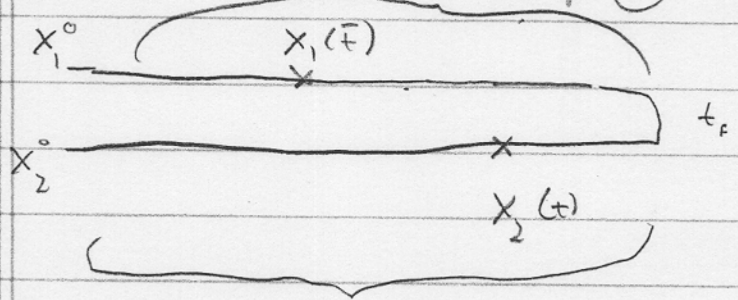
Last Time:

$$\int dx_1^0 dx_2^0 \, \rho \int_{x_1^0 x_2^0} Dx_1 Dx_2 \, e^{iS_1 - iS_2} \qquad x_2(t) \, x_1(\bar{t})$$

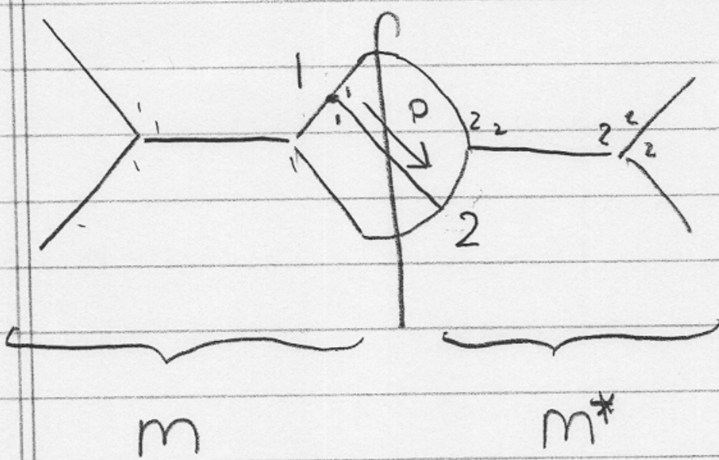
Amp ⓦ insertion at $\bar{t}$




Conj Amp ⓦ insertion at $t$

So

Example: $T = 0$




Cut Lines:

$$\Theta(p^0)\, 2\pi\, \delta(P^2 + m^2)$$

$$= \frac{2\pi}{2E_p}\, \delta(p^0 - E_p)$$

$$G^>(P) = G_{21}(P) = \int e^{iP\cdot X} \langle \hat{\phi}(x)\, \hat{\phi}(0) \rangle$$

Use $\hat{\phi}(x) = \sum_p a_p \frac{e^{iP\cdot X}}{\sqrt{2E_p}} + a_p^+ \frac{e^{-iP\cdot X}}{\sqrt{2E_p}}$

Find

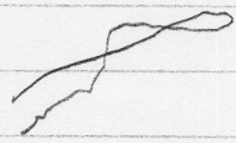
$$G^> = \underbrace{(1 + n_p)}^{\langle a a^+ \rangle} \frac{2\pi}{2E_p}\, \delta(p^0 - E_p) + \underbrace{n_{-p}}^{a^+ a} \frac{2\pi}{2E_p}\, \delta(p^0 + E_p)$$

absorption of particle

$$\xRightarrow[T=0]{} \frac{2\pi}{2E_p}\, \delta(p^0 - E_p)$$

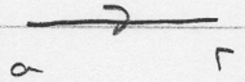
Then I introduced:

$$x_r = \frac{x_1 + x_2}{2} \qquad x_a = x_1 - x_2$$



small in classical limit

Then saw good things



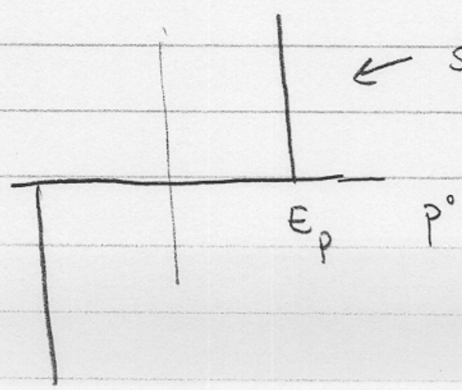

$$iG_{ra} = \langle x_r x_a \rangle = \Theta(t) \langle [\hat{x}(t), \hat{x}(\bar{t})] \rangle$$

