted radius is model dependent (dynamic polarization effects.)

There are two solutions to the analysis.

a_r = 0.4070 fm:

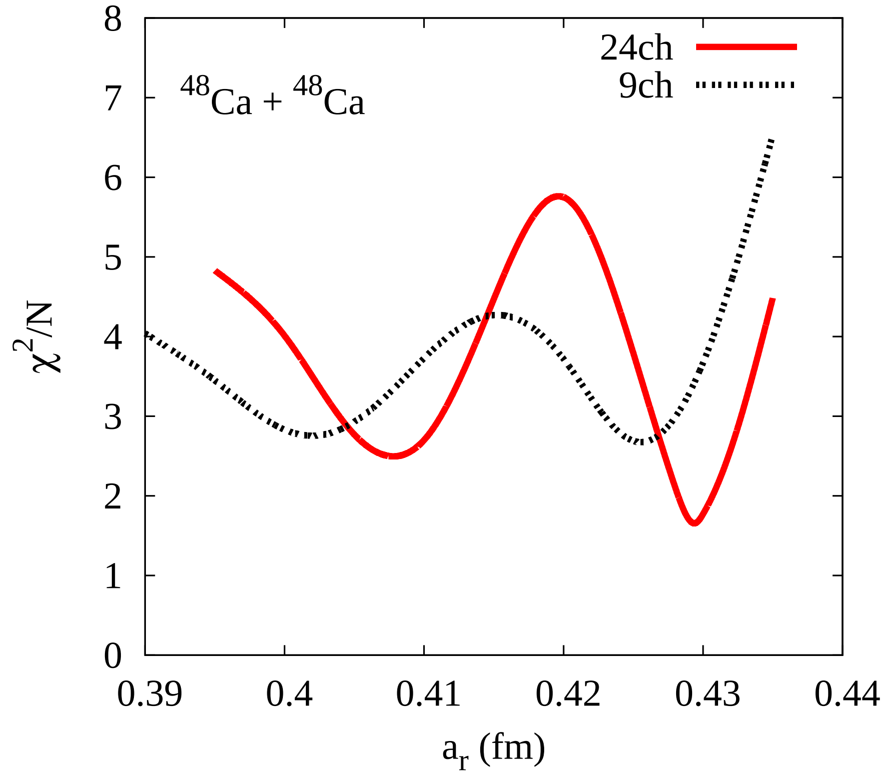
Ch9: $\langle r^2 \rangle^{1/2}$ = 3.55 fm.

Ch24: $\langle r^2 \rangle^{1/2}$ = 3.53 fm.

a_r = 0.4295 fm:

Ch9: $\langle r^2 \rangle^{1/2}$ = 3.57 fm.

Ch24: $\langle r^2 \rangle^{1/2}$ = 3.56 fm.



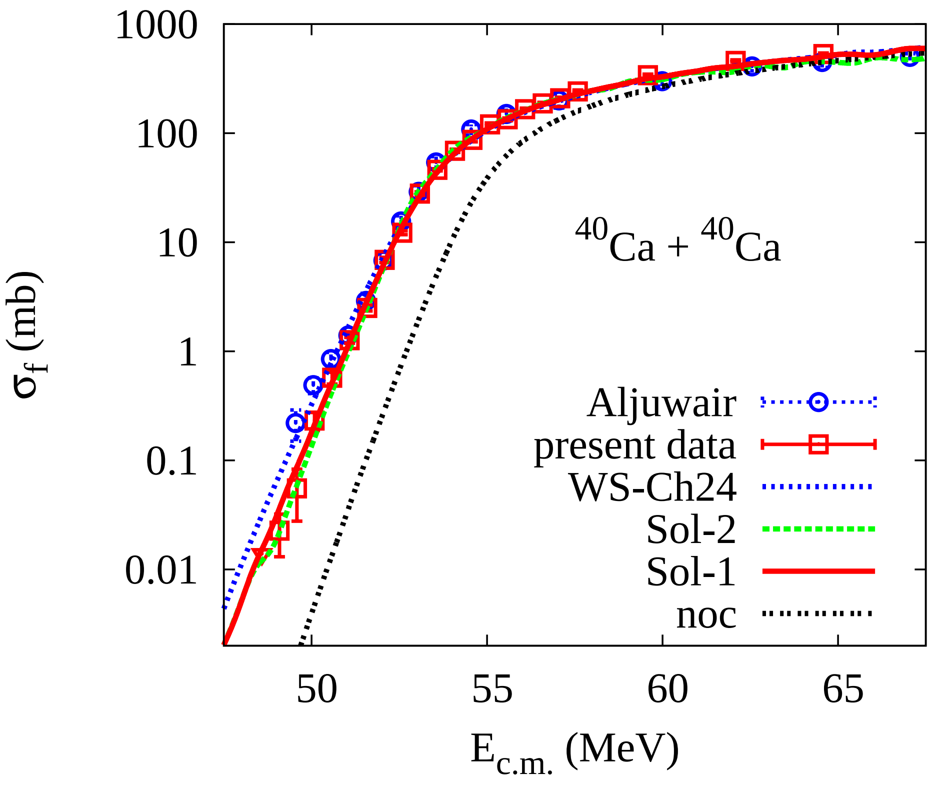
Compare: point matter rms = 3.53(3) fm.

