

# **Extension of mean-field theory for spectroscopy**

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0. Quantities calculated
1. Generalities
2. Performance assessment
3. Theories considered
4. Results

## **Quantities calculated**

Low-lying excitations:

2+

3-

Yrast levels

Charge radii

Separation energies

Pairing gaps

## **Bibliography**

G.F. Bertsch, C.A. Bertulani, W. Nazarewicz, N. Schunck, and M. Stoitsov, PR C 79 034306 (2009).

J.P. Delaroche, M. Girod, J. Libert, H. Goutte, S. Hilaire, S. Peru, N. Pillet, G.F. Bertsch, PRC 81 014303 (2010).

A. Mukherjee, A. Alhassid, G.F. Bertsch, PRC 83 014319 (2011).

L.M. Robledo and G.F. Bertsch, arXiv:1107.3581

## **Generalities**

### Characteristics of good theories

- need only a small set of parameters
- have wide predictive power
- have intrinsic criteria for limits of validity

### Goals in this work

- apply theory globally (but with exceptions...)
- quantitative assessment of performance
- predictions to be tested by FRIB and elsewhere

## Performance Metric:

Mean error and rms deviation about the mean, on a logarithmic scale.

$$R_i = \log \left( \frac{x_{th}^i}{x_{exp}^i} \right)$$

$$\bar{R} = \frac{1}{N} \sum_i^N R_i$$

$$\sigma_x \equiv \langle (R_x - \bar{R}_x)^2 \rangle^{1/2}.$$

# Extensions of self-consistent mean-field theory for spectroscopy

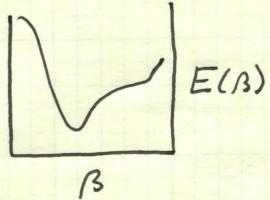
Generator Coordinate Methods

Collective Hamiltonian  
GOA

Discrete-basis Hill-Wheeler

Quasiparticle RPA

Generic GCM

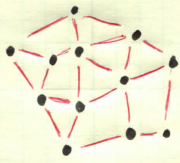
.....  $\Rightarrow$  

Collective Hamiltonian

$E(\beta) \Rightarrow V(\beta)$


$\leftarrow \bullet \rightarrow \Rightarrow \frac{\partial}{\partial \beta} \frac{1}{2M\beta} \frac{\partial}{\partial \beta}$

Discrete Basis HW



$\bullet$  configuration  
 $-$  interaction, overlap

QRPA



## How to do GCM

$\hat{H}$  = the many-body Hamiltonian, usually approximated by an EDF.

$\hat{Q}_i$  = a set of one-body operators

I) Minimize  $\langle \psi_\lambda | \hat{H} - \sum_i \lambda_i \hat{Q}_i | \psi_\lambda \rangle$  to find  $\psi_\lambda$

II find expectation values  $q_i = \langle \psi_\lambda | \hat{Q}_i | \psi_\lambda \rangle$

$$V(q) \equiv V(\lambda(q)) = \langle \psi_\lambda | \hat{H} | \psi_\lambda \rangle$$

This is the potential energy surface.

5DCH work:  $\hat{N}, \hat{Z}, r^2 Y_{20}, r^2 Y_{22}, \hat{J}_x$

octupole study:  $\hat{N}, \hat{Z}, r^2 Y_{20}, r^3 Y_{30}$

# The CEA/DAM global survey (HFB/GCM/5DCH)

PRC 81,014303 (2010)

Abstract    References    Citing Articles (8)    **Supplemental Material**

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## EPAPS

- README.TXT
- 5dch.txt
- heading\_5dch-table-eng.doc

Computed spectroscopic observables for 1712 nuclei:

- yrast energies up to  $J=6$
- excited  $0^+$ , first and second yrare  $J=2$
- $B(E2)$  values for many of the transitions
- $E0$  matrix elements
- deformations, including triaxiality