$$iG_{ra} = \frac{-i}{-\omega^2 + \omega_0^2 - i\varepsilon\omega^0} \overset{\text{FTheory}}{\Rightarrow} \frac{-i}{-(p^0)^2 + E_p^2 - i\varepsilon p^0} = \frac{-i}{p^2 + m^2 - i\varepsilon p^0}$$

Spectral Density:

$$\rho(\omega) = -2\,\mathrm{Im}\, G_R(P) = \frac{2\pi}{2E_p}\delta(p^0 - E_p) - \frac{2\pi}{2E_p}\delta(p^0 + E_p)$$

So




single particle pole

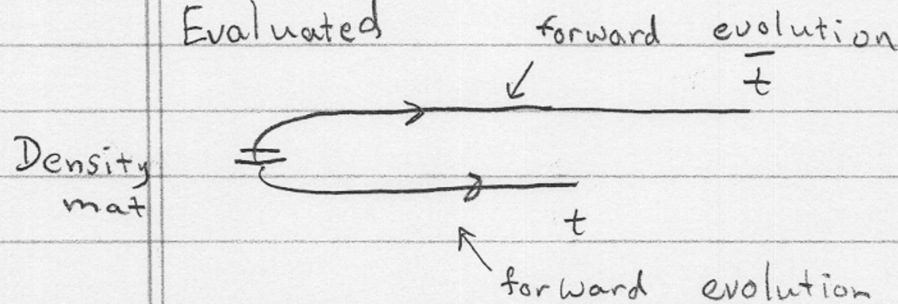
We also evaluated:

$$G_{rr} = \langle x_r(t)\, x_r(\bar{t}) \rangle = \frac{1}{2} \langle \{ x(t), x(\bar{t}) \} \rangle$$

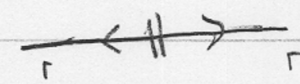
Find

$$G_{rr}(P) = \left( \frac{1}{2} + n(p^0) \right) \rho(p^0)$$

Evaluated



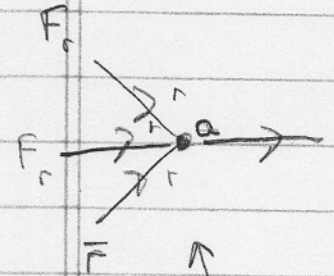

Hard to draw in Feynman Diagrams:



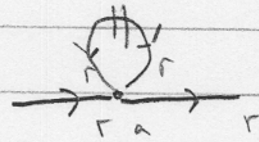

$$\equiv G_{rr}(\omega) = \left( \frac{1}{2} + n(p^0) \right) \rho(P)$$

## Interactions:

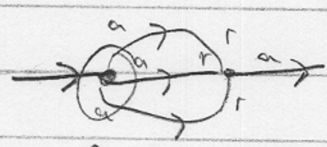
$$\frac{1}{4!} x_1^4 - \frac{1}{4!} x_2^4 = \overbrace{\frac{1}{3!} x_r^3 x_a}^{\text{Classical}} + \overbrace{\frac{1}{3} x_a^3 x_r}^{\text{Quantum}}$$



non linear response
classical



Thermal + vacuum
fluctuation contribution
to retarded response



Quantum Correction to $G_R$

Deriving Boltzman: Free theory

Yesterday we showed that the symmetrized

$$\left(\frac{-\partial^2}{\partial x^2} + m^2\right) G_{rr}(X, Y) = 0$$

$$G_{rr}(X, Y) \left(\frac{-\overleftarrow{\partial}^2}{\partial y^2} + m^2\right) = 0$$

Now introduce a wigner transform

$$G(\bar{X}, P) = \int ds\, e^{-iP\cdot s}\, G(X + \tfrac{s}{2}, \bar{X} - \tfrac{s}{2})$$

