TEXAS A&M UNIVERSITY COMMERCE

C.A. Bertulani

Reactions at Intermediate Energies

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Rutherford is (was) wrong

Aguiar, Aleixo, Bertulani, PRC 42, 2180 (1990)

$$L = L^{LO} + L^{NLO} + L^{N^2LO} + \cdots$$

$$L^{LO} = \frac{1}{2}\mu c^2\left(\frac{v}{c}\right)^2 - \frac{Z_1Z_2e^2}{r}$$

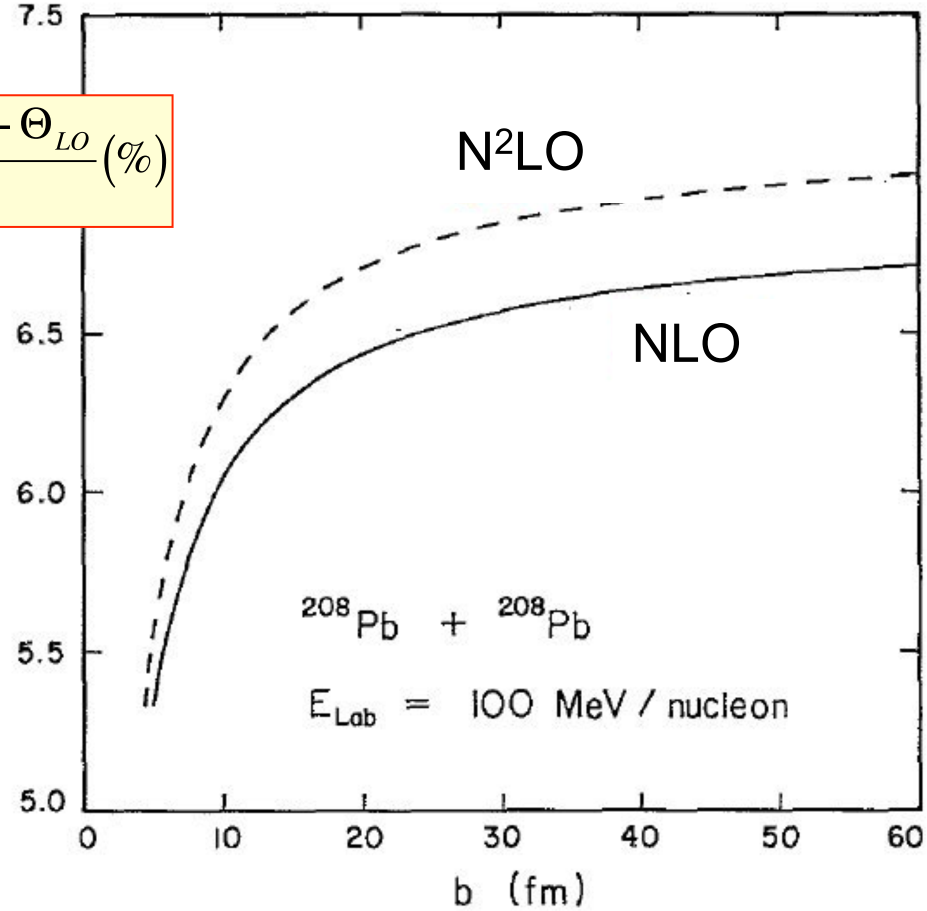
$$L^{NLO} = \frac{\mu^4c^2}{8}\left[\frac{1}{m_1^3} - \frac{1}{m_2^3}\right]\left(\frac{v}{c}\right)^4 - \frac{\mu^2Z_1Z_2e^2}{2m_1m_2r}\left[\left(\frac{v}{c}\right)^2 + \left(\frac{\mathbf{v}\cdot\mathbf{r}}{cr}\right)^2\right]$$

$$L^{N^2LO} = \frac{\mu c^2}{512}\left(\frac{v}{c}\right)^6 + \frac{Z_1Z_2e^2}{16r}$$

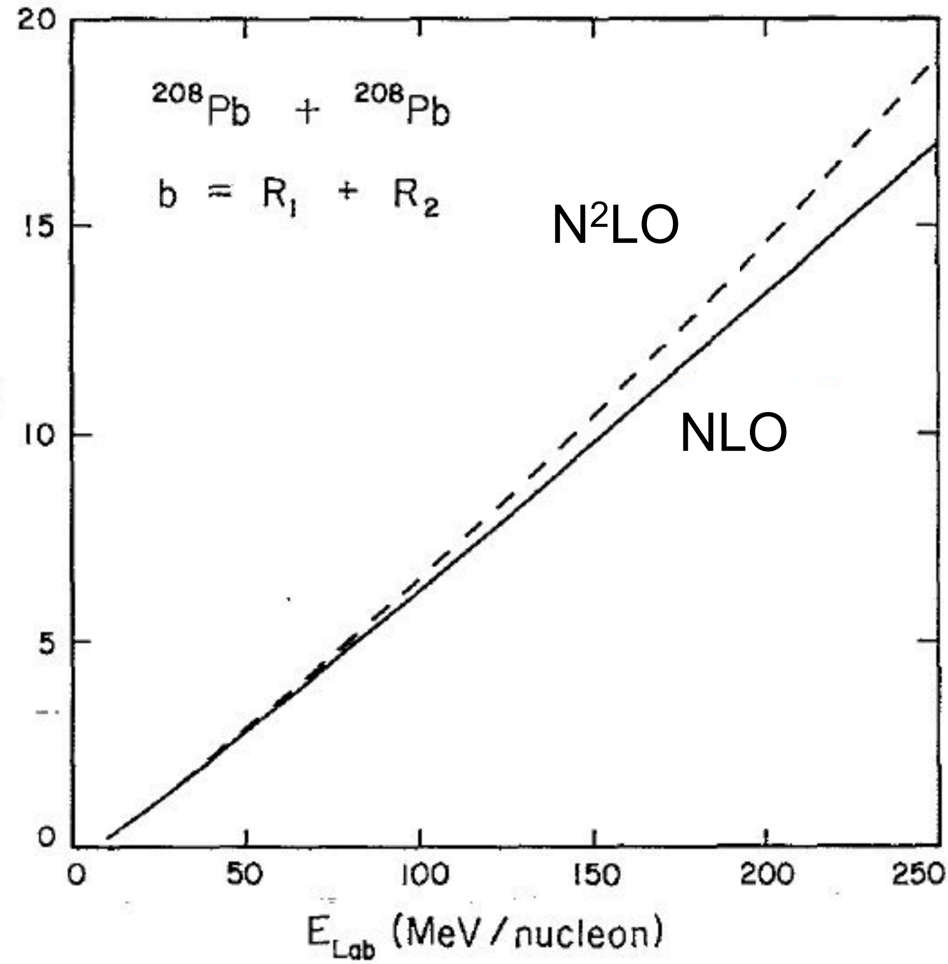
$$\times\left[\frac{1}{8}\left\{\left(\frac{v}{c}\right)^4 - 3\left(\frac{v_r}{c}\right)^4 + 2\left(\frac{v_rv}{c}\right)^2\right\} + \frac{Z_1Z_2e^2}{\mu c^2r}\left\{3\left(\frac{v_r}{c}\right)^2 - \left(\frac{v}{c}\right)^2\right\} + \frac{4Z_1^2Z_2^2e^4}{\mu^2c^4r^2}\right]$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega}$$

$$\frac{\Theta\,(E,b)-\Theta_{LO}}{\Theta_{LO}}(\%)$$

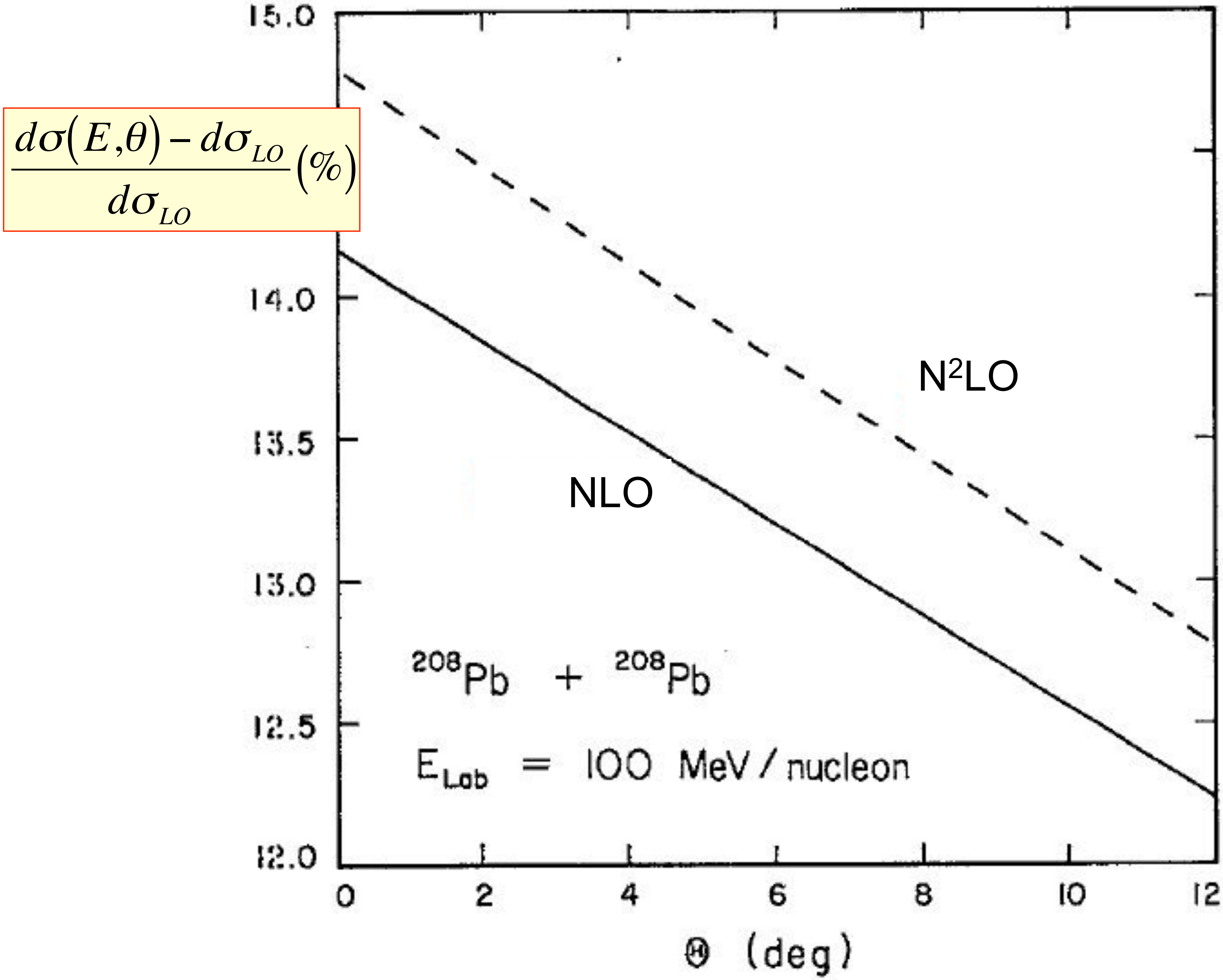


Deviations from Rutherford



important for elastic scattering: experimental data often reported as

$$\frac{d\sigma_{elast}}{d\sigma_{Ruth}}$$


$\frac{d\sigma(E,\theta) - d\sigma_{LO}}{d\sigma_{LO}}(\%)$
15.0
14.0
13.5
13.0
12.5
12.0
0
2
4
6
8
10
12
Θ (deg)
N^2LO
NLO
$^{208}Pb + {}^{208}Pb$
E_{Lab} = 100 MeV / nucleon

Coulomb excitation: orbital integrals with retardation

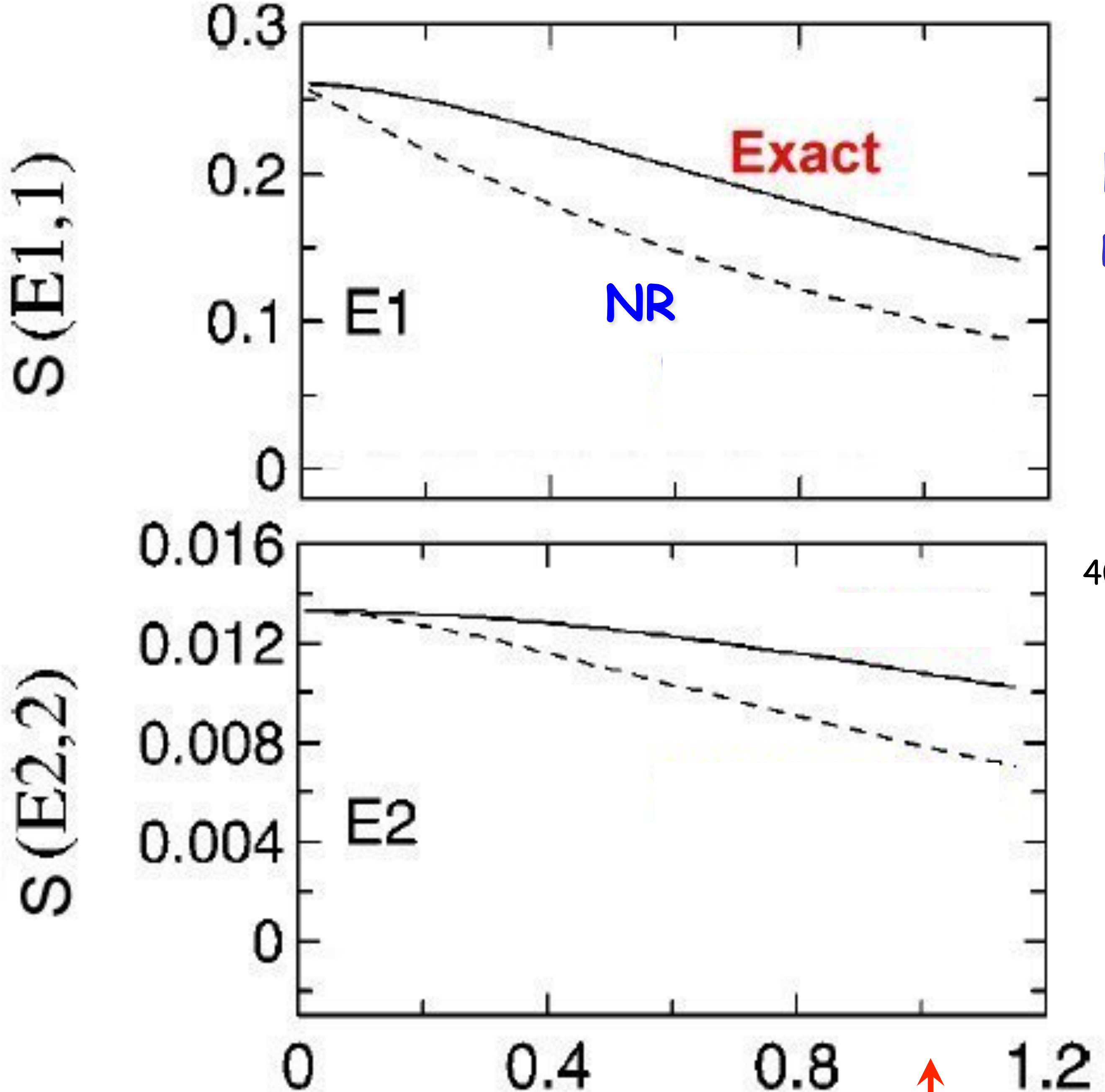
Aleixo, Bertulani, NPA 505, 448 (1989)

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{elast} \times P_{exc}$$

$$= \sum_{\pi\lambda\mu} \left|S\left(\pi\lambda\mu\right)\right|^2 \left|M_{fi}\left(\pi\lambda,-\mu\right)\right|^2$$

$$M_{fi}\left(\pi\lambda\mu\right) = \left\langle f \left| \mathrm{EM\,Operator}(\lambda\mu) \right| i \right\rangle$$