44 102 -1001.112 0.290 24. 4.84 4.81 5.32 -1004.872 -3.760 (

46 42 -699.649 0.000 0. 4.37 4.31 4.19 -701.419 -1.770 (

46 44 -730.992 0.000 0. 4.37 4.32 4.22 -733.013 -2.021 (

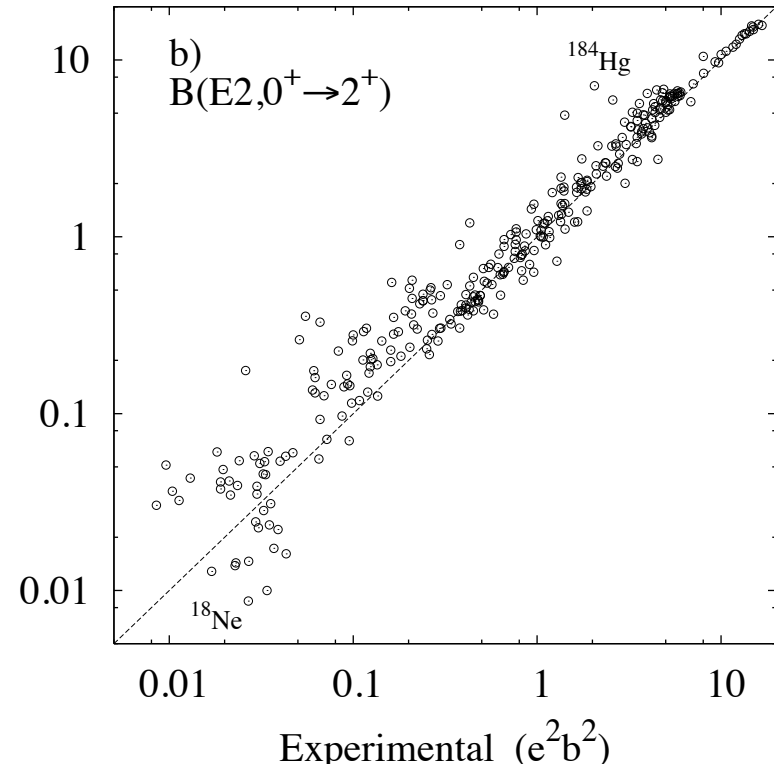
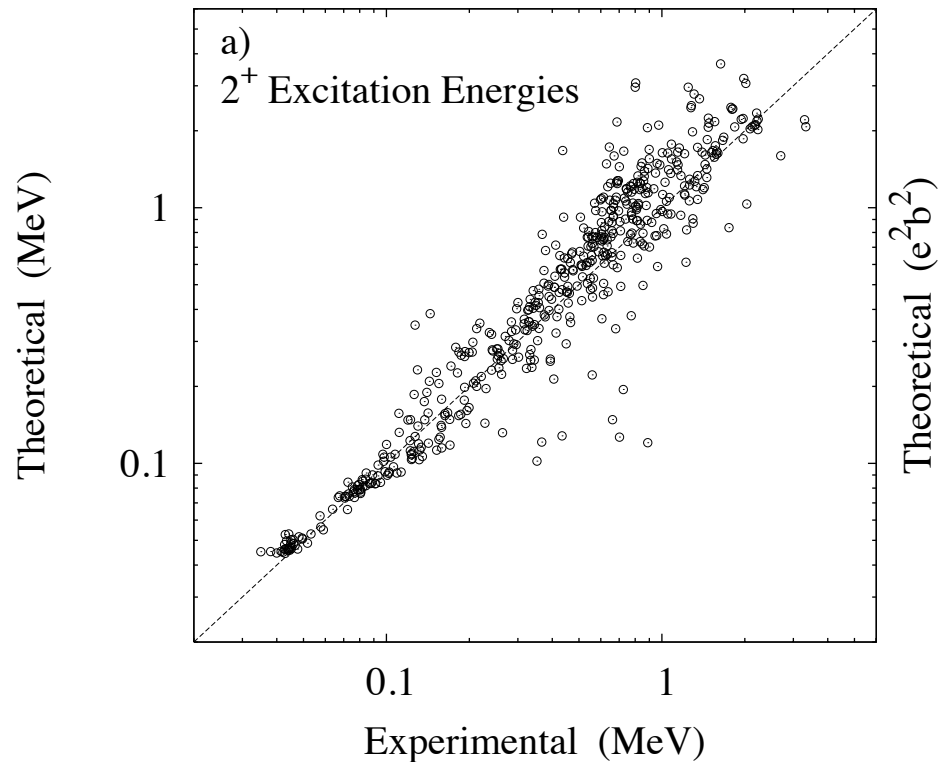
46 46 -761.066 0.106 0. 4.38 4.33 4.26 -763.027 -1.961 (

46 48 -789.873 0.000 0. 4.38 4.33 4.28 -790.963 -1.090 (

46 50 -817.521 0.000 0. 4.38 4.33 4.30 -816.719 0.802 (

46 52 -836.390 0.000 0. 4.40 4.35 4.34 -838.888 -2.498 (

# Global assessment of accuracy: first excited J=2 state



$E(\text{exp}) = E(\text{theory}) \pm 40\%$  over 2 order of magnitude

Many other observables



## Metrics for global performance

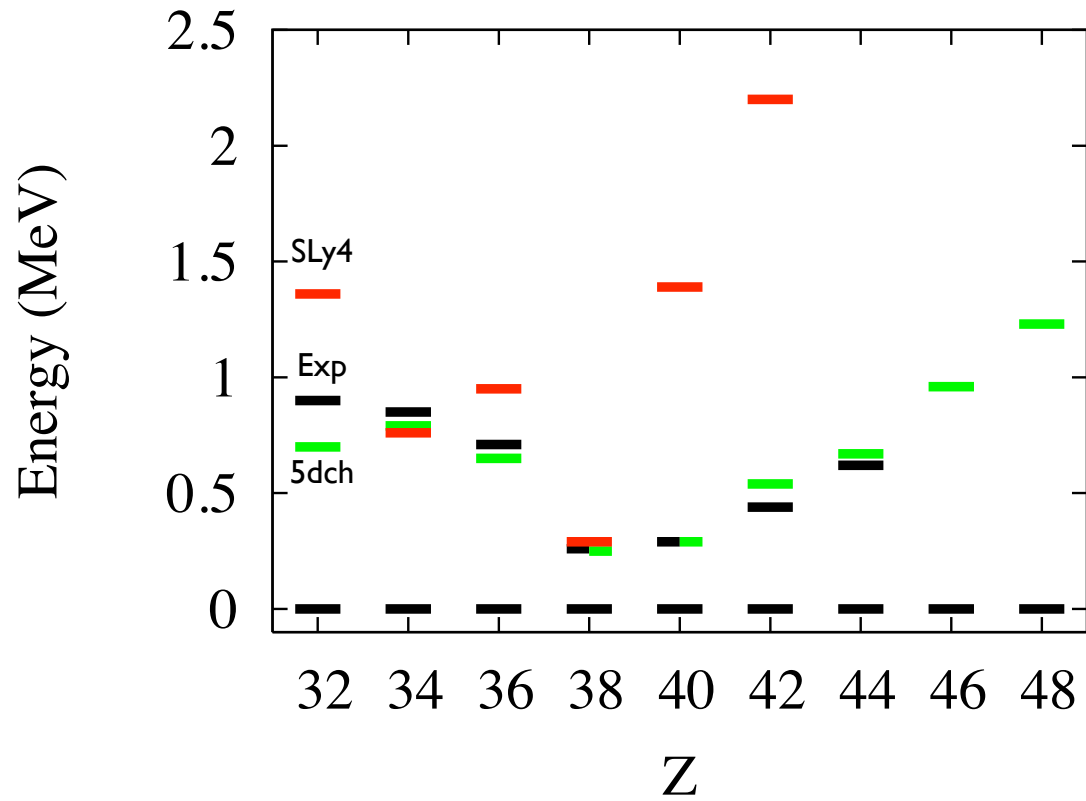
$$R_i = \log \left( \frac{x_{th}^i}{x_{exp}^i} \right) \quad \sigma_x \equiv \langle (R_x - \bar{R}_x)^2 \rangle^{1/2}.$$