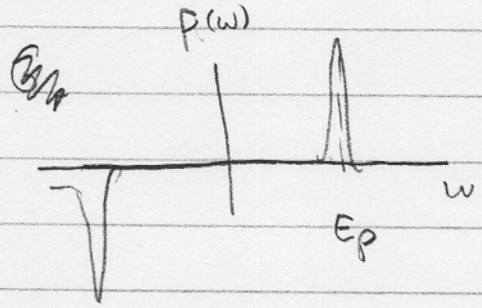
So

$$\left.\begin{aligned} &\left(\frac{\partial_x^2 + \partial_y^2}{2} + m^2\right) G(X, Y) = 0 \\ &(\partial_x^2 - \partial_y^2)\, G(X, Y) = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} &(P^2 + m^2)\, G(X, P) = 0 \\ &2P^\mu \frac{\partial}{\partial x^\mu} G(X, P) = 0 \end{aligned}$$

Recall free:

$$G_{rr}(P) = \left(\frac{1}{2} + n(p^0)\right) \Bigg\{ \underbrace{\frac{2\pi}{2E_p}\, \delta(p^0 - E_p) - \frac{2\pi}{2E_p}\, \delta(p^0 + E_p)}_{\rho(\omega)} \Bigg\}$$

Non-equilibrium




assume

$$G_{rr} \simeq \left(\frac{1}{2} + n_p(x)\right) \frac{2\pi}{2E_p} \delta(p^0 - E_p) + \left(\frac{1}{2} + \bar{n}_{-p}(x)\right) \frac{2\pi}{2E_p} \delta(p^0 + E_p)$$

I.e.

$$\frac{\frac{1}{2} + n_p(x)}{2E_p} \simeq \int_{E_p - \text{bit}}^{E_p + \text{bit}} \frac{dp^0}{2\pi} G_{rr}(X, P)$$

So

Free streaming

$$\frac{2p^\mu}{2E_p} \frac{\partial}{\partial x^\mu} n_p(X) = 0 \Rightarrow \boxed{\left(\partial_t + v_p \frac{\partial}{\partial x}\right) n_p(X) = 0}$$

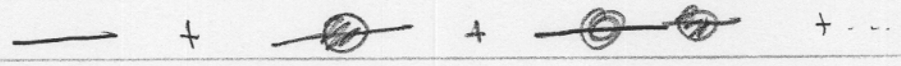
Then this $\left(\partial_t + v_p \frac{\partial}{\partial x}\right) \bar{n}_p = 0$

(1) When momentum (wave number) is large compared ~~to~~ compared inhomogeneities wave packets move along geodesics

Now Consider the unfree case

$$(-\partial_x^2 + m^2)\, G_{ss'}(x,y) + \int \Pi_{s\bar{s}}(x,z)\, G_{\bar{s}s'}(z,y) = \mathbb{1}_{ss'}$$

Eg.

$G =$ 

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \qquad \Pi = \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{pmatrix}$$

Find

$$G^> \sim (1+n)\,\frac{2\pi}{2E_p}\,\delta(p^0 - E_p)$$

$$2p^\mu \frac{\partial}{\partial x^\mu} G_{rr} = \Pi^< G^> - \underbrace{G^<}_{n_p \frac{2\pi}{2E_p}\delta(p^0 - E_p)} \Pi^>$$

Skip

$$2p^\mu \frac{\partial}{\partial x^\mu} n_p = -n_p \Pi^> + \Pi^<(1+n_p)$$

E.g. Photons

$$i\mathcal{M}_{fi}(\vec{k}) = \langle \vec{k} f | \, i\int_X J_\mu(x) A^\mu(x) \, | i \rangle$$

$$A_\mu(x) = \sum_{k,\lambda} e^{ik\cdot x} \frac{a}{\sqrt{2E_k}} \varepsilon_\mu + e^{-ik\cdot x} \frac{a^\dagger}{\sqrt{2E_p}} \varepsilon^*_\mu$$