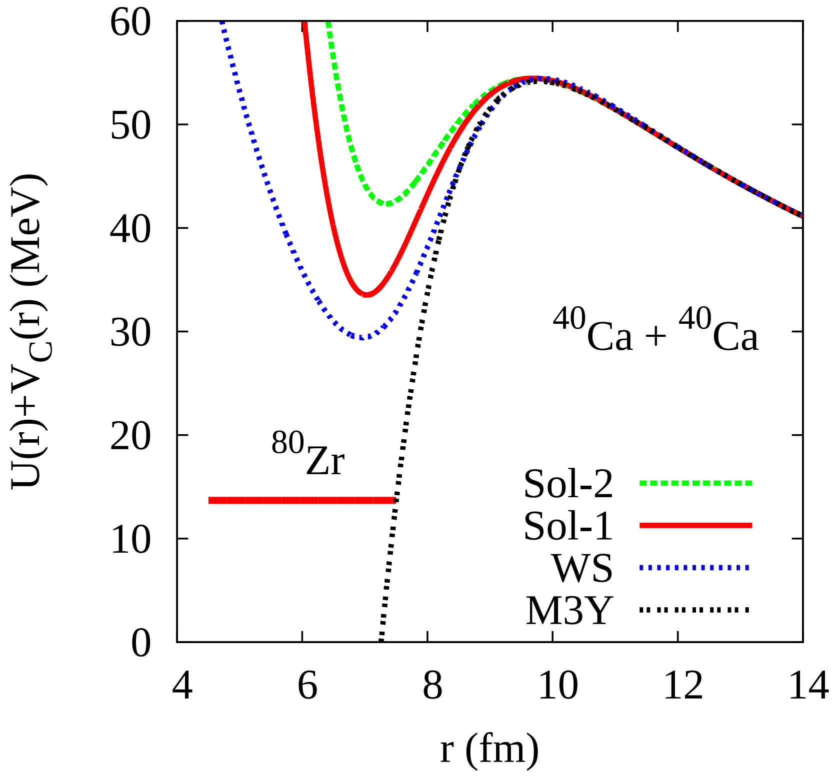
Which solution should I choose? The one with the smallest χ^2/N.

Dynamic polarization effect: fewer channels implies a larger radius.

Similar analysis of ^{40}Ca+^{40}Ca experiment.

Coupling to the low-lying 2^+, 3^-, 5^- states, and mutual excitations.





Also two solutions of the analysis.

Stefanini et al. (unpublished)

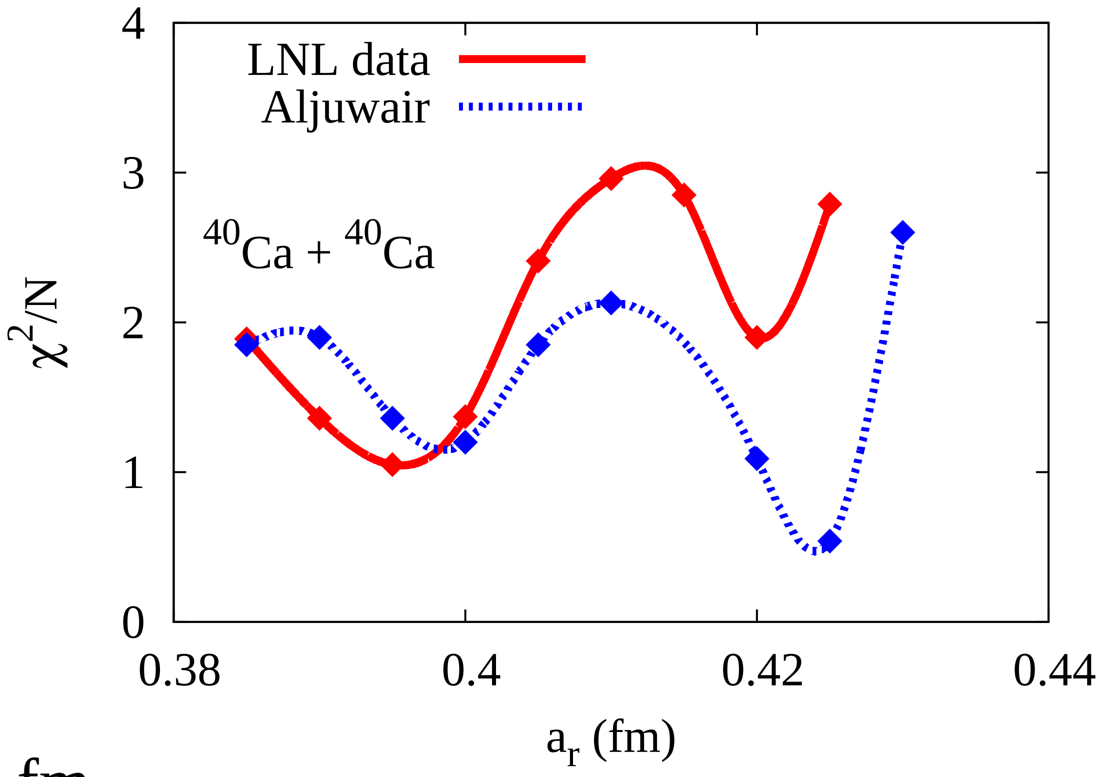
Also two solutions:

a_r = 0.395 fm:

$\langle r^2 \rangle^{1/2}$ = 3.38 fm.

a_r = 0.42 fm:

$\langle r^2 \rangle^{1/2}$ = 3.41 fm.



Compare point-protons: 3.38 fm,

Which solution should I choose?

A larger radius is justified by the dynamic polarization effect.

Table 1: Nuclear structure input for ^{40}Ca and ^{48}Ca. The $B(E\lambda)$ values are from ENDSF, Nat. Nuclear Data Center (BNL). Values marked with * are from Ref. [1]. Eff 2PH: effective parameters for the $(0^+, 2^+, 4^+)$ two-phonon quadrupole excitations.