$$S\left(\pi\lambda\mu\right) = \text{orbital integrals}$$

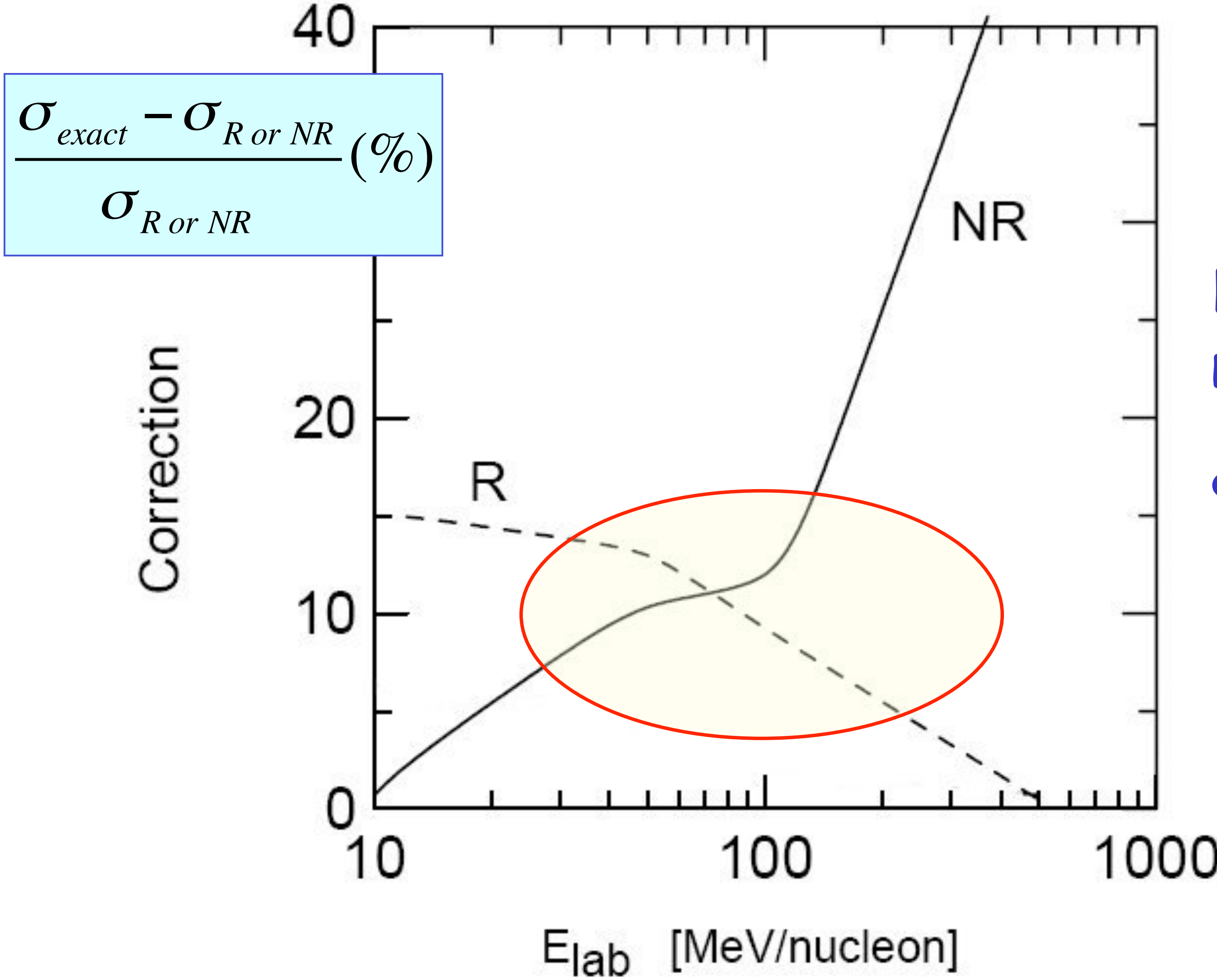


$$\xi = \frac{E_x b}{\gamma \hbar v}$$

Deviations from non-relativistic

^{40}S (100 MeV/nucleon) + *Au*

Corrections important large b's, large E_x's



Deviations from non-relativistic

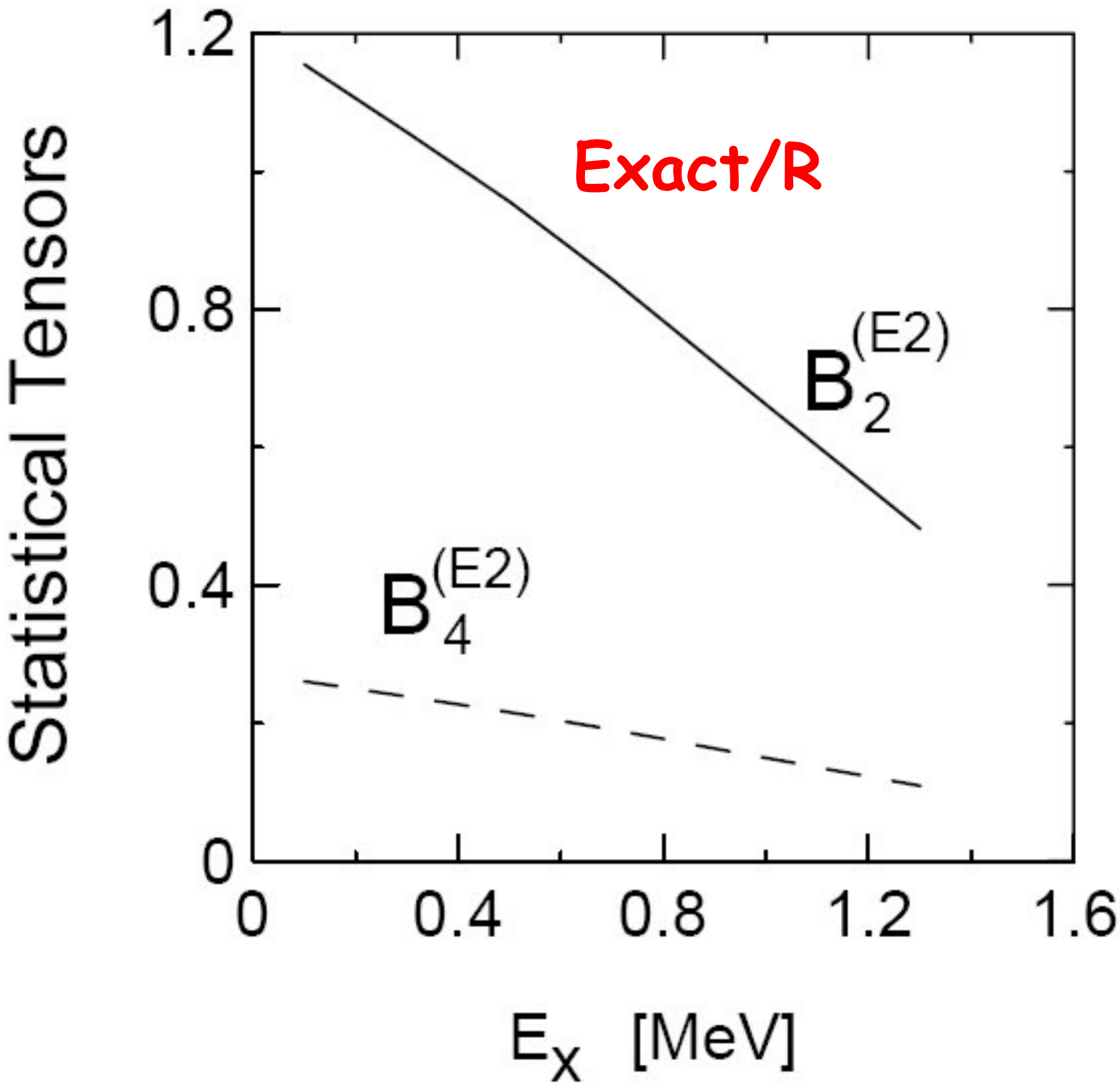
& from relativistic

^{40}S (100 MeV/nucleon) + Au

E_x = 0.89 MeV

De-excitation by γ-ray emission

$$W_\gamma(\theta_\gamma) = 1 + \sum_{\kappa=2,4} B_\kappa Q_\kappa(E_\gamma)\, P_\kappa(\cos\theta_\gamma)$$



Deviations from from relativistic theory

^{38}S (100 MeV/nucleon) + Au

We all know that:

• Relativitiy obviously important at GANIL, GSI, MSU and RIKEN
(we are talking dynamics)

• 'Rather' easy to include for Coulomb interaction
(transformation properties of E/M fields well known)

How about nuclear interaction?

• Transformation properties of nucleus-nucleus potentials not exactly known

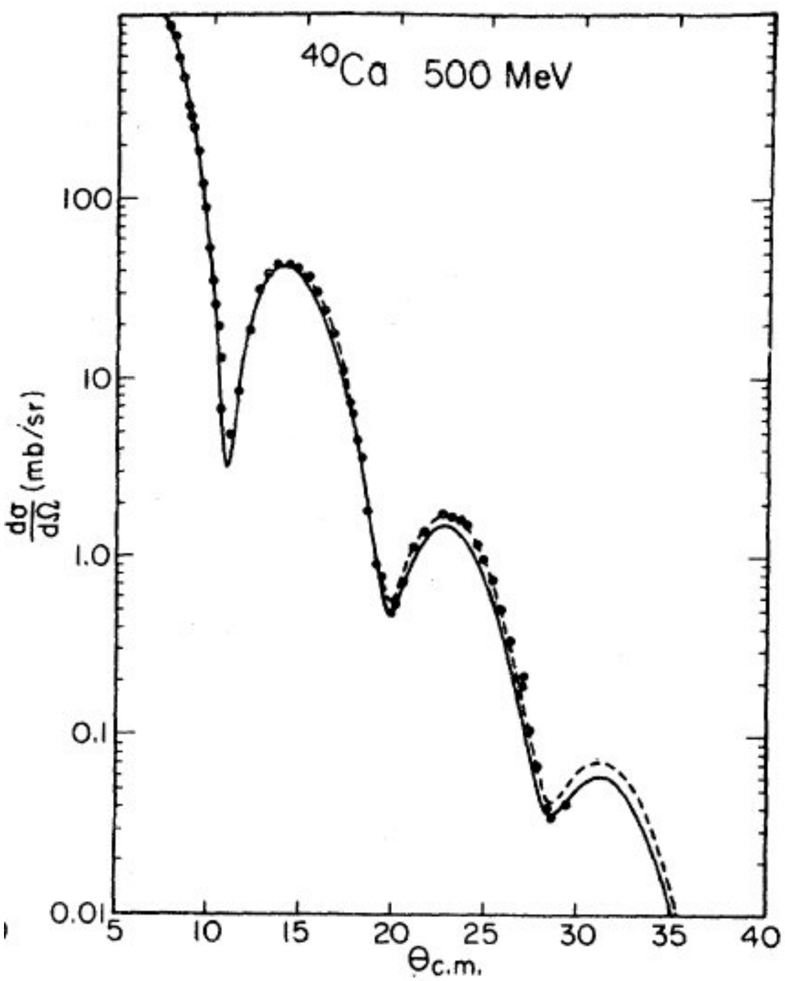
• Solution has to be based on QFT (QM + relativity)

• Can we save our DWBA, CC, or CDCC knowledge for something practical?

Clue: Proton-nucleus scattering at intermediate energies

- meson exchange, two-nucleon interaction
- mean field approximation, U_0 (ω exchange), U_S (2π exchange)

$$\left[E - V_C - U_0 - \beta\left(mc^2 + U_S\right)\right]\Psi = -i\hbar c\alpha\cdot\nabla\Psi$$



non-relativistic reduction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U_{cent} + \left(\frac{\hbar}{2mc}\right)^2 \frac{1}{r}\frac{d}{dr}U_{SO}\,\sigma\cdot\mathbf{L}\right]\phi = E\phi$$

$$U_{cent} = m^*\left(U_0 + U_S\right) + \cdots$$

$$m^* = 1 - \frac{U_0 - U_S}{2mc^2} + \cdots$$

$$U_{SO} = U_0 - U_S + \cdots$$

Arnold, Clark, PLB 84, 46 (1979)

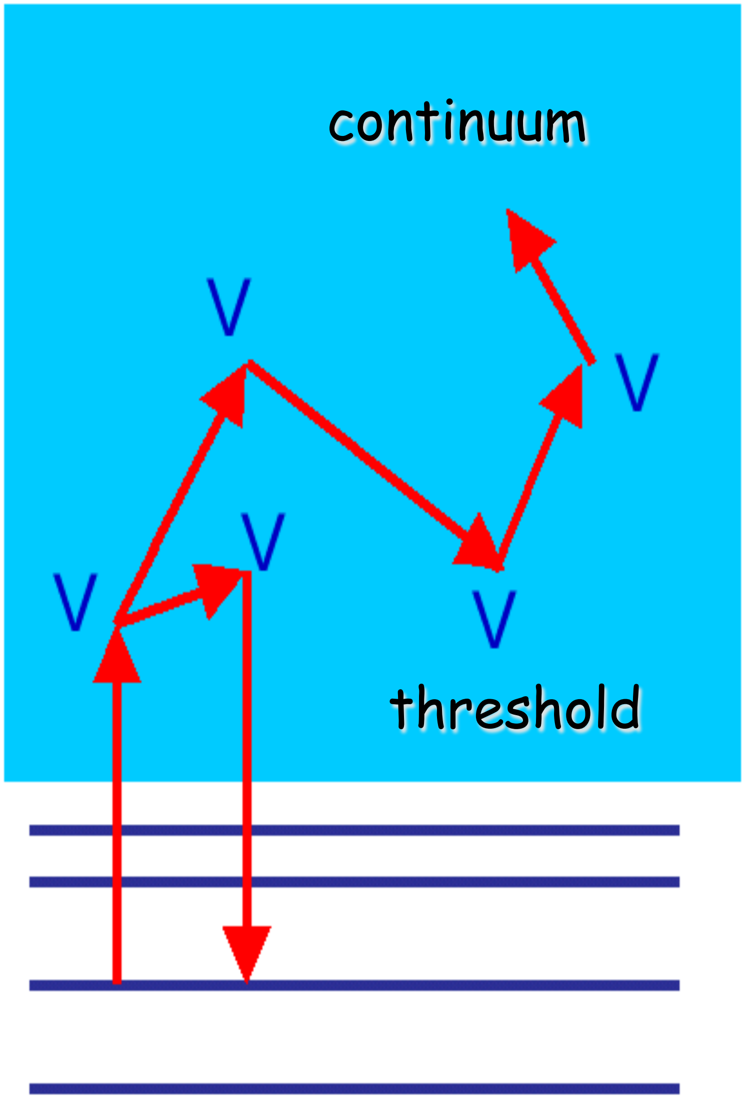
Continuum (CDCC)

$$\left|\varphi_b\right\rangle = \left|E_b, J_b M_b\right\rangle$$

$$\left|\varphi_{jJM}^{(c)}\right\rangle = \int \Gamma_j(E)\left|E, JM\right\rangle dE$$

$$\int \Gamma_i(E)\Gamma_j(E)\,dE = \delta_{ij}$$

continuum discretization



$$V_{\alpha\beta}(\mathbf{R}) = \left\langle \phi_\alpha(\mathbf{r})\middle| U(\mathbf{R},\mathbf{r}) \middle| \phi_\beta(\mathbf{r})\right\rangle$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - E\right]\chi_\alpha(\mathbf{R}) = -\sum_{\beta=0}^{N} V_{\alpha\beta}(\mathbf{R})\chi_\beta(\mathbf{R})$$

Coupled-channels

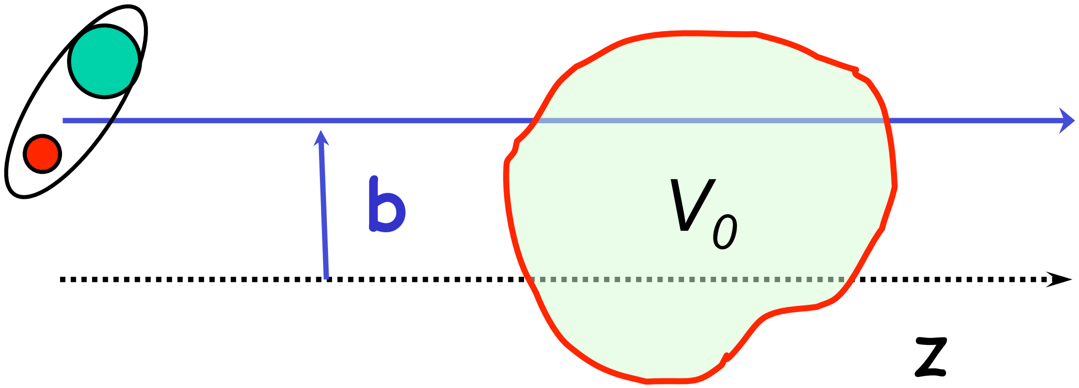
Bertulani, Canto, NPA 539, 163 (1992)
$^{11}Li + ^{208}Pb$ (100 MeV/nucleon)

Relativistic-CDCC

Bertulani, PRL 94, 072701 (2005)

$$\left[\nabla^2 + k^2 - U\right]\Psi(\mathbf{R},\mathbf{r}) = 0$$

$$U = V_0\left(2E - V_0\right)$$



$$\Psi(\mathbf{R},\mathbf{r}) = \sum_\alpha S_\alpha(\mathbf{b},z)\, e^{ik_\alpha z}\, \phi_\alpha(\mathbf{r}),$$

$$\mathbf{R} = (\mathbf{b}, z)$$

$$U \approx 2V_0 E$$

$$\nabla^2 S << ik_z \partial_z S$$

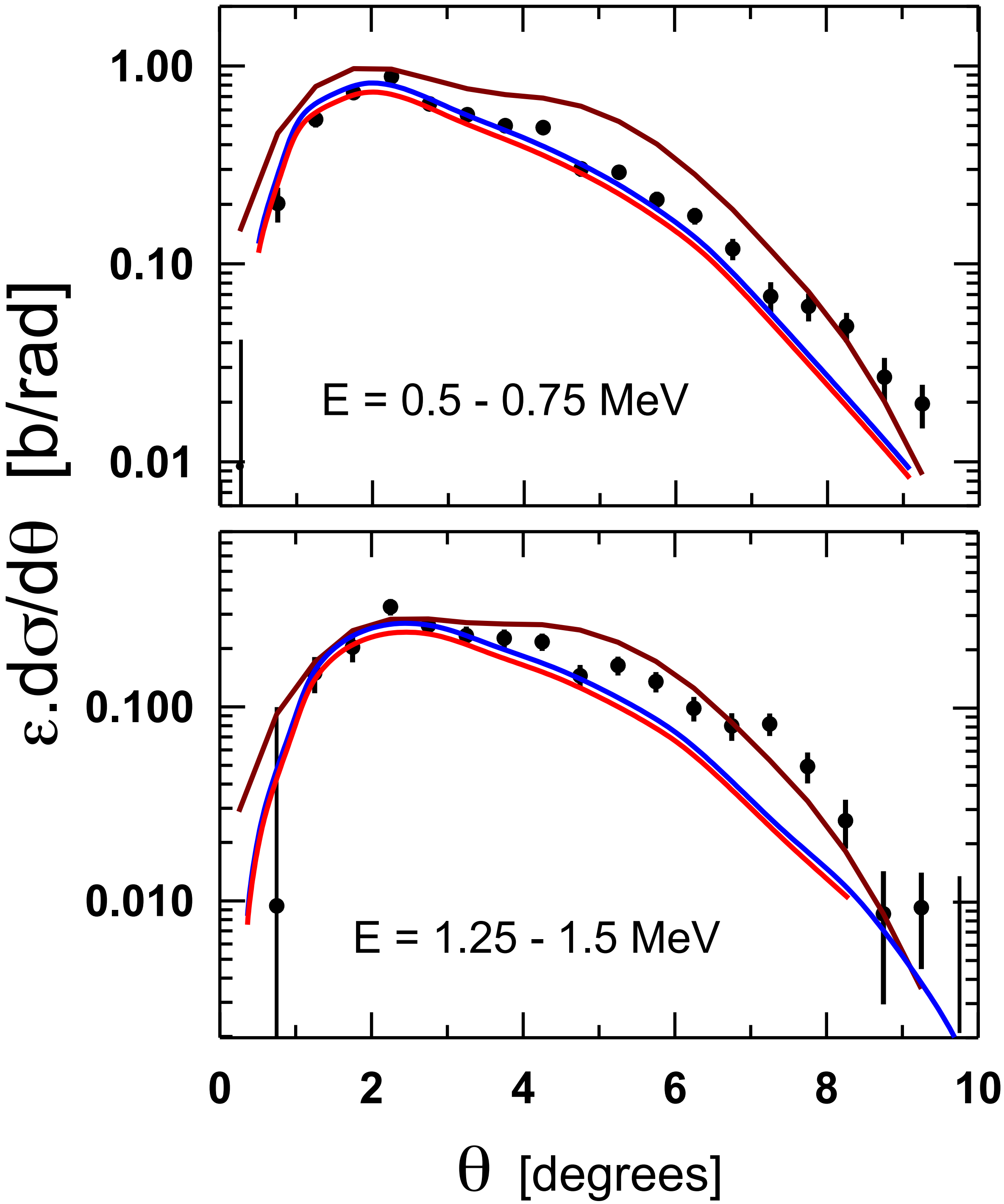
$$iv\partial_z S_\alpha(\mathbf{b},z) = \sum_\beta V_{\alpha\beta}(\mathbf{b},z) S_\beta(\mathbf{b},z)\, e^{i(k_\beta - k_\alpha) z}$$

$$f_\alpha(\mathbf{Q}) = -\frac{ik}{2\pi}\int d\mathbf{b}\, e^{i\mathbf{Q}.\mathbf{b}}\left[S_\alpha(\mathbf{b}, z=\infty) - \delta_{\alpha,0}\right]$$

$$\mathbf{Q} = \mathbf{K}'_\perp - \mathbf{K}_\perp \qquad \alpha = jlJM$$

V_0 = time-like component of 4-vector → Relativistic CDCC = Lorentz invariant

Pb(^{8}B,p^{7}Be) at 50 MeV/nucleon



DATA: Kikuchi et al, 1997

LO

Bertulani, Gai, NPA 626, 227 (1998)