Observable	Number	$\bar{R}$	$\sigma$
$E(2_1^+)$	513	0.11	0.35
$B(E2; 2_1^+ \rightarrow 0_1^+)$	311	0.20	0.42
$R_{42}$	480	0.03	0.14
$R_{62}$	427	0.08	0.21
$E(2_2^+)$	352	0.19	0.30
$E(0_2^+)$	317	0.31	0.36
$\langle 0_2^+   r_p^2   0_1^+ \rangle$	87	2.1	1.9

The CEA/DAM global survey (HFB/GCM/5DCH), on the N=Z line

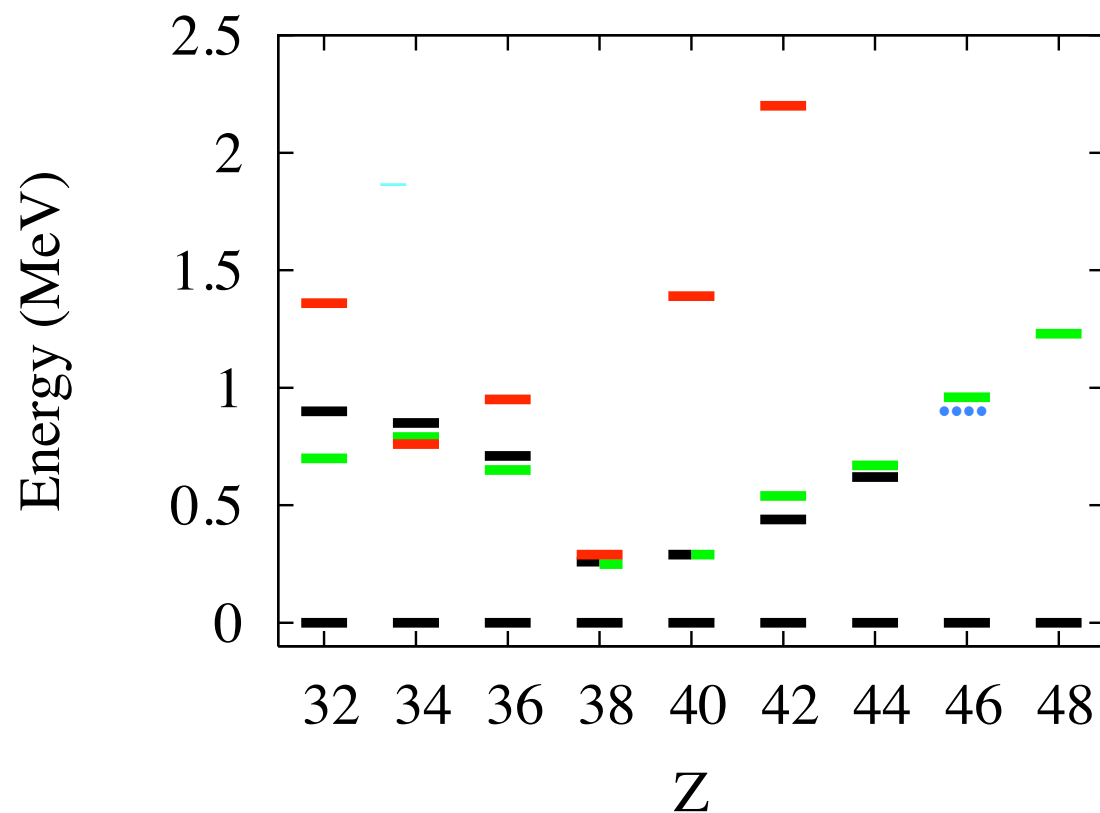
PRC 81,014303 (2010)