Nucleus	I^π	E_x (MeV)	$B(E\lambda)$ (W.u.)	$\frac{(\beta R)_C}{\sqrt{4\pi}}$ (fm)	$\frac{(\beta R)_N}{\sqrt{4\pi}}$ (fm)
^{40}Ca	2_1^+	3.905	2.26(14)	0.138*	0.125*
	3^-	3.737	27(4)	0.465*	0.315*
	5^-	4.491		0.344*	0.175*
	Eff 2PH	5.269	41(5)	0.416	0.416
^{48}Ca	2_1^+	3.832	1.71(9)	0.126*	0.190*
	3^-	4.507	5.0(8)	0.250*	0.190*
	5^-	5.146		0.049*	0.038*
	Eff 2PH	4.849	4.7(29)	0.15	0.15

[1] Esbensen and Videbaek, PRC 40, 126 (1989), analysis of ^{16}O+^{A}Ca scattering data.

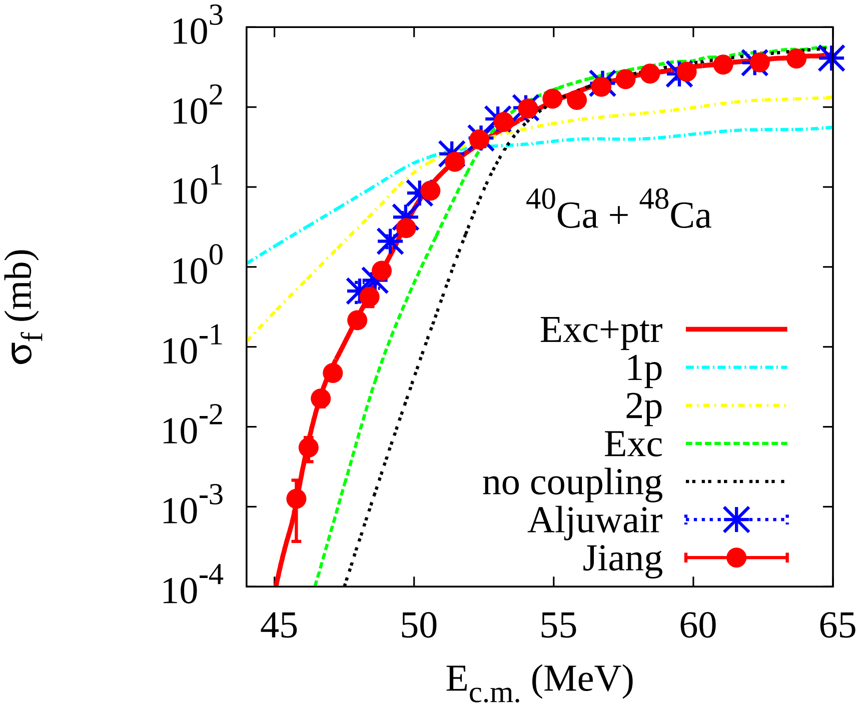
Note that ^{40}Ca is much softer that ^{48}Ca.

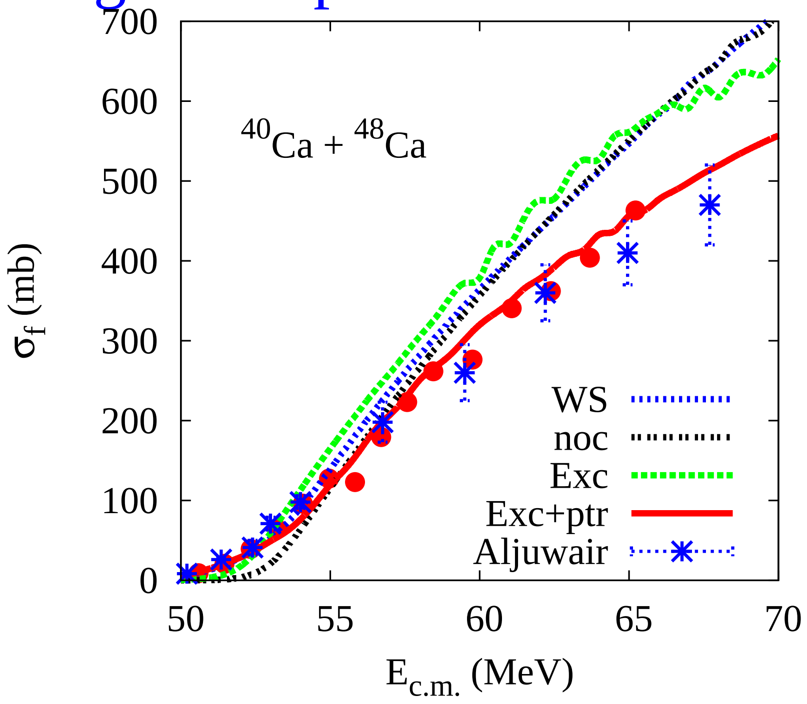
Coulomb and nuclear parameters are not (always) identical.

^{40}Ca+^{48}Ca experiment, Jiang et al, PRC 82, 041601 (2010).

Include one- and two-proton transfers with positive Q-values.

Adjust the two-proton transfer strength to reproduce data.