All orders

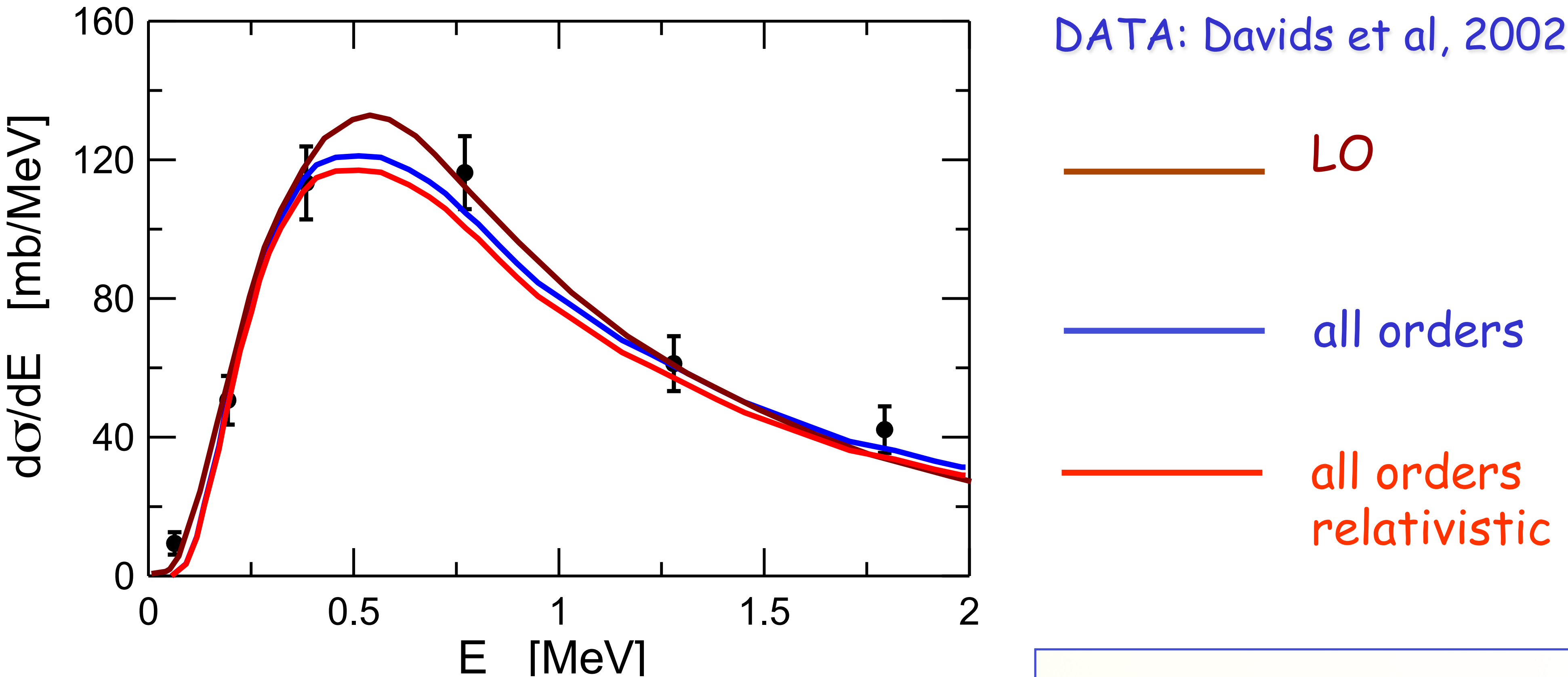
All orders relativistic

Bertulani, PRL 94, 072701 (2005)

V_0 = Coulomb + nuclear with relativistic corrections

5-7% effect

Pb(^{8}B,p^{7}Be) at 83 MeV/nucleon



V_0 = Coulomb + nuclear with relativistic corrections

4-10% effect

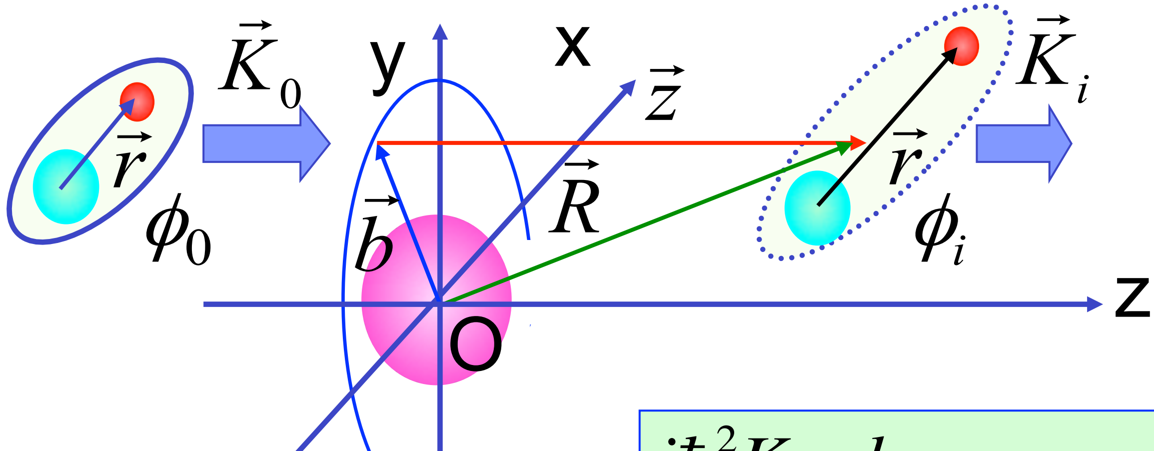
Transition: low to high energies

Eikonal scattering waves $\hat{S}_i(K_i,\vec{R})$

$$\psi^{E-CDCC} = \sum_i \hat{\phi}_i(\vec{r})\ \hat{S}_i(b,z)\exp(i\vec{K}_i\cdot\vec{R})$$

$$K_i = \sqrt{2\mu_R(E-\varepsilon_i)}/\hbar,$$

Energy conservation



- Boundary condition

$$\hat{S}_i(b,z) \xrightarrow{z\to-\infty} \delta_{i,0}$$

$$\Delta\hat{S}_i(b,z)\cong 0 \ \Rightarrow\ \frac{i\hbar^2K_i}{\mu_R}\frac{d}{dz}\hat{S}_i^{(b)}(z) = \sum_{i'}\mathrm{F}_{ii'}^{(b)}(z)\ \hat{S}_{i'}^{(b)}(z)e^{i(K_{i'}-K_i)z}$$

Eikonal scattering amplitude transformed into QM form

$$f_{i,0}^{\mathrm{E}} = \sum_L f_L^{\mathrm{E}} \equiv \sum_L \frac{2\pi}{iK_i}\sqrt{\frac{2L+1}{4\pi}}i^m Y_{Lm}(\Omega)[S_{i,0}^{b(L;i)}-\delta_{i,0}]$$

Hybrid scattering amplitude is given by

$$f_{i,0}^{\mathrm{H}} \equiv \sum_{L=0}^{L_{\mathrm{C}}} f_L^{\mathrm{Q}} + \sum_{L=L_{\mathrm{C}}+1}^{L_{\max}} f_L^{\mathrm{E}}$$

Ogata., et al, PRC68, 064609 (2003)

Relativistic CDCC

Form factor of non-rel. E-CDCC

$$F_{c'c}^{(b)}(Z) = \left\langle \Phi_{c'} \middle| U_{1A} + U_{2A} \middle| \Phi_c \right\rangle_{\mathbf{r}} e^{-i(m-m')\phi} = \sum_{\lambda} F_{c'c}^{(b);\lambda}(Z)$$

Lorentz tranform of form factor and coordinates

$$F_{c'c}^{(b);\lambda}(Z) \rightarrow f_{\lambda,m'-m} \gamma F_{c'c}^{(b)\lambda}(\gamma Z)$$

$$f_{\lambda,m'-m}^{\mathrm{Coul}} = \begin{cases} 1/\gamma & (\lambda=1,\ m'-m=0) \\ \gamma & (\lambda=2,\ m'-m=\pm 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$f_{\lambda,m'-m}^{\mathrm{nucl}} = 1$$

Assumptions

- ✓ Point charges for 1, 2 and A
- ✓ Neglecting far-field ($r_i > R$) contribution
- ✓ Correction to nuclear form factor

Ogata, Bertulani, PTP 121 (2009), 1399
PTP, 123 (2010) 701

Reaction

$^{208}Pb(^{8}B,\ ^{7}Be+p)$ at 250 A MeV and 100 A MeV

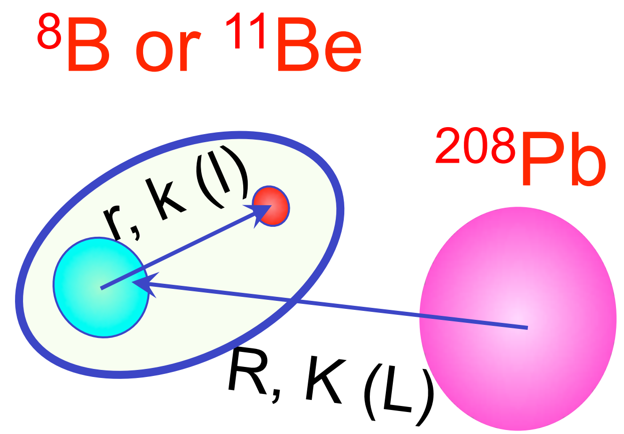
$^{208}Pb(^{11}Be,\ ^{10}Be+n)$ at 250 A MeV and 100 A MeV

Projectile wave function and distorting potential

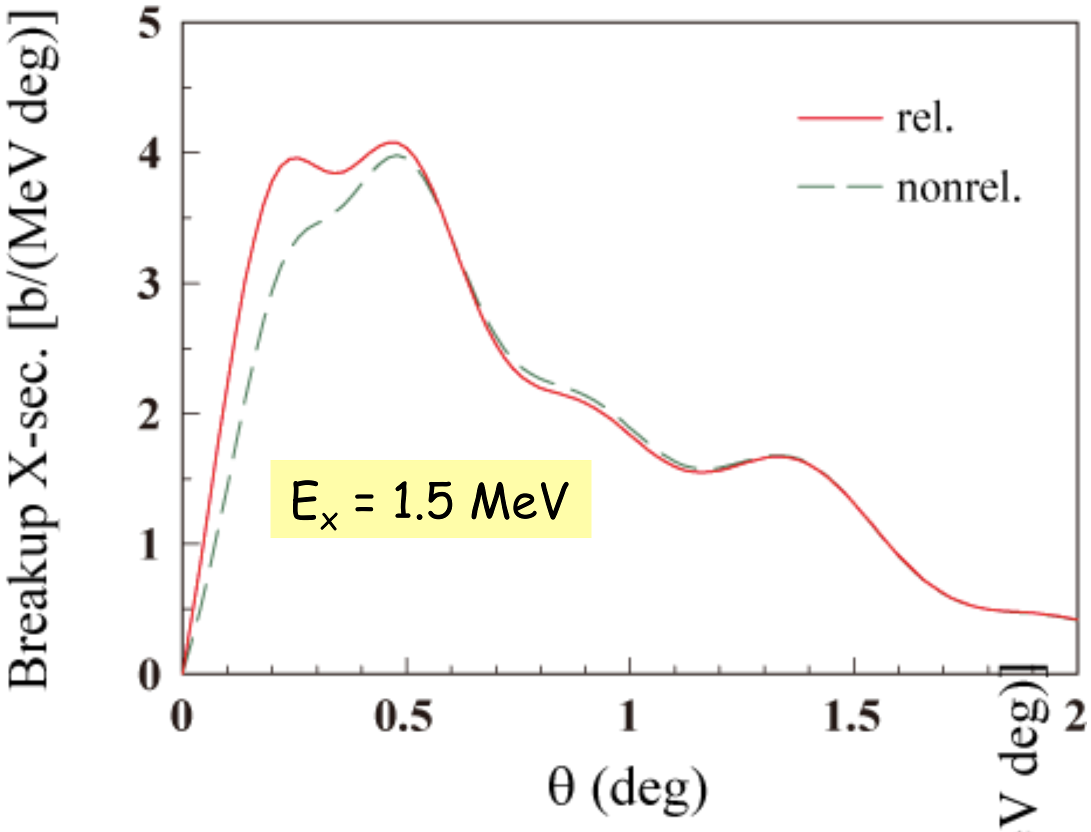
Standard Woods-Saxon

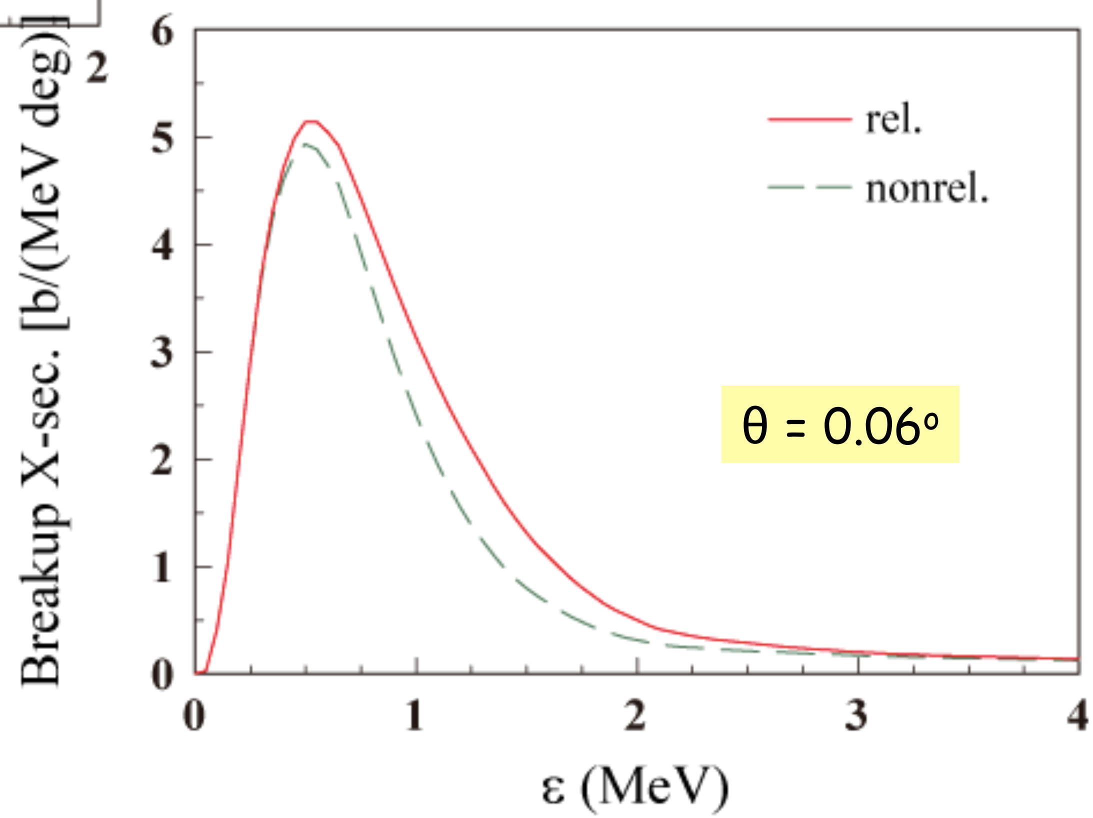
Modelspace

^{8}B	^{11}Be
$l_{max}= 3$	$l_{max}= 3$
$N_s=20,\ N_{p\text{-}d}=10$,	$N_{s,p}=20,\ N_d=10$,
$N_f=5$	$N_f=5$
$\varepsilon_{max}=10$ MeV	$\varepsilon_{max}=10$ MeV
$r_{max}= 200$ fm	$r_{max}= 200$ fm
$R_{max}= 500$ fm	$R_{max}= 450$ fm
$N_{ch} = 138$	$N_{ch} = 166$