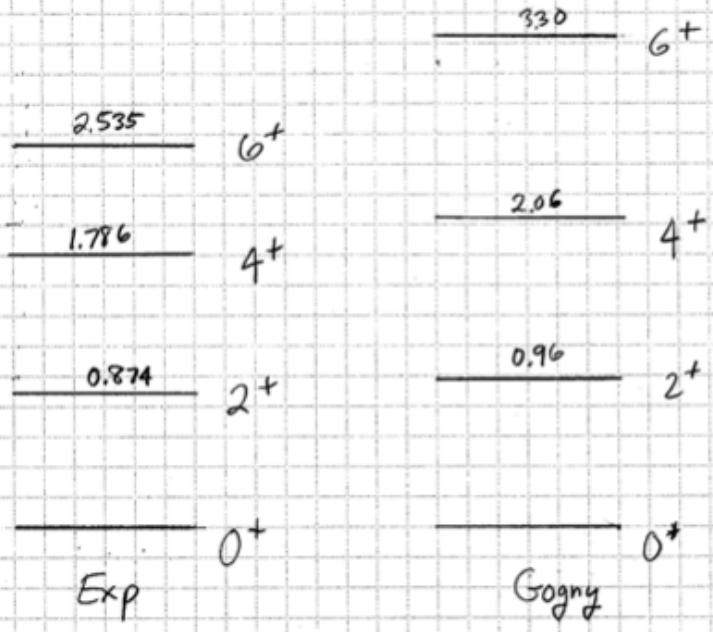
2+ Excitation energies



## Evidence for a spin-aligned neutron-proton paired phase from the level structure of $^{92}\text{Pd}$

B. Cederwall<sup>1</sup>, F. Ghazi Moradi<sup>1</sup>, T. Bäck<sup>1</sup>, A. Johnson<sup>1</sup>, J. Blomqvist<sup>1</sup>, E. Clément<sup>2</sup>, G. de France<sup>2</sup>, R. Wadsworth<sup>3</sup>, K. Andgren<sup>1</sup>, K. Lagergren<sup>1,4</sup>, A. Dijon<sup>2</sup>, G. Jaworski<sup>5,6</sup>, R. Liotta<sup>1</sup>, C. Qi<sup>1</sup>, B. M. Nyakó<sup>7</sup>, J. Nyberg<sup>8</sup>, M. Palacz<sup>2</sup>, H. Al-Azri<sup>3</sup>, A. Algora<sup>9</sup>, G. de Angelis<sup>10</sup>, A. Ataç<sup>11</sup>, S. Bhattacharyya<sup>2,†</sup>, T. Brock<sup>3</sup>, J. R. Brown<sup>3</sup>, P. Davies<sup>3</sup>, A. Di Nitto<sup>12</sup>, Zs. Dombrádi<sup>7</sup>, A. Gadea<sup>9</sup>, J. Gál<sup>7</sup>, B. Hadinia<sup>1</sup>, F. Johnston-Theasby<sup>3</sup>, P. Joshi<sup>3</sup>, K. Juhász<sup>3</sup>, R. Julin<sup>14</sup>, A. Jungclaus<sup>15</sup>, G. Kalinka<sup>7</sup>, S. O. Kara<sup>11</sup>, A. Khaplanov<sup>1</sup>, J. Kownacki<sup>5</sup>, G. La Rana<sup>12</sup>, S. M. Lenzi<sup>16</sup>, J. Molnár<sup>7</sup>, R. Moro<sup>12</sup>, D. R. Napoli<sup>10</sup>, B. S. Nara Singh<sup>3</sup>, A. Persson<sup>1</sup>, F. Recchia<sup>16</sup>, M. Sandzelius<sup>1,†</sup>, J.-N. Scheurer<sup>17</sup>, G. Sletten<sup>18</sup>, D. Sohrler<sup>7</sup>, P.-A. Söderström<sup>8</sup>, M. J. Taylor<sup>3</sup>, J. Timár<sup>7</sup>, J. J. Valiente-Dobón<sup>10</sup>, E. Vardaci<sup>12</sup> & S. Williams<sup>19</sup>





92Pd

## **Octupole Excitations**

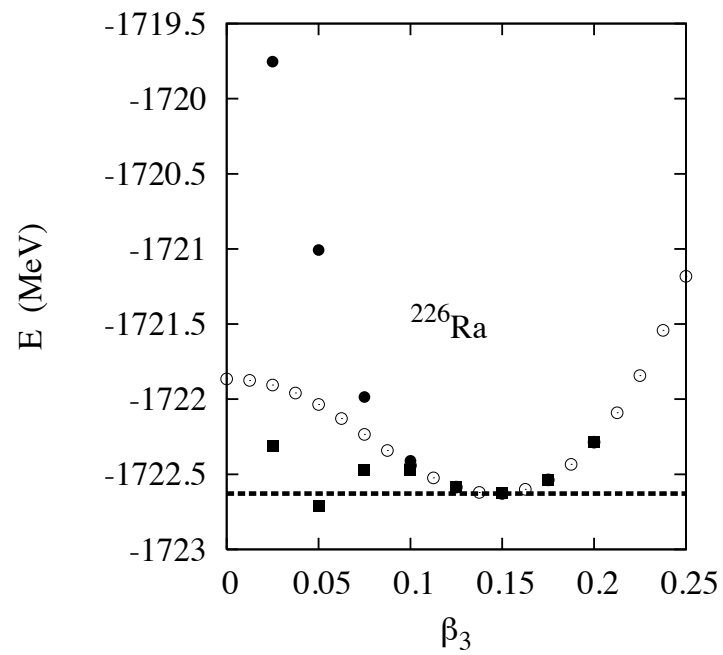
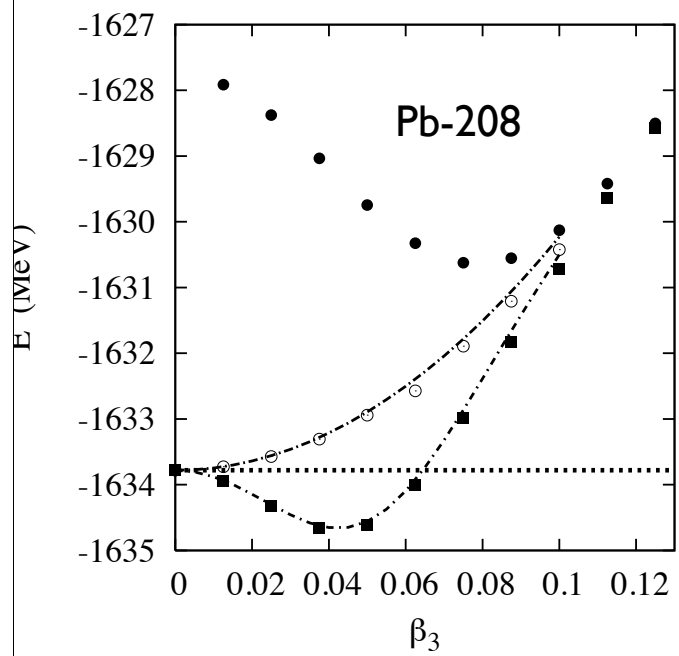
Robledo and Bertsch, arXiv:1107.3581