So

$$i\mathcal{M}_{fi} = i\sqrt{1+n} \int_X \langle f | J_\mu(x) | i \rangle \frac{e^{-ik\cdot x}}{\sqrt{2E_k}} \varepsilon^{\mu *}$$

$$(2\pi)^3 \frac{dP}{d^3k} = \frac{1}{Z} \sum_{fi} |\mathcal{M}_{fi}|^2 e^{-\beta E_i}$$

Using

$$\int_X \int_Y = \int_{\bar{X} = \frac{X+Y}{2}} \int_{X-Y}$$

$\sum \varepsilon_\nu \varepsilon^*_\mu \to g_{\mu\nu}$

We

$$(2\pi)^3 \frac{dP}{d^3k} = \int d^4\bar{X} \int_{X-Y} e^{-ik(X-Y)} \langle J_\nu(Y) J_\mu(X) \rangle g^{\mu\nu}$$

$$\times \frac{1+n_k}{2E_k}$$

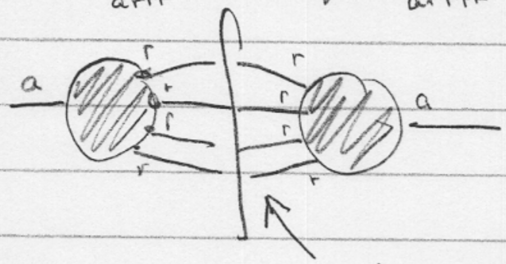
$$(2\pi)^3 \frac{dP}{d^4\bar{X}\, d^3k} = \frac{1}{2E_k} \underbrace{\int_{x-y} e^{-iK(x-y)} \langle J_\nu(y)\, J_\mu(x) \rangle g^{\mu\nu}}_{\equiv \Pi^<(K)}$$

$$\times \quad (1+n_k)$$

$$(2\pi)^3 \frac{dP}{d^4\bar{X}\, d^3k} = \frac{\Pi^<(K)}{2E_k} \times (1+n_k)$$

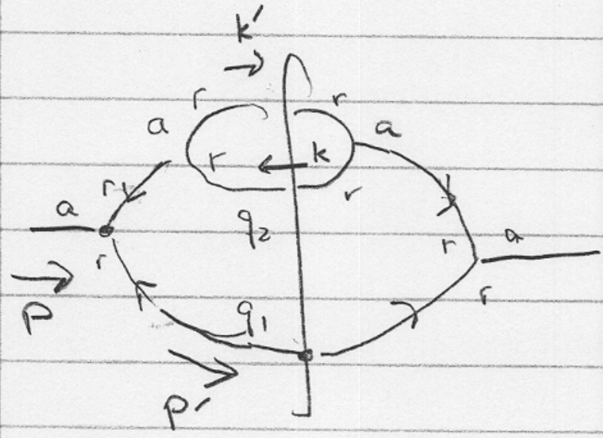
We have seen that $\Pi^>$ and $\Pi^<$ are squared matrix elements: (Simon-Caron-Huot)

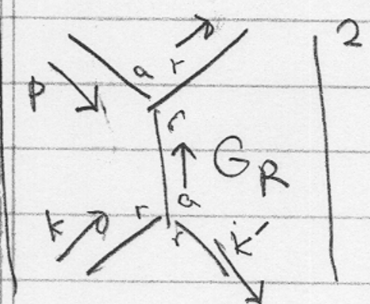
$\mathcal{M}_{arrr}$ $\qquad$ $\mathcal{M}^*_{arrrr}$

$\Pi^> =$ 

cut lines $= G^>(p) = (1+n_p)\frac{2\pi\delta(p^0-E_p)}{2E}$

$+ n_{-p}\frac{2\pi}{2E_p}\delta(p^0+E_p)$

$n_p \Pi^> =$ 

$= \int_{kp'k'}$ 


~ Feynman Graph
But "exchange gluon"
is retarded

$\times \; n_p n_k (1+n_{p'})(1+n_{k'})$

QCD at last:

Use KT

$$\left[\partial_t + v_p \frac{\partial}{\partial x}\right] f = C[f]$$

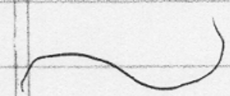
Scales:

Hard Particles: $p \sim T$ almost onshell - Carry most of energy

$$p^+ \sim T = p^0 + p^z$$
$$p^- \sim g^2 T = p^0 - p^z$$
$$p_\perp \sim gT \approx$$

$$p^2 = p^+ p^- + p_\perp^2 \sim O(g^2 T^2)$$

Soft Fields (Off shell)



$$p \sim gT$$

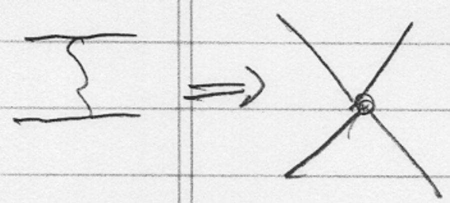
$$p^2 \sim g^2 T^2$$

Magetic Sector ~ non perturbative

$$p \sim g^2 T$$

Interactions:

① Hard Collisions $\sim T$

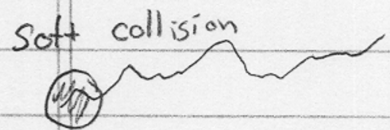


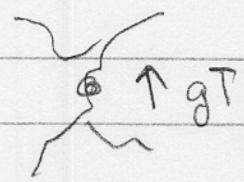
Randomizing Collisions

$$(\Delta p)^2 \sim \frac{t}{t_{coll}} T^2 \sim t\, T^3 g^4$$

$t_{d} \sim \frac{1}{T}$ $\qquad$ $t_{coll} \sim \frac{1}{g^4 T}$

② Random Walk - Interactions w/ soft fields



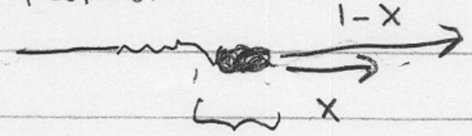




$t_d \sim \frac{1}{gT}$ $\qquad$ $t_{coll} \sim \frac{1}{g^2 T}$

$$(\Delta p)^2 = \frac{t}{t_{coll}} (\delta p)^2 \simeq \frac{t}{\frac{1}{g^2 T}} (gT)^2 \sim t\, \overbrace{g^4 T^3}^{\text{same}}$$

## (3) Collinear Brem

Random walk causes brem:




$\} \; \theta \sim q_\perp \sim gT$

Rate

$$\Gamma \sim \frac{\alpha}{t_{soft}} \sim \frac{g^2}{1/g^2 T} \sim g^4 T$$

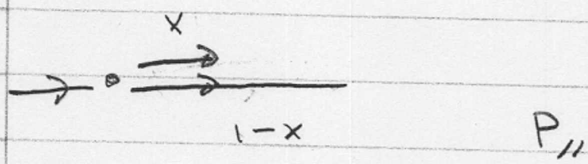
$$t_{coll} \sim \frac{1}{\Gamma} \sim \frac{1}{\alpha^2 T}$$

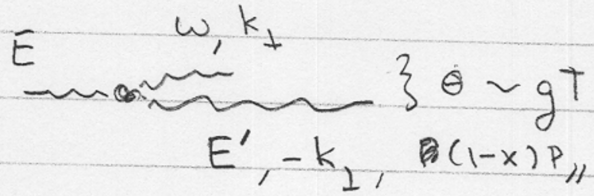
also same

So

$$(\Delta p)^2 \sim \frac{t}{t_{coll}} (\delta p)^2 \sim \frac{t}{\frac{1}{g^4 T}} (T^2) \sim T^3 g^4 t$$

# Collinear Bremm : Formation Times




take $x$ small
$\omega \simeq xP$

$$\Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{E - (E' + \omega)}$$

$$E' = \sqrt{(1-x)^2 P_{//}^2 + k_\perp^2} \simeq (1-x) P_{//} + \frac{k_\perp^2}{2P_{//}(1-x)}$$