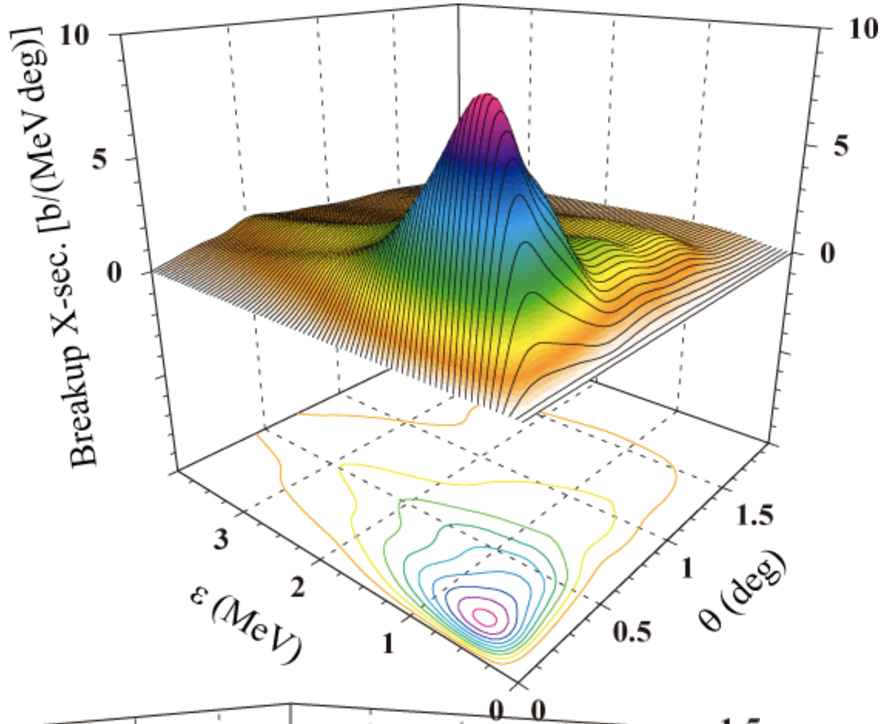
Pb(^{8}B,p^{7}Be) at 250 MeV/nucleon



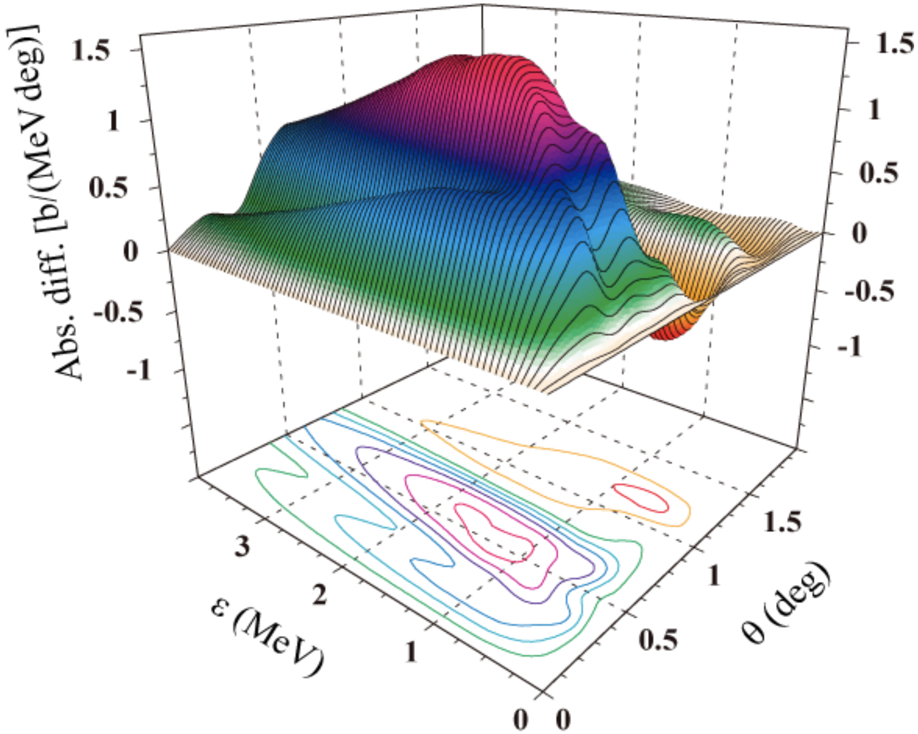


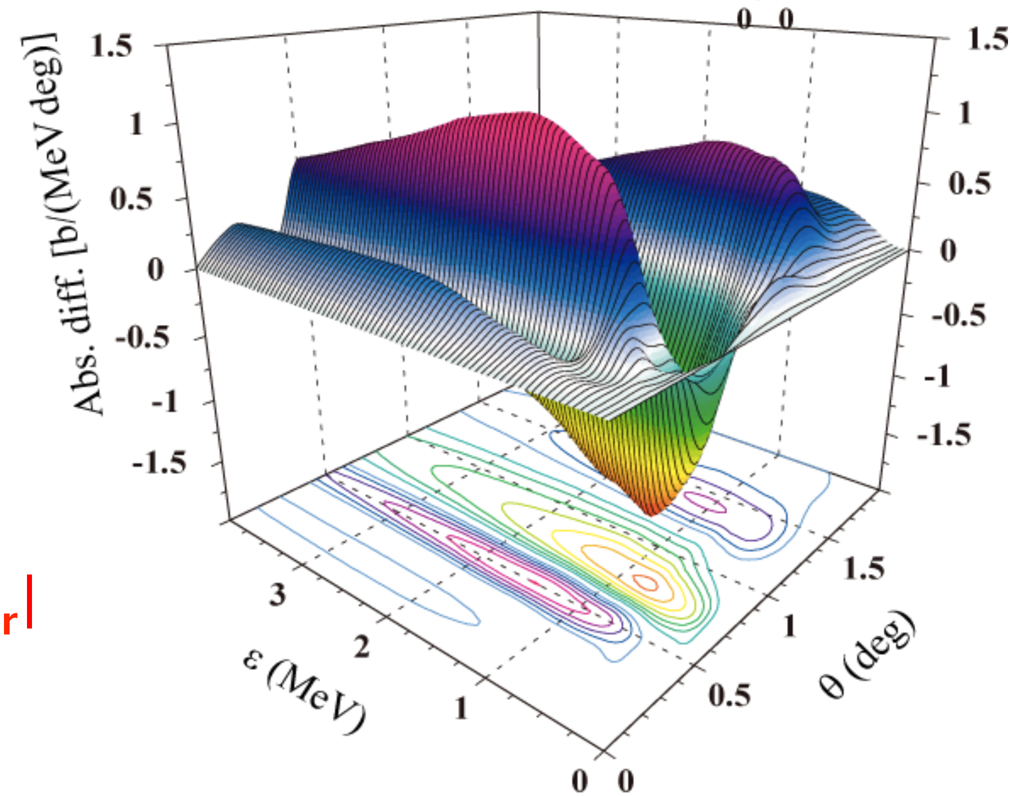
Pb(^{8}B,p^{7}Be) at 250 MeV/nucleon

all orders



$|\sigma_{all} - \sigma_{NR}|$





$|\sigma_{all} - \sigma_{no\text{-}nuclear}|$

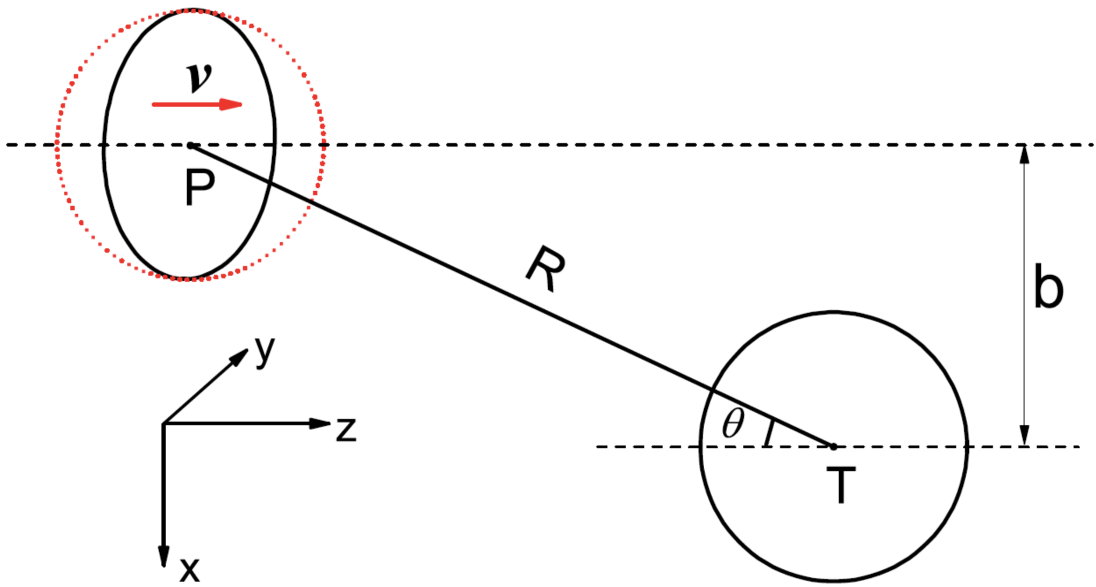
Relativistic MF nucleus-nucleus potential

Long, Bertulani, PRC 83, 024907 (2011).

σ, ω, ρ and γ exchange

$$E = \int d^3r \sum_a \bar{\psi}_a(-i\gamma\cdot\nabla + M)\psi_a$$
$$+\frac{1}{2}\sum_{\phi=\sigma,\omega,\rho,\gamma}\int d^3r d^3r' \sum_{ab}\bar{\psi}_a(\mathbf{r})\bar{\psi}_b(\mathbf{r}')\Gamma_\phi(\mathbf{r},\mathbf{r}')D_\phi(\mathbf{r}-\mathbf{r}')\psi_a(\mathbf{r})\psi_b(\mathbf{r}')$$

$$\Gamma_\phi(\mathbf{r},\mathbf{r}') = -g_\sigma(\mathbf{r})g_\sigma(\mathbf{r}')$$
$$\Gamma_\omega(\mathbf{r},\mathbf{r}') = -(g_\omega\gamma^\mu)_{\mathbf{r}}\cdot(g_\omega\gamma_\mu)_{\mathbf{r}'}$$
$$\Gamma_\rho(\mathbf{r},\mathbf{r}') = -(g_\rho\gamma^\mu\vec{\tau})_{\mathbf{r}}\cdot(g_\rho\gamma_\mu\vec{\tau})_{\mathbf{r}'}$$
$$\Gamma_\gamma(\mathbf{r},\mathbf{r}') = \frac{e^2}{4}\left[\gamma^\mu(1-\tau_z)\right]_{\mathbf{r}}\cdot\left[\gamma_\mu(1-\tau_z)\right]_{\mathbf{r}'}$$



$$D_\phi = \frac{1}{4\pi}\frac{e^{m_\phi|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$
$$D_\gamma = \frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

Lorentz transform

$$x_p = x_t + b, \qquad y_p = y_t$$
$$z_p = \gamma(z_t + R\cos\theta)$$

$$E(A_t, A_p, v) = E(A_t) + E(A_p, \mathrm{v}) + \mathrm{E}\,(A_t, A_p, \mathrm{v})$$

$$\mathrm{E}\,(A_t, A_p, \mathrm{v}) = \sum_{\phi=\sigma,\omega,\rho,\gamma} \int d^3r \int d^3r' \sum_{ab} \overline{\psi}_{t,a}(\mathbf{r}) \overline{\psi}_{p,b}(\mathbf{r}')\, \Gamma_\phi(\mathbf{r},\mathbf{r}')\, D_\phi(\mathbf{r}-\mathbf{r}')\, \psi_{t,a}(\mathbf{r}) \psi_{p,b}(\mathbf{r}')$$

Ex: σ and ω contributions

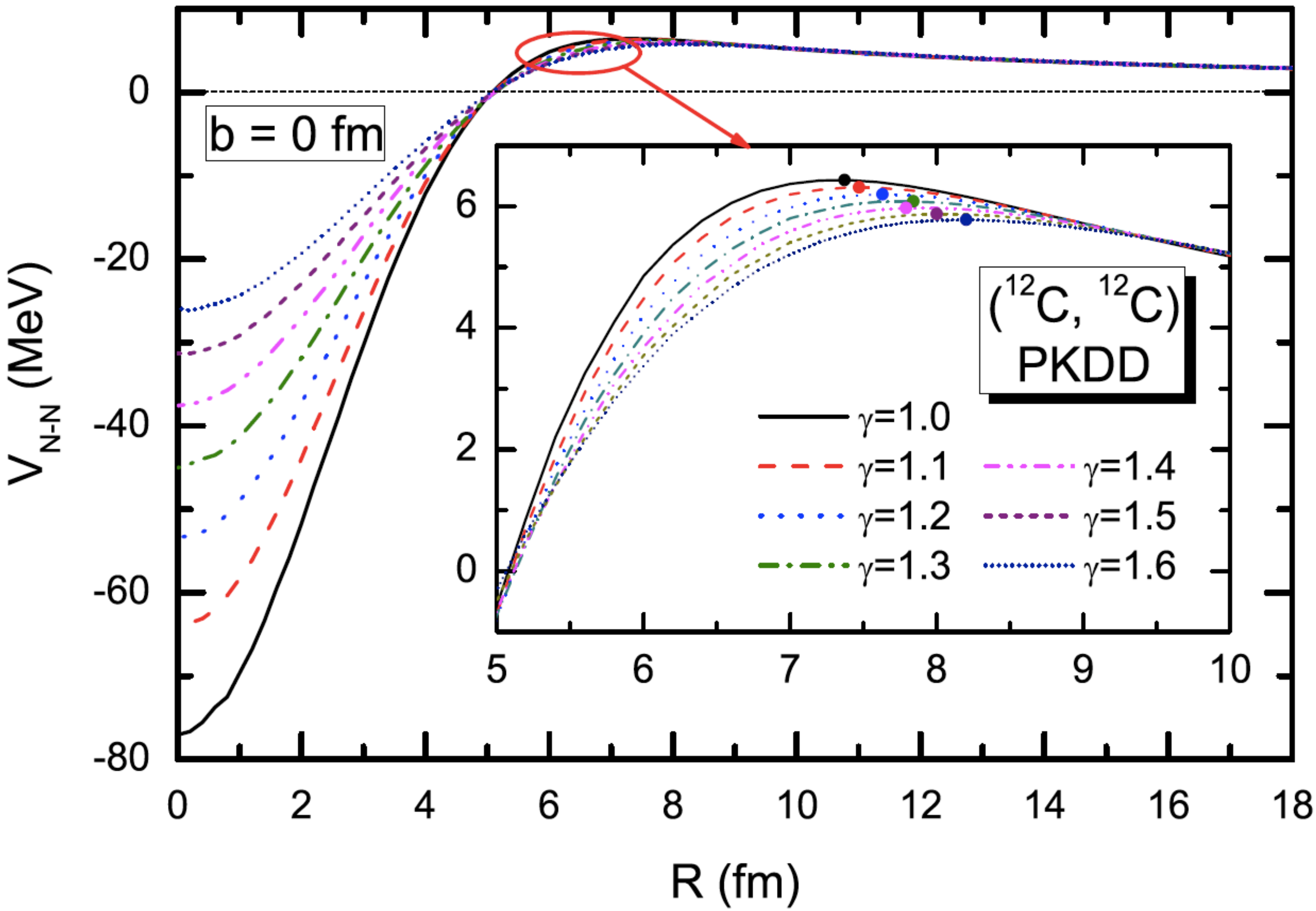
$$\mathrm{E}_\sigma = -\frac{1}{\gamma} \int d^3r_t \int d^3r_p'\, g_\sigma(\mathbf{r}_t)\, \rho_{s,t}(\mathbf{r}_t)\, D_\sigma(\mathbf{r}-\mathbf{r}')\, \rho_{s,p}(\mathbf{r}_p') g_\sigma(\mathbf{r}_p')$$

$$\mathrm{E}_\omega = \int d^3r_t \int d^3r_p'\, g_\omega(\mathbf{r}_t)\, \rho_{b,t}(\mathbf{r}_t)\, D_\omega(\mathbf{r}-\mathbf{r}')\, \rho_{b,p}(\mathbf{r}_p') g_\omega(\mathbf{r}_p')$$

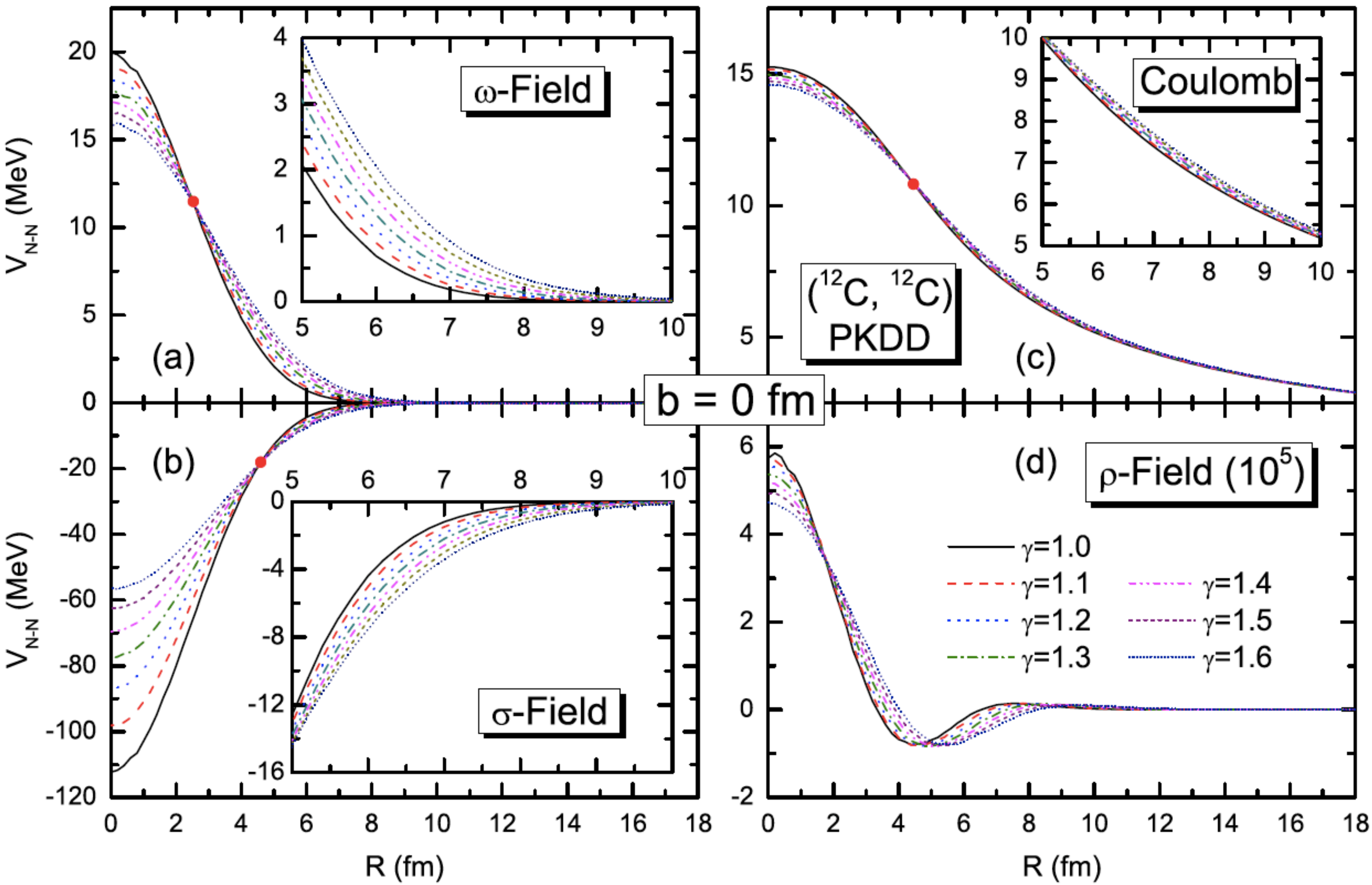
$$\rho_s(\mathbf{r}) = \sum_a \overline{\psi}_a(\mathbf{r}) \psi_a(\mathbf{r}), \qquad \rho_b(\mathbf{r}) = \sum_a \overline{\psi}_a(\mathbf{r}) \gamma^0 \psi_a(\mathbf{r})$$