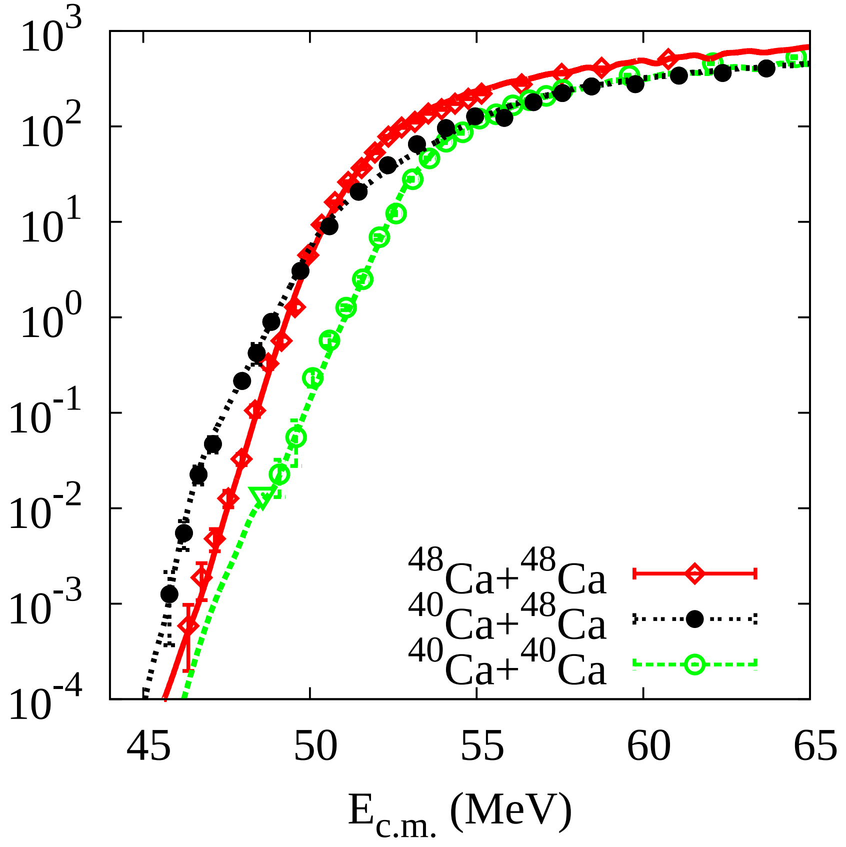
Two-proton transfer gives a large enhancement of subbarrier fusion and has ”a good effect” (suppression) on fusion at high energies.

Systematics of the fusion of calcium isotopes.

^{48}Ca+^{48}Ca, Stefanini et al.
Phys. Lett. B679, 95 (2009).

^{40}Ca+^{40}Ca, Stefanini et al.
(unpublished.) *^{40}Ca is soft.*

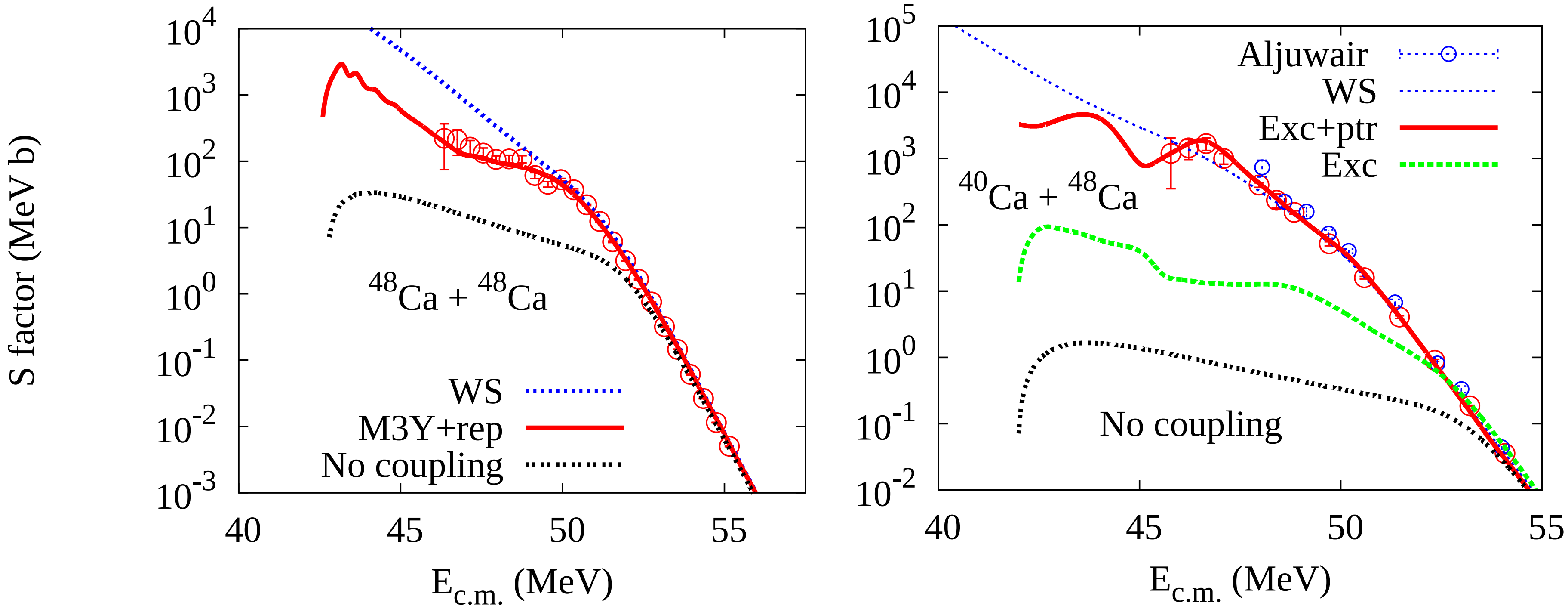
^{40}Ca+^{48}Ca, Jiang et al,
Phys. Rev. C 82, 041601 (2010).
Enhanced compared to the ^{48}Ca+^{48}Ca data.