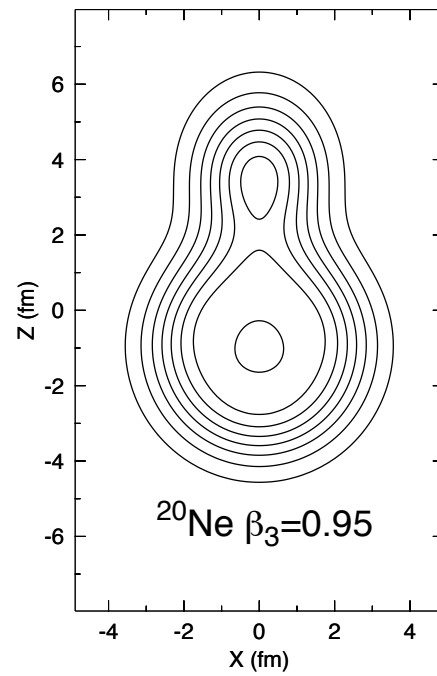
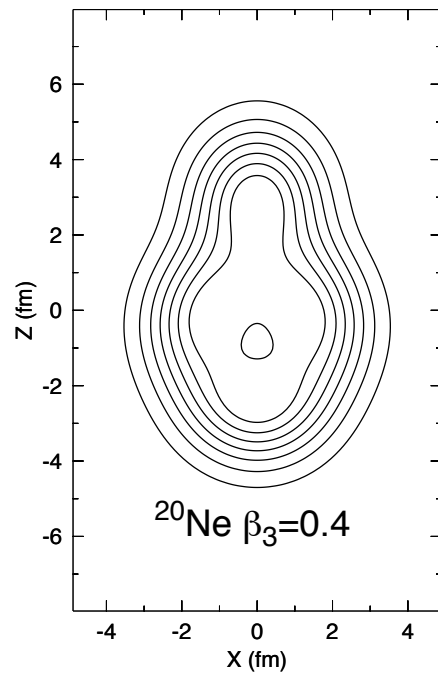
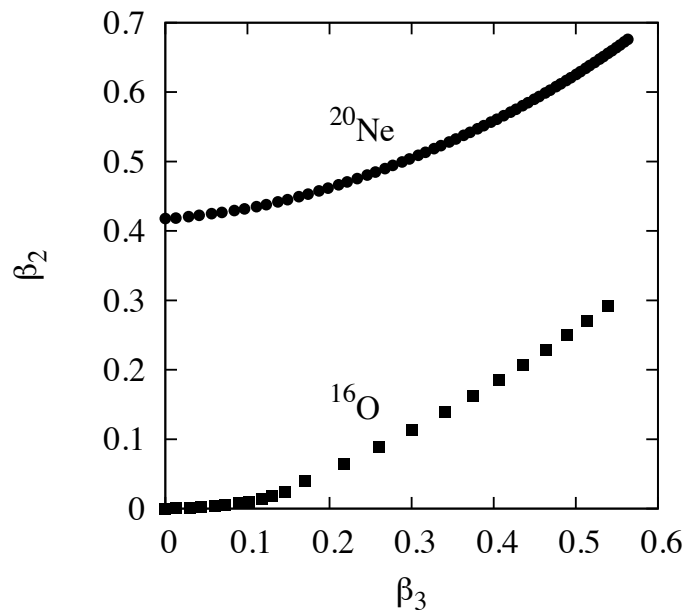
HFB + discrete basis Hill-Wheeler

Parity projection

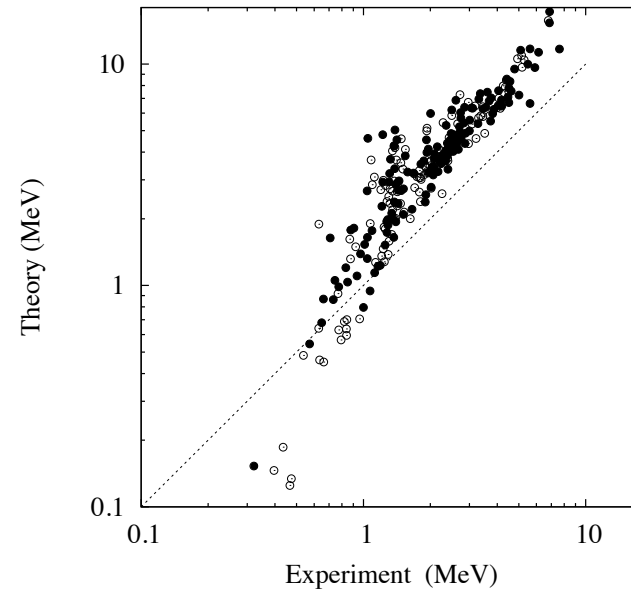
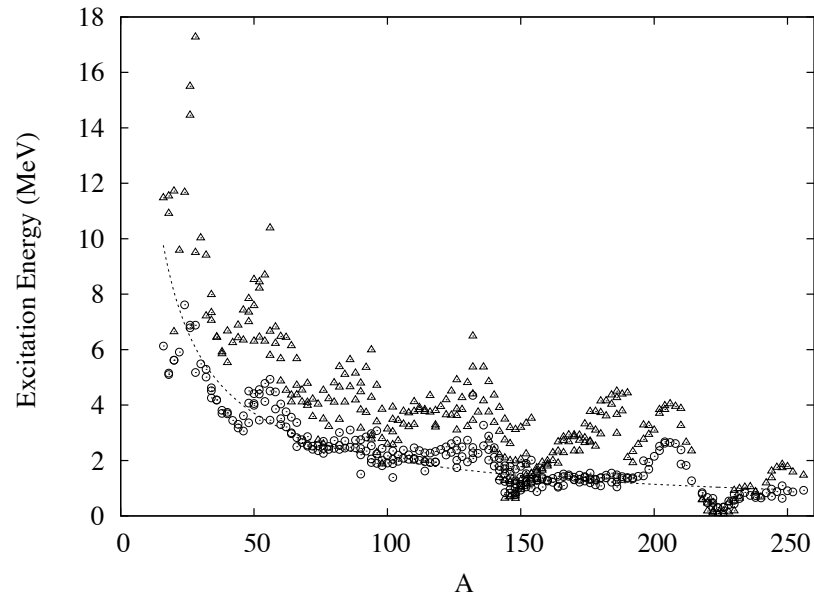
Axially symmetric octupole excitations only.