Projectile densities boosted to the target frame

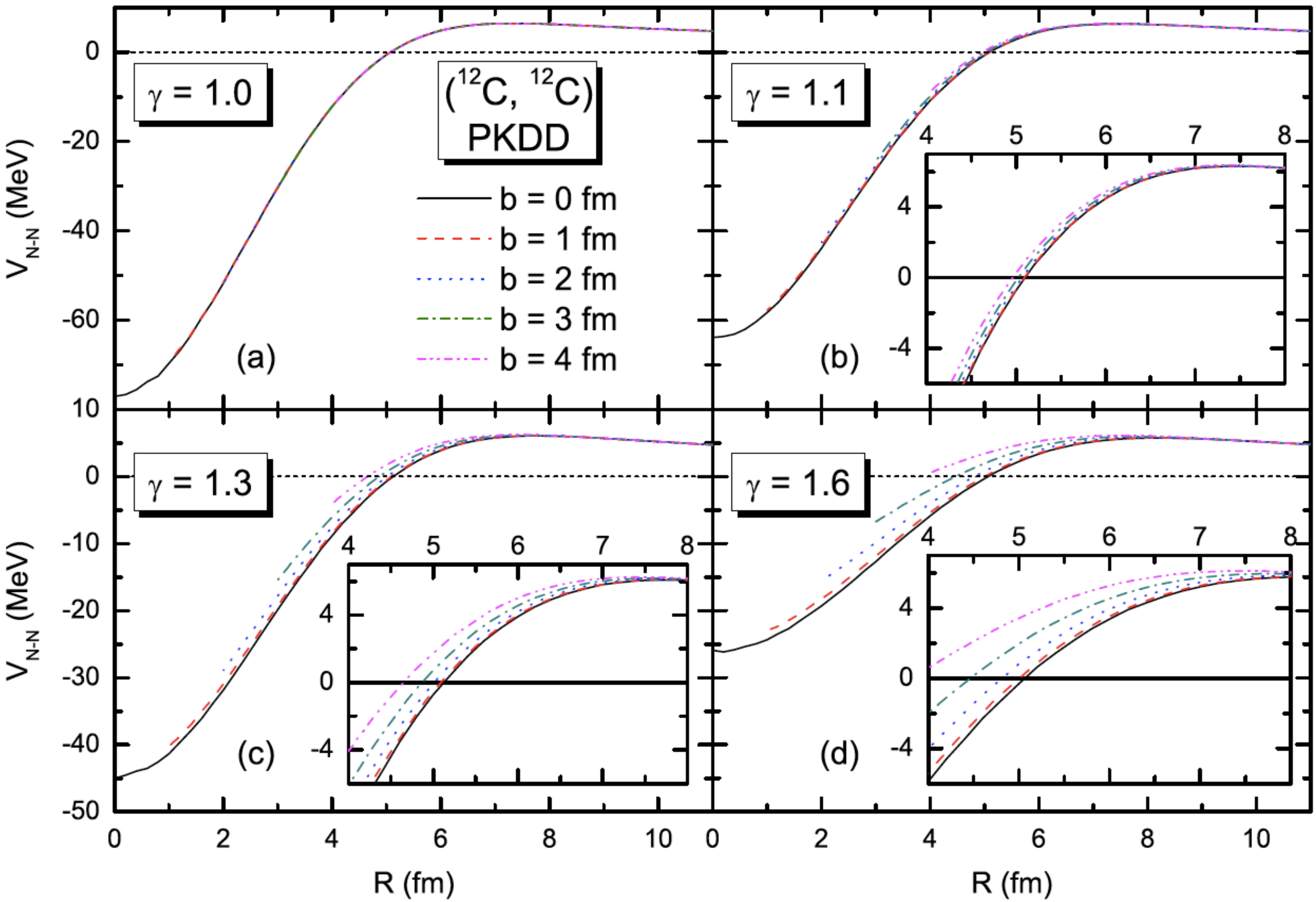
Results for ^{12}C + ^{12}C

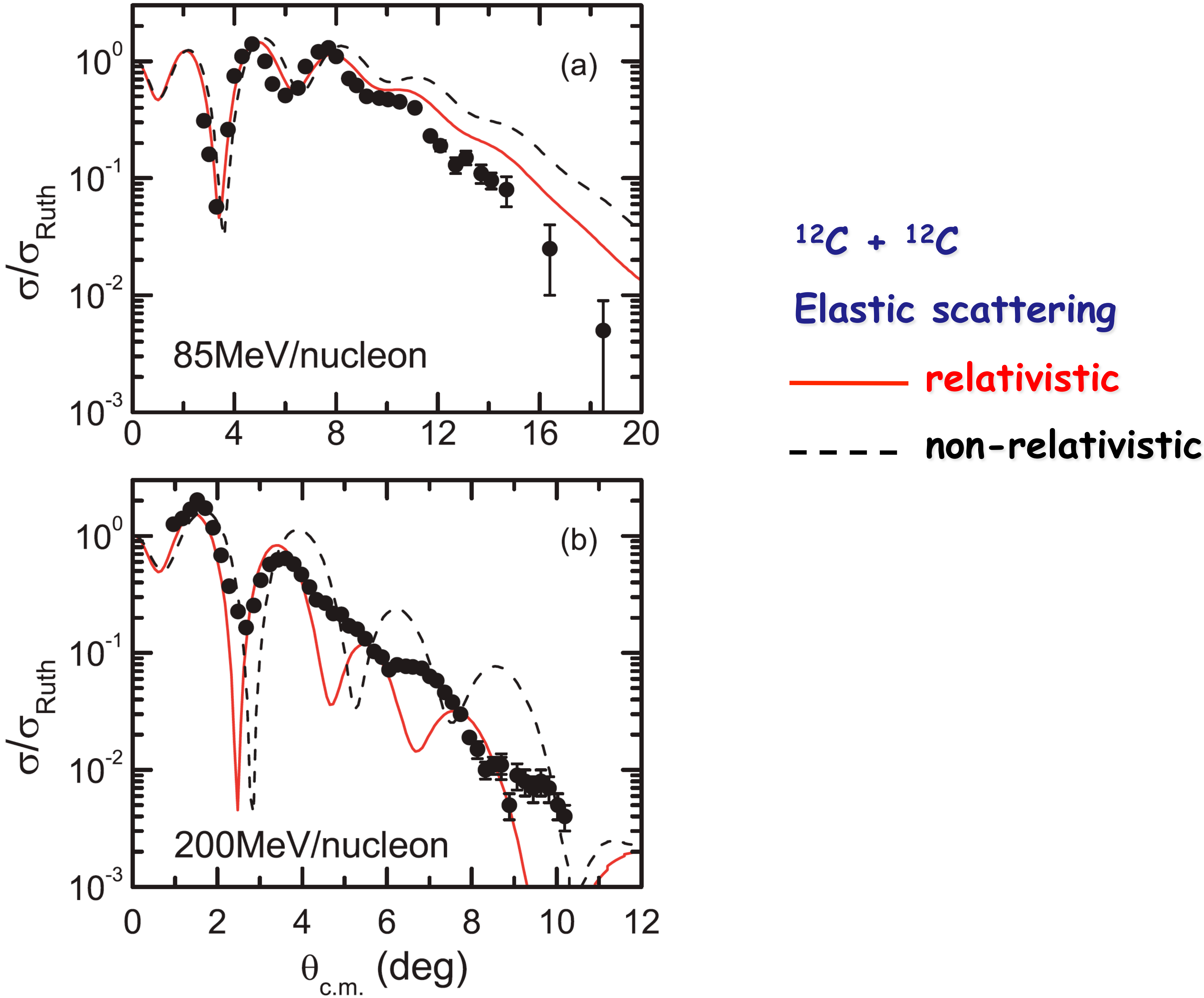


Contribution of different fields



Dependence on energy and impact parameter




(a)
85MeV/nucleon
(b)
200MeV/nucleon
σ/σRuth
θc.m. (deg)
12C + 12C
Elastic scattering
relativistic
non-relativistic

Medium effects in σ_{NN}

$$\langle \mathbf{k}|G|\mathbf{k}_0\rangle = \langle \mathbf{k}|V_{NN}|\mathbf{k}_0\rangle - \int \frac{d^3k'}{(2\pi)^3} \frac{\langle \mathbf{k}|V_{NN}|\mathbf{k'}\rangle Q(\mathbf{k'})\langle \mathbf{k'}|G|\mathbf{k}_0\rangle}{E(\mathbf{k'}) - E_0 - i\varepsilon}$$

$$E(\mathbf{P},\mathbf{k}) = e(\mathbf{P}+\mathbf{k}) + e(\mathbf{P}-\mathbf{k})$$

e = single-particle energies

E_0 = E on-shell

$$Q(\mathbf{P},\mathbf{k}) = 1, \quad \text{if} \quad k_{1,2} > k_F$$
$$= 0, \quad \text{otherwise}$$

$$\mathbf{k}_{1,2} = \mathbf{P} \pm \mathbf{k}$$

In real calculations:

$$\overline{Q}(P,k) = \frac{\int d\Omega\, Q(\mathbf{P},\mathbf{k})}{\int d\Omega}$$

$$e(p) = T(p) + v(p)$$

$$v(p) = \langle p|v|p\rangle = \mathrm{Re} \sum_{q \le k_F} \langle pq|G|pq - qp\rangle$$

- e depends on v
- v depends on G
- G depends on v

$\Longrightarrow$ Solve self-consistently (Brueckner theory)

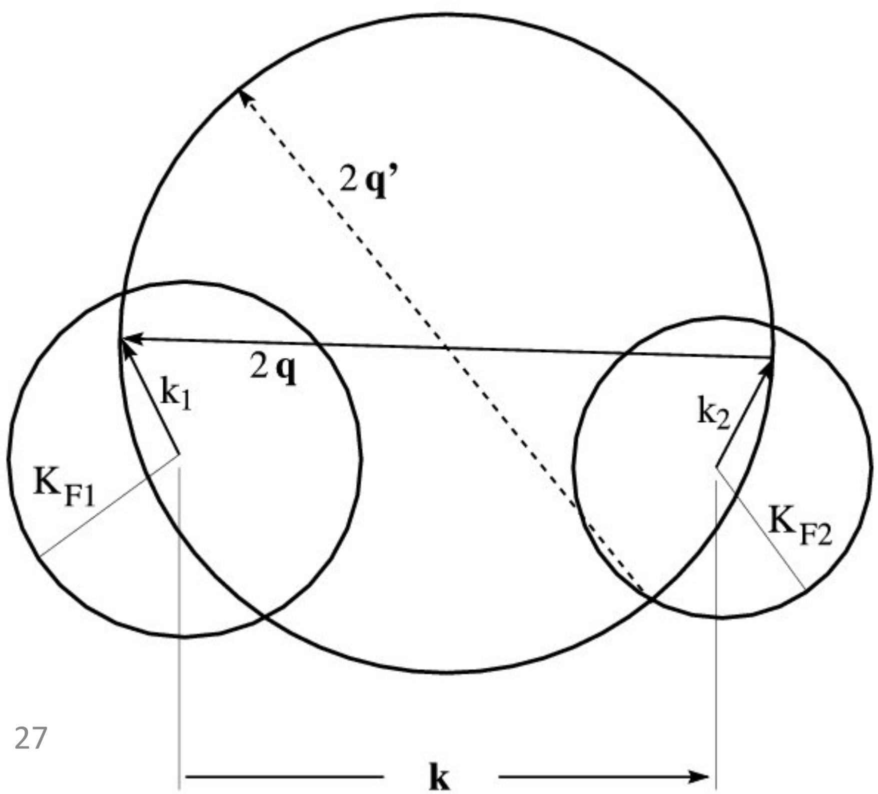
Geometric approximation + LDA

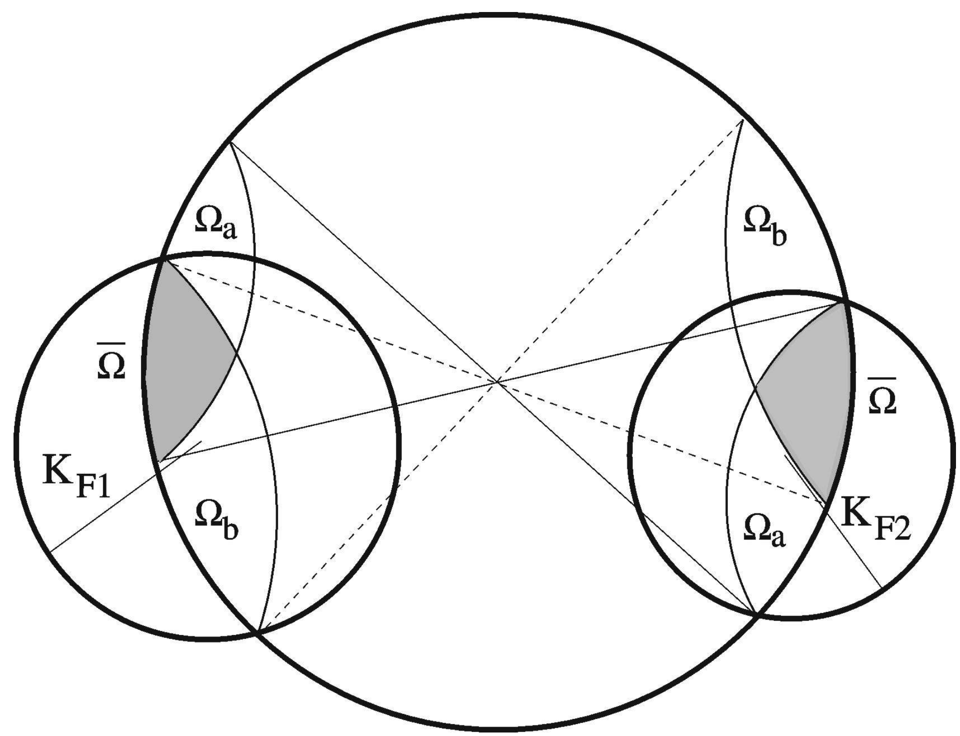
CB, Phys. Rep. (1991), JPG 27, L67 (2001)

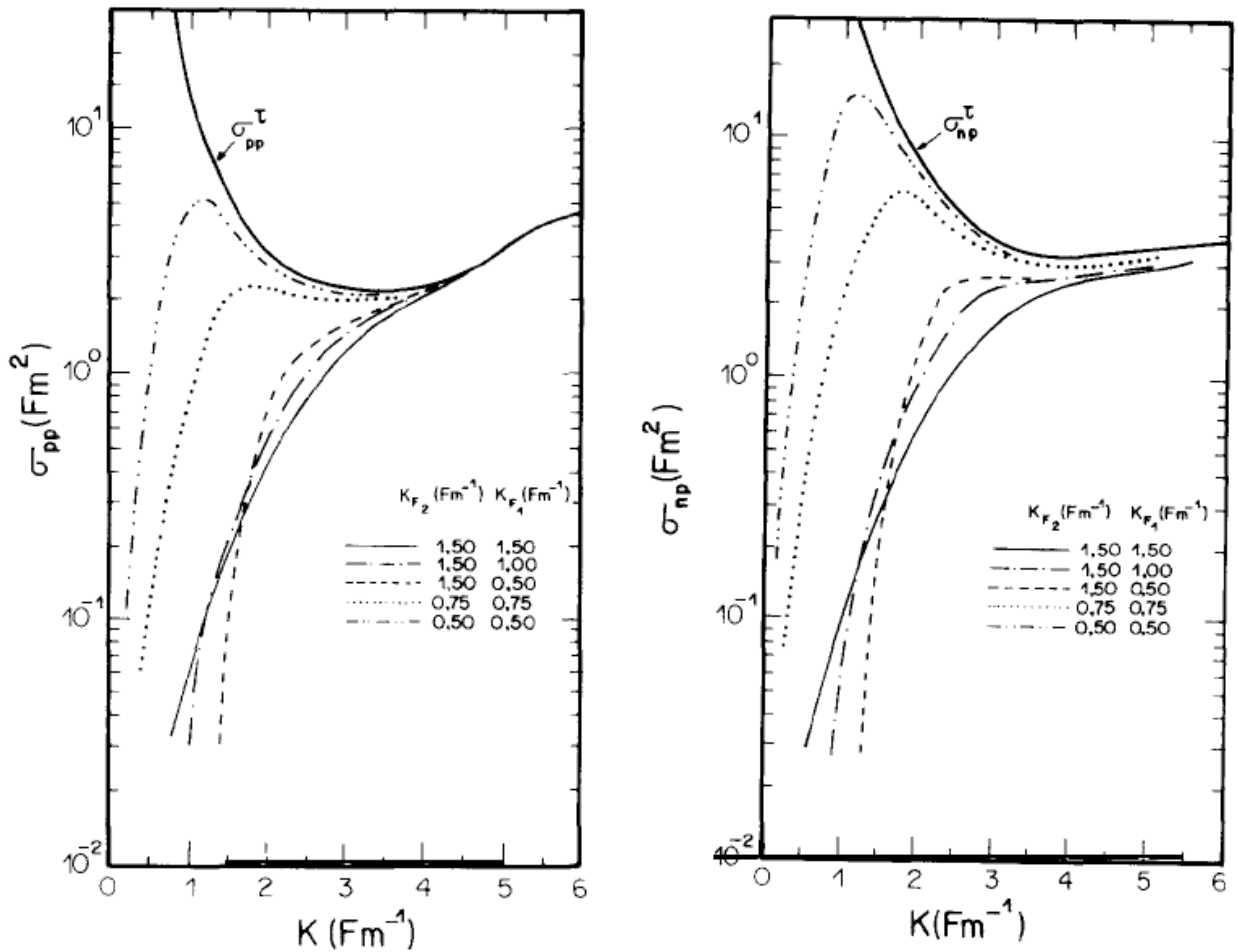
CB, De Conti, PRC C 81, 064603 (2010)

$$\overline{\sigma}_{NN}(E) = \int \frac{d^3k_1 d^3k_2}{\left(4\pi k_{1F}^3/3\right)\left(4\pi k_{2F}^3/3\right)} \frac{2q}{k} \sigma_{NN}(q) \frac{\Omega_{Pauli}}{4\pi}$$

$$\Omega_{Pauli} = 4\pi - 2\left(\Omega_a + \Omega_b - \overline{\Omega}\right)$$ = analytic