Large effect of one- and two-proton transfer with positive Q-values on the fusion of the asymmetric ^{40}Ca+^{48}Ca system.

The hindrance sets in at a lower energy.

S factors for ^{48}Ca+^{48}Ca and ^{40}Ca+^{48}Ca fusion.



Relatively weak coupling for ^{48}Ca+^{48}Ca: Hindrance occurs.

Strong coupling effects in the fusion of ^{40}Ca+^{48}Ca

due to two-proton transfer and a softer ^{40}Ca.

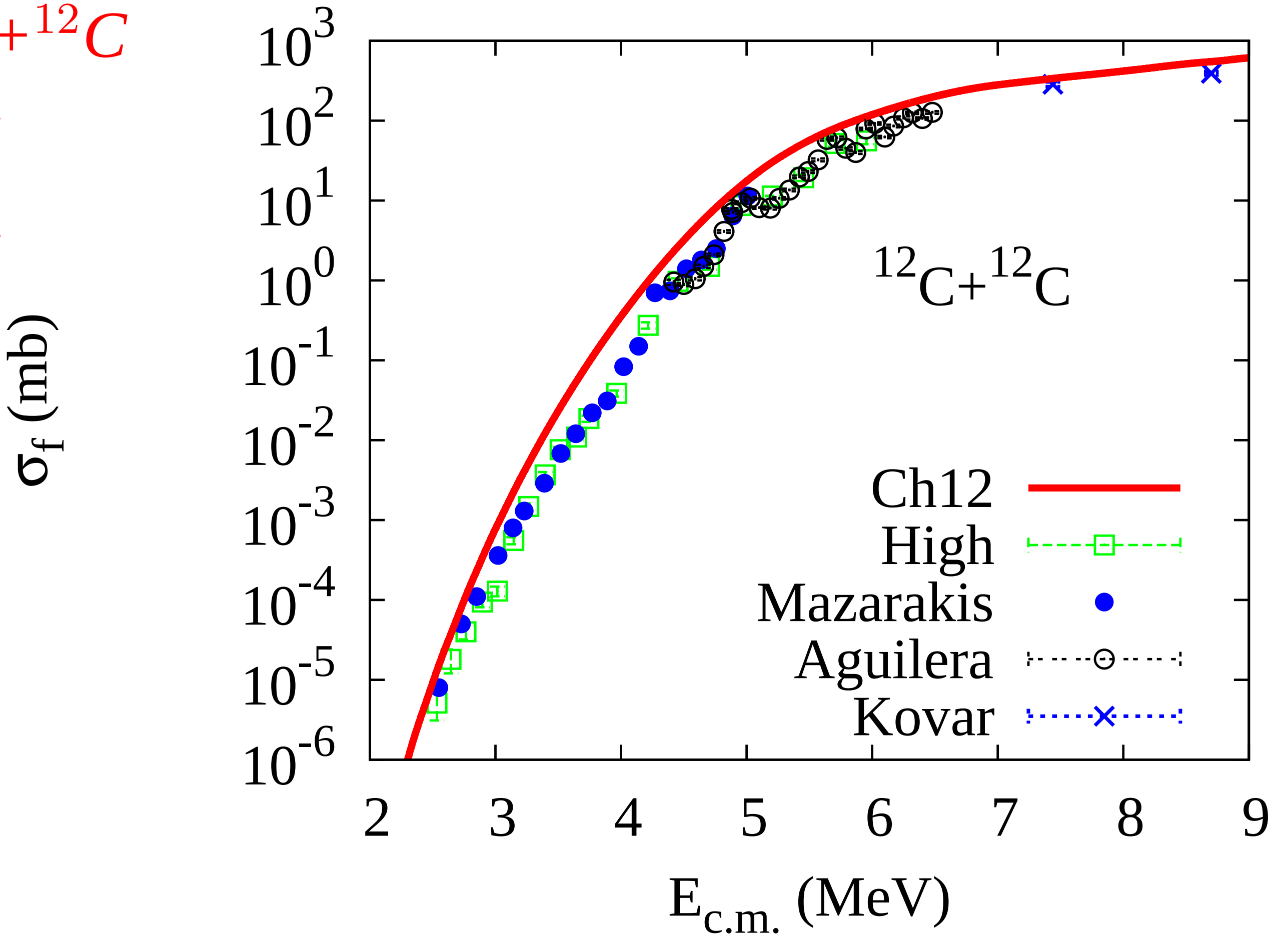
The hindrance sets in at a lower energy.

Coupled-channels calculations of ^{12}C+^{12}C fusion,

H. Esbensen, X. Tang, and C. L. Jiang.

The goal is to put constraints on the extrapolation

of the measured ^{12}C+^{12}C fusion cross sections to very low energies.



Problems:

Structures (peaks) in data.

Large experimental uncertainties up to 30%.

Closed channels:

use decaying states when $E_{c.m.} \leq E_x$ (Whittaker function.)

Entrance channel potential.

^{12}C density = *point-proton density.* R = 1.696 fm, a = 0.52 fm.