$$\omega = \sqrt{x P_{//}^2 + k_\perp^2}$$

$$\simeq x P_{//} + \frac{k_\perp^2}{2 x P_{//}}$$

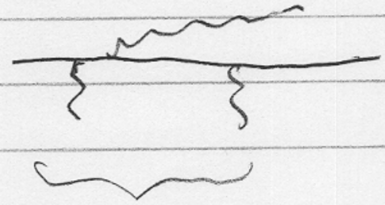
So

$$\Delta t_{form} \sim \frac{\hbar}{P_{//} - \left[(1-x)P_{//} + \frac{k_\perp^2}{2P_{//}(1-x)} + x P_{//} + \frac{k_\perp^2}{2P_{//}(\cancel{x})}\right]}$$

$$\Delta t_{form} \simeq \cancel{\hbar} \frac{2x(1-x)P_{//}}{k_\perp^2} \underset{x\ \text{small}}{\simeq} \frac{\omega}{k_\perp^2}$$

$$\Delta t_{form} \sim \frac{T}{(gT)^2} \sim \frac{1}{g^2 T}$$

$t_{form} \sim \frac{1}{g^2 T}$

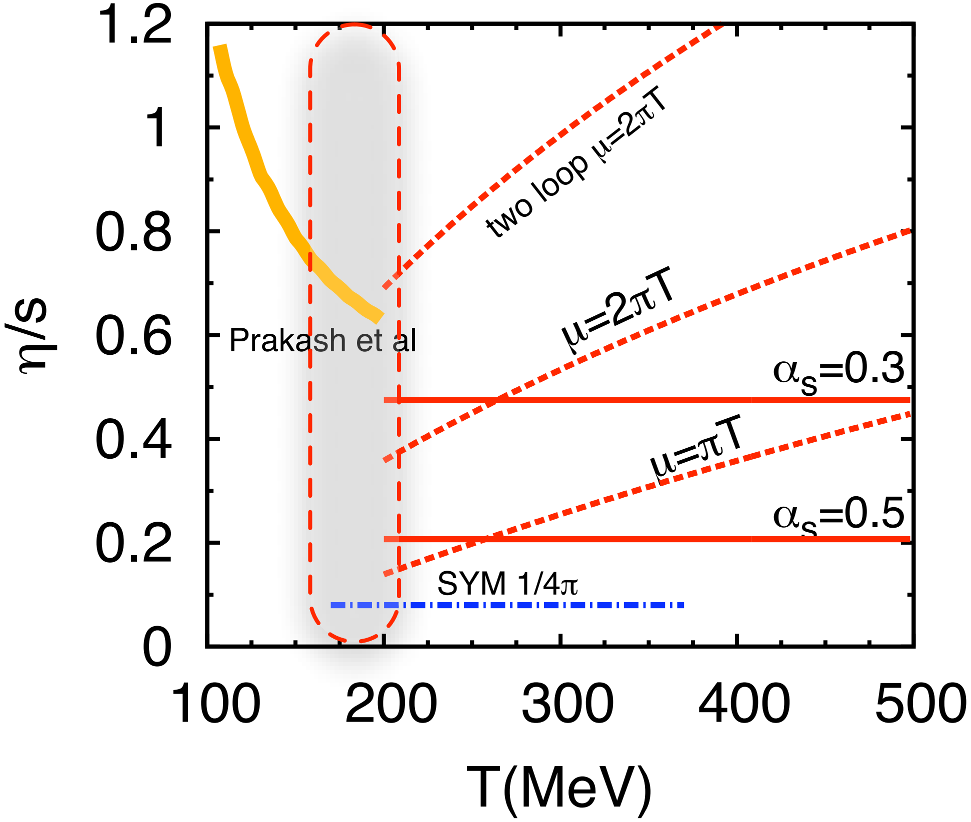


$t_{coll} \sim \frac{1}{g^2 T}$

← So these two scattering events are phase coherent

- Need to solve a kind of Schrödinger Eqn to determine the emission rate compliation from D. A. Teaney, “Viscous Hydrodynamics and the Quark Gluon Plasma,” arXiv:0905.2433 [nucl-th].

# Hadrons QGP




$$0.36 \left(\frac{\eta/s}{0.3}\right) \left(\frac{1\,\mathrm{fm}}{\tau_o}\right) \left(\frac{300\,\mathrm{MeV}}{T_o}\right) \ll 1$$