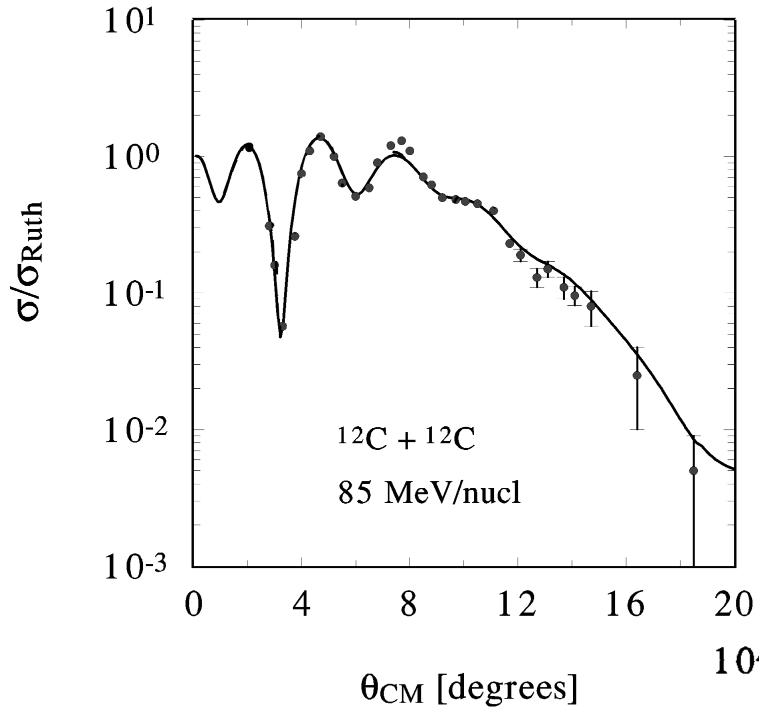


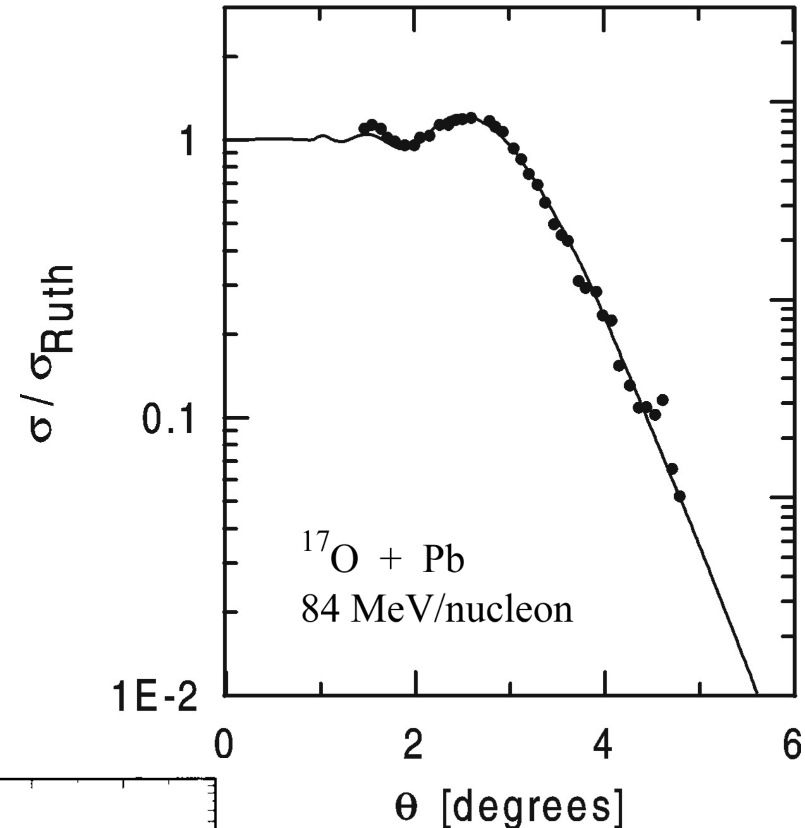


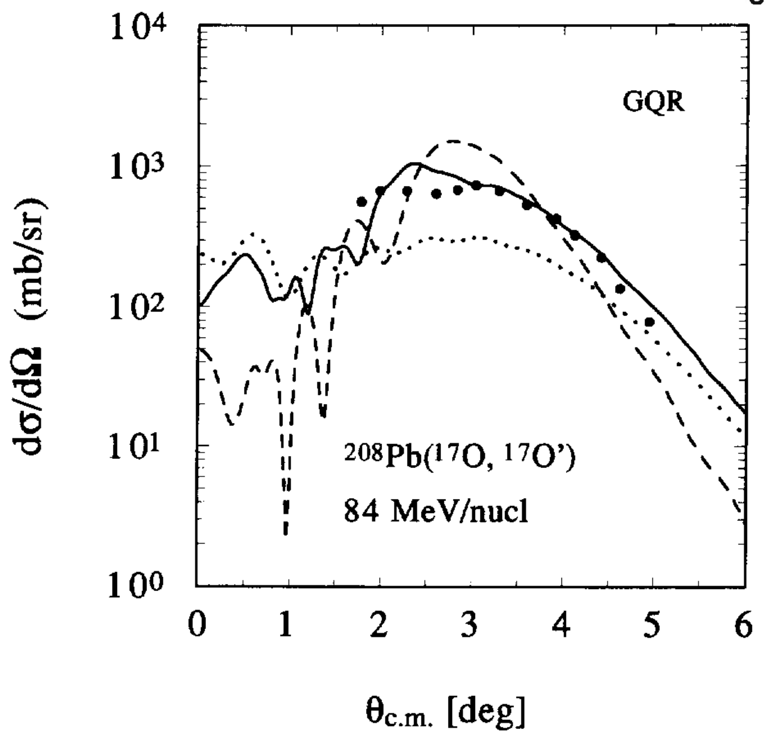
Hussein, Rego, Bertulani, Phys. Rep. 201 (1991) 279

Nucleus-nucleus elastic and inelastic scattering

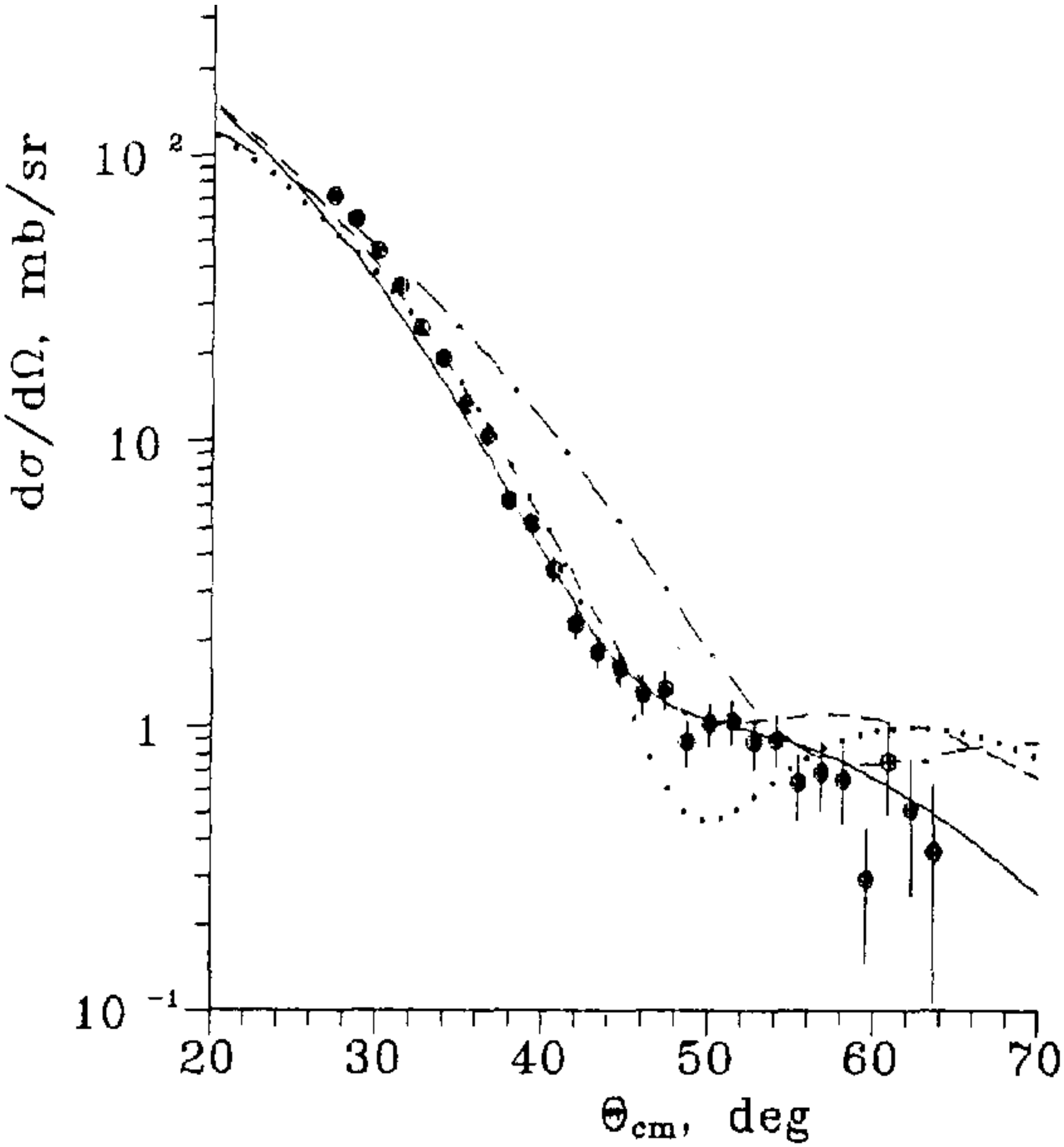
Bertulani, Sagawa, PLB 300 (1993) 205
NPA 588 (1995) 667







Elastic Scattering with ME in σ_{NN}

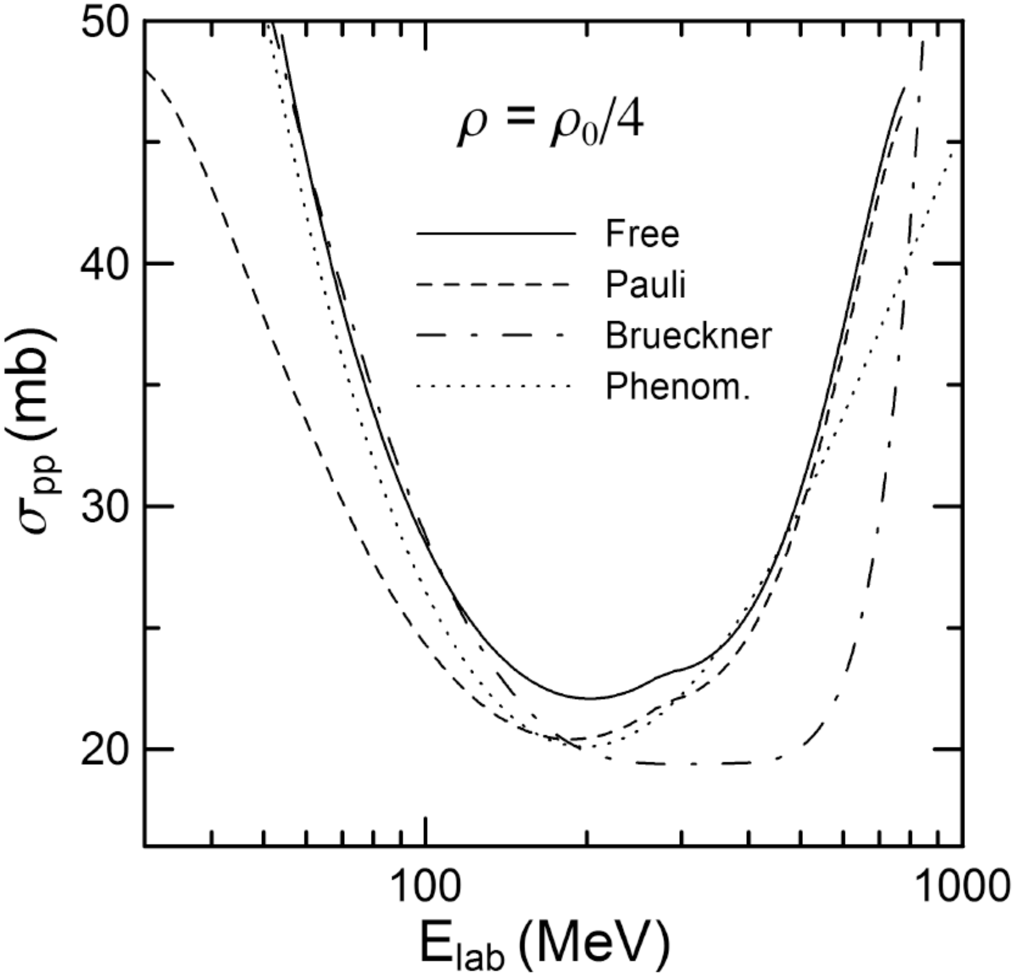


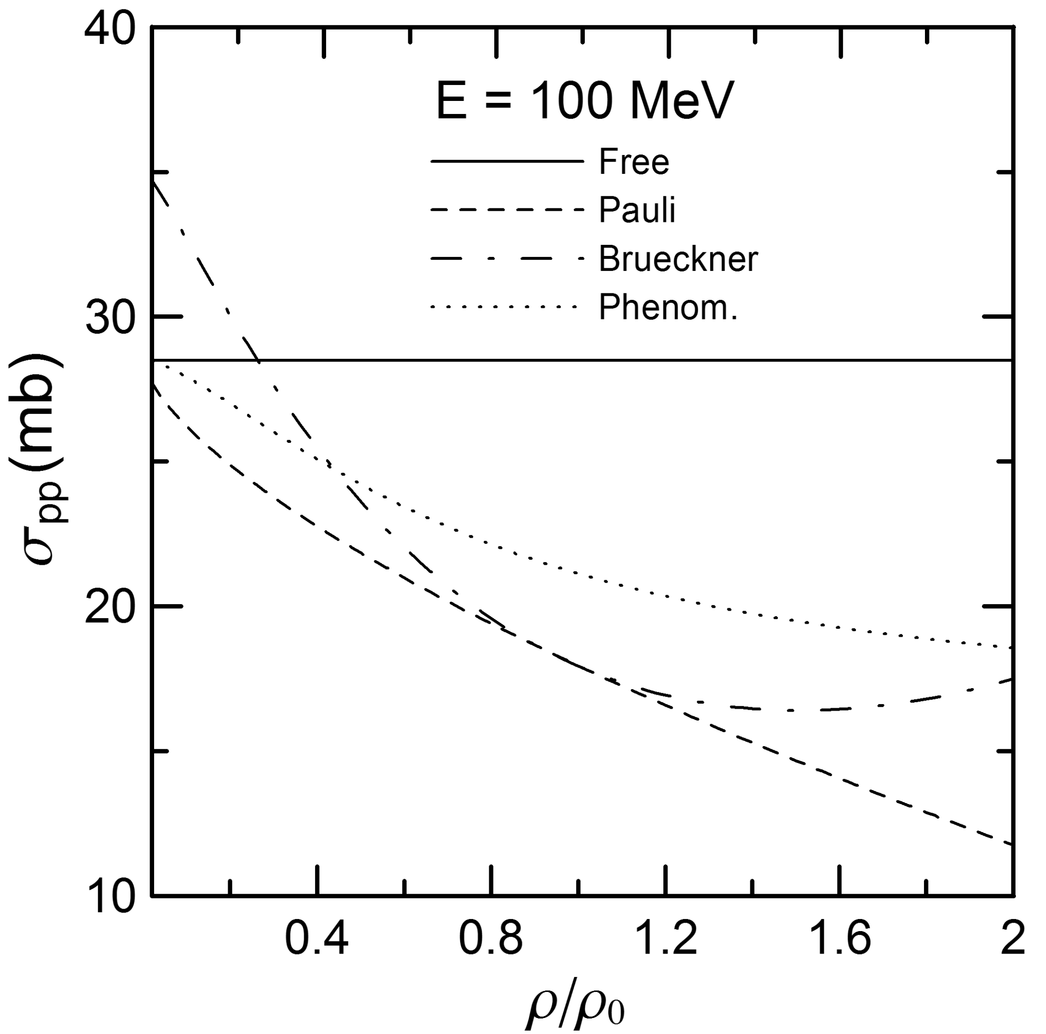
p + ^{8}He

72 MeV/nucleon

Chulkov, Bertulani, Korshenninikov NPA 587, 291 (1995)