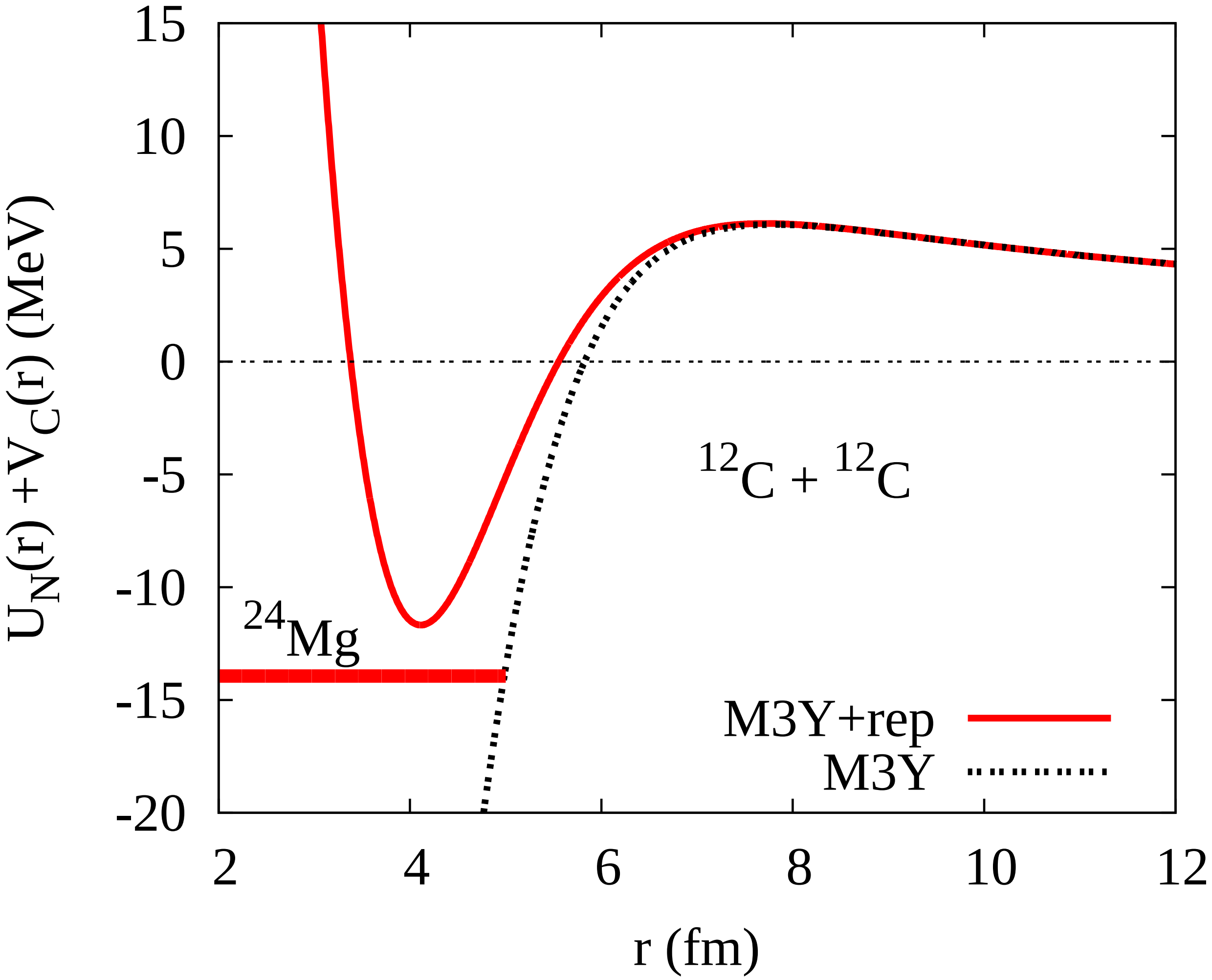
Repulsion:

K=234 MeV,

a_r = 0.426 fm.

V_{CB} = 6.12 MeV.

V_{min} = -11.67 MeV.



Nuclear structure input.

Table 2: Properties of E2 and E3 transitions to the low-lying states in ^{12}C. The intrinsic quadrupole moment was extracted from the lowest E2 transition.

Nucleus	State	E$_x$ (MeV)	Transition	B(Eλ) (W.u.)	β_λ^C
^{12}C	0_1^+	0		Q_0 =-19.5 fm^2	0.570
	2^+	4.439	E2: $0_1^+ \rightarrow 2^+$	4.65(26)	0.570
	0_2^+	7.654	E2: $2^+ \rightarrow 0_2^+$	8.0(11)	0.236
	3^-	9.641	E3: $0_1^+ \rightarrow 3^-$	12(2)	0.90(7)