## Examples of energy surfaces





### 3- excitation energies

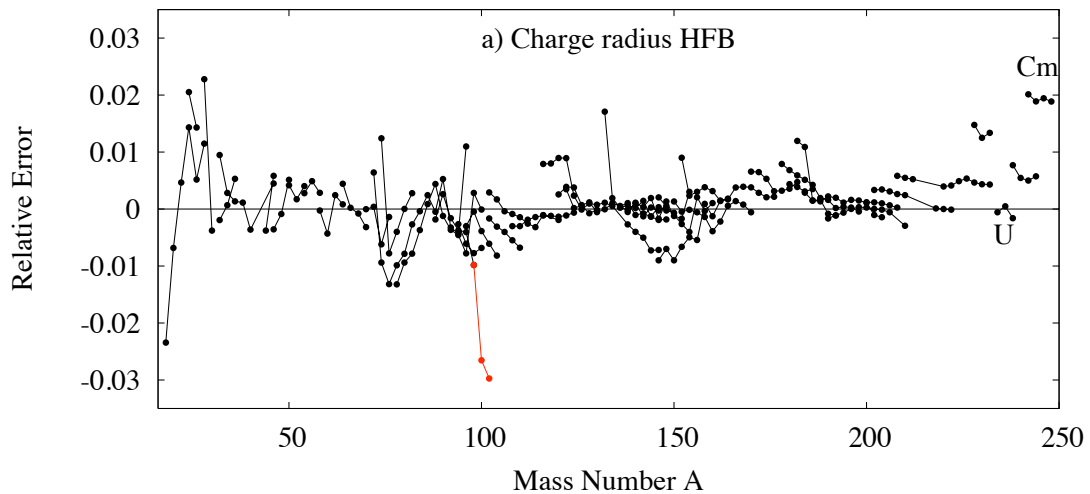


Selection	Number	HW		MAP	
		$R_e$	$\sigma_e$	$R_e$	$\sigma_e$
all	284	0.45	0.40		
$\beta_3 = 0$	277	0.55	0.23	0.59	0.22
$\beta_3 = 0$ , def.	59	0.62	0.32	0.75	0.26
$\beta_3 = 0$ , sph.	196	0.52	0.19	0.53	0.17

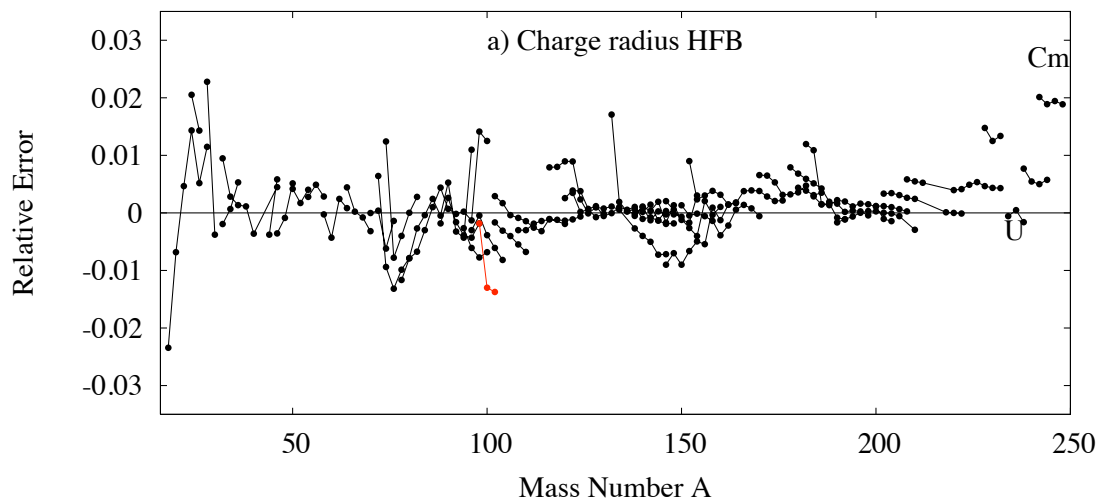


# Charge radii

Experimental data from Angeli, ADNDT 87 (2004)



HFB



## Performance on charge radius

TABLE II. Comparison of calculated charge radii with experiment:  $\bar{\epsilon}$  is the mean of  $\epsilon$  [see Eq. (18)];  $\sigma$  is its rms dispersion about the average. Three hundred thirteen nuclear radii were included in the comparison as in Fig. 6. In the column “HFB (new)” we use the modern value  $r_p = 0.875$  fm for the proton charge radius [48].

Theory	$\bar{\epsilon}$	$\sigma$
HFB	0.001	0.006
HFB (new)	0.005	0.007
CHFb+5DCH	0.006	0.007
Finite surface	0.0000	0.012