Medium effects in σ_{NN}



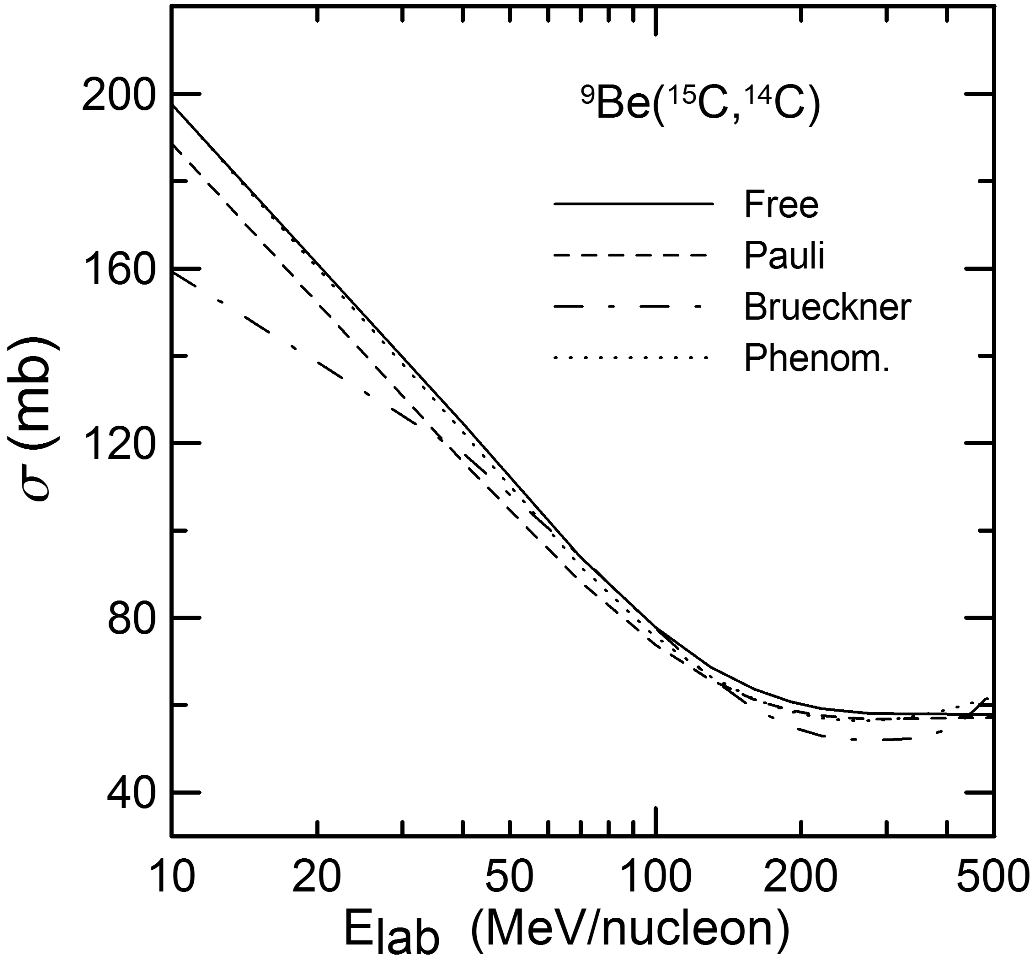


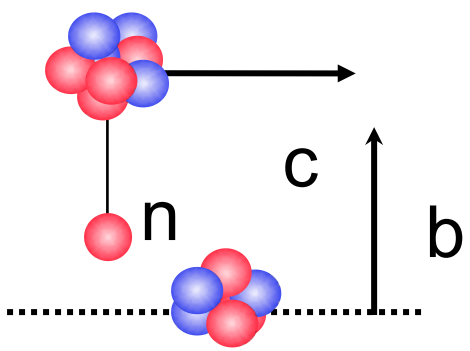
Bertulani, De Conti,
PRC C 81, 064603 (2010)

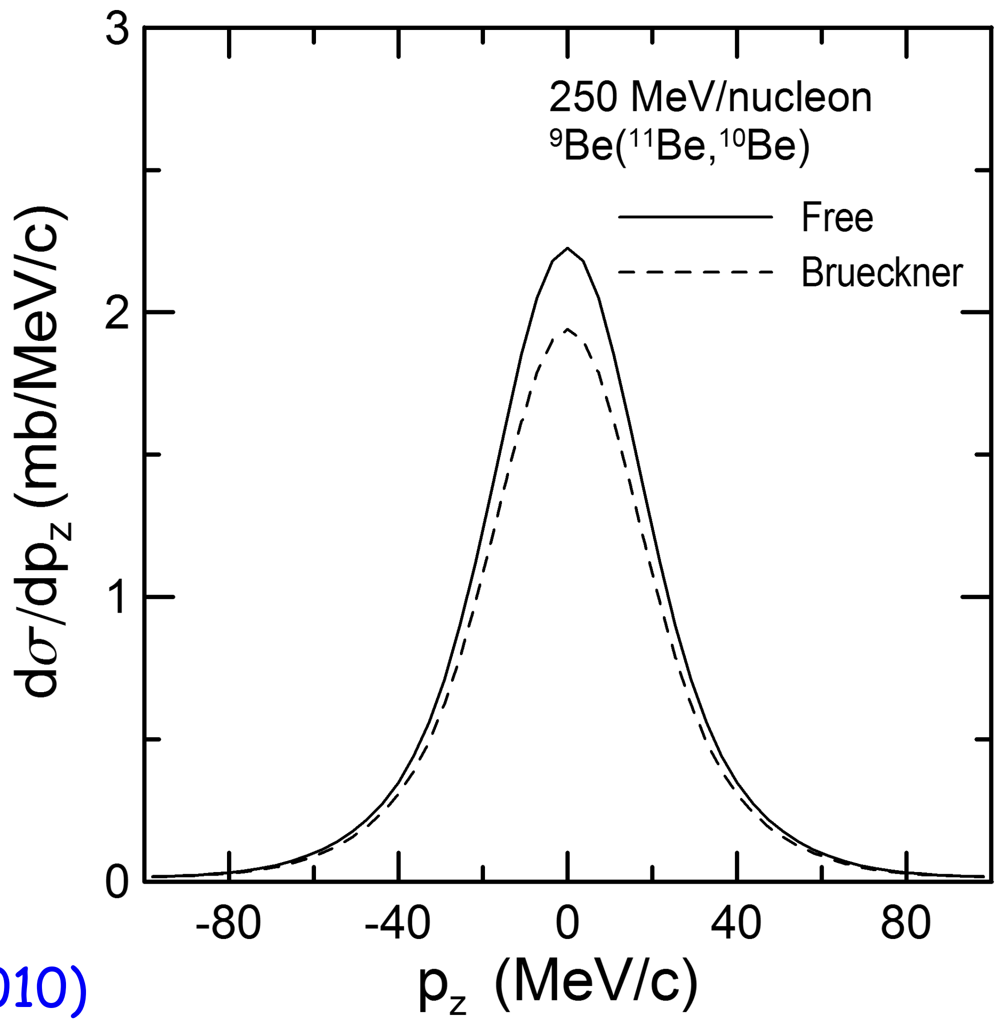
Medium effects in knockout reactions

and

momentum distributions



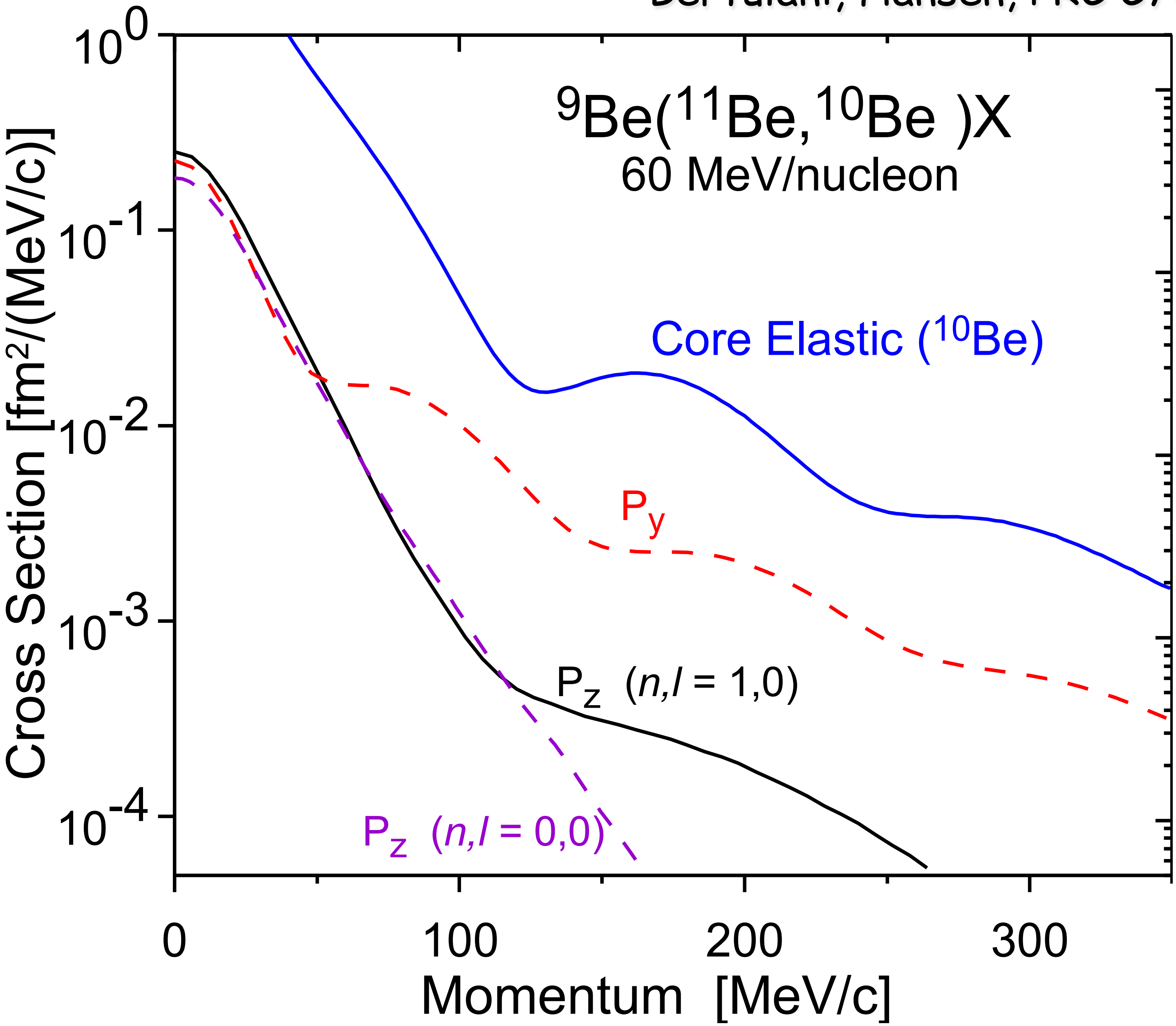




Bertulani, De Conti, PRC C 81, 064603 (2010)

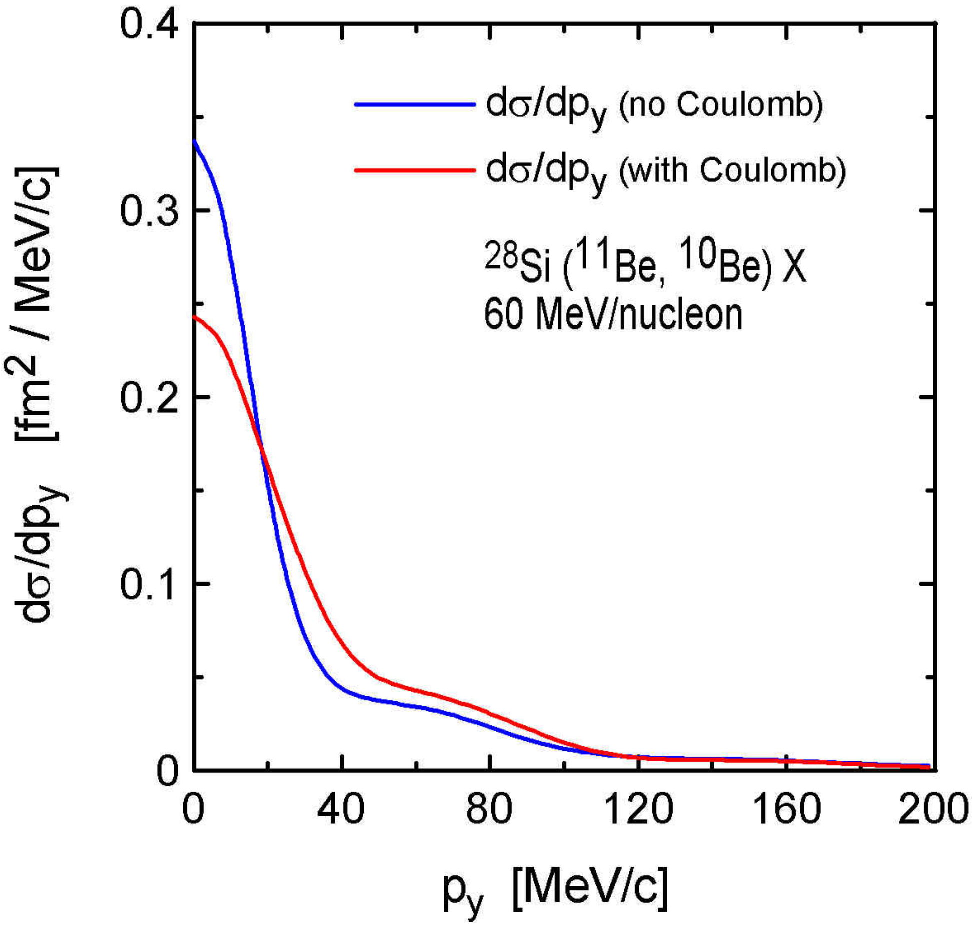
Final state core-target scattering

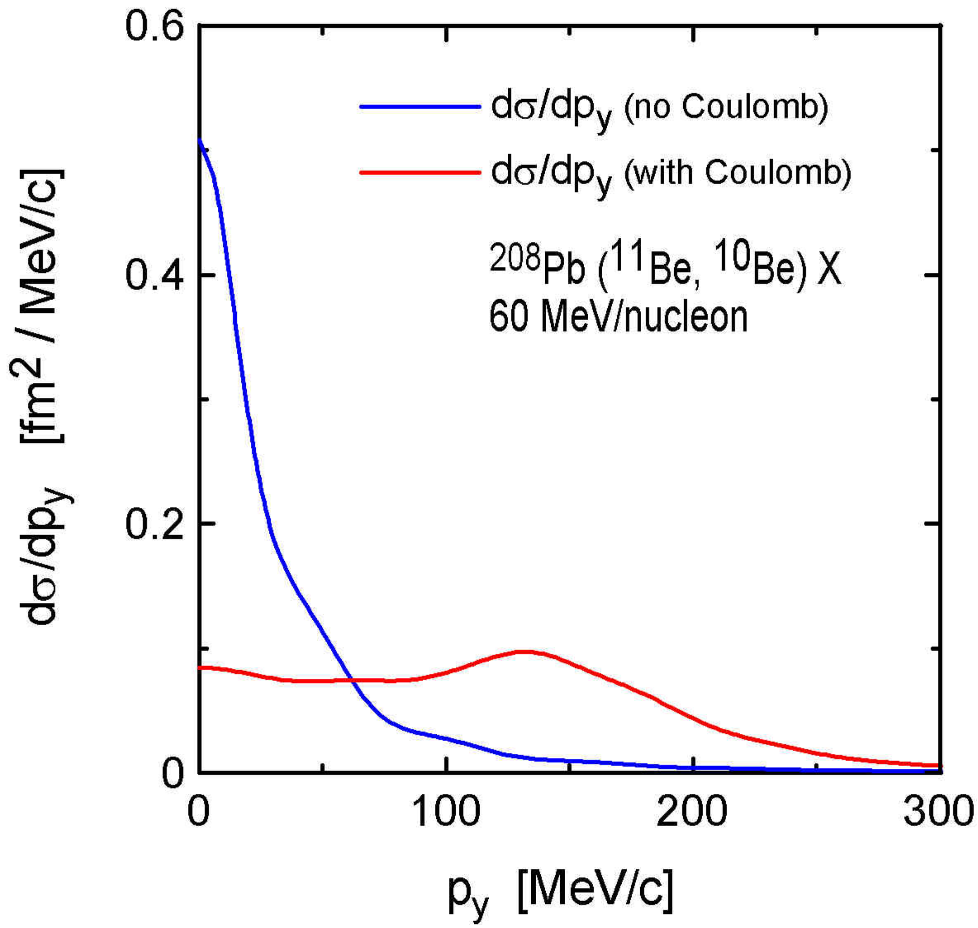
Bertulani, Hansen, PRC C70, 034609 (2004)