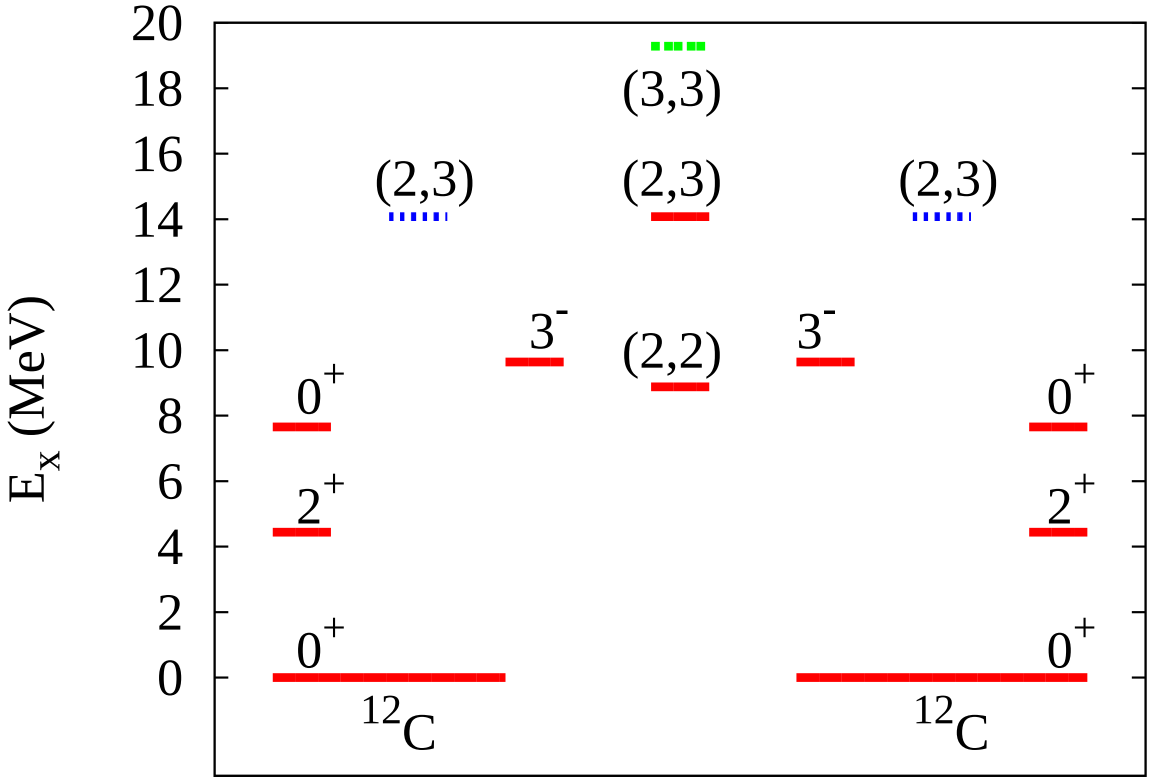
β_3 extremely large. Matrix element in ^{13}C is quenched: $\beta_3^{\rm eff} = \sqrt{\frac{3}{7}}\,\beta_3$.

Coupling scheme.

Assume independent modes of excitation.

$\beta_2 = 0.57$

$\beta_3 = 0.90(7)$



Ch10: all red states

Ch12: add (2,3)

ph1: all one-phonon excitations (Ch5).

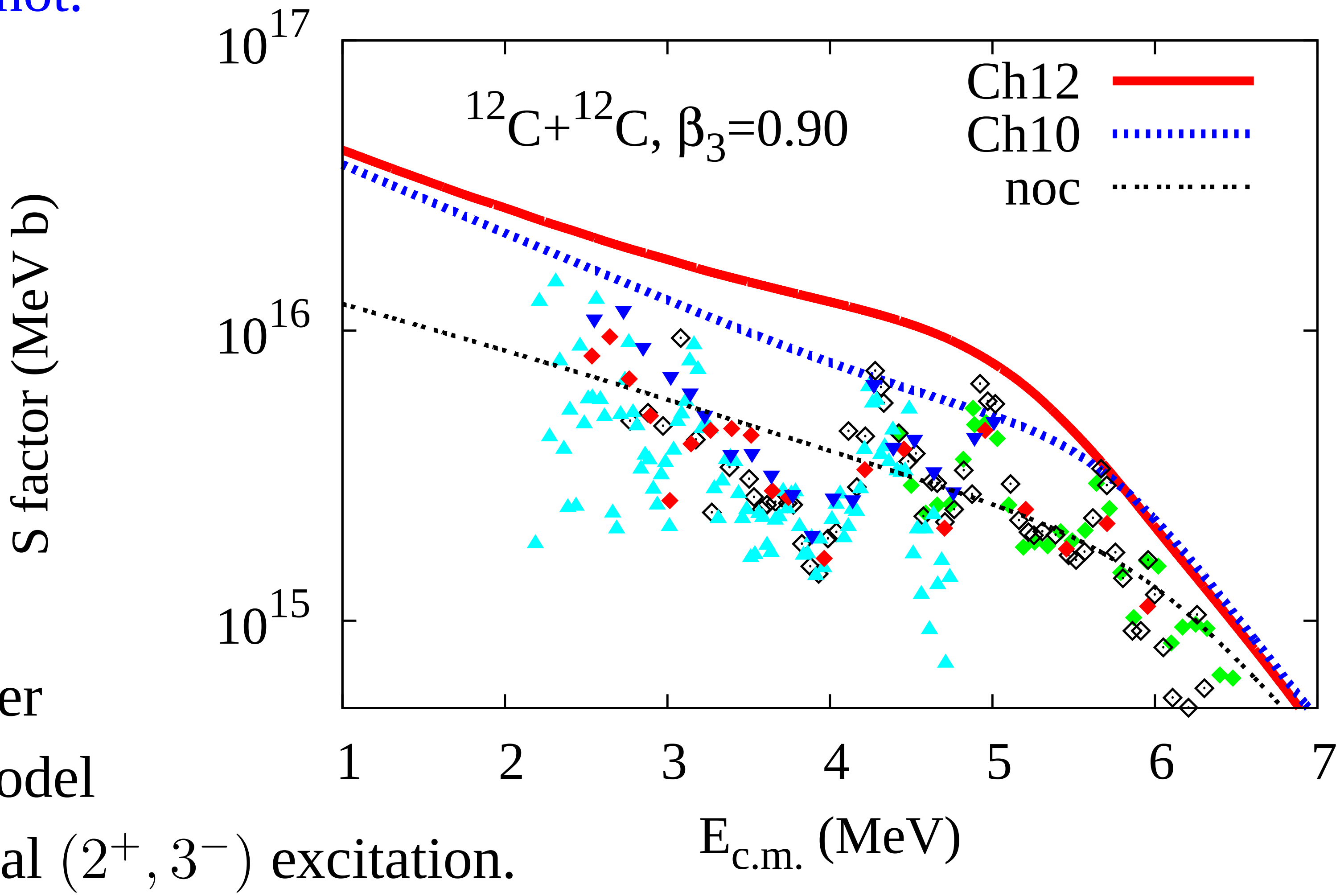
Ignore the $(3^-,3^-)$ excitation. The energy is too high.