## **Separation Energies**

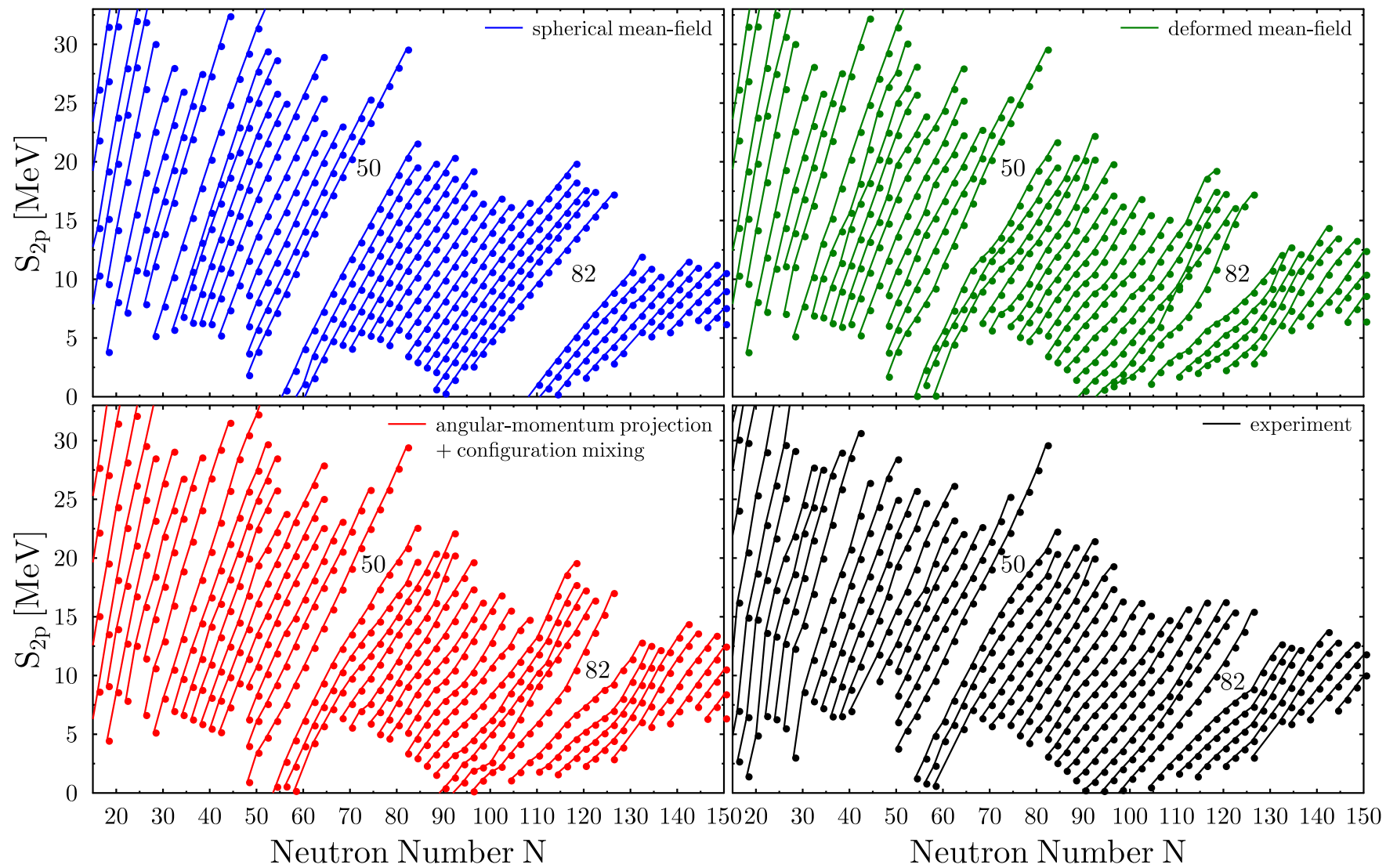


FIG. 2: Two-proton separation energies for isotonic chains.

## Mutually enhanced magicity

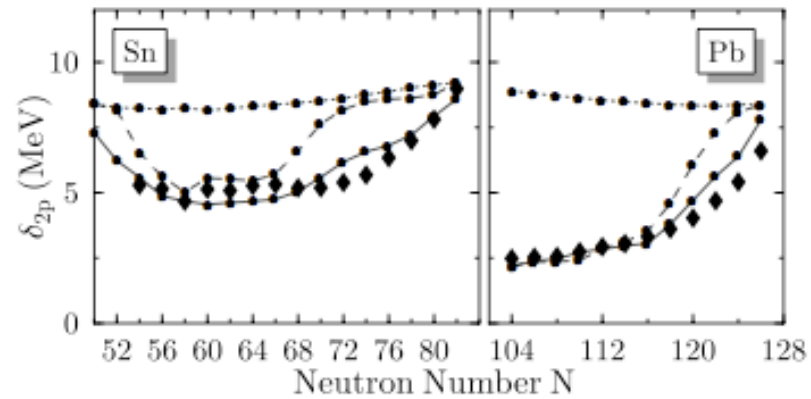
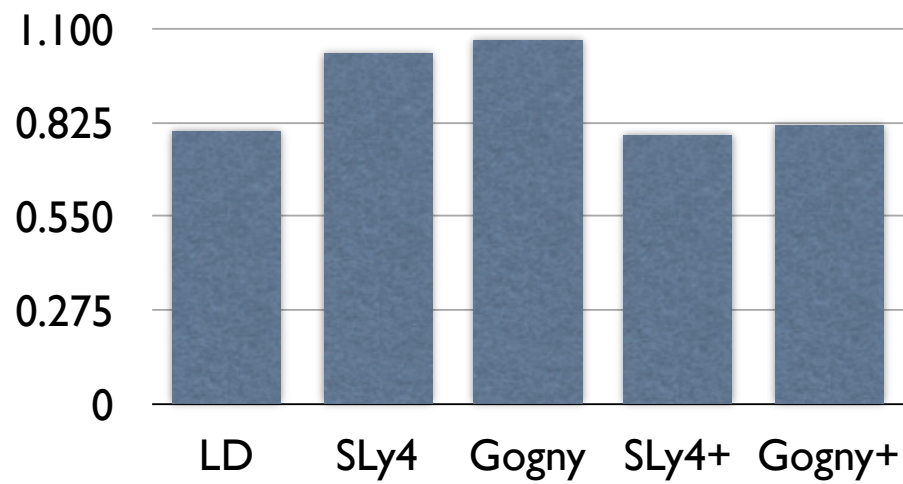