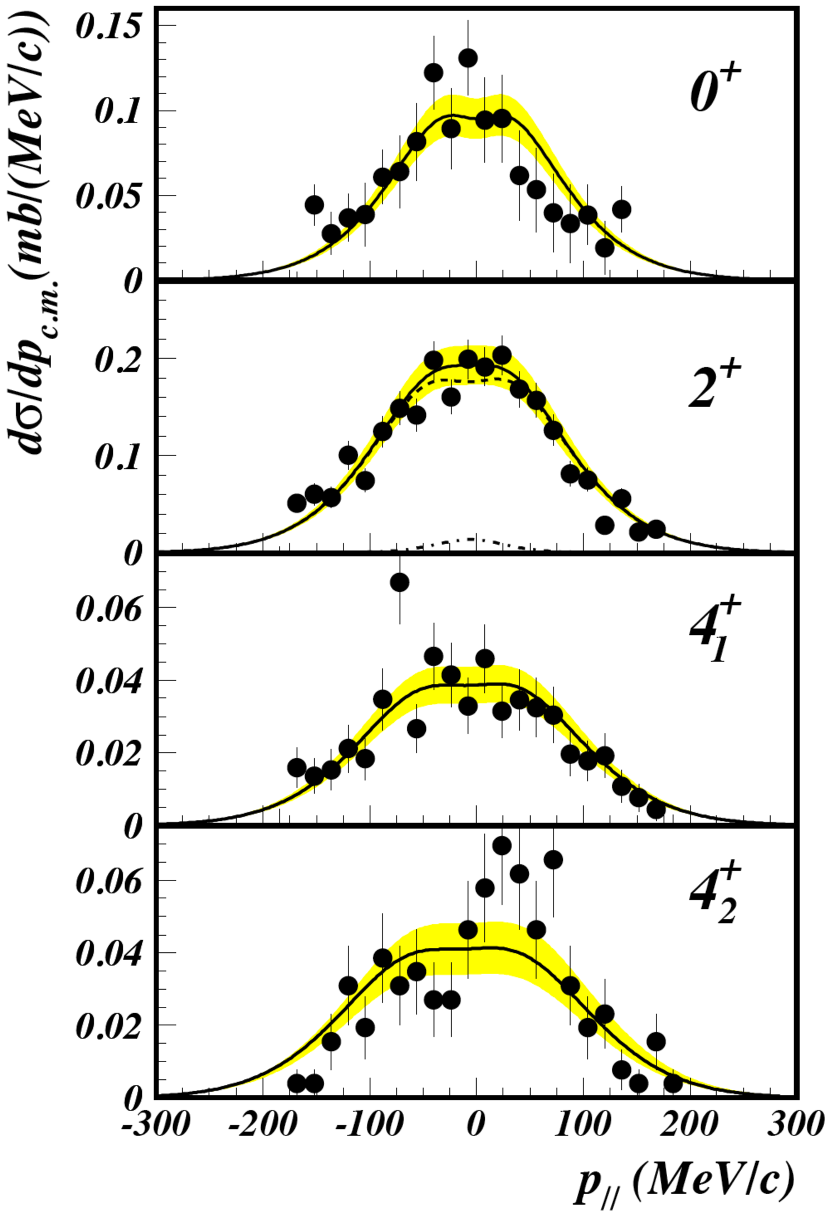
Coulomb reacceleration

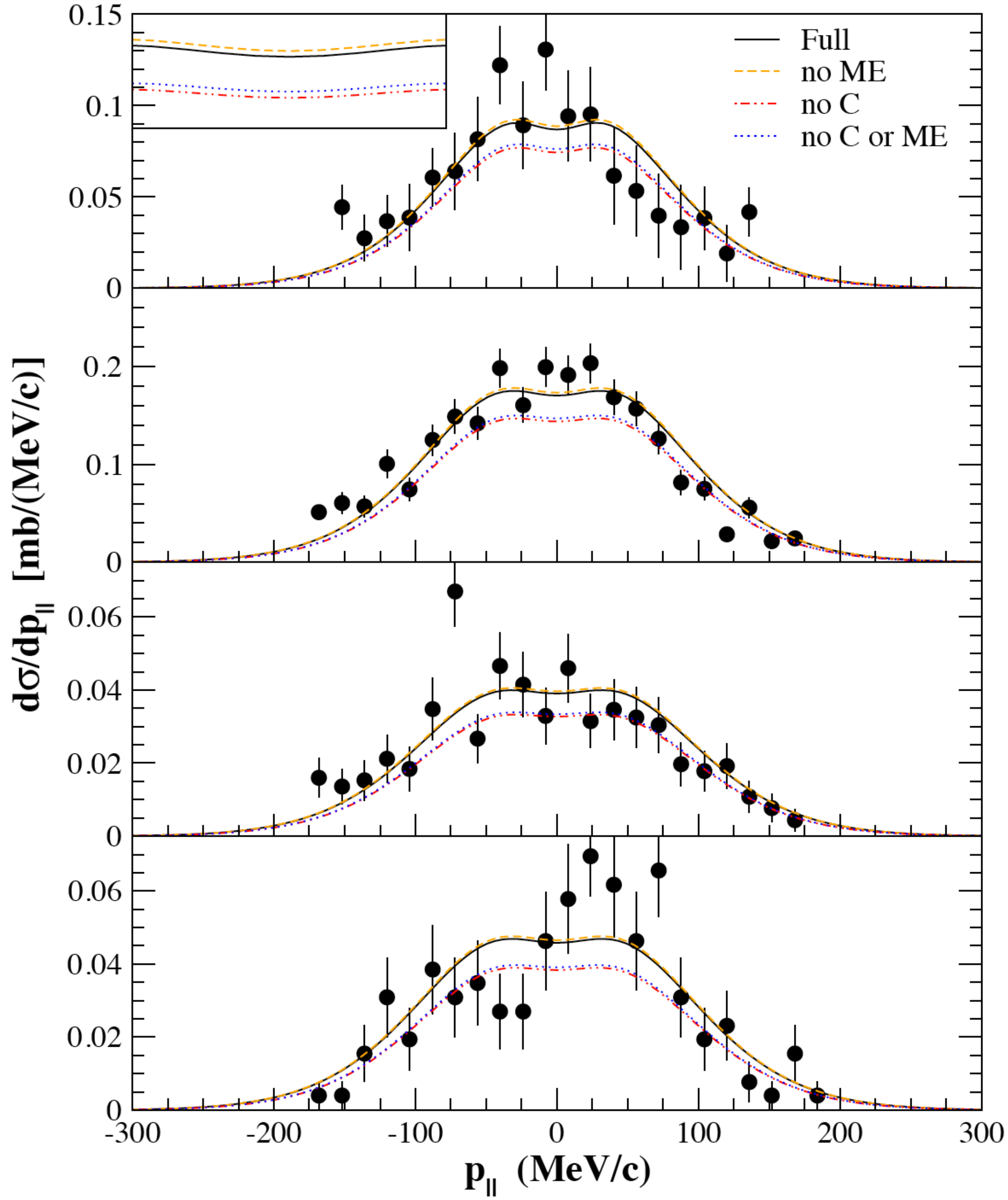
Bertulani, Hansen, PRC C70, 034609 (2004)





Data: Banu et al, PRC 84, 015803 (2011)



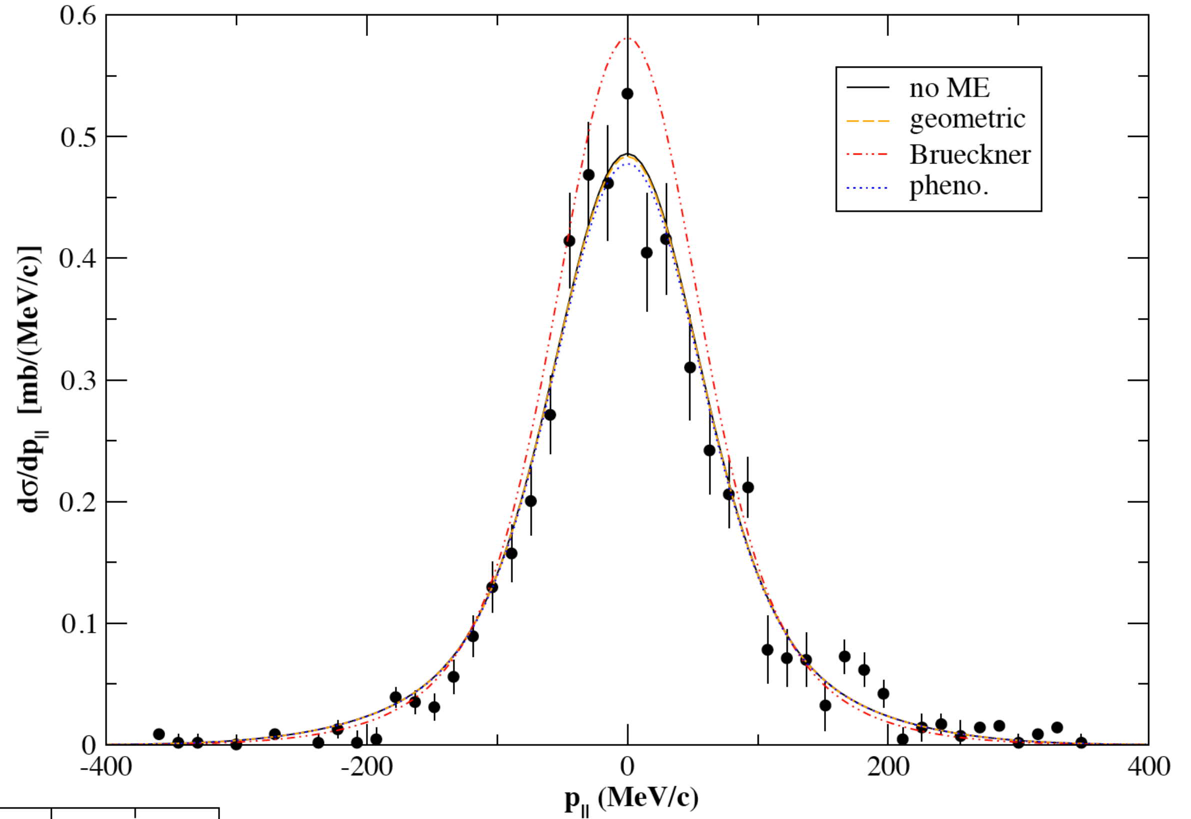


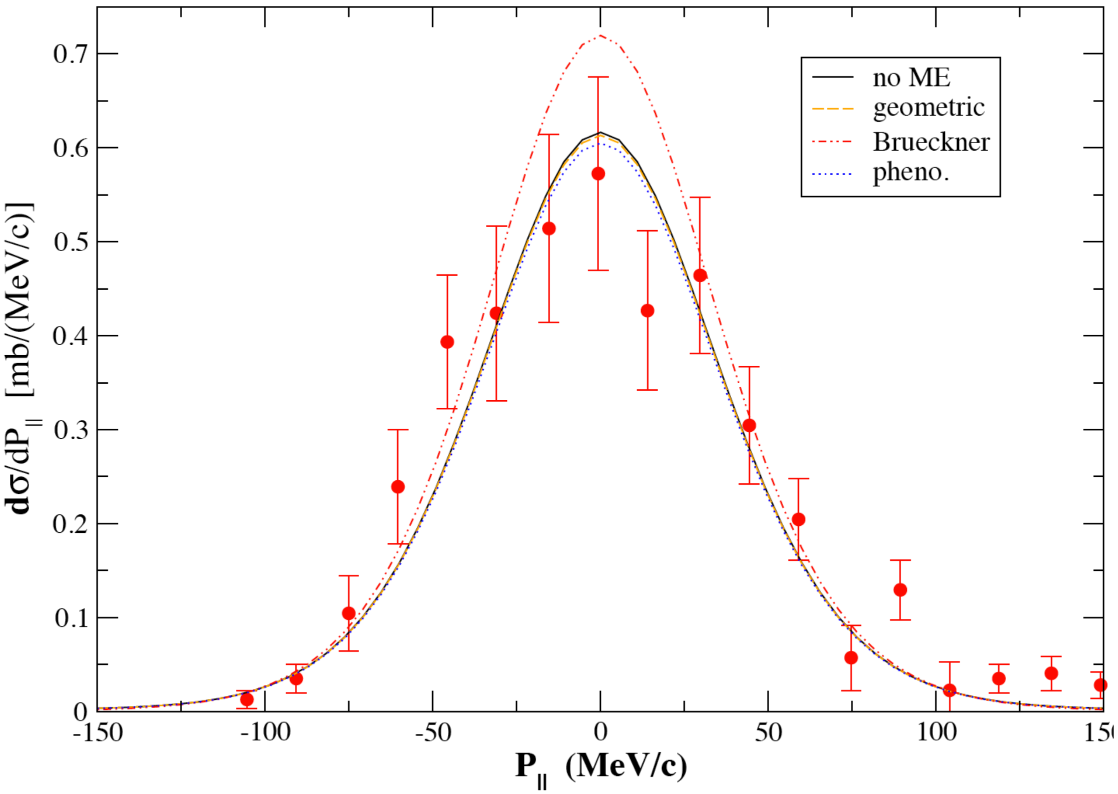
$^{12}C(^{23}Al,^{22}Mg)X$ @ 50 MeV/u

Karakoc, Bertulani, to be published

Data: Kanungo et al,
PRL 102, 152501 (2009)

$^{12}C(^{24}O,^{23}O)X$ @ 920 MeV/u





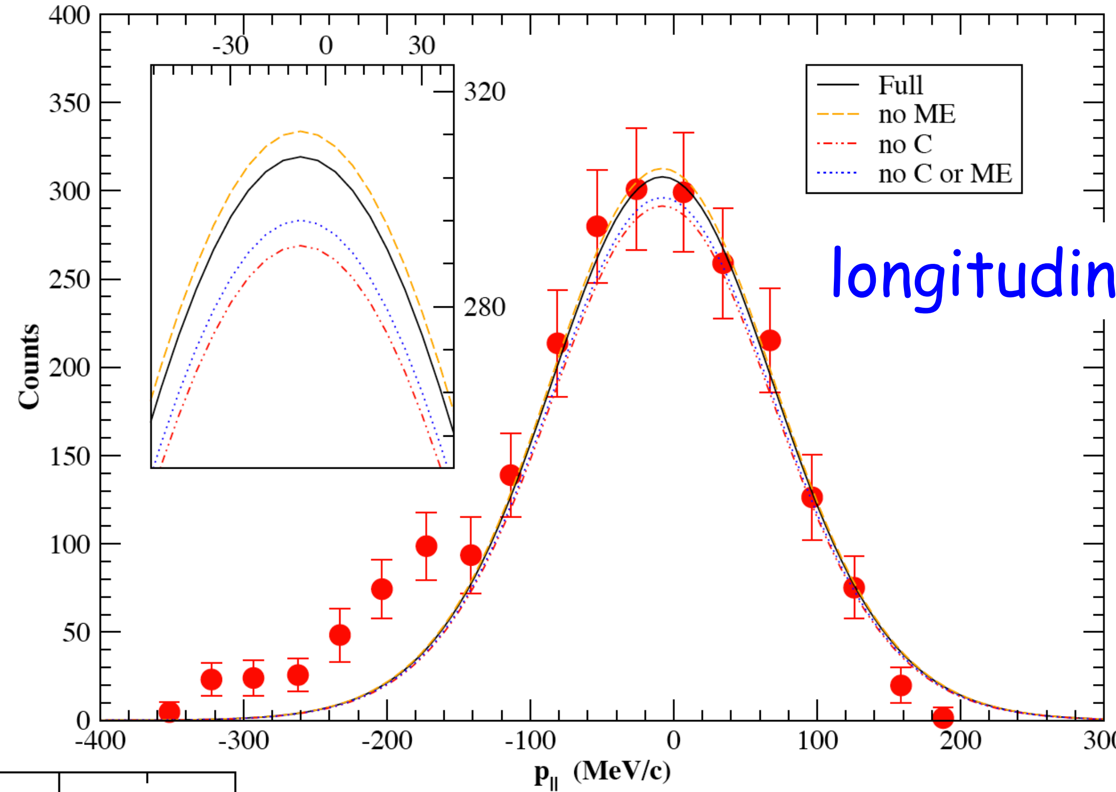
Data: Kanungo et al,
PLB 685, 253 (2010)

$^{12}C(^{33}Mg,^{32}Mg)X$ @ 898 MeV/u

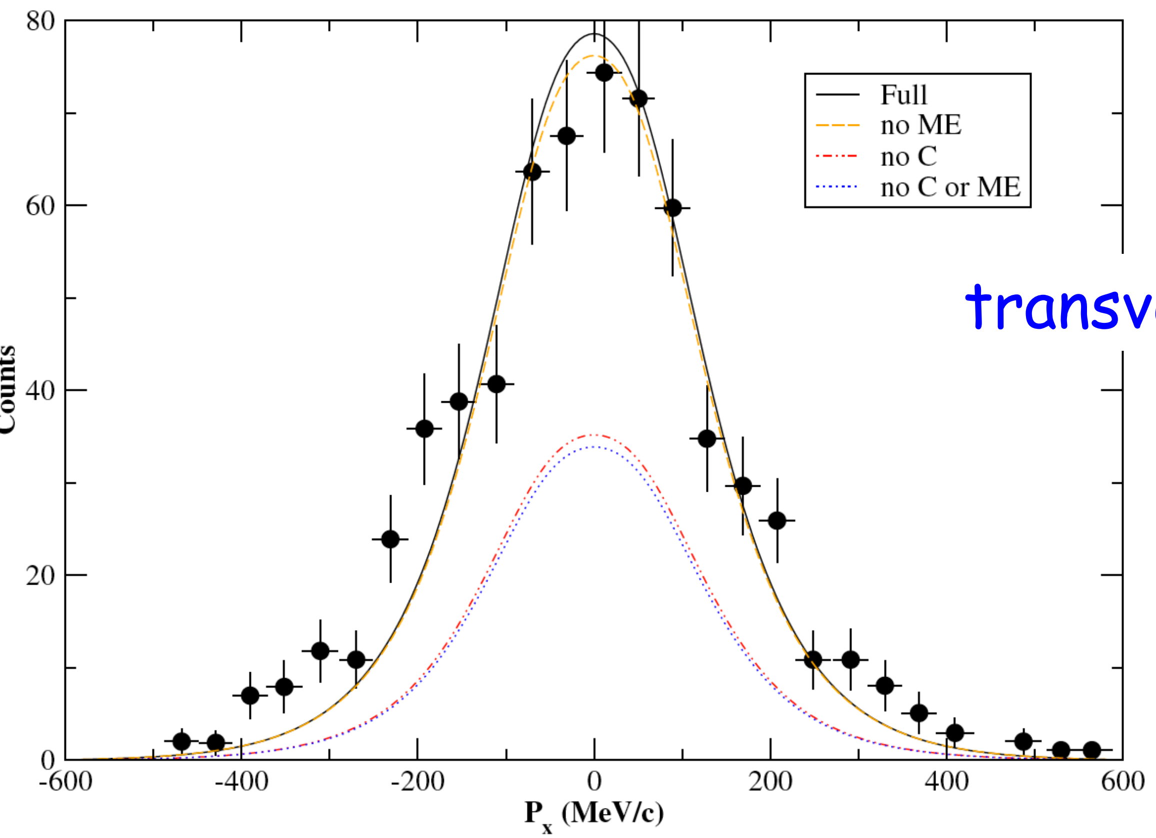
Karakoc, Bertulani, to be published

Data: Jeppesen et al, NPA 739, 57 (2004)

$^{9}Be(^{15}O,^{14}N)X$ @ 56 MeV/u



longitudinal



transverse

Data: Lecouey et al, PLB 672, 6 (2009)

$^{12}C(^{17}C,^{16}B)X$ @ 35 MeV/u

Karakoc, Bertulani, to be published

OUT OF TIME