Model of mutual $(2^+,3^-)$ excitation in same nucleus is questionable.

Sensitivity to the mutual $(2^+, 3^-)$ excitation in ^{12}C.

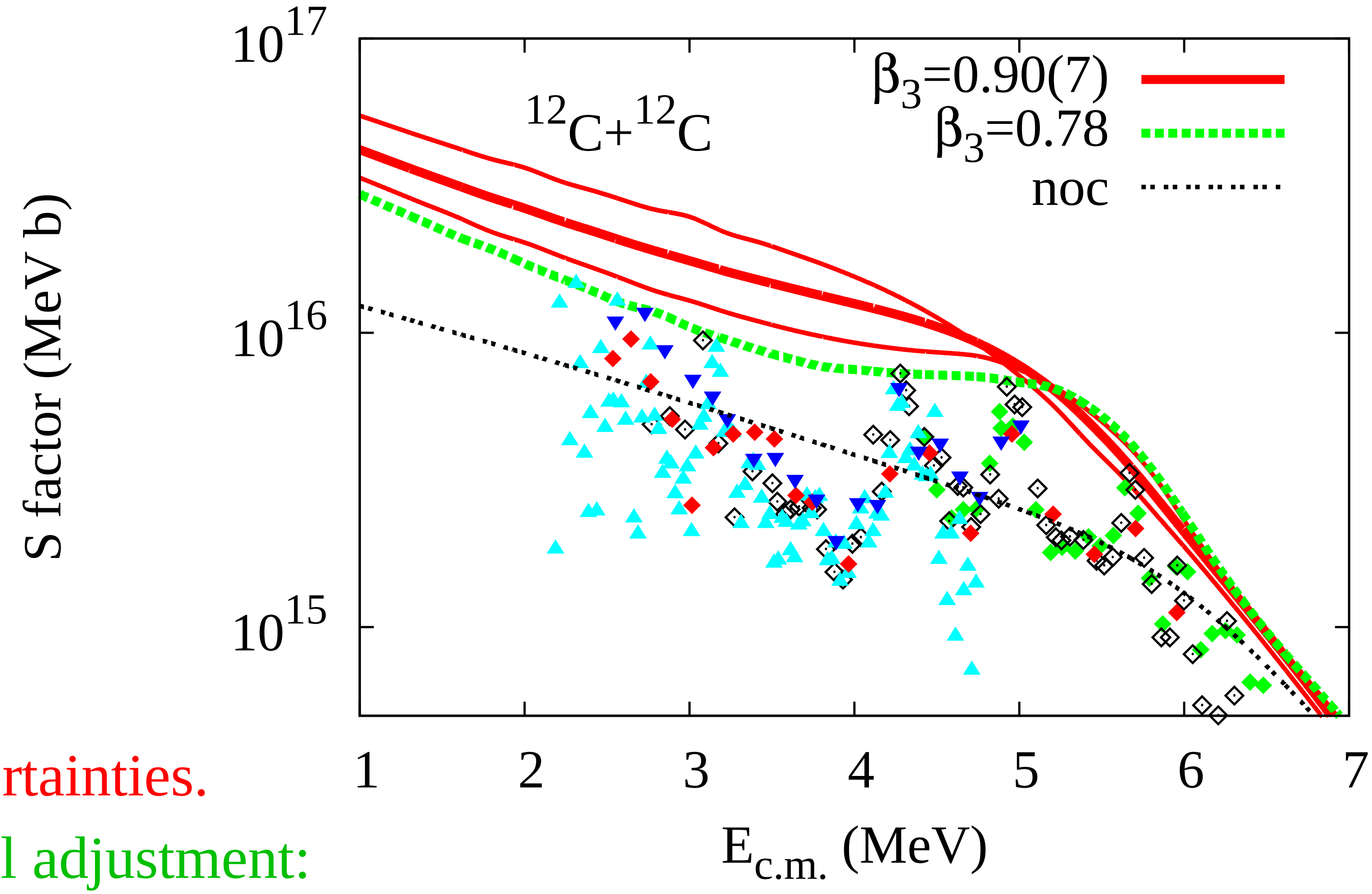
Ch12 includes this excitation, one in each nucleus.

Ch10 does not.



Need a better structure model of the mutual $(2^+, 3^-)$ excitation.

Sensitivity to $\beta_3 = 0.90 \pm 0.07$ in Ch12 calculations.



Large uncertainties.

Make small adjustment:

set $\beta_3 = 0.78$ to match the peaks.

Predict extrapolated value: S(1 MeV) = 3.0 10^{16} MeV b.

Conclusions.

- The fusion hindrance at low energies is *a general phenomenon* - the data are suppressed compared to CCC based on conventional WS potentials, - they have a steeper logarithmic slope.

- The data can be reproduced by adjusting the M3Y+rep potential. Apart from structure, essentially only two parameters: R and a_r.

- Extracted radii can be slightly larger than expected - due to dynamic polarization of states not included.

- Large uncertainty in the prediction of the ^{12}C+^{12}C fusion rate - due to large β_3-value and large sensitivity to $(2^+, 3^-)$ excitation.

- Conjecture: Coupled-channels calculations with IWBC should provide an upper limit for the ^{12}C+^{12}C fusion. Structures in the data are due to the low level density of the compound system.