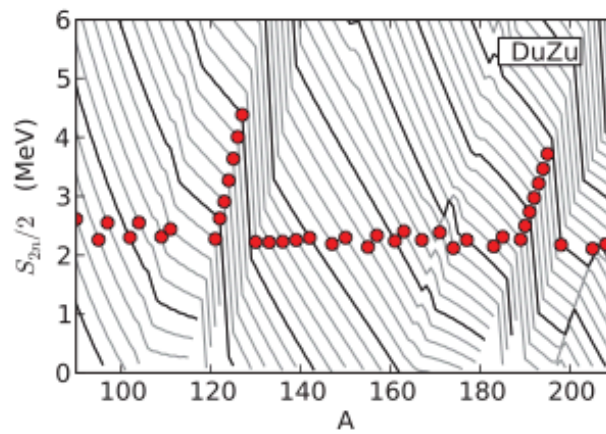
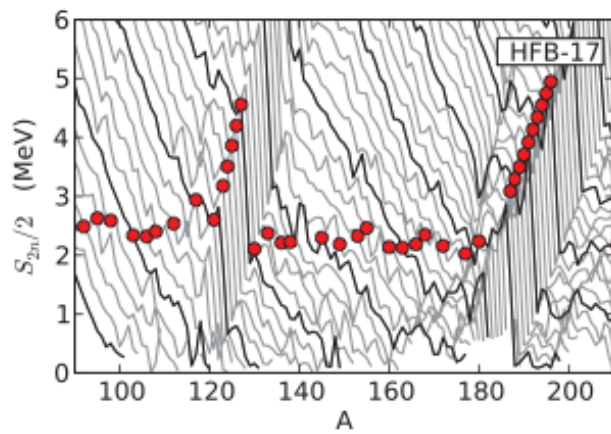
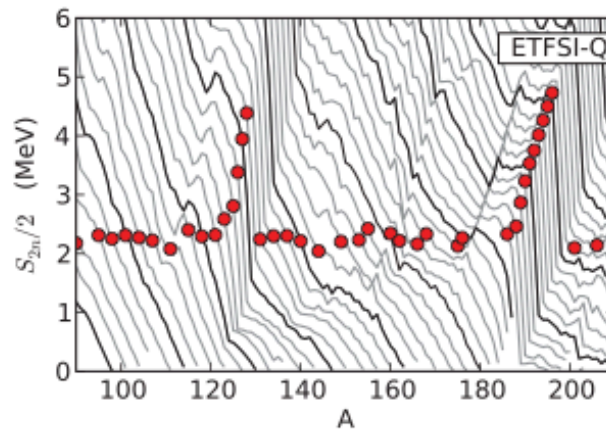
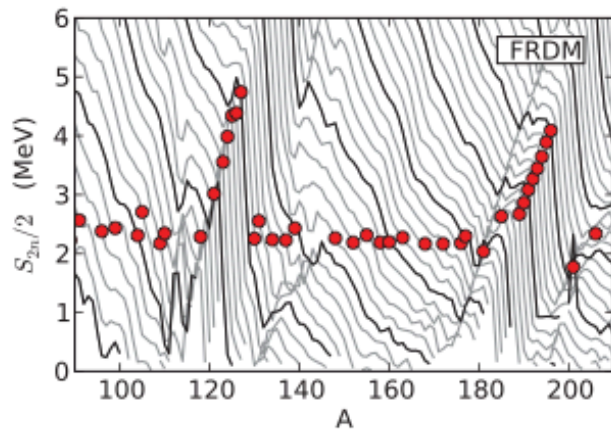
FIG. 2. Two-proton gaps, Eq. (3), for Pb and Sn isotopic chains. Theoretical curves are the following: spherical mean field (short dashed lines); mean field allowing for static deformations (long dashed lines); present theory (solid lines). Experimental values [1] are shown as diamonds.

S<sub>2n</sub> rms residuals (MeV)



# r-Process nucleosynthesis

Arcones and Martinez-Pinedo, PRC83



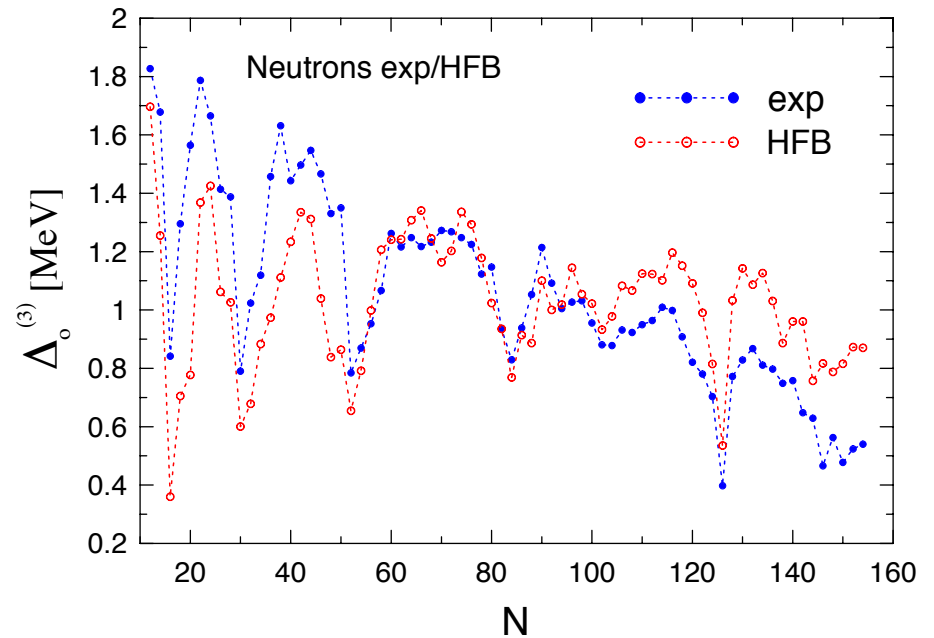
## Odd-even mass staggering

aka pairing gaps

$$\Delta_o^{(3)}(N) = \frac{1}{2} (2E(N, Z) - E(N - 1, Z) - E(N + 1, Z))$$

rms residuals for 443 nuclei (MeV)

Theory	$\sigma$	projected
Constant	0.31	
$c/A^\alpha$	0.24	
HFB	0.27	0.23
HF+BCS	0.28	0.24



Phenomenology is hard to beat!